Partial Correlation Network analysis

Is IBEX intercorrelated ?

The project tries to capture interdependencies across the IBEX 35 analysing its closing returns during the last seven years and evaluate it using the LASSO estimation technique in partial correlation networks and Garch residuals

Carles Sans Fuentes

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1. Executive Summary

This final degree was an opportunity for me to investigate in one of the most interesting fields from economics: the analysis of real financial data with statistics for the understanding of the stock market. The methods of data analysis, specifically econometrics, statistics or data analysis have evolved exponentially during these last decades because of the recent availability of huge amounts of information. Thanks to computer science and the huge amounts of available data (which is also called Big Data), the evaluation of these large quantity of information and the extraction of meaningful results can be accomplished cost effectively. This assessment given by data analysts provides valuable information to public and private companies to better manage their resources. The scope of this final degree project is to learn how to use one of the tools used in Big Data analysis as well as applying it to the project: Partial Correlation Networks with a multivariate time series analyzed with the LASSO estimation technique.

Firstly, the features of the instruments used in the project are explained. Specifically, the statistical models and tools used to produce illustrated network graph and tables that displays the correlations among different elements of the data set. This has also been performed with Garch (1,1), which is a model that fits quite well financial data. Finally, empirical data analysis has been made in order to conclude the project with some findings that can give interesting results for the understanding of the data set used.

The project results indicate the Partial Correlation networks by Joint Sparse Regressions Models approach presented by Peng et al. (2009) is an effective analysis tool. This method does assume data sparsity.. Additionally, conditional temporary dependence across different variables in time has proved to work well under different data sets, and also in the data from this project.

Out of the many tools and techniques available, this project implemented only the LASSO estimation technique in partial correlation networks using returns on financial data and Garch residuals. For this reason, the scope of this project is limited. There different areas and analysis that could be done to the same data, so that it has just been taken some of them that are considered relevant. Also, the choice of giving values to the parameter lambda is also limited since it has been analyzed only eight of them. For these reasons, the project is limited and deeper analysis on the data could be done in order to be more complete.

The purpose of this project is to report an analysis about a multivariate time series data set and understanding how the tools described above are used and implemented.

2. Network analysis

2.1 Introduction

In a 2005 New York Times article, Matthew Lesko said that *"Information is the currency of today's world. Those who control the information are the most powerful people on the planet and the ones with the most bulging bank accounts*[1]*."*

In the last decades, thanks to the development of computer processing power and the Internet network, huge loads of information are available on the Internet. In both the private and public sector, the analysis of large data sets with computer software, such as the effect of subvention policies from one state to another one, the percentage rate

[1] "Nothing is Secret Anymore!" - The Confessions of a Millionaire Information Broker" by Matthew Lesko. Copyright © 2005 Shareware123 Network

change of unemployment from Spain by the creation of jobs in France, the difference of the incomes provided by different firms related to the decision choice of the worker or the devaluation of a currency because of macro news, is a tool which is becoming more and more important.

Because of this huge amount of existing and available data the large quantity of different data sets presents a new challenge for analysists. “Infobesity” or “Infoxication” are trending words used to describe this phenomenon. Hence, the ability extract and analyze useful data from these sets, is becoming a valuable skill set. The potential for big data analysis in the fields such as biotechnology, economics or physics has a huge potential to lead to many previously unattainable innovations and breakthroughs.

The power of information relies on the usage that is given to it. Storing information does not give value for the society per se. Big Data sets must be analyzed in order to be profitable, and this is one of the challenges that this project pursues.

Currencies exchange rates, stock market returns of Dow Jones, average income per capita in poor countries or demand for fuel in Spain are some examples of different available data sets. Finding interconnections within the same data set or looking for interdependencies across markets can yield valuable results.

For this reason, the field Network Analysis is being developed during this last decade. The aim of Network Analysis is to identify hidden interdependencies within datasets---multivariate time series. This type of analysis is a challenging task, even with the computing available today.

Statistians and econometricians have different tools to overcome the challenges associated with large amounts of data, also called "information overload"[2]. One such tool is (insert name), which identifies hidden interconnections through the visual analysis of graphs and networks.

2.2 Networks

Graphs

In mathematics, we use graphs for the representation of different data. The purpose of this is to be able to evaluate and illustrate in an easier way the findings of the data collected. Essentially, a graph is a representation of nodes connected by lines. Nevertheless, a graph can have different characteristics. Depending on what you want to assess with your data, graphs can provide information among a great number of features. In this project, though, I am only going to explain those characteristics that concerns my analysis of the data in order to understand better the connections, taking into account the advantages but also the limitations of the data analyzed in this way .

$$G = (\vartheta, \varepsilon)$$

A graph is composed of two elements: Vertices (ϑ) or nodes and edges (ε) or lines. A vertice is a representation of an element or different elements in a graph, i.e. a currency in the stock market. Edges can connect different nodes in a graph. By doing this, there exists some kind of relation between these two vertices in the graph. The set of edges is defined as $E \subseteq V \times V$ $(i,j) \in E \Longleftrightarrow$ i and j are connected by an edge.

Characteristics of Graphs

A graph can be sorted (classified) on several features. The ones which are significant for the scope of this project are *directionality, weight and color.*

[2] Toffler 1970

Directionality of the edges: directionality defines the nature the relation between two vertices. When two vertices are connected to each other by an edge, this stands for a relation. In undirected graphs, the direction of this relation it is not taken into account and it is supposed that there is double causation, whereas in directed graphs, this relation is represented by an arrow instead of a line. This arrow can give direction in one way or in both, depending on the existing association between the vertices. There also exist mixed graphs, in which directed and undirected edges are represented.

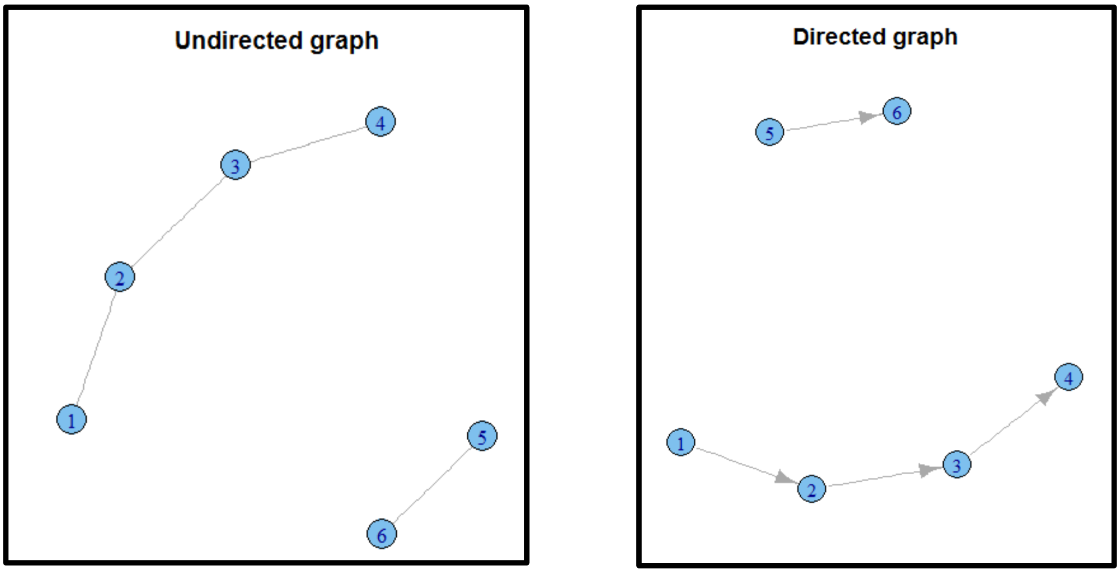

Figure 1. Undirected and directed network graphs

Weight of the edges: A weighted graph assigns a number (weight) to the edge connecting two vertices. These numbers might refer to lengths, capacities or costs of it. On the other hand, an unweighted graph gives the same weight to each edge of the network. This means that all edges across the graph are equally meaningful, e.g. the correlation of different currencies.

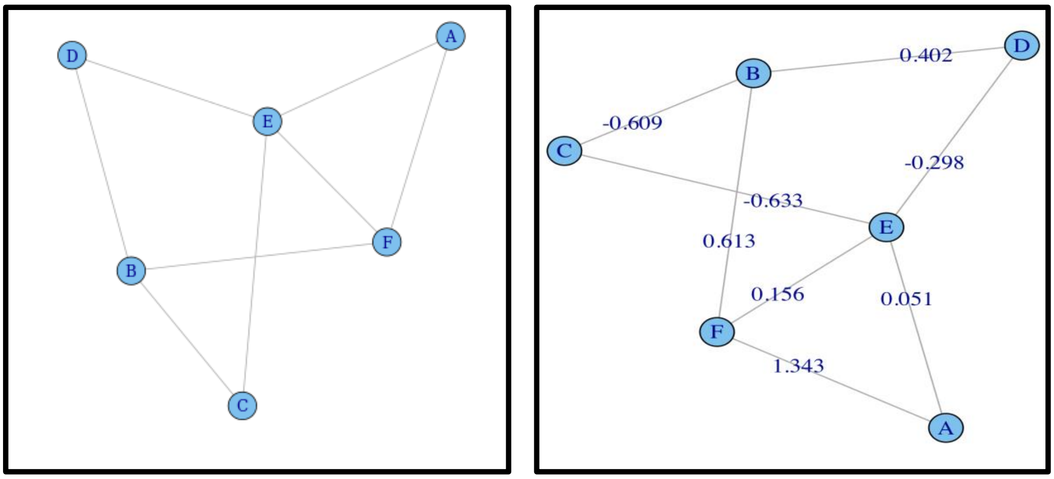

Figure 2. Weighted and unweightednetwork graphs

Color of vertices and edges: Colors in graphs are used to distinguish easily different parts of the graphs that you are interested in examining. Thanks to this, you can classify and ease the analysis of data; for example to distinguish different features within the same graph, i.e. to differentiate regions.

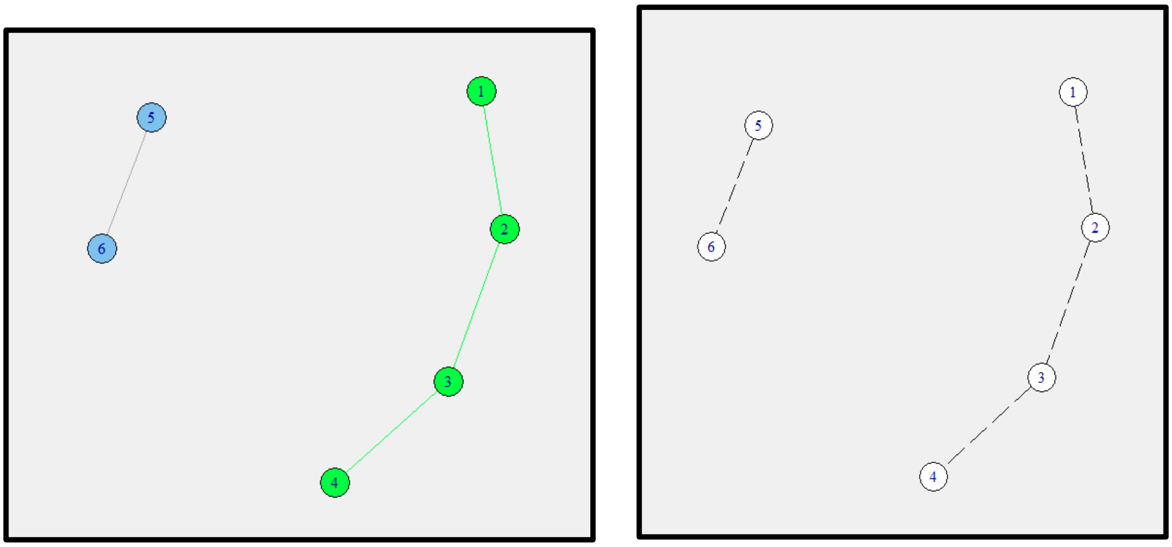

Figure 3. Colored and uncolored weighted networks graphs

Adjacency Matrix

Sometimes, graphs are not so easy to understand and it can be difficult to provide an accurate explanation of it. What it is shown in a graph can be also stored in a Matrix. From this Matrix, we can analyze in more detail the information of the graph. This Matrix is called the Adjacency matrix. If the graph has n vertices, the Adjacency matrix is defined to be a |n| x |n| matrix such that if vertices i and j are connected, by an edge, then the value of A_{ij} in the matrix is non- zero[3].

In the case of the graph being undirected, the Adjacency matrix will be totally symmetric, whether it is weighted or not. Moreover, in the Adjacency matrix, it is shown the existing weight and correlation of between two variables in the matrix. As an example to understand it better, see figure 4 below.

