Hump-shape Uncertainty, Agency Costs and Aggregate Fluctuations*

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Abstract

Previously measured uncertainty shocks using the U.S. data show a hump-shape time path: Uncertainty rises for two years before its decline. Current literature on the effects uncertainty on macroeconomics, including housing, has not accounted for this observation. Consequently, the literature on uncertainty and macroeconomics is divided on the effects and the propagation mechanism of uncertainty on aggregate fluctuations. This paper shows that when uncertainty rises and falls over time, then the output displays hump-shape with short expansions that are followed by longer and persistent contractions. And because of these longer and persistent contractions in output, uncertainty is, on average, counter-cyclical. Our model builds on the literature combining uncertainty and financial constraints. We model the time path of uncertainty shocks to match empirical evidence in terms of shape, duration and magnitude. In our calibrated models, agents anticipate this hump-shape uncertainty time-path once a shock has occurred. Thereby, agents respond immediately by increasing investment (i.e. pre-cautionary savings), but face a substantial drop in investment, consumption and output as more uncertain times lie ahead. With persistent uncertain periods, both risk premia and bankruptcies increase which cause a further deterioration in investment opportunities. Besides, we show that accounting for hump-shape uncertainty measures can result in a large quantitative effect of uncertainty shock relative to previous literature.

Keywords: agency costs; credit channel; hump-shaped uncertainty shocks; time-varying uncertainty.

JEL Codes: E4, E5, E2

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1 Introduction

This paper combines uncertainty shocks that rise and fall over time with an agency cost model to provide a further explanation for the observed cyclical fluctuations in output and consumption in the U.S. We model uncertainty (i.e. risk) shocks, changes in the standard deviation around a constant mean, corresponding to the empirical work of Jurado, Ludvigson and Ng (2015) (*Macro Uncertainty*) and Ludvigson, Ma and Ng (2016) (*Financial Uncertainty*): We model the time path of uncertainty shocks to match empirical evidence in terms of shape, duration and magnitude. These previously measured uncertainty shocks using the U.S. data show a hump-shape time path: Uncertainty rises for two years before its decline. Current literature on the effects uncertainty on macroeconomics, including housing, has not accounted for this observation. Consequently, the literature on uncertainty and macroeconomics is divided on the effects and the propagation mechanism of uncertainty on aggregate fluctuations. The models examining the effects of uncertainty in the presence of financial constraints, such as Dorofeenko, Lee and Salyer (2008, henceforth DLS), Chugh (2016), Dmitriev and Hoddenbagh (2015) and Bachmann and Bayer (2013) find uncertainty shock plays quantitatively small role in explaining aggregate fluctuations. Whereas Christiano, Motto and Rostagno (2014), however, find the effect of uncertainty shock on aggregate variables is quantitatively large.[1] A common theme on all of these aforementioned literature on uncertainty, however, is that a risk shock is characterized by an immediate one time peak after the innovation (i.e. non-hump shape).

This paper shows that when uncertainty rises and falls over time, then the output displays hump-shape with short expansions that are followed by longer and persistent contractions. And because of these longer and persistent contractions in output, uncertainty is, on average, countercyclical. Our model builds on the literature combining uncertainty and financial constraints as in DLS and Bansal and Yaron (2004). Our first calibration exercise builds on DLS as a benchmark, and incorporates a modified Bansal and Yaron (2004) uncertainty structure while the second calibration exercise includes the preferences due to Greenwood, Hercovwitz and Huffman (1988). Our model's uncertainty propagation mechanism is, however, different from other models examining the effects of uncertainty in the presence of financial constraints. Unlike other studies that find immediate adverse effects of uncertainty on investment and output following uncertainty shocks that peak immediately after the innovation, we examine the impact of an unexpected shock that does not peak immediately but rises before it falls. In our calibrated models, agents anticipate this hump-shape uncertainty time-path

[1] Some other works that find a large uncertainty effect are Bloom (2009), Bloom, Alfaro and Lin (2016) and Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), and Leduc and Liu (2015). There are other works that find a mixed results such as Gilchrist, Sim, and Zakrajsek (2014), who find a small impact on output and consumption but a large impact on investment.

once a shock has occurred. Thereby, agents respond immediately by increasing investment (i.e. precautionary savings), but then substantially reduce investment and consumption (and thus output) as more uncertain times lie ahead. With persistent uncertain periods, both risk premia and bankruptcies increase which cause a further deterioration in investment opportunities. A hump shape time-varying uncertainty accounts for the majority of the variation in the credit channel variables, although the results are sensitive to the presence and the magnitude of agency costs. In the absence of agency costs, uncertainty shocks cause expansions because there are no adverse effects for households. However, in this case, the shocks do not explain any variation in real ($<1\%$ in output and consumption) and financial ($<3.5\%$ in the risk premium, the bankruptcy rate and the relative price of capital) variables. Conversely, the more severe the agency friction, i.e. the higher the monitoring costs associated with the friction, the more important uncertainty shocks are. We also show that accounting for hump-shape uncertainty measures can result in a large quantitative effect of uncertainty shock relative to previous literature. We find hump-shaped risk shocks account for 5% of the variation in output and 10% and 16% of the variation in consumption and investment, respectively. Finally, we also analyze the role of the relative risk aversion parameter and uncertainty. We find the relation between explained variation in output and consumption and uncertainty is monotonic - a higher coefficient of relative risk aversion is associated with higher precautionary savings, the associated initial expansion in output is greater and the subsequent contraction is not as severe.

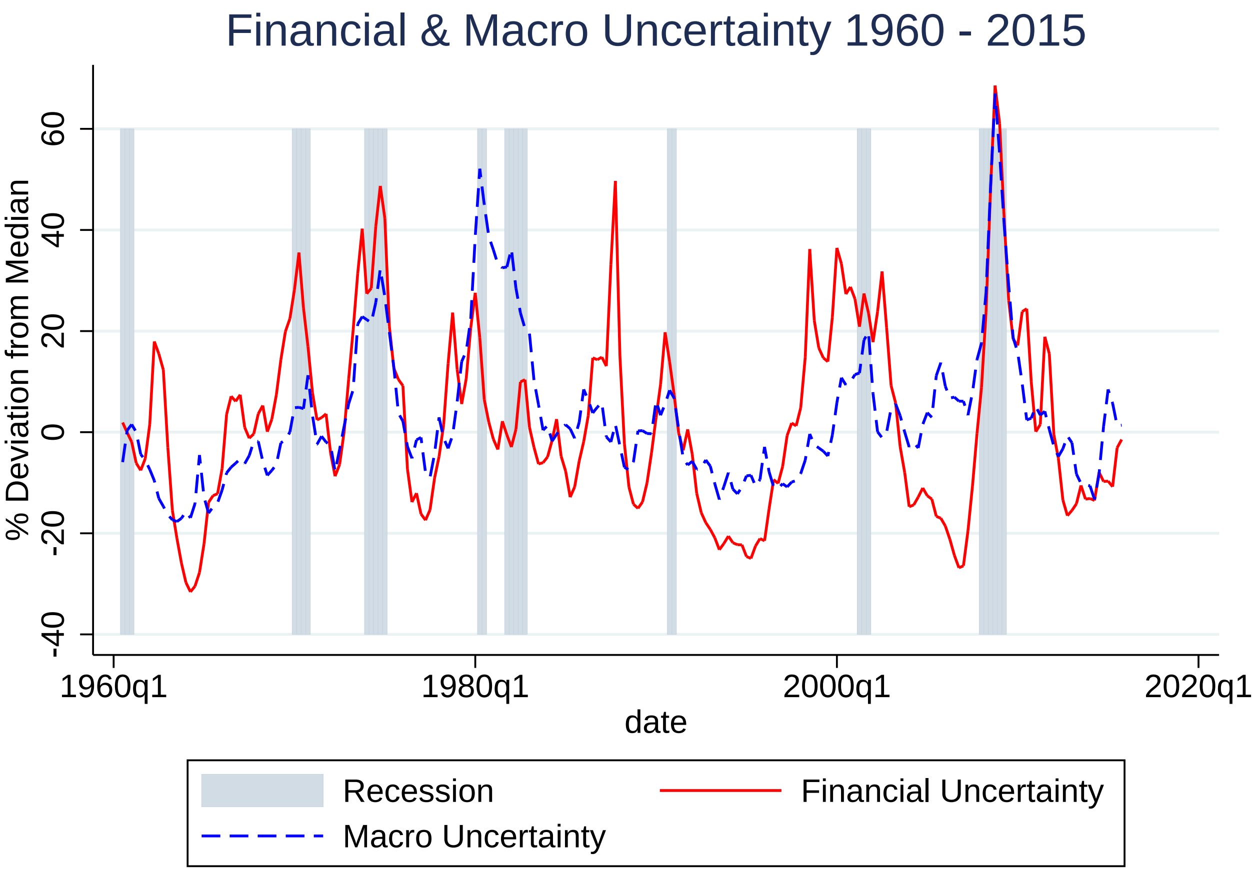
2 Motivation

2.1 Data

Figure 1 shows the *Financial Uncertainty* and *Macro Uncertainty* measures proposed by Jurado et al. (2015) and Ludvigson et al. (2016) from the period 1960 to 2015.

