

# The Zeeman-Effect



# Some Basics

- Quantum numbers (qn)

$n = 1, 2, 3, \dots$  main qn (energy states -shell)

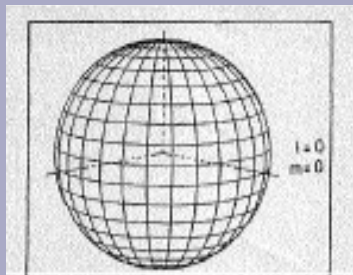
$l = 0, 1, \dots (n-1)$  orbital angular momentum (s, p, d, ...-subshell)

$m_l = -l, \dots, +l$  magnetic qn, projection of  $l$  (energy shift)

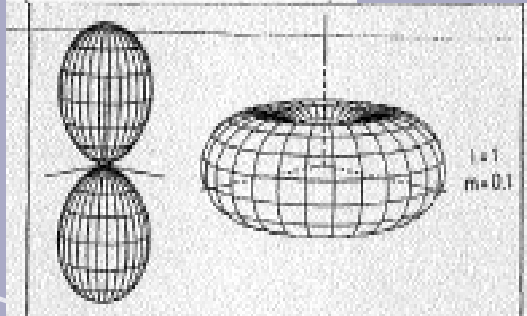
$m_s = \pm 1/2$  spin angular momentum

$j = |l-s| \leq j \leq l+s$  total angular momentum (if coupled, interactions of spin-orbit dominate over  $B$ )

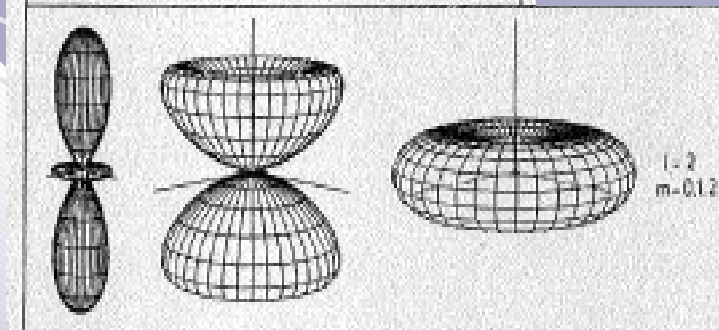
# Some Basics



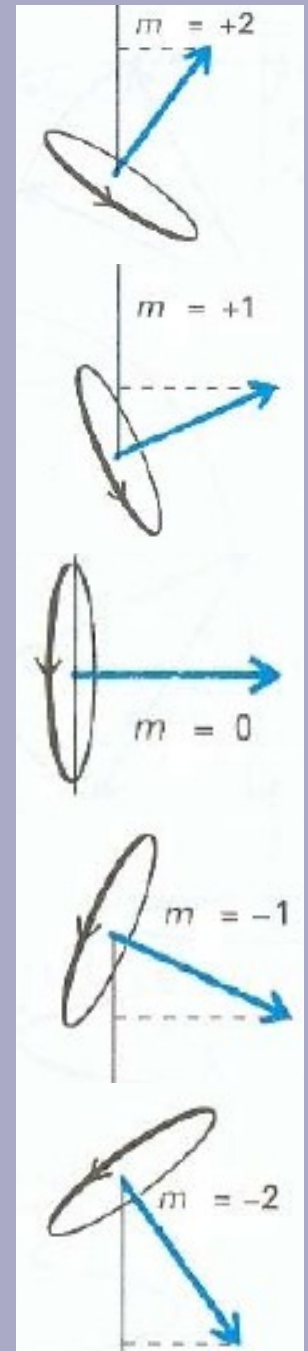
- $l = 0, m_l = 0$



- $l = 1, m_l = 0, 1$



»  $l = 2, m_l = 0, 1, 2$



# Some Basics

- more than 1 e<sup>-</sup>

$$n^{2S+1}L_J$$

$$L = \sum l_i \quad (S, P, D, \dots)$$

$$S = \sum s_i$$

$$J = \sum j_i \quad (\text{with } m_j)$$

Eigenvalues:

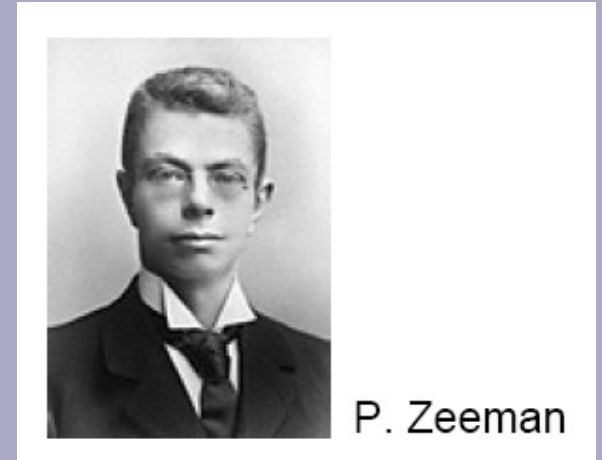
$$L = \hbar [l(l+1)]^{1/2}$$

$$S = \hbar [s(s+1)]^{1/2}$$

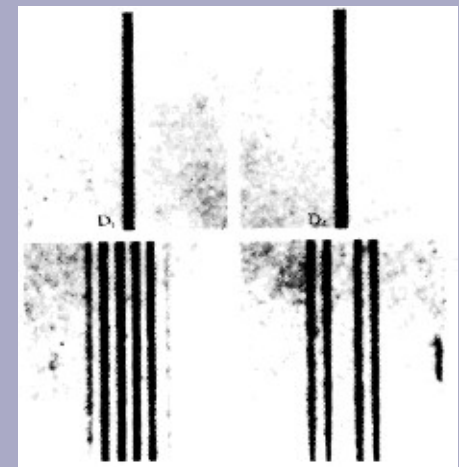
$$J = \hbar [j(j+1)]^{1/2}$$

# Zeeman Effect

- discovered 1896 different splitting patterns in different spectral lines
- splitted lines are polarized
- splitting proportional to magnetic field  
 $\Delta\lambda \sim g_L \lambda^2 B$