	1	2	3	4	5	6	7
1	0	8	0	1	0	10	0
2	8	0	1	15	0	1	0
3	0	1	0	0	0	0	0
4	1	15	0	0	1	5	0
5	0	0	0	1	0	7	0
6	10	1	0	5	7	0	1
7	0	0	0	0	0	1	0

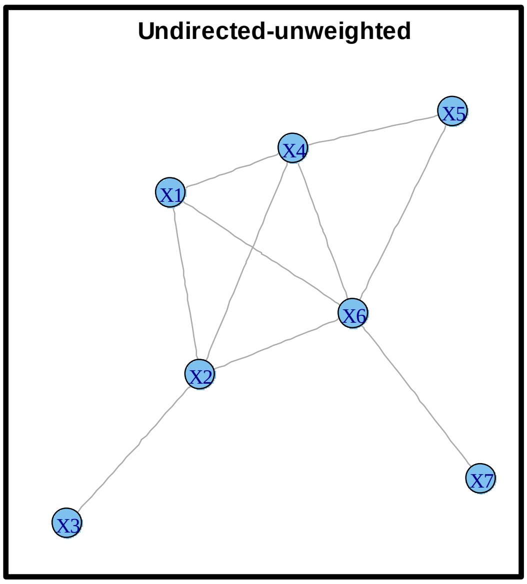

Figure 4. Adjacency matrix and undirected-unweighted graph

[33] The elements in the Diagonal of the Adjacency matrix are exactly zeros. For this reason, a node cannot

2.3 Network Analysis of Multivariate Time Series

One of the most important objectives of network analysis is to represent large amounts of data gathered to ease the forward interpretation of it. In this project, the large amount of data taken into account is a Multivariate Time Series.

A multivariate time series is defined as a compilation of values from different variables taken in successive periods of time, i.e. the compilation of n data values of M variables for T intervals of time.

$$y_t = \begin{bmatrix} y_{t1} \\ y_{t2} \\ \vdots \\ y_{tN} \end{bmatrix} \; for\ t = 1, .., T$$

In the expression above, y_t denotes a variable M for sequential data time from 1 until T. Here we have only written one variable but there can be many more .e.g. the daily sales of a bakery during one year for bread, croissants, muffins and donuts. In this case, the Multivariate time series has M=4 and T=365, so that n=M x T= 4 x 365.

White noise

In order to get a good analysis of the data, we should take into account a concept that is going to be relevant for the scope of this project. White noise is a random signal characterized by spectrum of discrete signals which are uncorrelated and have mean equal to zero and finite variance. A white noise process is a series of observations of some variables that have these characteristics. In this case, it is assumed that the white noise is identically, and normally distributed. This particular case is called to be a Gaussian White Noise[4]. The white noise is defined as:

[4] Diebold, Frank (2007). *Elements of Forecasting* (Fourth ed.).

$$E(y_t) = 0 \; for \; all \; t$$

$$Var(y_t) = \sigma^2 \; for \; all \; t$$

$$Cov(y_t, y_{t-s}) = 0 \; for \; t \neq s \; and \; s \neq 0$$

2.4 Partial correlation network

Types of Networks

Now that I have discussed different types of graphs to represent a network, it is important to point out the existence of difference of types of networks that can be shown also in a graph. A list of them is provided below:

- Partial Correlation Network: Dempster (1973), Lauritzen (1996), Meinshausen & Bühlmann (2006), Peng, Wang, Zhou, Zhu (2009)
- Granger Network: Billio, Getmanksi, Lo, Pellizzon (2012), Diebold and Yilmaz (2011)
- Long Run Partial Correlation Network: Davis, Zhang, Zheng (2012), Brownlees & Barigozzi (2012)
- Tail Dependence Networks: Hautsch, Schaumburg, Schienle (2011)

Partial correlation networks measures conditional temporary dependence across different variables in time. Granger Causality networks try to predict values of some variables based on previous data but only with dynamic linkages. Each of these two systems has its own pros and cons. In order to join these two abilities in one network, long partial correlation networks are being developed which have proven effective in both cases. Finally, Tail dependence networks evaluate how likely is that the boundary will be surpassed given that another threshold of the same data has already surpassed it.

For the purpose of this project, it is going to be learnt how partial correlation networks work, and I will perform further analysis with it. The usage of the other networks is beyond the scope of this project. Nevertheless, deeper analysis could be done by using these other models.

Definition

As it has been already mentioned, partial correlation networks measure temporary dependence across different variables in time. Y_t denotes a multivariate white noise process (defined in the previous part).

This partial correlation network can be represented in a graph following these two conditions:

1) Each element of y_t is represented as a node in the graph, i.e. several vertices in a graph.
2) An edge connecting two different vertices i and j gives partial correlation between these two given all the other elements of the graph.

In this project, the edges connecting vertices in a network graph are unweighted and undirected because in partial correlation networks, one is not interested in finding causality[5]. For this reason, I being partially correlated with j has the same meaning as saying that j is partially correlated with i (we imply both directions). Moreover, the algorithm illustrating our network graph does not take into account weights. It just assigns zero or one, the former showing partial independence (no correlation) and the latter partial dependence (correlation).

[5] For assessing causality we would use Granger causality (already explained in different types of networks). In this project we are only considering Intertemporal relations.

Partial correlation (ρ^{ij}) measures *cross-sectional linear dependence* between the elements y_{ti} and y_{tj} considering all the other variables:

$$\rho^{ij} = Cor\left(y_{ti,}y_{tj}|\{y_{tk}: k \neq i,j\}\right)$$

Relating it to its graphical representation, partial correlation networks are also formalized as the following:

$$\mathcal{E}_{PC} = \left\{\{i,j\} \in \vartheta x \vartheta | p^{ij} \neq 0\right\}$$

This formula simply means that there exists partial correlation between i and j belonging to the network given that the correlation between them is different from zero. This is really useful, since it gives the opportunity to provide good explanations for big data that otherwise it would be difficult to interpret. Nevertheless, it should be taken into account that sometimes these graphs representation of the network could be difficult to interpret because of its simplicity[6]. Later on, I will discuss the pros and cons of the partial correlation network, in order to understand better the problems and mistakes that can arise from our assessment of the data if they are not taken into account.

Understanding Linear Regressions

Partial Correlation Networks have a relation with linear regression models. To better understand this, consider the following example:

$$y_{1t} = c + \alpha_{12}y_{2t} + \alpha_{13}y_{3t} + \alpha_{14}y_{4t} + \mu_{1t}$$

$$y_{2t} = c + \alpha_{22}y_{2t} + \alpha_{23}y_{3t} + \alpha_{24}y_{4t} + \mu_{2t}$$

[6] Simplicity in this case stands for the explanation that the simplicity of these gaphs can give misleading reasonings about the data.

Here we have a multivariate time series with only two regressors. Suppose α_{13} is not zero i.e. one and three are partially correlated. Then, the partial correlation of y_{ti} and y_{tj} is comparatively described as linear correlation of its residuals of y_{ti} and y_{tj} , which are obtained from the regression of this two elements given all the other variables. For this reason, if we consider that there is a correlation path[7] between vertices i and j, then they are correlated. Intuitively, the residuals of two variables are randomly displayed so that if there is a correlation between these residuals, there must exist a correlation between these variables.

Characterizing the Partial Correlation Network

Partial correlation networks are defined as the inverse of the covariance matrix of the white noise process, which is also called the Concentration Matrix.

The white process y_t has a variance denoted by Σ. Also, k_{ij} indicates the (i,j) element of K = Σ−1. Then, $p^{ij} = \frac{-k_{ij}}{\sqrt{k_{ii}} k_{jj}}$

This suggests that the long run partial correlation network is totally represented by K. If k_{ij} is different from zero, then there is a link between vertices i and j. For this, the long partial correlation network can be expressed as the estimation of a concentration matrix.

Pros and Cons of Partial Correlation Networks

As with each type of network, partial correlation networks have their characteristics and its limitations.

The advantages of using Partial Correlation networks are essentially the possibility of being represented as a graph and the ease of interpretation. Of course, these are not

[7] A path stands for the sequence of adjacent edges accross the network (A and B are correlated and B and C as well; then the path of A to C is A-B-C).

advantages specifically of our model but of all networks and graph representation of networks. Even with this, however, the interpretation might be a challenge. The usage of the tools for interpreting could be misleading, due to the fact that real data is highly complex. We try to simplify it with different techniques explained in the following section: the sparsity of networks.

A specific feature of partial correlation networks is that when the data is distributed as white noise process (independently and identically distributed), no correlation between two elements implies conditional independence, and correlation between two elements implies conditional dependence. This is a strong assumption, and the data I am going to use for doing my analysis does not follow exactly this process of the white noise distribution, so that some of the conclusions may not be accurate enough because of the tools used.

Also, as it has already been said, this model only works for contemporaneous dependencies so that it does not depend on data related to the past i.e. data from the last week. This problem can be solved by using other models such as Granger causality or the Long Run Partial Correlation network, mentioned previously. Nevertheless, these other networks are beyond the scope of this project and so I will not consider them. In order to try to minimize these problems when evaluating the data, I will try to use also some kind different types of residuals that take into account previous data for the better evaluation of it.

2.5 Sparse Network Estimation

In order to ease the interpretation of the data, I will use some tools that will help to derive from a highly connected network a simpler network which is easier to interpret and has fewer connections. Hence, the concept of sparsity is going to be explained in this section.

Sparse: Definition and assumption

An important assumption of the partial correlation network is that of data being sparse. It is said that the data is sparse when each vertex has a connection with few others. If we consider the whole world as a network, we could say intuitively that a major part of the world is interconnected. For instance, concerning the law of one price[8], we could see that there exists a quite direct correlation among all the currencies of the world. Not amazingly, we could see that this data is all interconnected among them, whereas it could be more difficult to find a direct correlation or a path between the African tribes that lives in the desert and yourself trying to find dependencies between you and them, or with the political conflict of Ukraine. The second example can be set such that this data has no connection between them or such that the correlation is so small that is negligible[9]. This means that not every node is connected with any other node of the dataset. For this reason, the data is sparse, and so the network is not complete.

The networks we have already spoken about are characterized by a matrix parameter e.g. K, A. In these matrices, we have already seen the existence of zero elements in it, which basically means that some data in the network has no correlation. As explained in the example above, it makes sense to think of some data being sparse. Hence, we are interested in estimation methods that deliver sparse estimator and so sparse networks.

[8] The law of one price establishes that one good should have the same price everywhere.

[9] Negliging dependencies between variables is the system we are going to explain in the section the LASSO estimation.

LASSO Estimation Technique

In 1996, Robert Tibshirani presented an innovative regression model called LASSO. This estimation technique has some particular features that are very useful for the estimation of partial correlation network model and the analysis of multivariate time series, which is the kind of data I am using in my project.

The LASSO estimation technique (Least Absolute Shrinkage and Selection Operation) is useful for obtaining estimations of partial correlation among elements of the multivariate time series creating a sparse network from our data. This model is a version of the least squares estimation model with the feature of simultaneously estimating some parameters on the regression while shrinking some others to zero.

When numbers are shrunk, the correlation between two numbers is set to zero, and so conditional independence is assumed. Totally the opposite is the case when a value different from zero is given. What happens in the latter case is that conditional dependence is assumed. Some problems can arise from this analysis:

1) When creating a sparse network, conditional independence is assumed when correlation between variables is omitted. This is only true under the assumption that the data is under a Gaussian model. The problem of it is that we can omit correlations that really exist, and then the estimated network does not show the true number of correlations. This is also called Type II error.

2) When creating a sparse network, conditional dependence is also assumed in correlation with variables that are not shrunk and so different from zero. This means there exists also the possibility of predicting correlations between data

that in fact do not exist, and that have not been shrunk. This is called Type I error.

In order to calculate the coefficients, the following formula is used:

$$\hat{\theta}_{\lambda}^{LASSO} = arc\ min_{\theta}\ \sum_{i=1}^{N}(y_i - \sum_j x_{i,j}\beta_j)^2 + \lambda \sum_{j=1}^{p}|\beta_j|\ ; \qquad \lambda \geq 0$$

Where:

- $X_{i,j}$ are predictor variables
- y_i are the responses
- $ß_j$ are the full least squares estimates

As it can be seen, it is similar to the least squares estimation formula with the second part concerning lambda (OLS taking into account the lambda parameter).

The lambda parameter has the following three properties, which are useful when doing the analysis:

1) If lambda value is zero, the expression turns out to be the same as the OLS estimators' expression. There is no shrinkage in the estimator and it is the same as the ordinary least squares test.
2) When lambda is small and takes a low value, the estimator parameter is going to be in between zero and the OLS estimator, because some numbers are going to be shrunk and others not. In this case, the lower the lambda, the more possibilities of committing Type I error (not rejecting correlations that does not exist).

3) When lambda takes high values, the data is going to be sparser, and so fewer correlations will be shown. By doing this, the probability of committing type II error is higher.

For these reasons, it is important to take different values of lambda and to assess properly which one is the most accurate for the analysis.