Uncertainty shocks as defined by Jurado et al. (2015) and Ludvigson et al. (2016) (i) raise between 30% and 73% relative to the median, (ii) exhibit a constant long-run mean and (iii) rise and fall over time with persistence. For example, during the Great Recession period, the *Financial Uncertainty* measure peaks after rising for 22 months (2006:12 - 2008:10) and peaks after rising for 11 months (2007:11 - 2008:10) after reaching the median during the great recession period. Other uncertainty shocks indicated by Ludvigson et al. (2015) peak after rising for 21 months in the late 1960s (relative to the median, from 1968:7 to the peak in 1970:4); for 26 months in the mid 1970s (1972:11 - 1975:1); for 23 months in the late 1970s (1978:4 - 1980:3); for 10 months in the mid 1980s (1986:3 - 1987:1) and

Figure 1: *Financial Uncertainty* and *Macro Uncertainty* measures from 1960 to 2015, expressed as percent deviation relative to the median.



Source: Jurado et al. (2015) and Ludvigson et al. (2016).

for 8 months in the early 1990s (1989:12 - 1990:8).[2] The *Macro Uncertainty* proxy rose (relative to the median) for 26 months (1972:10 - 1974:12), for 18 months (1978:11 - 1980:5) and for 17 months (2007:5 - 2008:10, with 2007:5, with the trough before the peak slightly above the median). Consequently, the *Financial Uncertainty* and *Macro Uncertainty* measures, depicted in Figure 1, strongly suggest that uncertainty is not characterized by jumps as in Bloom (2009) but these measured uncertainty shocks show a hump-shape time path.

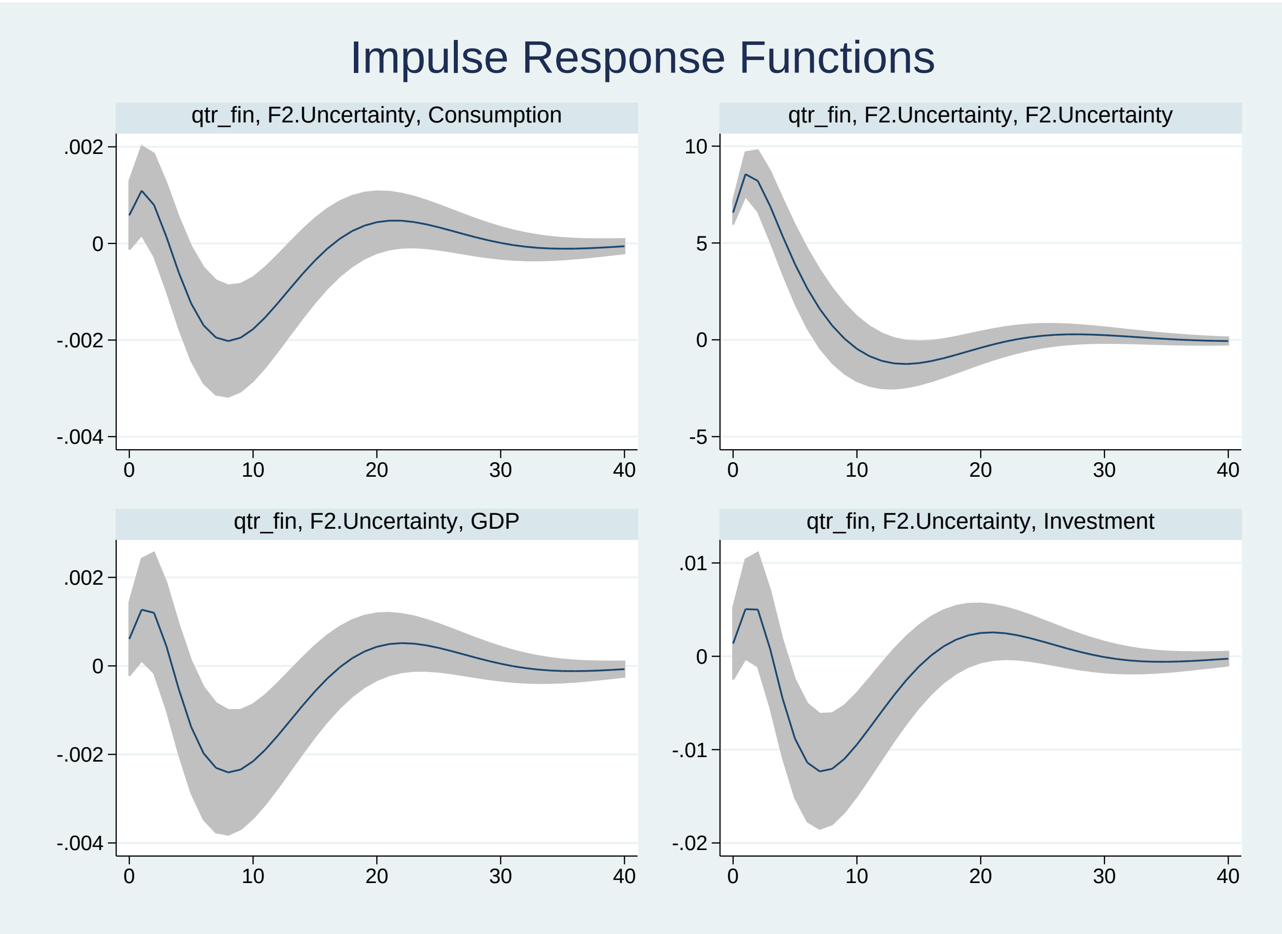
2.2 Empirical Evidence

To show corresponding hump-shapes for output, consumption and investment, we take a simplistic approach to examining the impact of uncertainty on these real variables, while avoiding a contemporaneous jump in uncertainty. We examine the impact of a shock to future uncertainty on today's

[2]On average, uncertainty peaks for these six shocks after increasing by 48.42%.

output, consumption, investment in a vector autoregression (VAR) model. In doing so, we thus ask, what is the impact on the variables of interest if the anticipated uncertainty is high in the future. We estimate the baseline specification of the VAR using data from 1960$Q3$ to 2013$Q4$ with two lags and the cyclical components of output, consumption and investment. Uncertainty is not HP-filtered and expressed as percentage deviation from the median. The results are highly similar if we use the cyclical component of HP-filtered uncertainty or the *Macro Uncertainty* measure. The vector of variables included in the VAR is given by $\begin{bmatrix} Uncertainty_{t+k} & GDP_t & Consumption_t & Investment_t \end{bmatrix}'$ with $k = 2$ in the baseline specification. Figure 2 shows the orthogonalized impulse response functions using this specification.

Figure 2: Impulse Response Functions of the VAR $\begin{bmatrix} Uncertainty_{t+2} & GDP_t & Consumption_t & Investment_t \end{bmatrix}'$ with two lags.



Note: Quarterly HP-filtered data from 1960 to 2015. Source: FRED and Ludvigson et al. (2016).

Financial Uncertainty induces hump-shaped responses in output, consumption and investment. However, as opposed to previous analyses, there are no immediate adverse effects if uncertainty is not restricted to jump unexpectedly from one period to another. Instead, a hump-shaped expansion precedes a pronounced contraction. These results are highly robust to different specifications and different orderings - as long as $2 \geq k \geq 8$, i.e. if uncertainty is high in the more distant future. If $k < 2$, the impulse responses show contractions in output, consumption and investment - in line with previous work that analyzes contemporaneous jumps in uncertainty.[3]

3 Model

Carlstrom and Fuerst (1997, henceforth CF) include capital-producing entrepreneurs, who default if they are not productive enough, into a real business cycles (RBC) model. In the CF framework, households and final-goods producing firms are identical and perfectly competitive. Households save by investing in a risk-neutral financial intermediary that extends loans to entrepreneurs. Entrepreneurs are heterogeneous produce capital using an idiosyncratic and stochastic technology with constant volatility. Unlike CF, DLS introduce stochastic shocks to the volatility (uncertainty shocks) of entrepreneurs' technology, such that uncertainty jumps to its peak and converges back to its steady state. While this approach remedies the procyclical bankruptcy rates following TFP shocks it introduces countercyclical bankruptcy rates, DLS is at odds with the measures from Jurado et al. (2015) and Ludvigson et al. (2016). In this paper, we alter the time path and the magnitude of the shocks introduced in DLS, such that they correspond more closely to the *Macro Uncertainty* and *Financial Uncertainty* measures. Following these changes, the model displays procyclical consumption, precautionary savings and an increase in output following an initial drop. Our model therefore explains the puzzling absence of precautionary savings following uncertainty shocks in the literature, as raised by Bloom (2014).

In the CF framework, the conversion of investment to capital is not one-to-one because heterogeneous entrepreneurs produce capital using idiosyncratic and stochastic technology. If a capital-producing firm realizes a low technology shock, it declares bankruptcy and the financial intermediary takes over production after paying monitoring costs. The timing of events in the model is as follows:

1. The exogenous state vector of technology and uncertainty shocks, denoted $(A_t, \sigma_{\omega,t})$, is realized.
2. Firms hire inputs of labor and capital from households and entrepreneurs and produce the final good output via a Cobb-Douglas production function.
3. Households make their labor, consumption, and investment decisions. For each unit of investment, the household transfers q_t units of the consumption goods to the banking sector.

[3]These results are robust to different lag lengths of the VAR. In a second specification, we also include lagged delinquency rates on business loans as a proxy for bankruptcies. The impulse response function of delinquencies is hump-shaped while the responses of output consumption and investment are highly similar for the second specification.

4. With the savings resources from households, the banking sector provide loans to entrepreneurs via the optimal financial contract (described below). The contract is defined by the size of the loan, i_t, and a cutoff level of productivity for the entrepreneurs' technology shock, $\bar{\omega}_t$.

5. Entrepreneurs use their net worth and loans from the banking sector to purchase the factors for capital production. The quantity of investment is determined and paid for before the idiosyncratic technology shock is known.

