# Semantic Holism in Scientific Language

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**Abstract**

Whether or not meaning is compositional has been a major issue in linguistics and formal philosophy of language for the last two decades. Semantic holism is widely and plausibly considered as an objection to the principle of semantic compositionality therein. It comes as a surprise that the holistic peculiarities of scientific language have been rarely addressed in formal accounts so far, given that semantic holism has its roots in the philosophy of science. For this reason, a model-theoretic approach to semantic holism in the language of science is presented here. This approach preserves compositionality to a large extent.

## 1 Introduction

What makes a semantics compositional? In the standard account, a semantics is compositional iff the meaning of each complex expression is determined by the meanings of the component expressions plus the way they are combined into the complex expression. (Hintikka and Sandu 2001, 49) give the following, more refined definition:

**Definition 1.** A semantics is compositional iff the applicability of a semantic predicate $A$ to a complex expression $E$ is completely determined by the applicability of all and sundry semantic attributes to the component expressions of $E$ plus the structure of E in terms of these component expressions.

A Tarski-type truth definition is compositional in the sense of definition 1. In such a definition, the truth-value of a sentence is determined by the values of the non-logical component expressions and the way these expressions are combined

with logical expressions. It goes without saying that the model-theoretic definition of truth, as given in standard accounts of formal logic, is of Tarski-type.

What makes a semantics holistic? The following definition is based on an explanation by Pagin (1997, 13):

**Definition 2.** A semantics for a language $L$ is holistic iff the semantic values of expressions in $L$ are determined together, by a totality of relations between expressions in $L$.

This definition is intended to imply that, in a holistic semantics, there is widespread interdependence between expressions that do not share any constituent parts. Therefore, no expression can get its meaning determined irrespective of other expressions (Pagin 1997, 13).

At first sight, semantic holism flatly contradicts the principle of semantic compositionality (Pagin 1997, 11). According to the latter principle, there is a clear order of determination between the semantic values of a certain set of (simple) expressions and the semantic values of another set of (complex) expressions. The principle of semantic holism, by contrast, says that the semantic values of expressions are determined together such that there are *mutual* dependencies between these values. This apparent contradiction serves as a crucial premise in an argument by Fodor and Lepore (1991), through which they aim to rule out inferential role semantics on the basis of the principle of semantic compositionality, a principle that appears essential to our capacity to understand newly encountered sentences.[1] Pagin (1997) showed, however, that there are at least two ways to harmonise semantic holism with the principle of semantic compositionality. We will refer to one of his proposals below.

The major contribution in the present paper is an attempt to analyse and to resolve the apparent tension between semantic holism and the principle of semantic compositionality for the language of science within a formal account. It is assumed, for rather intuitive reasons, that this language contains *theoretical terms* in the sense that the meaning of such terms is introduced through the axioms of a scientific theory. The standard way of using model-theoretic semantics for explicating the notion of non-logical truth will be found incompatible with the thesis of the existence of theoretical terms. Should we therefore dismiss model-theoretic semantics for the analysis of scientific language outright? This move will appear unnecessary.

An alternative and yet model-theoretic semantics for theoretical terms and theoretical sentences will be expounded here, through which the thesis of semantic holism in scientific language gains, we contend, clarity and elucidation. One

[1] According to inferential role semantics, the meaning of an expression $\alpha$ is to be identified with or supervenes on the inferential relations of sentences having occurrences of $\alpha$ (see, e. g., Block 1986).

further result obtained is the confirmation of the compatibility thesis regarding semantic holism and the principle of semantic compositionality for a wide range of theoretical sentences. The paper will finally attempt to answer some open questions with which readers of Quine have been left and, therefore, comes up with a new proposal of how to draw the distinction between meaning constitutive and non-meaning constitutive sentences.[2]

The notion of a meaning constitutive sentence is recognisable and of fundamental importance in such major works in the philosophy of science, as Duhem (1906/ 1991), Poincaré (1902/ 1952), and Feyerabend (1962). Duhem's demonstration that a scientific hypothesis in physics cannot be tested in isolation from its theoretical context is joined with and motivated by semantic considerations, according to which it is physical theories that are giving meaning to physical concepts (1906/ 1991, 183). Poincaré (1902/ 1952, 90) literally claims that certain scientific propositions acquire meaning only through the adoption of conventions. Clearly, Poincarean conventions are meaning constitutive sentences. Feyerabend's critique of the principle of meaning invariance in theory evolution is derived from a contextual theory of meaning, which says that a scientific term gets its meaning determined only in the context of a scientific theory.

These rather informal ideas about the semantics of scientific concepts and propositions seem incompatible with the style of formal semantics that evolved in logic and linguistics with the work of Tarski. The Tarskian picture leaves no room for meaning constitutive sentences since it assumes that sub-sentential expressions are meaningful prior to the semantic valuation of the whole range of sentences. There is thus, in addition to the tension between semantic holism and compositionality, another, closely related tension between the presumed existence of meaning constitutive sentences and the compositional semantics of a Tarski-type truth definition. The formal account of semantic holism that results from the present investigation goes hand in hand with an attempt to clarify the notion of a meaning constitutive sentence within the model-theoretic framework.

