

Momentum Measurement in Magnetic Field

Momentum is determined by measurement of **track curvature** $\kappa = 1/\rho$ in B field:

Measure **sagitta** s of the track. For the momentum component transverse to B field:

$$p_T = qB\rho$$

Units: $p_T[\text{GeV}] = 0.3B[\text{T}]\rho[\text{m}]$

$$\frac{L/2}{\rho} = \sin\frac{\theta}{2} \approx \frac{\theta}{2} \text{ (for small } \theta) \Rightarrow \theta \approx \frac{L}{\rho} = \frac{0.3B \cdot L}{p_T}$$

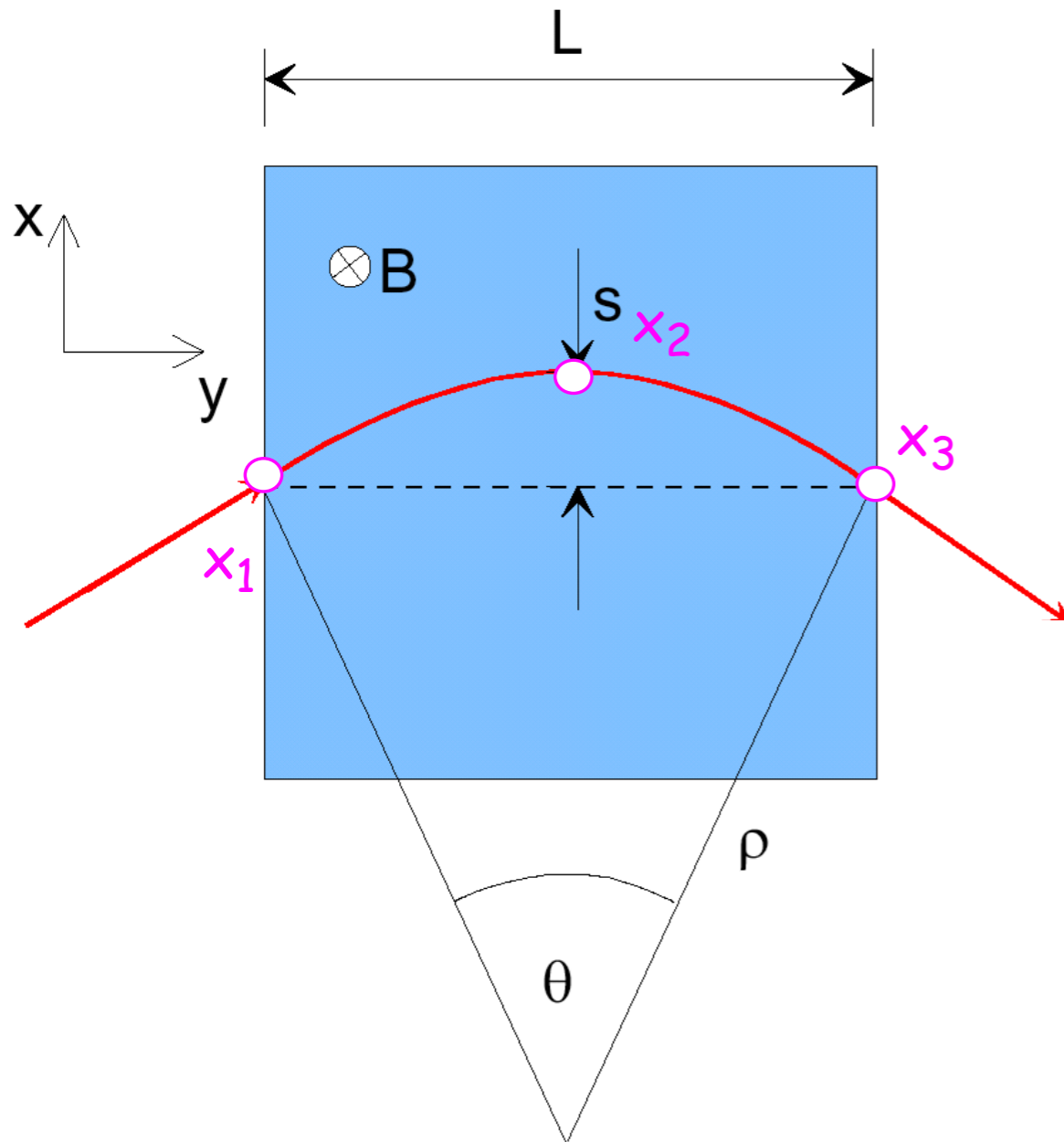
$$s = \rho \left(1 - \cos\frac{\theta}{2} \right) \approx \rho \left(1 - \left(1 - \frac{1}{2} \frac{\theta^2}{4} \right) \right) = \rho \frac{\theta^2}{8} \approx \frac{0.3L^2B}{8 p_T}$$

For the simple case of **three measurements**:

$$s = x_2 - (x_1 + x_3)/2 \Rightarrow ds = dx_2 - dx_1/2 - dx_3/2$$

with $\sigma_x \approx dx_i$ uncorrelated error of single measurement:

$$\sigma_s^2 = \sigma_x^2 + \frac{\sigma_x^2}{4} \cdot 2 = \frac{3}{2}\sigma_x^2$$



Relative Momentum Error

For 3 points the relative momentum resolution is given by: $\frac{\sigma(p_T)}{p_T} = \frac{\sigma_s}{s} = \sqrt{\frac{3}{2}} \sigma_x \cdot \frac{8p_T}{0.3BL^2}$

- degrades **linearly** with **transverse momentum**
- improves **linearly** with increasing **B field**
- improves **quadratically** with **radial extension** of detector

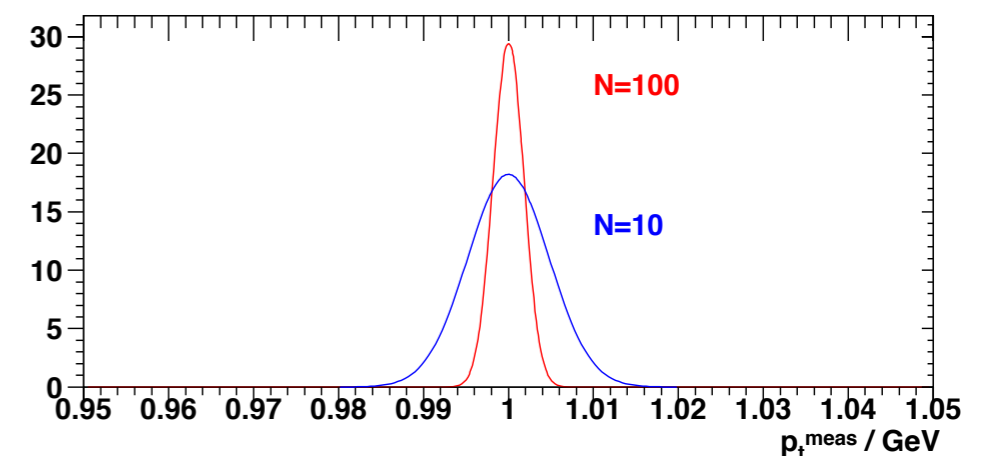
In the case of N equidistant measurements according to **Gluckstern** [NIM 24 (1963) 381]:

$$\frac{\sigma(p_T)}{p_T} = \frac{\sigma(\kappa)}{\kappa} = \frac{\sigma_x \cdot p_T}{0.3BL^2} \sqrt{\frac{720}{(N+4)}} \quad (\text{for } N \geq 10, \text{ curvature } \kappa = 1/\rho)$$

Example: For $p_T = 1\text{GeV}$, $L = 1\text{m}$, $B = 1\text{T}$, $\sigma_x = 200\mu\text{m}$ and $N = 10$ one obtains:

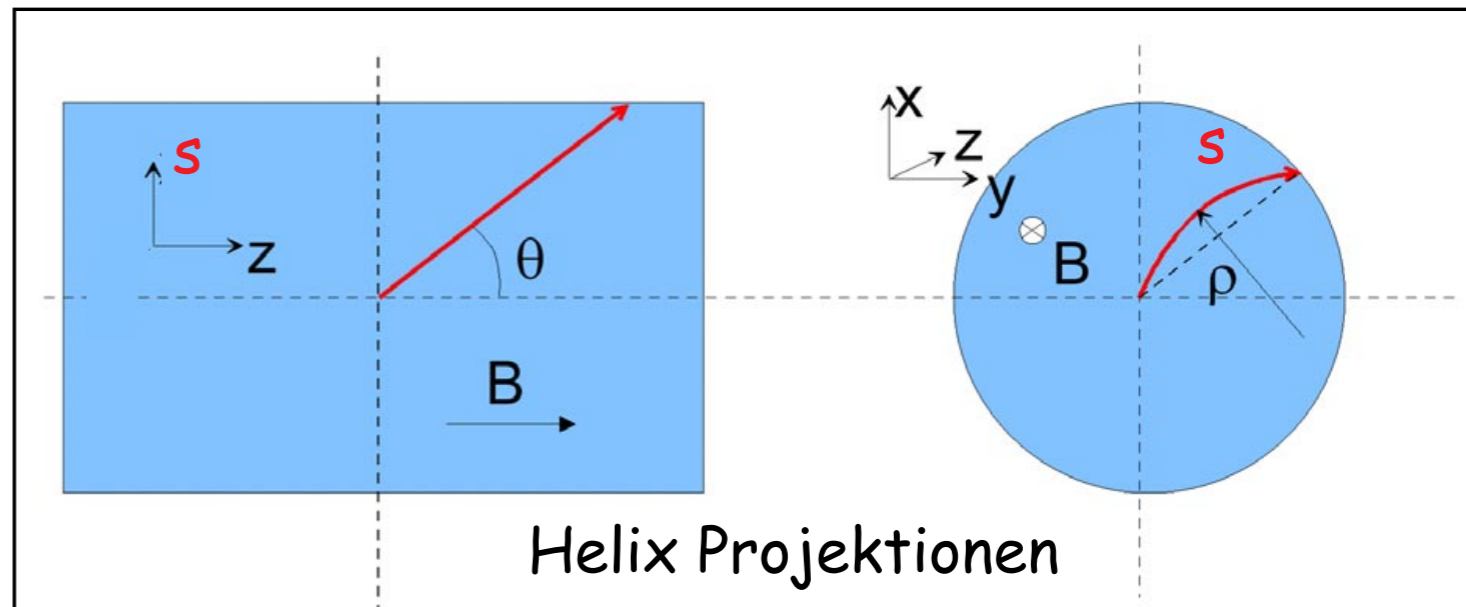
$$\frac{\sigma(p_T)}{p_T} \approx 0.5\% \quad \text{for a sagitta } s \approx 3.8\text{cm}$$

Important track detector parameter: $\frac{\sigma(p_T)}{p_T^2}$ (%/GeV)



Measurement of Total Momentum

In general one is interested in measuring the total momentum. The longitudinal component is derived from the measured polar angle θ with respect to the B-field axis. In the projection plane arc-length s versus z the trajectory is a straight line.



- in the case of N equidistant measurements with error :

$$\sigma_{\theta}^{Det} = \frac{\sigma(z)}{L} \sqrt{12 \frac{N-1}{N(N+1)}}$$

- in most cases the contribution from the transverse component dominates

$$\left(\frac{\sigma(p)}{p} \right)^2 = \left(\frac{\sigma(p_T)}{p_T} \right)^2 + \cot^2(\theta \sigma_{\theta}) \simeq \left(\frac{\sigma(p_T)}{p_T} \right)^2$$

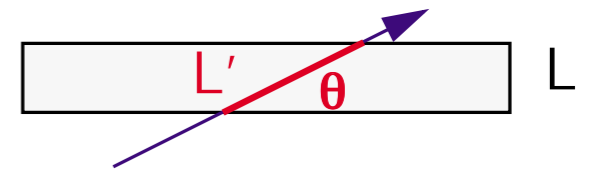
- all contributions discussed so-far result from the intrinsic detector resolution. In addition one has to consider the effect of multiple scattering.

Contributions from Multiple Scattering

The contribution to the momentum error from MS is given by:

$$\left. \frac{\sigma(p_T)}{p_T} \right|_{MS} = \frac{\sigma^{MS}(s)}{s} = \frac{\frac{L' 13.6 \times 10^{-3}}{\sqrt{3}} \frac{z}{p\beta} \sqrt{\frac{L'}{X_0}}}{0.3BL^2 z / (8p_T)} = \frac{0.2}{\beta B \sqrt{LX_0} \sin\theta} \quad \text{with} \quad \begin{array}{l} L' = L / \sin\theta \quad \text{total path} \\ p_T = p \sin\theta \end{array}$$

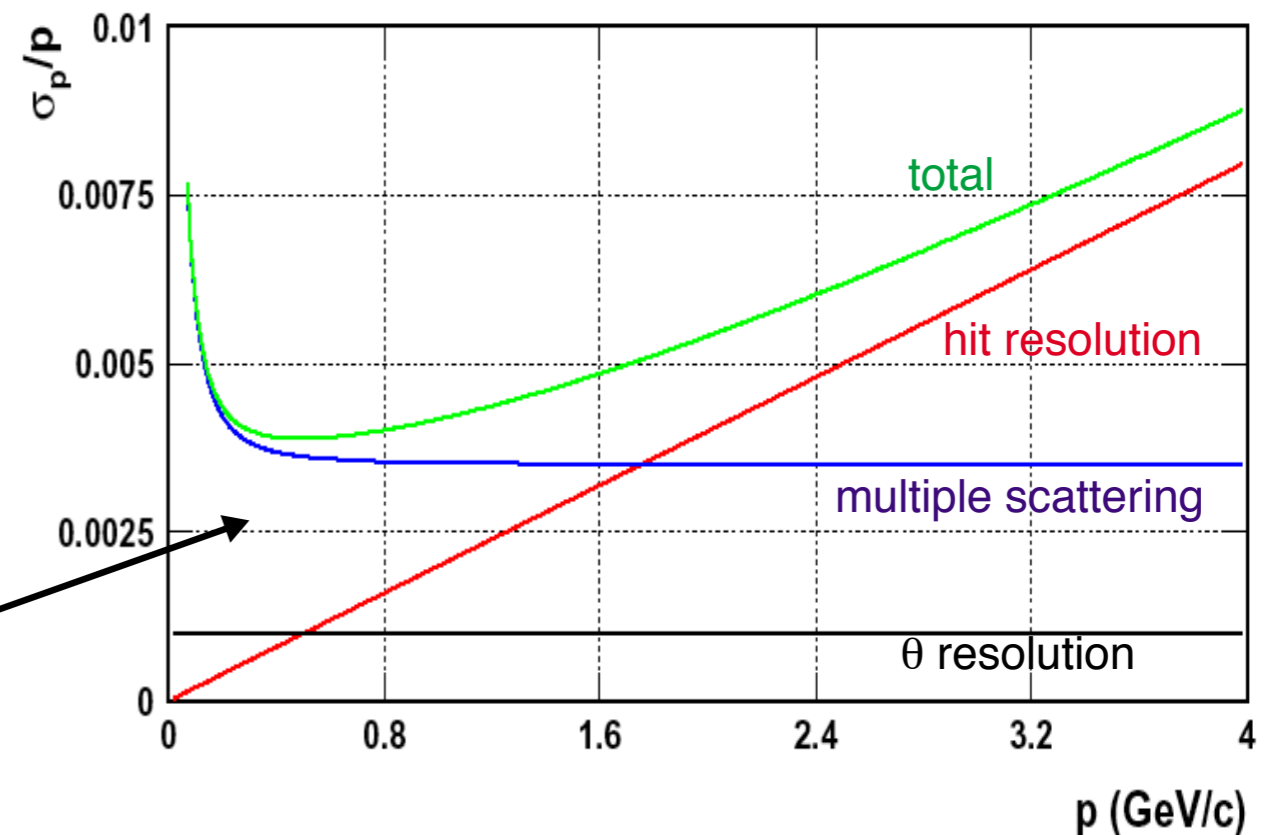
for $\beta \rightarrow 1$ this part is momentum independent!



The combined total momentum error is:

$$\left(\frac{\sigma_p}{p} \right)^2 = \left(\sqrt{\frac{720}{N+4}} \frac{\sigma_x p \sin\theta}{0.3BL^2} \right)^2 + \left(\frac{0.2}{\beta B \sqrt{LX_0} \sin\theta} \right)^2 + (\cot\theta \sigma_\theta)^2$$

Example for momentum dependence of individual contributions

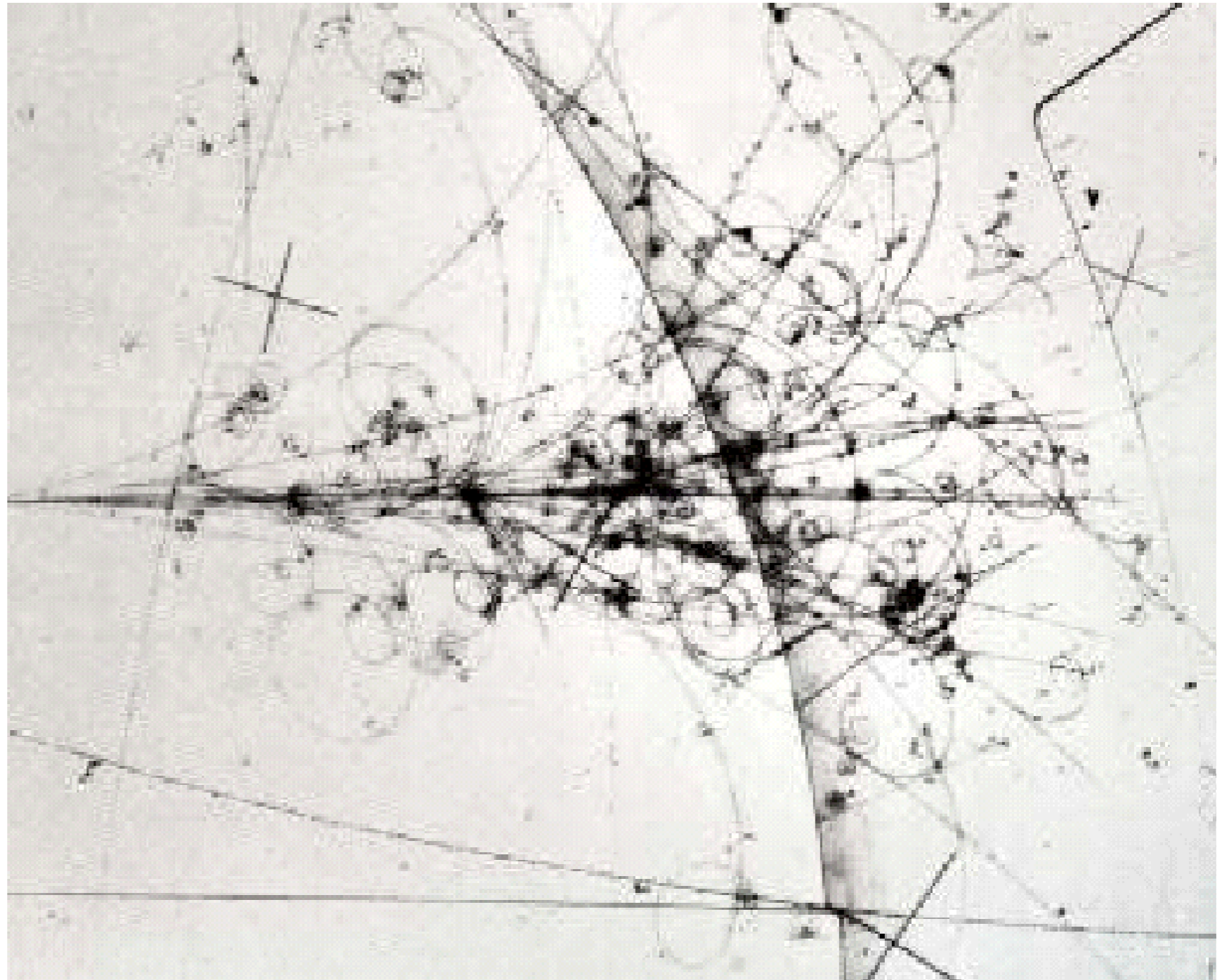


First Track Detectors

BEBC bubble chamber

Until \approx 1970:

- optical measurements using
 - bubble chambers
 - emulsions
 - spark chambers
- manual reconstruction
- can handle only very low data rates



Gas Detectors

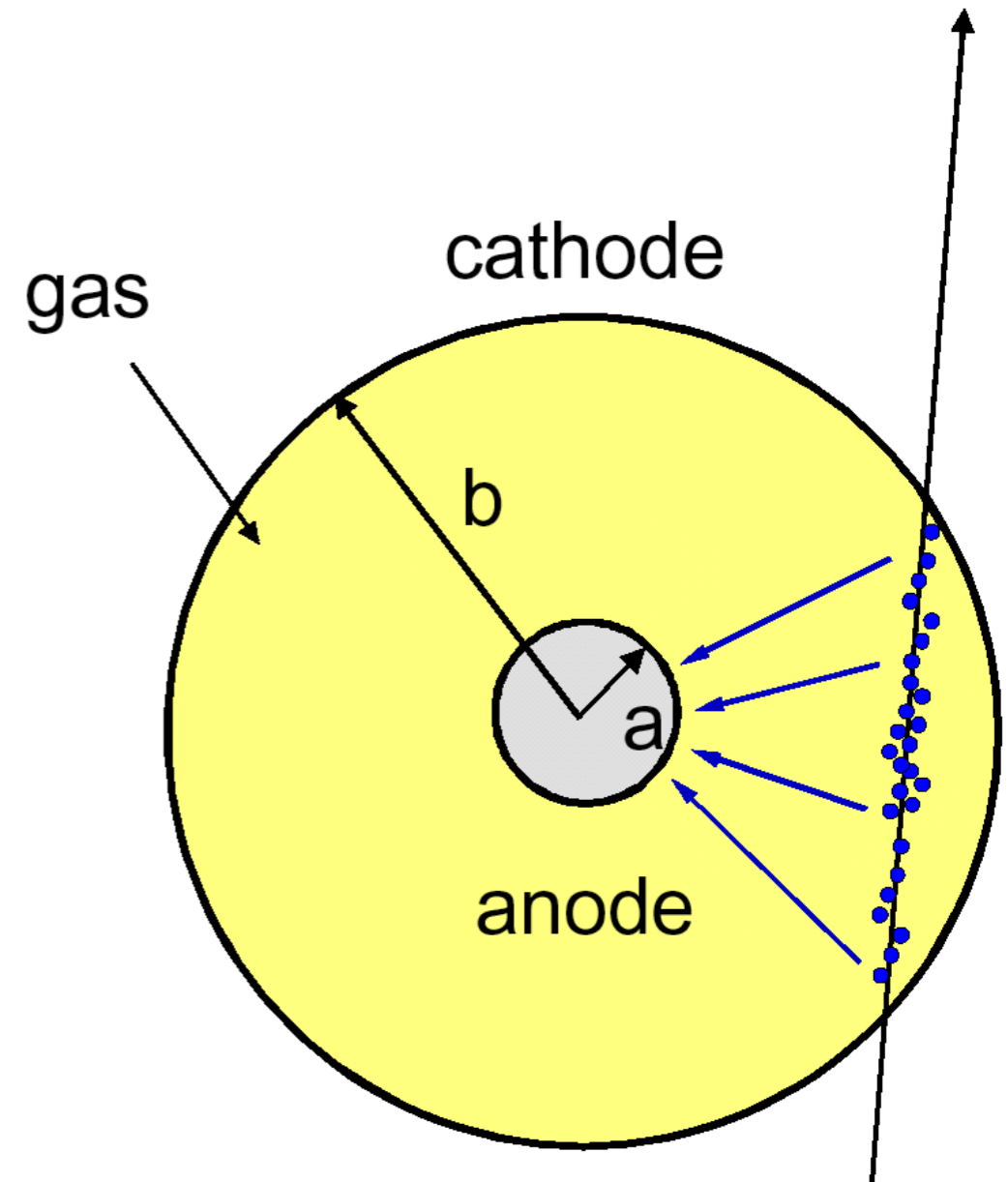
Criteria for optimal momentum resolution:

- many measurement points
- large detector volume
- very good single point resolution
- as little multiple scattering as possible

Gas detectors provide a good compromise and are used in most experiments. However:

- per cm in Argon only **ca. 100** electron-ion pairs are produced by ionisation (see next page)
- this has to be compared with the noise of a typical amplifier of **$\approx 1000 e^-$**

⇒ **a very efficient amplification mechanism is required**



Primary and Total Ionisation Yield in Gases

Gas	Density ρ [g/cm ³]	I_0 [eV]	W [eV]	n_p [cm ⁻¹]	n_T [cm ⁻¹]
H ₂	8.99 x 10 ⁻⁵	15.4	37	5.2	9.2
He	1.78 x 10 ⁻⁴	24.6	41	5.9	7.8
N ₂	1.23 x 10 ⁻³	15.5	35	10	56
O ₂	1.43 x 10 ⁻³	12.2	31	22	73
Ne	9.00 x 10 ⁻⁴	21.6	36	12	39
Ar	1.78 x 10 ⁻³	15.8	26	29	94
Kr	3.74 x 10 ⁻³	14.0	24	22	192
Xe	5.89 x 10 ⁻³	12.1	22	44	307
CO ₂	1.98 x 10 ⁻³	13.7	33	34	91
CH ₄	7.17 x 10 ⁻⁴	13.1	28	16	53
C ₄ H ₁₀	2.67 x 10 ⁻³	10.8	23	46	195

avg. ionisation pot. / shell elect.
 average energy loss./ion pair
 number primary electron-ion pairs
 total number electron-ion pairs

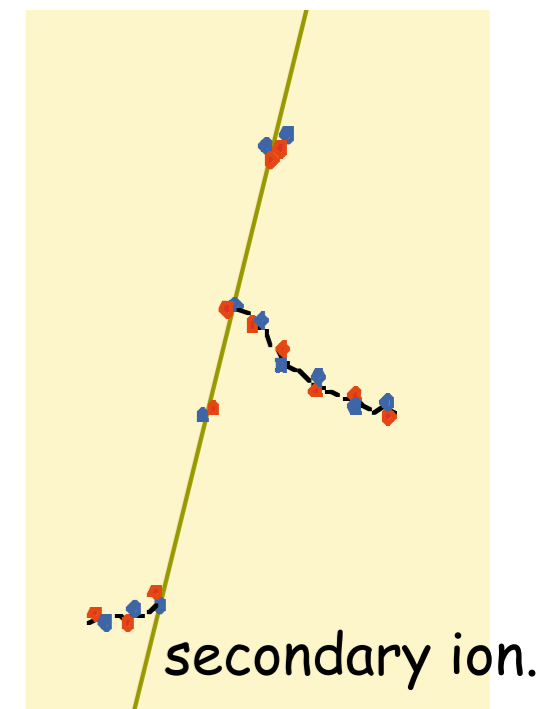
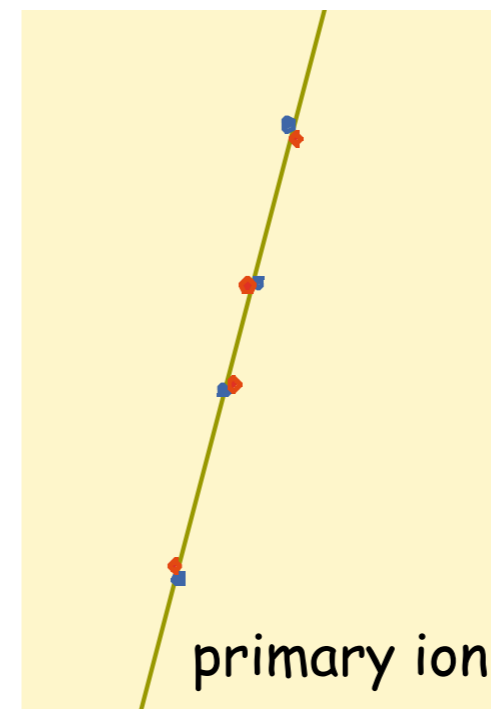
@ STP

Total number of produced electron-ion

pairs:
$$n_T = \frac{\Delta E}{W_i} = \frac{\frac{dE}{dx} \Delta x}{W_i} \quad \text{with}$$

- ΔE = total energy loss in Δx and
- W_i = average energy loss per produced ion pair:

$$n_T \approx 2 \dots 7 \cdot n_p$$



Signal Formation in a Condenser

Simple case of a condenser in a gas tight box:

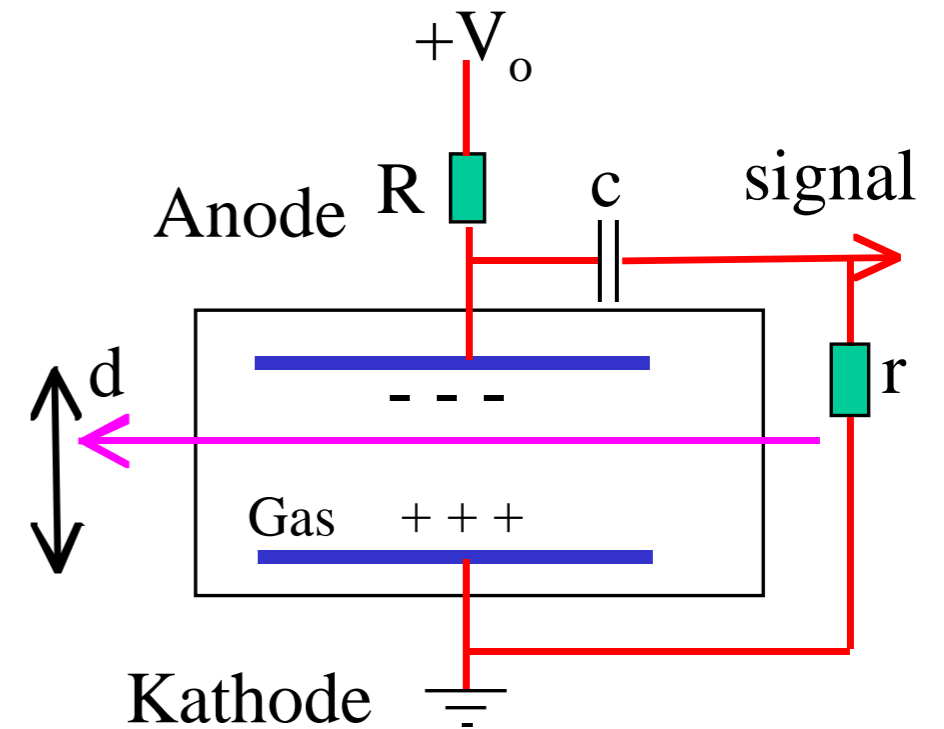
- electric field $E = V_0 / d$
- capacity C
- accumulated charge $Q_0 = C V_0$
- number of produced electron-ion pairs N
- if $RC \gg t_{\text{drift}} \Rightarrow$ charge on condenser plates will be reduced by $N|q|$