6. The idiosyncratic technology shock of each entrepreneur $\omega_{j,t}$ is realized. If $\omega_{j,t} \geq \bar{\omega}_t$ the entrepreneur is solvent and the loan from the bank is repaid; otherwise the entrepreneur declares bankruptcy and production is monitored by the bank at a cost proportional to the input, μi_t.

7. Solvent entrepreneur's sell their remaining capital output to the bank sector and use this income to purchase consumption c_t^e and (entrepreneurial) capital z_t. The latter will in part determine their net worth n_t in the following period.

3.1 The Impact of Uncertainty Shocks: Partial Equilibrium

The optimal contract is given by the combination of i_t and $\bar{\omega}_t$ that maximizes entrepreneurs' return subject to participating intermediaries. Financial intermediaries make zero profits due to free entry

$$\max_{i_t, \bar{\omega}_t} q_t i_t f(\bar{\omega}_t; \sigma_{\omega,t}) \tag{1}$$

subject to

$$q_t i_t g(\bar{\omega}_t; \sigma_{\omega,t}) \geq i_t - n_t. \tag{2}$$

Net worth is defined as

$$n_t = w_t^e + z_t(r_t + q_t(1 - \delta(u_t))), \tag{3}$$

Entrepreneurs' share of the expected net capital output is

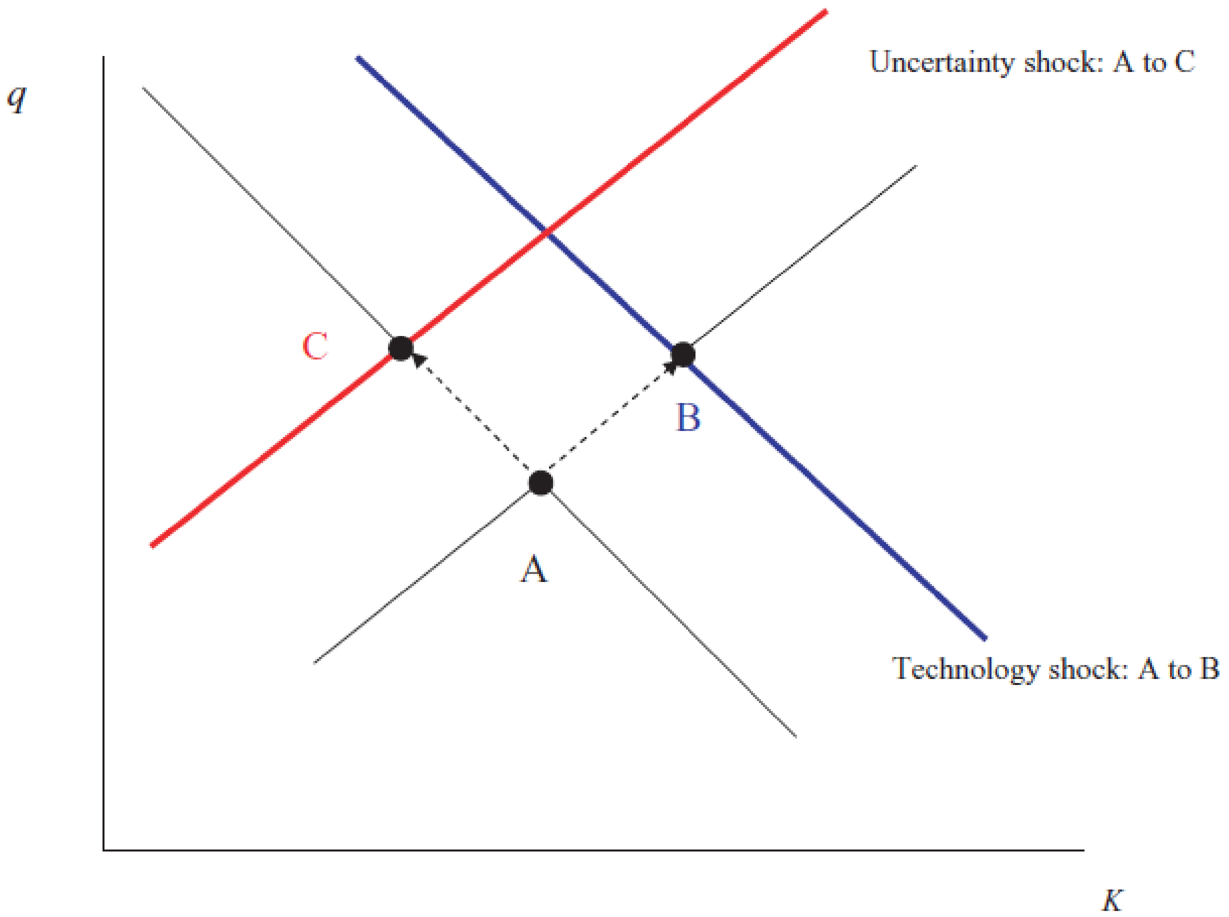
$$f(\bar{\omega}_t; \sigma_{\omega,t}) = \int_{\bar{\omega}_t}^{\infty} \omega \tilde{\phi}(\bar{\omega}_t; \sigma_{\omega,t}) d\omega - [1 - \tilde{\Phi}(\bar{\omega}_t; \sigma_{\omega,t})]\bar{\omega}_t \tag{4}$$

and the lenders' share of expected net capital output

$$g(\bar{\omega}_t; \sigma_{\omega,t}) = \int_0^{\bar{\omega}_t} \omega \tilde{\phi}(\bar{\omega}_t; \sigma_{\omega,t}) d\omega + [1 - \tilde{\Phi}(\bar{\omega}_t; \sigma_{\omega,t})]\bar{\omega}_t - \tilde{\Phi}(\bar{\omega}_t; \sigma_{\omega,t})\mu. \tag{5}$$

To understand the impact of an uncertainty shock, consider the uncertainty shock in partial equilibrium. For this analysis, q and n are assumed to be fixed while i and $\overline{\omega}$ are chosen. In this setting, uncertainty shocks adversely affect the supply of investment as follows. As σ_ω increases, the default threshold $\overline{\omega}$ and lenders' expected return fall. From the incentive compatibility constraint of entrepreneurs' problem (1), it can be seen that investment has to fall. The effect of an uncertainty shock is summarized graphically, and contrasted with an aggregate technology shock, in Figure 3 (taken from DLS).

Figure 3: The partial equilibrium impact of an uncertainty shock.



Note: Uncertainty adversely affects capital supply, in contrast to TFP shocks that affect capital demand. Source: DLS.

Whether these results carry over in general equilibrium depends on how the shock is modeled. They are not overturned following a jump in uncertainty, as analyzed in DLS, or if uncertainty reaches its peak quickly. In this case, bankruptcies, the associated agency costs, the risk premium and the price of capital increase. The return to investing falls, saving/investing is less attractive, so investment and output drop while households substitute into consumption. These results are overturned, however, following a shock that is hump-shaped if the peak is sufficiently far in the future.

3.2 Modeling Hump-Shaped Uncertainty Shocks

We allow for humps in uncertainty by modifying a subset of equations due to Bansal and Yaron (2004), such that a latent x_t variable affects $\sigma_{\omega,t}$:

$$\log(\sigma_{\omega,t+1}) = (1-\rho_{\sigma_\omega})\log(\bar{\sigma}_\omega) + \rho_{\sigma_\omega}\log(\sigma_{\omega,t}) + \widetilde{\varepsilon}_{t+1} \quad (6)$$

$$\widetilde{\varepsilon}_{t+1} = \varphi_\sigma \varepsilon_{\sigma,t+1} + x_{t+1} \quad (7)$$

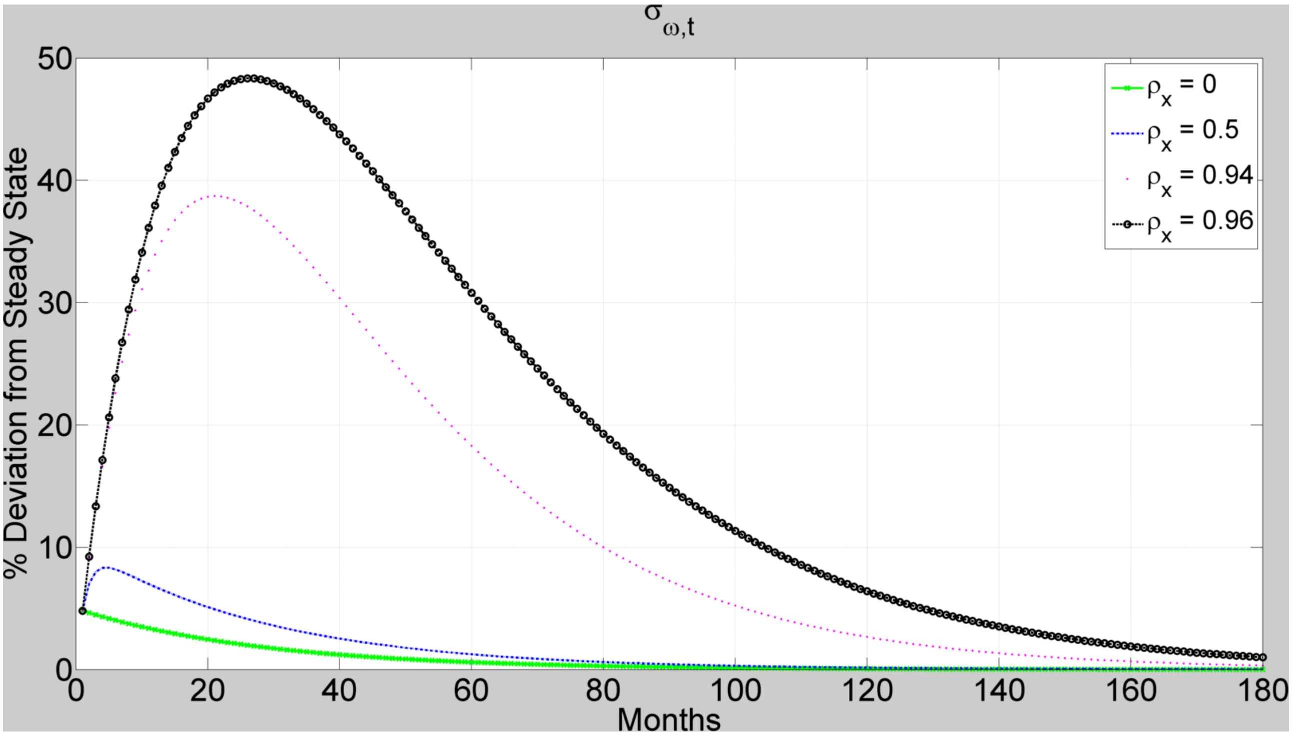
$$x_{t+1} = \rho_x x_t + \varphi_x \varepsilon_{x,t+1} \quad (8)$$

