Chapter 5

Design of Springs

5.1 Introduction

- A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed.
- The various important **applications of springs** are as follows :

1. To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, air-craft landing gears, shock absorber, sand vibration dampers.
2. To apply forces, as in brakes, clutches and spring loaded valves.
3. To control motion by maintaining contact between two elements as in cams and followers.
4. To measure forces, as in spring balances and engine indicators.
5. To store energy, as in watches, toys, etc.

5.2 Types of Springs

- Though there are many types of the springs, following, according to their shape, are important from the subject point of view.

1.Helical springs.

- The helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile loads.
- The cross-section of the wire from which the spring is made may be circular, square or rectangular.
- The two forms of helical springs are ***compression helical spring*** as shown in following Fig. (*a*) and ***tension helical spring*** as shown in Fig. (*b*).

closely coiled helical springs

- ❖ The helical springs are said to be closely coiled when the spring wire is coiled so close that the plane containing each turn is nearly at right angles to the axis of the helix and the wire is subjected to torsion.
- ❖ In other words, in a closely coiled helical spring, the helix angle is very small, it is usually less than 10°.
- ❖ The major stresses produced in helical springs are shear stresses due to twisting.
- ❖ The load applied is parallel to or along the axis of the spring.

open coiled helical springs

- ❖ In ***open coiled helical springs*****,** the spring wire is coiled in such a way that there is a gap between the two consecutive turns, as a result of which the helix angle is large.

Advantages of the helical springs:

(*a*) These are easy to manufacture.

(*b*) These are available in wide range.

(*c*) These are reliable.

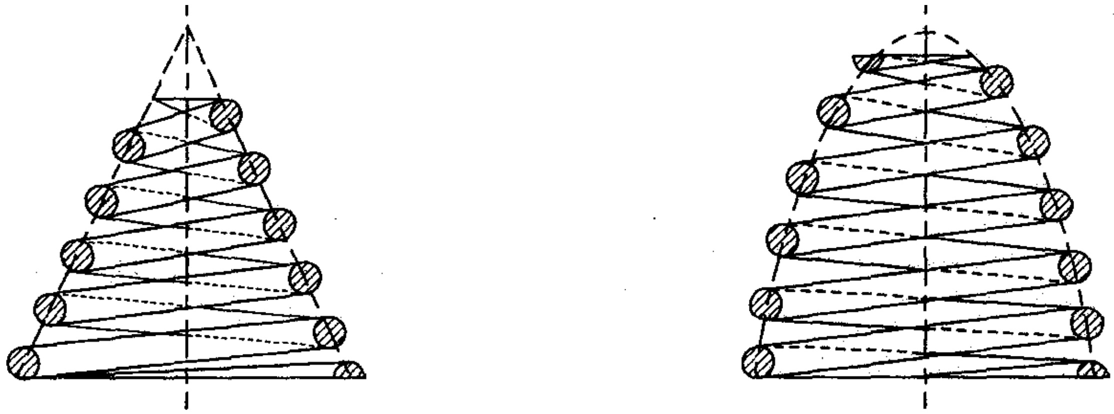
(*d*) These have constant spring rate.

(*e*) Their performance can be predicted more accurately.

(*f*) Their characteristics can be varied by changing dimensions.