## 2 Confirmation and Semantic Holism

Confirmation holism is a well-known doctrine that needs to be reviewed here only very briefly. It goes back to Duhem, who observed that a scientific hypothesis has, in general, observational implications only if joined with further sentences of a scientific theory. As a consequence of this, there are several op-

[2]Hintikka and Sandu (2001) advanced game-theoretic semantics and IF (independence-friendly) logics as non-compositional kinds of semantics. IF logics differ from classical logic in the interpretation of the quantifiers. The present proposal to account for semantic holism does not concern the interpretation of the quantifiers. Nor is game-theoretic semantics envisioned as a solution.

tions available for revising the truth-value assignment to sentences if a scientific hypothesis fails to pass an experimental test. Retracting this hypothesis is only one choice among others.

Confirmation holism by itself is not a thesis about the semantics of scientific language. One can very well maintain that the meaning of theoretical sentences is determined precisely in a non-holistic fashion, even though such sentences cannot be tested individually. If, however, confirmation holism is joined with some sort of verificationist account of meaning, then a holistic conception of meaning with respect to non-observational sentences is implied (Pagin 2006, 214). The original verificationist account of sentence meaning set forth in the era of logical positivism was not upheld for a long time. Yet, there are ramifications of it which are considered viable alternatives to standard truth-conditional semantics by at least some philosophers. Famously, Dummett (1978, 1991) suggested replacing truth-conditions as the basis of sentence meaning by assertibility conditions along the lines of intuitionistic logic. Putnam's internal realism, in which truth-conditions are replaced by the requirement of justification under ideal epistemic conditions, is inspired by this kind of semantics.

We may indicate briefly the reasons why one may wish to replace the truth-conditional conception of sentence meaning by some other, allegedly superior conception. The battery of arguments advanced by Dummett and Putnam may best be encapsulated in the proposition that truth-conditional semantics leads to semantic agnosticism in the sense that we would be unable to grasp the proper meanings of our linguistic expressions if this semantics were to be adopted. Language learning proceeds, so Dummett argues, on the basis of assertibility conditions and is not guided by truth-conditions. In short, it is acquisition considerations concerning the meaning of linguistic expressions that motivate a deviation from standard truth-conditional semantics. Such considerations may remain forceful even in the face of counter-arguments to certain elaborations of the assertibilist account of meaning.

What are the consequences of adopting a broadly verificationist, or assertibilist, account of sentence meaning for the semantics of scientific hypotheses, given the holistic limitations of confirming them? As a first approximative answer to this question, we may set forth the following proposition:

**Proposition 1.** The semantic value of a scientific hypothesis is determined only in the context of a corresponding scientific theory or a set of such theories.

This variant of semantic holism will be elaborated here. It differs from total, or global, holism according to which the whole system of our affirmed, non-observational sentences is the smallest meaningful unit expressible in scientific language, a formulation that has been maintained famously by Quine (1961). Therefore, the kind of holism to be investigated here is of a rather moderate or relative type. The distinction between relative and global holism as well as

the distinction between an axiom, or postulate, and a hypothesis of a scientific theory will be dealt with in Section 8.

One word on Quine's view of the subject matter. A rather concise formulation of the semantic consequences he draws from confirmation holism can be found in, among other writings, Quine (1972). There, he repudiates the view that a single theoretical sentence can play the role of an individually meaningful knowledge claim. In his (1969b, 80n, 89) Quine insists on a verificationist account of sentence meaning. Holism is reported as a doctrine that concerns the semantics of theoretical sentences in Quine (1981). In like manner, the present paper is concerned with the semantics of theoretical sentences, but it is intended to be a bit more formal about the semantic consequences of confirmation holism than Quine's and numerous other contributions to meaning holism have been.

## 3 Asserting Theoretical Sentences

Let us begin with considering the formal notation for asserting a sentence in Frege's Begriffsschrift before we move on to the corresponding model-theoretic notation. Frege, famously, introduced a sign for indicating that a particular sentence is asserted and another sign for indicating that a sentence has content. If you put an assertion sign (Behauptungszeichen) in front of a sentence $\phi$, then you indicate that $\phi$ is claimed to be true. Making such a claim presupposes that $\phi$ has a content that can be judged. This is indicated by the content stroke (Inhaltsstrich). The assertion sign is composed of the judgement stroke (Urteilsstrich) and the content stroke (Inhaltsstrich). Thus,

(1) $\vdash\!\!\!- \ \phi$

was considered to be a proper formulation of the claim that $\phi$ is a true individually meaningful sentence. Things became more complicated with the rise of model-theoretic semantics as initiated by Tarski. The question we are concerned with in this section is, what kind of model-theoretic notation is capable of translating Frege's assertion sign?

It has been observed that model-theoretic semantics tells us what it is for a sentence to be true in a (model-theoretic) structure but is of little help when it comes to understanding the truth of a sentence in the representational sense of truth. So, arguably, model-theoretic semantics is best viewed as interpretational semantics in the sense that it tells us what it is for a sentence to be true only with respect to a particular interpretation of the non-logical symbols of the language system and the specification of the universe in which the variables are interpreted. This view has been articulated, among others, by Etchemendy (1999). Likewise, Carnap pointed out that the model-theoretic apparatus of

predicate logic is apt to clarify the notions of logical truth and logical consequence but makes no contribution to understanding the notion of factual truth (Carnap 1973, 98n).

