Comparison of Harmonic Spin Matching Schemes using Orbit Bumps in the FCC-ee

Y. Wu[1], D. P. Barber[2], F. Carlier[3], L. van Riesen-Haupt[1], E. Gianfelice-Wendt[4], T. Pieloni[1]

[1]École Polytechnique Fédérale de Lausanne (EPFL)
[2]Deutsches Elektronen Synchrotron (DESY)
[3] European Organization for Nuclear Research (CERN)
[4] Fermilab

Energy calibration in FCC-ee

EPFL

- FCC-ee will operate on 4 centre-of-mass energies

  Z^0 bosons (91 GeV), WW pairs (160 GeV), Higgs bosons (240 GeV) and top quark pairs (350-365 GeV)
- High-precision centre-of-mass energy calibration
  - basis for precise measurements of the standard model particle properties
  - make it possible for the new rare process detection
  - precise measurements in FCC-ee will contribute to the measurements in FCC-hh

The current precision targets for the energy calibration: 4 keV at Z mass and 100 keV at W mass

the most promising way to achieve this target: **resonant depolarization**

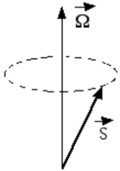
FCC collaboration, "FCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design Report Volume 2", in European Physical Journal: Special Topics, 228, pp. 261-623, 2019.

Ensure a sufficient spin polarization level (at least $5-10\%$)

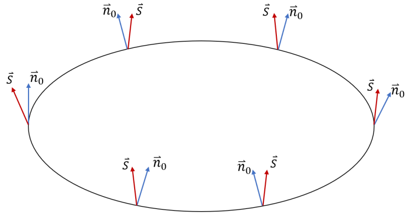
1. Estimate the achievable polarization under various lattice conditions
   e.g. misalignments+field errors
2. Use special structure to improve polarization
   e.g. closed orbit bumps

Thomas-BMT equation

$$\frac{\mathrm{d}\vec{S}}{\mathrm{d}t} = \vec{\Omega}_{\mathrm{BMT}} \times \vec{S}$$



$\hat{n}_0(s)$: one-turn periodic solution of the T-BMT equation on the closed orbit
the precession axis for arbitrary spins on the closed orbit



- Spins on the closed orbit precess around $\hat{n}_0$ for ν_0 turns in every revolution
- ν_0: closed orbit spin tune
- $\nu_0 = a\gamma$ in the perfectly aligned flat ring without solenoids
- $\nu_0 \neq a\gamma$ in general

- Sokolov-Ternov (ST) effect
  - spin-flip during synchrotron radiation emission
  - e-/e+ beams are gradually polarized in the rings
  - $P_{ST} \approx 92.4\%$ in a uniform field, less than P_{ST} in non-uniform fields
- Radiative depolarization
  - spin diffusion: a large number of stochastic photon emissions result in a random walk of $|\hat{S} \cdot \hat{n}|$.
  - total polarization level of a beam is decreased

ST effect + radiative depolarization ⇒ equilibrium polarization

Effective model for error seeds creation

- Use an effective model to simulate **residual orbits after lattice correction**
- Random small errors generated from truncated Gaussian distributions (truncated at 2.5 σ)

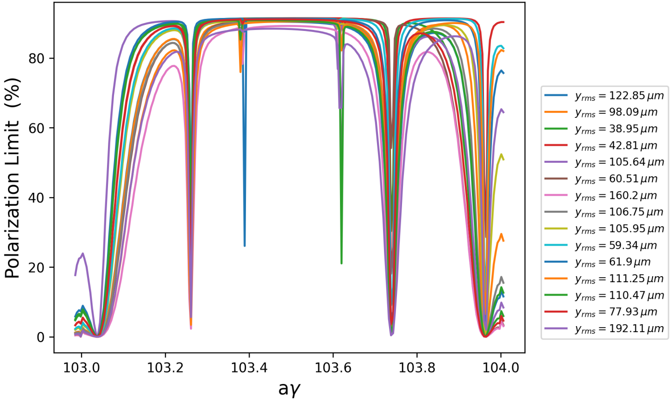
Type	$\sigma_{\Delta X}$ (nm)	$\sigma_{\Delta Y}$ (nm)	$\sigma_{\Delta Z}$ (nm)	$\sigma_{\Delta PSI}$ (μrad)	$\sigma_{\Delta THETA}$ (μrad)	$\sigma_{\Delta PHI}$ (μrad)
Arc quadrupole	120	120	120	2	2	2
Arc sextupole	120	120	120	2	2	2
Dipoles	120	120	120	2	0	0
IR quadrupole	120	120	120	2	2	2
IR sextupole	120	120	120	2	2	2