$$\varepsilon_{x,t}, \varepsilon_{\sigma,t} \overset{i.i.d.}{\sim} N(0,1), \rho_\sigma, \rho_x \in [0,1). \quad (9)$$

$\widetilde{\varepsilon}_{t+1}$ is a composite term that enables uncertainty to jump (innovations in the first term $\varphi_{\sigma,t+1}\varepsilon_{\sigma,t+1}$), as in DLS, or increase over time corresponding to the empirical proxies (innovations via the latent variable x_{t+1}). Figure 4 plots the time series of $\sigma_{\omega,t}$ using different persistence parameters $\rho_x = [0, 0.5, 0.94, 0.96]$. The horizontal axis measures time in monthly periods, while the vertical axis shows the percentage deviation from the steady state. Setting $\rho_x = 0$ induces a jump in uncertainty, as analyzed in DLS. The larger ρ_x, the more pronounced the hump in $\sigma_{\omega,t}$ and the longer uncertainty rises before it peaks.

In the benchmark case with $\rho_x = .96$, uncertainty peaks after rising for 25 months, corresponding to the empirical evidence. We match the innovation relative to the steady state using the average increase of an uncertainty shock relative to the long-run mean: We set $\varphi_x = 0.048$ such that $\sigma_{\omega,t}$ increases by 48% relative to the steady state. Our 48% relative increase compares with previous papers as follows. In Bloom (2009) and Bloom, Alfaro and Lin (2016), who use two-state Markov chains to examine the impact of uncertainty, σ_ω increases by 100%; in Bloom, Floetotto, Jaimovich, Saporta-Eksten and Terry (2012), σ_ω increases between 91% and 330%. Leduc and Liu (2015) introduce an increase of 39.2% relative to the steady state. Christiano, Motto and Rostagno (2014), use a combination of un- and anticipated innovations over a sequence of eight quarters, and their magnitude of these innovations is between 2.83% and 10% per period. DLS use a 1% innovation, Chugh (2016) and Bachmann and Bayer (2013) examine increases of about 4%, while Dmitriev and Hoddenbagh (2015) use a 3% innovation. These differences are partially driven by differences in measurement; see also Strobel (2015). Not surprisingly, greater innovations in uncertainty are associated with a greater role of uncertainty in terms of variation explained. One special case is the model of Christiano et al. (2014) who introduce a news component to their shocks: Sims (2015) points out potential issues in using *news* and variance decompositions. Lee, Salyer and Strobel (2016) show that the *news* component plays a prominent role regarding the importance of uncertainty.

Figure 4: Modeling Uncertainty Shocks using different persistence parameters.



Note: The horizontal axis shows monthly periods, while the vertical axis shows the percentage deviation from the steady state. The case with $\rho_x = 0$ corresponds to a jump in uncertainty as analyzed in DLS. The higher ρ_x, the more pronounced the hump in uncertainty. In the benchmark case with $\rho_x = .96$, σ_ω peaks after rising 25 months, corresponding to the empirical evidence. ρ_{σ_ω} is set to $0.9^{1/3}$.

3.3 The Impact of Uncertainty Shocks: General Equilibrium

In order to unambiguously identify the change in the impact of a risk shock that is due to its hump-shape, we insert the shock described in the previous section in a framework identical to DLS. For this reason, the model's exposition is confined to the agents' optimization problems. The representative household's objective is to maximize expected utility by choosing consumption c_t, labor h_t and savings k_{t+1}, i.e.

$$\max_{\{c_t,k_{t+1},h_t\}_0^\infty} E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(c_t) + \nu(1-h_t) \right] \tag{10}$$

subject to

$$w_t h_t + r_t k_t \geq c_t + q_t i_t \tag{11}$$

$$k_{t+1} = (1-\delta)k_t + i_t \tag{12}$$

with w_t the wage and r_t the rental rate of capital. These are equal to their marginal products, as the representative final-good's producing firm faces a standard, static profit maximization problem[4]

$$\max_{K_t, H_t, H_t^e} A_t K_t^{\alpha_K} H_t^{\alpha_H} (H_t^e)^{1-\alpha_K-\alpha_H} - r_t K_t - w_t H_t - w_t^e H_t^e, \tag{13}$$

with $K_t = k_t/\eta$ and $H_t = (1-\eta)h_t$, where η represents the fraction of entrepreneurs in the economy. Total Factor Productivity (TFP) A_t follows an autoregressive process of order one in logs,

$$\log(A_{t+1}) = \rho_A \log(A_t) + \varphi_A \varepsilon_{A,t+1} \tag{14}$$

with $\varepsilon_{A,t} \overset{i.i.d.}{\sim} N(0,1)$. The problem of entrepreneurs is given by

$$\max_{\{c_t^e, z_{t+1}\}_0^\infty} E_0 \sum_{t=0}^{\infty} (\gamma\beta)^t c_t^e \tag{15}$$

subject to

$$\begin{aligned} n_t &= w_t^e + z_t(r_t + q_t(1-\delta)) & (16)\\ z_{t+1} &= n_t\left[\frac{f(\bar{\omega}_t, \sigma_{\omega,t})}{1 - q_t g(\bar{\omega}_t, \sigma_{\omega,t})}\right] - \frac{c_t^e}{q_t}. & (17) \end{aligned}$$

The entrepeurs are risk neutral and supply one unit of labor inelastically. Their net worth is defined by sum of labor income w_t^e, the income from capital $z_t r_t$ plus the remaining capital $z_t q_t(1-\delta)$. At the end of a period, entrepreneurial consumption is financed out of the returns from the investment project, which implies the law of motion (17). As the equilibrium conditions are described in DLS, we will not list them in this section.

3.4 Calibration

We calibrate the model for the monthly frequency. Otherwise, the frequency of uncertainty would be too low relative to the empirical counterparts. Table 1 shows the benchmark calibration of the key parameters. The household's monthly discount rate of 0.9975 implies an annual risk free rate of about 3%. Following DLS, we set $\overline{\sigma}_\omega = 0.207$, which implies an annual risk premium of 1.98%. The slight increase in the risk premium, which is 1.87% in DLS, is due to changes associated with the monthly calibration. The default threshold ϖ targets an annual bankruptcy rate of 3.90%, as in DLS.

[4]When solving the model, we follow DLS and assume the share of entrepreneurs labor $(1-\alpha_K-\alpha_H)$ is approximately zero.

Table 1: Benchmark calibration for the monthly frequency.

Parameter	Function	Value	Rationale / Source (see also discussion in the text)
β	Discount rate	0.9975	Monthly calibration
α	Capital's share of production	0.36	DLS
μ	Monitoring costs	0.25	DLS
δ	Depreciation rate	0.2/3	DLS
$\overline{\sigma}_\omega$	Steady state uncertainty	0.207	Steady State Risk Premium
ρ_{σ_ω}	Persistence parameter uncertainty	$0.9^{1/3}$	DLS
ρ_x	Persistence parameter hump component	0.96	Jurado et al (2015), Ludvigson et al (2016)
φ_σ	Innovation in uncertainty (jump)	0.01	DLS
φ_x	Innovation in uncertainty (hump)	0.048	Jurado et al (2015), Ludvigson et al (2016)
ϖ	Steady state default threshold	0.557	Steady State Bankruptcy Rate