To do justice to our plain and representational understanding of truth within the framework of model-theoretic semantics, the notion of an *intended interpretation* has been invoked sensibly (see, e. g., Carnap 1973, 101). An intended interpretation of a formal system represents the meaning of the non-logical symbols. It can be made explicit by so-called rules of designation which assign either an intensional or an extensional interpretation to these symbols by means of expressions of a metalanguage, where every intensional interpretation determines an extensional one uniquely. Of course, the domain in which the variables are interpreted must also be specified. With the help of such a notion, the assertion of a sentence $\phi$ can be given the following notation:

(2) $\quad \mathfrak{A}_{\mathfrak{i}} \models \phi.$

where $\mathfrak{A}_{\mathfrak{i}}$ stands for the intended interpretation of the language. If a formal system has no intended interpretation, then it is not sensible to use sentences of that system for the purpose of making assertions, apart from logically true ones.

Whether or not $\mathfrak{A}_{\mathfrak{i}} \models \phi$ holds true is precisely determined by the recursive definition of truth in model-theoretic semantics. According to this definition, the truth-value of $\phi$ is determined by the interpretation of the non-logical symbols occurring in that sentence and, if there are any variables occurring in that sentence, by the domain in which the variables are interpreted. Which further sentences or whole theories are held true in the language does not affect the truth-value assignment to $\phi$. In other words, the semantic value of $\phi$ is determined by the semantic values of its component expressions plus the way these expressions are combined to form $\phi$. But then, we have to conclude that $\phi$ has a precisely determined meaning quite independent of the theoretical context in which it is asserted. This conclusion does seem to rule out semantic holism - regarding the truth predicate of theoretical sentences - in a straightforward manner. Or, to put it more carefully, the above notation for asserting the truth of a sentence has no resources to account for holistic interdependencies among the semantic values of the expressions of a language $L$.

Model-theoretic semantics gives us a non-holistic account of the meaning of theoretical sentences, provided the notion of an intended interpretation is used to explicate the representational dimension of language. This is the simple result of the preceding considerations, a result that poses a serious problem for the philosophy of Quine for the following reasons. First, formal logic serves in Quine's philosophy as an omnipresent framework, which is supposed to account for truth and inference not only in mathematics but also in the natural

sciences. In particular, the model-theoretic definition of truth is used by Quine in a context wider than that of pure mathematical logic (see, e. g., Quine 1992, 84–86). Second, the notion of an intended interpretation is also used by Quine himself.[3] Third, more importantly, there seems to be no other way to bridge the gap between interpretational (model-theoretic) semantics and representational (model-theoretic) semantics than to invoke the notion of an intended interpretation.[4]

A simple and straightforward solution to the problem just diagnosed would be to remain content with a purely syntactic understanding of scientific language and hence refrain from a model-theoretic analysis of that language. This solution would presumably have been favoured by Neurath but is hardly desirable from a logician's point of view. We may therefore seek for a semantic understanding of scientific language which is not only compatible with semantic holism but also apt to account for it.

# 4 Indirect Interpretation of Theoretical Terms

The model-theoretic account of semantic holism, which has been developed elsewhere in greater detail, is based on Carnap's notion of an indirect interpretation of theoretical terms by postulates. In his (1939, 65-69) Carnap importantly distinguishes between two kinds of indirect interpretation of a term, one by a definition and another by a law of nature. Later on in his (1956) and (1975) he used the term 'postulate' to refer to those sentences which are not definitions but do nevertheless have the function of bestowing meaning on the theoretical terms. It is of particular relevance to our present concerns that Carnap views the interpretation of a theoretical term by postulates as being analogous to the

[3]So he does in Quine (1969a, 53n). In the present paper, semantic holism is discussed independently of any indeterminacy of translation. Extending the discussion to include the latter claim is not necessary since semantic holism needs no support from the indeterminacy thesis. This should be obvious from the way semantic holism has been introduced above. Furthermore, Quine (1981) expounds semantic holism independently of the indeterminacy thesis. In his (1969b, 78–80) he employs semantic holism, as derived from the non-translatability of theoretical sentences into observational ones, to establish the indeterminacy of translation but not vice versa.

[4]The notions of interpretational and representational semantics are borrowed from Etchemendy (1999). One is doing representational semantics when a determinate understanding of the non-logical symbols is being assumed, however implicit this understanding may be. This contrasts with interpretational semantics, where no such determinate understanding or interpretation is being assumed.

interpretation of a defined term by a definition. We shall now seek to explain what the analogy consists in.

The notion of an indirect interpretation is to be understood against the background of a direct interpretation. The interpretation of a non-logical symbol is direct iff it consists of an assignment of an extension or an intension to that symbol and is given by one or several expressions of the metalanguage. By contrast, the interpretation of a non-logical symbol is indirect if it is given by one or several sentences of the object language which have the status of being non-logical axioms in the calculus. Carnap's explanation of the logical status of postulates mainly concerns their syntactic function: Postulates are, according to Carnap, non-logical axioms which are taken to be valid in the calculus and which therefore can be used in every derivation. An intuitive understanding of postulates is not required in order to justify their use in the calculus. The theoretical part of the calculus is rather 'free floating' and is connected with the empirical world only by the interpretation of observational terms (see again Carnap 1939, 67–69).