\Rightarrow resulting voltage pulse: V reduces by $\Delta V = N|q|/C$

Example:

$N \approx 120$ for Argon, $C = 10 \text{ pF} \Rightarrow$

$$\Delta V = \frac{120 \cdot e}{10 \text{ pF}} = \frac{120 \cdot 1.6 \times 10^{-16} \text{ C}}{10 \times 10^{-9} \text{ F}} \simeq 2 \mu\text{V} \quad \text{too small !}$$



C = chamber capacity

c = pulse forming and coupling capacitor

R = resistor of power supply

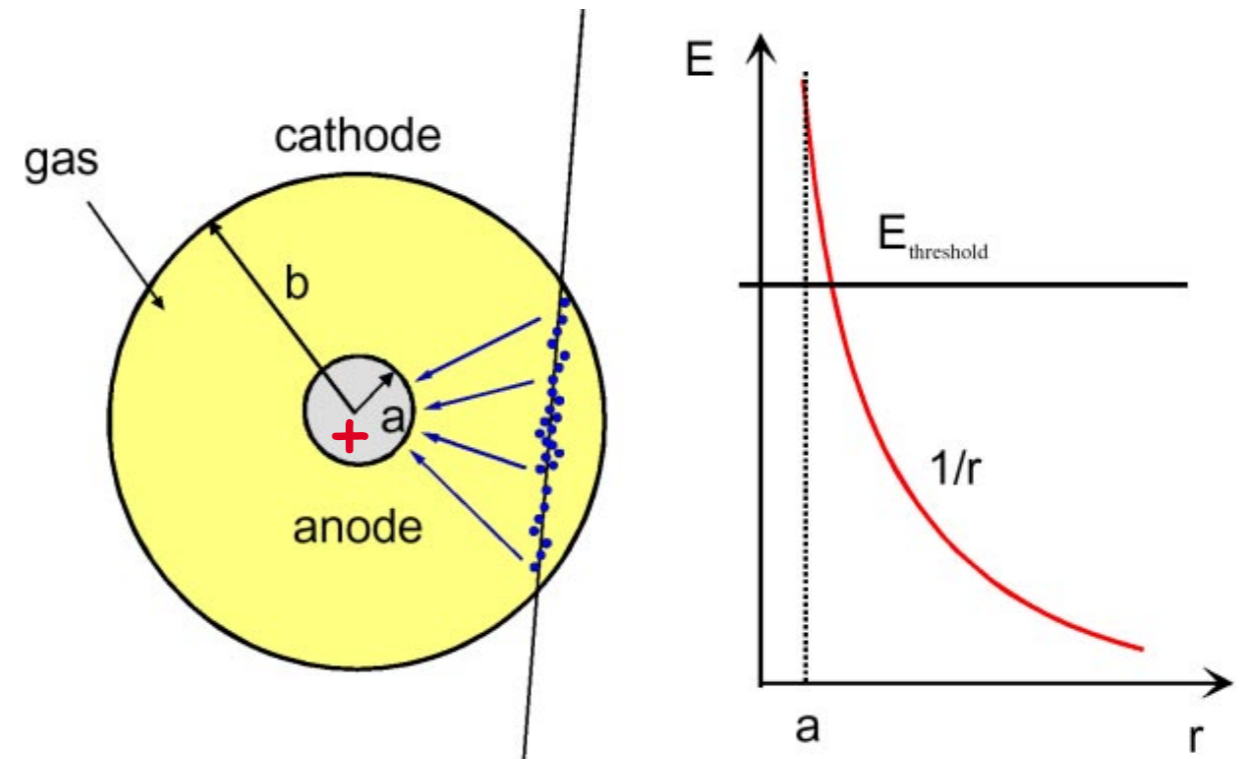
r = pulse forming resistor

Gas Amplification

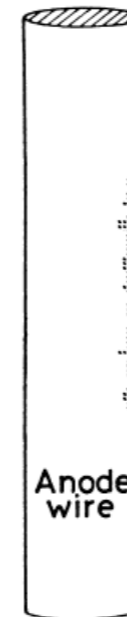
For cylindrical geometry:

$$E(r) \propto \frac{1}{r} \quad \text{and} \quad V(r) \propto \ln \frac{r}{a}$$

- the primary electrons drift towards the positive anode
- due to $1/r$ dependence the electric field close to very thin wires reaches values of $E > \text{kV/cm}$
- \Rightarrow in between collisions with atoms electrons gain enough energy to ionize further gas molecules
- \Rightarrow **exponential** increase in number of electron-ion pairs very close (few μm) to the wire



25 μm



simulated avalanche



First Townsend Coefficient

Number of electron-ion pairs created per unit length in the avalanche per electron is given by the **first Townsend coefficient** α :

- relation to cross section for ionisation:

$$\alpha = \sigma_{ion} \cdot \frac{N_A}{V_{Mol}}$$

- number of produced ions: $n(x) = n_0 \cdot \exp(\alpha(E) \cdot x)$

- the **gas gain** is given by:

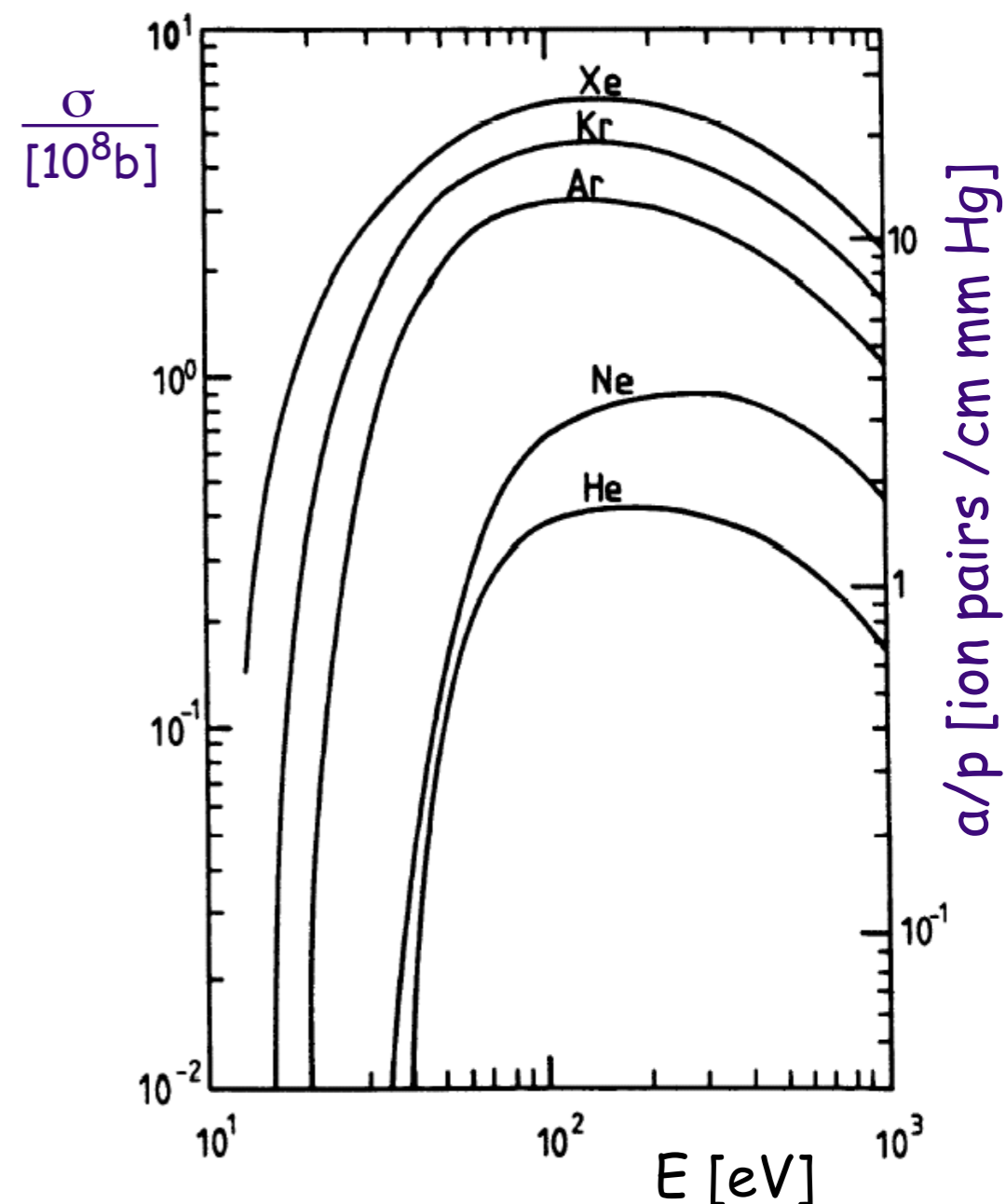
$$A = \frac{n}{n_0} = \exp\left[\int_a^{r_c} \alpha(r) dr\right]$$

with a anode diameter and r_c distance to wire where avalanche starts

- example: Argon and $E = 100\text{eV}$:

$$\sigma = 3 \times 10^{-16} \text{cm}^2 \Rightarrow \alpha^{-1} \approx 1 \mu\text{m}$$

Raether limit: avalanche size limited to $A < \sim 10^8$ or $\alpha \cdot x < 20$



Second Townsend Coefficient

- In the avalanche many atoms are excited. When they return to their ground state UV photons are emitted.
- UV photons create more electrons through photo electric effect with gas atoms or when they hit the metallic electrodes
- The exponential growth of the gas amplification with increasing HV starts to get damped when the number of ionization processes by UV photons gets significant

- start of the avalanche n_0 primary electrons
- after amplification An_0
- number of photo electrons $An_0\gamma$ with $\gamma \ll 1$ (2. Townsend coefficient)
- which get amplified to $A^2n_0\gamma$ etc.

- in total there are:
$$A_\gamma n_0 = n_0 A \sum_{n \geq 0} (A\gamma)^n = \frac{n_0 A}{1 - A\gamma}$$

where A_γ denotes the gas amplification including photons

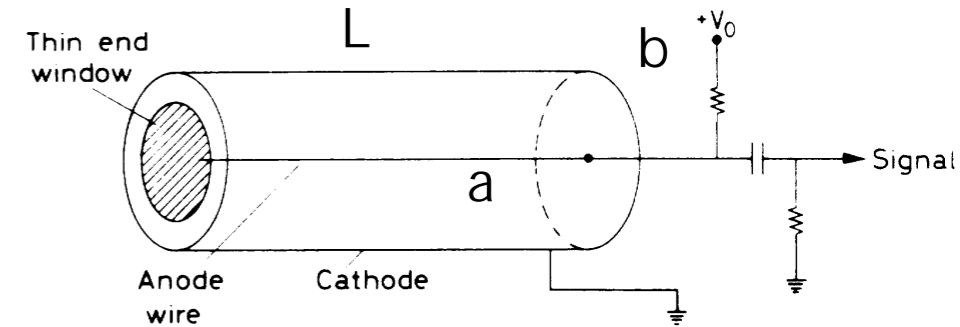
- for $A\gamma \rightarrow 1$ the expression diverges, i.e. the signal no longer depends on the amount of primary ionization: Geiger Müller region

Pulse Formation in Cylindrical Chamber

In such a cylindrical geometry the following relations hold:

$$\phi(r) = \frac{CV_0}{2\pi\epsilon L} \ln\left(\frac{r}{a}\right) \quad E(r) = \frac{CV_0}{2\pi\epsilon L} \frac{1}{r} \quad C = \frac{2\pi\epsilon L}{\ln(b/a)}$$

The stored electric energy is: $W = \frac{1}{2}CV_0^2$

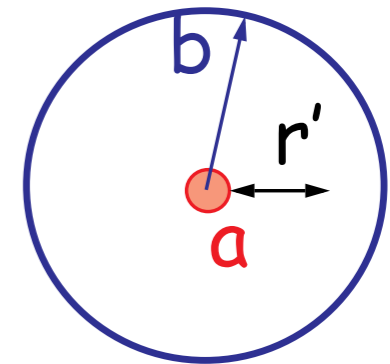


When the charge q moves a distance dr this causes a change of potential energy by:

$$dW = q \frac{d\phi(r)}{dr} dr \equiv CV_0 dV \Rightarrow dV = \frac{q}{CV_0} \frac{d\phi(r)}{dr} dr$$

At the position r' the electrons induce a voltage of:

$$\Delta V^- = \frac{-q}{CV_0} \int_{a+r'}^a \frac{d\phi(r)}{dr} dr = \frac{-q}{CV_0} \int_{a+r'}^a \frac{-CV_0}{2\pi\epsilon L} \frac{dr}{r} = \frac{-q}{2\pi\epsilon L} \ln \frac{a+r'}{a}$$



Similarly the voltage induced by the moving ions at the position r' amounts to:

$$\Delta V^+ = \frac{+q}{CV_0} \int_{a+r'}^b \frac{d\phi(r)}{dr} dr = \frac{+q}{CV_0} \int_{a+r'}^b \frac{-CV_0}{2\pi\epsilon L} \frac{dr}{r} = \frac{-q}{2\pi\epsilon L} \ln \frac{b}{a+r'}$$

The resulting total voltage is given by : $\Delta V = \Delta V^- + \Delta V^+ = -q/C$

Contribution of Ions and Electrons to the Signal

The contribution of electrons and ions to the signal is very different in the case of gas amplification, since this occurs very close to the anode wire at a distance: $r' = k \cdot \lambda$ where k is the number of mean free path-lengths λ , which are required for avalanche formation.

As a consequence the drift path of the ions is much longer. For example in the case $b = 1\text{cm}$, $a = 10\mu\text{m}$, $r' = 2\mu\text{m}$

$$\frac{\Delta V^+}{\Delta V^-} = \frac{\ln \frac{b}{a+r'}}{\ln \frac{a+r'}{a}} \approx \frac{\ln 10^3}{\ln 1.2} \approx 40$$

The time dependence in this case is given by ($r' \approx 0$): $V(t) = V^+(t) = -\frac{q}{2\pi\epsilon L} \ln \frac{r(t)}{a}$

The solution of the problem is to find the relation $r(t)$

The mobility is given by: $\mu \equiv \frac{v_{drift}}{E(r)} = \frac{1}{E(r)} \frac{dr}{dt}$

For cylindrical geometry: $\frac{dr}{dt} = \mu E(t) = \mu \frac{CV_0}{2\pi\epsilon L} \frac{1}{r} \Rightarrow r dr = \mu \frac{CV_0}{2\pi\epsilon L} dt$

Time Evolution of the Signal

Integration of $r dr = \mu \frac{CV_0}{2\pi\epsilon L} dt$ yields:

$$\int_{r(0)=a}^{r(t)} r dr = \mu \frac{CV_0}{2\pi\epsilon L} \int_0^t dt \Rightarrow r(t) = \left(a^2 + \frac{\mu CV_0}{\pi\epsilon L} t \right)^{1/2}$$

$$V(t) = \frac{-q}{2\pi\epsilon L} \ln \frac{r(t)}{a} = \frac{-q}{4\pi\epsilon L} \ln \left(1 + \frac{\mu CV_0}{\pi\epsilon L a^2} t \right) = \frac{-q}{4\pi\epsilon L} \ln \left(1 + \frac{t}{t_0} \right) \quad \text{with} \quad t_0 = \frac{a^2 \ln(b/a)}{2\mu V_0}$$

The total drift time T follows from $r(T) = b$

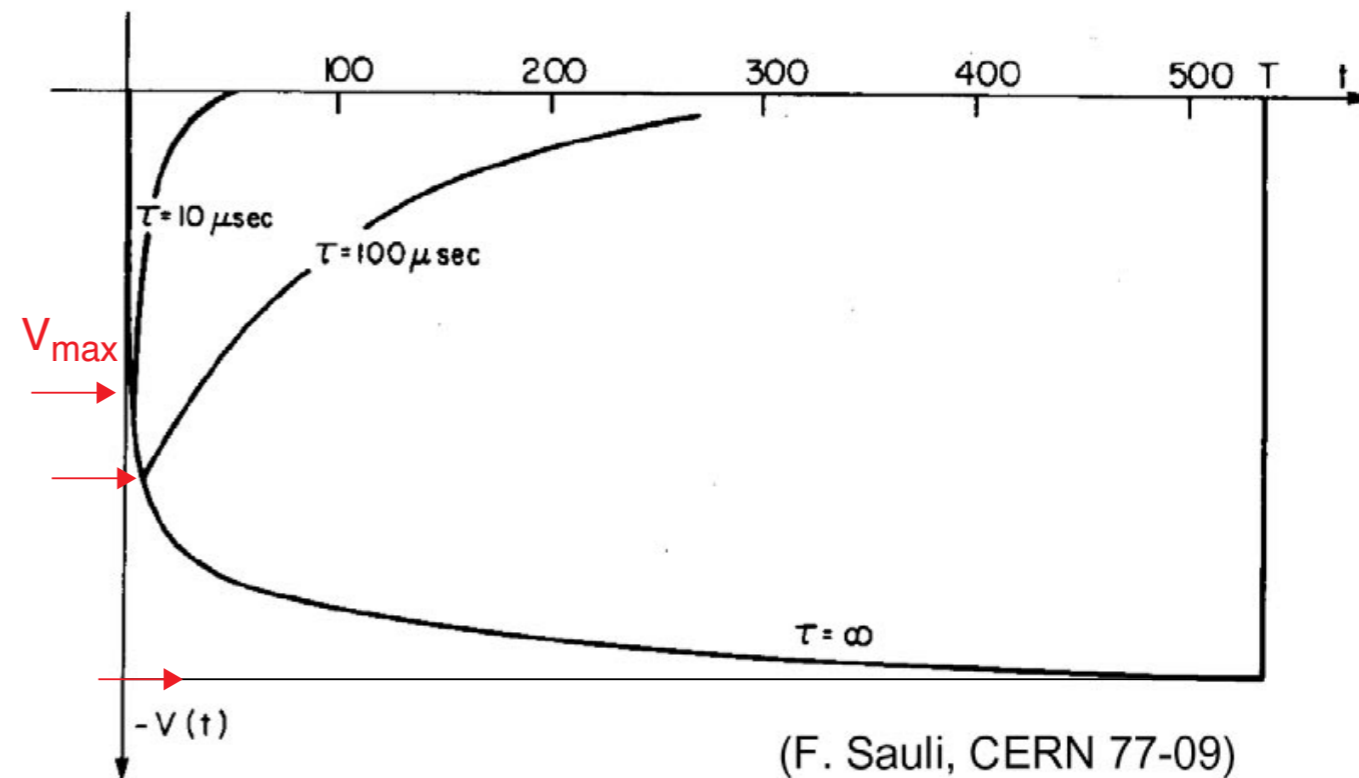
$$T = \frac{t_0}{a^2} (b^2 - a^2) \approx \frac{b^2}{a^2} t_0$$

Example:

$\mu = 1.5 \text{ cm}^2 \text{ s}^{-1} \text{ V}^{-1}$, $V = 1500 \text{ V}$, $a = 10 \mu\text{m}$, $b = 1 \text{ cm}$
and $t_0 = 1.5 \times 10^{-9} \text{ s}$

$$\Rightarrow T = 1.5 \times 10^{-3} \text{ s}$$

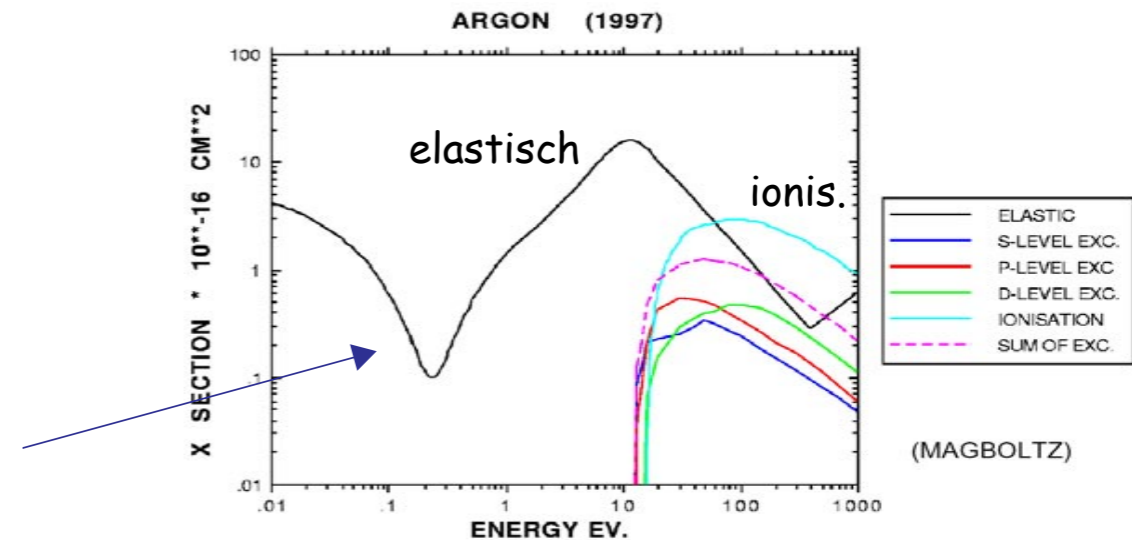
Usually it is necessary to differentiate the signal (which makes it shorter) by the electronics to reduce the dead time.



Considerations on the Choice of Gas

In principal avalanche formation occurs in every gas. However the gas should have properties which lead to:

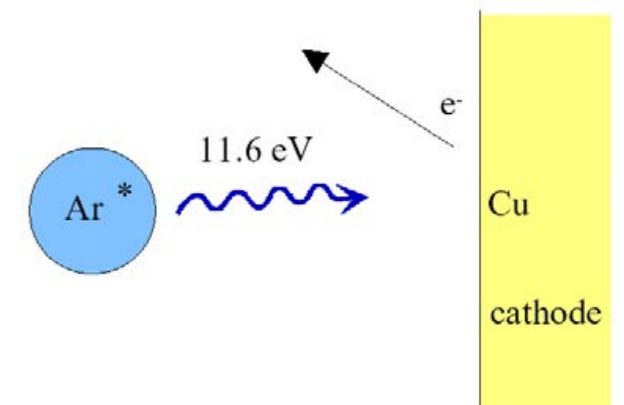
- low operation voltage
- stable operation also at high gas gain
- good rate capability
- sufficient longevity
- fast recovery



The main component of a chamber gas usually is a **noble gas**, since it is mono-atomic and the electrons mainly experience elastic **collisions with little energy transfer**. This enables gas gain already at low voltages. For price reasons the choice in most cases is Argon.

Stable operation in most cases requires the presence of **additives**. In the avalanche Ar atoms get excited, which emit **UV photons** when they return to their ground state ($E > 11.6$ eV). When these photons reach the cathode made from Al or Cu (≈ 7.7 eV excitation potential) electrons are liberated by the photo electric effect which drift back to the anode and create another avalanche.

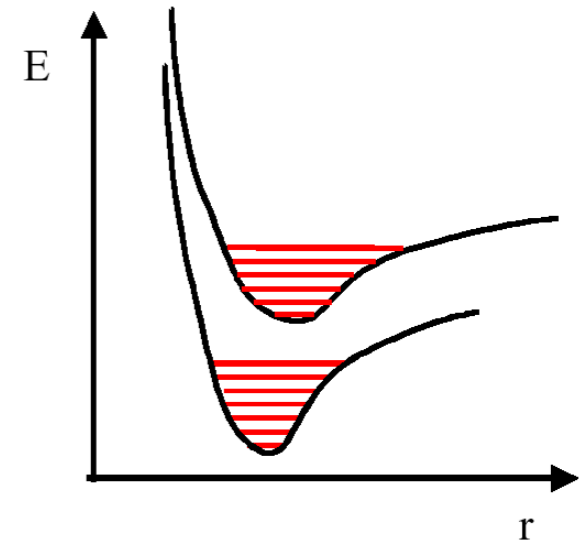
This would lead to „**breakdown**“ already at relatively low voltages for a detector with pure Ar.



Additives to the Chamber Gas: Quencher

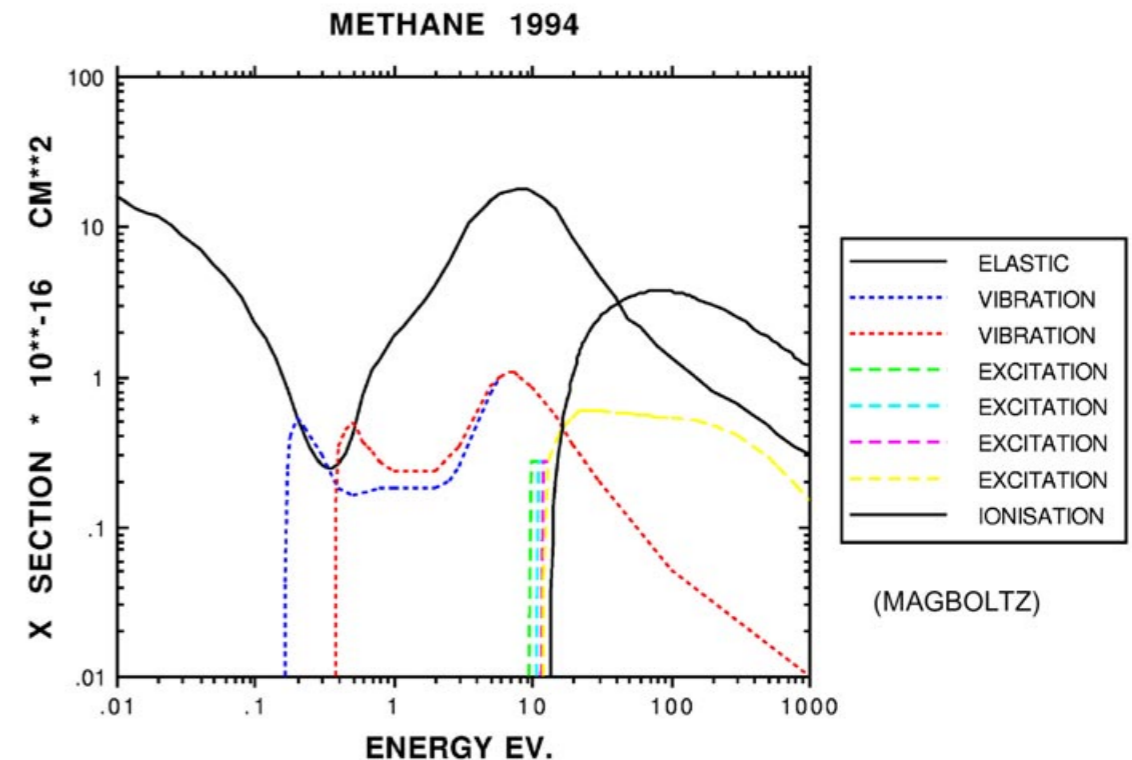
To solve this problem one usually adds a **poly-atomic** component to the gas.