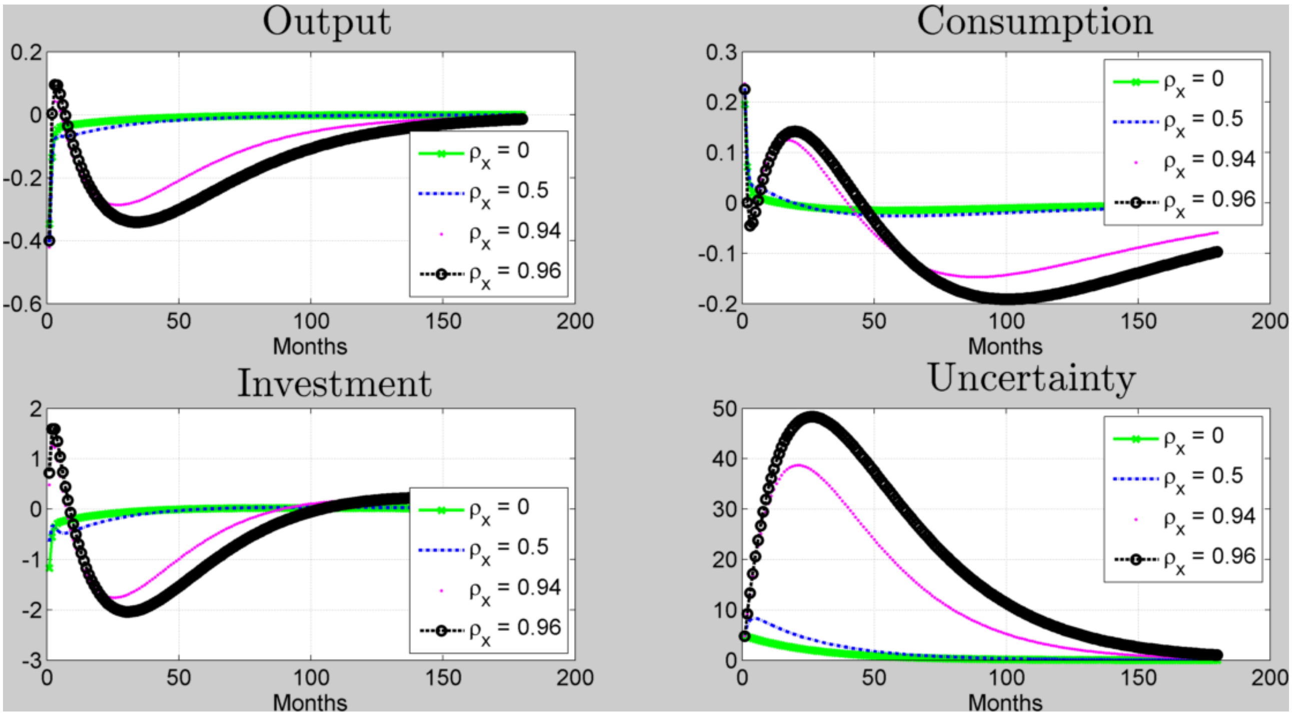
3.5 Cyclical Behaviour

Because of the assumption on entrepreneurs' productivity, first order approximation of the equilibrium conditions does not impose certainty equivalence. Instead, uncertainty (time-varying second moment) appears in the policy function as a state variable. Figures 5 and 6 show the impulse response functions following jumps and humps in uncertainty, i.e. the impulse response function for different values of $\rho_x = [0, 0.5, 0.94, 0.96]$.

If $\rho_x = 0$, uncertainty jumps to its peak and an immediate drop in investment and output ensues, which is expected from the partial equilibrium analysis. Household consumption counterfactually increases as households substitute into consumption. The larger ρ_x, the longer the shock takes to peak. Interestingly, there is a threshold value of ρ_x that is necessary to induce prevautionary savings. For instance, $\rho_x = 0.5$ is insufficient to overcome the partial equilibrium results and to induce precautionary savings. However, values of ρ_x corresponding to the uncertainty proxies overturn the partial equilibrium results: An uncertainty shocks is followed by an increase in investment and a hump-shape response of output following the initial drop. Moreover, in line with the data, consumption is procyclical. The intuition is that immediately after the shock, agency costs are still moderate relative to future periods so investment demand increases, which raises output following the initial drop. While households also substitute into consumption, entrepreneurs greatly reduce consumption after an uncertainty shock because of the increase probability of default and because the higher price of capital results in an increase in investment. The intuition of the model can also be seen in the context of the agency friction. Figure 7 shows the impulse responses for different values of μ.

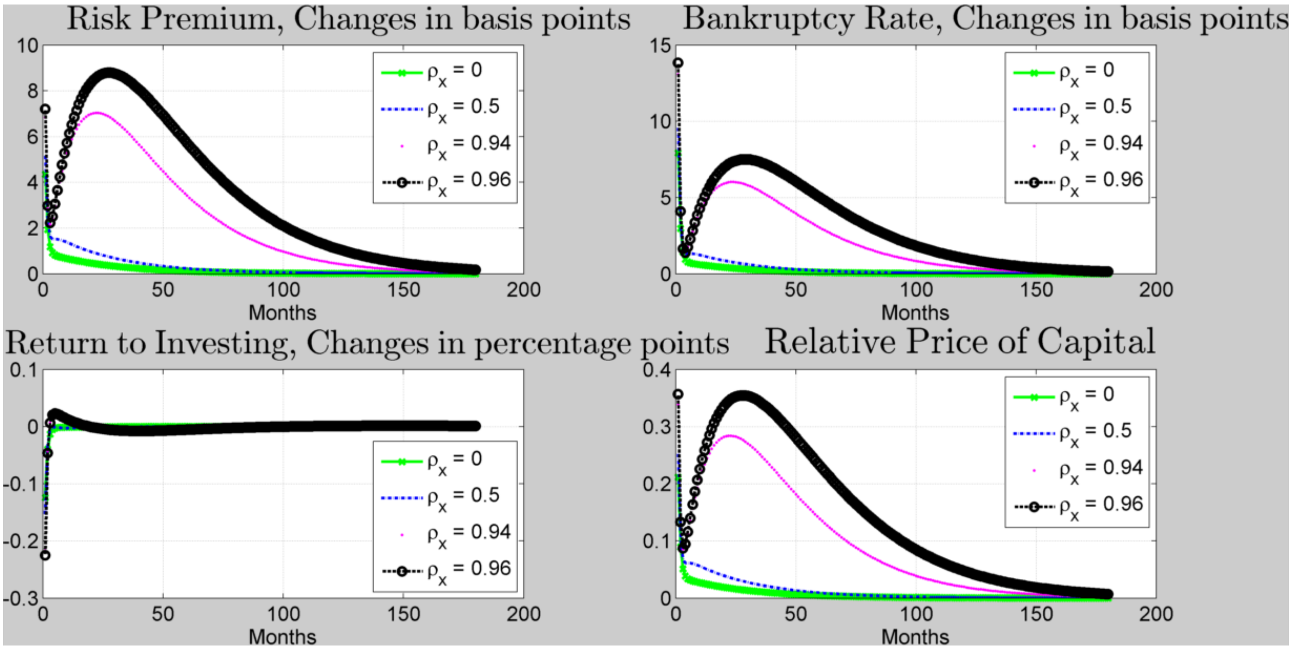
Without agency friction, $\mu = 0$ (actually, for computational reasons, $\mu = 0.0001$), the relative

Figure 5: Impulse responses of output, household consumption and investment following an uncertainty shock for different persistence parameters, $\rho_x = [0, 0.5, 0.96, 0.979]$.



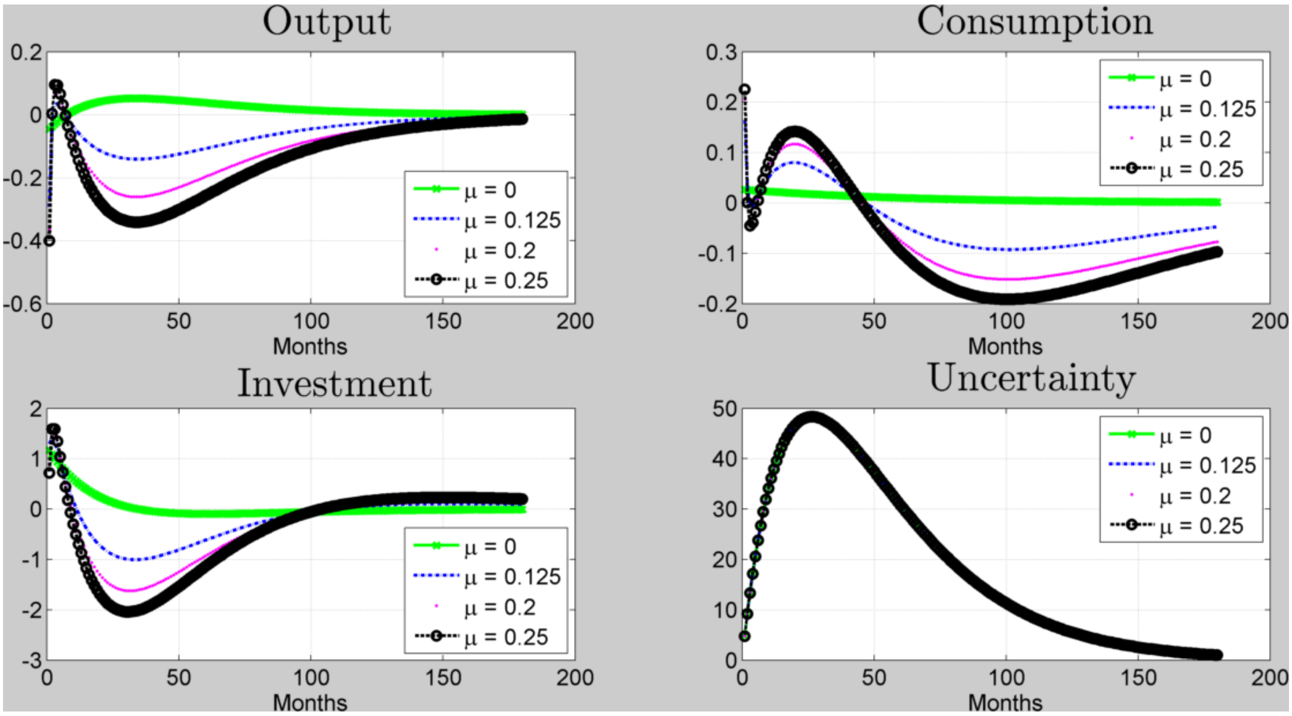
Note: The horizontal axis shows monthly periods, the vertical axis shows the percentage deviation from the steady state.