We will now, in addition to Carnap's syntactic explanation, move the focus onto the semantic function of postulates. Some insight regarding this may be derived from an explanation of the semantic function of a definition. By Beth's definability theorem, as given in standard accounts of mathematical logic, we know that, if a symbol $P$ is defined by a set $\Phi_d$ of sentences in a language $L(V)$, there is one and only one $L(V \cup \{P\})$ structure that expands a given $L(V)$ structure $\mathfrak{A}$ such that $\Phi_d$ is satisfied. Moreover, we can say that $\Phi_d$ does impose a constraint on the intended interpretations of the language $L(V \cup \{P\})$. For, obviously, once a symbol $P$ is defined by a set $\Phi_d$ of sentences in a language $L(V)$, an interpretation of $L(V \cup \{P\})$ that does not satisfy $\Phi_d$ cannot be regarded as an intended interpretation of $L(V \cup \{P\})$. Henceforth, we will call a structure for a language *admissible* iff it satisfies the explicit constraints imposed on the interpretation of that language. In light of this, we can explain the interpretation of a term by a definition as follows:

**Explanation 1.** A set $\Phi_d$ of sentences that defines a non-logical symbol $P$ in a language $L(V)$ does impose a constraint on the intended interpretations of $L(V \cup \{P\})$. This means, in terms of model-theoretic semantics, every admissible structure of the language $L(V \cup \{P\})$ must satisfy $\Phi_d$. With respect to a given $L(V)$ structure $\mathfrak{A}$, there is one and only one $L(V \cup \{P\})$ structure that expands $\mathfrak{A}$ such that $\Phi_d$ is satisfied.

How can this explanation be modified to account for the semantic function of postulates? First, there is no one-to-one correspondence between a symbol and a set of sentences that interprets that symbol. Rather, one and the same postulate usually contributes to the interpretation of several theoretical terms. Second, the introduction of theoretical terms by postulates may be accompanied

by the introduction of another, theoretical domain of interpretation, in addition to the domain of interpretation for the observational language $L(V_o)$. In the case of Carnap's dual-level conception a domain of (mathematical) theoretical entities is assumed for the interpretation of the theoretical terms. Third, it should not be assumed that the interpretation of theoretical terms results in a unique determination of the extension of these terms. Taking these differences into account when observing the semantic similarities between definitions and postulates may result in the following explanation of the semantic function of postulates:

**Explanation 2.** A set $\Phi_{TC}$ of postulates that interprets a set of theoretical terms $V_t$ on the basis of a language $L(V_o)$ does impose a constraint on the intended interpretations of the language $L(V_o, V_t)$. This means, in terms of model-theoretic semantics, every admissible $L(V_o, V_t)$ structure must satisfy $\Phi_{TC}$. The admissible $L(V_o, V_t)$ structures may have two domains of interpretation, one observational domain $D_o$ and a domain of theoretical entities $D_t$. With respect to a given $L(V_o)$ structure $\mathfrak{A}$, there may be several $L(V_o, V_t)$ structures that extend $\mathfrak{A}$ and satisfy $\Phi_{TC}$.

In Carnap's dual-level conception of scientific language, on which we are aiming to elaborate, the interpretation of the observational language $L(V_o)$ is assumed to be determined uniquely by rules of designation reflecting the intended interpretation of $L(V_o)$. There is thus a twofold constraint on the *admissible* interpretations of $L(V_o, V_t)$, one by the interpretation of $L(V_o)$ and another by the postulates $\Phi_{TC}$. In other words, an $L(V_o, V_t)$ structure is admissible iff it satisfies the postulates and extends the intended interpretation of $L(V_o)$ to include an interpretation of the theoretical terms $V_t$.

Based on such an understanding of an admissible structure, truth rules for theoretical sentences may be set up as follows. The symbol $\Phi_{TC}$ stands for a set of T- and C-postulates, where the T-postulates contain only $V_t$ terms as non-logical symbols, while the C-postulates contain both $V_o$ and $V_t$ terms. Let $\mathfrak{A}_o$ be designating the intended interpretation of the observational language. Let $\mathrm{MOD}(\Phi_{TC})$ be designating the set of $L(V_o, V_t)$ structures that satisfy the postulates $\Phi_{TC}$ and $\mathrm{EXT}(\mathfrak{A}_o, V_t, D_t)$ the $L(V_o, V_t)$ structures that extend $\mathfrak{A}_o$ to include an interpretation of the $V_t$ symbols, where these symbols are allowed to have argument positions that are interpreted in $D_t$. Every extension of a given $L(V_o)$ structure $\mathfrak{A}$, by definition, must agree with this structure on the interpretation of the $V_o$ symbols. A theoretical sentence is one in which at least one theoretical term occurs. Then, the truth-value assignment to such a sentence is defined as follows:

**Definition 3.** $\mathbf{S}_{\mathfrak{a}}$ designates the set of $L(V_o, V_t)$ structures that are admissible under an interpretation of the $V_t$ symbols by the postulates $\Phi_{TC}$. It is defined

as follows:

$$\mathbf{S}_{\mathfrak{a}} := \begin{cases} MOD(\Phi_{TC}) \cap EXT(\mathfrak{A}_o, V_t, D_t) & \text{if } MOD(\Phi_{TC}) \cap EXT(\mathfrak{A}_o, V_t, D_t) \neq \emptyset, \\ EXT(\mathfrak{A}_o, V_t, D_t) & \text{if } MOD(\Phi_{TC}) \cap EXT(\mathfrak{A}_o, V_t, D_t) = \emptyset. \end{cases}$$