Energy scan with multiple error seeds

Setting 1

$\sigma = 120\,\mathrm{nm}$ for x,y,z misalignments

$\sigma = 2\,\mu\mathrm{rad}$ for angular deviations



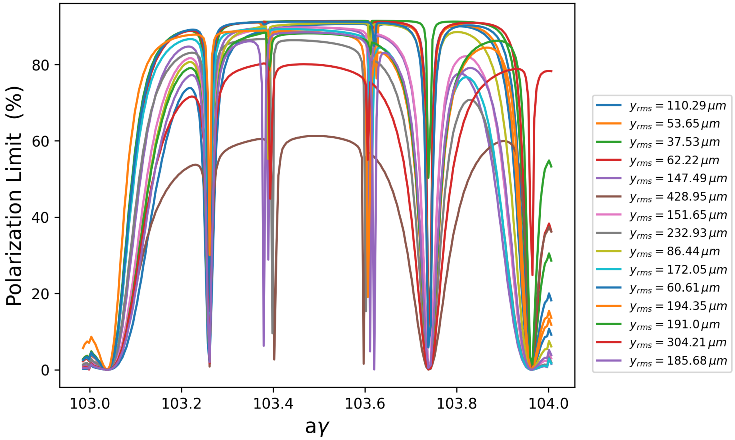
First order energy scan showing equilibrium polarization levels near Z energy

Energy scan with multiple error seeds

Setting 2

$\sigma = 200\,\mathrm{nm}$ for x,y,z misalignments

$\sigma = 2\,\mu\mathrm{rad}$ for angular deviations



Harmonic spin matching (HSM)

Conventional lattice correction + harmonic spin matching

- Perfectly aligned flat ring $\rightarrow$ $\hat{n}_0(s)$ vertical $\rightarrow$ small spin diffusion
- Misaligned ring $\rightarrow$ $\hat{n}_0(s)$ not vertical $\rightarrow$ stronger spin diffusion
- Random vertical quadrupole misalignments are difficult to control
- HSM: use multiple vertical orbit correctors to create an additional controllable $\hat{n}_0$ tilt and reduce $(\delta \hat{n}_0)_{\mathrm{rms}}$ $\rightarrow$ elevate polarization
- Use closed vertical bumps to avoid disturbing the orbits outside bumps

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D. P. Barber, et al. "High spin polarization at the HERA electron storage ring." Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 338.2-3 (1994): 166-184.

Harmonic bump composition

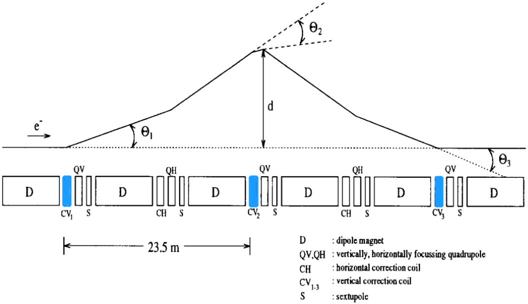
1st corrector: give a vertical kick

2nd corrector: kick y back to initial value at the 3rd corrector

3rd corrector: kick y' back to initial value

Kicks of 2nd and 3rd correctors are adjusted to make the bump **CLOSED**

one independent variable for each bump



a schematic of a bump in HERA*

*D. P. Barber, et al. "High spin polarization at the HERA electron storage ring." Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 338.2-3 (1994): 166-184.

Three correction schemes

1. HERA formalism (used in HERA)

   D. P. Barber, et al. A general harmonic spin matching formalism for the suppression of depolarisation caused by closed orbit distortion in electron storage rings. No. DESY–85-044. DESY, 1985.