Figure 6: Impulse responses of the risk premium, the bankruptcy rate, return to investment and the relative price of capital following an uncertainty shock for different persistence parameters, $\rho_x = [0, 0.5, 0.96, 0.979]$.



Note: The horizontal axis shows monthly periods, the vertical axis shows the percentage deviation from the steady state, unless indicated otherwise.

Figure 7: Impulse responses of output, consumption and investment following an uncertainty shock for different magnitudes of monitoring costs, μ=[0, 0.125, 0.2, 0.25].



Note: The horizontal axis shows monthly periods, the vertical axis shows the percentage deviation from the steady state.

price of capital q_t is unity. An uncertainty shock, then, induces an expansion in output, consumption and investment because there are no adverse effects for households. Although the bankruptcy rate increases, there are no adverse effects. Instead, households benefit from a more productive investment opportunity. If there are agency costs ($\mu > 0$), the relative price of capital q_t increases, as well as risk premia and bankruptcy rates. In this case, households incur adverse effects of bankruptcies because of the monitoring costs $\tilde{\Phi}(\mu_\omega, \sigma_\omega)\mu$. Unlike shocks that jump, however, hump-shaped shocks increase investment by around 1.7% relative to the steady state, despite the price-increase associated with the agency friction - overturning the partial equilibrium effects. Since expectations are rational, households know that uncertain times of relatively poor investment opportunities are ahead, so they substantially increase saving as soon as they learn about the shock. As shown in Figure 7, the magnitude of the initial increase is inversely related to the size of μ - the higher the monitoring costs, the more capital is destroyed. Without agency friction, consumption does not increase by much in order to invest more. With agency friction and adverse effects of uncertainty for the households, consumption increases the more the greater μ. The initial increase in investment leads to an expansion in output. However, the subsequent deterioration of conditions in the credit channel leads to a drop in investment and to a contraction in output.

Table 2 presents the model's correlation coefficients. The model produces procyclical consumption

Table 2: Correlation coefficients of consumption, investment, bankruptcy rate and uncertainty with output.

	Correlation with y			
Uncertainty Shock	c	i	BR	σ_ω
Jump	-0.72	0.95	-0.99	-0.69
Hump	**0.19**	0.88	-0.97	-0.95

Note: BR refers to the bankruptcy rate. The autocorrelation coefficient of uncertainty is $\rho_{\sigma_\omega} = 0.9^{1/3}$, as in DLS, while uncertainty peaks after rising for 25 months, i.e. $\rho_x = 0.96$.

and investment if uncertainty is not restricted to jump; the degree of procyclicality of consumption depends on the persistence of the latent variable, i.e. on how long the shock takes to peak. The bankruptcy rate and uncertainty are strongly countercyclical for both types of risk shocks.

4 GHH Preferences and Variable Capital Utilization

The previous section uses the framework of DLS to emphasize the impact of hump-shaped uncertainty shocks: precautionary savings, a hump-shape response of output with a short expansion that is followed by a longer and persistent contraction as well as mildly procyclical consumption. However, the initial drop in output that precedes the short expansion is much stronger compared to the VAR evidence, while the procyclicality of consumption is senstive to the persistence parameter of the latent variable. To remedy these features, we modify the model by using the preferences due to Greenwood, Hercowitz and Huffman (1988), to eliminate effects of labor supply due to changes in consumption, and allow for variable capital utilization. The representative household thus chooses the capital utilization rate u_t, which is impacts the depretation rate $\delta(u_t)$, with $\delta'(u_t), \delta''(u_t) > 0$. The problem is given by

$$\max_{\{c_t, k_{t+1}, u_t, h_t\}_0^\infty} E_0 \sum_{t=0}^{\infty} \beta^t (1-\iota)^{-1} \big[c_t - \chi \frac{h_t^{1+\theta}}{(1+\theta)}\big]^{1-\iota} \tag{18}$$

subject to

$$\begin{aligned} w_t h_t + r_t(u_t k_t) &\geq c_t + q_t i_t && (19) \\ k_{t+1} &= (1-\delta(u_t))k_t + i_t && (20) \\ \delta(u_t) &= \delta_0 + \delta_1(u_t - 1) + \frac{\delta_2}{2}(u_t - 1)^2. && (21) \end{aligned}$$

The coefficient of relative risk aversion is given by ι, while $1/\theta$ corresponds to the intertemporal elasticity of substitution in labor supply. χ is the relative importance of leisure. The problem of the

Table 3: Benchmark calibration for the monthly frequency with GHH preferences and variable capital utilization.

Parameter	Function	Value	Rationale / Source (see also discussion in the text)
ι	Coefficient of relative risk aversion	1	Greenwood et al. (1988)
$1/\theta$	Intertemporal elasticity of substitution in labor supply	0.8	Greenwood et al. (1988)
χ	Relative importance of leisure	9.8930	Household works 1/3 of his time
δ_0	Steady state rate of capital depreciation	0.02/3	DLS
δ_1	Normalize steady state capital utilization	0.0108	Capital utilization is unity in steady state
δ_2	Sensitivity of capital utilization	0.2	Schmitt-Grohe and Uribe (2008)*

*Schmitt-Grohe and Uribe (2008) estimate $\delta_2 = 0.11$ but with a relatively large standard error of 0.26. We set δ_2 slightly higher to restrict capital utilization a bit more given the monthly calibration.

final-goods' producing firms is given by

$$\max_{u_t K_t, H_t, H_t^e} A_t (u_t K_t)^{\alpha_K} H_t^{\alpha_H} (H_t^e)^{1-\alpha_K-\alpha_H} - r_t(u_t K_t) - w_t H_t - w_t^e H_t^e. \tag{22}$$

The problem of the entrepreneurs and the optimal contract remain unchanged for the most part, except for the depreciation rate, $\delta(u_t)$. The calibration of the additional parameters is standard and displayed in Table 3. The set of equations determining the equilibrium properties are displayed in the Appendix.

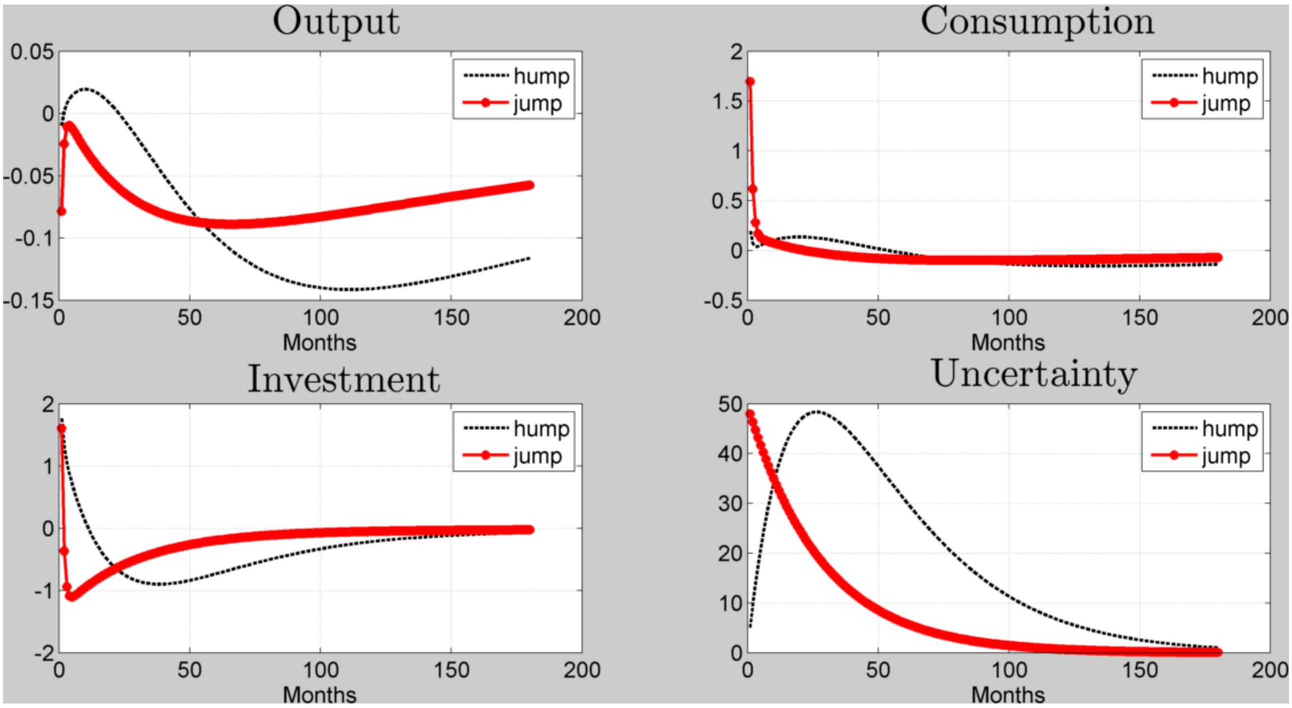
4.1 Cyclical Behaviour

While the previous section examined an identical economy as DLS, we now consider the impact of the different types of uncertainty shock using the preferences due to Greenwood et al. (1988) to remedy the shortcomings discussed above. We examine the impact on output, investment and household consumption following an innovation that is comparable in magnitude (a 48% innovation). Consider first the impact of the hump-shaped shock displayed in Figure 8. The results are quite robust, although the initial adverse impact on output is much smaller while the brief ensuing expansion is (relative to the DLS framework) more pronounced and persistent. However, as shown in Table 4 below, the procyclicality of consumption is not senstive anymore to uncertainty's time to peak. In contrast, the results change following a jump in uncertainty. Most notably, while output drops as expected, investment increases in the first period, mitigating the drop in output, but then drops persistently below its steady state and output falls again. This is not due to precautionary savings, however. If there is a jump in uncertainty, the amount of capital destroyed immediately after the shock is much larger compared to a hump-shaped shock (total costs of default following a jump are more than twice as large after the first two years, which also shows in the bankruptcy rate in Figure 9).[5] Labor, however,

[5]Not surprisingly, the marginal product of capital increases following a jump while it initially drops (and is positive only in later periods) following a hump.

is not substituted for capital (labor perfectly comoves with output given the GHH preferences) but the capital stock is simply replaced. This is also reflected by the marginal product of capital, which increases following a jump in uncertainty whereas it falls following a hump in uncertainty.