- these gases have many non-radiative vibrational and rotational states over a wide range of energies
- the molecules easily absorb the UV photons in the range $\approx 100\text{-}200\text{ nm}$
- they de-excite by collisions or dissociation
- because $\tau_{\text{emission}} \gg \tau_{\text{collision}}$ the UV photons are very efficiently eliminated or **quenched**



Typically hydrocarbon components are being used:

- Methane CH_4
- Ethane C_2H_6
- Isobutane iC_4H_{10}
- or alcohol vapours



Drift and Diffusion in Gases

In a field free region:

Due to multiple collisions electron-ion pairs assume a thermal energy distribution. According to the law of equal partition the average energy at room temperature is given by:

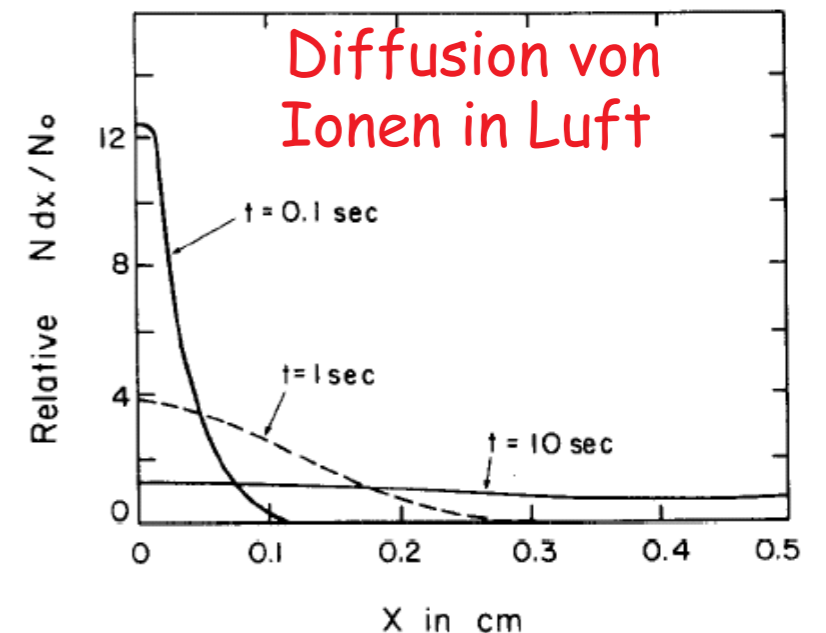
$$\epsilon_T = \frac{3}{2}kT \approx 0.035 \text{ eV}$$

- they follow a Maxwell-Boltzmann distribution: $F(\epsilon) = \text{const} \cdot \sqrt{\epsilon} \cdot \exp(-\epsilon/kT)$
- a point-like charge distribution will diffuse due to multiple scattering into the neighboring space:

$\frac{dN}{N} = \frac{1}{\sqrt{4\pi Dt}} \exp\left(-\frac{x^2}{4Dt}\right) dx$ is the fraction of charge located in dx at the distance x at the time t

- D is the so-called **diffusion coefficient**
- linear diffusion: $\sigma_x = \sqrt{2Dt}$
- volume diffusion: $\sigma_{vol} = \sqrt{3}\sigma_x = \sqrt{6Dt}$
- mean free path of diffusion: $\lambda(\epsilon) = \frac{1}{N\sigma(\epsilon)}$

where $N = N_0\rho/A$ is the number of molecules/volume

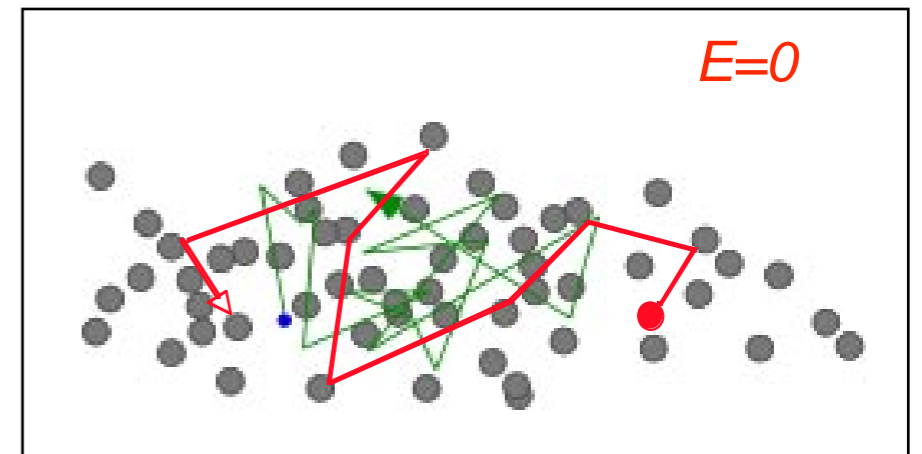


Motion of Charge Carriers in electric Field

In an electric field a directional drift parallel to the field direction is superimposed on the irregular motion of diffusion. Definition of drift velocity:

$$\vec{v}_{\text{drift}} = \mu(E) \cdot \vec{E} \cdot \frac{p_0}{p} \quad \text{with:}$$

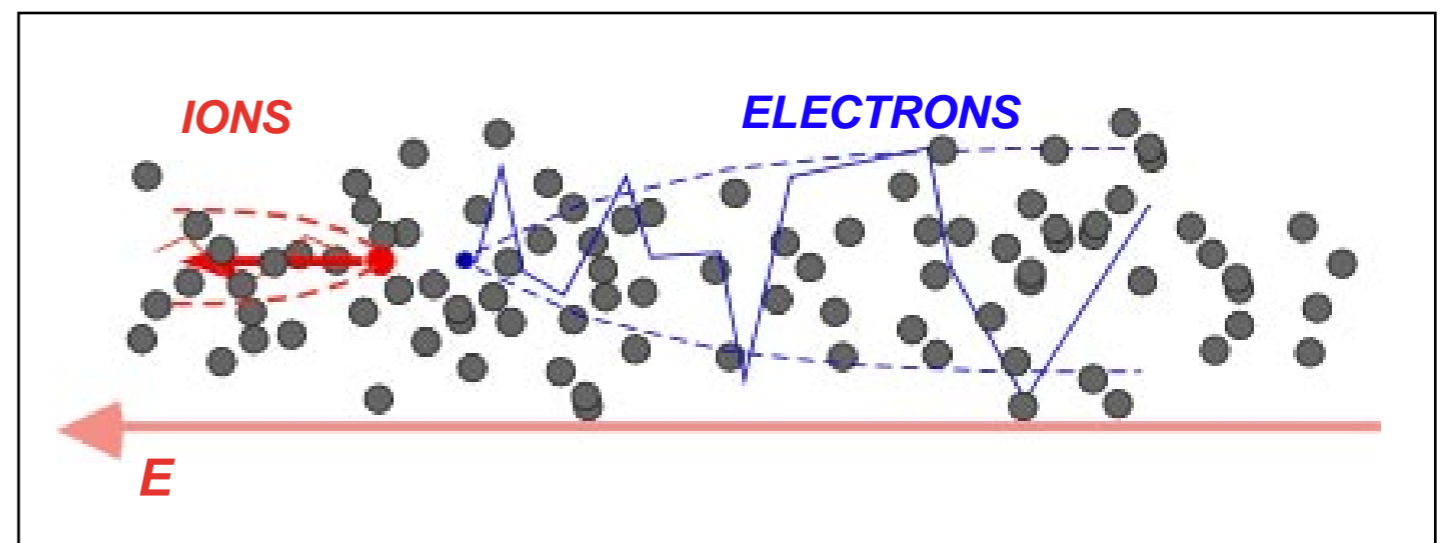
- $\mu(E)$ energy dependent mobility of charge carriers
- \vec{E} electric field
- $\frac{p_0}{p}$ normalized pressure (to STP)



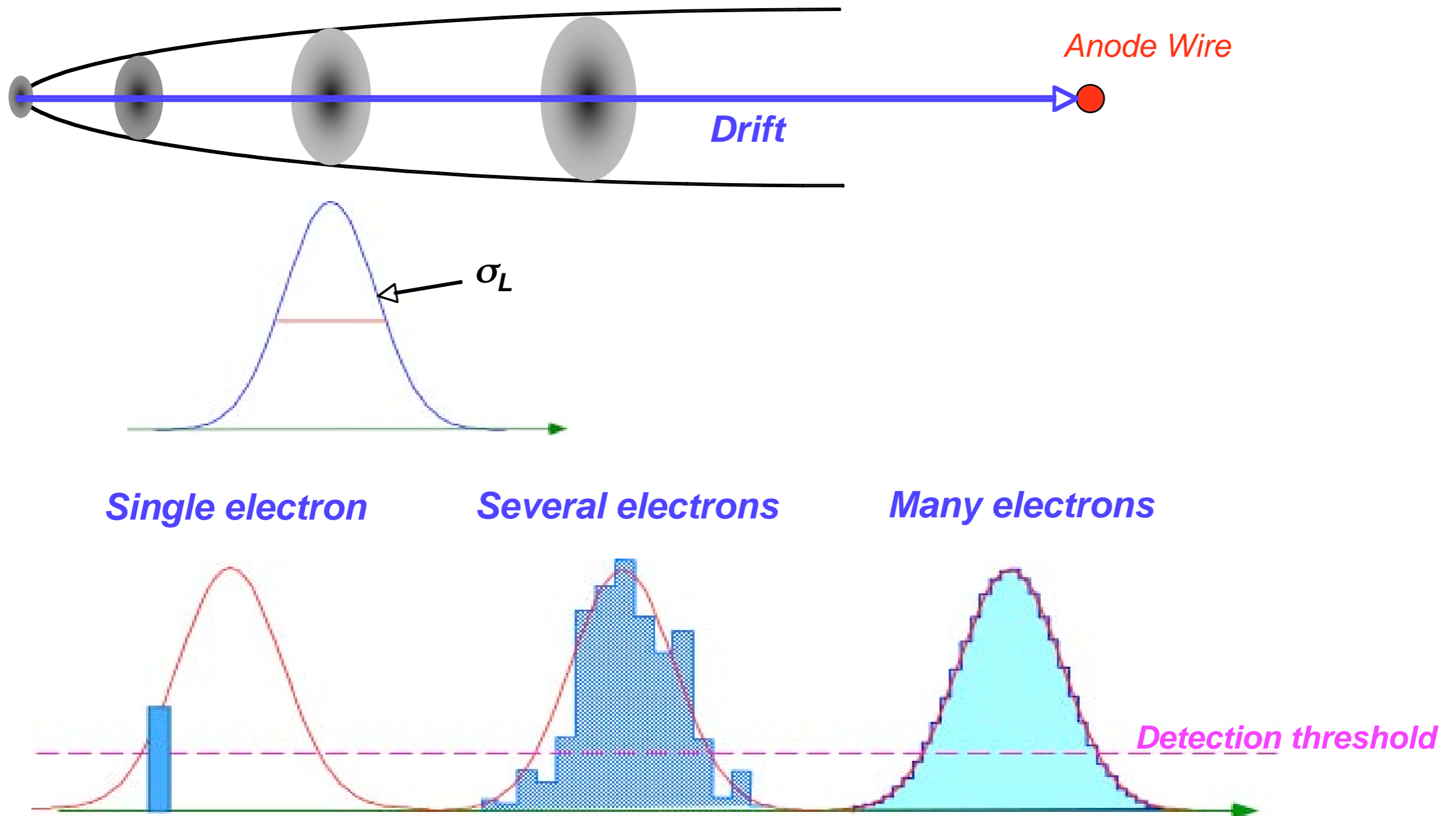
Figures taken from
F. Sauli, CERN

Valid under the assumption:

- no recombination
- no attachment



Impact of Diffusion on Position Resolution



Recombination and Attachment

Recombination:

Ions and electrons created in the primary ionization process recombine before they are detected. This leads to an effective decrease of the density of positive ions n^+ :

$$\frac{dn^+}{dt} = \alpha n^+ n^- \text{ with recombination coefficient } \alpha$$

Values of α for O_2 or CO_2 are up to 10^{-6} (10^{-7}) $cm^{-3} s^{-1}$ in the case of recombination with negative ions (electrons)

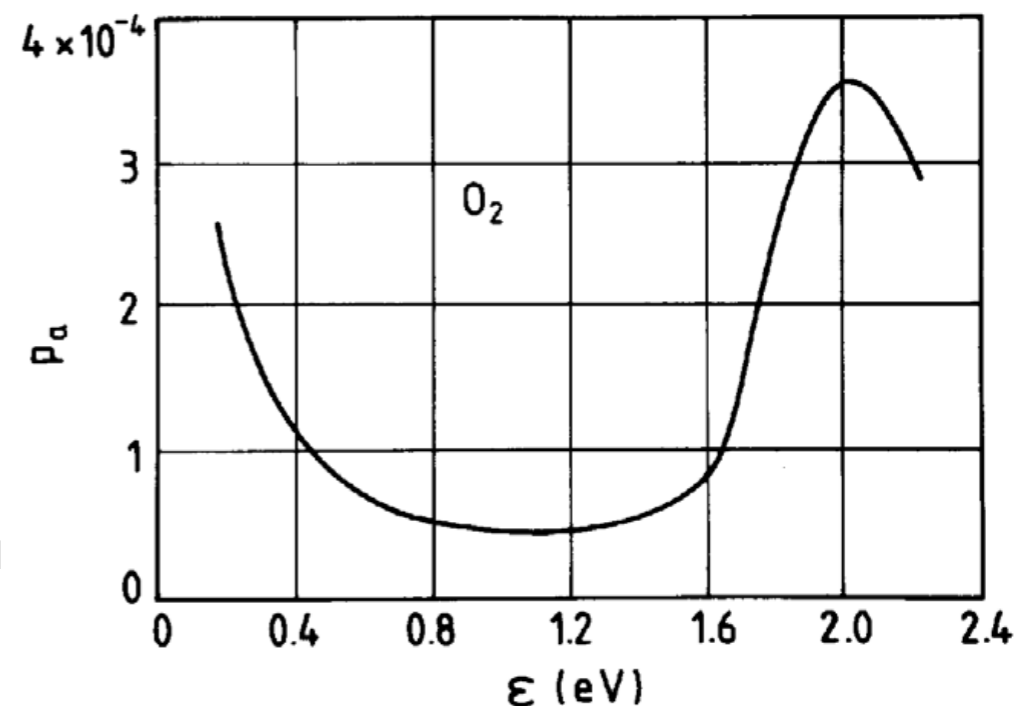
Electron attachment:

For poly-atomic gases (no noble gases !) low energy electrons ($\approx eV$) can get attached to the molecule after a collision

- p_a probability per collision
- n_s number of collisions per time
- $t_a = 1/(p_a n_s)$ mean time until attachment

-> necessity for gas cleaning !

Anlagerungswahrscheinlichkeit pro Stoß



Elektronegative Gase (ohne E-Feld):

Gas	p_a	$n_s [s^{-1}]$	$t_a [ns]$
CO_2	6.2×10^{-9}	2.2×10^{11}	7.1×10^5
O_2	2.5×10^{-5}	2.1×10^{11}	190
H_2O	2.5×10^{-5}	2.8×10^{11}	140
Cl_2	4.8×10^{-4}	4.5×10^{11}	5

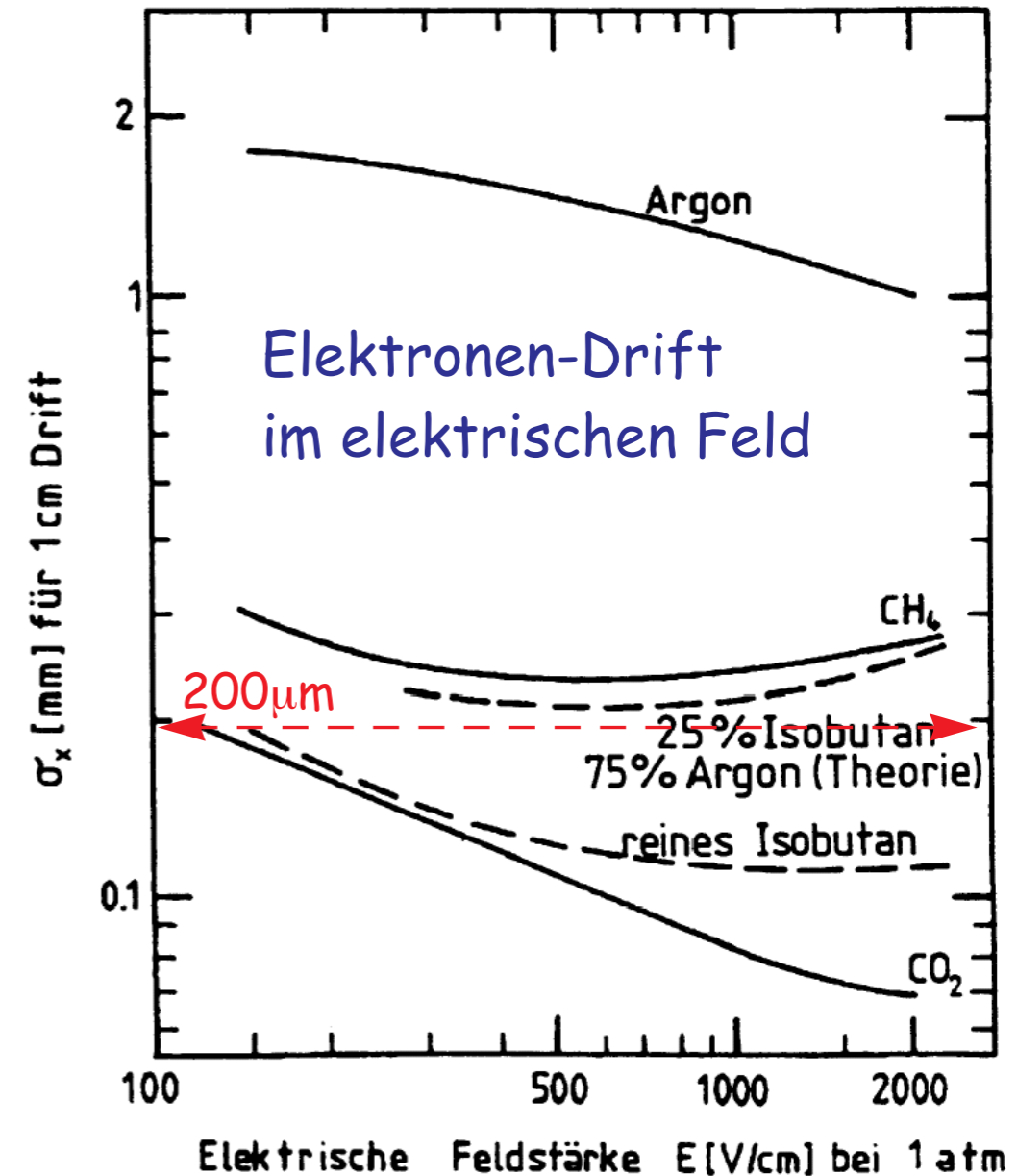
Ion- and Electron-Drift for several Gases

- mean free path
- diffusion coefficient
- mobility

von Ionen für einige Gase bei Normalbedingungen:

Gas	A	$\lambda_{Ion}[\text{cm}]$	$D_{Ion}[\text{cm}^2/\text{s}]$	$\mu_{Ion}\left[\frac{\text{cm/s}}{\text{V/cm}}\right]$
H ₂	2.02	1.8×10^{-5}	0.34	13.0
He	4.00	2.8×10^{-5}	0.26	10.2
Ar	39.95	1.0×10^{-5}	0.04	1.7
O ₂	32.00	1.0×10^{-5}	0.06	2.2
H ₂ O	18.02	1.0×10^{-5}	0.02	0.7

- mobility of electrons is $\approx 10^3$ times larger compared to ions
- mean free path of electrons $\lambda_e \sim 6 \times \lambda_{ion}$



Transverse extension of an electron swarm after 1 cm drift as a function of the applied electric field strength.

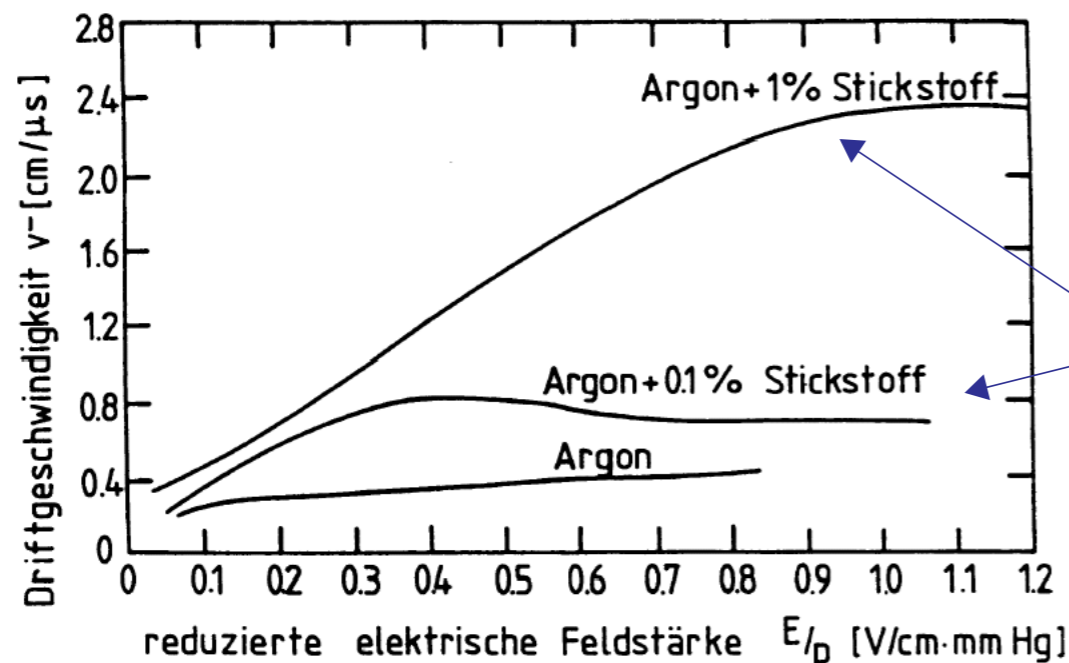
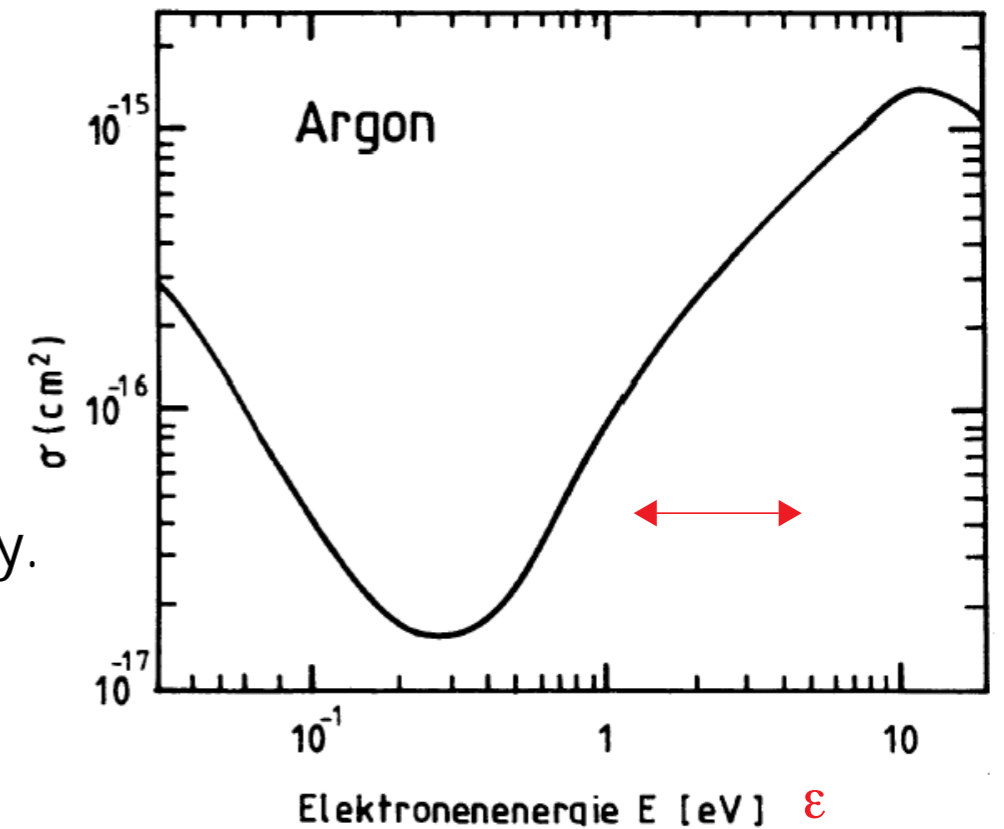
Ramsauer Effect

Due to their larger mean free path between collisions electrons can gain much more energy between collisions in the electric field than ions.

Drift velocity $\vec{v}_{\text{drift}} = \frac{e}{m} \cdot \vec{E} \cdot \tau(\vec{E}, \varepsilon)$, where the time τ

between the collisions depends on the collision cross section.

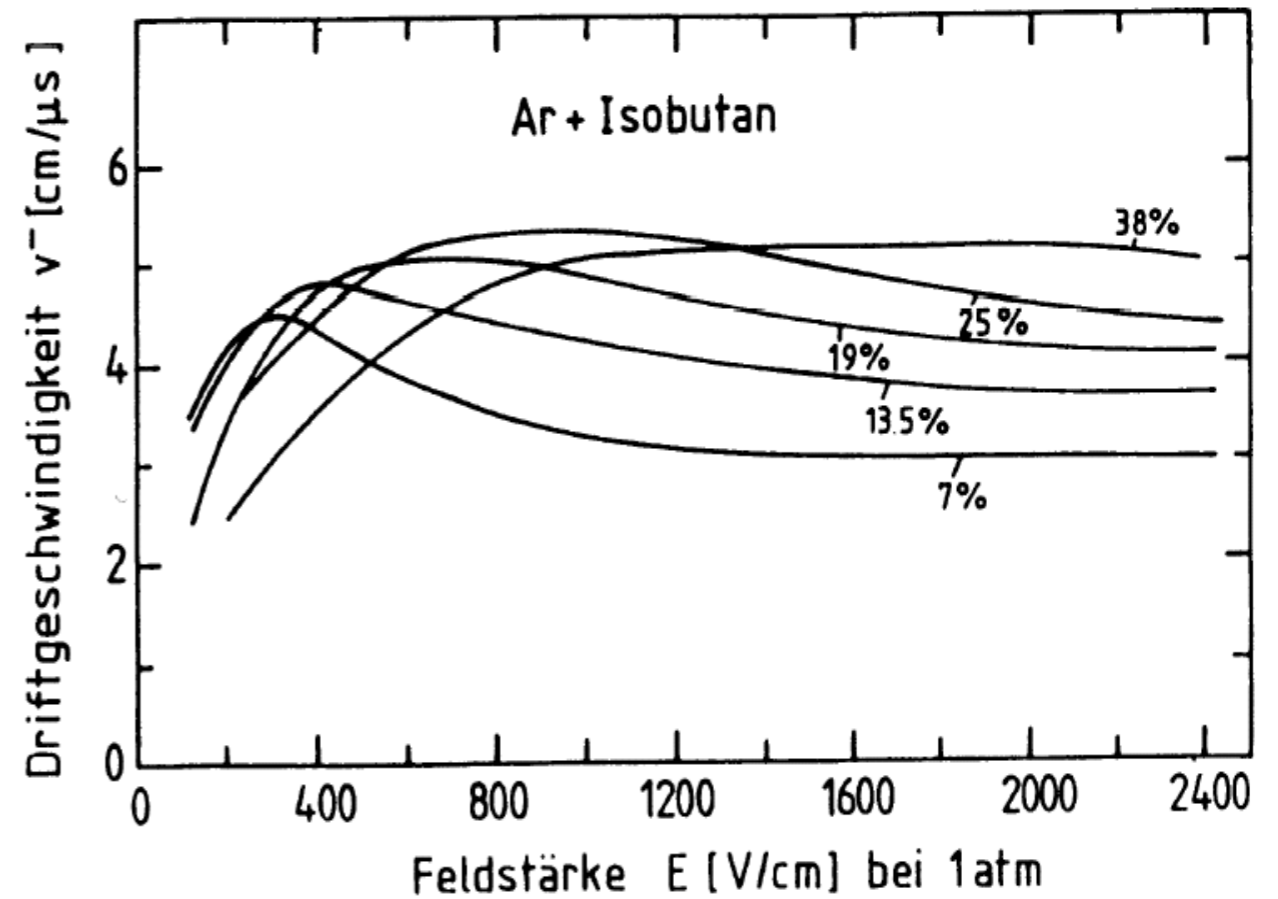
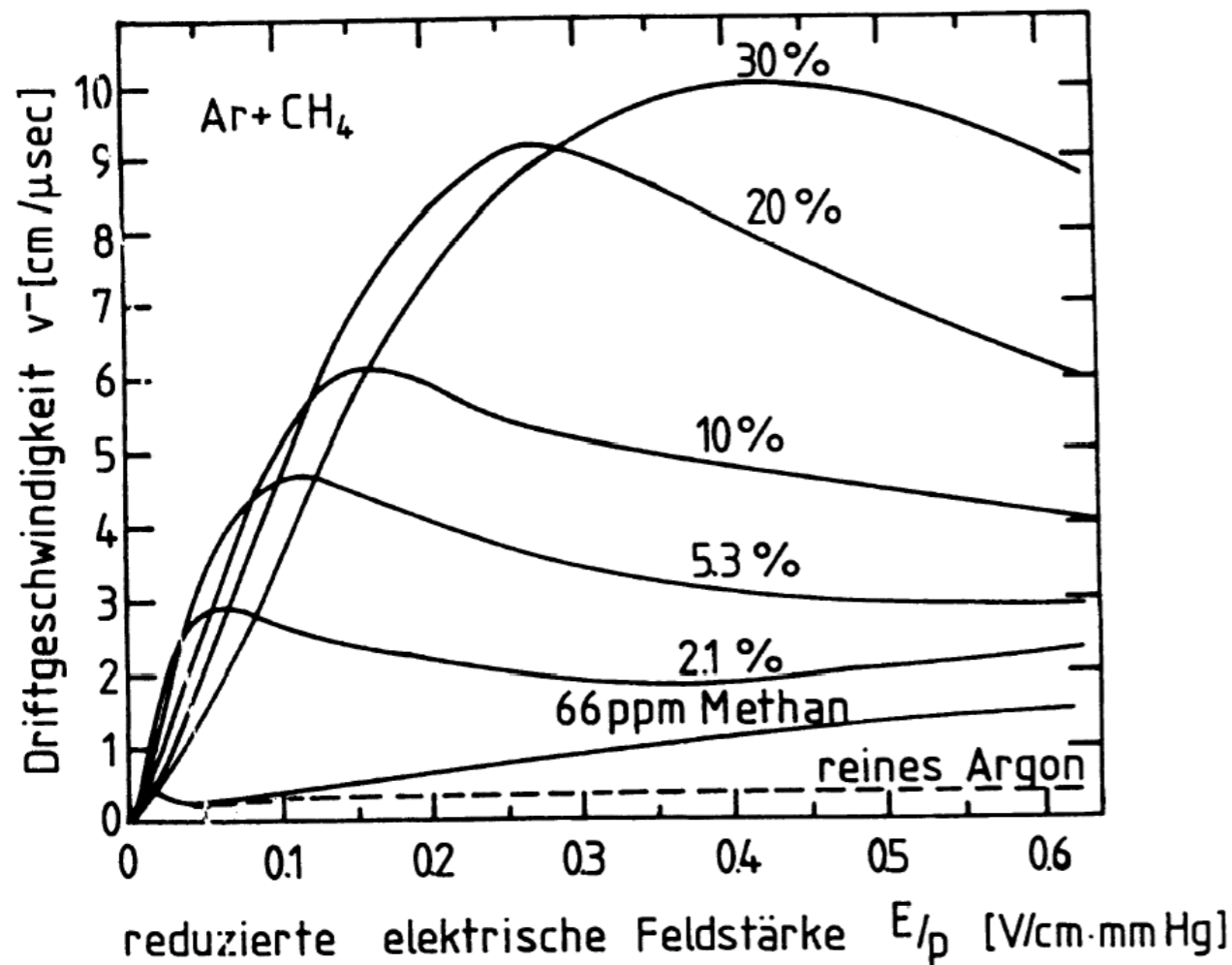
Wavelengths of electrons in eV range roughly correspond to dimensions of molecules in noble gases \Rightarrow quantum mechanical interference effects lead to a **strong variation** of cross section σ with electron energy.



As a consequence already tiny impurities which lead to a small shift in the **mean energy ε** can cause a **drastic** change of the drift velocity of gases.