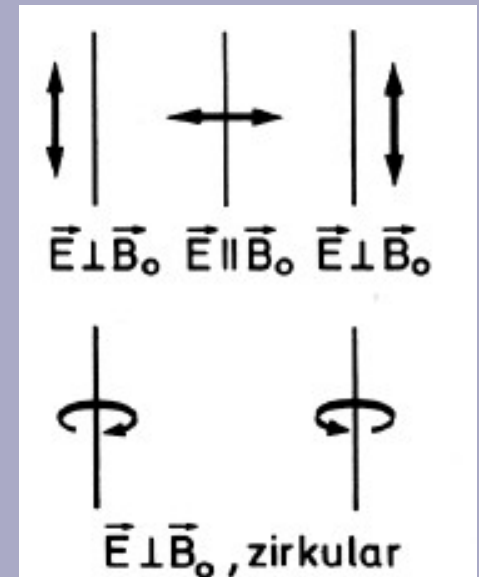


- Interaction of atoms with external magnetic field due to magnetic moment  $m_j$



# Zeeman Effect

- normal Zeeman effect:  
 $S = 0, L = J \neq 0$
- split in 3 components  
components with  $\Delta m_j = 0$  ( $\pi$ )  
components with  $\Delta m_j = -1$  and  $+1$  ( $\pm \sigma$ )
  - 2 possibilities:
    - $B$  parallel to line of sight  
→ two  $\sigma$  circular polarized
    - $B$  perpendicular to line of sight  
→ two  $\sigma$  polarised vertical to field  
one  $\pi$  polarised horizontal to field



# Zeeman Effect

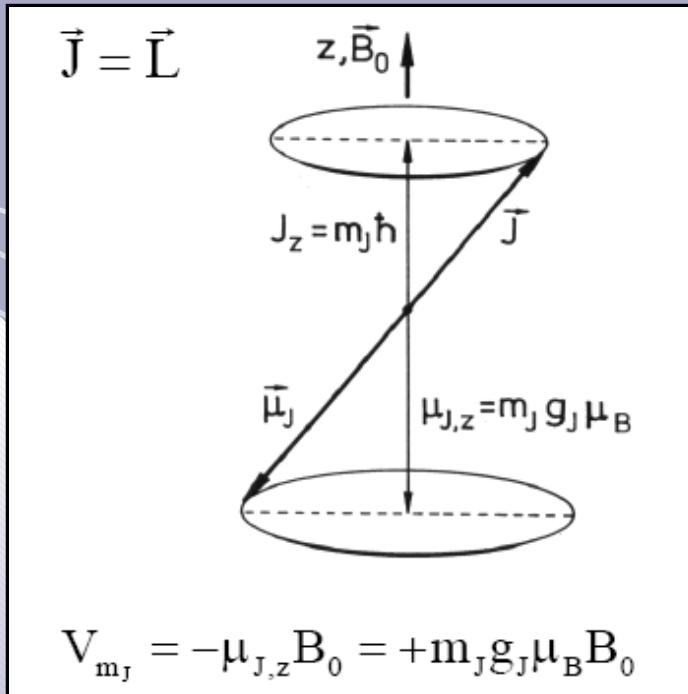
$$V = - \vec{\mu} \cdot \vec{B}$$

• interaction with external field

$$\vec{\mu}_j = g_j \frac{e_0}{2m_e} \vec{j}$$

– magnetic momentum of Atom

$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

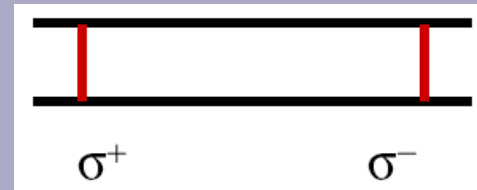
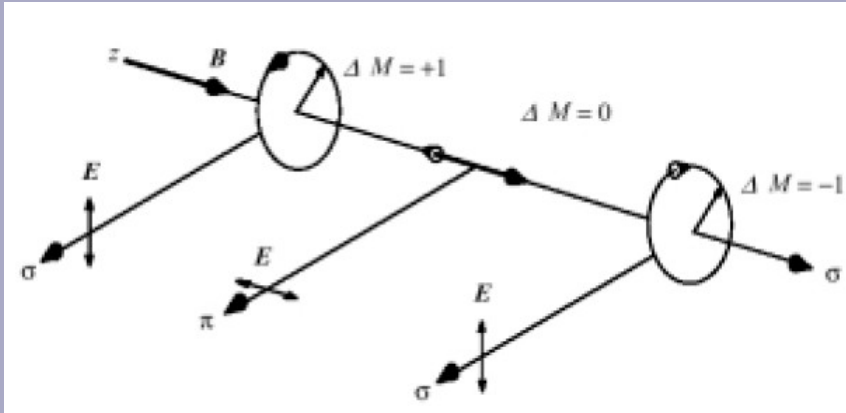


