Reliability, Redundancy, and Resiliency

- Lecture #09 – September 26, 2023
- Review of probability theory
- Component reliability
- Confidence
- Redundancy
- Reliability diagrams
- Intercorrelated failures
- System resiliency
- Resiliency in fixed fleets

Review of Probability

- Probability that A occurs

$$0 \leq P(A) \leq 1$$

- Probability that A does not occur

$$P(\bar{A})$$

- Sum of all probable outcomes

$$P(A) + P(\bar{A}) = 1$$

Review of Probability

- Probability of both A and B occurring

$$P(A) \cap P(B) = P(A)P(B)$$

- Probability of either A or B occurring

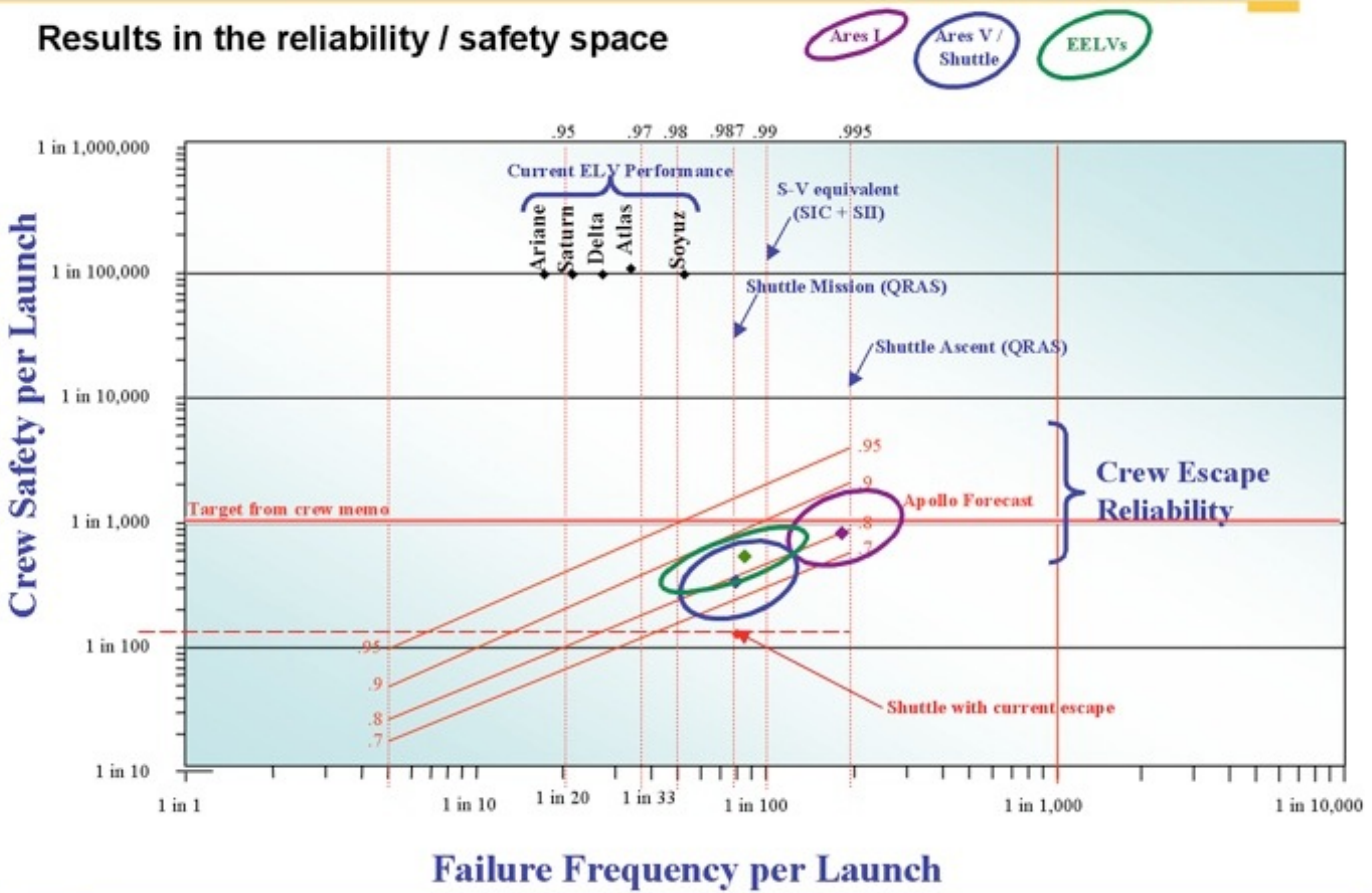
$$P(A) \cup P(B) = 1 - P(\bar{A})P(\bar{B})$$