2. Conical and volute springs.

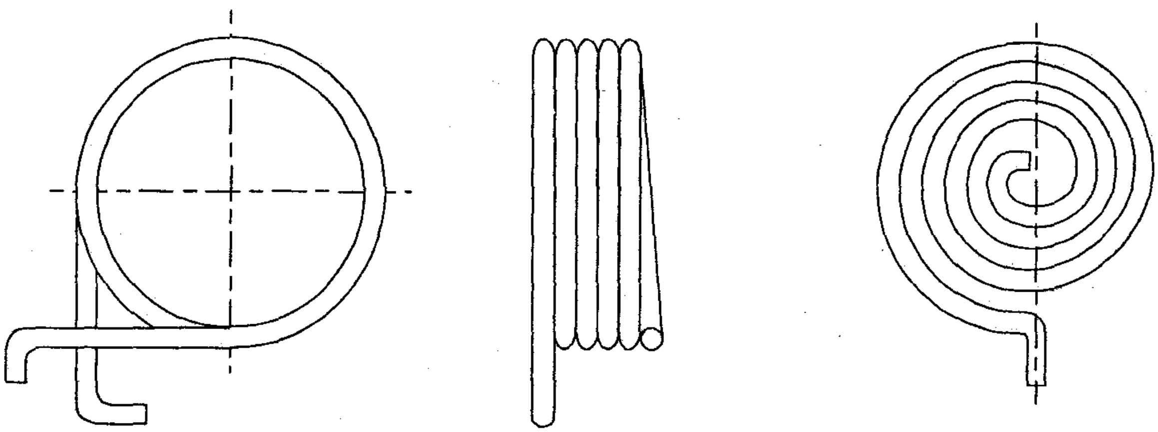
- ❖ The conical and volute springs, as shown in following Fig. are used in special applications where a telescoping spring or a spring with a spring rate that increases with the load is desired.
- ❖ The conical spring, as shown in Fig. (*a*), is wound with a uniform pitch whereas the volute springs, as shown in Fig. (*b*), are wound in the form of paraboloid with constant pitch and lead angles.
- ❖ The springs may be made either partially or completely telescoping.
- ❖ In either case, the number of active coils gradually decreases. The decreasing number of coils results in an increasing spring rate.
- ❖ This characteristic is sometimes utilised in vibration problems where springs are used to support a body that has a varying mass.
- ❖ The major stresses produced in conical and volute springs are also shear stresses due to twisting.



(*a*) Conical spring. (*b*) Volute spring.

3. Torsion springs.

- These springs may be of ***helical*** or ***spiral*** type as shown in following Fig.
- The **helical type** may be used only in applications where the load tends to wind up the spring and are used in various electrical mechanisms.
- The **spiral type** is also used where the load tends to increase the number of coils and when made of flat strip are used in watches and clocks.
- The major stresses produced in torsion springs are tensile and compressive due to bending.



(*a*) Helical torsion spring. (*b*) Spiral torsion spring.

4. Laminated or leaf springs.

- The laminated or leaf spring (also known as ***flat spring*** or ***carriage spring***) consists of a number of flat plates (known as leaves) of varying lengths held together by means of clamps and bolts, as shown in following Fig.
- These are mostly used in automobiles.
- The major stresses produced in leaf springs are tensile and compressive stresses.

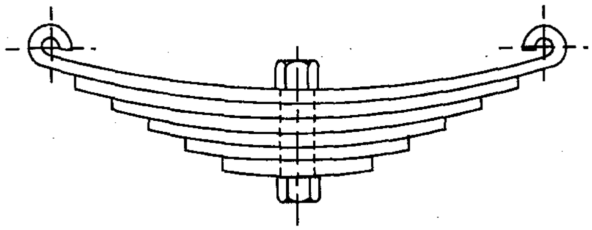


Fig. 23.4. Laminated or leaf springs.

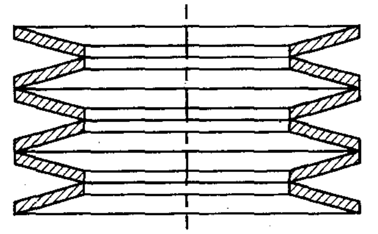


Fig. 23.5. Disc or bellevile springs.

5. Disc or bellevile springs.

- These springs consist of a number of conical discs held together against slipping by a central bolt or tube as shown in Fig.
- These springs are used in applications where high spring rates and compact spring units are required.
- The major stresses produced in disc or bellevile springs are tensile and compressive stresses.

6. Special purpose springs.

- These springs are air or liquid springs, rubber springs, ring springs etc.
- The fluids (air or liquid) can behave as a compression spring.
- These springs are used for special types of application only

5.3 Material for Helical Springs

- The material of the spring should have high fatigue strength, high ductility, high resilience and it should be creep resistant.
- It largely depends upon the service for which they are used *i.e*. severe service, average service or light service.
- ***Severe service*** means rapid continuous loading where the ratio of minimum to maximum load (or stress) is one-half or less, as in automotive valve springs.
- ***Average service*** includes the same stress range as in severe service but with only intermittent operation, as in engine governor springs and automobile suspension springs.
- ***Light service*** includes springs subjected to loads that are static or very infrequently varied, as insafety valve springs.
- The springs are mostly **made** from **oil-tempered carbon steel** wires containing 0.60 to 0.70 per cent carbon and 0.60 to 1.0 per cent manganese.
- Music wire is used for small springs.
- Non-ferrous materials like phosphor bronze, beryllium copper, monel metal, brass etc., may be used in special cases to increase fatigue resistance, temperature resistance and corrosion resistance.