**Definition 4.** $\nu(\phi)$ is the function that assigns truth-values to theoretical sentences of $L(V_o, V_t)$. It is defined as follows:

i) $\nu(\phi) := T$ iff for every structure $\mathfrak{A} \in \mathbf{S}_{\mathfrak{a}}$, $\mathfrak{A} \models \phi$ holds

ii) $\nu(\phi) := F$ iff for every structure $\mathfrak{A} \in \mathbf{S}_{\mathfrak{a}}$, $\mathfrak{A} \models \phi$ does not hold

iii) $\nu(\phi)$ is indeterminate iff there is at least one structure $\mathfrak{A}_1$ for which $\mathfrak{A}_1 \models \phi$ holds true and at least another structure $\mathfrak{A}_2$ for which $\mathfrak{A}_2 \models \phi$ does not hold true, where both $\mathfrak{A}_1$ and $\mathfrak{A}_2$ are members of $\mathbf{S}_{\mathfrak{a}}$, the set of admissible structures.

The idea lying behind these rules is rather simple: A theoretical sentence is true iff it is true in every admissible structure. A theoretical sentence is false iff it is false in every admissible structure. And a sentence has no determinate truth-value iff it is true in, at least, one admissible structure and false in, at least, another structure being also admissible.

The third clause of the definition indicates that truth-value gaps may occur at the theoretical level of the language because the interpretation by postulates does not result in a unique interpretation of theoretical terms. The most convenient way to deal with such indeterminate sentences appears to be the super-valuation logic originally developed by van Fraassen (1969). Taking recourse to this logic is convenient and beneficial because the logical axioms and inference rules of classical logic remain valid there.[5]

[5]For a closer examination of how truth-value gaps at the theoretical level are dealt with, see Andreas (2008). One may wonder whether a paraconsistent logic would also be a means to deal with indeterminate sentences. Paraconsistent logics, however, are advanced in view of situations in which a proposition and its negation can be derived, shown to be true, or are simply maintained. No such situation necessarily arises in the case of the present system as long as the set of postulates is consistent with the set of true observation sentences. Introducing a logic that allows to reason with inconsistencies would be a further step that is not required by the occurrence of truth-value gaps.

The three-valued logics of Łukasiewicz and Kleene, which are adjusted to the occurrence of truth-value gaps, come with a deviation from the inferential systems of classical logic. Hence, if preservation of logical axioms and inference rules of classical logic is considered a desideratum, supervaluation logic, or a corresponding modal version of that logic, is the first choice to account for truth-value gaps.

# 5 Semantic Holism in the Framework of Model Theory

How may the preceding elucidation of Carnap's doctrine of indirect interpretation of theoretical terms contribute to a model-theoretic account of semantic holism? Now, the truth-rules in definition 4 give rise to another model-theoretic translation of (1), the formal notation for asserting a sentence in Frege's *Begriffsschrift*. For the assertion of a theoretical sentence $\phi$, we propose to adopt the following notation:

(3) $\mathbf{S}_{\mathfrak{a}} \models \phi$

In words: For all structures $\mathfrak{A} \in \mathbf{S}_{\mathfrak{a}}$ it holds that $\mathfrak{A} \models \phi$, where $\mathbf{S}_{\mathfrak{a}}$, the set of admissible structures, is defined by definition 3. This notation can be seen to differ in two major respects from what has been assumed to be the model-theoretic standard notation. First, in place of a single intended interpretation there is now a range of interpretations which equally qualify as being intended ones. Second, sentences of the object language are allowed to determine which interpretations of the language are the intended ones.

In what sense is the present semantics of theoretical terms and sentences a holistic one? Now, unlike standard semantics, where the valuation of theoretical terms is taken as being determined directly in an intended interpretation, the values of such terms are determined indirectly through the axioms of a scientific theory. Notably, these axioms establish *conceptual relations* among the theoretical symbols such that certain *systems* of valuations are admissible - those in which the axioms are true - and others - those which do not satisfy any axiom - not admissible. Because these conceptual relations are part of the definiens in the definition of the set of admissible structures, it holds that the theoretical symbols get their semantic values determined not independently of one another. Hence, we have a case of (moderate) holism in the sense of definition 2.

Think of the ideal gas law as a simple example of a conceptual relation between theoretical terms. Any valuation of the symbols for pressure, volume and temperature not satisfying this law is not admissible for empirical systems being an intended application of the ideal gas law. The value of the pressure function is thus not determined independently of the values of the temperature and the volume function, where amounts of gas serve as the empirical arguments of these functions. The values of these function may in turn depend on the values of further theoretical and non-theoretical symbols, depending on the choice of measurement. Hence, we have widespread interdependence between the semantic values of expressions that do not share any constituent parts, as was required by definition 2.

What about compositionality? The surprising result is that compositionality - in the sense of definition 1 - still holds for theoretical sentences. For the semantic value of a theoretical sentence is determined through the semantic values of its component expressions, with the qualification that any admissible valuation of the component expressions needs to be considered. Hence, we can say that the admissible systems of valuation of the component expressions determine the semantic value of a theoretical sentence. The determination of logically complex expressions through the semantic values of simple expressions is less straightforward than in standard semantics, but compositionality still holds.