3. Regression Models

3.1 Markov processes

During the last decades, different models have appeared in the prediction of data using different forecasting model techniques. There exist different models for Markov processes. A Markov process is a stochastic process that follows the Markov properties:

Markov properties

Let me denote Y_t as an stochastic[10] model with an associated filtration[11] (F_t). The basic properties of a Markov process are:

- Measurability: $E[Y_t \mid F_t] = Y_t$ if Y is F_t measurable for $t \geq 0$
- Linearity: $E[Y + Z \mid F] = E[Y \mid F] + E[Z \mid F]$
- Conditional expectation: $E[Y \mid F] = E[\, E[Y \mid G] \mid F]$ when $G \supseteq F$
- Taking out what is known: $E[ZY \mid F] = Z\, E[Y \mid F]$ when Z is F-measurable
- Independence $E[Y \mid F] = E[Y]$ if $\sigma(Y)$ and F are independent

Measurabilty simply says that if the function F_t is measurable, Y_t can be perfectly predicted. Linearity also implies that thanks to the assumption of measurability,if F is

[10] A stochastic process is a sequence of random variables used to describe linear time series.

[11] A filtration is a order set S_j values subject to the premise that when the values of the indexed set of S_i, if i≤j, then $S_i \subseteq S_j$.

measurable and Y and Z depend on F, they can also be perfectly described. Conditional expectation simply says that if G is equal or inside F, then the conditional expectation on it is the same as the expectation on F. If Z is known and F measurable, the expectation of ZY given F is the same as Z multiplied by the expectation of Y given F. Finally, if independence is assumed the conditional expectation of Y on F should be the expectation of Y because Y and F are independent.

I am not going to explain the maximum likelihood estimator for the following models since it is not going to be useful for the scope of this project, but it is important to know it for the further forecasting of values that can be found in the references.

3.2 Black-Scholes Model

Markov processes are often used in financial data, specifically in calculating returns. One of the most used models as a Markov process has been the Black-Scholes Model. In 1973, the theorem was published by Fischer Black and Myron Scholes for the first time.

Let me present an example to understand better the model. Let S_t be the prices of the Dow Jones. Then, the log prices of the Dow Jones can be defined as $X_t = \log(S_t)$, $t \geq 0$. Now, the return prices are defined as $R_t = \log(S_t / S_{t-1})$, $t \geq 1$.

Let W_t be a marginal density $f(W_t)$ such that it follows a white noise process with variance σ^2. This model makes the assumption that W_t is independent from F_{t-1} from the returns. Then, the mean of the returns is given by the expectation of Y_t plus the independent shock W_t following the assumption also that it is a white noise process:

$$R_t = \mu + W_{t,} \qquad \text{t} = 1,2 \ldots W_t \sim fw$$

Under this model, it is also assumed that because the returns are described as previously said, the log of the prices are also calculated such that:

$$X_{t+1} = X_t + \mu + W_{t+1} \qquad t \geq 0$$

This means that the log-prices follow a Markov process plus the expected value and an independent shock.

Criticism

The Black-Scholes model has gained a lot of critics since it appeared. The main reason is that several assumptions taken have not been well represented with actual financial data. For instance, the differences in the log-prices are assumed to be uncorrelated, whereas real data presents some correlation across different log-prices. Another problem presented is the assumption concerning prices being normally distributed. It is unusual to see data following this distribution since the presence of outliers makes the data fail to fit in the model (this can be simply seen by making some test i.e. the Kolmogorov-Smirnov test). Finally, variances are not constant through time and it is shown that there are fluctuations on it. The inconsistency is that the model takes into account that in periods of big changes, volatility tends to be high and vice versa. Nevertheless, it also happens in actual data that there exist periods of really high volatility in times where no big changes exist. This is also called as “volatility clustering”. This volatility can be proved by analyzing the squares of the log prices, but doing this goes beyond the scope of the project.

3.3. The ARCH model

Despite of the problems presented by the Black Scholes model, it is still frequently used in financial time series due to its simplicity. Of course, readjustments to this model have been made in the last decades in order better fit the data in the model. That's why new readjusted models have been presented. Two of these models are going to be discussed in this project and also in this section: the ARCH and the GARCH model. This model of ARCH, first developed by Engle (1982) solves different problems presented by the Black-Scholes model. For instance, it captures more outliers since this model implies heavy tails on the distribution of the returns, and it also captures the "volatility clustering".

Let me denote the ARCH model as a multiplicative model:

$R_t = X_t W_t, for\ t > 1$

Where X_t which refers to the volatility that depends on R_t as expressed below:

$$X_t = \sqrt{\alpha + \lambda R_{t-1}^2}$$

As it is also considered as a Markov process, it has the same properties as X_t being F measurable and the condition of independence of W_t , but there have some differences comparing it with the Black Scholes model. One of them is that the ARCH model is composed of two components: X_t is predictable but W_t is not. Also, in the ARCH model R_1 remains unspecified. There are several ways to solve this, but one of the most used is to make a presumption on the first observation.

If the volatility is described as in the formula above, the models for R_t are called *Auto Regressive conditional heteroschedasticity (ARCH),* concretely in this case named as ARCH (1)[12]. The parameters α and λ gives stationarity and finite moments to the ARCH process. Nevertheless, the explanation for further orders goes beyond the scope of

[12] There exist more orders depending on the lag of the returns (t-k).

this project and it is not going to be assessed here. I will simply not that the second order would have alpha bigger than zero and lambda between zero and one.

3.4. The GARCH model

The ARCH (1) presented previously have several improvements compared with the Black-Scholes model. Nonetheless, there are still some problems that can be remedied with the *Generalized Auto Regressive conditional heteroschedasticity* Garch model by Bollerslev (1986): the squares of the log prices for the volatilty clustering are not exactly well displayed by the formulas shown in the previous section. The Garch model improves this calculation of the volatility clustering and it gives more flexibility to further orders of the model (not only for the order 1, which is the one I will be focus on).

Consider the same multiplicative model from before: $R_t = X_t W_t, for\ t > 1$, where W_t is distributed as a Gaussian process with heavy tails. However, there is a difference in the volatility of the log prices:

$$X_t = \sqrt{\alpha + \lambda R_{t-1}^2 + \beta X_{t-1}^2}$$

The parameters α, λ and β are constrained in order to assure stationarity and finite moments. The parameter α is bigger than zero, and the sum of the parameters λ and β must be between zero and one. This model with the volatility calculated as shown above is called the Garch (1,1). There also exist more orders for Garch defined as Garch (p,q), but this goes beyond the scope of the project.

Pros and Cons of Garch

The Garch model includes advantages and drawbacks that must be taken into account once the analysis of data is completed. The primary advantages of it are the ones presented by the ARCH model plus a better estimation and flexibility of the volatility clustering. It is quite easy to measure and to understand volatility from the historical values, and the results got gives further proof that the model works pretty well under financial time series such as exchange rates and stock price returns.

It is quite easy to measure and understand volatility from the historical values; the results give further proof that the model works pretty well under financial time series—such as exchange rates and stock price returns.

On the other hand, the reliance of these models on the volatility of the historical values is high. And we do not know how completely we can explain the future returns based on past returns, since different things can happen in the future that could affect the forecasting. Also, the assumption of price movements being stable and Gaussian distributed with heavy tails is an assumption that should be even more flexible.

4. Data Analysis of Actual data

4.1. Introduction

Once the different networks and utilities of the models have been explained, it is time to apply it with actual data. The aim of this project is to provide a good explanation applied to a data set, taking into account the advantages and the drawbacks of the different models used. With a real multivariate time series data, I am going to draw different estimated partial correlation networks for different values of lambda using the LASSO estimator for white noise process residuals and the Garch(1,1) residuals of the log returns of the prices of IBEX 35. From these results I will extract conclusions; such as the stronger correlations in the market, or the sector degree of interdependencies.

The purpose of the project is to understand and see a real application of a data set by creating estimated partial correlation networks and conducting data analysis. Notice that I do not know the true number of edges or the true number of correlations between the different variables of the data set; hence all the conclusions derived from it should be compared with other different networks and models than the ones used in this project in order to get a deeper analysis of the data and ensure the conclusions extracted from it. Moreover, it is important to bear in mind that the graph plotted is going to be undirected, colored and unweighted, as described in the previous sections of the project, with the limitations and the advantages that could be derived from it.

In order to show empirically the methodology of the project explained above, the data set must satisfy some characteristics for the network graph to be plotted. At first, the assumptions for the partial correlation network must be satisfied: the partial correlation network works under a white multivariate white noise process. Moreover, the data should be normalized[13] if data must be compared.

Furthermore, the usage of this partial correlation network's graph for more than 100 variables would give difficulties for the assessment of the elements, so that the optimal usage of this network graph has some limits concerning variables. For this case, it may be better to cluster different variables and then split them up deeper analysis in each clustered variable i.e. if you were interested in drawing the relations inside the country in the consumption of different segments, clustering first the metallurgy, agriculture, services and so many other elements. Then, if correlation exists among some segments, looking further into it and separate these clustered variables into simpler ones.

[13] For the comparison of data with the true results, as the true results are normalized in the model, data compared to it should be normalized as well for the better judgment of the results.

Once the advantages and the drawbacks have been described, the analysis of 32 firms from the IBEX 35 is a good number of variables for the usage of these tools and getting proper results from it.

4.2 Software

The software that it is going to be used in this project is R. R is a free programming language for statistical purposes, such as analyzing data sets. The purpose of it is to help me to analyze the data and plot some graphics for the easier evaluation of it. MATLAB or GNU Octave could also be used for the purpose of this project, but R is as good as the other two. All of them enable you to work with data frames or matrices, and display different data sheet helping you in the purpose of ordering, comparing and calculating different data and quick and easy way.

Packages

In order to be able to get the results we have got in R, there are some packages that are needed to be downloaded and installed in your computer. These ones are the following:

- Library(moments)
- Library(lattice)
- Library(MASS)
- Library(tseries)
- Library(igraph)

4.3 Explanation of the data

The data used for the purpose of this project are the closure of historic prices of the 32 firms that have been listed at least in IBEX 35 for some time from the 1st of January of 2007 until the thirty first of December of 2013 (this data can be found in www.invertia.com) . This means I am going to have a matrix of 32 variables comprising each of them 1784 observations. The firms analyzed are going to be the following ones: (coloured differently concerning the sector they belong to):

Financial Institution and Real Estate Company (Red Colour):

- POP (Banco Popular)
- SAB (Banco Sabadell)
- BKT (Bankinter)
- BBVA (Banco Bilbao Vizcaya Argentaria)
- BME (Bolsas y Mercados Españoles)
- SAN (Banco Santander)
- MAP (Mapfre)

Petroleum and Energy (Blue Colour)

- ABG (Abengoa)
- REP (Repsol)
- GAS (Gas Natural)
- ENG (Enagas)
- ENE (Endesa)
- IBE (Iberdrola)
- REE (Red Eléctrica Corporación)

Raw materials, Industry &Building companies (Green Color)

- ANA (Acciona)
- ACE (Acerinox)
- MTS (Arcelor Mittal)
- ACS (Actividades de Construcción y Servicios)
- FCC (Fomento de Construcciones y Contratas)
- FER (Ferrovial)
- OHL (Obrascón Huarte Lain)
- SCYR (Sacyr Vallehermoso)
- TRE (Técnicas Reunidas)

Consumer goods sector (yellow colour):

- GRF (Grifols)
- ITX (Inditex)
- EBRO (Ebro Foods)

- VIS (Viscofan)

Consumer service sector, technology and telecommunications (orange colour):

- ABE (Abertis Infraestructuras)
- IDR (Indra)
- JAZ (Jazztel)
- TEF (Telefónica)
- TL5 (Telecinco)

The abbreviations are important to be kept in mind since they are going to be the ones appearing on the graphs in the results section. This division is not going to be done in the script but just in the graphs. They will be modified using the command tkplot, which enables the user to change some elements of the graph, such as the color of the vertices, edges or the position of the edges in the graph to make it more visual and to ease to evaluation. Also, I am going to color the edges. The way I am going to do it is just colouring the edge as with the color of the vertices that has more edges.

4.4. Procedure of the data

There have been two ways of proceeding in this project in order to get the estimated networks. However, they are quite similar since the only difference relies on the way the different residuals are calculated. These two scripts are going to be attached on the annexes section of the project.

Script one: Calculating the partial correlation network graph

Firstly, the closure prices of the firms in the ibex 35 gathered in the file ibex_done.txt are read using the command read.table in R. This table is called in the script IBEX, and it contains all the data observations. To this table, two calculations are done for the following analysis in order to get the returns: logarithms have been applied to IBEX and then returns have been applied for lag one[14] using the command diff while creating a

[14] Lag one is X_{t-1}, for all $t \geq 1$ of X (length of X), starting from observation one.

matrix with all these values. From this matrix called returns, I have deleted[15] the first element of each variable for irrelevance in the data analysis. Take into account that this matrix called returns has no labels. For this reason, a vector with all the abbreviations for each company has been created for the forward analysis of the matrix.