- **$J$  and  $\mu_J$  antiparallel**  
 since  $S=0$  and  $L=J \rightarrow g_J = 1$   
 $\rightarrow$  only 3 values of  $\Delta m$  possible:  
 $0, \pm 1 \rightarrow \pi, \pm \sigma$

$$\mu_B = (e\hbar)/(2m_e) \quad \text{Bohr magneton}$$

$$\rightarrow V = m_J \mu_B B_0$$

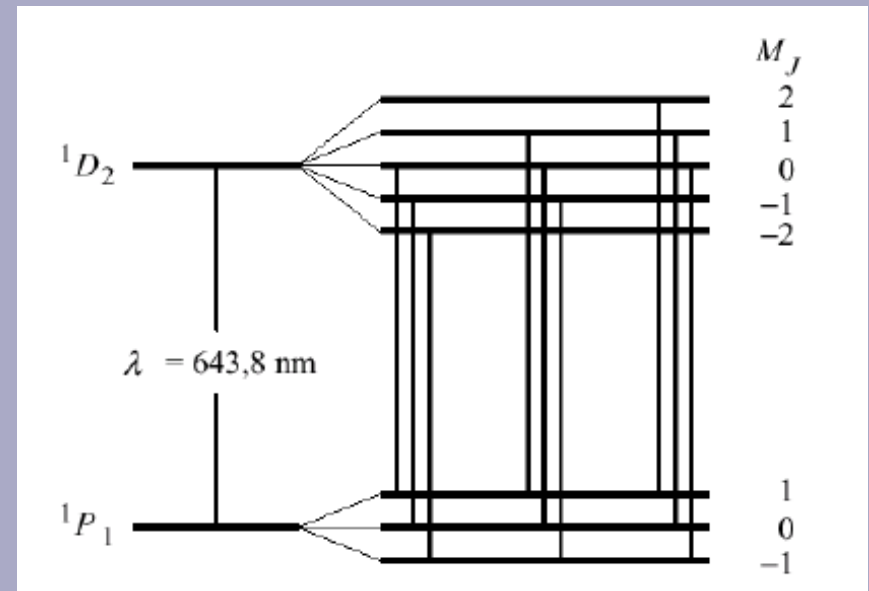
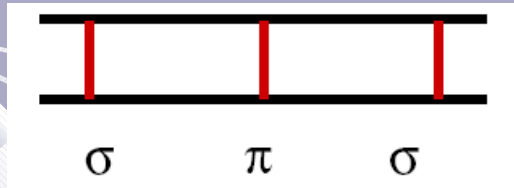
# Zeeman Effect



- Selection rules:

$$\Delta L = \pm 1$$

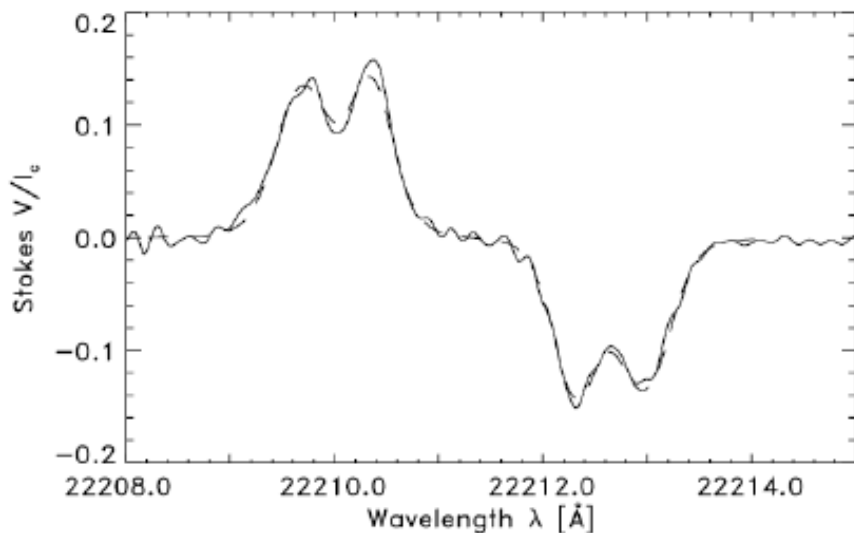
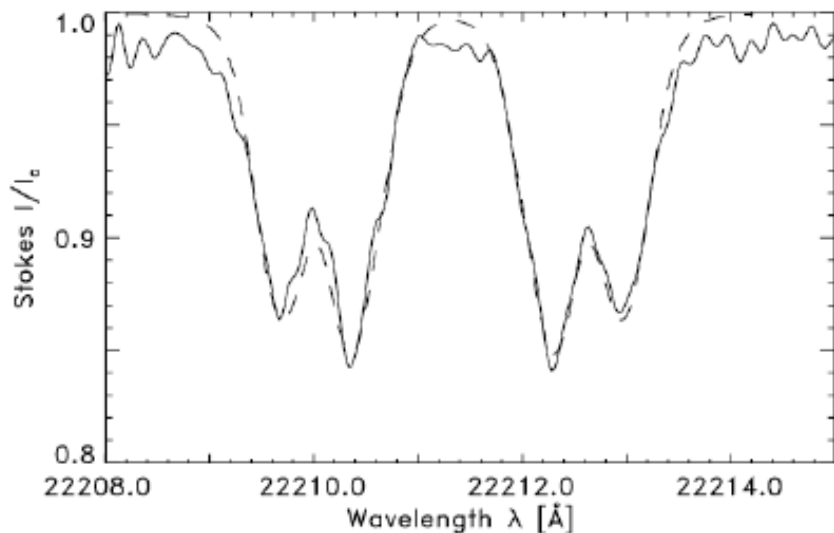
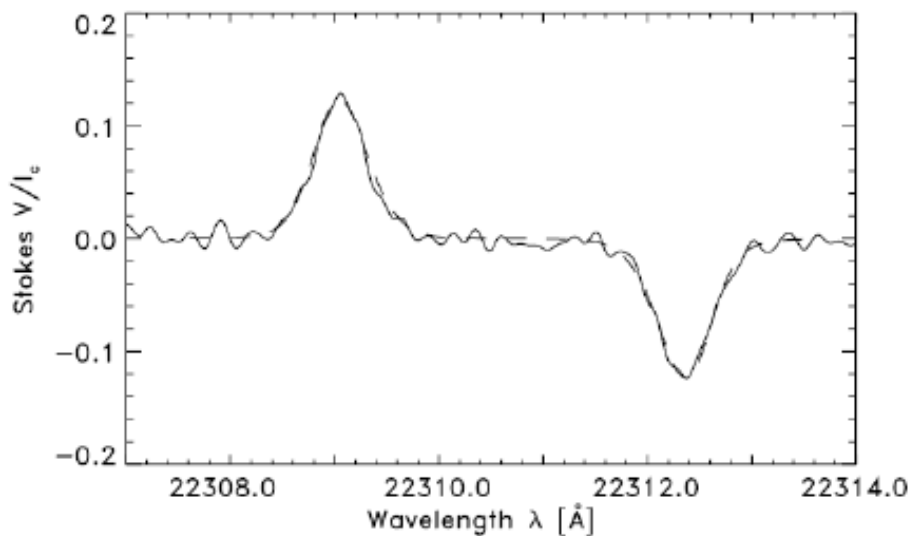
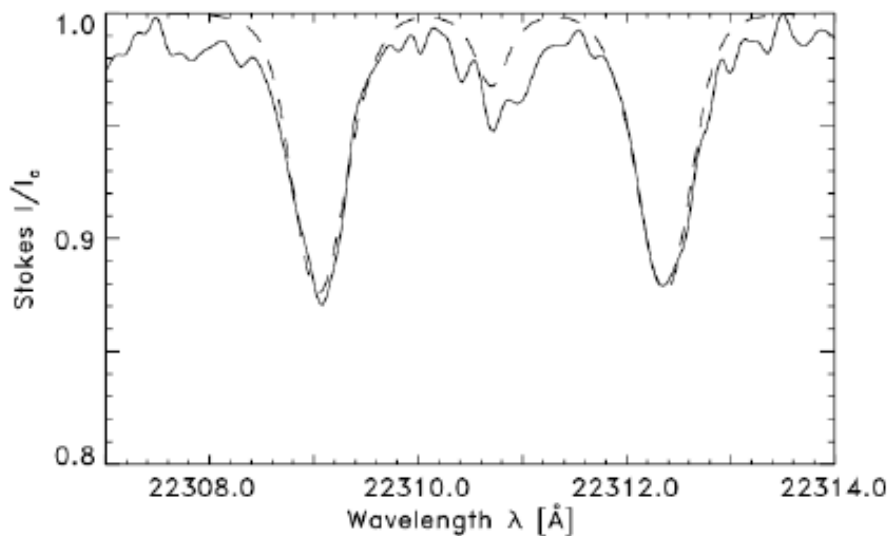
$$\Delta M = 0, \pm 1$$



- fine-structure depending on  $n, j$



# Fully Split Titanium Lines at $2.2\mu\text{m}$



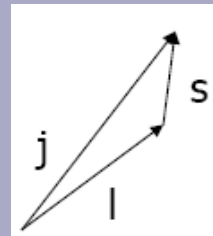
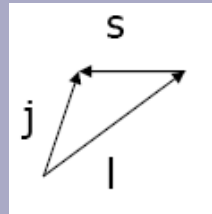
# Anormal Zeeman Effect

- anormal Zeeman effect (LS – coupling):  
 $S \neq 0, L \neq 0 \rightarrow$  usual case

$$\vec{j} = \vec{l} + \vec{s}$$

$$j = 1 + 1/2 = 3/2$$

$$j = 1 - 1/2 = 1/2$$

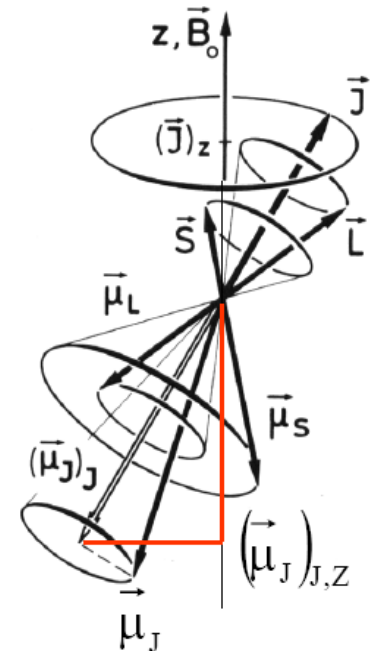


$$\vec{\mu}_L = -g_L \frac{\mu_B}{\hbar} \vec{L}$$

$$\vec{\mu}_S = -g_S \frac{\mu_B}{\hbar} \vec{S}$$

$$\vec{\mu}_J = \vec{\mu}_L + \vec{\mu}_S$$

- $J$  and  $\mu_J$  not antiparallel
- $g_J \neq 1$
- $\rightarrow \Delta E = g_J \Delta m_J \mu_J B$
- more different energy levels
- $\rightarrow$  more components possible