Figure 8: Impulse responses of output, household consumption and investment following a shock (jump and hump) to uncertainty.



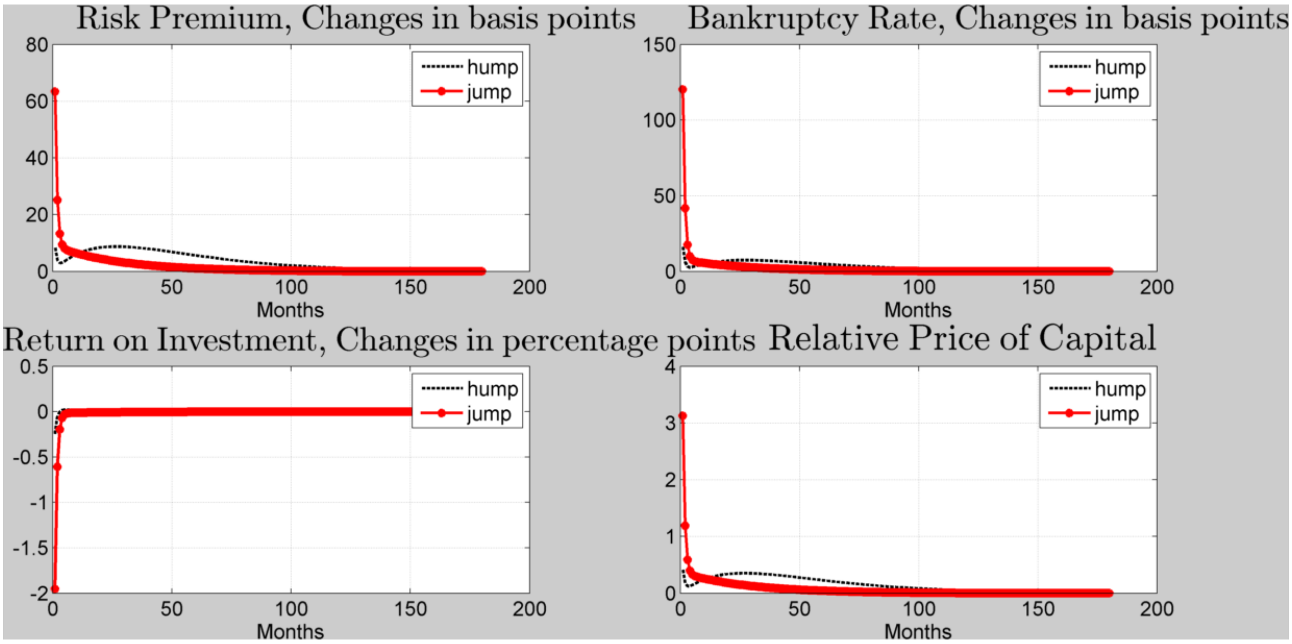
Note: Uncertainty increases by 48% for both types of uncertainty shock. The horizontal axis shows monthly periods, the vertical axis shows the percentage deviation from the steady state.

Table 4 presents a further analysis of the equilibrium characteristics. The model produces procyclical consumption and investment for TFP and risk shocks. As discussed above, this is partially due to the GHH preferences, which are also the reason for perfect comovement of labor and output. As opposed to Carlstrom and Fuerst (1997), the bankruptcy rate is countercyclical for both types of uncertainty shocks. This similarity conceals the fact that following a hump-shaped uncertainty shock, the bankruptcy rate is initially procyclical and becomes countercyclical only later on. The reason is that uncertainty starts to rise while output still expands due to the initial increase in investment. While uncertainty rises, investment and output decrease. In contrast, if uncertainty jumps to its peak, output immediately decreases.[6] Similary, although the correlation between output and uncertainty for the two shocks is not that different, the dynamics are following a shock are.

The correlation of the bankruptcy rate and output implied by the model is considerably higher

[6]Note also that the hump-shaped movements in bankruptcy rates observed in the data are absent if uncertainty moves from steady state to peak from one period to another - simply because of the time path of $\sigma_{\omega,t}$ and the log-normality assumption of entrepreneurs' productivity. Conversely, they are, to a large extent, present by construction following a hump-shaped shock.

Figure 9: Lending channel variables following a shock (jump and hump) to uncertainty.



Note: Uncertainty increases by 48% for both types of uncertainty shock. The horizontal axis shows monthly periods, the vertical axis shows the percentage deviation from the steady state, unless indicated otherwise.

compared to the data, while the correlation between output and consumption is fairly close to the data for hump-shaped uncertainty shocks. The correlation between output and investment is considerably lower compared to the data because investment is the main driver of the dynamics and leads output. The relative volatilities implied by both types of uncertainty shock are too high for consumption, investment and uncertainty and much lower for hours and bankruptcies, compared to the data. Thus, even though uncertainty, in this model, accounts for the majority of the variation in bankruptcies, uncertainty shocks are not sufficient to explain the observed relative variation. Considering the simplicity of the model and that uncertainty on its own is an unlikely source of bankruptcies, this finding is not too surprising.

Table 5 shows the relation between the coefficient of relative risk aversion ι and the role of uncertainty, which is (inversely) monotonically related. The higher ι,the more agents save as a precaution. The associated initial expansion in output is therefore also greater the higher ι, and the subsequent contraction is not as severe; the volatility of consumption is much smaller the higher ι, and for very high values of ι consumption negatively deviates from the steady state. For this reason, the (unconditional) variation in output, consumption and investment due to the hump-shape shock is the lower the more risk-averse households are - the initial impact is larger but subsequently the overall variation is diminished.

Finally, to assess the relative importance of TFP and uncertainty in the context of the agency

Table 4: Business Cycle Characteristics.

		Volatility relative to $\sigma(y)$				*Correlation with* y			
Shock	$\sigma(y)$	c	i	BR	σ_ω	c	i	BR	σ_ω
TFP	0.20	0.65	4.66	0.003	-	0.96	0.93	0.23	-
Risk Jump	0.00046	1.48	11.60	0.42	15.12	0.55	0.40	-0.13	-0.33
Risk Hump	0.038	0.98	10.66	0.14	17.65	0.89	0.46	-0.27	-0.28
U.S. Data	2.04	0.47	4.03	14.08	7.36	0.78	0.87	-0.81	-0.09*

Note: *BR* refers to the bankruptcy rate. For this analysis, the innovations to the shocks are such that uncertainty jumps by 1%, as in DLS, and increases over time up to 48% as suggested by the empricial evidence. TFP is highly persistent with an autocorrelation coefficient of $0.9^{1/3}$, and subject to an innovation of 1%. Although the model is calibrated and simulated for 10,000 months, we present quarterly statistics by computing the three month averages. The U.S. Figures for output, consumption, investment and labor are from Dorofeenko et al. (2016). The statistics for bankruptcies and uncertainty are based on own calculations. For the bankruptcy rate, we use use quarterly data from 1987Q1 to 2013Q4 and the delinquency rate as a proxy. For uncertainty, we use the *Financial Uncertainty* measure from 1960 until 2015. *The correlation between output and *Macro Uncertainty* from 1960Q3 to 2014Q2 is -0.21. Source: FRED, Jurado et al. (2015) and Ludvigson et al. (2016).

Table 5: Variance Decomposition: Hump-shaped uncertainty shocks, TFP shocks and the coefficient of relative risk aversion ι.

	y		c		i	
ι	*TFP*	*Hump*	*TFP*	*Hump*	*TFP*	*Hump*
1	94.92	5.08	89.16	10.84	82.40	17.60
2	96.27	3.73	93.31	6.69	87.03	12.97
10	96.88	3.12	95.93	4.07	91.96	8.04
20	96.67	3.33	96.06	3.94	93.27	6.73

Note: *Hump* refers to uncertainty shocks that are hump-shaped. For emphasis, this analysis omits jumping uncertainty shocks.