\Rightarrow necessity of gas cleaning

v_{drift} vs E-Field for various Argon-Mixtures



- details in the energy dependence of the cross section (Ramsauer minimum) lead to a characteristic maximum in the dependence on the electric field
- for stable operation one often tries to operate at the maximum: $\frac{dv_{\text{drift}}}{dE} = 0$
- typical values for drift velocities are: $v_{\text{drift}} \approx 2\text{-}10 \text{ cm}/\mu\text{s} = 20\text{-}100 \mu\text{m}/\text{ns}$

Drift in E- und B-Fields: Langevin Equation

The presence of magnetic field drastically modifies the drift properties of electrons. The equation of motion: $m\ddot{\vec{x}} = q\vec{E} + q \cdot \vec{v} \times \vec{B} + m\vec{A}(t)$, where $m\vec{A}(t)$ represents a time dependent stochastic force, which results from the collisions with the gas molecules. In the time average this force can be described by a frictional force which is proportional to the velocity $-m\vec{v}/\tau$, where τ is the mean time between collisions. Integration yields:

$$\vec{v}_{\text{drift}} = \frac{\mu}{1 + \omega^2\tau^2} \left(\vec{E} + \frac{\vec{E} \times \vec{B}}{B} \omega\tau + \frac{(\vec{E} \cdot \vec{B}) \cdot \vec{B}}{B^2} \omega^2\tau^2 \right) \text{ for the stationary case.}$$

$\mu = e \cdot \tau / m$ mobility of charge carriers

$\omega = e \cdot B / m$ cyclotron frequency

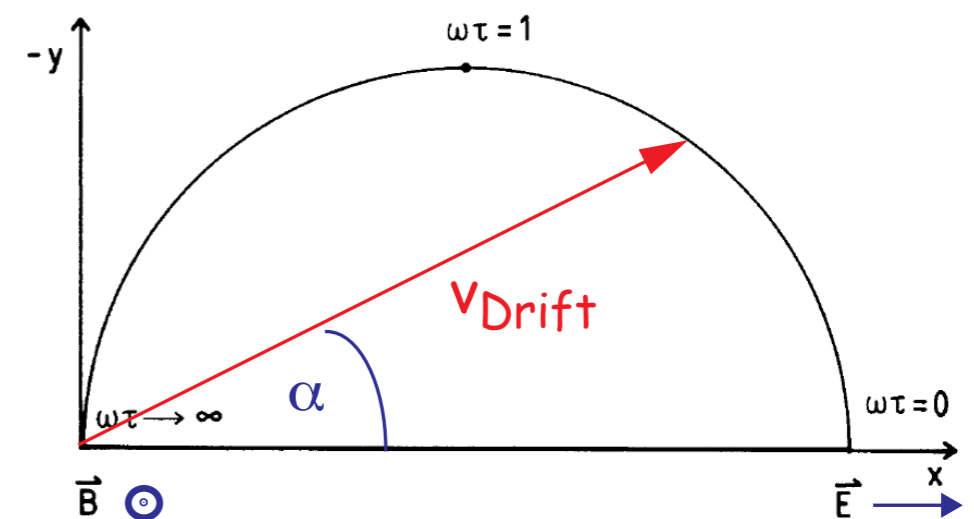
For the frequently realized case $\vec{E} \perp \vec{B}$:

- modulus of drift velocity:

$$|\vec{v}_{\text{drift}}| = \frac{\mu E}{\sqrt{1 + \omega^2\tau^2}}$$

- Lorentzangle α between drift direction and direction of the magnetic field:

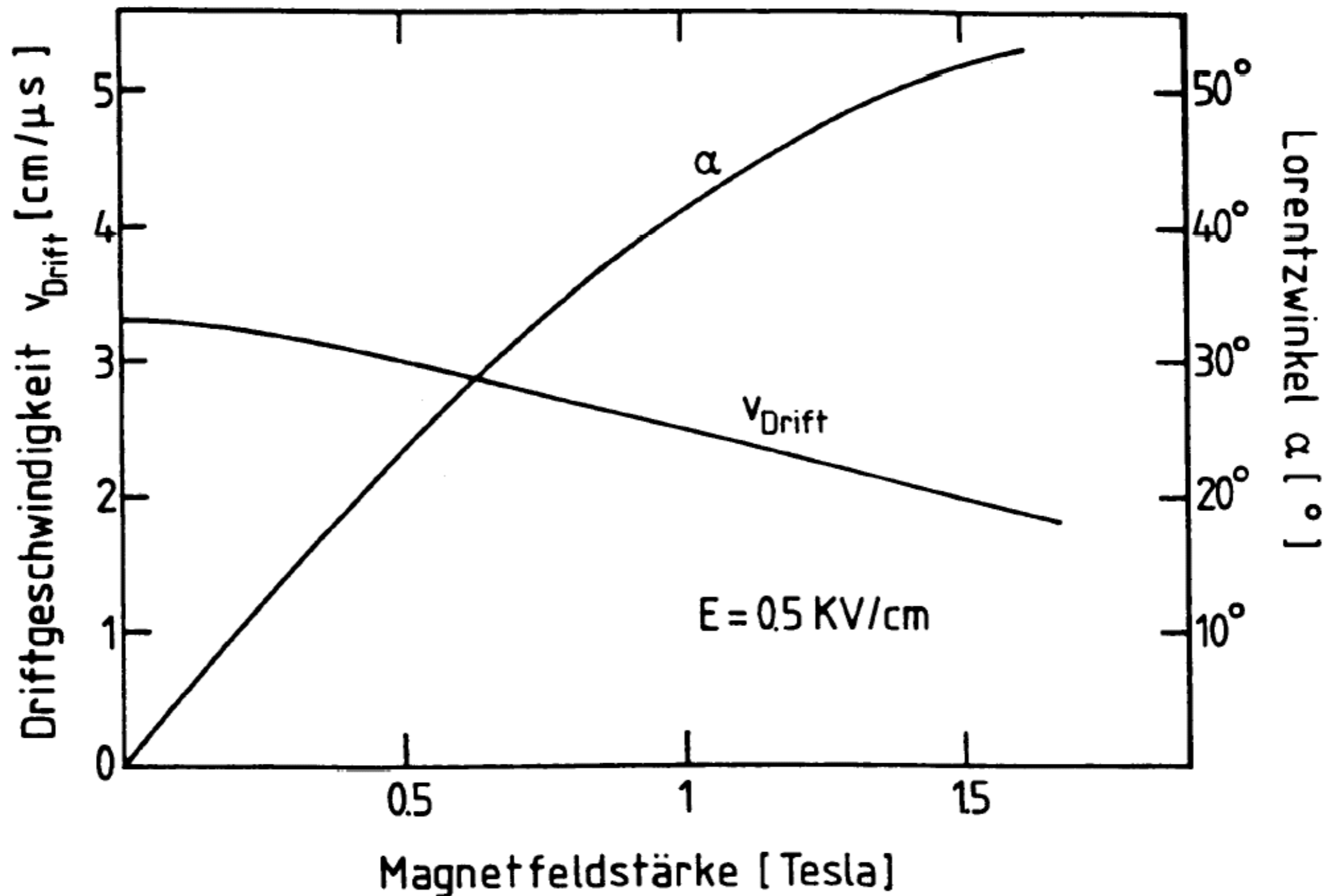
$$\alpha \approx v_{\text{drift}} \frac{B}{E} \text{ for } \alpha \ll 1$$



$$\tan \alpha = \omega\tau = \frac{1}{\sqrt{\frac{1}{v_{\text{Drift}}^2} \frac{E^2}{B^2} - 1}}$$

B-Field Dependence of v_{drift} and α

Example: Argon (67.2%) Isobutane (30.3%) Methylal (2.5%)



Diffusion in Electric and Magnetic Fields

The magnetic field also reduces the diffusion coefficient of electrons transversely to the direction of the magnetic field.

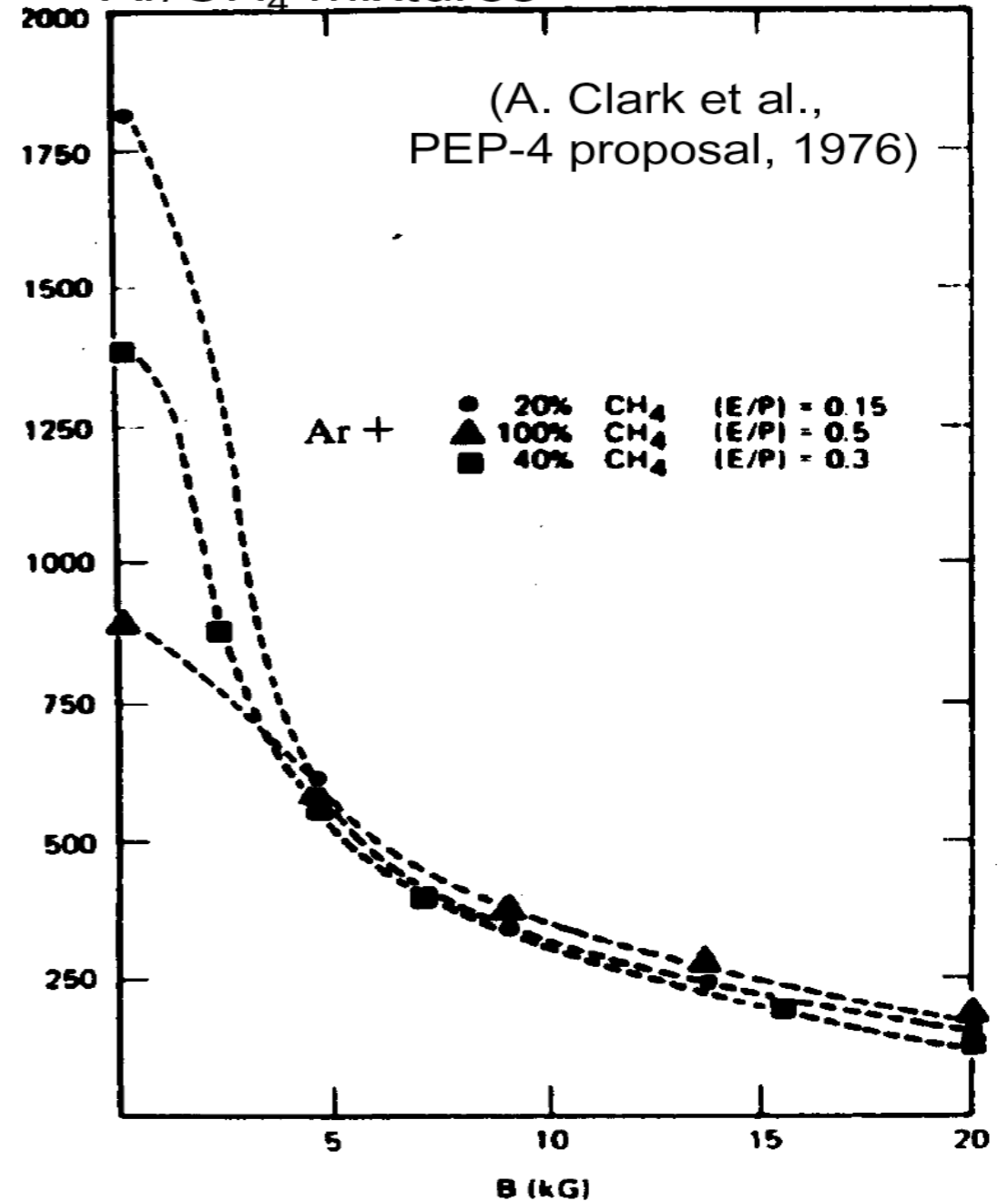
For $\vec{B} = (0, 0, B_z)$ i.e. $\vec{E} \parallel \vec{B}$ the diffusion coefficient gets modified to:

$$D_z = D$$

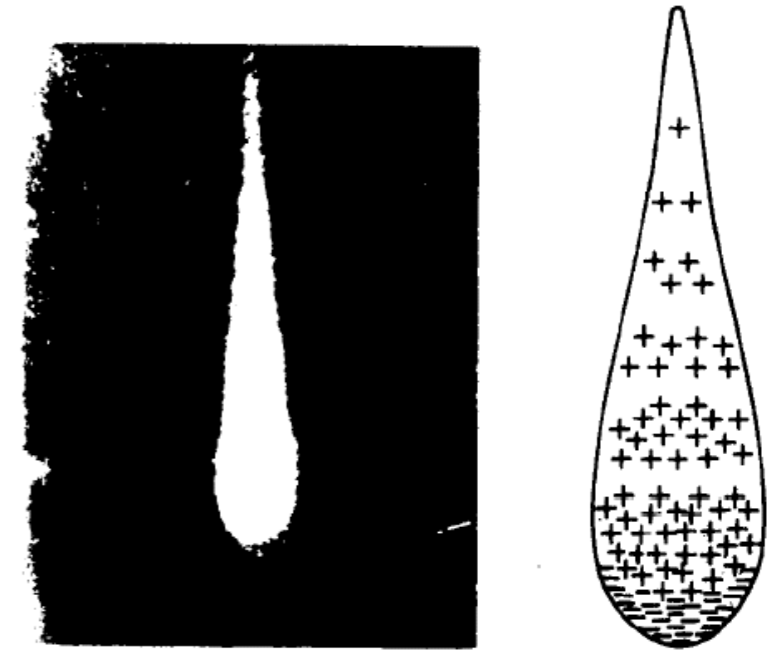
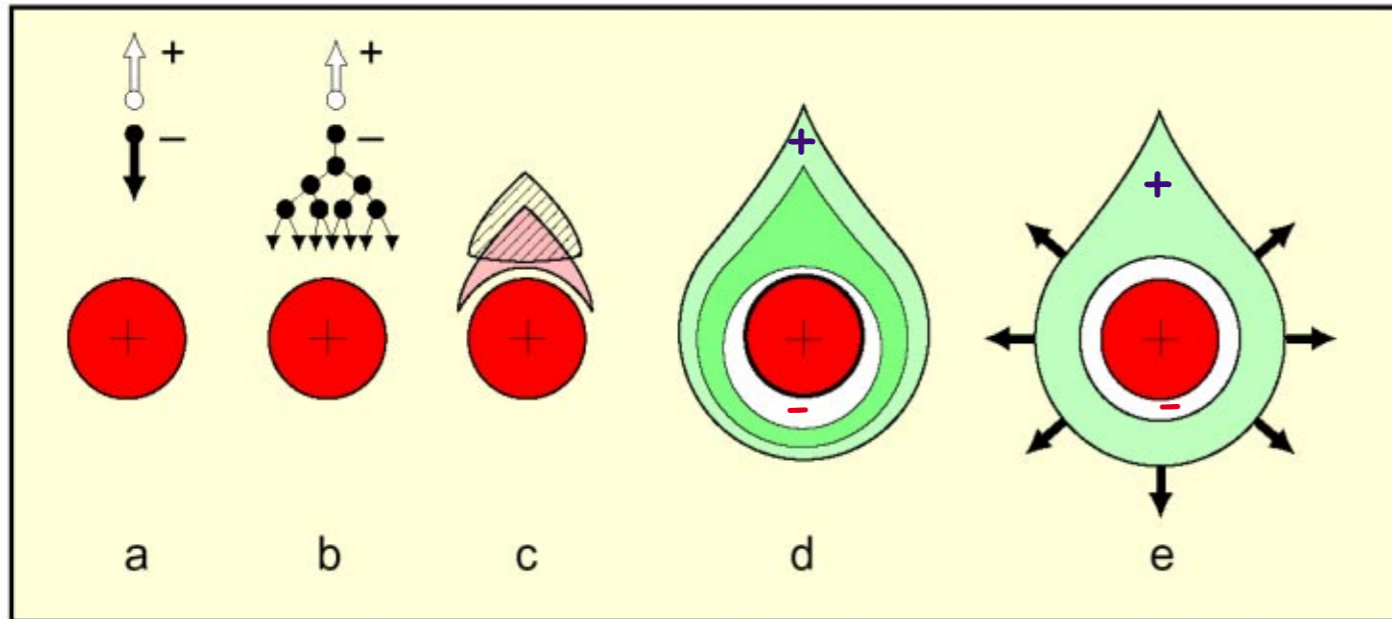
$$D_x = D_y = \frac{D}{1 + \omega^2 \tau^2}$$

This effect is for instance exploited in a TPC, in order to reduce the effect of transverse diffusion during the long drift time.

Transverse diffusion σ (μm) for a drift of 15 cm in different Ar/CH₄ mixtures



Avalanche Formation

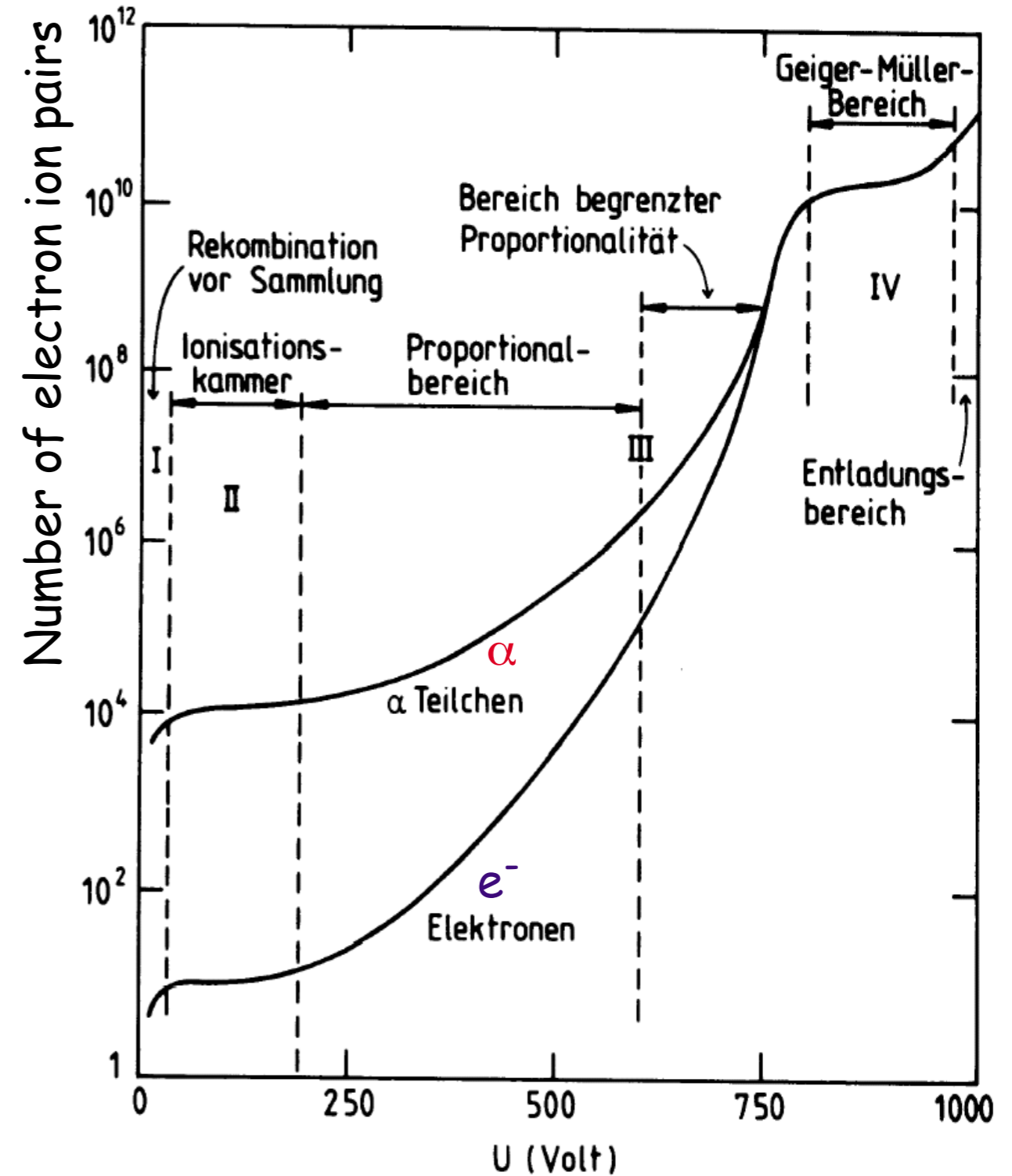


picture taken with
cloud chamber

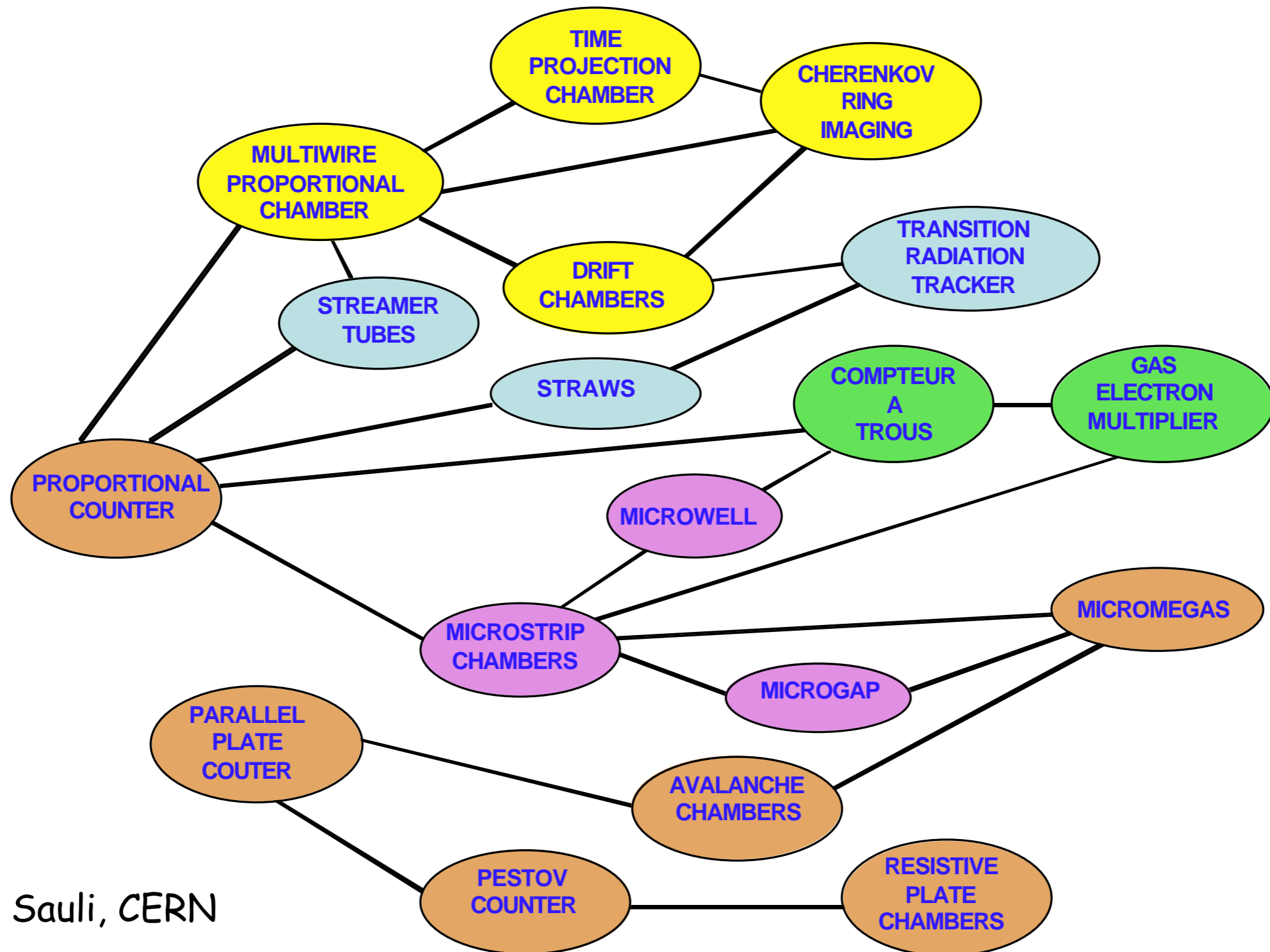
- due to transverse diffusion a droplet like avalanche develops around the anode
- electrons are collected very fast (≈ 1 ns)
mobility of electrons ≈ 1000 times larger than for ions
- the cloud of positive ions remains and slowly drifts towards the cathode

Modes of Operation of Gas Detectors

- **Ionisation chamber:**
complete charge collection, but no charge amplification.
- **Proportional counter:**
above threshold voltage multiplication starts. Detected signal proportional to original ionization → energy measurement (dE/dx) possible. Secondary avalanches have to be quenched. Gain $\approx 10^4-10^5$
- **Region of limited proportionality:**
or streamer mode: strong photo emission → secondary avalanches. Needs efficient quencher or pulsed HV. Gain upto $\approx 10^9$, hence simple electronics sufficient.
- **Geiger-Müller counter:**
massive photo emission. Full length of anode wire affected. Stop discharge by HV break down. Strong quenchers needed.



Family Tree of Gaseous Detectors



Fabio Sauli, CERN

Multi Wire Proportional Chamber

Generalize principal of proportional tube to large area detector.

Multi Wire Proportional Chamber: **MWPC**

George Charpak 1968

- anode wires act as independent detectors
- capacitive coupling of negative signal from anode wire where avalanche is formed to neighbours is small compared to pulse, which is generated by ions drifting towards cathode
- furthermore development in electronics: possibility to read many channels in parallel → 10^6 tracks per second
⇒ Breakthrough in detector development

Nobelprize for physics 1992

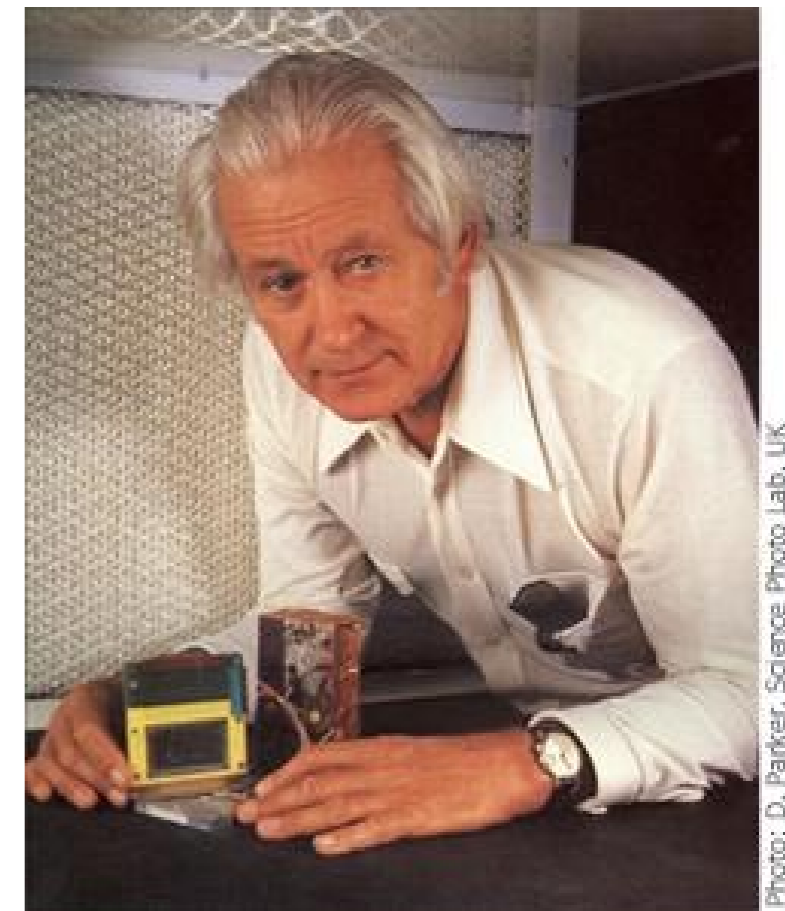
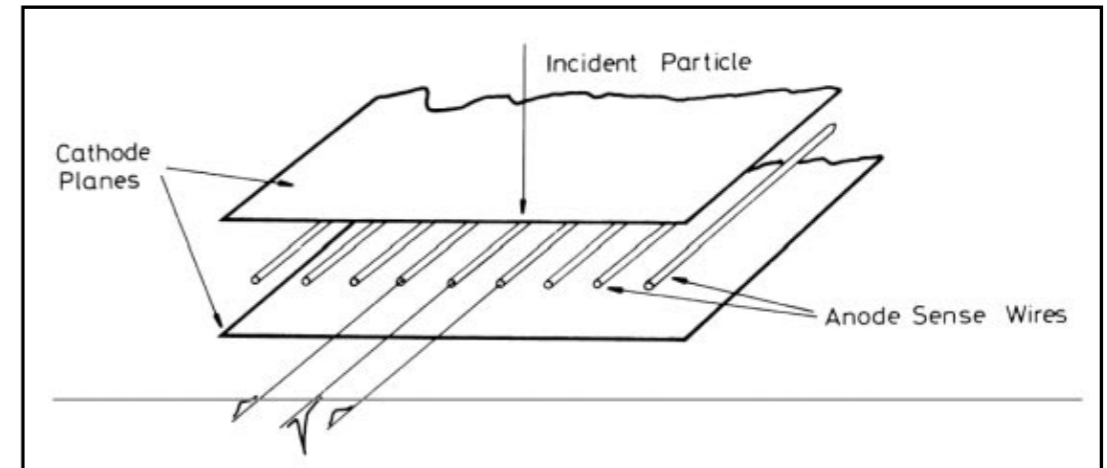


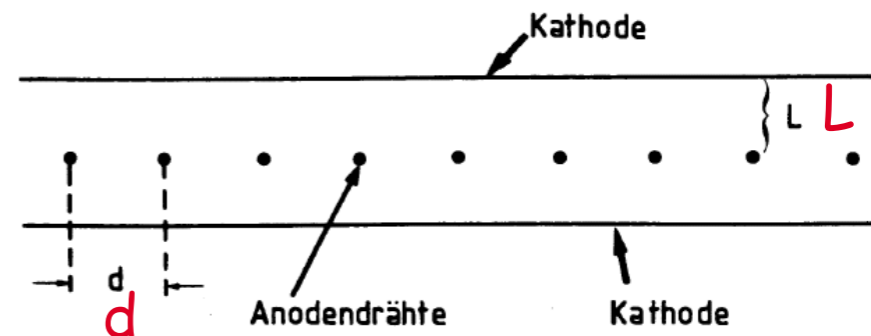
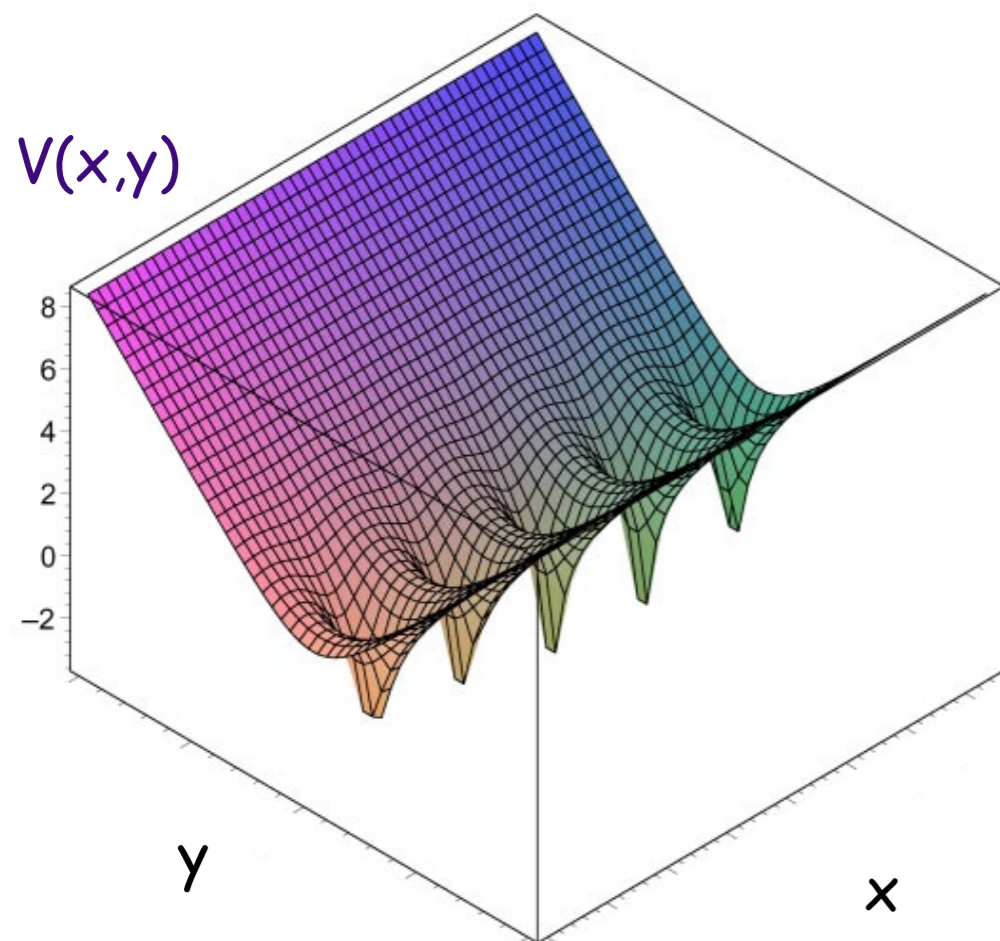
Photo: D. Parker, Science Photo Lab, UK

MWPC

Use of gold plated tungsten wires with diameter 15-30 μm as anode wires. Chamber walls made from glas fiber material (rigid, low mass). Thin metal foil acting as cathode (typically $d \approx 50\mu\text{m}$). Typical dimensions: $d = 2\text{mm}$, $L = 4\text{mm}$.