- The helical springs are either cold formed or hot formed depending upon the size of the wire.
- Wires of small sizes (less than 10 mm diameter) are usually wound cold whereas larger size wires are wound hot.
- The strength of the wires varies with size, smaller size wires have greater strength and less ductility, due to the greater degree of cold working.

5.4 Terms used in Compression Springs

- The following terms used in connection with compression springs are important from the subject point of view.

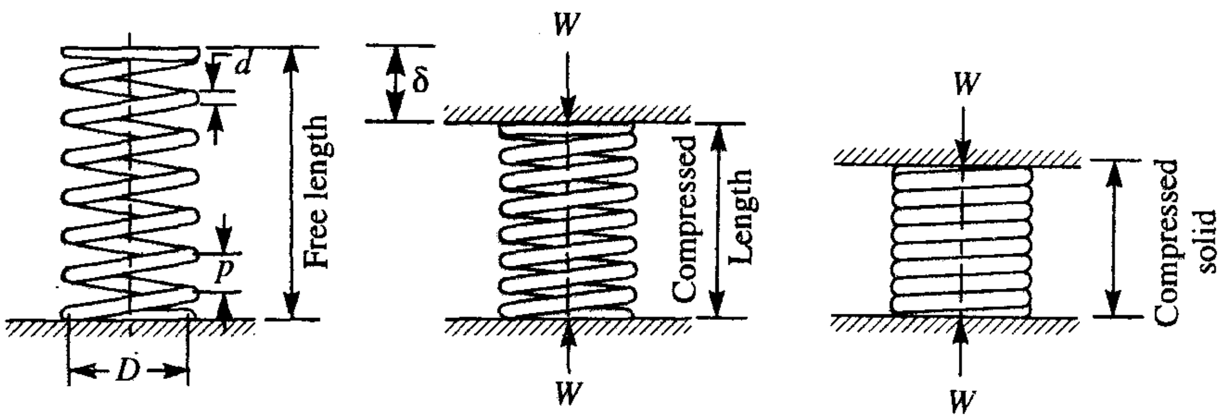
1.Solid length.

- When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be ***solid*.**
- The solid length of a spring is the product of total number of coils and the diameter of the wire.
- Mathematically, Solid length of the spring,

$$L_S = n'. d$$

where n' = Total number of coils, and

d = Diameter of the wire.



2. Free length.

- The free length of a compression spring, as shown in above Fig. is the length of the spring in the free or unloaded condition.
- It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coils (when fully compressed).
- Mathematically,

Free length of the spring,

L_F = Solid length + Maximum compression + Clearance between adjacent coils (or clash allowance)

$$\boldsymbol{L_F = n'.d + \delta max + 0.15\ \delta max}$$

3. Spring index.

- The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire.
- Mathematically,

Spring index, $\boldsymbol{C = D / d}$

where D = Mean diameter of the coil, and

d = Diameter of the wire.

4. Spring rate.

- The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring.
- Mathematically,

Spring rate, $\boldsymbol{k = W / \delta}$

where W = Load, and

δ = Deflection of the spring.

5. Pitch.

- The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state.
- Mathematically,

$$\text{Pitch of the coil,} \quad p = \frac{\text{Free length}}{n' - 1}$$

5.5 Stresses in Helical Springs of Circular Wire

- Consider a helical compression spring made of circular wire and subjected to an axial load W, as shown in following Fig. (a).