From here, the partial correlation matrix has been created using the space.joint command. When you do so, as the LASSO estimation technique is used and a value must be given to lambda. Taking different values of lambda associates different degrees of error type I and II. Thus, optimization and forward evaluation must be done taking into account which lambda has been used. There are different methods for the optimization of lambda such as BIG or AIC, but those are beyond the scope of the project. The values concerning lambda that are going to comprised and assessed in the analysis are 0, 0.05, 0.01, 0.25, 0.5, 1, 1.5 and 2.

Ones the value of lambda has been chosen, the sparse joint regression is created. Then, I introduce to this matrix the vector of the values called matres (which has been generated before), which will be the abbreviations of the companies listed appearing to the graph. Finally, I attempt to define the graph as undirected, and using the tkplot, colours of the edges, as well as the position of the variables are going to be modified for better understanding of it for some values of lambda. Remember, the packages previously mentioned must be charged for the software to run some of the commands used in the script.

[15]The return of observation one cannot be given since it is unknown the previous price and so the log price.

Script two: partial correlation network graph of the Garch (1,1) residuals

As I have already explained in the script one, the returns of the log prices of the file ibex_done.txt must be created, eliminating the first value of each element of the returns (which is NA).

Then, using the garch command I generate the Garch(1,1) values of each variable (each firm from this project). From here, residuals can be taken in order to do generate as before partial correlation matrix using the command space.joint. However, a matrix must be created before, with the collection of residuals from each variable and eliminating the first element again because of its lack of significance. In this case, the matrix is called matres.

As before, the partial correlation network is created and using the graph.adjacency command and later on the tkplot, I will modify the colors and the position of the edges and vertices, as well as its size to ease the interpretation for some values of lambda.

Problem of Garch

From this second analysis, two variables (ABG and ENE) have presented some problems (NaNs have been produced). Because the problem is in a really small percentage of the data (it only happens in the first observation of ABG and in the first 20 observations of ENE out of 1784 observations per variable), I have decided to still use Garch (0,1) for these two variables, which is less restrictive about the assumptions of the model. The assumptions of Garch (1,1) have not been explained in this project, since it goes beyond the scope of the project and only an idea of the model is intended to know for the understanding of the analysis.

This problem arises because Garch(1,1) assumes from the formula that the sum of parameters a1 and b1 must be lower than one otherwise it cannot be well calculated.

In these two cases, the sum of the parameters was surpassing the number one but just by a some thousandth. For this reason, I used Garch (0,1) for this two cases, which is less restrictive in terms of the parameters and it works as well for the purpose of the project. Moreover, it gives the same results as using Garch(1,1) and setting the NaNs to zero. Nevertheless, it should be taken into account that the data does not fit properly the assumption of being well Garch (1,1) modelled. Hence, other models should also be tested or restrict a little bit different our data into this model.

Difference between analysing Standardized and not Standardized Garch

When considering the analysis of partial correlations network, Garch must be properly analyzed and understood. It is important to identify that there exist a different approach to Garch by standardizing or not. When doing so, Garch shows in the same degree of importance variances for the different variables, and so it is way to compare it with the returns got before.

On the other hand, standardizing the results may not give more importance to periods of high volatility. Therefore, the effect on volatility along the time and the analysis of it can be biased or hidden by not taking into account this factor. This is the reason why I decided not to standardize the Garch residuals for my analysis approach, since I found more interesting to test for importance in volatilities rather than looking whether the correlations are similar. The optimal result would be to do both cases, but for the scope of the project I decided to do Garch without standardizing. I want to outline it does not exist one unique way of approaching an analysis, but it is important to understand the way it is done for the forward understanding of the data partial correlation network and performing an optimal evaluation.

4.5 Results

The analysis of the results is going to be divided in two separate parts. First, I am going to evaluate the normal partial correlation network of the returns of the data. Then I am going to the same for the Garch (1,1) Residuals.

In the partial correlation network, I am going to thoroughly assess visually and numerically the results of the data. (The graphics and the tables are displayed in the annex).

Partial Correlation network

It can be seen in tables and in figures from 5 and 8 to 15, that even when lambda is equal to two, there exist a lot of correlations in the partial correlation network. The most important ones I am going to assess each time are the top 5 correlations of firms. This might mean that the strength of its correlation is really high since otherwise they would have disappeared while lambda is increased.

Once the data is starting getting sparse (lambda equals to 0.05), most of the companies still have more than 25 correlations. The ones having more correlations at this point are ACE ACS, MTS, SAB, BKT, GRF and ITX, with 29 and 28 correlations. Nevertheless, this is not really significant since the number of correlations shrunk is really low and a lot of correlations that do not exist are still represented in the graphic. Setting lambda to 0.1, it shows that the number of existing correlations among different firms is reduced averaged on 22, having companies such as SCYR with 27 and ABE or ANA with 26 correlations respectively.

Setting 0.25 or 0.5 as the value of lambda gives a similar partial correlation network as the one explained above: ABE, ACS, BBVA, OHL and TL5 are the ones with more correlations across the IBEX. Still, the average number of correlations among these firms is still really high near to seventeen. A good answer to this is typically the stock market of Spain is normally moved all together, since it depends on other markets, macro news and at the end on the Spanish environment as well as how well the concerned companies are doing.

When lambda is one, the existing correlations from firms mentioned above are even stronger: ABE, BBVA, MAP, OHL and SAN are the most interesting and the strongest cases. . For the values of 1.5 and 2, not much the things change from before. The strong correlations are concentrated in BBVA, with 28 correlations in the market, followed by SAN with 20. FER, ANA, and MAP also have significant correlations.

It is not only important to analyze the firms with more conditional dependences, but also necessary to pay attention to the average number of correlations in the stock market. Even for the biggest lambda analyzed, the average number of correlations that exist is almost nine. This means that on average one firm depends on another nine. Moreover, it is important to think that with lambda equal twoa lot of existing dependencies might be shrunk, so that the market has even more dependencies that the ones previously said. I want to remark again I am now speaking about intertemporary dependencies, so that I am examining the number of correlations at one single point of time for each firm to create and see these dependencies. Having this clear, the most relevant conclusion that can be extracted from it is that IBEX 35 is presumably moved all at the same time. This means that when one firm has positives returns, the probability all the other firms having positive returns is higher than being negative and vice versa: let me take BBVA or SAN. Only by having their returns positive, the probability of both of them having positive returns and the other firms having negative returns is practically impossible. They they are the base of the interconnections across this market, and what it happens to them affects the whole economy because of large number of existing correlations from them with other firms. Intuitively, this conclusion also makes sense, since the banks here in Spain, as well as the Construction companies or the state companies have been the ones driving the economy, not only during the previous years but during the last decades. Looking at figure 5, it can be also be seen the data quite sparsed (lambda equal to seven), but with interesting results. Interdependencies within sectors are high for those concerning financial Institutions and Real Estate (Red Colour) and Petroleum and Energy (blue Colour) companies, whereas in the other sectors they are practically inexistent for this level of sparsity. Nevertheless, lower level of lambda shows that raw materials, Industry &Building (Green Color) companies are even more correlated than the other mentioned sectors among themselves. The firms composing the yellow sector (Consumer goods companies) seem not have interdependencies within its yellow segment neither with the other segments. This means that an impact on this sector does not affect much or nothing to other firms from IBEX. The firms composing this sector are VIS, EBRO, ITX and GRF.

In an empirical way, they are also affected by macro facts and by the impact of the big companies in the market, but their behaviour and it is growth shows to be consistent with their performance more than being driven by the state of the economy. For instance, ITX has managed to grow during the financial crisis until now thanks to the good allocation of its resources and understanding of the market.

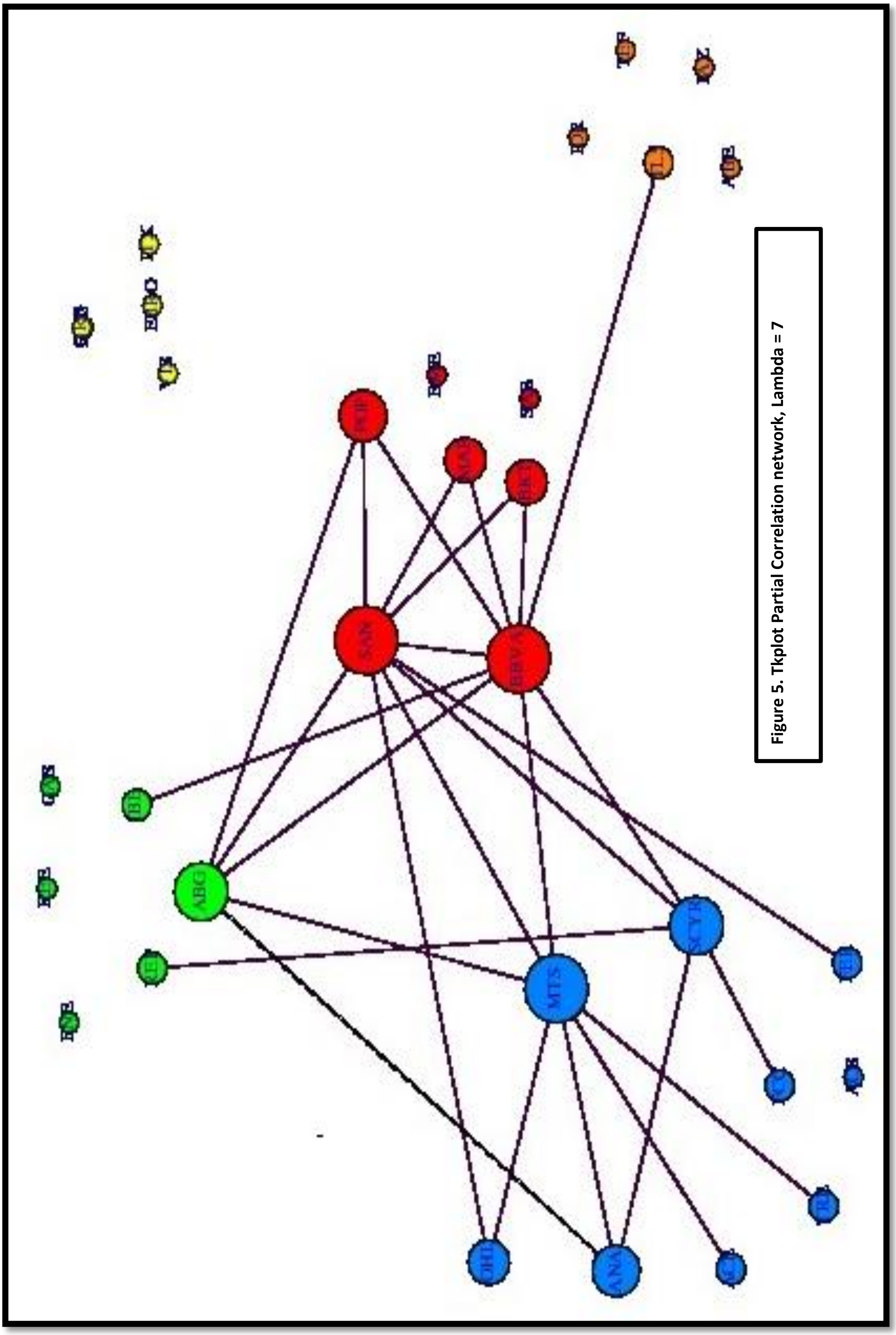

Figure 5. Tkplot Partial Correlation network, Lambda = 7

Garch partial correlation network

Intuitively, garch residuals should give similar results as the ones found from returns. Interestingly, this is not exactly like this; it can be seen that interdependencies in this case are much lower than in the partial correlation network with the returns (see figures 16-23 and tables from the Garch partial correlation networks in the annex).

When lambda equals to two, no correlations exist among the firms. As I have previously explained in the LASSO estimation technique section, this value assigned to lambda is probably too big for this analysis and the true network has correlations that are not shown for this value. As expected, the first interdependency found in this correlation network is BBVA and SAN for lambda associated to 1,5. For lambda equal to one, BBVA, SAN and ACS are the ones having the most interdependencies in the market. Not surprisingly, interdependencies when lambda equals to 0,25 or 0,5 are higher than with larger lambda: ABE, ACS, BBVA and SAN are the most significant ones. It is important to take into account that the average number of interdependencies in this case for lambda equals to 0.25 is almost 14 per firm. For lower values of lambda, such as 0.1 or 0.05, it is interesting to see that there exist so many other firms with high number of interconnections and so being for the current value of lambda more relevant. BME, ACS, FERR and TL5 are the ones with more correlations in both cases. For lambda equals to 0.1, it exists though a really high number of correlations as well, which is near 18 per firm.

When lambda equals to zero, a perfect interconnected network is created without existing any sparsity at all.

In summary, it is important to understand the interconnections, but also what GARCH does. GARCH does also take into account previous variance and error terms from data. By having this in mind, the meaning of correlations between data also infers some meaning on past data. Hence, the biggest effects of firms on the market in this case are from BBVA,SAN and ACS. Other firms such as ABE, FERR or TL5 have a significant effect in the market concerning its number of interconnections on it, but they are not as significant as the effect that the banks have on it. Also, the effect of some construction companies such as ACS is quite big in the stock market. The economic growth of Spain, before the crisis was based on the construction and this sector still plays an important role in the Spanish economy and its development.