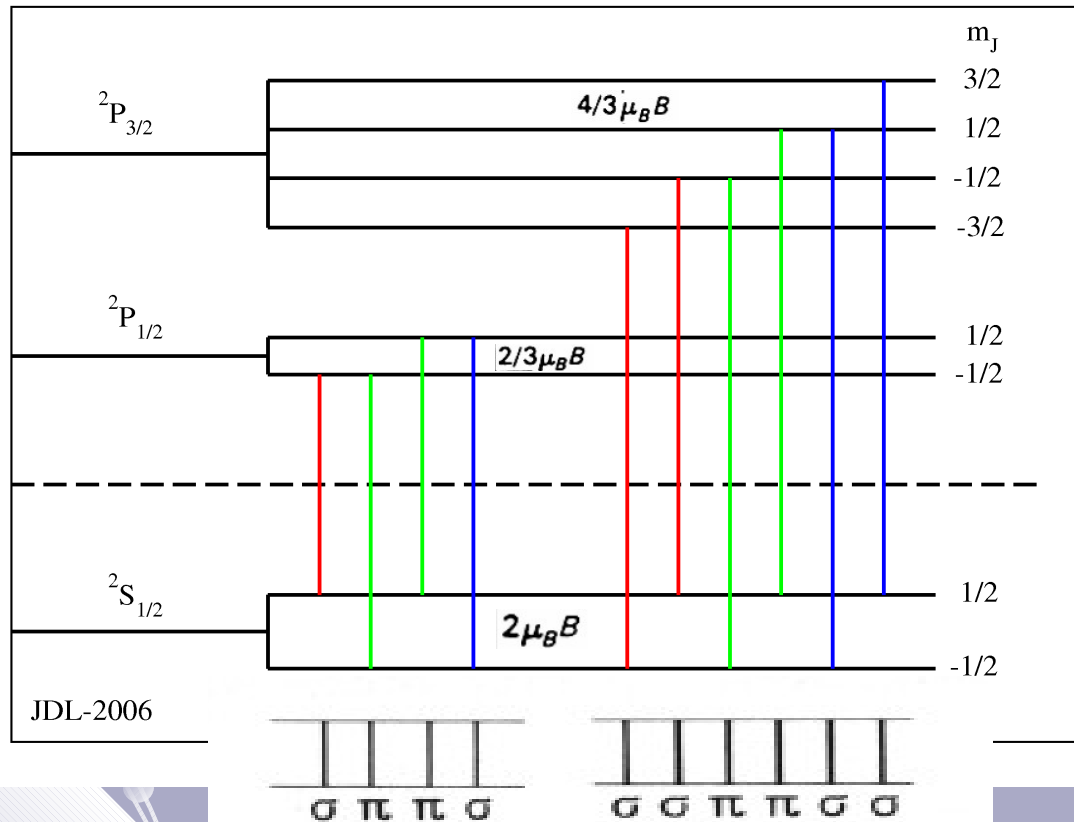


# Anormal Zeeman Effect

$$n^{2S+1}L_J$$

Zeeman components for sodium D lines

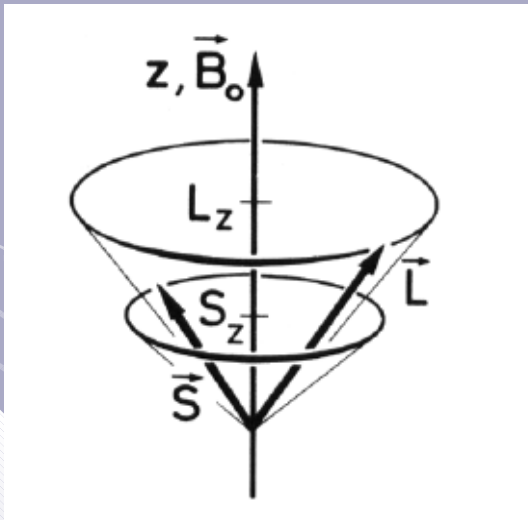
green: pi components, red & blue: sigma components



# Paschen-Back Effect

- for most transitions and  $B < 50000 \text{ G}$  ( $5 \text{ T} =$  upper limit for main sequence stars) lines are in Zeeman regime

- if  $|\vec{B}_0| > |\vec{B}_J|$   $L$  and  $S$  decouple  $\rightarrow$  Paschen-Back effect



$$V_{m_S, m_L} = (m_L + 2m_S)\mu_B B_0$$
$$\Delta E = (\Delta m_L + 2\Delta m_S)\mu_B B_0$$