2. Rossmanith-Schmidt scheme (used in PETRA)

   R. Rossmanith and R. Schmidt, Compensation of depolarizing effects in electron-positron storage rings. Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 236.2 (1985): 231-248.

3. LEP method (Deterministic) (used in LEP)

   R. W. Assmann, Optimierung der transversalen Spin-Polarisation im LEP-Speicherring und Anwendung für Präzisionsmessungen am Z-Boson. Diss. Munich U., 1994.

HERA formalism

$$\delta \hat{n}_0 = \alpha \hat{m} + \beta \hat{l}$$

$\hat{l}(s), \hat{n}_0(s), \hat{m}(s)$ are periodic spin axes that form a right-hand coordinate system. Expand α and β to Fourier series

$$(\alpha - i\beta)(s) = -i\frac{C}{2\pi}\sum_k \frac{f_k}{k - \tilde{\nu}} e^{i2\pi ks/C}$$

f_k: Fourier coefficients, related to the closed orbit and perturbing fields

Make additional orbit corrections using orbit bumps to reduce the rms tilt by minimizing the Fourier coefficients

Simplified HERA formalism

- Extract n_0 direction at the end of all elements
- Expand $n_{0x}(s) + in_{0z}(s)$ into Fourier series
- Minimize target coefficients using four bumps

$$n_{0x} + in_{0z} \approx \sum_{k=-N}^{N} c_k \cdot e^{i2\pi ks/C}$$

$$c_k = \frac{1}{C}\int (n_{0x} + in_{0z}) \cdot e^{-i2\pi ks/C} ds \approx \frac{\sum_{j=1}^{M} [n_{0x}(s_j) + in_{0z}(s_j)] \cdot e^{-i2\pi ks_j/C}}{M}$$

Response matrix

Each closed bump can be represented using a single variable, and each bump has an independent and linear contribution to the Fourier coefficients

$$\mathbf{MK} = \mathbf{C}$$

K: amplitudes (the first kick value) of the bumps
C: real and imaginary parts of the required harmonics coefficients
$[c_{0\text{real}}, c_{0\text{imag}}, c_{1\text{real}}, c_{1\text{imag}}]$

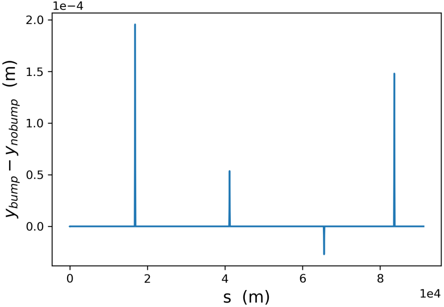
If the harmonics 0 and 1 of a misaligned lattice is **A**, the bumps should generate $-\mathbf{A}$, and bump amplitudes can be estimated via inversing matrix

$$\mathbf{K} = \mathbf{M}^{-1}(-\mathbf{A})$$

Computing the response matrix

- Search for all possible locations in a perfectly aligned lattice
- Add one bump in one possible location
- Adjust 2nd and 3rd kicks to make y and yp be all 0 outside the bump
- Analyze its contribution to the target harmonics
- Change the bump position and redo the analysis
- Select four bumps that build a matrix with the largest determinant
- Add four bumps into a misaligned machine with estimated bump amplitudes, and match the orbits to make bumps closed

Changes after adding bumps



- How does orbit change?
  - Orbits outside of the bumps are unaffected.
  - max $0.2\,\mathrm{mm}$ Δy within bumps is at the level of rms orbit of the effective lattice
- How does vertical dispersion change?
  - $(\eta_y)_{\mathrm{rms}}$ 7.602 mm → 7.675 mm, 0.96% increase
  - $[\Delta\eta_y(s)]_{\mathrm{rms}} = 0.4\,\mathrm{mm}$
- How does vertical emittance change?
  - ε_y 0.703 pm → 0.719 pm, 2.16% increase