Table 6: Variance Decomposition: Hump-shaped uncertainty shocks, TFP shocks and the monitoring costs μ.

		y			c			i			RP			BR	
μ	*TFP*	*Hump*	*Jump*	*TFP*	*Hump*	*Jump*	*TFP*	*Hump*	*Jump*	*TFP*	*Hump*	*Jump*	*TFP*	*Hump*	*Jump*
0	99.82	0.14	0.04	99.89	0.09	0.02	95.97	0.64	3.39	96.59	1.76	1.65	98.13	0.70	1.18
0.125	98.27	1.24	0.49	93.06	3.04	3.90	79.61	8.45	11.94	0.52	34.34	65.14	0.95	11.49	87.56
0.2	95.59	3.20	1.21	85.45	6.76	7.79	79.14	12.66	8.20	0.47	37.53	62.00	0.91	13.40	85.68
0.25	93.19	4.99	1.82	79.91	9.72	10.36	75.79	16.18	8.02	0.44	39.62	59.93	0.89	14.73	84.37

Note: *BR* refers to the bankruptcy rate, *RP* refers to the risk premium. *Hump* and *Jump* refer to uncertainty shocks that are hump-shaped or jump, respectively.

friction, Table 6 shows the variance decomposition based on different values of μ. Without agency friction, neither type of uncertainty shock matters. Introducing the friction and setting $\mu = 0.125$, uncertainty overall plays a small role for output (1.7%), a non-negligible role for consumption (7%) and it quantitatively matters for investment (20%). Lending-channel variables, in turn, are much more strongly affected, with productivity shocks accounting for less than one percent of the variation in risk premium and bankruptcy rates. Unsurprisingly, the importance of uncertainty for financial variables remains high as monitoring costs ($\mu = 0.25$) double. However, in terms of real variables, uncertainty accounts for 7% of the variation in output, 20% and 25% of the variation in consumption and investment, respectively. In comparison to each other, hump-shaped shaped uncertainty accounts for the lion's share in real variables, explaining 5%, 10% and 16% of the total variation in output, consumption and investment, respectively. The lending-channel variables are more strongly affected by unexpected changes in uncertainty, with 85% of the variation in bankruptcy rates and 60% of the variation in the risk premium due to shocks that jump.

5 Conclusion

We model uncertainty shocks that rise and fall over time. This approach to modeling uncertainty shocks is based on empirical evidence due to Jurado et al. (2015) and Ludvigson et al. (2016). Hump-shaped uncertainty shocks result in a different propagation mechanism compared to previous work combining uncertainty and financial accelerator models. The model's propagation mechanism resembels business cycles if uncertainty is combined with an agency friction. Changes in the investment supply drive these dynamics. We find that uncertainty shocks, calibrated corresponding to the data, play a non-negligible role for the variation in real and financial variables. Using a conservative calibration, they explain 5% of the variation in output, 10% of the variation in consumption and 16% in investment. However, the relative volatility and the correlation in the data suggest that uncertainty is not sufficient to explain the variation in bankruptcy rates. Nevertheless, the contraction and subsequent sluggish recovery due to hump-shaped uncertainty shocks resemble features observed in the recent crisis: there is an expansion, followed by a contraction and a sluggish recovery. We foresee further research in the following line. First, agents in the model anticipate the time path of uncertainty after a shock has occurred. Thus, replacing rational expectations with a learning mechanism could be an interesting extension. Moreover, examining the impact of uncertainty shocks in the context of both equity and debt finance, as in Covas and den Haan (2012), might provide further insights into the choice between different sources of external finance.

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A Appendix

A.1 Optimality Conditions

The final goods' production firm's production function is given by

$$y_t = A_t(k_t u_t)^{\alpha}((1-\eta)h_t)^{1-\alpha} \tag{23}$$

The aggregate resource constraint is

$$y_t = (1-\eta)c_t + \eta c_t^e + \eta i_t \tag{24}$$

The aggregate law of motion is given by

$$K_{t+1} = (1-\delta(u_t))K_t + \eta i_t(1-\widetilde{\Phi}\mu) \tag{25}$$

which is equivalent to

$$k_{t+1} = (1-\delta(u_t))k_t + i_t(1-\widetilde{\Phi}\mu) \tag{26}$$

with

$$\delta(u_t) = \delta_0 + \delta_1(u_t - 1) + \frac{\delta_2}{2}(u_t - 1)^2 \tag{27}$$

The household's problem is described in the text. The intertemporal optimality conditions of the household are as follows:

- Intratemporal optimality

$$\chi h_t^{\theta} = A_t(1-\alpha)(k_t u_t)^{\alpha}((1-\eta)h_t)^{-\alpha} \tag{28}$$

- Intertemporal optimality

$$q_t(c_t - \chi h_t^{1+\theta}/(1+\theta))^{-\iota} = \beta E\{(c_{t+1} - \chi h_{t+1}^{1+\theta}/(1+\theta))^{-\iota}(A_{t+1}\alpha(k_{t+1}u_{t+1})^{\alpha-1}((1-\eta)h_{t+1})^{1-\alpha}u_{t+1} + q_{t+1}(1-\delta(u_{t+1})))\} \tag{29}$$

- The stochastic discount factor is given by

$$m_{t,t+1} = E\{\beta\frac{(c_{t+1} - \chi h_{t+1}^{1+\theta}/(1+\theta))^{-\iota}}{(c_t - \chi h_t^{1+\theta}/(1+\theta))^{-\iota}}\} \tag{30}$$

- The return on investment is

$$R^k_{t,t+1} = \frac{(A_{t+1}\alpha(k_{t+1}u_{t+1})^{\alpha-1}((1-\eta)h_{t+1})^{1-\alpha}u_{t+1} + q_{t+1}(1-\delta(u_{t+1})))}{q_t} \tag{31}$$

- The optimal level of capital utilization is

$$u_t = 1 + (\frac{A_t\alpha(k_t u_t)^{\alpha-1}((1-\eta)h_t)^{1-\alpha}}{q_t} - \delta_1)/\delta_2 \tag{32}$$

- The risk premium is

$$riskpr = qR^k_{t,t+1} - 1 = q\frac{\bar{\omega}_t}{1-(1-qg(\bar{\omega}_t,\sigma_{\omega,t}))} - 1 = \frac{\bar{\omega}_t}{g(\bar{\omega}_t,\sigma_{\omega,t})} - 1 \tag{33}$$

The optimal contract, the solution to the problem (1) subject to participating lenders, determines investment and the default threshold. Entrepreneurs with $n_t > 0$ borrow $i_t - n_t$ units of consumption and pay back $\frac{(1+r^k)(i_t-n_t)}{i_t} \equiv \bar{\omega}_t$ which is possible if $\omega_t \geq \bar{\omega}_t$. The first order necessary conditions are given by

$$q_t = \frac{1}{(1-\tilde{\Phi}(\bar{\omega}_t;\sigma_{\omega,t}))\mu + \tilde{\phi}(\bar{\omega}_t;\sigma_{\omega,t})\mu f(\bar{\omega}_t,\sigma_{\omega,t})/\frac{\partial f(\bar{\omega}_t,\sigma_{\omega,t})}{\partial\bar{\omega}_t}} \tag{34}$$

which governs the default threshold $\bar{\omega}$ as a function of q as well as the leverage ratio

$$i_t = \frac{1}{1-q_t g(\bar{\omega}_t,\sigma_{\omega,t})}n_t \tag{35}$$

which determines investment $i(\bar{\omega}(q), q, n)$ and ensures incentive compatibility.

Entrepreneurs' intertemporal efficiency results from maximizing entrepreneurial consumption, which is linear in c_t^e.

$$q_t = \beta\gamma E_t[q_{t+1}(1-\delta(u_{t+1})) + A_{t+1}\alpha(u_{t+1}k_{t+1})^{\alpha-1}((1-\eta)h_{t+1})^{1-\alpha}(\frac{q_{t+1}f(\bar{\omega}_{t+1},\sigma_{\omega,t+1})}{1-q_{t+1}g(\bar{\omega}_{t+1},\sigma_{\omega,t+1})})] \quad (36)$$

The law of motion of entrepreneurs' capital z_{t+1} is given by the equations

$$\eta n_t = z_t[q_t(1-\delta(u_t)) + A_t\alpha(u_tk_t)^{\alpha-1}((1-\eta)h_t)^{1-\alpha}] \quad (37)$$

$$z_{t+1} = \eta n_t[\frac{f(\bar{\omega}_t,\sigma_{\omega,t})}{1-q_tg(\bar{\omega}_t,\sigma_{\omega,t})}] - \eta\frac{c_t^e}{q_t} \quad (38)$$

A.2 Computation of the Steady State

In order to solve the model, I set the steady state default threshold as the inverse of the log-normal distribution for a given target default, which is 0.039/12

$$\varpi = \tilde{\Phi}^{-1}(0.039/12, \overline{\sigma}_\omega) \quad (39)$$

Given ϖ, the steady state bankruptcy rate is the log-normal distribution with $\mu_\omega = -\overline{\sigma}_\omega^2/2$.

Entrepreneurs' share of net capital output is the partial expectation of a log-normally distributed variable

$$f(\varpi;\overline{\sigma}_\omega) = \Phi(\frac{-\log(\varpi) - \overline{\sigma}_\omega^2/2 + \overline{\sigma}_\omega^2}{\overline{\sigma}_\omega}) - (1-\tilde{\Phi}(\varpi;\overline{\sigma}_\omega))\varpi \quad (40)$$

where Φ denotes the normal distribution.

The lenders' share is

$$g(\varpi;\overline{\sigma}_\omega) = 1 - f(\varpi;\overline{\sigma}_\omega) - \tilde{\Phi}(\varpi;\overline{\sigma}_\omega)\mu \quad (41)$$

The steady state relative price of capital is

$$q = \frac{1}{1-\tilde{\Phi}(\varpi;\overline{\sigma}_\omega)\mu + \widetilde{\phi}(\varpi;\overline{\sigma}_\omega)\mu f(\varpi;\overline{\sigma}_\omega)/(\tilde{\Phi}(\varpi;\overline{\sigma}_\omega)-1)} \quad (42)$$

which in turn determines γ, which prevents self-financing entrepreneurs

$$\gamma = \frac{1 - qg(\varpi; \overline{\sigma}_\omega)}{qf(\varpi; \overline{\sigma}_\omega)} \tag{43}$$

and steady state risk premium

$$riskpr = \frac{\varpi}{g(\varpi; \overline{\sigma}_\omega)} - 1 \tag{44}$$