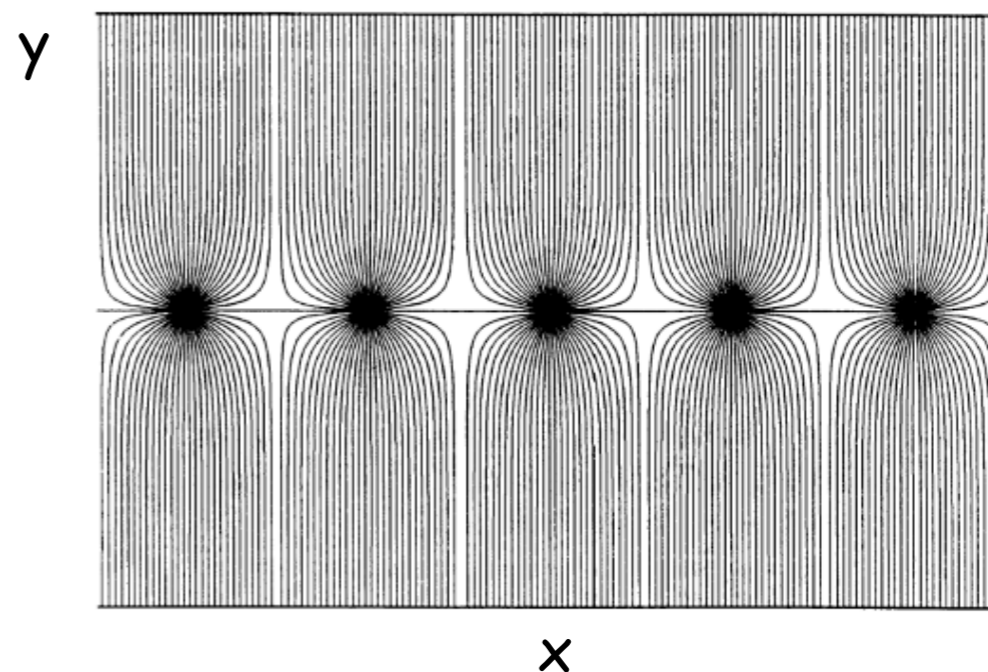
Electrostatic potential in a planar MWPC given by:

$$V(x, y) = -\frac{q}{4\pi\epsilon_0} \ln \left\{ 4 \left[\sin^2 \left(\frac{\pi x}{d} \right) + \sinh^2 \left(\frac{\pi y}{d} \right) \right] \right\}$$



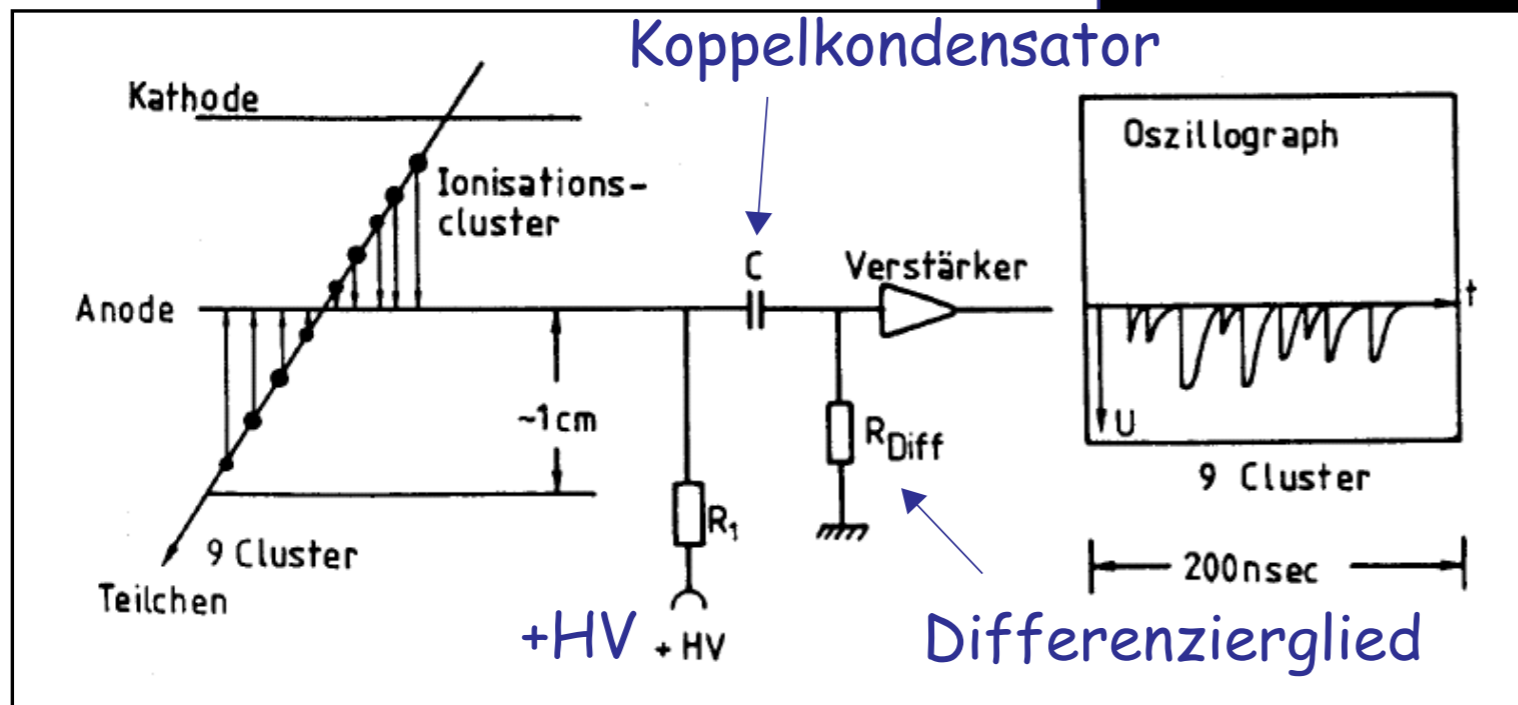
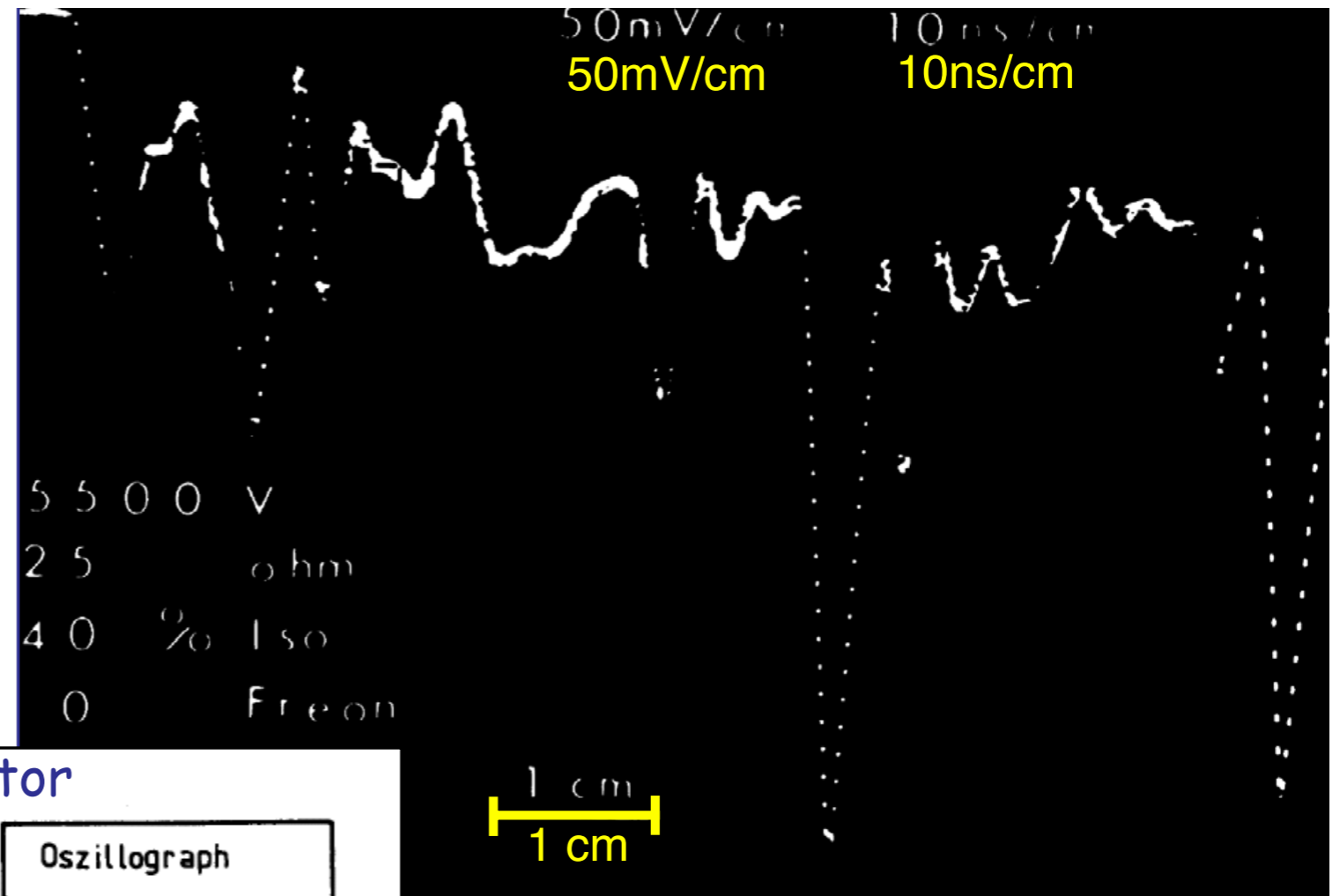
resolution $\sigma = d/\sqrt{12}$

electric field lines



Signals in a MWPC

Using suitable electronics the avalanches caused by single **ionization cluster** can be made visible with an oscilloscope



Plateau Curve

Readout of MWPCs is typical digital, i.e. all signals surpassing a given threshold are recorded.

The detection efficiency then depends on:

- applied high voltage
- threshold value
- selected duration of time window

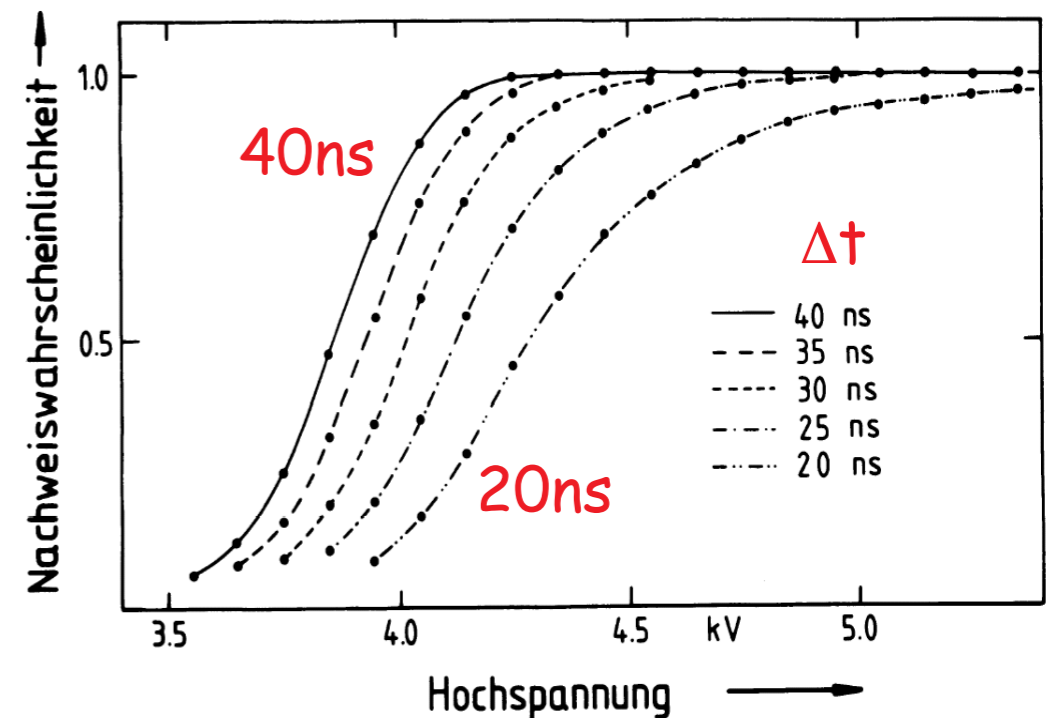
=> chose HV so, that \approx all signals are above threshold. i.e. $\epsilon \approx 100\%$, but no continuous discharge anywhere in the detector

The actual choice of parameters depends on:

- level of noise
- sensitivity of electronics
- required time resolution

Ionisation/cm $1/\lambda$ Dicke für 99% Effizienz: 4.6λ

	Encounters/cm	$t_{99}(\text{mm})$	Free electrons/cm
He	5	9.2	16
Ne	12	3.8	42
Ar	25	1.8	103
Xe	46	1.0	340
CH ₄	27	1.7	62
CO ₂	35	1.3	107
C ₂ H ₆	43	1.1	113



Mechanical Considerations

- Requirements on mechanical tolerances for stable operation are quite high

- the gas gain M depends on variations of anode diameter a and gap size L

$$\frac{\Delta M}{M} \approx 3 \frac{\Delta a}{a} \quad \text{resp.} \quad \frac{\Delta M}{M} \approx 12 \frac{\Delta L}{L} \quad \Rightarrow \text{effective shortening of plateau due to gap variations}$$

- For digital readout the resolution perpendicular to wire directions is: $\sigma(x) = d/\sqrt{12}$

- Wire length and wire spacing can only be changed within certain limits

- **Wire sagging** is caused by forces acting perpendicular to the wires

- electrostatic deflection

- gravitation

- Range of stability: $\frac{d}{l} \geq 1.5 \times 10^{-3} V[kV] \sqrt{\frac{20}{T[g]}}$ with wire length l , high voltage V and wire tension T

longer wires have to be mechanically supported (fixed), which however causes local inefficiencies

Pros and Cons of MWPCs

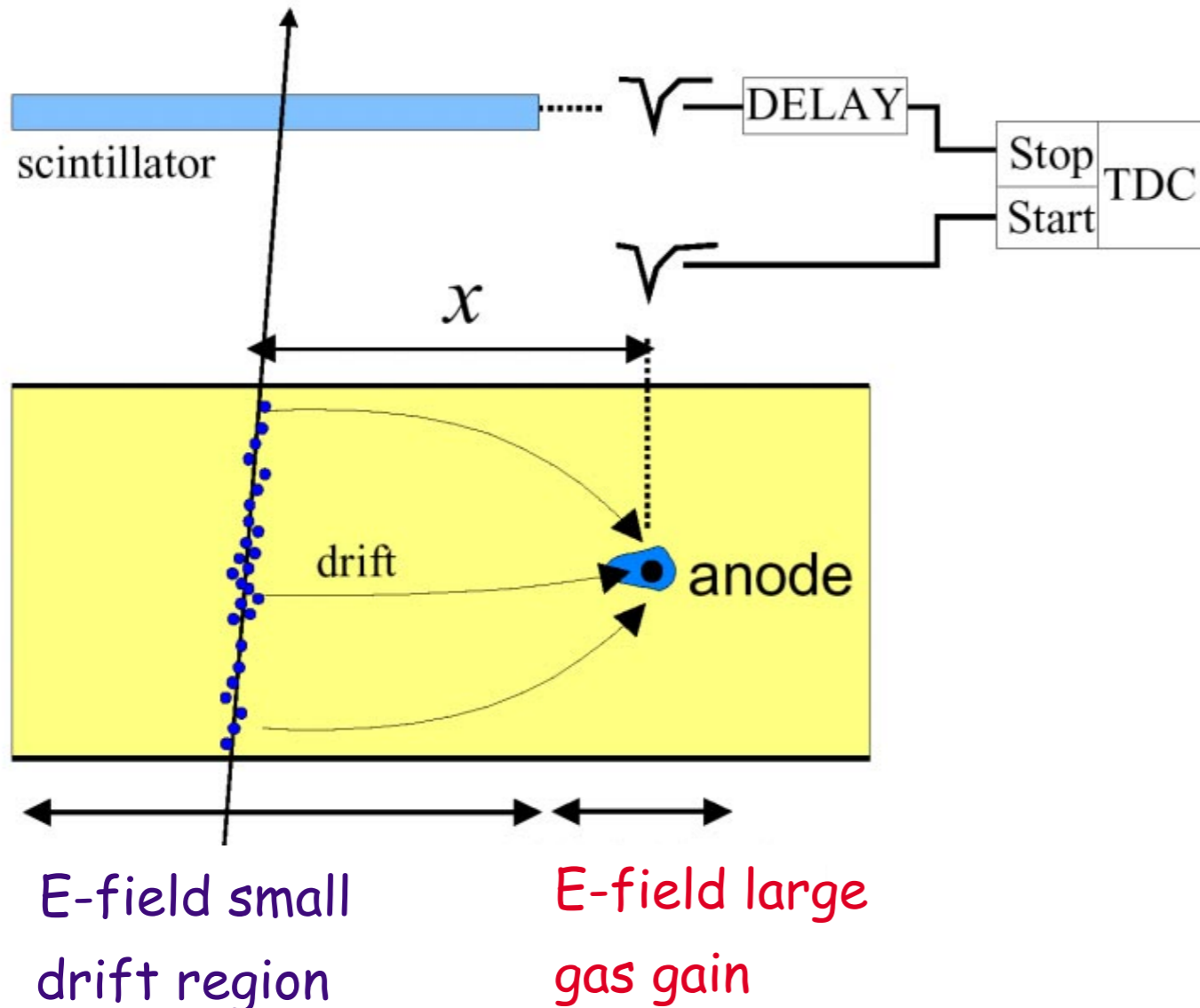
- Advantages

- relatively easy to equip large areas
- planar and cylindrical geometries can be realized
- simple coordinate reconstruction using digital readout
- good rate capability
- operation in magnetic field without problem (small drift distances)
- fast signal ($t \approx 20-50$ ns)
=> can be used for triggering

- Disadvantages

- limited position resolution $\sigma(x) = d/\sqrt{12}$
i.e. typically $\approx 0.5-1$ mm
- large tracking volumes need a large number of readout channels
- => not suited for optimal momentum resolution
- for very high rates gain loss due to space charge effects

Principal of a Driftchamber



TDC: Time to Digital Converter

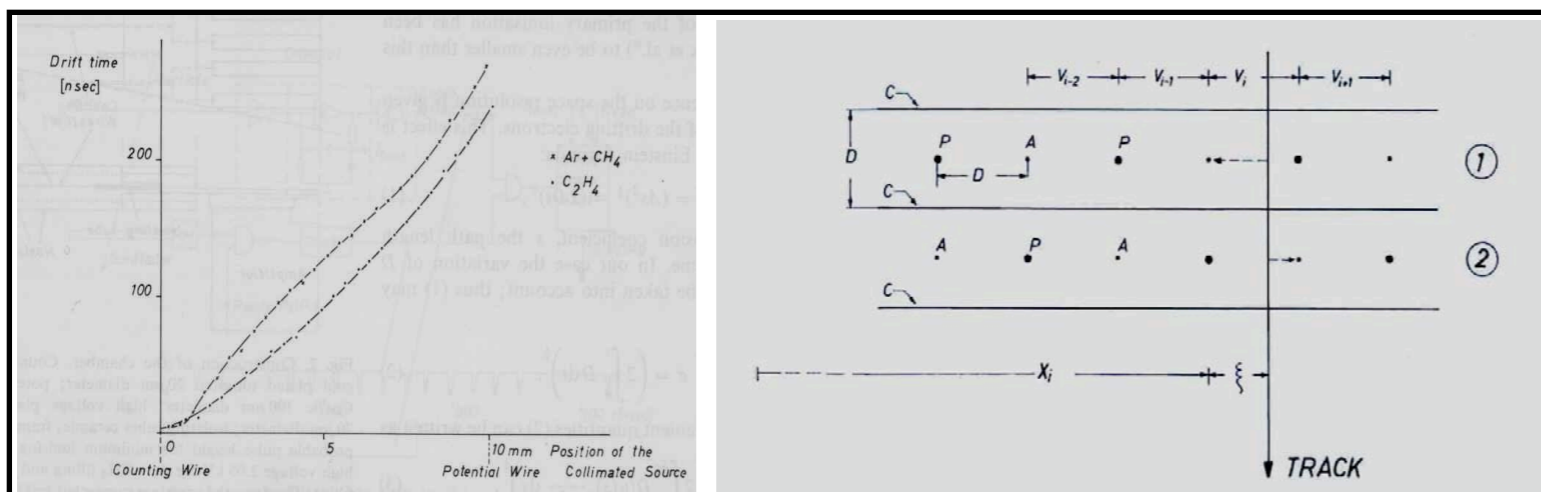
Measure arrival time t_1 of electrons at anode wire relative to reference t_0 .

- external definition of time reference t_0 (here by fast scintillator signal)
- x -coordinate given by:

$$x = \int_{t_0}^{t_1} v_D(t) dt$$

- if drift velocity v_D constant over full drift distance: $x = v_D(t_1 - t_0) = v_D \Delta t$
- advantage of drift chambers: much **larger sensitive volume** per read out channel

First Drift Chamber Phys. Inst. Heidelberg 1971

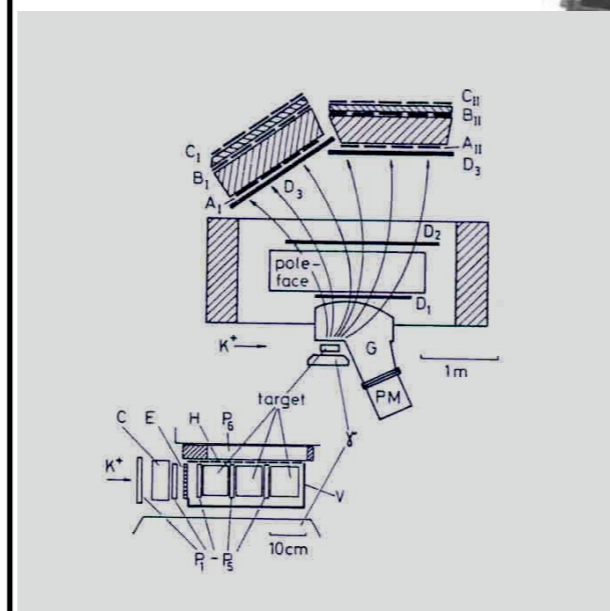
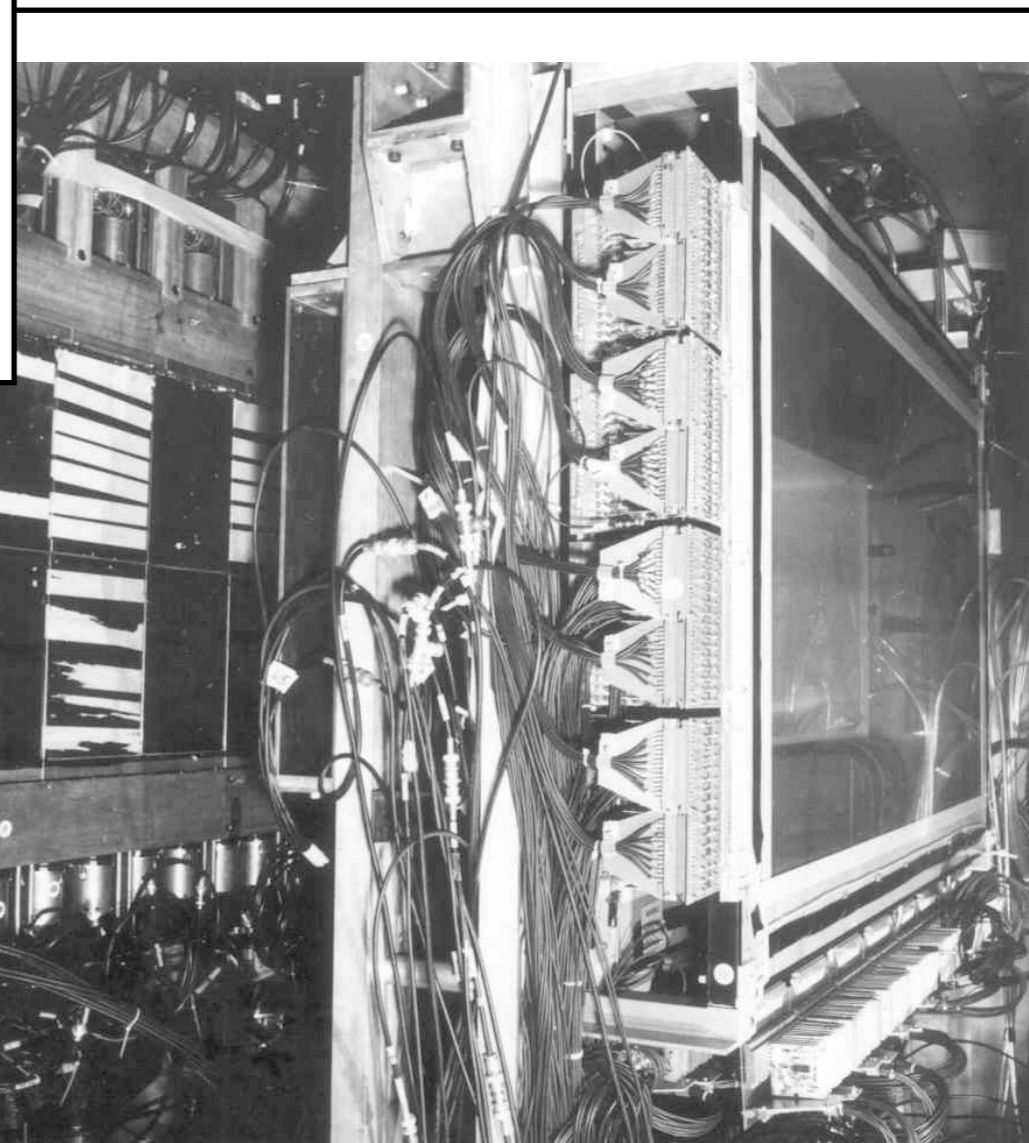


THE MULTIWIRE DRIFT CHAMBER A NEW TYPE OF PROPORTIONAL WIRE CHAMBER*

A. H. WALENTA, J. HEINTZE and B. SCHÜRLEIN

I. Physikalisches Institut der Universität Heidelberg, Heidelberg, Germany

Received 27 November 1970



Signal Readout

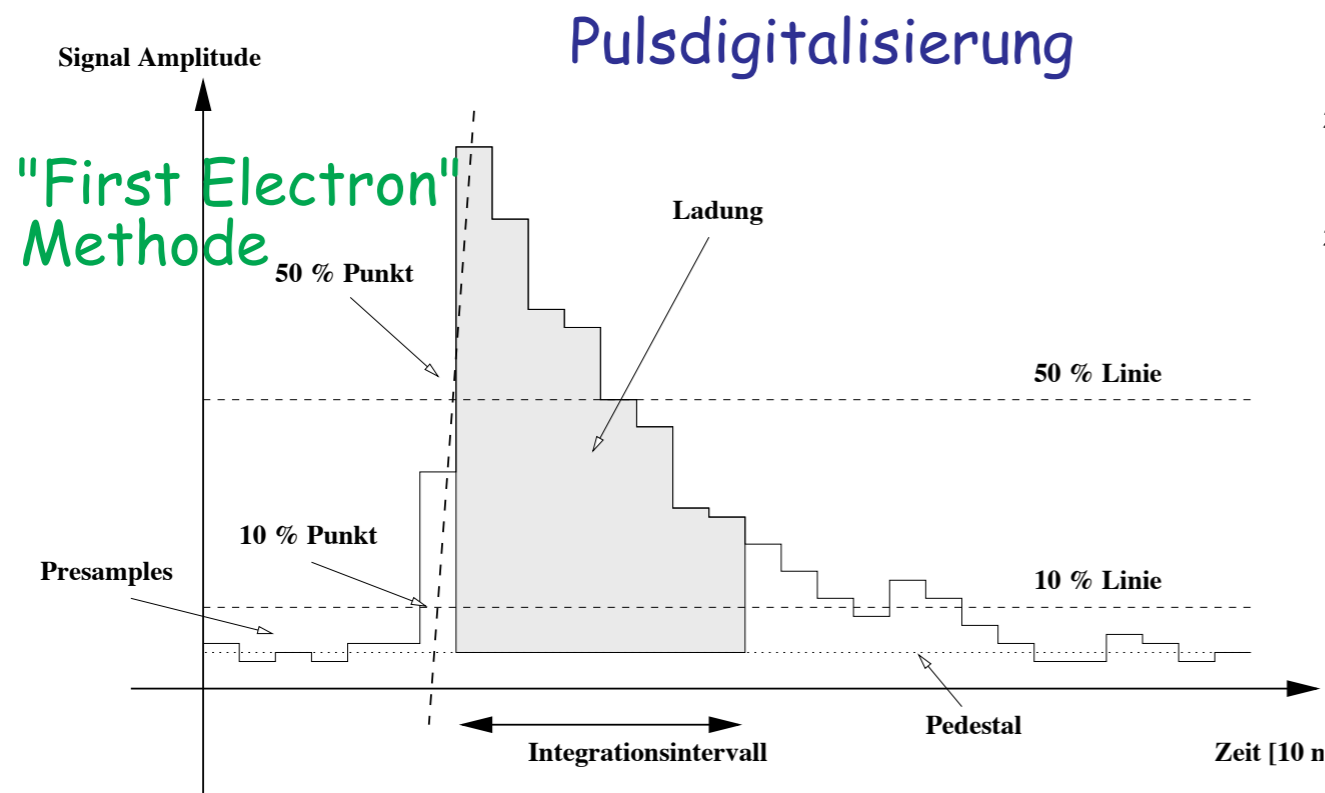
Since the number of channels in a drift chamber usually does not exceed a few thousands, one can afford more complex readout electronics

In most cases not only time, but also pulse shape is measured.