$$= 1 - [1 - P(A)][1 - P(B)]$$

$$= P(A) + P(B) - P(A)P(B)$$

Baseline Results

Results in the reliability / safety space



Simple Overview of Abort Reliability

$$P_{survival} = P_{launch} \cup P_{abort}$$

$$P_{survival} = 1 - \left(\bar{P}_{launch} \cap \bar{P}_{abort}\right)$$

$$P_{survival} = 1 - \left[\left(1 - P_{launch}\right)\left(1 - P_{abort}\right)\right]$$

$$P_{abort} = 1 - \frac{1 - P_{survival}}{1 - P_{launch}}$$

$$P_{survival} = 0.999; \;\; P_{launch} = 0.97$$

$$P_{abort} = 1 - \frac{1 - 0.999}{1 - 0.97} = 0.9667$$

Effect of Successive Trials

- Any trial has possible results A and $\bar{A}$ (e.g., heads/tails)
- Possible outcomes of two trials:
  - Both $A \implies P = P(A)^2$
  - First A, then $\bar{A} \implies P = P(A)P(\bar{A}) = P(A)[1 - P(A)]$
  - First $\bar{A}$, then $A \implies P = P(\bar{A})P(A) = [1 - P(A)]P(A)$
  - Both $\bar{A} \implies P = P(\bar{A})^2 = [1 - P(A)]^2$
  - All possible outcomes: $P = P(A)^2 + 2P(A)[1 - P(A)] + [1 - P(A)]^2 = 1$

General Probability in Successive Trials

- For N trials:

$$P_{0\,fail} = P(A)^N$$

$$P_{1\,fail} = NP(A)^{N-1}[1 - P(A)]$$

$$P_{2\,fail} = \frac{N(N-1)}{2} P(A)^{N-2}[1 - P(A)]^2$$

$$P_{3\,fail} = \frac{N(N-1)(N-2)}{2(3)} P(A)^{N-3}[1 - P(A)]^3$$

$$P_{K\,fail} = \boxed{\frac{N!}{K!(N-K!)}} P(A)^{N-K}[1 - P(A)]^K$$

Combinations of K out of N

Expected Value Theory

- Probability of an outcome does not determine value of the outcome
- Define $E(A)$ as the value associated with an outcome of A
- Combine probabilities and values to determine expected value of outcome

$$EV = P(A)E(A) + P(\bar{A})E(\bar{A})$$

- If rolling a die,

$$EV(roll) = P(1)E(1) + P(2)E(2) + P(3)E(3) + P(4)E(4) + P(5)E(5) + P(6)E(6)$$
$$= (1/6)(1) + (1/6)(2) + (1/6)(3) + (1/6)(4) + (1/6)(5) + (1/6)(6) = 3.5$$

Expected Value Example

- Maryland State Lottery - pick six numbers out of 49 (any order)

$$P(win) = \left(\frac{49!}{6!43!} \right)^{-1} = 1/13{,}983{,}816$$

- Assume $10,000,000 jackpot

$$EV = P(win)\,E(win) + P(loss)E(loss)$$

$$EV = \left(7.151 \times 10^{-8}\right)\left(\$10^{7}\right) + (1)(-\$1) = -\$0.39$$

How Long Do You Have to Play to Win?