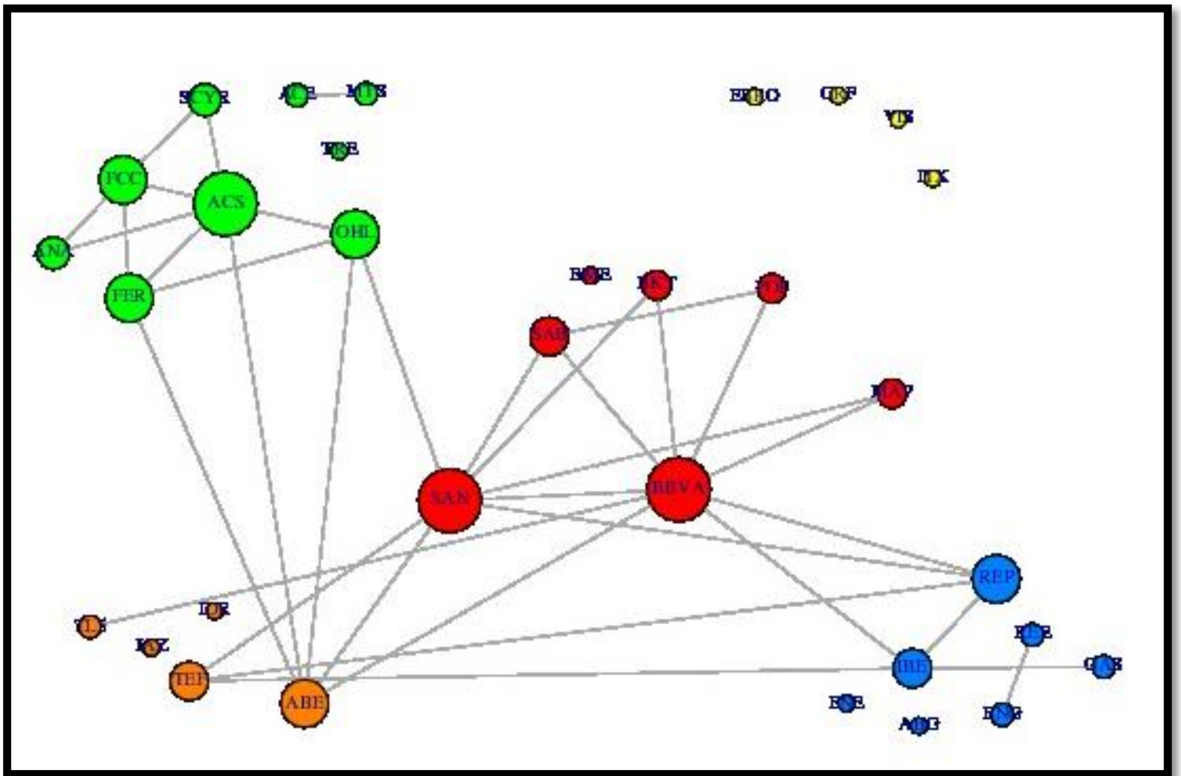

Figure 6. Tkplot Partial Correlation Network, lambda =1, Garch

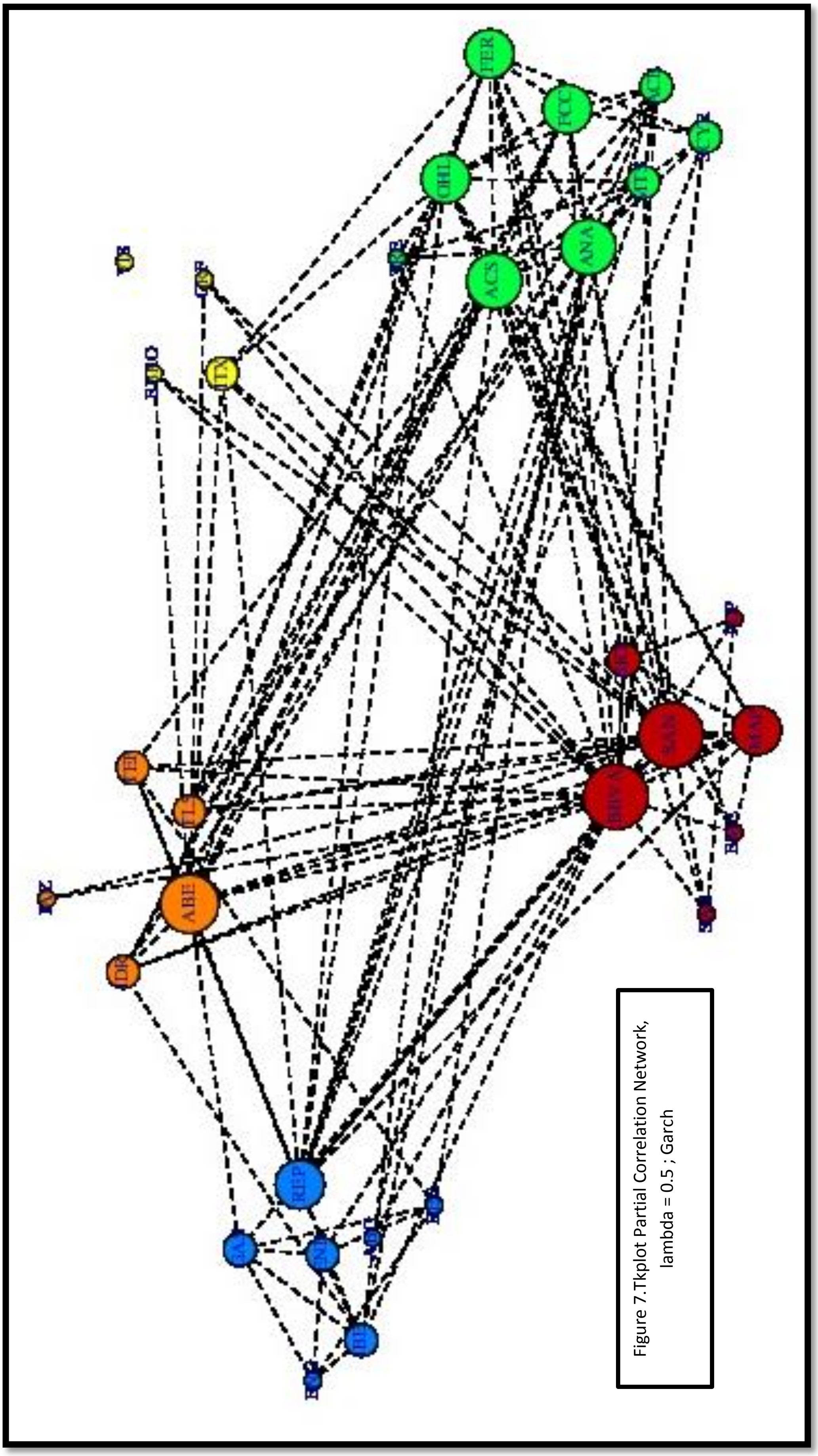

Figure 7.Tkplot Partial Correlation Network, lambda = 0.5 ; Garch

In figures 6 and 7, the interesting finding assessed in the previous section where the partial correlation network with returns gets boosted: financial Institutions and Real Estate (Red Colour), Petroleum and Energy (blue Colour) and Raw materials, Industry &Building (Green Color) companies seem to have a large amount of internal correlations (concerning its sector). The consumer goods (yellow colour) and Consumer service sector and technology and telecommunications (orange colour) companies do not seem to be really correlated within them or with the other firms of the market.

5. Conclusions of the project

- The Spanish stock market is very intercontemporary correlated. This means that basically IBEX is moved all together. Intuitively, the Spanish economy relies on macro news and information. The world, being every day more globalized, depends basically on the big stock markets such as Dow Jones. This drives the whole economy to trends such as growth or falls from the economy and also to the stagnation of the small economies. Spain, being part of Europe, depends a lot on these factors and so it is mainly influenced by these big investors and on how the way the world economy is developed. The economies increase and decrease together.
- The existing intercorrelations evaluated for Garch residuals and the returns of IBEX shows similar results in both cases. Difference between the normal returns and adjusting for Garch is that Garch takes into account much more volatility periods, giving to them more importance than other periods. Nevertheless, data

shows that no matter if the Spanish stock market is in high or low periods of volatility, the same stock market behaves in the same or similar way.

- Banks, specifically SAN and BBVA, are the pillars of the connections of the IBEX 35. Building Companies have important effects on the Spanish economy along these years. A big impact on some of the other companies, such as bankruptcy will also affect the whole market, but not in the same level as if it was a bank.

- Intercorrelations among the different sectors described in the explanation of the data yielded interesting results. On the one hand, financial Institutions and Real Estate (Red Colour), Petroleum and Energy (blue Colour) and Raw materials, Industry &Building (Green Color) companies seem to have quite a lot of intercorrelation within their sector (i.e. the fall of one of them also leads to the fall of the others). Consumer goods (yellow colour) and Consumer service, technology and telecommunications (orange colour) sectors do not seem to be really correlated within them, or with other firms. This does not mean that these correlations do not exist, but they are weaker than others in the market. I am explaining the most significant correlations in the stock market, but this does not deny the possibility of these ones having correlations with other firms in the market. It just says that these correlations are not as strong or even significant as the ones I am describing.

- Taking into account that the probability of estimating correctly the true number of correlation networks is near to impossible, I can try to think whether this analysis makes sense intuitively. As far as I am concerned, it gives an answer which is quite interesting and that it could be taken into account for different financial purposes. Also, This project's analytical results are supported by recent significant events in the Spanish economy: the bubble of construction or the financial crisis. These events have affected the market quite heavily, and that it has been reflected in the stock market and in my analysis.

- At a micro level, BBVA and SAN are the ones having more interconnections in the market for lambdas considered more significant and the network being more sparsed. Intuitively and empirically, it is a logical result since these banks have also a large number of shares or other credit concessions from other firms of the IBEX, and the unexpected or expected effect on these two banks concerning its profits, being the biggest ones in Spain, would have a big effect on all the firms that have deals with them. This effect also exists vice versa, but it is much lower.

- The limitations of this project are that I have only studied intercontemporary correlations within the market, but not the correlation effect of past events on the following returns for time being bigger than one day. Nevertheless, expectations in the market are normally quite quickly applied to the price and this one is affected really quickly by the news that is happening and its possible outcomes. Nevertheless, deeper work on this part could be done for the better evaluation of the data and the better confirmation of my conclusions., Garch (1,1) has shown to be not the best model for the data and other models could be implemented.

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8. Annexes

Annex 1. Script Partial Correlation Network

Annex 2. Script Partial Correlation Network (Garch)

Annex 3. Partial correlation network graphs for different values of lambda

Annex 4. Partial correlation network graphs for different values of lambda (Garch)

Annex 5. Table of partial correlation networks for lambda =7

Annex 6. Table of partial correlation networks for lambda =1, (Garch)