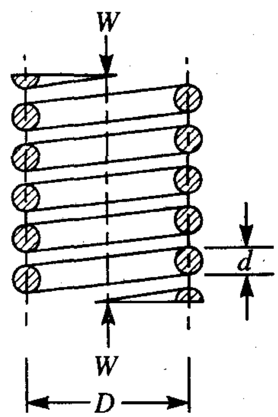
Let

D = Mean diameter of the spring coil,

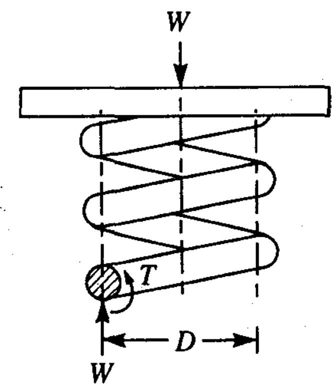
d = Diameter of the spring wire,

n = Number of active coils,

G = Modulus of rigidity for the spring material,
W = Axial load on the spring,
τ = Maximum shear stress induced in the wire,
C = Spring index = D/d,
p = Pitch of the coils, and
δ = Deflection of the spring, as a result of an axial load W.



(*a*) Axially loaded helical spring.



(*b*) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

Now consider a part of the compression spring as shown in Fig. 23.10 (*b*). The load W tends to rotate the wire due to the twisting moment (T) set up in the wire. Thus torsional shear stress is induced in the wire.

A little consideration will show that part of the spring, as shown in Fig. 23.10 (*b*), is in equilibrium under the action of two forces W and the twisting moment T. We know that the twisting moment,

$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\therefore \quad \tau_1 = \frac{8W.D}{\pi d^3} \quad \text{...(}i\text{)}$$

In addition to the torsional shear stress (τ_1) induced in the wire, the following stresses also act on the wire :

1. Direct shear stress due to the load W, and
2. Stress due to curvature of wire.

We know that direct shear stress due to the load W,

$$\tau_2 = \frac{\text{Load}}{\text{Cross-sectional area of the wire}}$$

$$= \frac{W}{\frac{\pi}{4} \times d^2} = \frac{4W}{\pi d^2}$$

We know that the resultant shear stress induced in the wire,

$$\tau = \tau_1 \pm \tau_2 = \frac{8W.D}{\pi d^3} \pm \frac{4W}{\pi d^2}$$

The *positive* sign is used for the inner edge of the wire and *negative* sign is used for the outer edge of the wire. Since the stress is maximum at the inner edge of the wire, therefore

Maximum shear stress induced in the wire,

$$= \text{Torsional shear stress} + \text{Direct shear stress}$$

$$= \frac{8W.D}{\pi d^3} + \frac{4W}{\pi d^2} = \frac{8\,W.D}{\pi d^3}\left(1 + \frac{d}{2D}\right)$$

$$= \frac{8\,W.D}{\pi d^3}\left(1 + \frac{1}{2C}\right) = K_S \times \frac{8\,W.D}{\pi d^3} \quad \text{...(iii)}$$

... (Substituting $D/d = C$)

where K_S = Shear stress factor $= 1 + \dfrac{1}{2C}$

From the above equation, it can be observed that the effect of direct shear $\left(\dfrac{8\,WD}{\pi d^3} \times \dfrac{1}{2C}\right)$

is appreciable for springs of small spring index C.

- ❖ Also we have neglected the effect of wire curvature in equation (iii).
- ❖ It may be noted that when the springs are subjected to static loads, the effect of wire curvature may be neglected, because yielding or the material will relieve the stresses.

Wahl's stress factor (K)

In order to consider the effects of both direct shear as well as curvature of the wire, a Wahl's stress factor (K) introduced by A.M. Wahl may be used. The resultant diagram of torsional shear, direct shear and curvature shear stress is shown in Fig. 23.11 (*d*).

∴ Maximum shear stress induced in the wire,

$$\tau = K \times \frac{8\,W.D}{\pi d^3} = K \times \frac{8\,W.C}{\pi d^2} \quad \text{...(iv)}$$

where $K = \dfrac{4C - 1}{4C - 4} + \dfrac{0.615}{C}$

The values of K for a given spring index (C) may be obtained from the graph as shown

