PLP 6404 Epidemiology of Plant Diseases
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Lecture 11: Disease progress in time: simple models

Prof. Dr. Ariena van Bruggen
Emerging Pathogens Institute and
Plant Pathology Department, IFAS
University of Florida at Gainesville

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Overview

- Review of lecture 3 (disease progress curves)
- Linear and exponential growth of capital (simple interest and compound interest)
- The monomolecular equation
- The logistic equation
- The Gompertz equation
- Summary

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Review of lecture 3: progress curves

- The most commonly considered disease progress curves:
  - linear (rare and only early on)
  - exponential (common in the beginning)
  - saturation or monomolecular curve (common)
  - logistic curve (common).
- Saturation in the monomolecular and logistic curves is caused by;
  - a lack of healthy plant tissue
  - unsuitable conditions for further infection in midseason.

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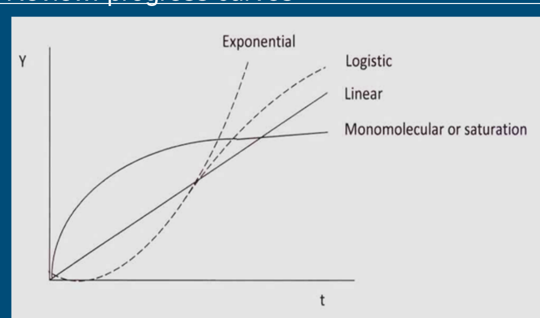
Review: progress curves



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Review: progress curves

- The monomolecular curve is close to linear in the very beginning and then curves down; it is similar to a curve for an enzymatic reaction described by Michaelis-Menten or Monod
- The logistic curve consists first of an exponential phase, followed by a very brief linear phase at the inflection point and then a saturation phase.

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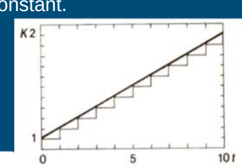
Review: simple interest, linear growth

- If interest = 5% annually, the annual rate of increase r equals 0.05
- If interest is put in a drawer at the end of the year, the total capital grows at a constant rate over years
- Kt = K0 * (1+ r*t) [N]
- The average growth rate is constant.
- dK / dt = r [N.T^{-1}]



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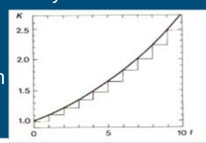
Review: compound interest, exponential growth

- If the interest is added to the capital annually, the interest earned will itself earn interest in subsequent years
- The growth of the capital accumulates stepwise at payment dates with compound interest
- $Kt = K0 * (1+r)^t$ [N]
- When the time unit is much smaller than a year
- $Kt = K0 * e^{r.t}$ [N]
- e = base of natural logarithm (~ 2.7)
- r = interest rate expressed as fraction



Review: compound interest, exponentail growth

- The growth rate of the capital is:
- dK / dt = r*K [N.T^{-1}]
- Taking logarithms on both sides of the equal sign:
- loge K_t = r*t + loge K_0 or loge K_t = loge K_0 + rt
- Loge K_t – loge K_0 = rt
- r = 1/t * loge K_t /K_0 [T^{-1}]
- r is relative growth rate = a constant

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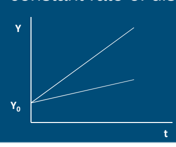
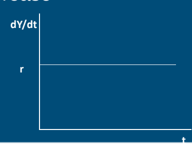
Linear disease progress curves

- Instead of K_0 and K_t as used by Zadoks when talking about growth of capital, other plant pathologists use Y_0 and Y_t for disease responses
- Linear equation:
  - integrated form: $y = y_0 + r\,t$
  - differential form: dy/dt = r
- constant rate of disease increase





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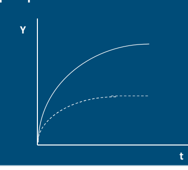
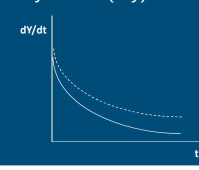
Monomolecular disease progress curves

