Section 3.2.3 – Amortisation Table & Reducing Balance Loans

VCAA "Dot Points"

Reducing balance loans (compound interest loans with periodic repayments), including:

- use of a first-order linear recurrence relation to model and analyse (numerically and graphically) the amortization of a reducing balance loan, including the use of a recurrence relation to determine the value of the loan or investment after n payments, including from first principles for n ≤ 5
- use of a table to investigate and analyse the amortisation of a reducing balance loan on a step-by-step basis, the payment made, the amount of interest paid, the reduction in the principal and the balance of the loan

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Reducing Balance Loans

When you take out a **loan** with a bank, or other financial institute, they will provide you access to funds and **charge you interest** for the service.

Just as a savings account gains interest over time, therefore making your savings grow. A **loan account** with be charged interest making your **debt grow**.

Example 1

James takes out a loan say for $1000 at 6% p.a. compounded annually and made no additional payments. His debt would continue to grow each and every year. The table below shows his growing debt over the first five years of such an arrangement.

Number of payments	Payment amount ($)	Interest paid ($)	Loan balance ($)
0	0	0	***$ 1,000.00***
1	$ -	$ 60.00	$ 1,060.00
2	$ -	$ 63.60	$ 1,123.60
3	$ -	$ 67.42	$ 1,191.02
4	$ -	$ 71.46	$ 1,262.48
5	$ -	$ 75.75	$ 1,338.23

Calculations

Loan balance (1) = Loan Balance (0) + Interest = $1000.00\left(1+\frac{6}{100}\right) = \$1{,}060.00$

Loan balance (2) = Loan Balance (1) + Interest = $1060.00\left(1+\frac{6}{100}\right) = \$1{,}123.600$

Loan balance (3) = Loan Balance (2) + Interest = $1{,}123.60\left(1+\frac{6}{100}\right) = \$1{,}191.02$

Loan balance (4) = Loan Balance (3) + Interest = $1{,}191.02\left(1+\frac{6}{100}\right) = \$1{,}262.48$

Loan balance (5) = Loan Balance (4) + Interest = $1{,}262.48\left(1+\frac{6}{100}\right) = \$1{,}338.23$

From the above table it can be seen that your debt has risen from an original **(V_0) of $1000** to a value of **$1338.23 after 5 years (V_5).**

Example 2

What would happen if James instead paid $237.40 at the end of each year in an attempt to pay off the reducing balance loan? Construct an amortization table for this new arrangement.
(**NB:** His loan is still for $1000 at 6% p.a. compounded annually)

Number of payments	Payment amount ($)	Interest paid ($)	Principal reduction ($)	Loan balance ($)
0	0	0	0	***$ 1,000.00***
1	$ 237.40	$ 60.00	$ 177.40	$ 822.60

1st payment

$Interest = 1000\left(\frac{6}{100}\right) = \mathbf{\$60.00}$

$Principal\ reduction = Payment\ amount - Interest\ paid$
$= \$237.40 - \$60 = \mathbf{\$177.40}$

$Loan\ balance\ after\ 1^{st} payment = starting\ balance - principal\ reduction$
$= \$1000.00 - \$177.40 = \mathbf{\$822.60}$

Number of payments	Payment amount ($)	Interest paid ($)	Principal reduction ($)	Loan balance ($)
0	0	0	0	***$ 1,000.00***
1	$ 237.40	$ 60.00	$ 177.40	$ 822.60
2	$ 237.40	$ 49.36	$ 188.04	$ 634.56

2nd payment

$Interest = 822.60\left(\frac{6}{100}\right) = \mathbf{\$49.36}$

$Principal\ reduction = Payment\ amount - Interest\ paid$
$= \$237.40 - \$49.36 = \mathbf{\$188.04}$

$Loan\ balance\ after\ 2^{nd}\ payment = balance\ after\ 1^{st}\ payment - principal\ reduction$
$= \$822.60 - \$188.04 = \mathbf{\$634.56}$

Number of payments	Payment amount ($)	Interest paid ($)	Principal reduction ($)	Loan balance ($)
0	0	0	0	$ 1,000.00
1	$ 237.40	$ 60.00	$ 177.40	$ 822.60
2	$ 237.40	$ 49.36	$ 188.04	$ 634.56
3	$ 237.40	$ 38.07	$ 199.33	$ 435.23

3rd payment

$Interest = 634.56\left(\frac{6}{100}\right) = \$38.07$

$Principal\ reduction = Payment\ amount - Interest\ paid$
$= \$237.40 - \$38.07 = \$199.33$

$Loan\ balance\ after\ 3^{rd}\ payment = balance\ after\ 2^{nd}\ payment - principal\ reduction$
$= \$634.56 - \$199.33 = \$435.23$