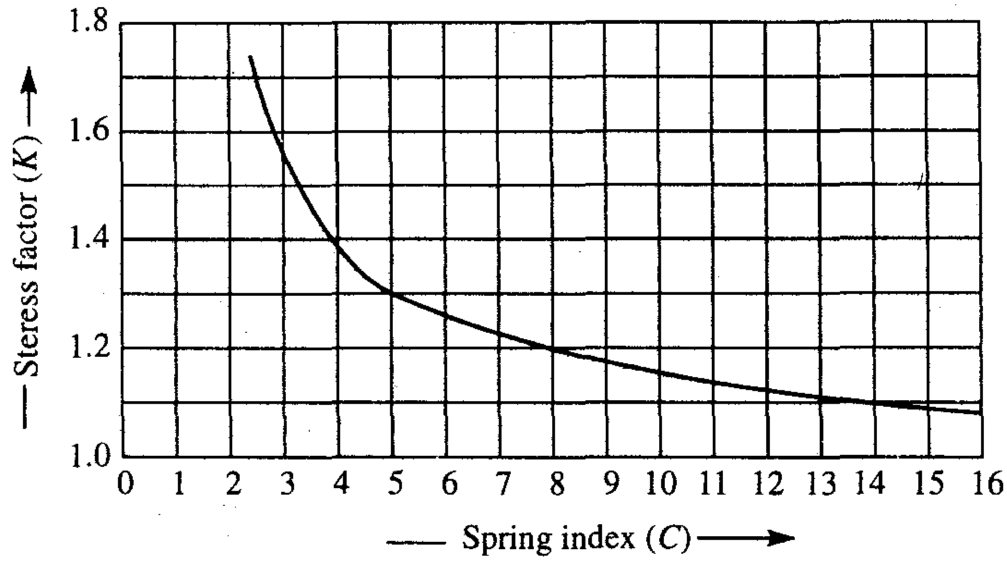


Fig. 23.12. Wahl's stress factor for helical springs.

We see from Fig. 23.12 that Wahl's stress factor increases very rapidly as the spring index decreases. The spring mostly used in machinery have spring index above 3.

Note : The Wahl's stress factor (K) may be considered as composed of two sub-factors, K_S and K_C, such that

$$K = K_S \times K_C$$

where K_S = Stress factor due to shear, and

K_C = Stress concentration factor due to curvature.

5.6 Deflection of Helical Springs of Circular Wire

In the previous article, we have discussed the maximum shear stress developed in the wire. We know that

Total active length of the wire,

$$l = \text{Length of one coil} \times \text{No. of active coils} = \pi D \times n$$

Let θ = Angular deflection of the wire when acted upon by the torque T.

$\therefore$ Axial deflection of the spring,

$$\delta = \theta \times D/2 \qquad ...(i)$$

We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G.\theta}{l}$$

$$\therefore \quad \theta = \frac{T.l}{J.G} \qquad ... \left(\text{considering } \frac{T}{J} = \frac{G.\theta}{l}\right)$$

where J = Polar moment of inertia of the spring wire

$$= \frac{\pi}{32} \times d^4, \; d \text{ being the diameter of spring wire.}$$

and G = Modulus of rigidity for the material of the spring wire.

Now substituting the values of l and J in the above equation, we have

$$\theta = \frac{T.l}{J.G} = \frac{\left(W \times \frac{D}{2}\right)\pi D.n}{\frac{\pi}{32} \times d^4 G} = \frac{16\, W.D^2.n}{G.d^4} \qquad \ldots(ii)$$

Substituting this value of θ in equation (i), we have

$$\delta = \frac{16\, W.D^2.n}{G.d^4} \times \frac{D}{2} = \frac{8\, W.D^3.n}{G.d^4} = \frac{8\, W.C^3.n}{G.d} \qquad \ldots (\because C = D/d)$$

and the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{G.d^4}{8\, D^3.n} = \frac{G.d}{8\, C^3.n} = \text{constant}$$

5.7 Surge in Springs

- When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire.
- A little consideration will show that in the beginning, the end coils of the spring in contact with the applied load takes up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils.
- In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end.
- This wave of compression travels along the spring indefinitely.
- If the applied load is of fluctuating type as in the case of valve spring in internal combustion engines and if the time interval between the load applications is equal to the time required for the wave to travel from one end to the other end, then resonance will occur.
- This results in very large deflections of the coils and correspondingly very high stresses. Under these conditions, it is just possible that the spring may fail.
- This phenomenon is called ***surge.***
- It has been found that the natural frequency of spring should be at least twenty times the frequency of application of a periodic load in order to avoid resonance with all harmonic frequencies upto twentieth order.
- The natural frequency for springs clamped between two plates is given by

$$f_n = \frac{d}{2\pi\, D^2.n}\sqrt{\frac{6\, G.g}{\rho}} \text{ cycles/s}$$

Where

d = Diameter of the wire,
D = Mean diameter of the spring,
n = Number of active turns,
G = Modulus of rigidity,
g = Acceleration due to gravity, and
ρ = Density of the material of the spring.