- Monomolecular equation:
  - integrated form: $y = 1- (1-y_0)\, e^{-rt}$
  - linearized form: $\ln[1/(1-y)] = \ln[1/(1-y_0)] + r\,t$
  - differential form: dy/dt = r(1-y)
- The absolute rate of disease increase (dy/dt) is proportional to the amount of healthy tissue (1-y)





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Monomolecular equation calculations

- Calculate the values of y_1, y_2, r_m, and Δt using a calculator
- Level of disease at time2, y_2:
  - $\ln [1/(1-y_2)] = \ln [1/(1 - y_1)] + r_m \Delta t$
- The monomolecular rate, r_m :
  - $r_m = \{\ln [1/(1-y_2)] - \ln [1/(1-y_1)]\} / \Delta t$
- The time interval, Δt :
  - $\Delta t = \{\ln [1/(1-y_2)] - \ln [1/(1-y_1)]\} / r$

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Exponential disease progress curve

- The exponentail equation (compound interest) is:
  - $y_t = y_0 * \exp(r * t)$.
- The "exp" in the equation is for "exponentiation"; the inverse is "natural logarithms" (to the base "e"; i.e., 2.71828); expressed as $\log_e$; or commonly as "ln".
  - integrated form: $y = y_0\, e^{rt}$ (e is base of natural log)
  - linearized form: $\ln(y) = \ln(y_0) + r\,t$
  - differential form: dy/dt = r y
- absolute rate of disease increase is proportional to the amount of disease

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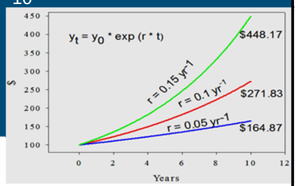
Exponential equation calculations

- Example exponential increase for 10 yr:
  - $y_t = y_0 * \exp(r * t)$.
- For $y_0 = 100$, $r = 0.10$, and $t = 10$:
  - $y_t = 100 * \exp(0.10 * 10) = 271.83$
- Or $271.83 = 100 * \exp(0.1*10)$
- Log-transformed): $\ln(y_t) = \ln(y_0) + r * t$;
  - $\ln(271.83) = \ln(100) + 0.1 * 10$
  - $5.60517 = 4.60517 + 1.0$



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Exponential equation calculations

- Sometimes we know the beginning amount of disease and the amount at a later time, but we may not know the exact value of "r".
- For $y_0 = 100$, $r = 0.10$, and $t = 10$:
- Solve the equation for r:
  - $y_t = y_0 * \exp(r * t) \rightarrow \ln(y_t) = \ln(y_0) + r * t$
  - $r = [\ln(y_t) - \ln(y_0)] / t$
  - $r = [\ln(271.83) - \ln(100)] / 10$
  - $r = [5.60517 - 4.60517] / 10$
  - $r = 0.10$

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Logistic disease progress curve

- Start with the exponential growth curve, but the healthy area may become more and more limited until the carrying capacity is reached
- Add the correction factor $(1-y)$ for the limit of available host tissue to the exponential rate equation
- The logistic rate equation:
  - $dy_t/dt = r\, y_t\, (1 - y_t)$
- The logistic model equation:
  - $y_t = 1/ [1 + B \exp(-r\, t)]$ where $B = (1-y_0)/ y_0$
- The logistic transformation equation:
  - $Y_t = \ln [y_t/(1-y_t)]$.

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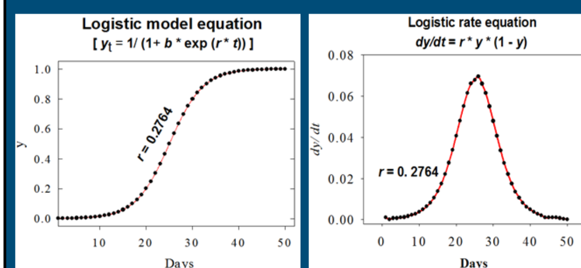
Integrated logistic curve and derivative



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Comparison of exponential and logistic equations

