

Wissensverarbeitung

- Knowledge Representation & Reasoning I -

Alexander Felfernig and Gerald Steinbauer
Institute for Software Technology
Inffeldgasse 16b/2
A-8010 Graz
Austria

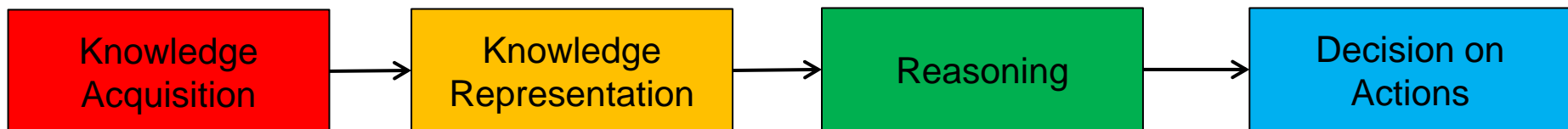
Motivation



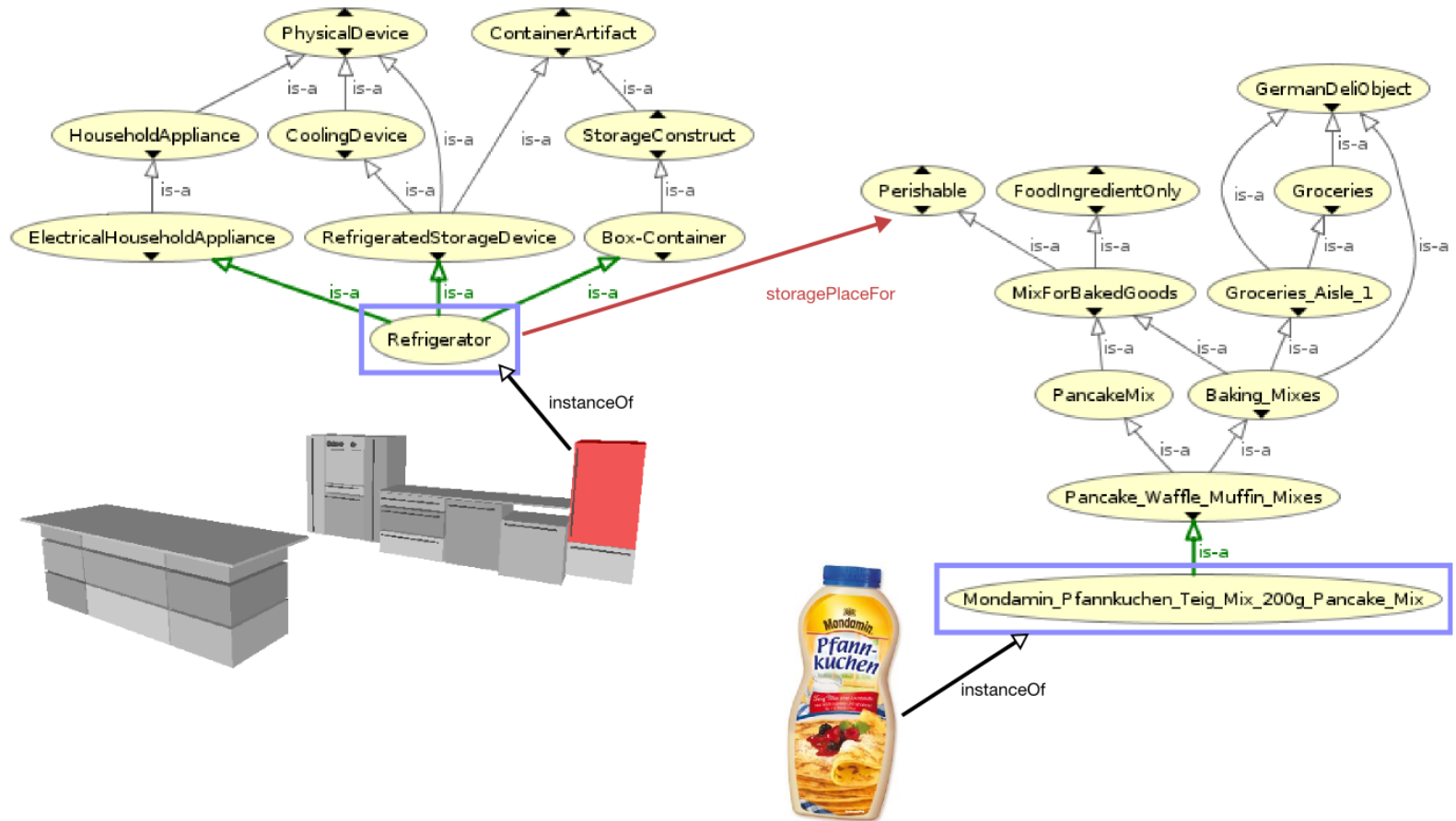
Robotic Roommates Making Pancakes

Artificial Intelligence and Knowledge Representation & Reasoning

- **Artificial Intelligence** (AI) can be described as:
 - the study of **intelligent behavior** achieved **through computational means**
- **Knowledge Representation & Reasoning** can be viewed as:
 - the study of how to **reason** (compute) with **knowledge** in order to decide what to do.

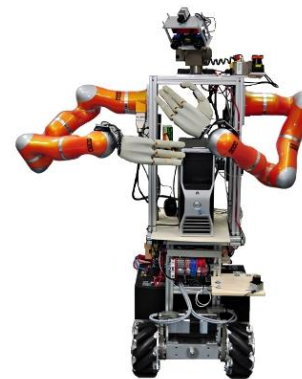


Use Case: Expected Object Location



Clear Need for proper KR & R

cYcorp



What is Knowledge?

- **easier question**: how do we talk about it?
- we say “John knows that ...” and fill the blank with a **proposition**
 - can be **true** / **false**, right / wrong
- **contrast**: “John fears that ...”
 - same content, different **attitude**
- other **forms** of knowledge:
 - know **how**, who, what, when, ...
 - **sensorimotor**: typing, riding a bicycle
- **belief**: not necessarily true and/or held for appropriate reasons
 - and **weaker** yet: “John suspects that ...”

What is Representation?

- **symbols** standing for things in the world



first aid



women

“John”



John

“John loves Mary”