Annex 1

SCRIPT PARTIAL CORRELATION NETWORK

```
### Libraries

library(MASS)

library(space)

library(igraph)

### Reading data script

IBEX=read.table(file="ibex_done.txt",header=T, sep="\t", dec=",", row.names=1,
colClasses= c("character",rep("numeric",32)))

IBEX.35=strptime(rownames(IBEX), "%d/%m/%Y")

#apply logs to data

log.ibex=log(IBEX)


###apply returns, lag = 1

N <- ncol(log.ibex)

T <- nrow(log.ibex)

returns <- matrix( 0 , T , N )

for( i in 1:N ){

	returns[2:T,i] <- diff(log.ibex[,i])*100

}

returns=returns[-1,]

####define vector with names

matres <- c( "ABG", "ABE", "ANA", "ACE", "ACS", "MTS", "POP", "SAB", "BKT", "BBVA",
"BME", "EBRO", "ENG", "ENE", "FCC", "FER", "GAS", "GRF", "IBE", "ITX", "IDR", "JAZ",
"MAP", "OHL", "REE", "REP", "SCYR", "SAN","TL5", "TEF", "TRE", "VIS")


#### fit network, lamda= 0

results0 <- space.joint(returns,lam1=0*T)
```

```
# get estimated partial correlation matrix
SigInv.hat0 <- results0$ParCor
A.hat0 <- 1*( SigInv.hat0 != 0 ) - diag(rep(1,N))
colnames(A.hat0) <- matres
network.hat0 <- graph.adjacency( A.hat0 , mode='undirected')

#### fit network, lamda= 0.05
results <- space.joint(returns,lam1=0.05*T)
# get estimated partial correlation matrix
SigInv.hat <- results$ParCor
A.hat <- 1*( SigInv.hat != 0 ) - diag(rep(1,N))
colnames(A.hat) <- matres
network.hat <- graph.adjacency( A.hat , mode='undirected')

#### fit network, lamda= 0.1
results1 <- space.joint(returns,lam1=0.1*T)
# get estimated partial correlation matrix
SigInv.hat1 <- results1$ParCor
A.hat1 <- 1*( SigInv.hat1 != 0 ) - diag(rep(1,N))
colnames(A.hat1) <- matres
network.hat1 <- graph.adjacency( A.hat1 , mode='undirected')

#### fit network, lamda= 0.25
results2 <- space.joint(returns,lam1=0.25*T)
# get estimated partial correlation matrix
SigInv.hat2 <- results2$ParCor
A.hat2 <- 1*( SigInv.hat2 != 0 ) - diag(rep(1,N))
colnames(A.hat2) <- matres
```

```
network.hat2 <- graph.adjacency( A.hat2 , mode='undirected')

#### fit network, lamda= 0.5
results3 <- space.joint(returns,lam1=0.5*T)
# get estimated partial correlation matrix
SigInv.hat3 <- results3$ParCor
A.hat3 <- 1*( SigInv.hat3 != 0 ) - diag(rep(1,N))
colnames(A.hat3) <- matres
network.hat3 <- graph.adjacency( A.hat3 , mode='undirected')

#### fit network, lamda= 1
results4 <- space.joint(returns,lam1=1*T)
# get estimated partial correlation matrix
SigInv.hat4 <- results4$ParCor
A.hat4 <- 1*( SigInv.hat4 != 0 ) - diag(rep(1,N))
colnames(A.hat4) <- matres
network.hat4 <- graph.adjacency( A.hat4 , mode='undirected')

#### fit network, lamda= 1.5
results5 <- space.joint(returns,lam1=1.5*T)
# get estimated partial correlation matrix
SigInv.hat5 <- results5$ParCor
A.hat5 <- 1*( SigInv.hat5 != 0 ) - diag(rep(1,N))
colnames(A.hat5) <- matres
network.hat5 <- graph.adjacency( A.hat5 , mode='undirected')

#### fit network, lamda= 2
results6 <- space.joint(returns,lam1=2*T)
```

```
# get estimated partial correlation matrix

SigInv.hat6 <- results6$ParCor

A.hat6 <- 1*( SigInv.hat6 != 0 ) - diag(rep(1,N))

colnames(A.hat6) <- matres

network.hat6 <- graph.adjacency( A.hat6 , mode='undirected')


#### plot

par( mfrow=c(2,2) )

plot(network.hat0 )

plot(network.hat )

plot(network.hat1 )

plot(network.hat2 )

plot(network.hat3 )

plot(network.hat4 )

plot(network.hat5 )

plot(network.hat6 )


par( mfrow=c(1,1) )

tkplot(network.hat4 )
```

Annex 2

SCRIPT PARTIAL CORRELATION NETWORK (Garch)

```
###Read the data

IBEX=read.table(file="ibex_done.txt",header=T, sep="\t", dec=",", row.names=1, colClasses=
c("character",rep("numeric",32)))

IBEX.35=strptime(rownames(IBEX), "%d/%m/%Y")

#apply logs to data

log.ibex=log(IBEX)


###apply returns, lag = 1

N <- ncol(log.ibex)

T <- nrow(log.ibex)

returns <- matrix( 0 , T , N )

for( i in 1:N ){

returns[2:T,i] <- diff(log.ibex[,i])*100

}

#Eliminate the first row of 0

returns=returns[-1,]

### Libraries

library(MASS)

library(space)

library(igraph)

library(tseries)
```

```
###DOING GARCH (1,1)

garch.ABG=garch(returns[,1], order=c(0,1),trace=F)

garch.ABE=garch(returns[,2], order=c(1,1),trace=F)

garch.ANA=garch(returns[,3], order=c(1,1),trace=F)

garch.ACE=garch(returns[,4], order=c(1,1),trace=F)

garch.ACS=garch(returns[,5], order=c(1,1),trace=F)


garch.MTS=garch(returns[,6], order=c(1,1),trace=F)

garch.POP=garch(returns[,7], order=c(1,1),trace=F)

garch.SAB=garch(returns[,8], order=c(1,1),trace=F)

garch.BKT=garch(returns[,9], order=c(1,1),trace=F)

garch.BBVA=garch(returns[,10], order=c(1,1),trace=F)


garch.BME=garch(returns[,11], order=c(1,1),trace=F)

garch.EBRO=garch(returns[,12], order=c(1,1),trace=F)

garch.ENG=garch(returns[,13], order=c(1,1),trace=F)

garch.ENE=garch(returns[,14], order=c(0,1),trace=F)

garch.FCC=garch(returns[,15], order=c(1,1),trace=F)


garch.FER=garch(returns[,16], order=c(1,1),trace=F)

garch.GAS=garch(returns[,17], order=c(1,1),trace=F)

garch.GRF=garch(returns[,18], order=c(1,1),trace=F)

garch.IBE=garch(returns[,19], order=c(1,1),trace=F)

garch.ITX=garch(returns[,20], order=c(1,1),trace=F)


garch.IDR=garch(returns[,21], order=c(1,1),trace=F)
```

```
garch.JAZ=garch(returns[,22], order=c(1,1),trace=F)

garch.MAP=garch(returns[,23], order=c(1,1),trace=F)

garch.OHL=garch(returns[,24], order=c(1,1),trace=F)

garch.REE=garch(returns[,25], order=c(1,1),trace=F)


garch.REP=garch(returns[,26], order=c(1,1),trace=F)

garch.SCYR=garch(returns[,27], order=c(1,1),trace=F)

garch.SAN=garch(returns[,28], order=c(1,1),trace=F)

garch.TL5=garch(returns[,29], order=c(1,1),trace=F)

garch.TEF=garch(returns[,30], order=c(1,1),trace=F)


garch.TRE=garch(returns[,31], order=c(1,1),trace=F)

garch.VIS=garch(returns[,32], order=c(1,1),trace=F)


####CREATE RESIDUALS


res.ABG <- garch.ABG$res

res.ABE <- garch.ABE$res

res.ANA <- garch.ANA$res

res.ACE <- garch.ACE$res

res.ACS <- garch.ACS$res


res.MTS <- garch.MTS$res

res.POP <- garch.POP$res

res.SAB <- garch.SAB$res

res.BKT <- garch.BKT$res

res.BBVA <- garch.BBVA$res
```

```
res.BME <- garch.BME$res
res.EBRO <- garch.EBRO$res
res.ENG <- garch.ENG$res
res.ENE <- garch.ENE$res
res.FCC <- garch.FCC$res

res.FER <- garch.FER$res
res.GAS <- garch.GAS$res
res.GRF <- garch.GRF$res
res.IBE <- garch.IBE$res
res.ITX <- garch.ITX$res

res.IDR <- garch.IDR$res
res.JAZ <- garch.JAZ$res
res.MAP <- garch.MAP$res
res.OHL <- garch.OHL$res
res.REE <- garch.REE$res

res.REP <- garch.REP$res
res.SCYR <- garch.SCYR$res
res.SAN <- garch.SAN$res
res.TL5 <- garch.TL5$res
res.TEF <- garch.TEF$res

res.TRE <- garch.TRE$res
res.VIS <- garch.VIS$res
```

```
###create matrix of residuals, lag = 1

matres=matrix(c( res.ABG, res.ABE, res.ANA, res.ACE, res.ACS, res.MTS, res.POP, res.SAB,
res.BKT, res.BBVA, res.BME, res.EBRO, res.ENG,res.ENE, res.FCC, res.FER, res.GAS, res.GRF,
res.IBE, res.ITX, res.IDR, res.JAZ, res.MAP, res.OHL, res.REE, res.REP, res.SCYR, res.SAN, res.TL5,
res.TEF, res.TRE, res.VIS), length(res.ABG),32)

matres=matres[-1,]

N <- ncol(matres)

T <- nrow(matres)

#######GARCH with names

titles <- c( "ABG", "ABE", "ANA", "ACE", "ACS", "MTS", "POP", "SAB", "BKT", "BBVA", "BME",
"EBRO", "ENG", "ENE", "FCC", "FER", "GAS", "GRF", "IBE", "ITX", "IDR", "JAZ", "MAP", "OHL",
"REE", "REP", "SCYR", "SAN", "TL5", "TEF", "TRE", "VIS")


#### fit network, lamda= 0

results00 <- space.joint(matres,lam1=0*T)    ######[1] "iter=1" Error in jsrm(Y.u, sig.u, n, p,
lam1, lam2) : NA/NaN/Inf in foreign function call (arg 5)

# get estimated partial correlation matrix

SigInv.hat00 <- results00$ParCor

A.hat00 <- 1*( SigInv.hat00 != 0 ) - diag(rep(1,N))

colnames(A.hat00) <- titles

network.hat00 <- graph.adjacency( A.hat00 , mode='undirected')


#### fit network, lamda= 0.05

solution <- space.joint(matres,lam1=0.05*T)

# get estimated partial correlation matrix

SigInv.solution <- solution$ParCor

A.solution <- 1*( SigInv.solution != 0 ) - diag(rep(1,N))

colnames(A.solution) <- titles

network.solution <- graph.adjacency( A.solution , mode='undirected')
```

```
#### fit network, lamda= 0.1
solution1 <- space.joint(matres,lam1=0.1*T)
# get estimated partial correlation matrix
SigInv.solution1 <- solution1$ParCor
A.solution1 <- 1*( SigInv.solution1 != 0 ) - diag(rep(1,N))
colnames(A.solution1) <- titles
network.solution1 <- graph.adjacency( A.solution1 , mode='undirected')

#### fit network, lamda= 0.25
solution2 <- space.joint(matres,lam1=0.25*T)
# get estimated partial correlation matrix
SigInv.solution2 <- solution2$ParCor
A.solution2 <- 1*( SigInv.solution2 != 0 ) - diag(rep(1,N))
colnames(A.solution2) <- titles
network.solution2 <- graph.adjacency( A.solution2 , mode='undirected')

#### fit network, lamda= 0.5
solution3 <- space.joint(matres,lam1=0.5*T)
# get estimated partial correlation matrix
SigInv.solution3 <- solution3$ParCor
A.solution3 <- 1*( SigInv.solution3 != 0 ) - diag(rep(1,N))
colnames(A.solution3) <- titles
network.solution3 <- graph.adjacency( A.solution3 , mode='undirected')

#### fit network, lamda= 1
solution4 <- space.joint(matres,lam1=1*T)
# get estimated partial correlation matrix
```

```
SigInv.solution4 <- solution4$ParCor

A.solution4 <- 1*( SigInv.solution4 != 0 ) - diag(rep(1,N))

colnames(A.solution4) <- titles

network.solution4 <- graph.adjacency( A.solution4 , mode='undirected')


#### fit network, lamda= 1.5

solution5 <- space.joint(matres,lam1=1.5*T)

# get estimated partial correlation matrix

SigInv.solution5 <- solution5$ParCor

A.solution5 <- 1*( SigInv.solution5 != 0 ) - diag(rep(1,N))

colnames(A.solution5) <- titles

network.solution5 <- graph.adjacency( A.solution5 , mode='undirected')


#### fit network, lamda= 2

solution6 <- space.joint(matres,lam1=2*T)

# get estimated partial correlation matrix

SigInv.solution6 <- solution6$ParCor

A.solution6 <- 1*( SigInv.solution6 != 0 ) - diag(rep(1,N))

colnames(A.solution6) <- titles

network.solution6 <- graph.adjacency( A.solution6 , mode='undirected')


#### fit network, lamda= 7

results7 <- space.joint(returns,lam1=7*T)

# get estimated partial correlation matrix

SigInv.hat7 <- results7$ParCor

A.hat7 <- 1*( SigInv.hat7 != 0 ) - diag(rep(1,N))

colnames(A.hat7) <- matres
```

```r
network.hat7 <- graph.adjacency( A.hat7 , mode='undirected')

#### plot
par( mfrow=c(1,1) )
plot(network.hat0 )
plot(network.hat )
plot(network.hat1 )
plot(network.hat2 )
plot(network.hat3 )
plot(network.hat4 )
plot(network.hat5 )
plot(network.hat6 )
tkplot(network.hat7)
```