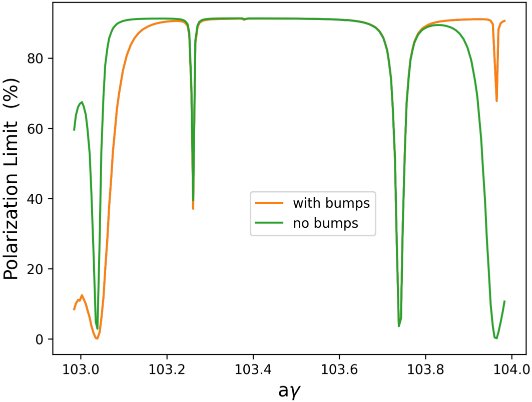
HERA formalism

At 45.82 GeV ($a\gamma = 103.983$)

	$(\delta n_0)_{\mathrm{rms}}$ (mrad)	Polarization (%)
no correction	2.28	10.68
a set of four random bumps	0.912	90.58
optimized four bumps	0.9	90.96
a set of eight random bumps	0.903	90.79

Not necessary to use optimized locations, but better to have a symmetric layout

Using 4 bumps which are optimized at 45.82 GeV ($a\gamma = 103.983$)



Rossmanith-Schmidt scheme

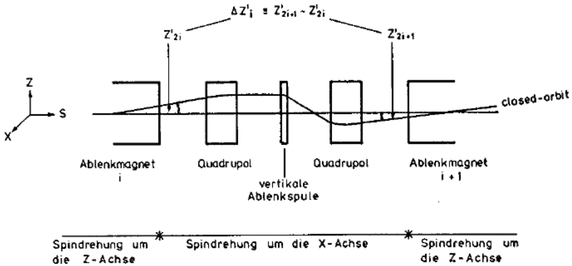
Assume that spin precessions around vertical direction only happen in bending magnets, and the radial perturbing fields on the closed orbit only exist between bending magnets

$$|\delta\vec{n}_0(s)| = \frac{1/c^2}{2(1-\cos 2\pi\nu)}\left[\left(\int_s^{s+L}\delta\Omega_x\cos\phi\mathrm{d}s\right)^2 + \left(\int_s^{s+L}\delta\Omega_x\sin\phi\mathrm{d}s\right)^2\right]$$

$$\delta\Omega_x = \frac{e}{m_0 c\gamma}(1+a\gamma)B_x \text{ and } \phi = \gamma a\alpha$$

$$\text{also } \frac{e}{m_0 c\gamma}\int_{s_{2i}}^{s_{2i+1}} B_x(s)\mathrm{d}s = -\Delta y_i^{'}$$

$$|\delta\vec{n}_0| = \frac{1/c^2}{2(1-\cos 2\pi\nu)}(1+\gamma a)\left[\left(\sum_{i=1}^N \sin(\gamma a\alpha_i)\Delta y_i^{'}\right)^2 + \left(\sum_{i=1}^N \cos(\gamma a\alpha_i)\Delta y_i^{'}\right)^2\right]$$



Rossmanith-Schmidt scheme

Expand $\Delta y'(\alpha)$ into Fourier series

$$\Delta y'(\alpha) = \sum_{k=1}^{\infty} (a_k \cos k\alpha + b_k \sin k\alpha)$$

$$\begin{matrix} a_k \\ b_k \end{matrix} = \frac{1}{N} \sum \Delta y_i'(\alpha_i) \begin{matrix} \cos k\alpha_i \\ \sin k\alpha_i \end{matrix}$$

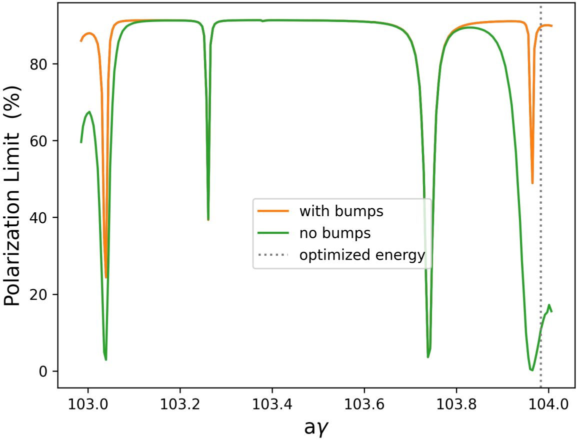
The harmonics which are adjacent to $a\gamma$ contribute most to the sum. FCC-ee (Z) operates between $a\gamma$ 103 and 104, so that a/b_{103} and a/b_{104} are to be suppressed using four closed bumps.