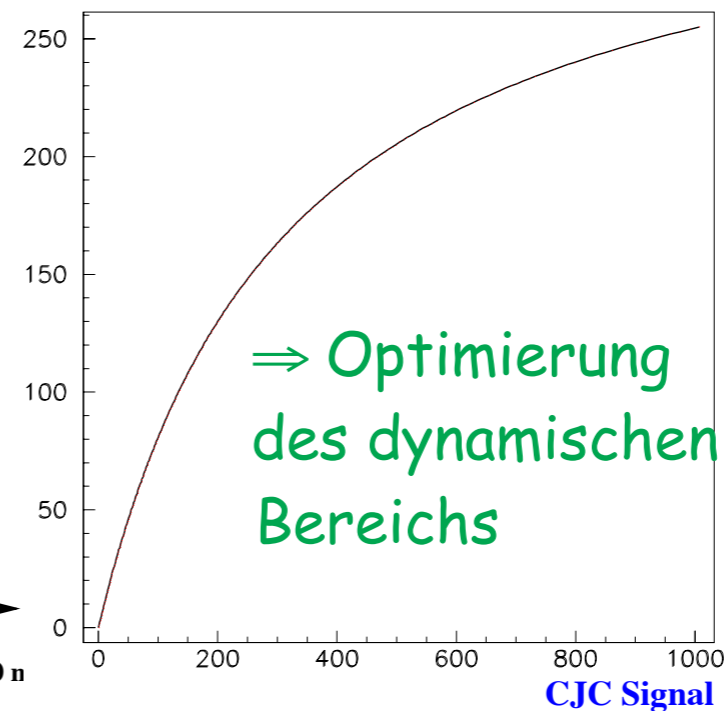
Advantages:

- better time determination
- measurement of dE/dx possible -> particle identification

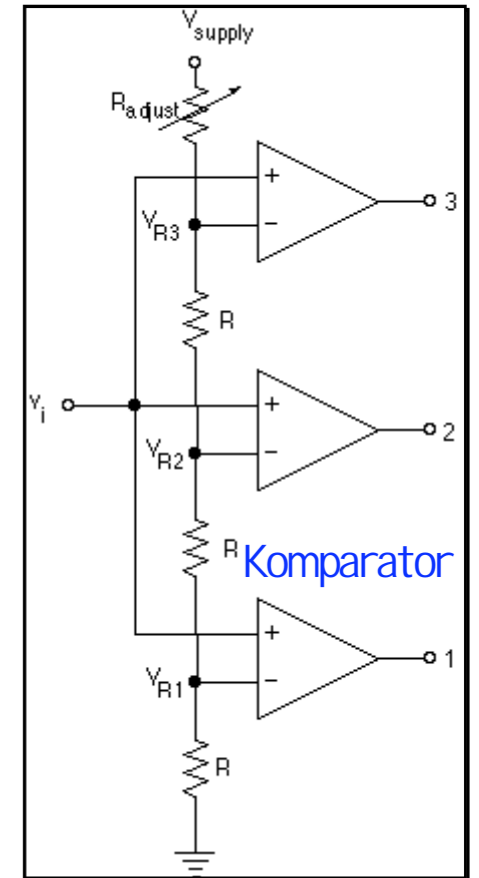
Very often so called **Flash Analog Digital Converter** (FADC) are used



Nichtlineare Kennlinie



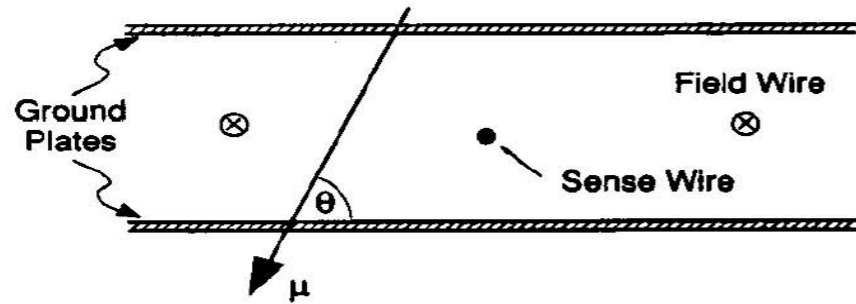
FADC-Digitalisierung



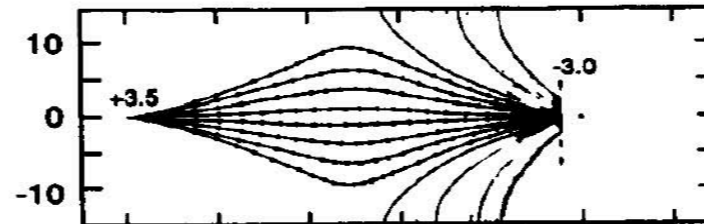
Prinzip eines 4-Bit FADC

Examples for Planar Cell Geometries

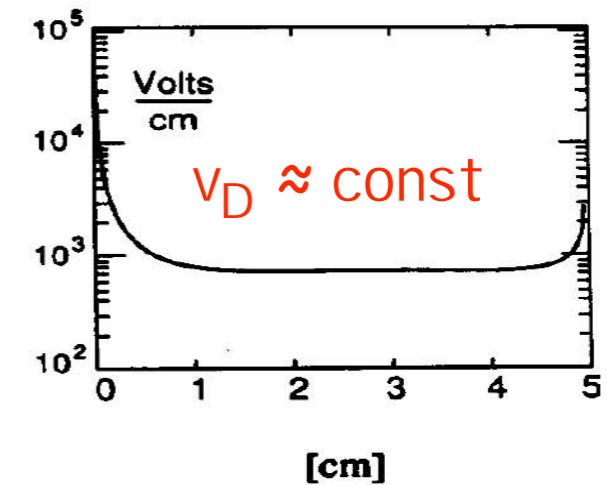
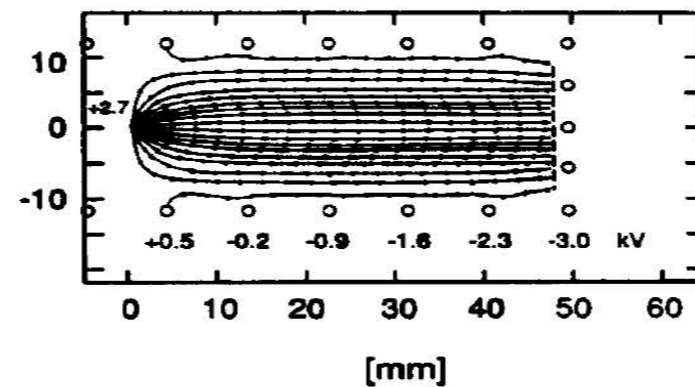
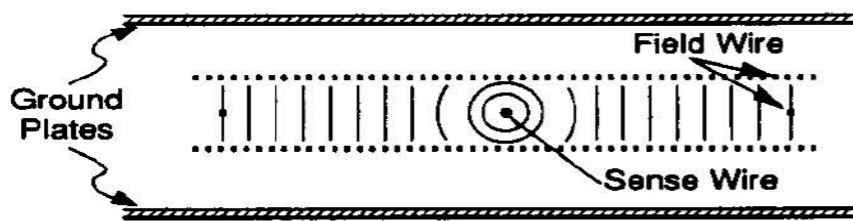
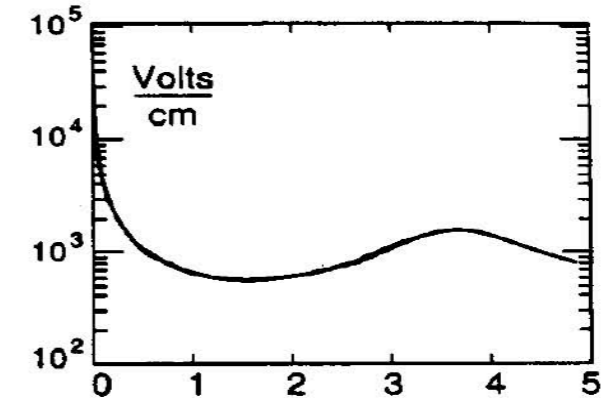
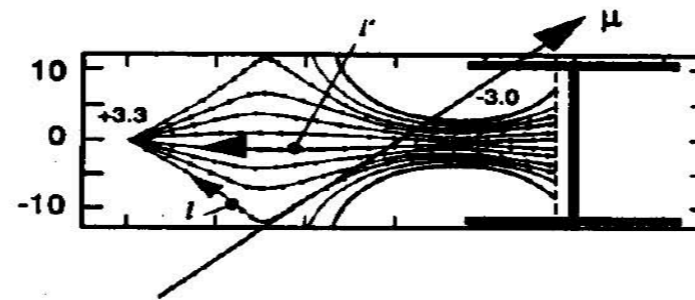
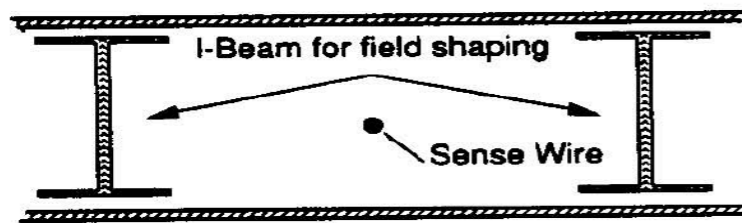
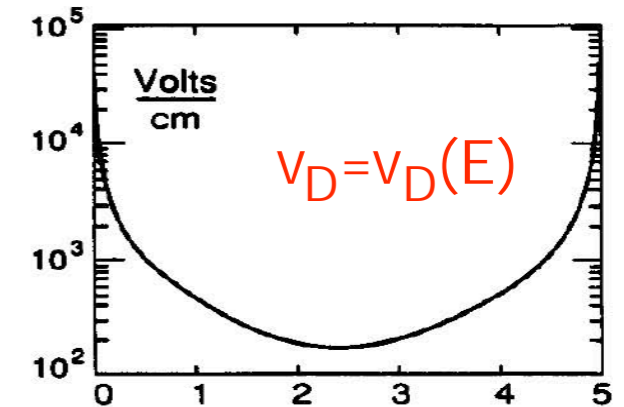
Geometrie



Feldlinienverlauf



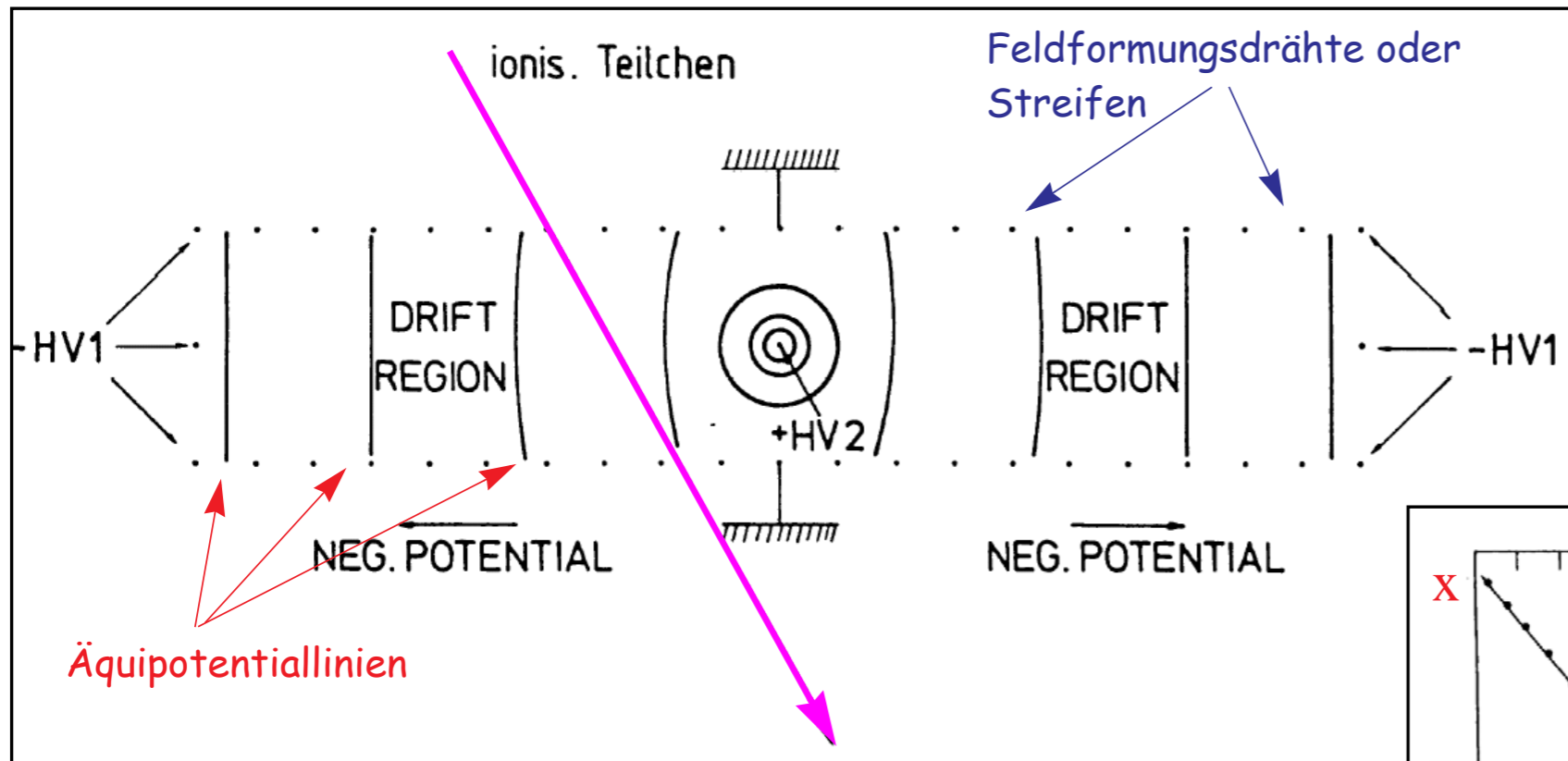
E-Feld



Zellgrösse kann sehr gross gewählt werden

Field Shaping

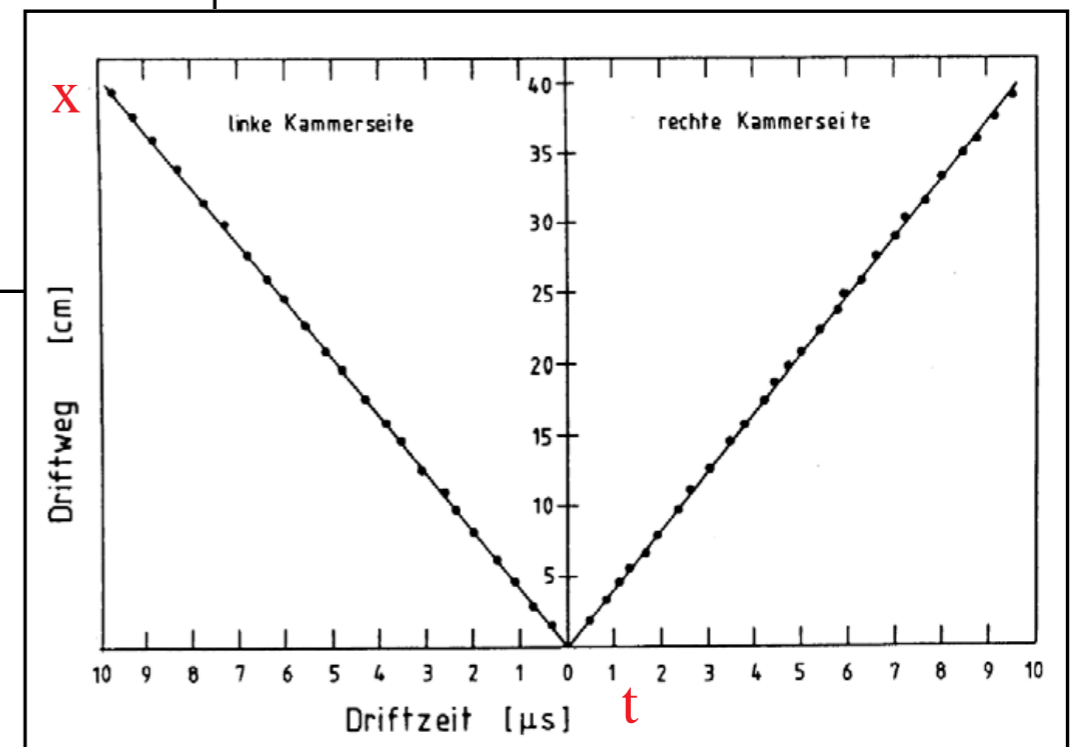
By suitable choice of geometry and potential of field forming wires one can reach constant E-field within the drift region and therefore **constant drift velocity** !



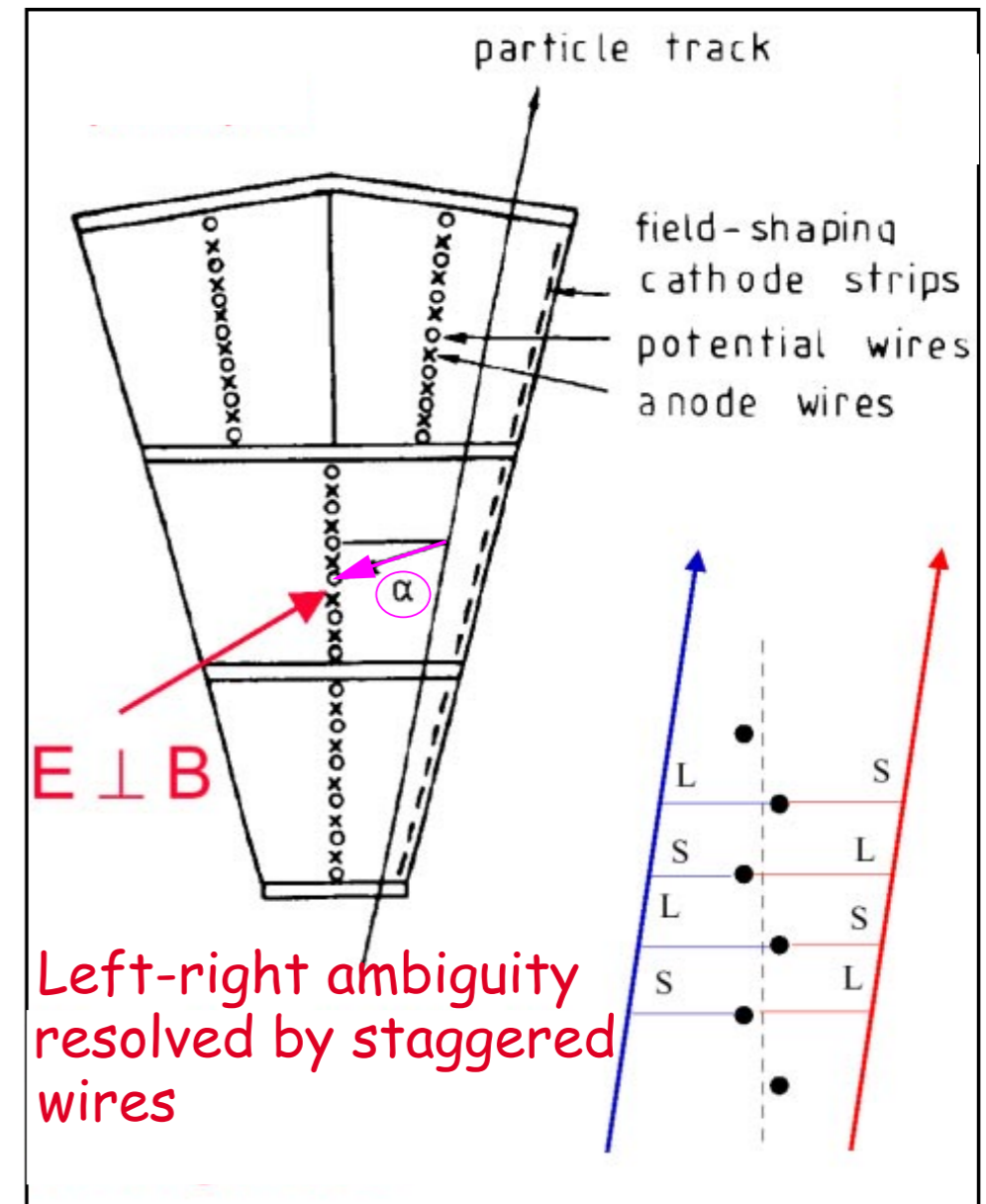
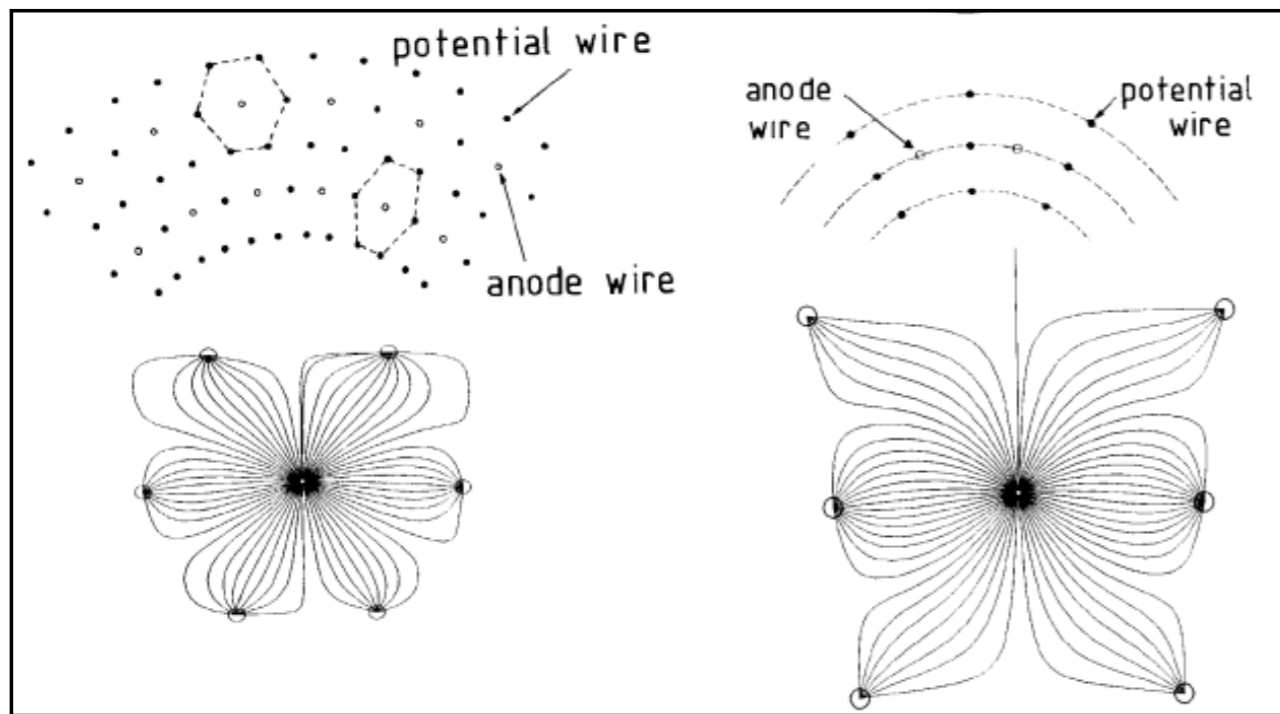
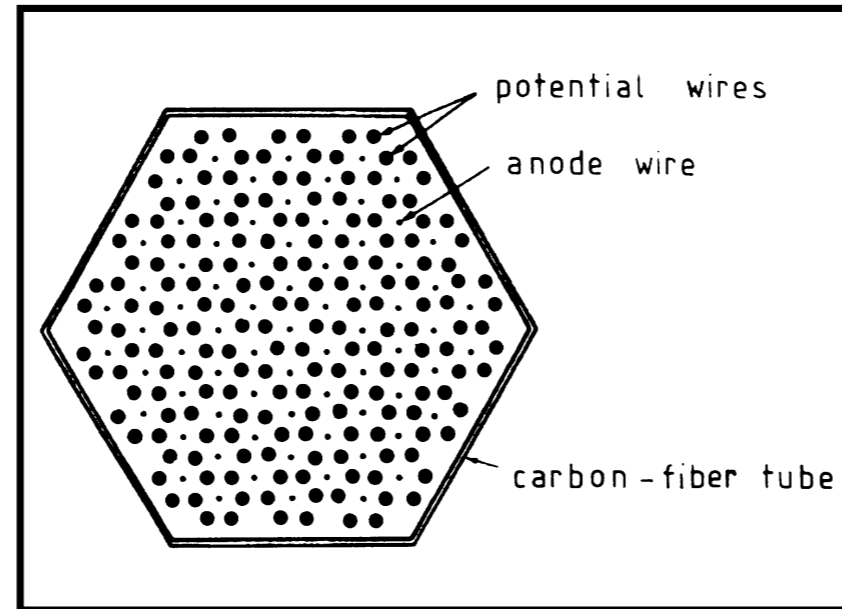
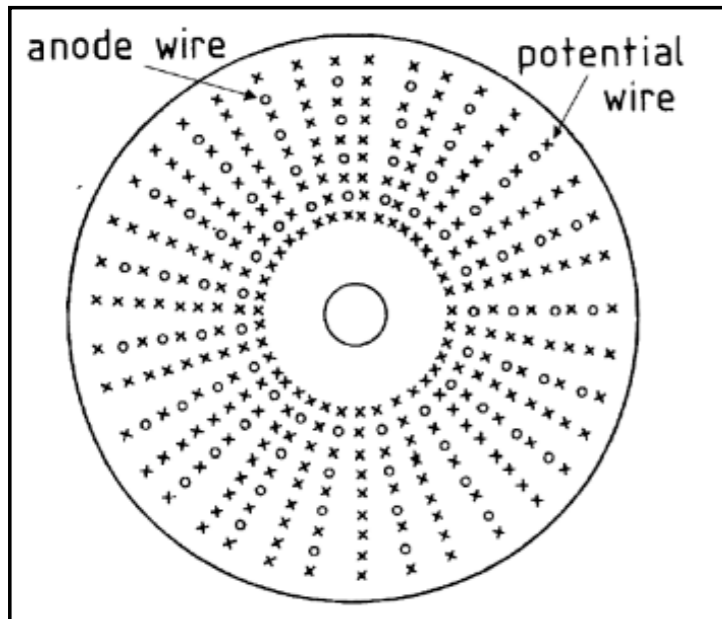
Space - drift time relation

Problem: for a single hit one can not determine on which side of the sense wire the particle passed

=> **left-right ambiguity**

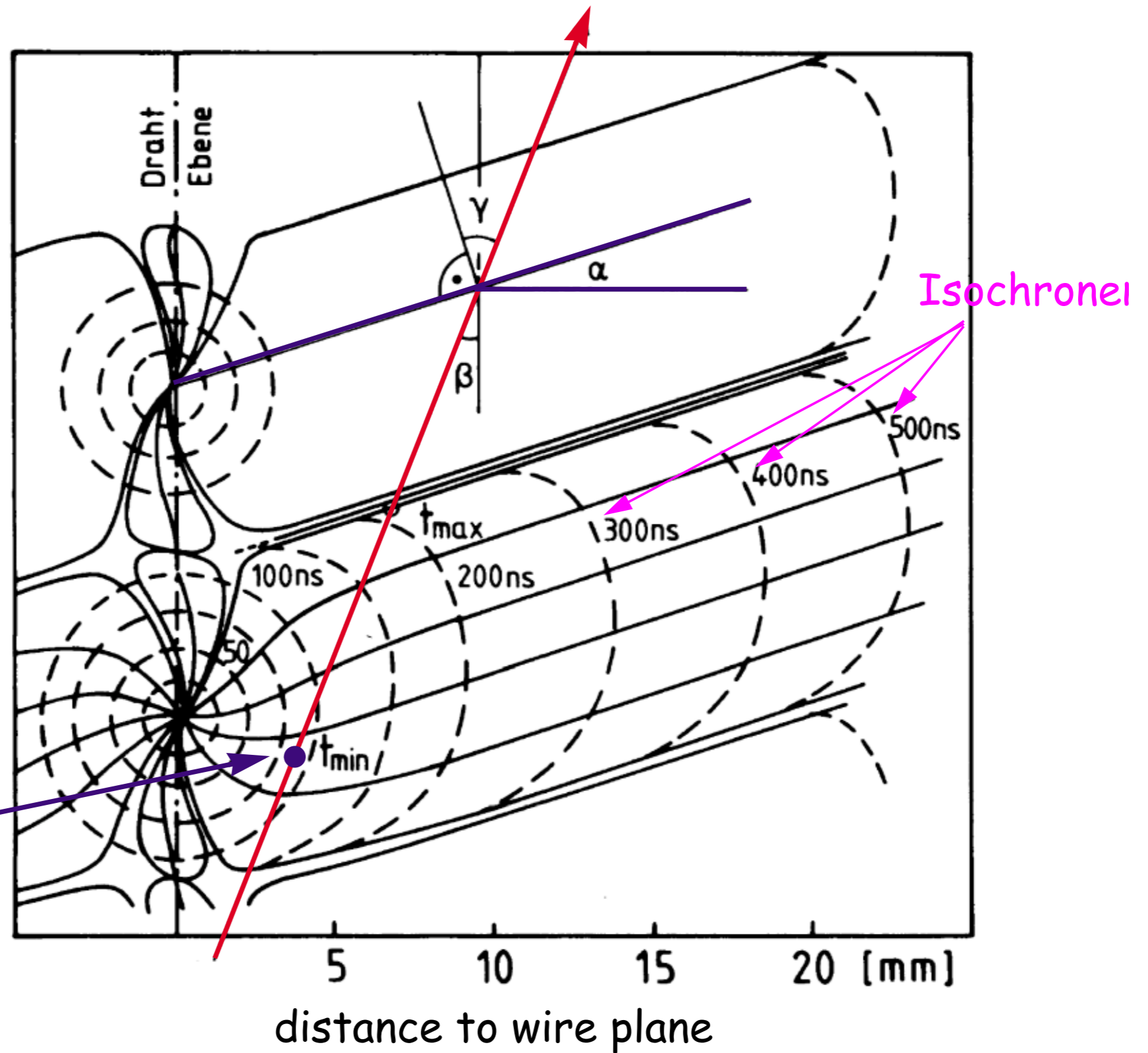


Examples of Cylindrical Driftchamber Geometries



cell of a "jet"-driftchamber

Lorentzangle and Isochrones



first electrons

Isochronen

Intrinsic Position Resolution

The intrinsic position resolution is influenced by three effects:

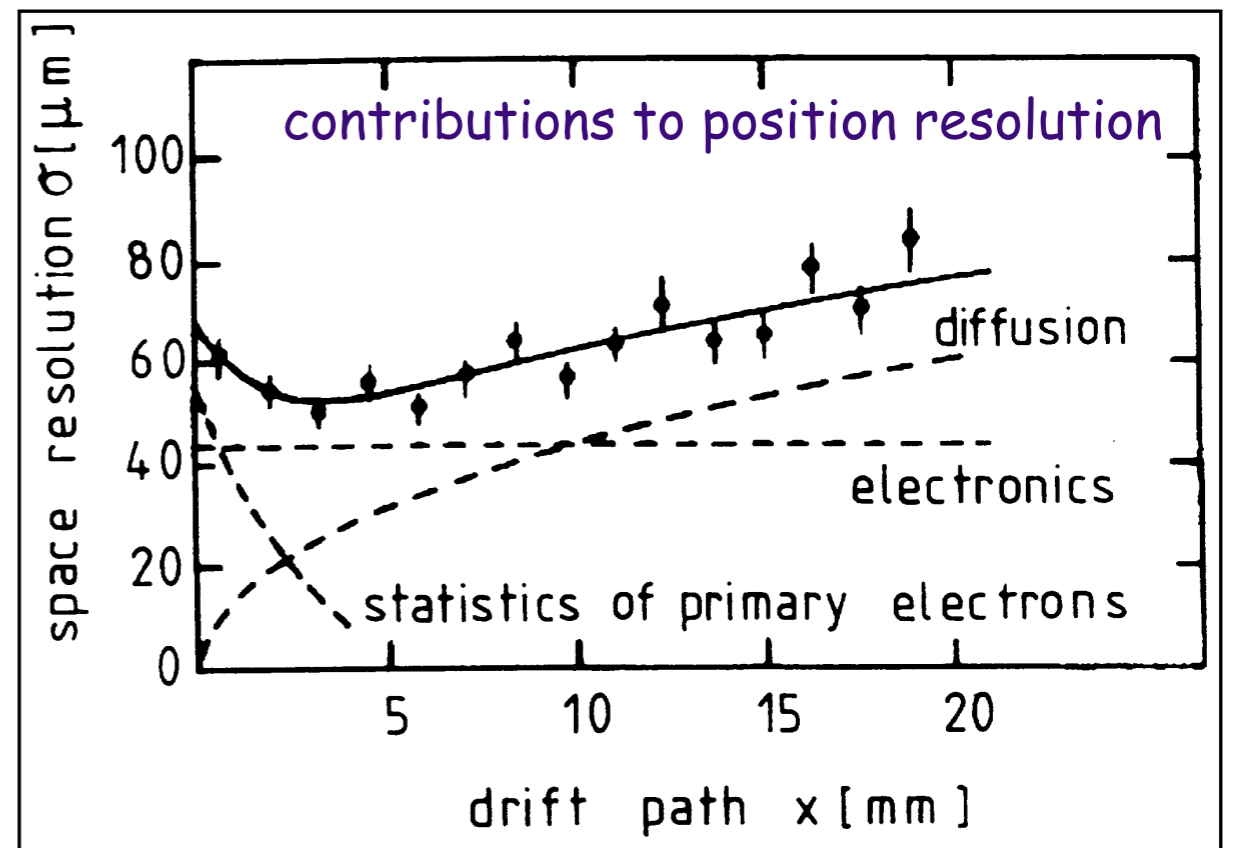
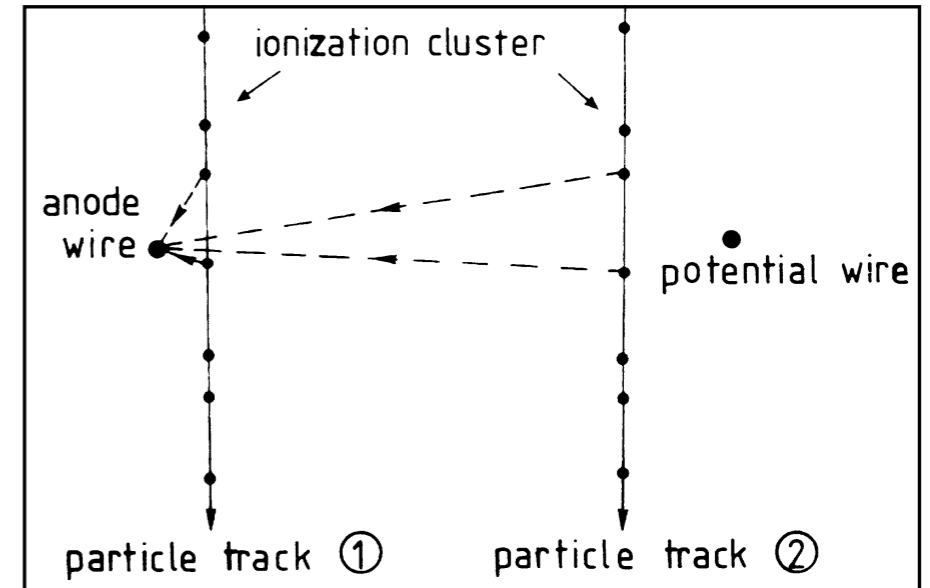
- **statistics of primary ionisation**: point of origin of primary cluster varies by $\approx 100\mu\text{m}$
- **diffusion of electron cloud** during it's drift to anode

$$\sigma = \frac{1}{\sqrt{n}} \sqrt{\frac{2Dx}{\mu E}}$$

- Lorentz effect

- limitations in time resolution of whole chain of **electronic signal processing**

- cabel
- pulse shaping
- definition of time refernce t_0 etc



Drift Chambers during Construction

H1 Central Jet Chamber

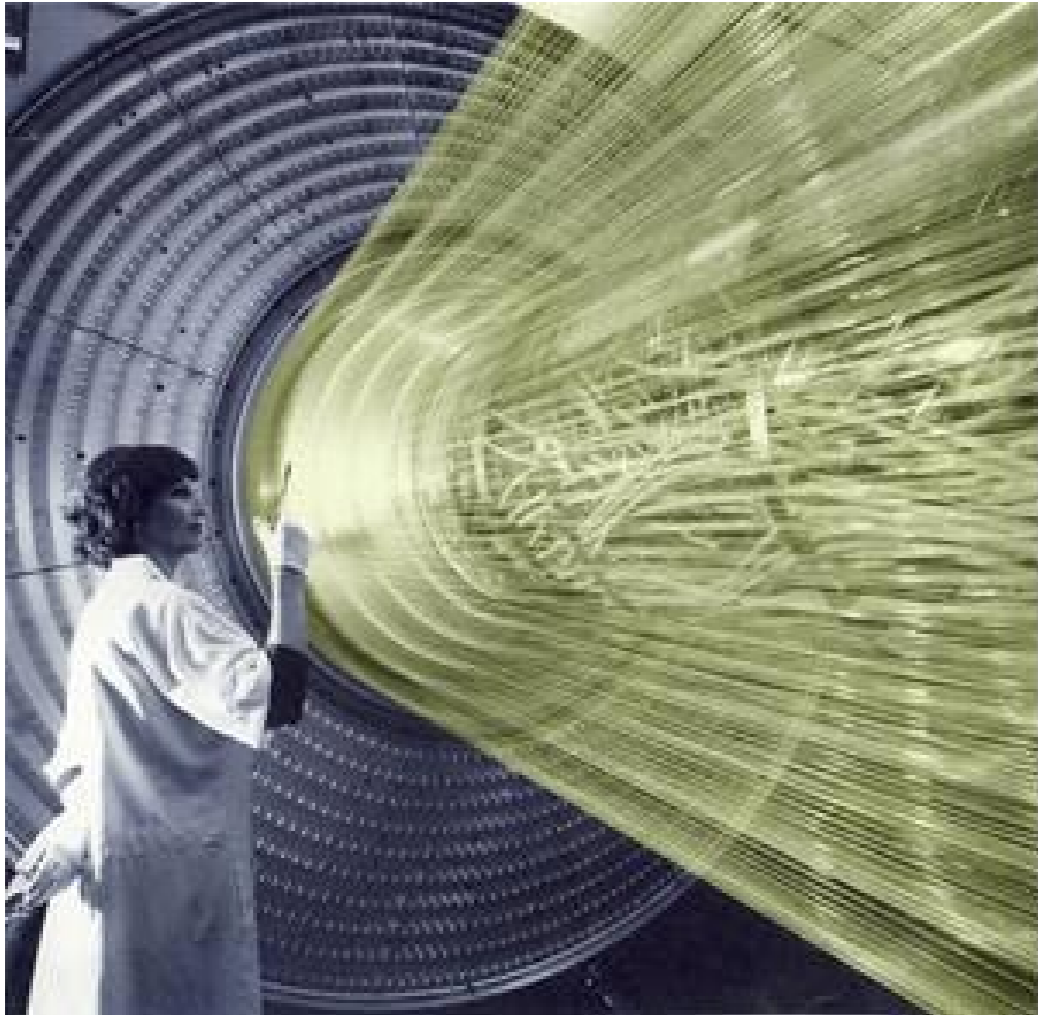
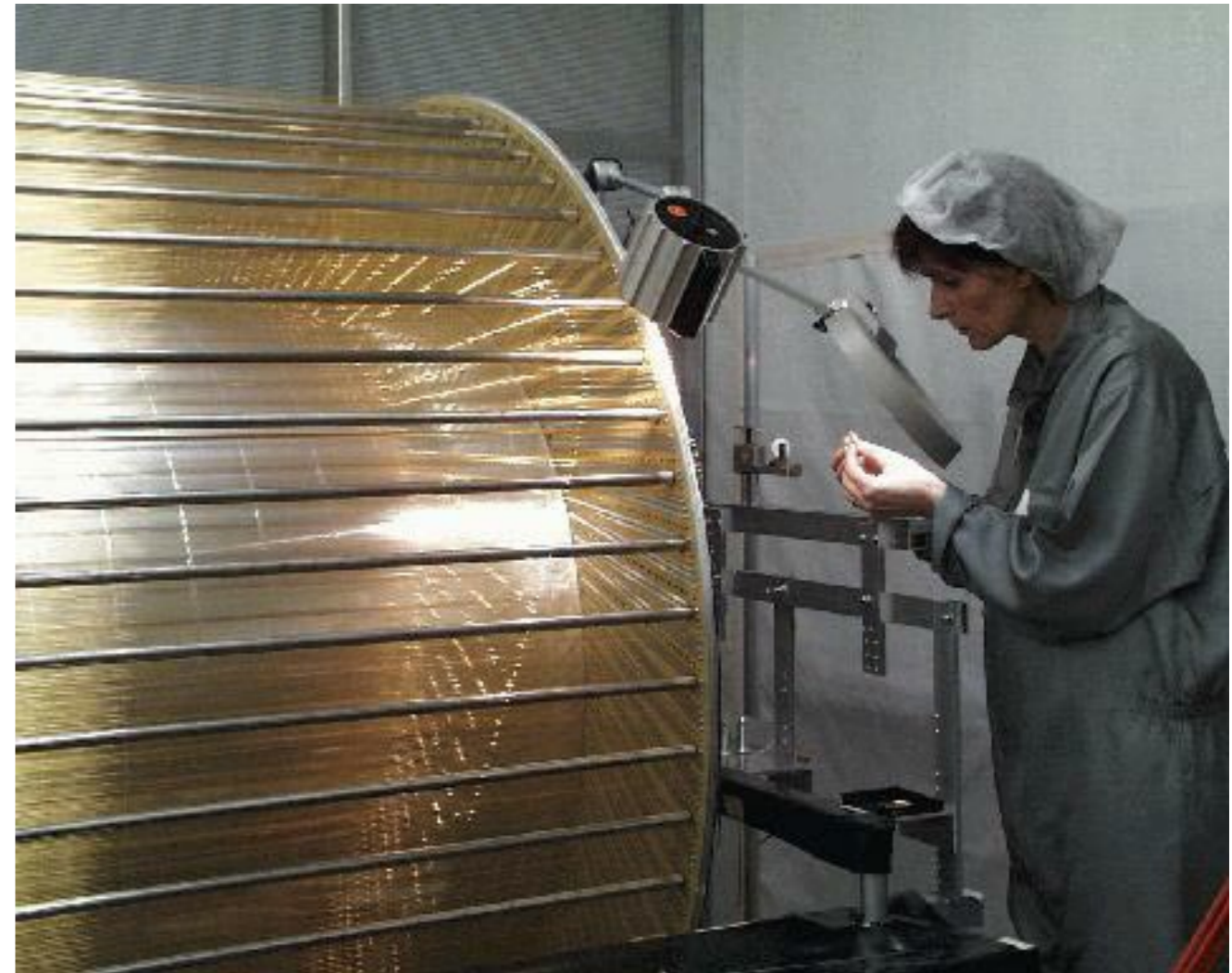
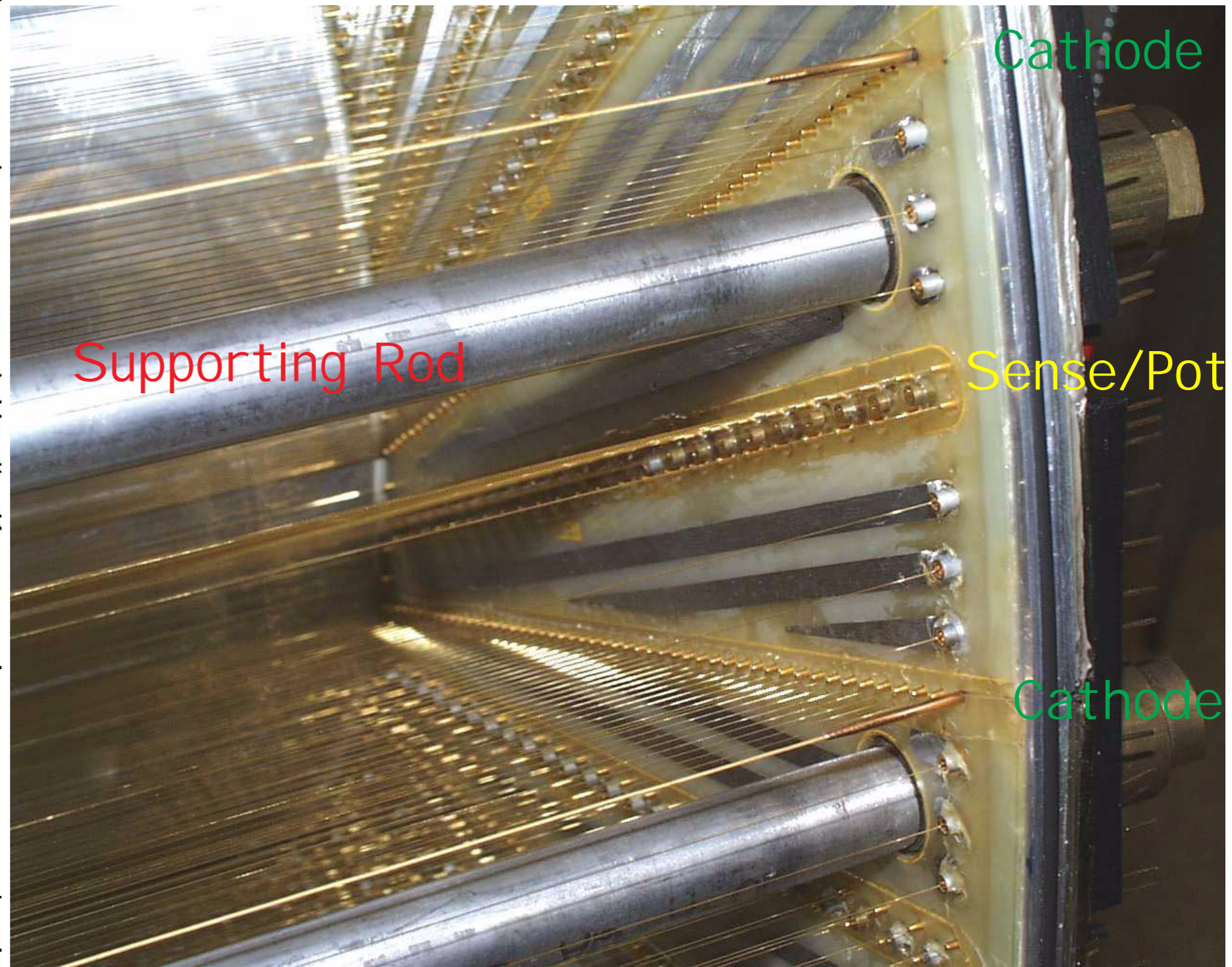
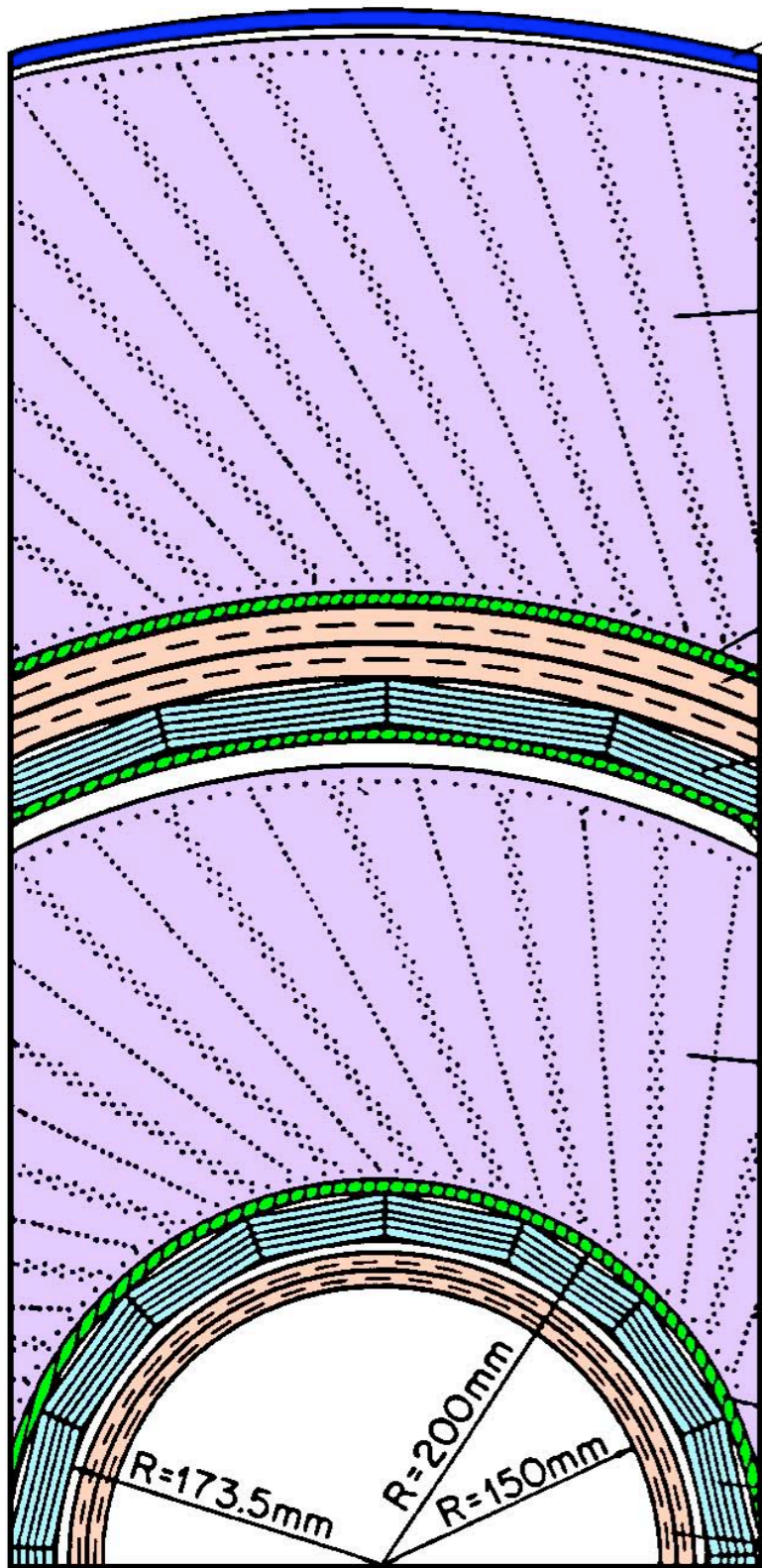


Photo: SLAC, USA



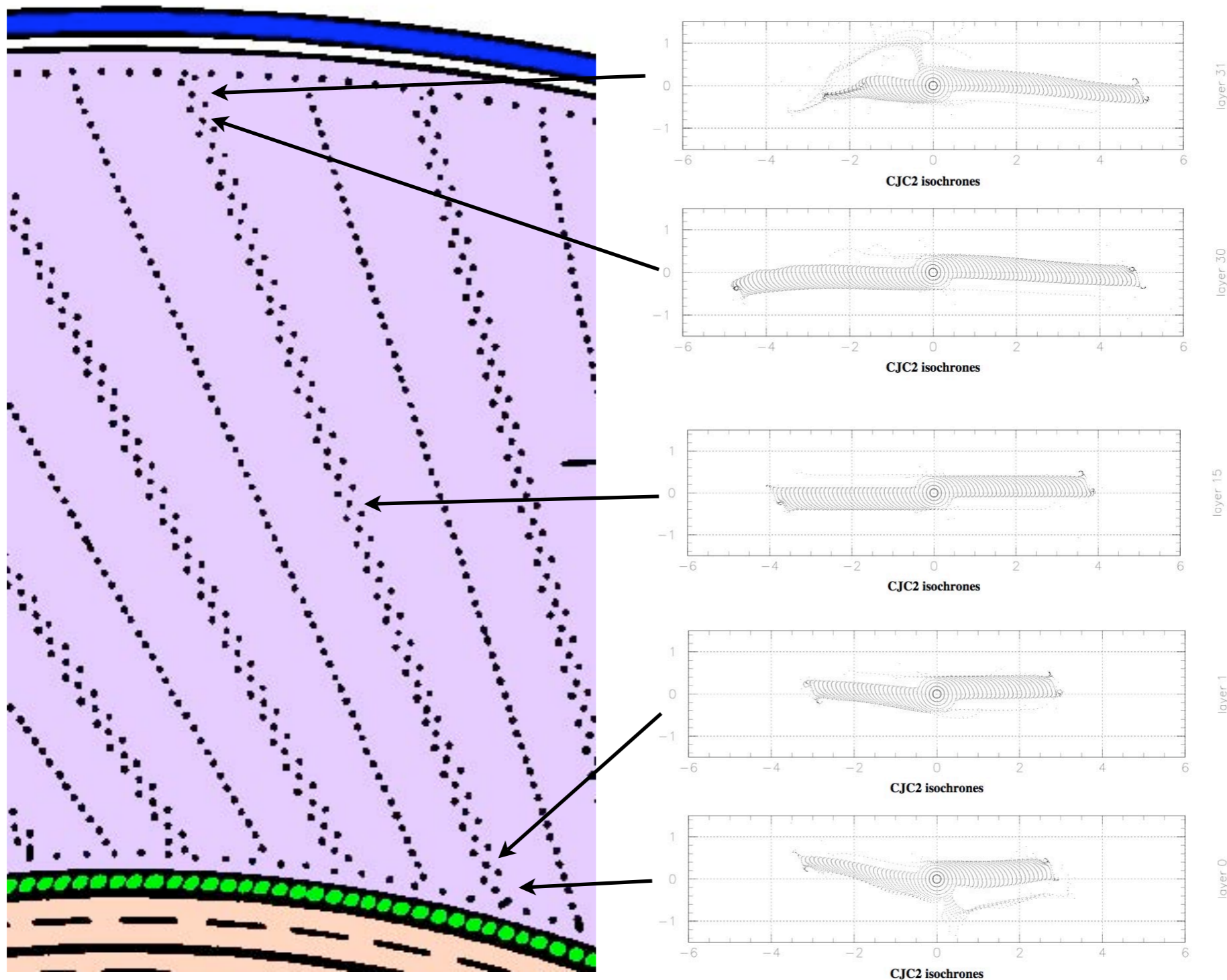
- ≈ 15000 wires
- total force from wire tension ≈ 6 tons

Central Drift Chamber of H1: CJC



Cells are tilted in order to compensate Lorentz angle

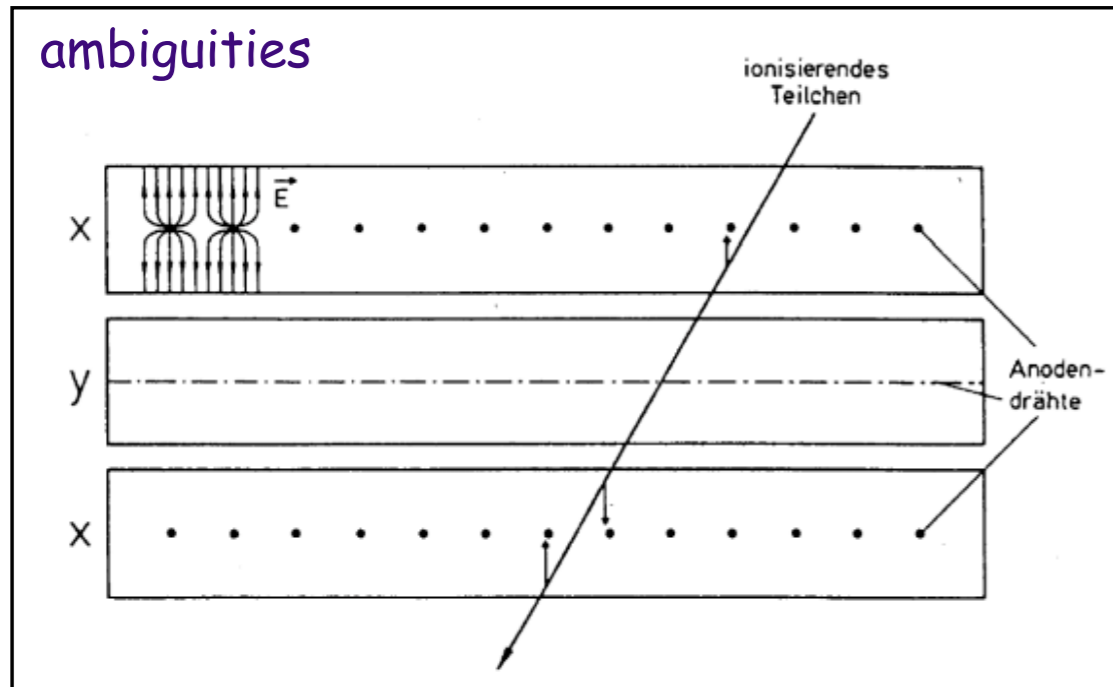
Deviations from the ideal Configuration in CJC2



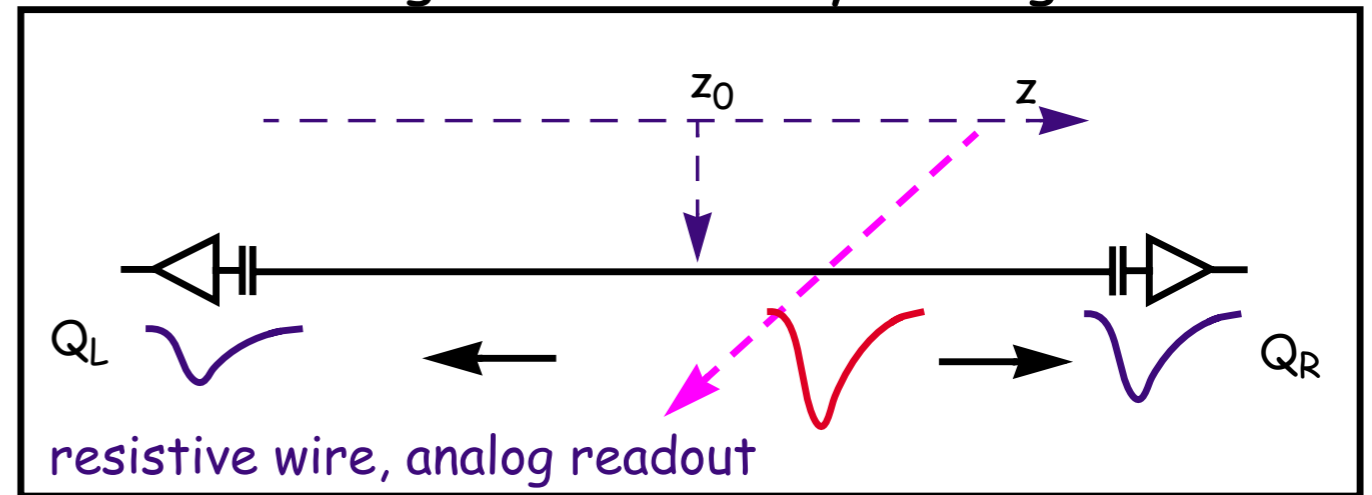
- electrostatic effects at the boundary lead to distortions at small and large radii
- such effects must be taken into account in calibration / reconstruction