❖ The **surge in springs may be eliminated** by using the following methods :

1. By using friction dampers on the centre coils so that the wave propagation dies out.

2. By using springs of high natural frequency.

3. By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.

5.8 Energy Stored in Helical Springs of Circular Wire

We know that the springs are used for storing energy which is equal to the work done on it by some external load.

Let W = Load applied on the spring, and

δ = Deflection produced in the spring due to the load W.

Assuming that the load is applied gradually, the energy stored in a spring is,

$$U = \frac{1}{2} W.\delta \qquad ...(i)$$

We have already discussed that the maximum shear stress induced in the spring wire,

$$\tau = K \times \frac{8\, W.D}{\pi\, d^3} \quad \text{or} \quad W = \frac{\pi\, d^3.\tau}{8K.D}$$

We know that deflection of the spring,

$$\delta = \frac{8\, W.D^3.n}{G.d^4} = \frac{8 \times \pi\, d^3.\tau}{8K.D} \times \frac{D^3.n}{G.d^4} = \frac{\pi\, \tau.D^2.n}{K.d.G}$$

Substituting the values of W and δ in equation (i), we have

$$U = \frac{1}{2} \times \frac{\pi\, d^3.\tau}{8\, K.D} \times \frac{\pi\, \tau.D^2.n}{K.d.G}$$

$$= \frac{\tau^2}{4\, K^2.G}\left(\pi\, D.n\right)\left(\frac{\pi}{4} \times d^2\right) = \frac{\tau^2}{4\, K^2.G} \times V$$

where V = Volume of the spring wire

= Length of spring wire × Cross-sectional area of spring wire

$$= \left(\pi\, D.n\right)\left(\frac{\pi}{4} \times d^2\right)$$

Note : When a load (say P) falls on a spring through a height h, then the energy absorbed in a spring is given by

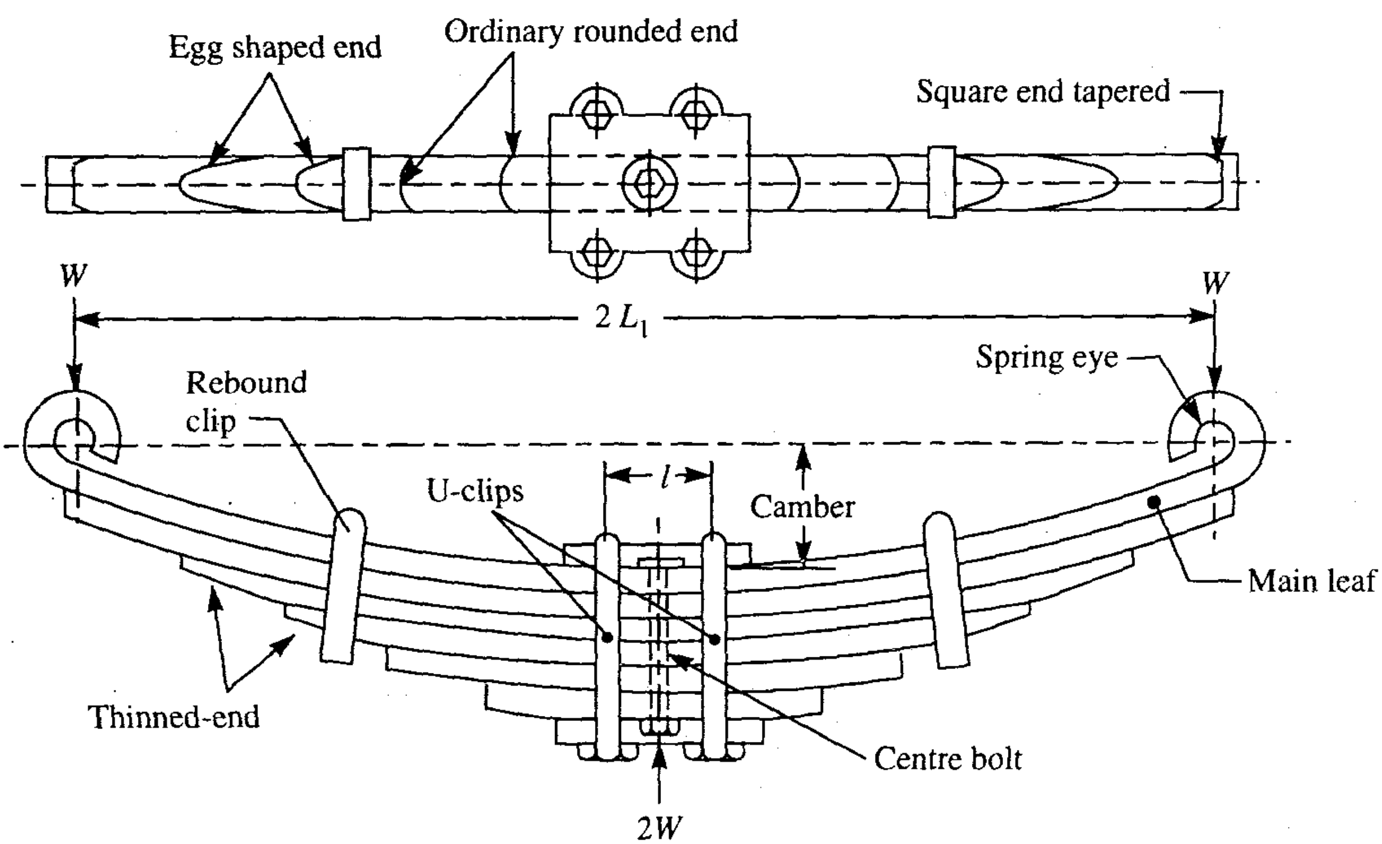
$$U = P(h + \delta) = \frac{1}{2} W.\delta$$