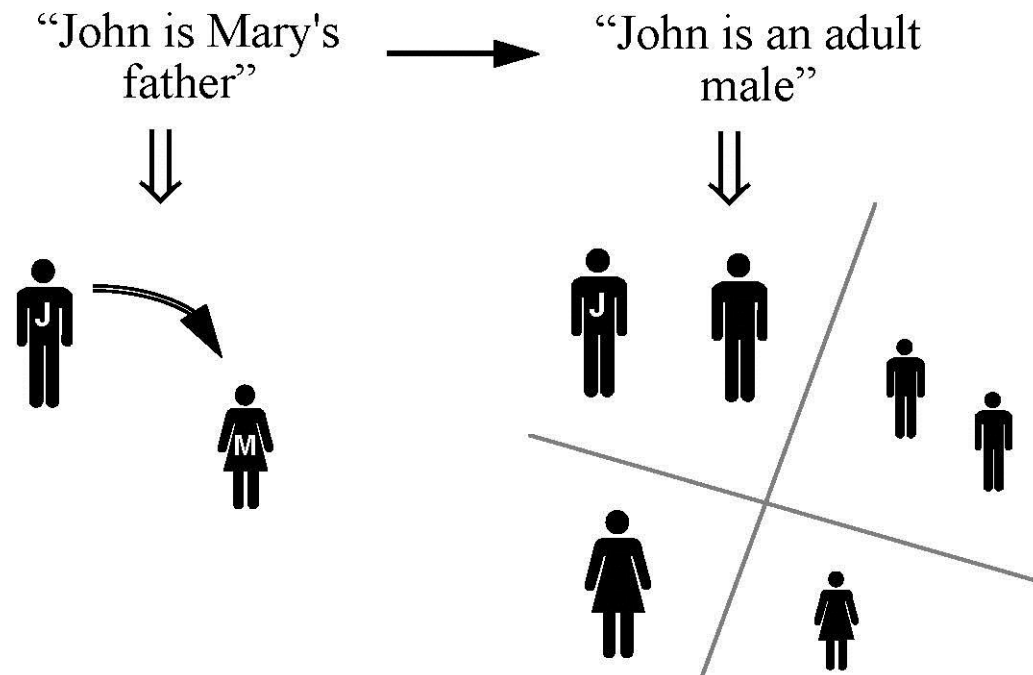


the proposition that John loves Mary

- **Knowledge Representation:**
 - symbolic **encoding** of propositions believed (by some agent)

What is Reasoning?

- **manipulation** of symbols encoding propositions to produce representations of new propositions
- analogy: arithmetic $1011 + 10 \rightarrow 1101$



Knowledge-Based Systems

EXAMPLE 1:

```
printColour(snow) :- !, write("It's white.").  
printColour(grass) :- !, write("It's green.").  
printColour(sky) :- !, write("It's yellow.").  
printColour(X) :- write("Beats me.").
```

EXAMPL 2:

```
printColour(X) :- colour(X,Y), !, write("It's "), write(Y), write(".").  
printColour(X) :- write("Beats me.").  
colour(snow,white).  
colour(sky,yellow).  
colour(X,Y) :- madeof(X,Z), colour(Z,Y).  
madeof(grass,vegetation).  
colour(vegetation,green).
```

Advantage

- knowledge-based system most suitable for **open-ended** tasks
 - can structurally isolate **reasons** for particular behavior
- **good** for
 - **explanation** and justification
 - “Because grass is a form of **vegetation**.”
 - informability: **debugging** the KB
 - “No the sky is not yellow. It's **blue**.”
 - **extensibility**: new relations
 - “Canaries are **yellow**.”
 - extensibility: new **applications**
 - returning a list of all the **white** things
 - **painting** pictures

KR Requirements

- the following **requirements** are desirable for KR approaches:
 - expressiveness
 - processibility
 - flexibility
 - modularity
 - understandability
 - representation of **uncertainty**

Expressiveness

- approach have to be able to represent relevant **issues**
- approach has to have enough **expressional power**
 - **facts**, e.g.: *tom* studies *cs* or: *ann* has *fever*.
 - **relations** between facts, for example, **rules** such as: if *tom* studies *cs* then *tom* knows about *oo programming languages* or: if *ann* has *fever* then she is *ill*

Processibility

- approach can **automatically** provide result in time
 - derive **new** knowledge systematically
 - **calculi** of formal logic
- approach **automatable**
 - problems solvable in **finite** time
 - **polynomial** behavior (“tractable reasoning”)
 - description as an **algorithms**
 - always **tradeoff** between processibility and expressiveness

Flexibility

- approach has to be **general**
- approach applicable in **different** application domains

Understandability

- used language **understandable**
- support **acquisition** of knowledge
- underlying knowledge bases **maintainable**

Modularity

- **structuring** mechanisms for knowledge bases
- knowledge **changes** over time
- **support** deletion and insertion of knowledge
- changes should have only **local** effects

Representation of uncertainty

- **vague information**
 - facts/results are **imprecise**, for example, *tom is big*
- **uncertain information**
 - we **assume** that *Ann is already in Graz*
- **uncertain relations**
 - *if I drink tap water in Ghana I **might** get diarrhea*
- ... altogether **challenges** for intelligent systems

Further challenges

- **inconsistent** knowledge
 - e.g. a robot believes to be at location A and B
- **wrong** knowledge
 - e.g. spinach contains a lot of iron
- **common-sense/default** reasoning
 - for example, “*birds typically fly*”