Annex 3

PARTIAL CORRELATION NETWORK GRAPHS FOR DIFFERENT VALUES OF LAMBDA

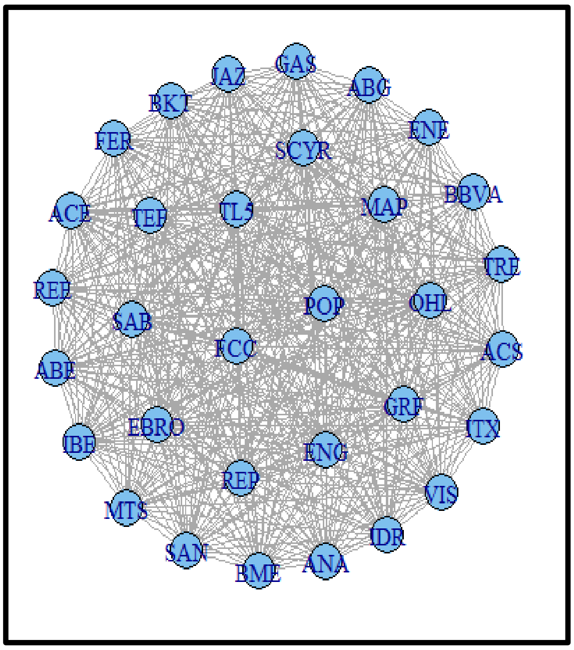

Figure 8. Partial correlation network, lambda =0

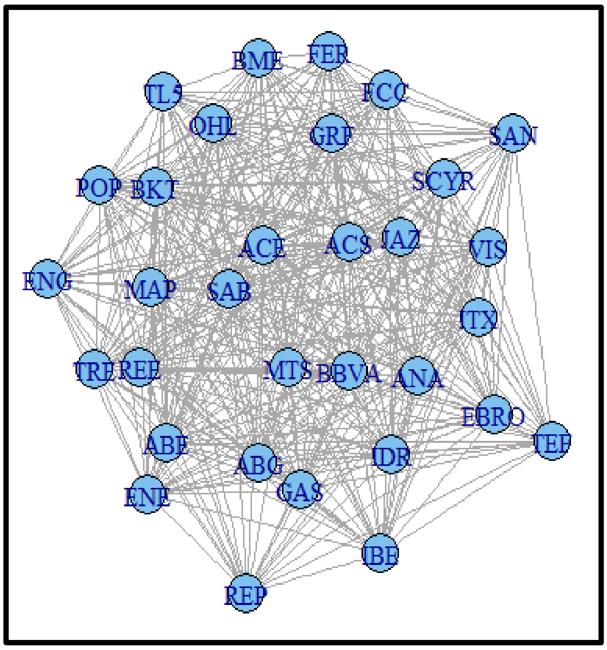

Figure 9. Partial correlation network, lambda =0.05

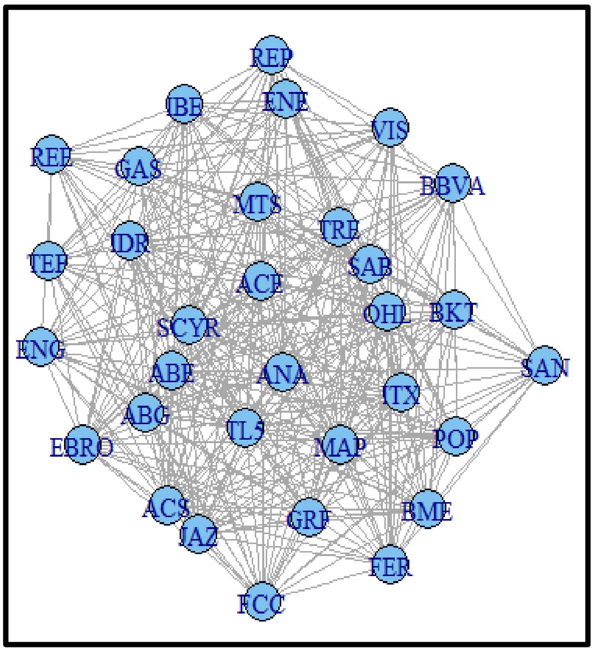

Figure 10. Partial correlation network, lambda =0.1

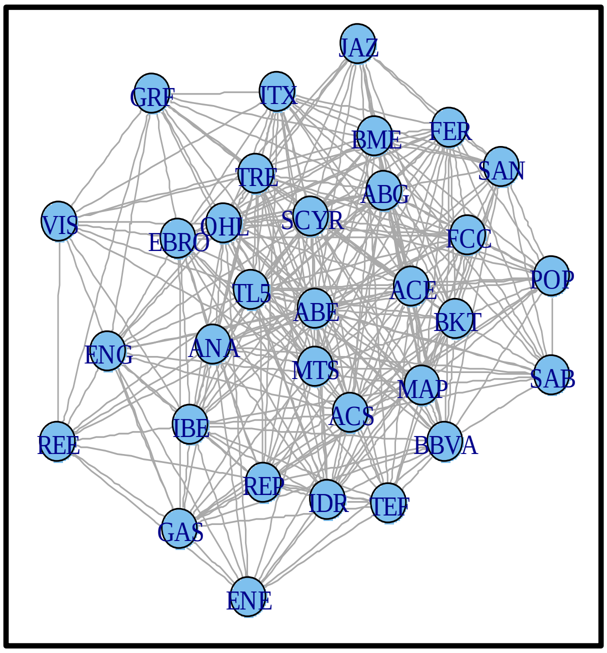

Figure 11. Partial correlation network, lambda =0.25

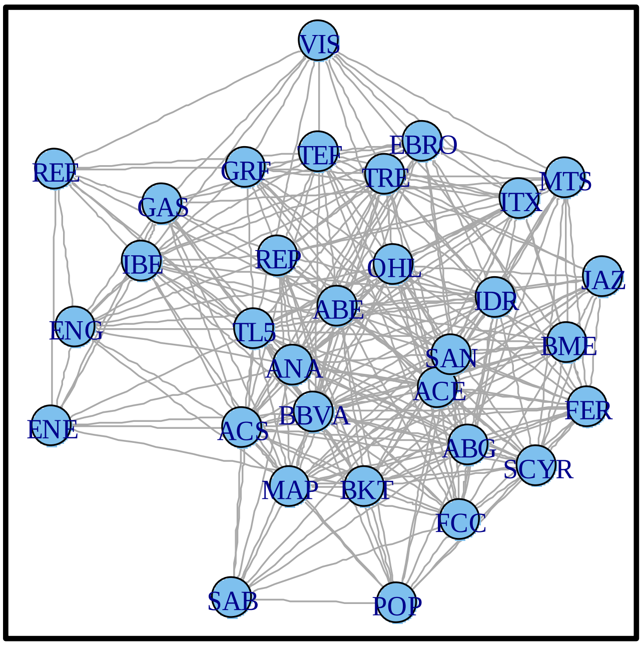

Figure 12. Partial correlation network, lambda =0.5

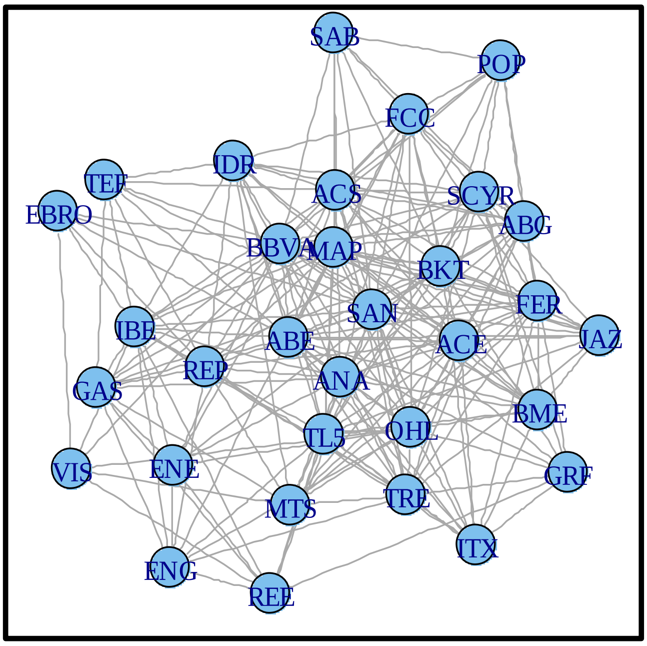

Figure 13. Partial correlation network, lambda =1

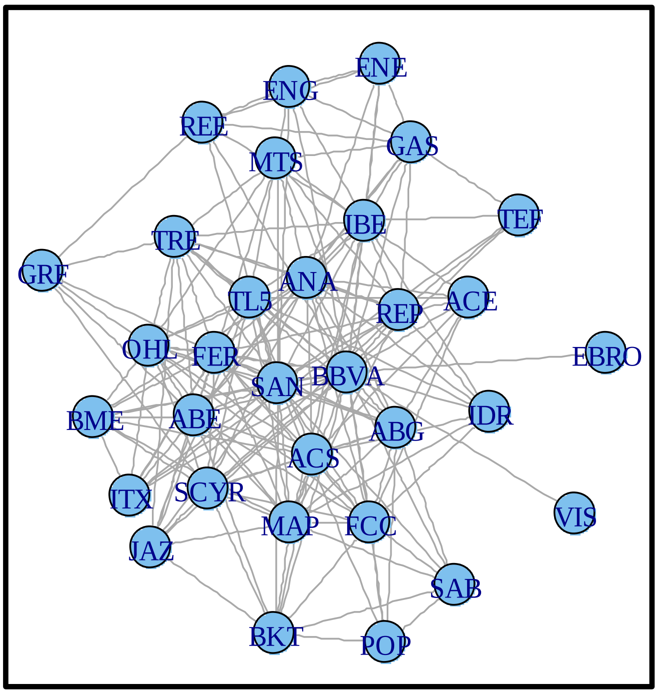

Figure 14. Partial correlation network, lambda =1.5

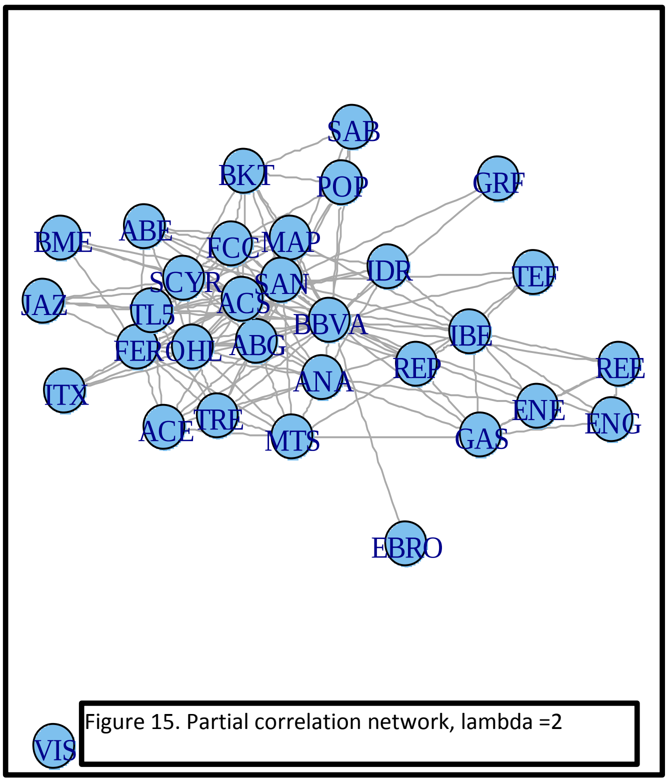

Figure 15. Partial correlation network, lambda =2

Annex 4

PARTIAL CORRELATION NETWORK GRAPHS FOR DIFFERENT VALUES OF LAMBDA (GARCH)

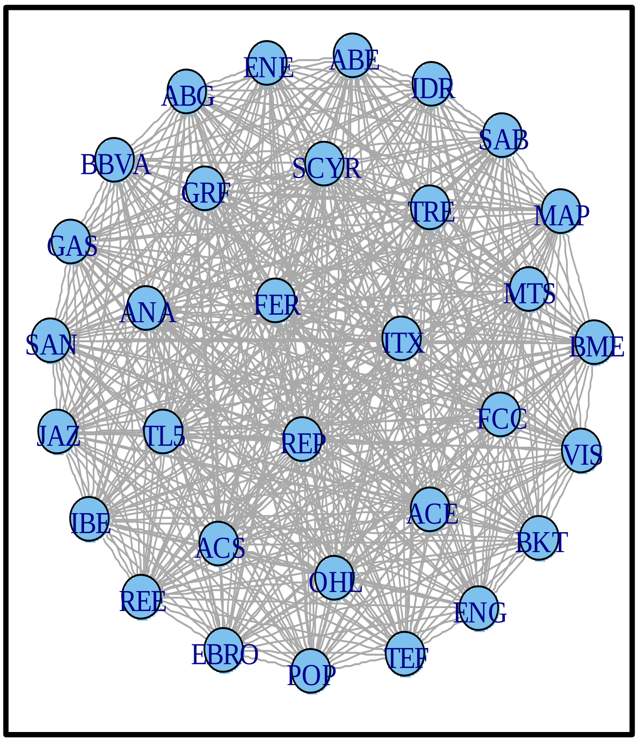

Figure 16. Partial correlation network, lambda =0, Garch (1,1)