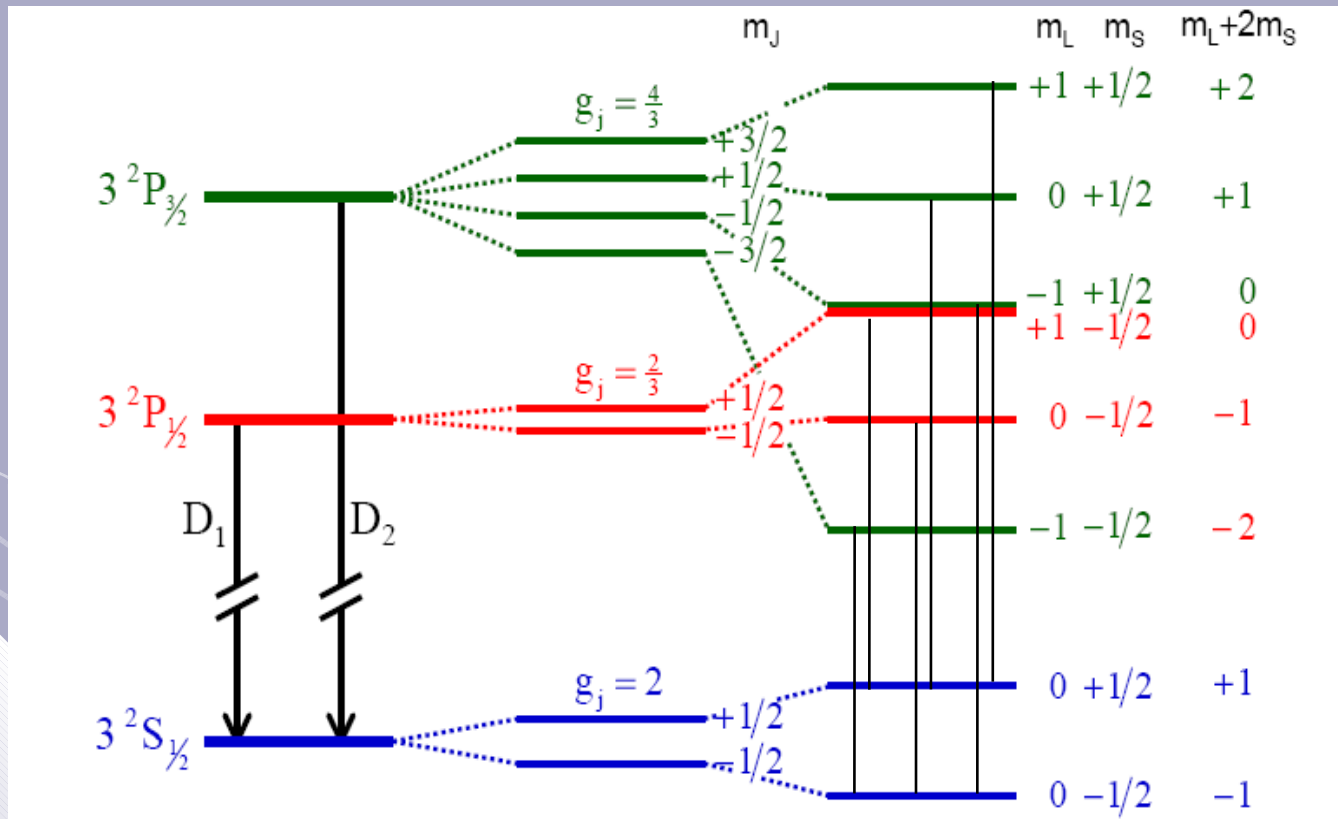
# Paschen-Back Effect

weak  
field

strong  
field

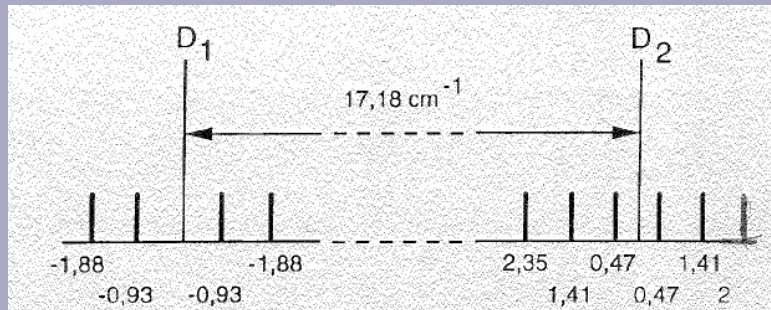
$$\Delta m_L = 0, \pm 1$$

$$\Delta m_S = 0$$



# Paschen-Back Effect

- fine structure splitting varies a lot in atoms, so a few lines may be in Paschen-Back regime at much smaller B value than others.



- Zeeman splitting for sodium D<sub>1</sub> and D<sub>2</sub> in a field of 3 T
- splitting for Li: 0.3 cm<sup>-1</sup>

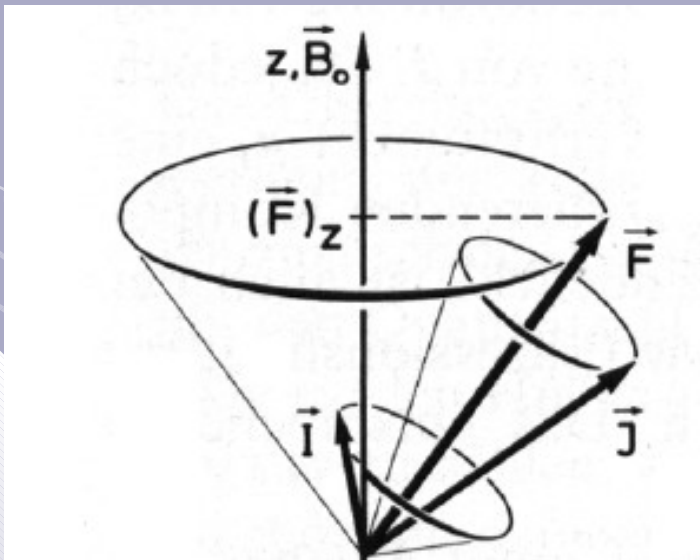
→ 3 T is a strong field for Li but a weak field for Na  
Na shows Zeeman splitting  
Li would show Paschen-Back splitting