Our Goals

- what are the goals we will tackle in the **course**
 - get to **know** example knowledge representations
 - **use** this representations for a concrete problem
 - be **aware** of the modelling issues of different representations
 - be able to **automatically** reason with different representations (manually and using tools)
 - **understand** the performance issues of different representations
 - be able to **manage** specific aspects of KR & R

Content

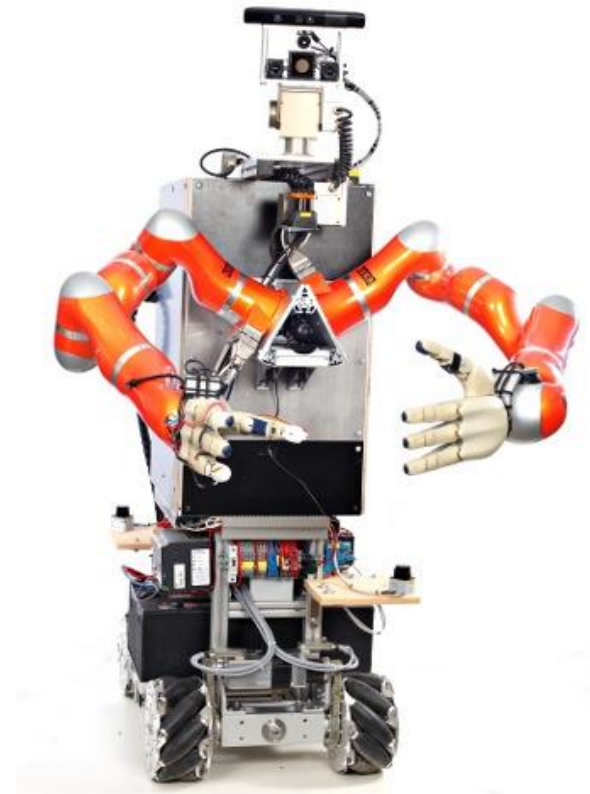
- **next** four units
- **foundations** of different representations
- **representing** knowledge with different representations
- automated **reasoning** with different representations
- special **issues** with different representations
- we **focus** on
 - First Order Logic (**FOL**)
 - Answer Set Programming (**ASP**)
 - Description Logic (**DL**)

Tool Support

- we will use **state-of-the-art** tools
- please **install** them for the lecture and the practical work
- FOL
 - Automated Theorem Prover – **Prover9** (GUI)
 - <https://www.cs.unm.edu/~mccune/mace4/>
- ASP
 - Potsdam Answer Set Solving Collection – **clingo** (command line)
 - <http://potassco.sourceforge.net/>
- DL
 - ontology editor - **Protégé** (GUI)
 - <http://protege.stanford.edu/>

Practical Work

- support **ROSIE** in setting the table



Logic-based Knowledge Representation

- **logic**: properties of the world (domain) are represented in the form of propositions and sentences.
- **syntax**: expressions (formulae) φ ;
admissible/syntactically correct sentences
- **semantics**:
 - meaning of the sentences
 - true sentences \rightarrow basis for logical consequences, for example, from $\varphi_1, \varphi_2, \dots, \varphi_n$ we can derive ψ .
 - inference systems (calculi): allow „calculations“
 - natural representation of facts and rules, for example:

```
from_Crete(Epimenides).  
 $\forall x(\text{from\_Crete}(x) \rightarrow \text{lies}(x)).$   
logical conclusion: lies(Epimenides).
```

Logic-based Knowledge Representation

- **reference language**
 - first order (predicate) logic
 - most **important** formalism of logic
 - **expressive**: all computable functions specifiable (Church's thesis)
 - simple, **natural** syntax and intuitive semantics
- **disadvantages**
 - **non-decidable** but decidable fragments
 - high computational **complexity** → further restrictions of expressivity needed

First Order Logic (FOL)

- also known as **Predicate Logic**
- popular in **AI**
- more **expressive** power than others
- FOL **handles**
 - objects
 - properties
 - relations



Syntax FOL

- FOL builds up **formulas**
- FOL comprises the following **vocabulary**
- **constant symbols**
 - refers to a single **object**
 - e.g. *Homer*
 - constant symbols need to be **interpreted**
 - can be used as a **name** for an object
 - e.g. *Bart*