Hence, we seem to have established another case where semantic compositionality and semantic holism are compatible. This case is of the type envisioned by Pagin (1997), viz., holistic interdependence between the semantic values of simple expressions and compositionality for logically complex expressions. Pagin exemplified this type of compatibility with inferential role semantics through assuming the existence of abstract functions mapping inferential roles of an expression to its meaning, where meanings are taken as primitive and are not further analysed. Here, the 'mechanism' of holistic interdependence is analysed and accounted for in a more concrete fashion, as the definitions 3 and 4 show how theoretical terms and theoretical sentences get their semantic values assigned to. Adopting Church's (1956, 6) explanation of what the meaning, or sense, of an expression is, viz., that what determines the extension of that expression, we can say that these definitions *represent* the meaning of theoretical terms and theoretical sentences. In both Pagin's and the present analysis our intuition that compositionality is essential to the semantics of logical symbols can be retained and even remains justified despite widespread holistic interdependencies.

Arguably, there are exceptions to the principle of compositionality. The semantics of postulates cannot be claimed to be compositional in any legitimate sense. For the theoretical symbols are nothing but meaningless uninterpreted symbols prior to the adoption of a set of sentences as postulates. It would therefore be wrong to say that the semantic values of postulates depend on the semantic values of their component expressions. Rather, the order of determination between component expressions and sentences is reversed in the case of postulates. It is the semantic values of the postulates that determine - besides the valuation of the non-theoretical symbols - the valuation of the (theoretical) component expressions.[6]

[6]Schurz (2005) has already shown that the semantics of scientific theories is non-compositional. His investigation is concerned with Lewis's (1970) account of theoretical terms. There, Lewis attempted a definition of theoretical terms which rests on the possibility of a unique determination of the extension of such terms. The present semantics of theoretical sentences remains, however, more faithful to Carnap's doctrine of partial interpretation, which does not allow for a unique determination.

It can be observed finally that the truth-value assignment to theoretical sentences is defined here such that proposition 1 holds for theoretical sentences not being considered as postulates. Since the range of admissible structures is determined partly through the set $\Phi_{TC}$ of postulates, the truth-value of a theoretical sentence depends on the context of a scientific theory.

# 6 A Simple Case Study

Let us demonstrate the difference between standard model-theoretic semantics and its present modification with a simple case study concerning the semantics of the time function in physics. Poincaré argued that propositions about temporal distances acquire meaning only through a semantic convention that defines the concept of equality of temporal distances. The argument, in short, is that there is no direct means of comparing succeeding temporal intervals, and any indirect method of comparison rests on some previously adopted standard of congruence. For a long time, the periods of a mechanical pendulum served as such a standard. More precisely, the statement that succeeding periods of the oscillating motion of a mechanical pendulum are of equal duration was used. Let us introduce some symbolic notation to analyse the logical structure of such a statement. $V_d$, the non-logical vocabulary, may be equated with the set $\{t, S, E\}$. These constants are intended to have the following meanings:

$t(x)$ - time function
$S(x, y)$ - event $x$ succeeds event $y$
$E(x)$ - $x$ is an event where the pendulum reaches an extremum on either side.

$V_o = \{E, S\}$, $V_t = \{t\}$. $D_o$ is a set of observational, spatiotemporal events, $D_t$ the set of real numbers. Now, a formalisation of the convention that succeeding periods of a mechanical pendulum are of equal duration may go as follows:

$$\begin{aligned}(4)\quad & \forall x \forall y \forall z (S(y,x) \wedge S(z,y) \wedge E(x) \wedge E(y) \wedge E(z) \wedge \\ & \neg \exists u (S(u,x) \wedge S(z,u) \wedge E(u) \wedge \neg(u = y)) \rightarrow t(y) - t(x) = t(z) - t(y))\end{aligned}$$

It is important to note here - and this point is the key motivation of the present investigation - that the semantic value of this statement determines, in part, the semantic values of the time function. If we attempted to use the recursive model-theoretic definition of truth to determine the semantic value of statement (4), this would not get us very far because the semantic values of the time function are completely indeterminate prior to the introduction of a standard of congruence for the measurement of time. Hence, in the case of the above

statement, which has the function of a semantic convention, compositionality fails to hold. Equivalent considerations apply to the semantics of other physical magnitudes, as temperature in phenomenological thermodynamics, force in classical mechanics, or electromagnetic field intensity in electrodynamics.

# 7 A Criterion for a Sentence's Being a Postulate

So far, the semantic function of postulates has been elucidated, but little has been said about how to draw the distinction between postulates and other theoretical sentences. For this we propose to adopt the following explanations:

**Explanation 3.** A theoretical sentence $\phi$ is a postulate iff i) it is held true, ii) neither acquired nor justified by inferential transformations of a set of sentences whose members are also held true, and iii) it is not logically true.

The notion of a theoretical sentence can be explained simply as follows:

**Explanation 4.** A sentence $\phi$ is called theoretical iff there is at least one theoretical term occurring in $\phi$.

Still, we need to explain the distinction between observational and theoretical terms:

**Explanation 5.** A symbol $\alpha$ is called observational iff the determination of its extension, or at least a part of its extension, can proceed in a direct manner. That means, there is a method of determining $\alpha$ not dependent on the validity of any inferential transformation.

**Explanation 6.** A symbol $\alpha$ is called theoretical iff every method of determining its extension rests on inferences. The validity of these inferences is dependent on general axioms which are neither true for logical reasons nor true by virtue of being a definition.