R. Schmidt, Polarisationsuntersuchungen am Speicherring PETRA. No. DESY-M-82-22. DESY, 1982.

Rossmanith-Schmidt scheme

$$45.82\,\mathrm{GeV}\ (a\gamma = 103.983)$$

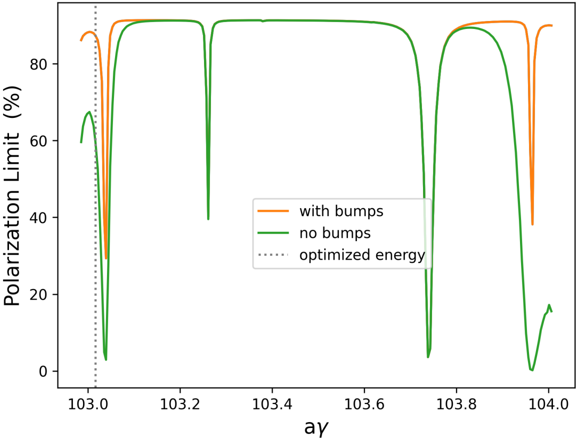
$$\delta n_0 : 2.28\,\mathrm{mrad} \Rightarrow 0.90\,\mathrm{mrad}\ ,\ P_{DK} : 10.68\% \Rightarrow 89.65\%$$



Rossmanith-Schmidt scheme

$$45.394\,\text{GeV}\ (a\gamma = 103.016)$$

$$\delta n_0 : 1.14\,\mathrm{mrad} \Rightarrow 0.98\,\mathrm{mrad}\ ,\ P_{DK} : 59.66\% \Rightarrow 87.35\%$$



Modified Rossmanith-Schmidt scheme

$$\int B_x(s)\mathrm{d}s \propto -\Delta y' \approx \sum_{i=1}^{N_{\mathrm{quad}}} k_{1i} y_i L_i + \sum_{j=1}^{N_{\mathrm{Vkicker}}} \mathrm{kick}_j$$

- Use y_{eff} to avoid the errors from thin lens approximation
  $y_{\mathrm{eff}} \approx \frac{1-\cos\sqrt{k}L}{\sqrt{k}L \sin\sqrt{k}L}(y_1 + y_2)$ $(k < 0)$ or $y_{\mathrm{eff}} \approx \frac{\cosh\sqrt{k}L - 1}{\sqrt{k}L \sinh\sqrt{k}L}(y_1 + y_2)$ $(k > 0)$
- If two BPMs are installed at both ends of each quadrupole
  Polarization 10.68% $\rightarrow$ 84.71%
- 3056 dipoles, 1856 quadrupoles $\rightarrow$ fewer BPMs required

LEP method

Assume radial fields in quadrupoles and proportional to the beam position. Analyze unweighted vertical BPM readings and minimize critical harmonics

$$a_k = \frac{1}{\pi} \sum_{i=1}^{N_{BPM}} y_i \cdot \Delta\theta_i \cdot \cos\left(k \cdot \theta_i\right)$$

$$b_k = \frac{1}{\pi} \sum_{i=1}^{N_{BPM}} y_i \cdot \Delta\theta_i \cdot \sin\left(k \cdot \theta_i\right)$$

If there is a BPM next to each quadrupole and functions properly

$$45.82\,\mathrm{GeV}\ (a\gamma = 103.983)$$

$$\delta n_0 : 2.28\,\mathrm{mrad} \Rightarrow 2.03\,\mathrm{mrad}\ ,\ P_{DK} : 10.68\% \Rightarrow 13.72\%$$

F. Sonnemann, Increase of spin polarization for energy calibration at LEP. Diss. Aachen, Tech. Hochsch., 1998.

Comparison

At 45.82 GeV ($a\gamma = 103.983$)