Syntax FOL

- **function symbols**
 - represent **intuitively** a *n*-ary function
 - take *n* **arguments** (objects)
 - **assign** exact one object *o* to the arguments,
 $o = f(o_1, \dots, o_n)$
 - e.g. *Marge = mother_of(Lisa)*
- **variable symbols**
 - stand for an object which will be **bound** later at evaluation of a formula
 - can be used in **quantifiers**
 - e.g. *x, y, z*

Syntax FOL

- **predicate symbols**
 - represent a n -ary **predicate**
 - take n **arguments** (objects)
 - if $n=1$ it represents the **property** of an object, e.g. *smug(Lisa)*
 - if $n>1$ it represents a **relation** of objects, e.g. *sibling(Maggie,Bart)*
 - if $n=0$ it represents a property **independent** to objects, similar to propositions, e.g. *the_simpsons_are_cool()*

Syntax FOL

- **terms**
 - refer to an **object**
 - each **constant** symbol c and **variable** symbol x is a term
 - if t_1, \dots, t_n are terms then $f(t_1, \dots, t_n)$ is a **term**, e.g. *father_of(Bart)*
- **atomic formulae**
 - if P is a n -ary **predicate** symbol and t_1, \dots, t_n are terms then $P(t_1, \dots, t_n)$ is an atomic formula (atom)

Syntax FOL

- **formulas**
 - each **atomic** formula is a formula in F
 - if φ_1 and φ_2 are **formulae** from F and x is a **variable** symbol, then also
 - $\varphi_1 \wedge \varphi_2$ (**conjunction**), $\varphi_1 \vee \varphi_2$ (**disjunction**)
 - $\varphi_1 \rightarrow \varphi_2$ (**implication**), $\varphi_1 \leftrightarrow \varphi_2$ (**equivalence**)
 - $\neg \varphi_1$ (**negation**)
 - $\forall x \varphi_1$ (**universal** quantifier, φ_1 area of binding)
 - $\exists x \varphi_1$ (**existential** quantifier)
 - e.g. $\forall x[\text{married}(x) \rightarrow \exists y(\text{spouse}(x,y))]$

Definitions FOL

- **free variables**

- a occurrence of a variable x in φ is named **free** if x is not bound by a quantifier (\forall, \exists)
- a variable x is named free in φ if it occurs freely at least **once** in φ
- a **formula** φ with free variables x_1, \dots, x_n is written as $a(x_1, \dots, x_n)$
- is there a **free** variable ?
 - $\text{loves}(\text{Mary}, x), p(a) \rightarrow q(a)$
 - $\exists x(p(x) \vee \forall y(p(x, y)))$
 - $\forall x(p(x) \vee \exists x(q(x)))$

Definitions FOL

- **sentence**
 - is a formula **without** free variables
 - it is a **closed** proposition
 - e.g. $\forall y \exists x \neg \text{likes}(x, y)$
- **universal sentence**
 - a sentence of the form $\forall x_1 \forall x_2 \dots \forall x_n \psi$ and ψ is **free** of quantifiers
- **literal**
 - L is an **atomic** formula ($\text{loves}(\text{Marge}, x)$) or its **negation** ($\neg \text{likes}(\text{Selma}, \text{Homer})$)
- **clause**
 - C is an **disjunction** of literals, $C = L_1 \vee \dots \vee L_n$

Semantics FOL

- defines the **meaning** of a formula
- defined over **interpretations** and **models**
- the interpretation of a vocabulary V needs
- a non-empty set D of objects (**domain**, **universe**)
- an **assignment** of all symbols over D
 - **constant** symbol c : object $I(c) \in D$
 - n-ary **function** symbol f : function $I(f): D^n \rightarrow D$
 - n-ary **predicate** symbol P : relation $I(P) \subseteq D^n$