Number of payments	Payment amount ($)	Interest paid ($)	Principal reduction ($)	Loan balance ($)
0	0	0	0	$ 1,000.00
1	$ 237.40	$ 60.00	$ 177.40	$ 822.60
2	$ 237.40	$ 49.36	$ 188.04	$ 634.56
3	$ 237.40	$ 38.07	$ 199.33	$ 435.23
4	$ 237.40	$ 26.11	$ 211.29	$ 223.94

4th payment

$Interest = 435.23\left(\frac{6}{100}\right) = \$26.11$

$Principal\ reduction = Payment\ amount - Interest\ paid$
$= \$237.40 - \$26.11 = \$211.29$

$Loan\ balance\ after\ 4^{th}\ payment = balance\ after\ 3^{rd}\ payment - principal\ reduction$
$= \$435.23 - \$211.29 = \$223.94$

Number of payments	Payment amount ($)	Interest paid ($)	Principal reduction ($)	Loan balance ($)
0	0	0	0	***$ 1,000.00***
1	$ 237.40	$ 60.00	$ 177.40	$ 822.60
2	$ 237.40	$ 49.36	$ 188.04	$ 634.56
3	$ 237.40	$ 38.07	$ 199.33	$ 435.23
4	$ 237.40	$ 26.11	$ 211.29	$ 223.94
5	$ 237.38	$ 13.44	$ 223.94	-$ 0.00

5th payment

$Interest = 223.94\left(\frac{6}{100}\right) = \$13.44$

$Principal\ reduction = Payment\ amount - Interest\ paid$
$= \$237.38^{*} - \$13.44 = \$223.94$

$Loan\ balance\ after\ 5^{th}\ payment = balance\ after\ 4^{th}\ payment - principal\ reduction$
$= \$223.94 - \$223.94 = \$0.00$

* *The final payment of 237.40 has been adjusted to account for rounding errors.*

Amortisation Summary

Number of payments	Payment amount ($)	Interest paid ($)	Principal reduction ($)	Loan balance ($)
0	0	0	0	***$ 1,000.00***
1	$ 237.40	$ 60.00	$ 177.40	$ 822.60
2	$ 237.40	$ 49.36	$ 188.04	$ 634.56
3	$ 237.40	$ 38.07	$ 199.33	$ 435.23
4	$ 237.40	$ 26.11	$ 211.29	$ 223.94
5	$ 237.38	$ 13.44	$ 223.94	-$ 0.00
Total	**$ 1,186.98**	**$ 186.98**	**$ 1,000.00**	

Exam Styled Questions – Multiple Choice

Question 1
(2016 VCAA Exam 1 Section A - Qn 22)

The first three lines of an amortisation table for a reducing balance home loan are shown below.
The interest rate for this home loan is 4.8% per annum compounding monthly.
The loan is to be repaid with monthly payments of $1500.

Payment number	Payment	Interest	Principal reduction	Balance of loan
0	0	0.00	0.00	250 000.00
1	1500	1000.00	500.00	249 500.00
2	1500			

The amount of payment number 2 that goes towards reducing the principal of the loan is

A. $486
B. $502
C. $504
D. $996
E. $998

B

Interest rate (% per month) $= \frac{4.8}{12} = 0.4\%\ p.m.$

Interest paid for 2nd payment $= 249500.00\left(\frac{0.4}{100}\right) = \$998.00$

$$Principal\ reduction = Payment\ amount - Interest\ paid$$
$$= \$1500.00 - \$998.00 = \$502.00$$

$\therefore$ **Option B**

Question 2
(2018 VCAA Exam 1 Section A - Qn 23)

Five lines of an amortisation table for a reducing balance loan with monthly repayments are shown below.

Repayment number	Repayment	Interest	Principal reduction	Balance of loan
25	$2200.00	$972.24	$1227.76	$230 256.78
26	$2200.00	$967.08	$1232.92	$229 023.86
27	$2200.00	$961.90	$1238.10	$227 785.76
28	$2200.00	$1002.26	$1197.74	$226 588.02
29	$2200.00	$996.99	$1203.01	$225 385.01

The interest rate for this loan changed immediately before repayment number 28.
This change in interest rate is best described as

A. an increase of 0.24% per annum.
B. a decrease of 0.024% per annum.
C. an increase of 0.024% per annum.
D. a decrease of 0.0024% per annum.
E. an increase of 0.00024% per annum.

A

Change in interest rate (% p.a.)
= 5.28 % p.a. (28^{th}) – 5.04 % p.a. (26^{th})
= +0.24 % p.a.

Calculate interest rate (% p.a.) for 26^{th} payment:

$$\text{solve}\left(\frac{x}{100}\cdot 230256.78=967.08,x\right)$$
$$x=0.420001$$
$$\left(x=0.42000066186976\right)\cdot 12 \qquad 12.\cdot x=5.04001$$

Calculate interest rate (% p.a.) for 28^{th} payment:

$$\text{solve}\left(\frac{x}{100}\cdot 227785.76=1002.26,x\right)$$
$$x=0.440001$$
$$\left(x=0.44000116600792\right)\cdot 12 \qquad 12.\cdot x=5.28001$$