- Exponential
  - integrated form: $y = y_0\, e^{rt}$
  - linearized form: $\ln(y) = \ln(y_0) + r\, t$
  - differential form: $dy/dt = r\, y$
- Logistic
  - integrated form: $y = \frac{1}{1 + \left(\frac{1-y_0}{y_0}\right) e^{-rt}}$
  - linearized form: $\ln[y/(1-y)] = \ln[(y_0/(1-y_0)] + rt$
  - differential form: $dy/dt = ry\,(1-y)$

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Logistic equation calculations

- The logistic transformation equation:
- $Y_t = \ln [y_t/(1-y_t)]$ is used to:
- calculate epidemic rates (r):
  - $r = [\text{logit}(y_2) - \text{logit}(y_1)] /(t_2 - t_1)$
- predict future disease (y_2):
  - $\text{logit}(y_2) = \text{logit}(y_1) + r\,(t_2 - t_1)$
- estimate initial disease (y_1):
  - $\text{logit}(y_1) = \text{logit}(y_2) - r\,(t_2 - t_1)$
- determine the time interval (Δt) between disease levels:
  - $(t_2 - t_1) = [\text{logit}(y_2) - \text{logit}(y_1)] / r$

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Logistic disease progress

- There is a limit to how fast an epidemic can go
- the latent period "*p*" sets the limit.
- When *p* is short, *r* is usually fast; when *p* is long, *r* is usually slow.
- The product of *p* r is* called the "explosiveness" of the epidemic.
- The limits of *p * r* are in the range of 0.0 to 6.0
- Examples:

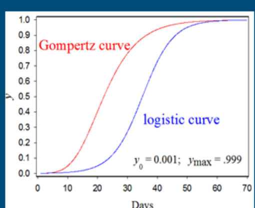
Examples:	*p*	*r*	*p* r*	Result
Late blight:	4	0.4	1.6	somewhat explosive
Bean rust	10	0.4	4.0	very explosive
Leaf spots	10	0.1	1.0	Not explosive
Wheat rust	10	0.6	6.0	Extremely explosive

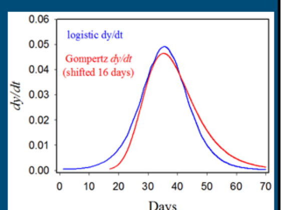
Gompertz disease progress curve

- Similar to the logistic curve, but assymmetric
- With a longer 'tail' than the logistic curve
- Mostly a better fit to real epidemic data than the logistic curve
- The Gompertz rate equation (differentiated form):
  - dy/dt = ry [-ln(y)]
- The Gompertz model equation (integrated form):
  - $y = y_0^{e^{-rt}}$
- The Gompertz transformation equation:
  - *Y* = -ln [-ln(*y*)].

Comparison of Gompertz and logistic curves

- Gompertz faster in the beginning and longer tail at the end
- The Gompertz rate curve is skewed to the left; the logistic rate curve is symmetric





Comparison of logistic and Gompertz equations

- Logistic
  - integrated form: $y = \frac{1}{1+\left(\frac{1-y_0}{y_0}\right)e^{-rt}}$
  - linearized form: $\ln[y/(1-y)] = \ln[y_0/(1-y_0)] + rt$
  - differential form: dy/dt = ry (1-y)
- Gompertz
  - integrated form: $y = y_0^{e^{-rt}}$
  - linearized form: $-\ln[-\ln(y)] = -\ln[-\ln(y_0)] + r\,t$
  - differential form: dy/dt = ry [-ln(y)]

Calculations with the Gompertz equation

- Epidemic rates for the Gompertz function are calculated with gompits, just like with logits.
- With **logits** (ln[y/(1-y)]) **:**
- $r_l = [\text{logit}\ (y_2) - \text{logit}\ (y_1)]\ /\ (t_2 - t_1)$.
- With **gompits** (-ln[-ln(y)]) **:**
- $R_g = [\text{gompit}\ (y_2) - \text{gompit}\ (y_1)]\ /\ (t_2 - t_1)$.

Summary

- Linear and exponential growth of capital (simple interest and compound interest)
- The monomolecular equation
- The logistic equation
- The Gompertz equation
- What are the characteristics of the curves?
- How do the equations differ?
- How do you calculate the epidemic rates?