# Hyperfine structure

- fields of  $B_0 \geq 0.1$  T often cause splitting
- magnetic momentum of a proton due to its spin:

$$\bar{\mu}_I = \frac{e_0}{2m_p} \bar{I}$$

$I$  ... spin of nucleus

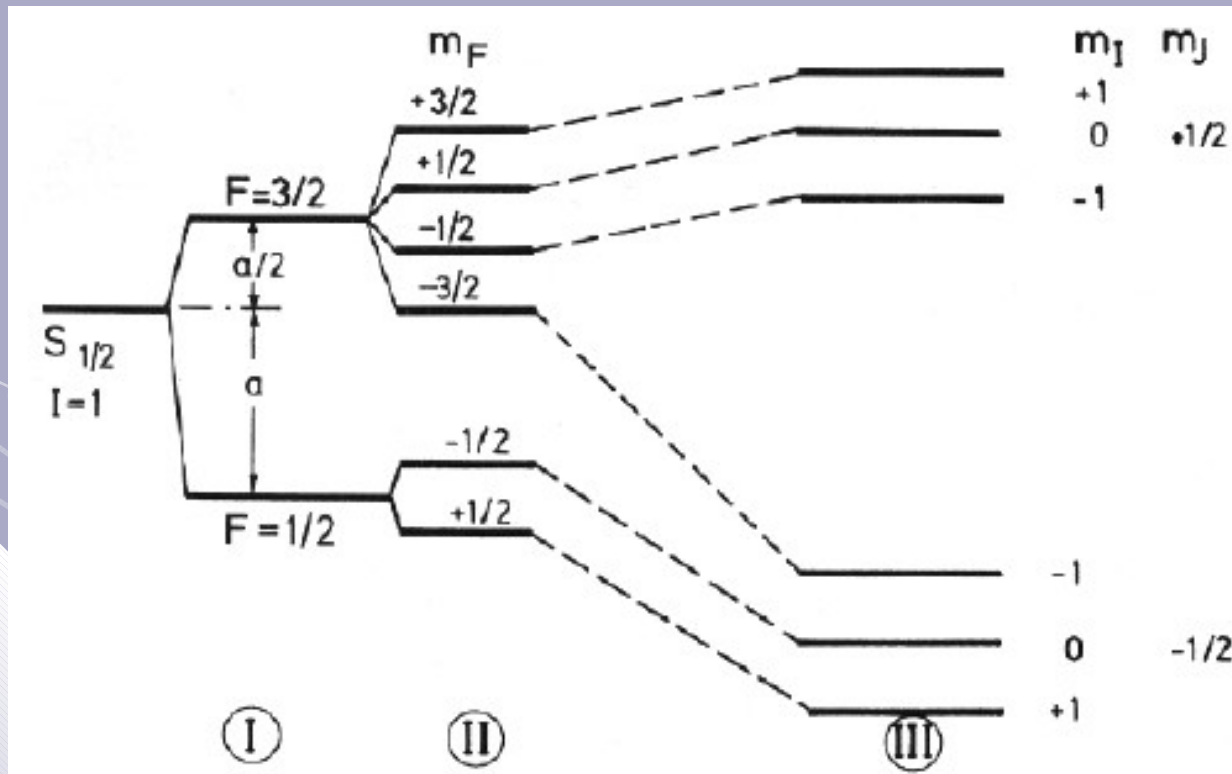


$\rightarrow F = J + I$  spin coupling

$m_F = -F, F-1, \dots, +F$

# Hyperfine structure

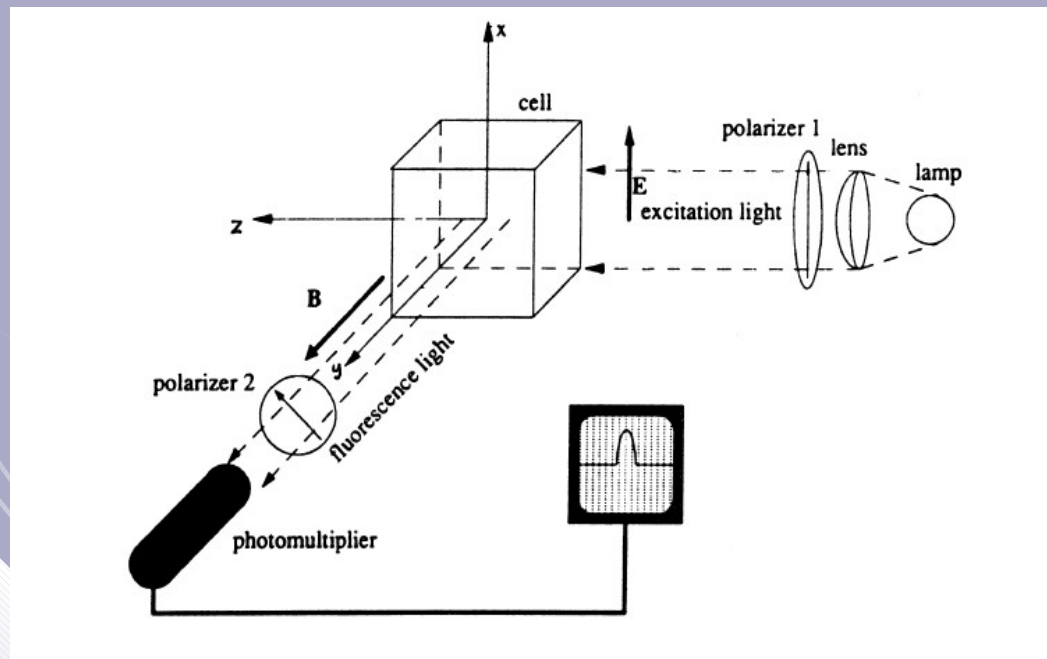
- for stronger fields  $\rightarrow$  lines in the Paschen-Back regime