Method	$(\delta n_0)_{rms}$ (mrad)	Polarization (%)
no correction	2.28	10.68
HERA formalism	0.90	90.96
Rossmanith-Schmidt scheme	0.90	89.65
Modified R-S scheme	1.01	84.71
LEP method	2.03	13.72

Many questions remained regarding all three schemes

Pros and cons

1. HERA formalism
   - Pro: systematic and rigorous mathematical derivation
   - Con: empirically setting the bumps will be inevitable
2. Rossmanith-Schmidt scheme
   - Pro: based on the acquisition of a more measurable quantity
   - Con: BPMs at both ends of each dipole/quadrupole $\rightarrow$ extra cost
   - Con: restricted by BPM misalignments and calibration errors
3. LEP method
   - Pro: based on the real observables
   - Con: restricted by BPM misalignments and calibration errors

Outlooks

- Model the lattice using multiple **larger** error seeds, and estimate the maximum acceptable orbits that guarantee a sufficient polarization
  (collaborate with the FCC tuning group)
- Complete the harmonic spin matching schemes
  (collaborate with HSM experts)
  - solve the remaining questions regarding the three schemes
  - find an effective method that relies on the analysis of real observables
  - test its effectiveness under different lattice conditions

Thank you!

Remained problems of HSM

- What's the harmonics that should be corrected in the simplified HERA formalism
- Whether there is a way to extract $\Delta y'$ information from vertical BPM readings in quadrupoles
- How to make LEP method work
- If it's possible to correct vertical resonance (not HSM)
- What will happen if the errors are much larger
- What will happen near other integers besides 103 and 104

Backup

	τ_{ST}	τ_{BKS}	τ_{dep}	
FCC-ee $(\Delta y)_{\mathrm{rms}} = 72\,\mu\mathrm{m}$	11 779 min	11 773 min	$4.26 \times 10^6\,\mathrm{min}$	90 min for 10% with wigglers (CDR)
HERA (26.7 GeV)	$\sim 43\,\mathrm{min}$	$\sim 40\,\mathrm{min}$	$\sim 10\,\mathrm{min}$	$\tau_{dk} \sim 8\,\mathrm{min}$
LEP	$\sim 310\,\mathrm{min}$ 46 GeV	—	$\sim 24\,\mathrm{min}$ 46.5 GeV	30 min for 10% no wigglers

in LEP $(\Delta y)_{\mathrm{rms}} = 530\,\mu\mathrm{m}$, $(\eta_y)_{\mathrm{rms}} = 13\,\mathrm{cm}$

Backup

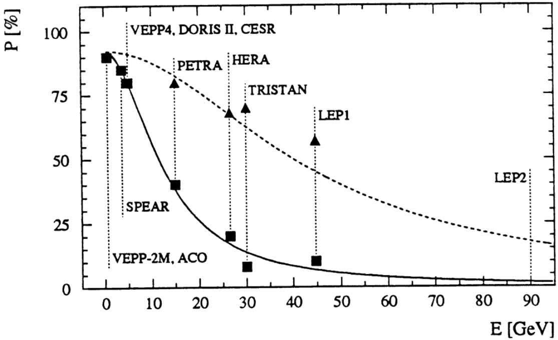
$$P(t) = P_{dk}\left[1 - e^{-t/\tau_{dk}}\right] + P_0 e^{-t/\tau_{dk}} \simeq P_0 e^{-t/\tau_{dep}}$$

Backup

Systematic errors of the average beam energy determination

- Energy dependent momentum compaction
- Vertical orbit distortions (radial fields)
- Longitudinal fields

Backup



Maximum measured polarization in different storage rings with HSM (triangles) and without HSM (squares)

R. W. Assmann, et al. Polarization Studies at LEP in 1993. No. CERN-ALEPH-PUB-94-135. CM-P00061204, 1994.

Possible problems with LEP method

- radial fields not only exist in quads
- the radial field seen by the particle is not fully proportional to the y position
- even if it's proportional, each quad has different strength (ky)
- how much spin rotates is an integration of radial field within the element (kyL)