where W = Equivalent static load *i.e.*, the gradually applied load which shall produce the same effect as by the falling load P, and

δ = Deflection produced in the spring.

5.9 Construction of Leaf Spring

- A leaf spring commonly used in automobiles is of semi-elliptical form as shown in following Fig.
- It is built up of a number of plates (known as leaves).
- The leaves are usually given an initial curvature or cambered so that they will tend to straighten under the load.
- The leaves are held together by means of a band shrunk around them at the centre or by a bolt passing through the centre.
- Since the band exerts a stiffening and strengthening effect, therefore the effective length of the spring for bending will be overall length of the spring *minus* width of band.
- In case of a centre bolt, two-third distance between centres of *U*-bolt should be subtracted from the overall length of the spring in order to find effective length.
- The spring is clamped to the axle housing by means of *U*-bolts.



- The longest leaf known as ***main leaf*** or ***master leaf*** has its ends formed in the shape of an eye through which the bolts are passed to secure the spring to its supports.
- Usually the eyes, through which the spring is attached to the hanger or shackle, are provided with bushings of some antifriction material such as bronze or rubber.
- The other leaves of the spring are known as ***graduated leaves*.**
- In order to prevent digging in the adjacent leaves, the ends of the graduated leaves are trimmed in various forms as shown in Fig
- Since the master leaf has to with stand vertical bending loads as well as loads due to sideways of the vehicle and twisting, therefore due to the presence of stresses caused by these loads, it is usual to provide two full length leaves and the rest graduated leaves as shown in Fig.
- Rebound clips are located at intermediate positions in the length of the spring, so that the graduated leaves also share the stresses induced in the full length leaves when the spring rebounds.

Materials for Leaf Springs

- The material used for leaf springs is usually a plain carbon steel having 0.90 to 1.0% carbon. The leaves are heat treated after the forming process. The heat treatment of spring steel produces greater strength and therefore greater load capacity, greater range of deflection and better fatigue properties.
- According to Indian standards, the recommended materials are :

  1. For automobiles : 50 Cr 1, 50 Cr 1 V 23, and 55 Si 2 Mn 90 all used in hardened and tempered state.

  2. For rail road springs : C 55 (water-hardened), C 75 (oil-hardened), 40 Si 2 Mn 90 (waterhardened)and 55 Si 2 Mn 90 (oil-hardened).

  3. The physical properties of some of these materials are given in the following table. All values are for oil quenched condition and for single heat only.