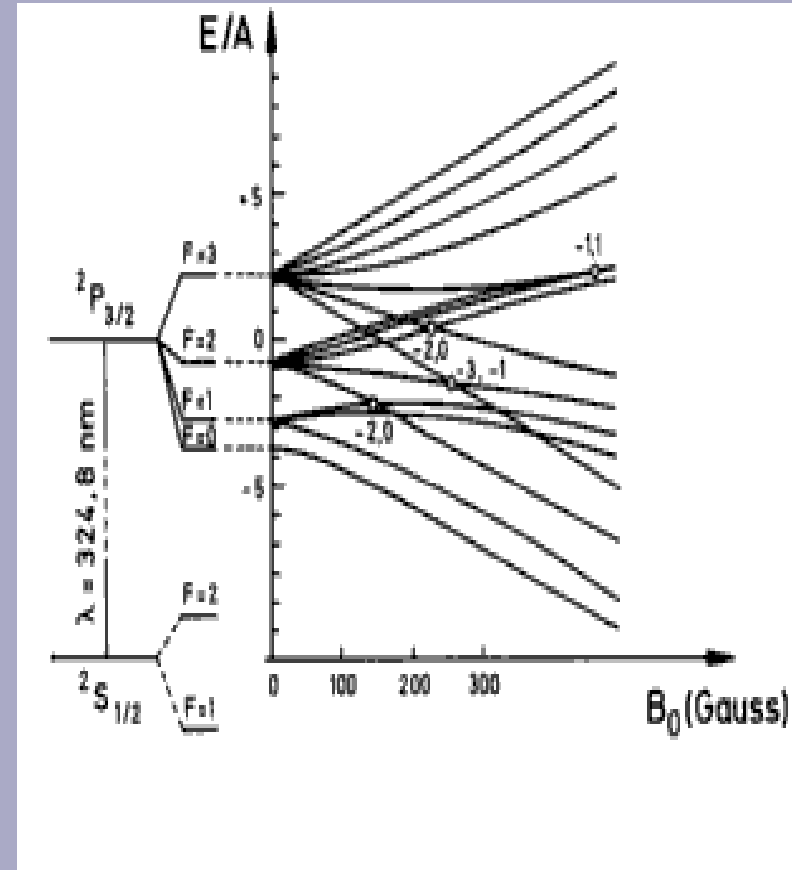
# Hanle effect

- classical explanation:  
mercury atoms in absorption-cell – polarized light excites  $e^-$   
damped oscillation until  $e^-$  is ground state (lifetime of upper state) –  
emits polarised light (fluorescence resonance) – depending on  
observing angle light is not always visible – if external  $B_0$  present  
==> intensity of fluorescence resonance changes (= **Hanle effect**)



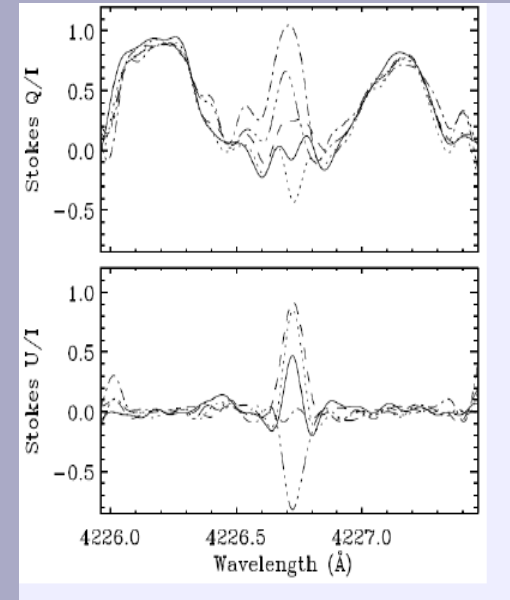
# Hanle effect

- quantum mechanical explanation:  
**level-crossing spectroscopy**
- a Zeeman sublevel (energy decreases with increasing  $B_0$ ) may cross with another magnetic sublevel from a lower state (energy increases with increasing magnetic field)
- if atoms excited coherently at fields corresponding to crossing points, changes of fluorescence intensity
- → “non-zero field level crossing” occurs in atoms that already show hyperfine structure without an external  $B_0$ .



# Hanle effect

- unpolarised light is scattered through  $\sim 90^\circ$   
scattered beam is linearly polarised  
perpendicular to scattering plane
- if scattering atom is in magnetic field  
 $J$  vector precesses with period of  $4\pi mc/eB$   
(Lamor frequency)
- if this frequency is comparable to the decay lifetime of upper  
state, the polarisation plane of re-emitted photon will be rotated  
from non-magnetic case
- Intensity of the fluorescent light proportional to  $B \Rightarrow$  allows  
detection of weak fields ( $<10\text{G}$ ) in situations with large-angle  
scattering



# Applications