Options for Readout of Second Coordinate

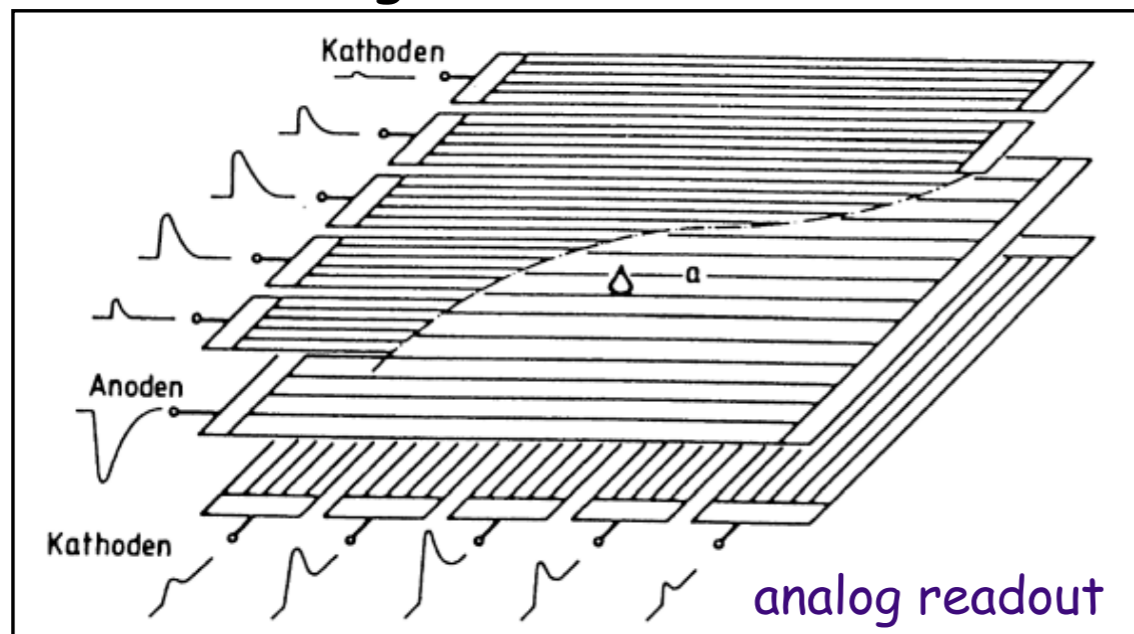
Crossed Planes



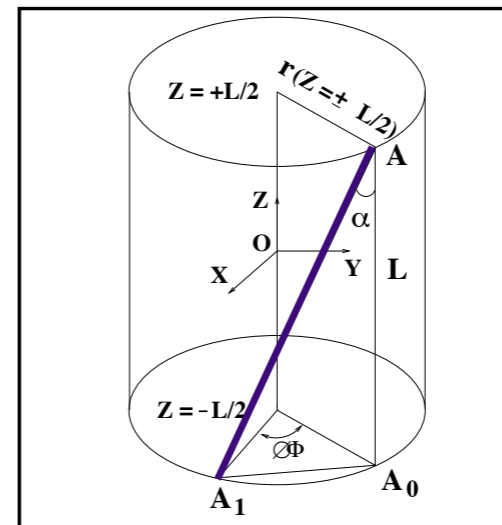
Charge Division, z-by-timing



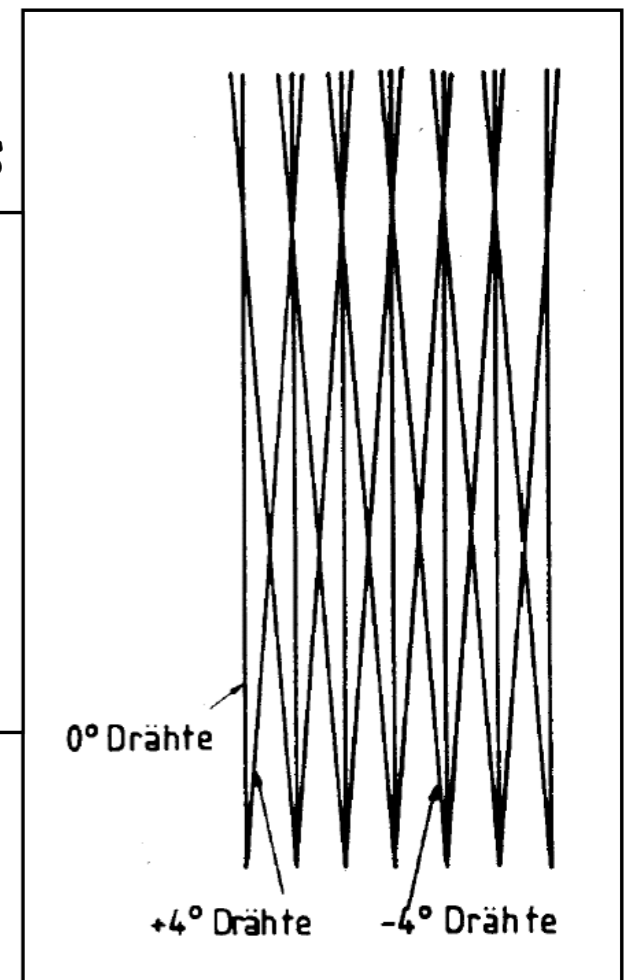
Segmented Cathodes



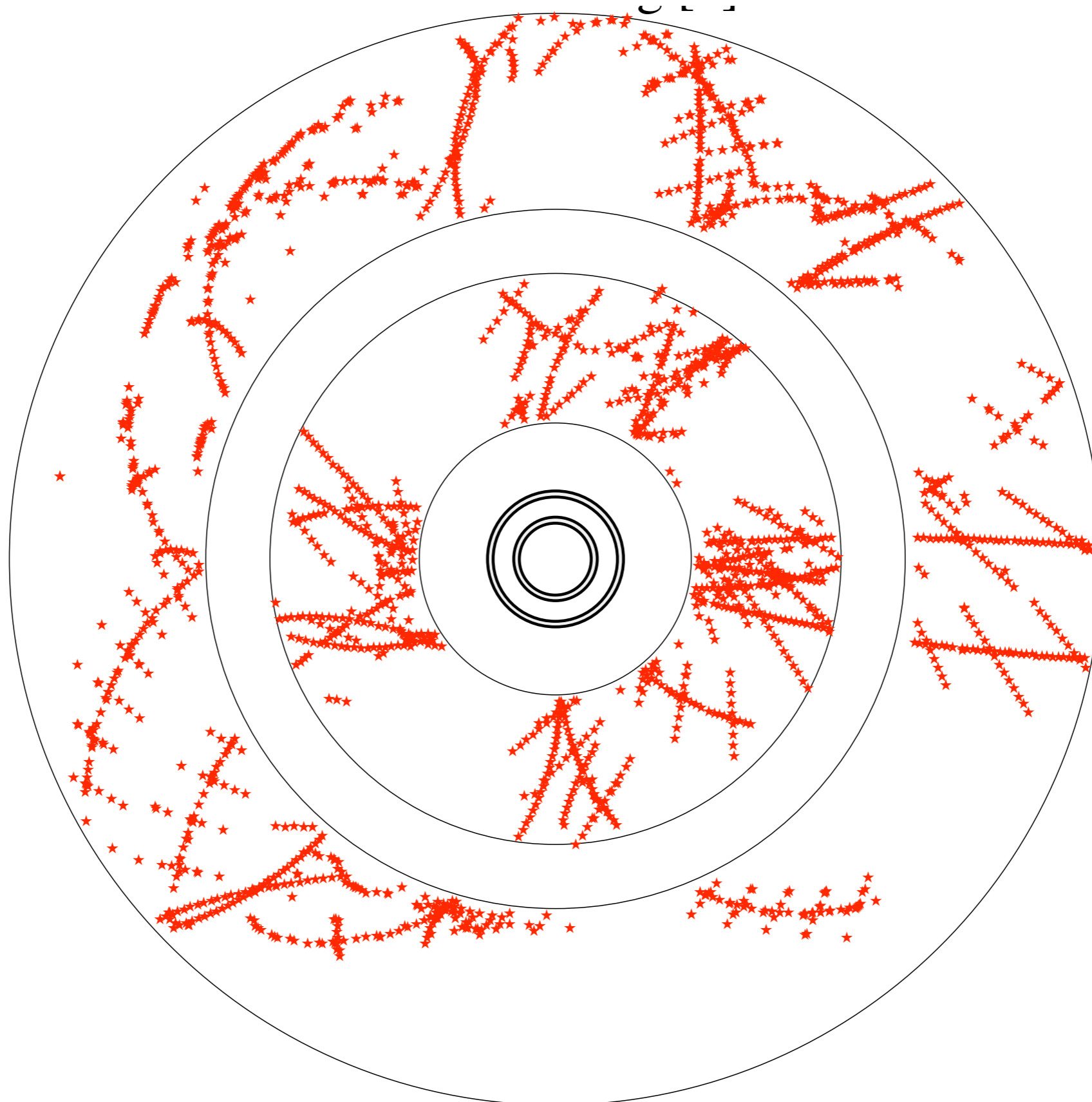
Stereo Wires

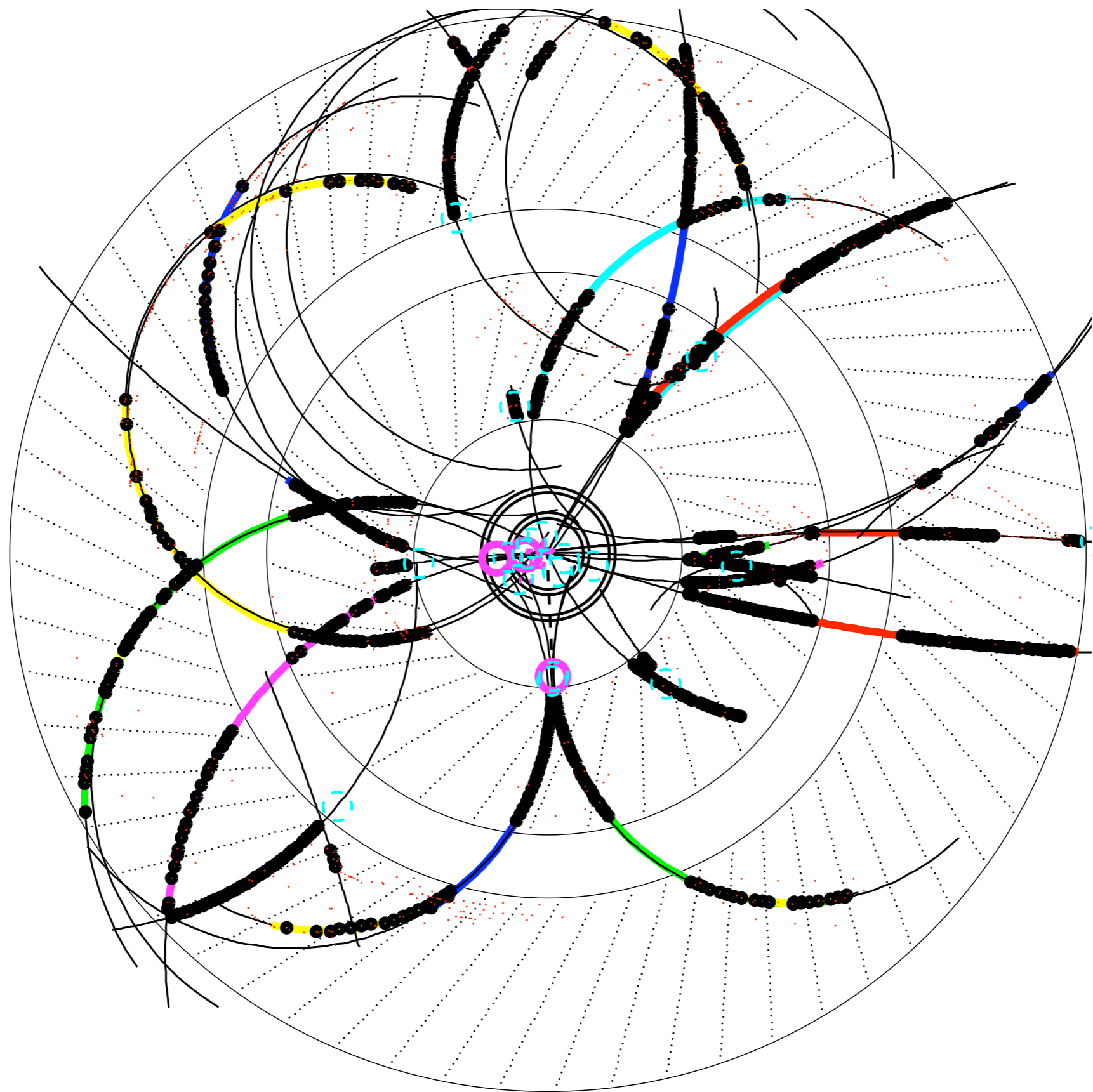


$$\rightarrow \sigma_z \approx 0.1 \text{ mm}$$

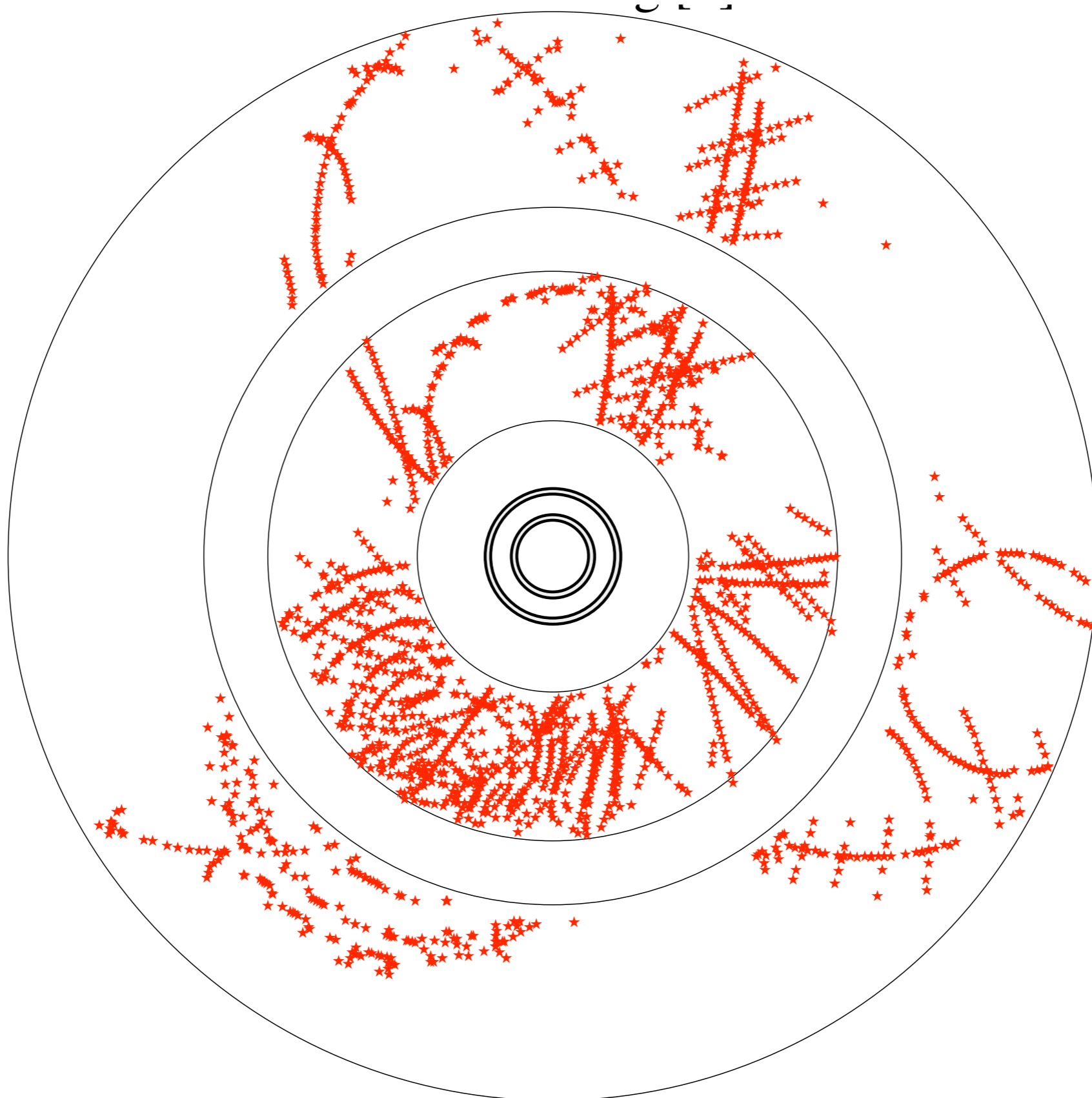


A simple Case

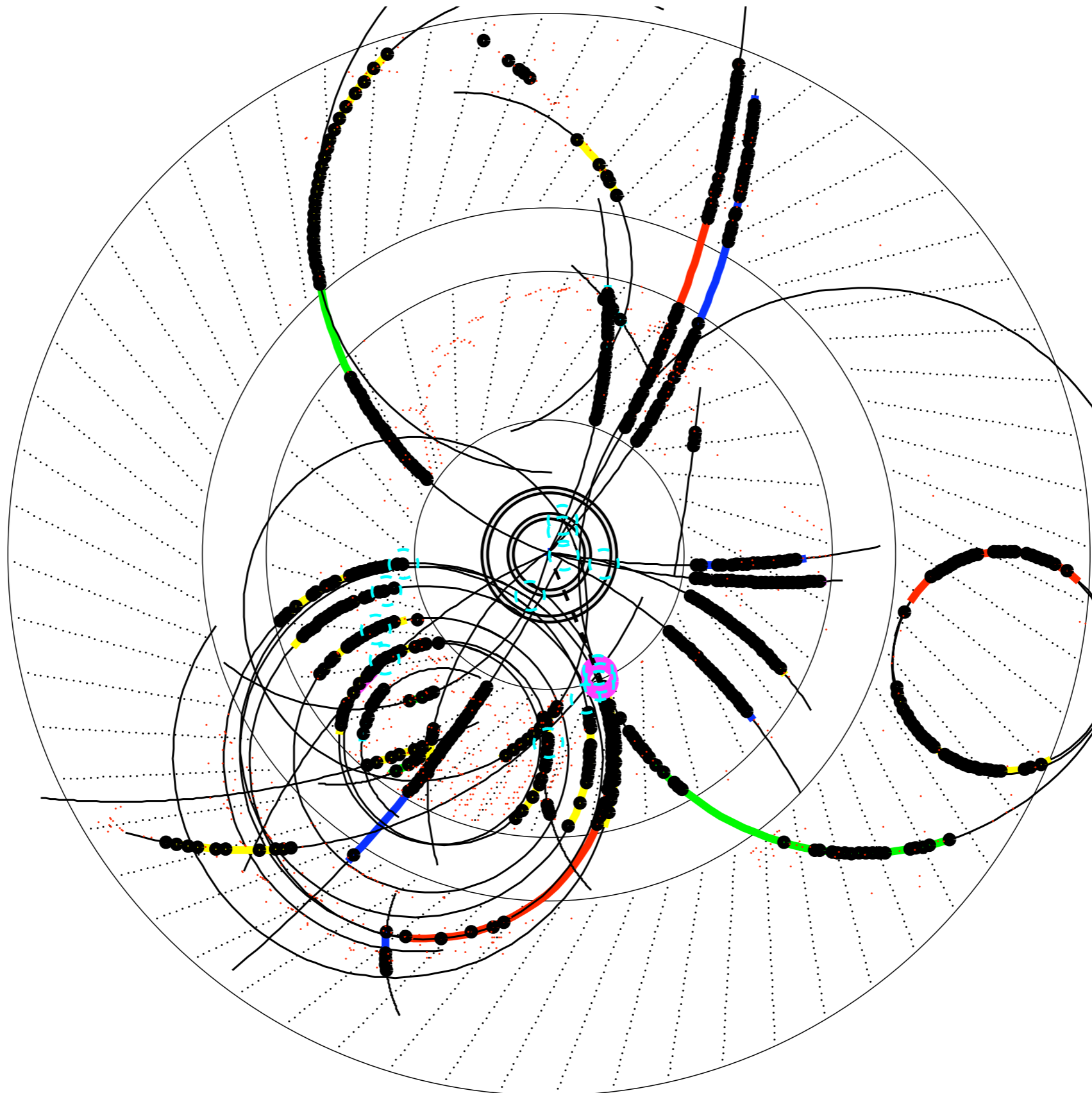




Somewhat more busy



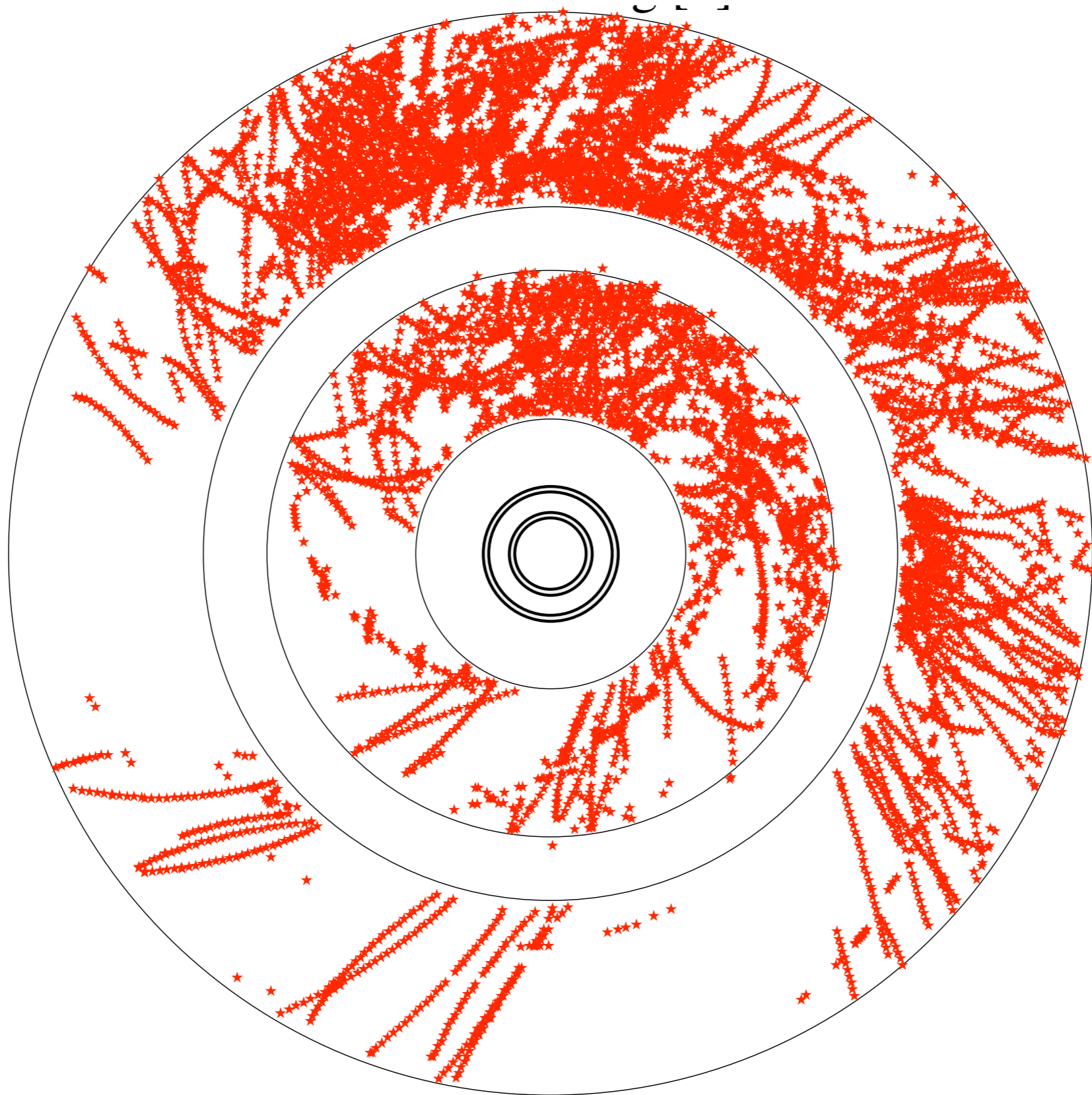
Event with „Looper“



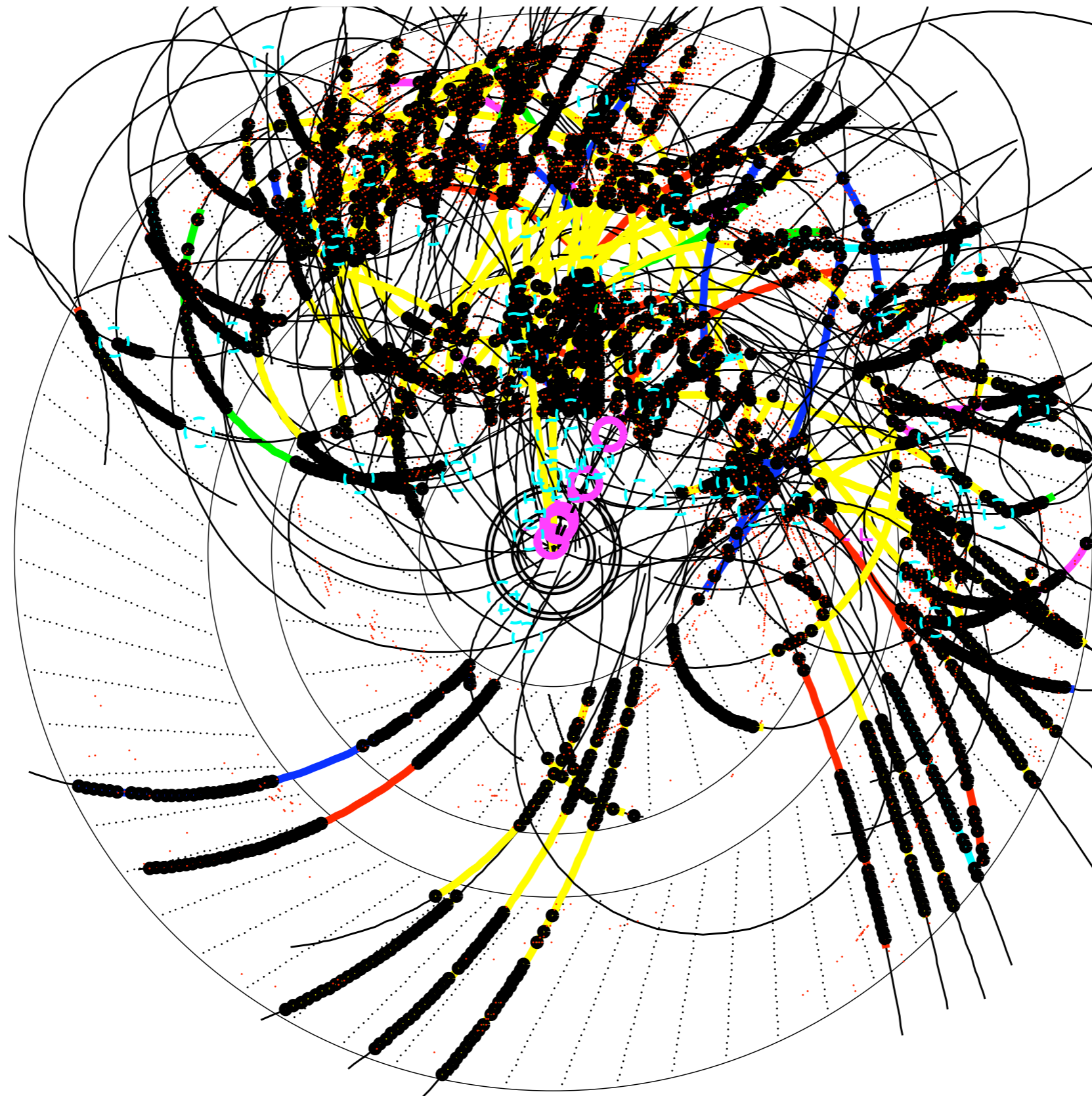
Example taken from
V. Blobel Uni HH

carsten.niebuhr@desy.de

Challenging ...



Do you find the missing Track ?



*Example taken from
V. Blobel Uni HH*

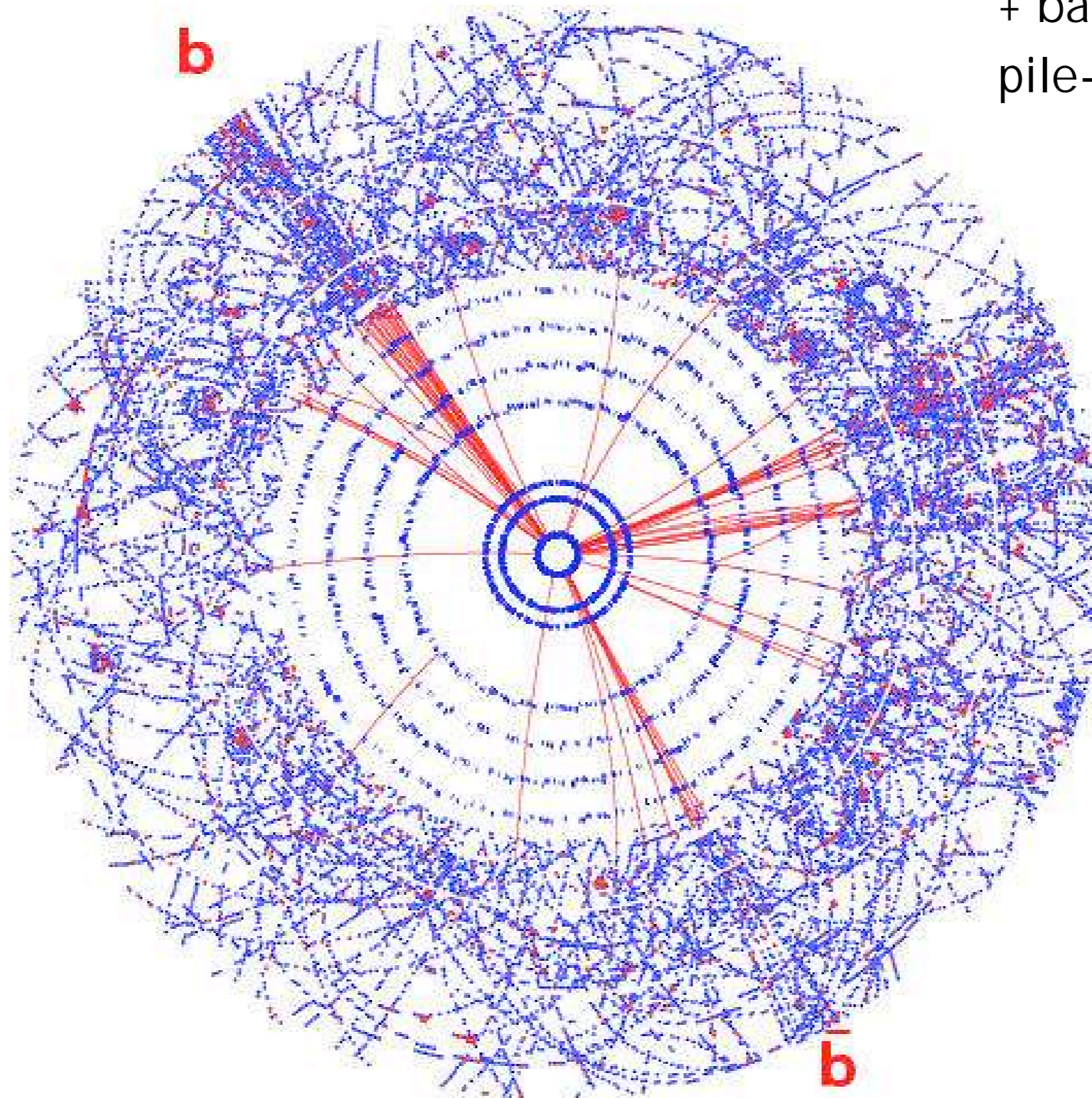
carsten.niebuhr@desy.de

Simulated Event in ATLAS

ATLAS

Higgs \rightarrow $b\bar{b}$

+ background from
pile-up



Methods of Pattern Recognition and Fitting

- **Global Methods of Pattern Recognition**
 - Template Matching
 - Fuzzy Radon Transform
 - Histogramming Methods
 - Neural Network Techniques
 - Hopfield neuron
 - Denby-Peterson method .
 - Elastic arms and deformable templates
- **Local Methods of Pattern Recognition**
 - Seeds
 - 2D Versus 3D propagation
 - Naive Track Following
 - Combinatorial Track Following
 - Use of Kalman Filter
 - Arbitration

Track fitting usually is based on minimization of the so called **chi² function**:

$$\chi^2 = \sum_{i=1}^N \frac{(m_i - f_{\vec{p}})^2}{\sigma_i^2}$$

- m_i are the individual measurements (hits)
- f_p is an expression describing the trajectory (e.g. helix), which depends on a set of parameters p

Properties: For normally distributed measurements this expression is proportional to the negative logarithm of the corresponding likelihood function. In the linear case the result is unbiased and has smallest variance.