Semantics FOL

- evaluation of a variable-free **term** t
 - if t is a **constant** symbol c then $I(t)=I(c)$
 - if t is of **form** $f(t_1, \dots, t_n)$ then
$$I(t)=I(f)(I(t_1), \dots, I(t_n))$$
- evaluation of a **sentence** φ
 - a sentence φ can be **evaluated** to *true* or *false* in I
 - if φ is **variable-free** atom $P(t_1, \dots, t_n)$ then
$$I(\varphi)=\textit{true}$$
 if the tuple $(I(t_1), \dots, I(t_n))$ is in $I(P)$

Semantics FOL

- evaluation of a **sentence** φ continued
- if φ is a **composition** of formulae φ_1 and φ_2 ($\varphi_1 \circ \varphi_2$) then $I(\varphi)$ is a propositional composition of $I(\varphi_1)$ and $I(\varphi_2)$ [$I(\varphi_1) \circ I(\varphi_2)$]
 - $I(\varphi_1)$ and $I(\varphi_2)$ are similar to **propositions**
 - evaluation uses the **truth table**
- if $\varphi = \forall x(\psi)$ then $I(\varphi) = \text{true}$ if $I(\psi[x/o]) = \text{true}$ for **all** $o \in D$
 - $\psi[x/o]$ is the resulting formula if all **occurrences** of free variable x in ψ are replaced by o

Semantics FOL

- evaluation of a **sentence** φ continued
 - if $\varphi = \exists x(\psi)$ then $I(\varphi) = true$ if $I(\psi[x/o]) = true$ for at **least** one $o \in D$
- evaluation of a FOL sentence φ is in general **undecidable**
 - e.g. **infinite** number of objects in D
 - no algorithm for **finite** time

FOL Definitions

- **model**: I is a model for φ (written as $I \models \varphi$)
iff $I(\varphi) = t$
- I is a model for a **set** of formulas S (written
as $I \models S$) iff $I(\varphi) = t$ for all $\varphi \in S$
- the set of **all models** for φ or S is named
 $Mod(\varphi)$ or $Mod(S)$
- a formula φ is a **tautology** if every
interpretation I is a model for φ ,
 $Mod(\varphi) = Int(\varphi)$
 - $((x \equiv y) \wedge (y \equiv z)) \rightarrow (x \equiv z)$

Elementary Properties FOL

- useful for proofs and reasoning
- $S \models \varphi \rightarrow \psi \Leftrightarrow S \cup \{\varphi\} \models \psi$ (**deduction theorem**)
- $S \models \varphi \Leftrightarrow S \cup \{\neg\varphi\} \models \perp$ (**proof by contradiction**)
- $\{\varphi, \varphi \rightarrow \psi\} \models \psi$ (**modus ponens**)
- $S \cup \{\varphi_1 \vee \varphi_2\} \models \psi \Leftrightarrow S \cup \{\varphi_1\} \models \psi$ and $S \cup \{\varphi_2\} \models \psi$ (**case analysis**)
- $\forall x \psi(x) \models \psi[x/t]$, t some term (**specialization**)

Elementary Properties FOL

- example (Epimenides Paradox)
- given
 - $\forall x(\text{from_crete}(x) \rightarrow \text{lies}(x))$
 - $\text{from_crete}(\text{Epimenides})$
- to prove
 - $\text{lies}(\text{Epimenides})$

Elementary Properties FOL

- **satisfiability**
 - if a sentence is valid is **undecidable** (Church-Theorem)
 - FOL is **semi-decidable**, there are an algorithm which terminated if φ is a valid sentence
- **logic inference systems** (Calculi)
 - algorithms to **derive** valid sentence
 - **finite** time

Logical KR

- use of **FOL**
- **domain theory**
 - a general **description** of the problem domain
 - e.g. **groups** in mathematics, geometry
- **axioms**
 - general **valid** sentences
- **facts**
 - **specific** knowledge
 - literals, e.g., *child(Bart, Homer)*,
¬likes(Moe, Bart)