These explanations essentially refer to the practice of scientific reasoning and to the affirmations made by speakers of the scientific community. Clause i) of explanation 3 may be qualified such that a postulate is held true by adherents of a research programme in the sense of Lakatos. Therefore, in scientific contexts, several systems of postulates might be necessary to accomplish a logical reconstruction. This would amount to there being several, partially overlapping linguistic frameworks in use within one and the same scientific community.

Paradigmatic examples of postulates, as the axioms of a certain axiomatisation of classical mechanics or the postulates of an axiomatisation of quantum mechanics qualify clearly as such according to the above explanations. Such axioms are neither definitions nor logical truths nor derived from other propositions. By

contrast, paradigmatic examples of scientific hypotheses - as opposed to the axioms - of a scientific theory, such as sentences reporting the values of a scientific quantity for particular empirical objects, are not classified as postulates. Assertions of such sentences are rather based on derivations from postulates and observational antecedent conditions. Certain sentences determining the metric of scientific concepts can be singled out as postulates, furthermore. Think, for example, of the proposition that water freezes at zero degrees Celsius under standard pressure.[7]

# 8 Relative and Methodological Holism

Quine (1981) distinguishes between two kinds of holism. What he calls moderate or relative holism already acknowledges that we cannot expect a scientific sentence to have its own separable meaning. But relative holism retains a distinction in kind between sentences that determine the structure and the meaning of the language and other sentences not having this function. Quine thinks that relative holism should be superseded by methodological holism, which is introduced in his (1981, 71n) as follows:

> The fourth move, to methodological holism, follows closely on this holism [i. e., relative holism, author]. Holism blurs the supposed contrast between the synthetic sentence, with its empirical content, and the analytic sentence, with its null content. The organizing role that was supposedly the role of analytic sentences is now seen as shared by sentences generally, and the empirical content that was supposedly peculiar to synthetic sentences is now seen as diffused through the system.

To whom shall we attribute relative holism? This question is difficult to answer. Carnap distinguishes sentences interpreting theoretical terms from other sentences in (1939, 1956, 1975), but in none of these writings are sentences of the former type considered analytic outright. Likewise, Poincaré's conventionalism seems to be a holism of the relative type, with the qualification that conventions are not assumed to be void of empirical content.

Independently of historical considerations, we can say that it is rather misleading to construe relative holism as relying on the analytic-synthetic distinction. This becomes evident in light of the present account of semantic holism. In this account, only sentences qualifying as postulates are assumed to determine the

[7]The notion of a theory is taken as primitive here and left without a formal explanation. Inspiration of how to make the individuation of theories as well as their interrelations precise may be gained from Balzer et al. (1987).

meaning of theoretical terms. And the distinction between postulates and other theoretical sentences must clearly not be equated with the analytic-synthetic distinction. Analyticity is therefore no requirement for a sentence to determine the meaning of non-logical symbols. Henceforth, we shall be using the term 'relative holism' in the wider sense such that this kind of holism is not understood as relying on the analytic-synthetic distinction.

It appears now as unnecessary to move from relative to methodological holism, for the present account of semantic holism expounds a holism of the relative type that does not rely on the analytic-synthetic distinction. It is furthermore doubtful whether methodological holism can be combined with a model-theoretic account of the meaning of the logical symbols in scientific language. If what has been said is correct and notation (3) is adopted for the assertion of theoretical sentences, then semantic holism gets integrated into a model-theoretic account of sentence meaning. But this notation essentially relies on the distinction between sentences interpreting theoretical terms and others not having this function. Therefore, notation (3) cannot be adjusted to the view that "the organising role is shared by sentences generally".

The difficulties inherent in Quine's supposedly model-theoretic account of scientific language have been observed already by Dummett (1978, 303–318). There, he claims that, under a holistic conception of scientific language, a model-theoretic account of sentence meaning ceases to have any explanatory value, where holism is understood as methodological holism. More precisely, Dummett's claim is that a proof of soundness of an axiomatisation of classical logic can only be seen as justifying inferential practice in science if a non-holistic conception of sentence meaning is assumed.

Particularly relevant to our discussion of semantic holism is that Dummett (1978, 303), charitably, interprets Quine as having a preference for a syntactic rather than a semantic approach to logic. Even though the attribution of such a preference to Quine can, to our mind, hardly be justified as an interpretation of his work, the reason why Dummett makes this attribution seems to be a proper one: Methodological holism disconnects model theory from scientific language so that a purely syntactic understanding of that language should be preferred consequently. To this we add that the connection between model theory and scientific language can be re-established if we remain adherent of relative holism.

# 9 Note on Incommensurability

Feyerabend (1962) expounds the view that meaning variance of a scientific concept in theory evolution results in different, mutually incommensurable concepts. In a reexamination of his own work, he explained that the notion of

incommensurability was intended to mean a deductive separation of theories in the sense that it is inadmissible to use one and the same calculus, or one and the same formal language, to express statements of different theories with incommensurable concepts (Feyerabend 1978, 179n). No claim is made there that incommensurability leads to serious problems of communication, a view that was contemplated upon by Kuhn for conflicting paradigms. It appears that the present semantics of theoretical terms supports the view that meaning variance of scientific concepts in theory evolution gives rise to deductive separation without having the consequence that one is bound to speak and to understand only one language. To show this, let us continue the simple case study concerning measurement of time from Section 6.