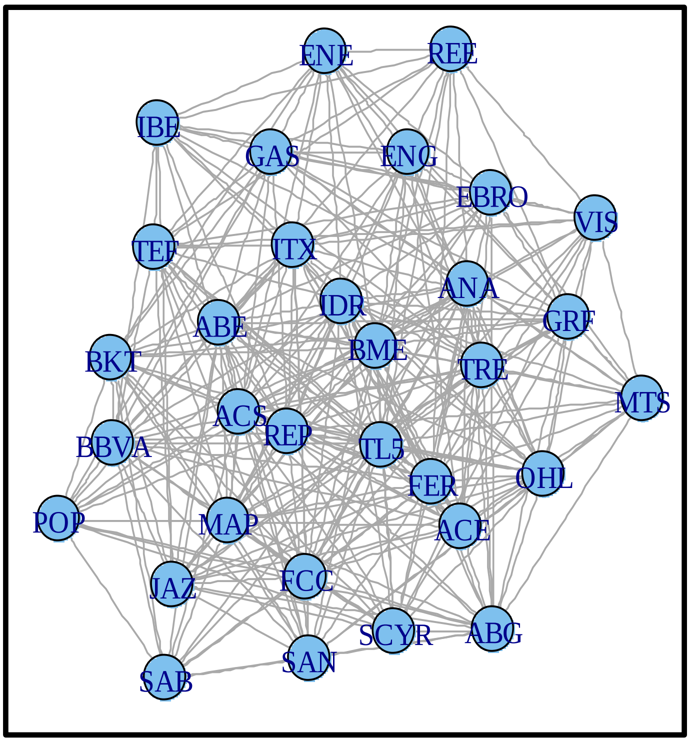

Figure 17. Partial correlation network, lambda =0.05, Garch (1,1)

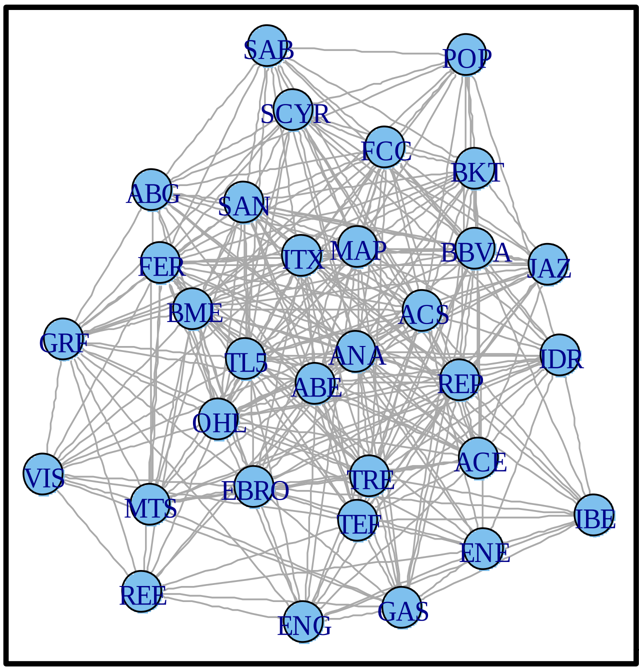

Figure 18. Partial correlation network, lambda =0.1, Garch(1,1)

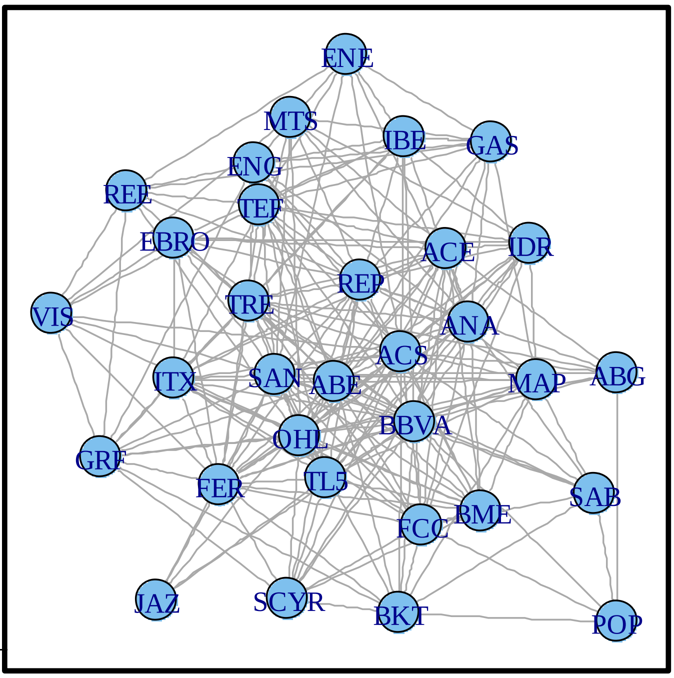

Figure 19. Partial correlation network, lambda =0.25, Garch(1,1)

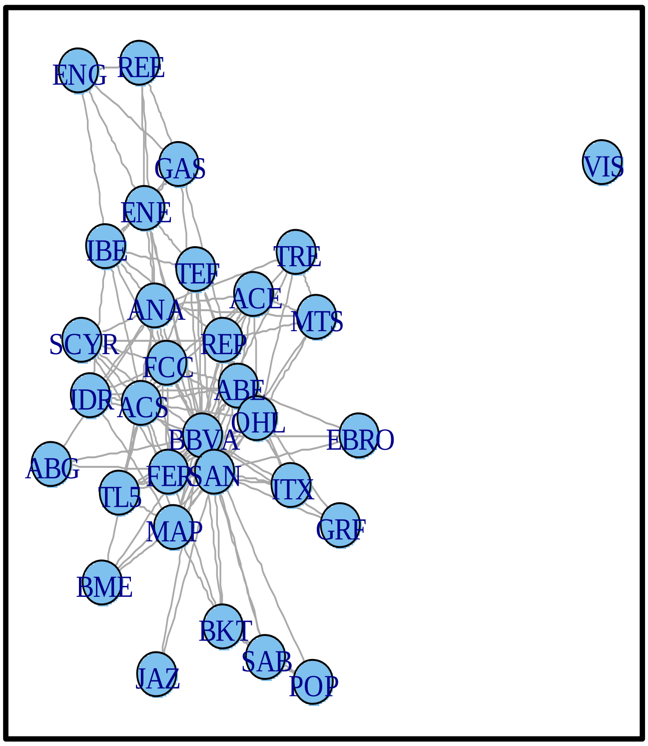

Figure 20. Partial correlation network, lambda =0.5, Garch(1,1)

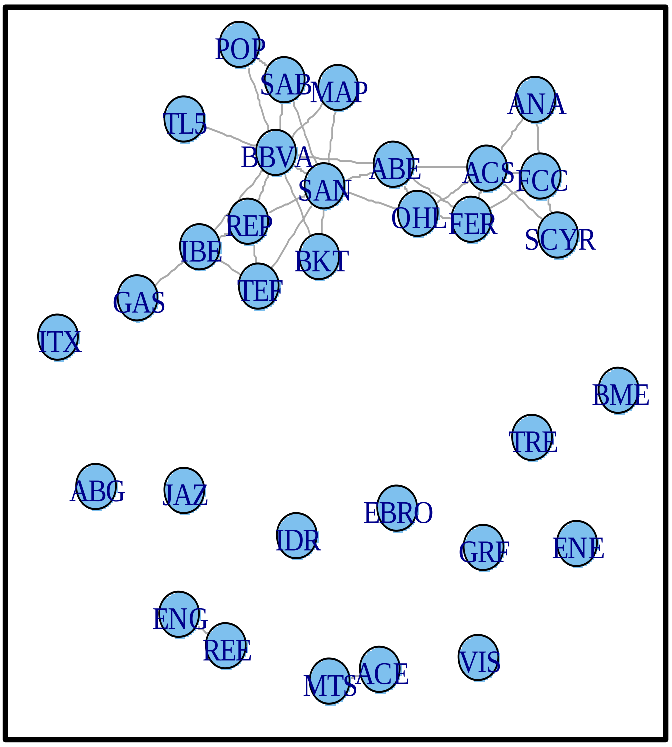

Figure 21. Partial correlation network, lambda =1, Garch(1,1)

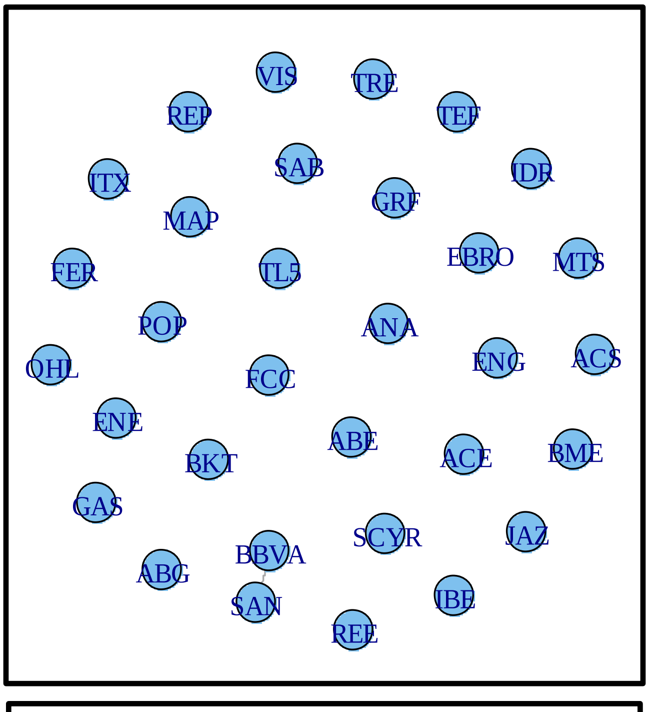

Figure 22. Partial correlation network, lambda =1.5, Garch(1,1)

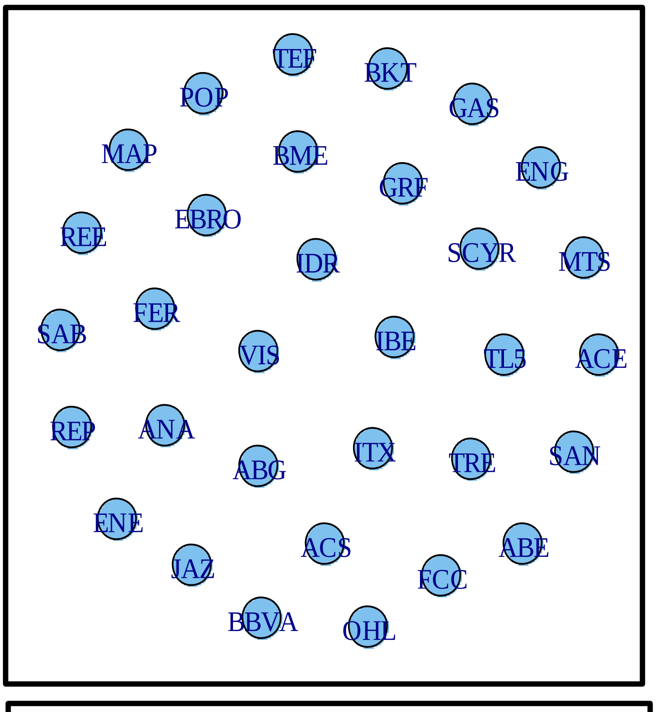

Figure 23. Partial correlation network, lambda =2, Garch(1,1)

Annex 5

TABLE OF PARTIAL CORRELATION NETWORKS FOR LAMBDA =7

LAMBDA =7	ABG	ABE	ANA	ACE	ACS	MTS	POP	SAB	BKT	BBVA	BME	EBRO	ENG	ENE	FCC	FER	GAS	GRF	IBE	ITX	IDR	JAZ	MAP	OHL	REE	REP	SCYR	SAN	TL5	TEF	TRE	VIS
ABG	0	0	1	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
ABE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ANA	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
ACE	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ACS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
MTS	1	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0
POP	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
SAB	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BKT	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
BBVA	1	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	1	1	0	0	0
BME	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EBRO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENG	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
FCC	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0

FER	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
GAS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
GRF	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
IBE	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ITX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
IDR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
JAZ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
MAP	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
OHL	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
REE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
REP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
SCYR	0	0	1	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0
SAN	1	0	0	0	0	1	1	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0
TL5	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
TEF	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
TRE	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
VIS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Total partial correlations	5	0	3	1	0	7	3	0	2	9	0	0	0	0	1	1	0	0	1	0	0	0	2	2	0	1	5	9	1	0	1	0

Annex 6

TABLE OF PARTIAL CORRELATION NETWORKS FOR LAMBDA =1, (GARCH)

LAMBDA =1	ABG	ABE	ANA	ACE	ACS	MTS	POP	SAB	BKT	BBVA	BME	EBRO	ENG	ENE	FCC	FER	GAS	GRF	IBE	ITX	IDR	JAZ	MAP	OHL	REE	REP	SCYR	SAN	TL5	TEF	TRE	VIS
ABG	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ABE	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0
ANA	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ACE	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ACS	0	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0
MTS	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
POP	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SAB	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
BKT	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
BBVA	0	1	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	1	0	0	0
BME	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EBRO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ENG	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
ENE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
FCC	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0

FER	0	1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
GAS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
GRF	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
IBE	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0
ITX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
IDR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
JAZ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
MAP	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
OHL	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
REE	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
REP	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1	0	0
SCYR	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SAN	0	1	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	1	0	0
TL5	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
TEF	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1	0	0	0	0
TRE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
VIS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Total partial correlations	0	5	2	1	6	1	2	3	2	9	0	0	1	0	4	4	1	0	4	0	0	0	2	4	1	4	2	8	1	3	0	0