Logical KR

- **general rules**
 - **universal** quantified sentences
 - $\forall x_1 \forall x_2 \dots \forall x_n (\varphi_1 \wedge \dots \wedge \varphi_n) \rightarrow \psi$
 - φ_i : **condition, premise**
 - ψ : **conclusion**
 - similar to **if-then** statement
- **universal quantified Horn Clauses**
 - all φ_i and ψ are **atoms**
 - subset of FOL, **reduced** expressiveness
 - e.g., $\forall x (\text{human}(x) \rightarrow \text{mortal}(x))$

Logical KR

- **universe of Discourse**
 - in FOL in general no **special** domain
 - in KR a particular **domain D** and **interpretation**
 - e.g., **arithmetic**: $\{0, 1, 2, 3\}$ (constants), $\{+, *\}$ (functions)

Methodology of Modeling

- what is a **good** way to model in logic
- **no** formal approach
- **clarity** of names, concepts
 - names of constants, predicates etc. are **meaningful**, e.g. *parent(x,y)*
- **disclose** relations
 - $\forall x(\text{senior}(x) \rightarrow \text{discount}(x))$
 - fact: *senior(Peter)*
 - **problem**: gender? why discount ?
 - $\forall x \forall y [(y = \text{gender}(x) \wedge \text{age}(x) > \text{age_limit}(y)) \rightarrow \text{discount}(x)]$

Methodology of Modeling

- **validation**
 - why a sentence gets **valid** ?
- **generality**
 - can one express a sentence more **generally** ?
- **requirement** of predicates
 - are **new** predicates necessary ?
 - **relation** to other predicates
 - description of super/sub **classes**

Approach

1. conceptualization

- decide what to **represent**
- **abstract** concept

2. choice of vocabulary

- **translation** of the abstract concept to FOL
- the resulting **vocabulary** is an ontology of the problem domain

3. coding of the domain theory

- **specify** all relations and rules

4. coding of the specific knowledge

Example – Alpine Club

- Tony, Mike and John belong to the Alpine Club. Every member of the Alpine Club who is not a skier is a mountain climber. Mountain climbers do not like rain, and anyone who does not like snow is not a skier. Mike dislikes whatever Tony likes, and likes whatever Tony dislikes. Tony likes rain and snow.
- Prove that the given sentences logically entail that there is a member of Alpine Club who is a mountain climber but not a skier.
- Suppose we had been told that Mike likes whatever Tony dislikes, but we had not been told that Mike dislikes whatever Tony likes. Prove that the resulting set of sentences no longer logically entails that there is a member of Alpine Club who is a mountain climber but not a skier.

Example – Simple Assignment Task

- suppose a simple **assignment** task
- we have n **persons**
- we have m **items** of l **types**, $l \leq m$
- each person **needs** one item of a type
- an item can only be assigned to a **single** person
- prove if a setup is **consistent** or not

Logical KR Issues I

- need further **axioms** for reduced interpretation
- **unique names assumption (UNA)**
 - **different** constants c_1, \dots, c_n refer to different objects
 - $c_1 \neq c_2, \dots, c_1 \neq c_n$ for all $i \neq j$
 - with UNA *Homer* \neq *Marge* is **valid**, but not in general

Logical KR Issue II

- **domain closure axiom (DCA)**
 - in general the domain is defined by the **constant** symbols c_1, \dots, c_n
 - to prevent the interpretation of **further** objects,
 $\forall x(x = c_1 \vee x = c_2 \dots \vee x = c_n)$
 - set of **constant** symbols C have to be finite
 - if C is finite and there are no function symbols then interpretation with DCA and UNA = **H-interpretation**
 - to limit a properties P to **certain** objects,
 $\forall x[P(x) \leftrightarrow (x = c_1 \vee x = c_2 \dots \vee x = c_n)]$

Logical KR Issue II

- sometimes **expected** sentences are not derivable
- solution **closed world assumption** (CWA): represent only positive and assume all facts (not derivable) to be false
- CWA is **non-monotonic**: addition of new facts may limit the number of derivable negative facts

Thank You!