It is a presumed insight of Poincaré that propositions about the duration of time acquire meaning only through the - implicit or explicit - introduction of a time metric that defines the equality, or congruence, of temporal distances between spatiotemporal events. Is there a unique meaning to the notion of congruence of temporal intervals? Several standards of congruence have been used in the history of physics. Huygens gave a precise description of a pendulum clock and a corresponding definition of the unit of one second. Therein, he assumed that one and the same pendulum clock with a constant and well defined length could be used at different latitudes, an assumption that was soon found not to agree with the presumed constancy of the rotation of the earth (Mach 1933, 149–155). As a consequence of this divergence, the statement that the earth rotates with constant velocity had been adopted as standard of temporal congruence. Finally, in the 20-th century, the unit of time has been defined in terms of the radiation of Caesium 133, as is well known. On the basis of this definition and corresponding clocks with a very high level of precision, slight variations in the angular velocity of the earth could be measured.

Using the words of Feyerabend, one can say that the history of measuring time in physics exhibits a certain degree of meaning variance with respect to the time metric in use. Moreover, Feyerabend seems correct when he points out that meaning variance leads to deductive separation of statements being connected to different theories or frameworks. One cannot use the same symbols for expressing propositions about temporal distances when different time metrics underlie these propositions. The deductive separation of such propositions has, however, not the consequence that one is bound to understand only one time metric. Historians of science as well as many scientists have rather acquired a multilingual language competency which allows them to move between different conceptual frameworks. With the help of some technical transformations, it is not difficult to construct an all encompassing language that contains different conceptual frameworks as sub-languages.

The formal semantics of Poincarean conventions being elaborated here is apt, we think, to analyse more severe cases of incommensurability which had been

a central issue in post-Kuhnian philosophy of science. The statement that the earth is at rest serves as semantic convention in the astronomy of Ptolemaeus, which is used to determine the semantic values of statements about the spatiotemporal positions of other heavenly bodies in our solar system. By contrast, the statement that the sun is at rest serves as an alternative semantic convention in Copernican astronomy. From the viewpoint of the present semantics, it is not sensible to ask which system of conventions represents the physical facts as they truly are. Rather, pragmatic criteria, as simplicity of theory formulation, empirical scope, and explanatory power, will be decisive for the adoption of one framework or another in case more than one framework proves empirically adequate. This view has been advocated by numerous authors before but most of the time without a proper semantics.

Note that the adoption of a system of conventions is not void of empirical, factual content since, in case the interpretation of the observational language cannot be extended to an $L(V_o, V_t)$ structure satisfying the postulates, the sentences being postulates are not assigned to the value true according to the definitions (3) and (4). The empirical content of a system of postulates, or conventions, is thus construed in a holistic fashion, which is in line with the ideas Poincaré and Carnap had about conventions and postulates respectively.

The view that a scientific language contains meaning constitutive sentences - in the sense that scientific terms are meaningless symbols prior to the affirmation of such sentences - has been taken here as the essence of Poincarean conventionalism. We did not argue for that view in detail but tried to render it plausible through giving it a semantic foundation using model-theoretic notions. The opponent of Poincarean conventions and Carnapian postulates has the burden to show how scientific statements get their semantic values assigned to and to show, moreover, how scientists come into a position to recognise an assignment as being correct if no method of determination may be introduced through conventions or postulates. These notions are taken synonymously here.[8]

# 10 Conclusion

Is meaning compositional in scientific language? The presumed existence of meaning constitutive sentences gives rise to the thesis of semantic holism and does therefore suggest a negative answer. On the other hand it must be noted that compositionality inheres in the standard truth definition in model-theoretic semantics. From a logician's point of view, this may appear to be an even stronger argument for compositionality than the foundational role that com-

[8]For a more recent exposition of conventionalism concerning spatiotemporal metrics, with particular consideration of the theory of special and general relativity, see Gruenbaum (1973).

positionality is supposed to play in our mastering a language. The present investigation attempted to resolve the tension between semantic holism and compositionality within the model-theoretic framework.

The core idea of the resolution is to let the meaning constitutive sentences determine the admissible interpretations of a scientific language with theoretical symbols. Then, the notion of truth of a theoretical sentence is defined as truth in all admissible interpretations. The semantic values of sub-sentential expressions thus continue to determine the semantic values of sentences, with the qualification that the order of determination is reversed for meaning constitutive sentences. Arguably, these sentences are exempted from the principle of compositionality. The range of exceptions is, however, very small in comparison to the large range of non-meaning constitutive sentences for which compositionality holds outright. At least, this is an empirical hypothesis concerning scientific languages which may be confirmed through applying the criteria that have been set up for a sentence's being meaning constitutive.

How does a scientific language differ from ordinary language? From the viewpoint of the present investigation, we are tempted to say that the existence of meaning constitutive sentences is what is distinct of a scientific language. Yet, ordinary language may be argued to contain such sentences as well. If one wishes to maintain some variant of the *descriptive picture* in the theory of proper names, one is well advised to view the proposition that Moses led the Israelites out of Egypt as constituting, in part, the meaning of the proper name "Moses". Furthermore, conceptual relations among predicates may also be recognisable in ordinary language. As the proposition that water is not combustible may be used to identify amounts of water, this proposition has a meaning constitutive function. To maintain the above characterisation of a scientific language, we propose to follow the view that scientific theorising is present in ordinary language already. Further research must show whether the results of the investigation have further non-intended applications in the formal semantics of languages being not explicitly scientific but resemble scientific languages in containing meaning constitutive sentences.

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