- Odds of losing one play

$$1 - 1/13{,}983{,}816 = 0.9999999285$$

- How many times do you have to play until you have a 50/50 chance of winning? How many times can you play and lose until your chance of a perfect record is only 50%?

$$(0.9999999285)^N = 0.5 \implies N = 9{,}692{,}842$$

- Playing twice a week, it would take 93,200 years

Utility Theory

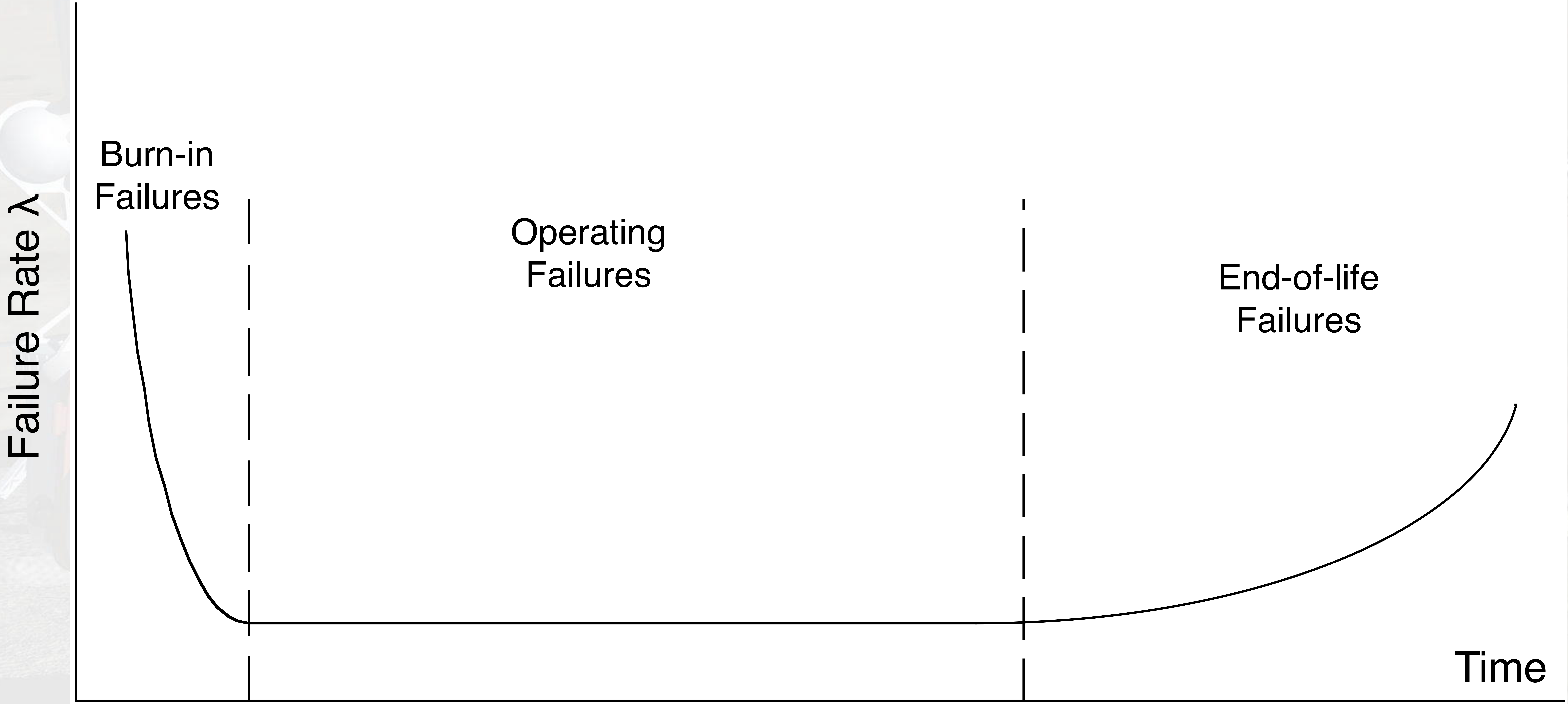
- Numerical rating from expected value calculations does not fully quantify utility
- Lottery example previously: utility of (highly unlikely) win exceeds negative utility of small investment: *risk proverse*

$$U(+\$10{,}000{,}000) \ggg U(-\$1)$$

- Imagine lottery where \$1000 buys 1:500 chance at \$1M - EV=(.998)(-\$1000)+(.002)(\$.999M)=\$1000
risk adverse

$$U(+\$1{,}000{,}000) \lll U(-\$1000)$$

Component Reliability



Reliability Analysis

- Failure rate is defined as fraction of currently operating units failing per unit time

$$\lambda(t) = -\frac{1}{R(t)}\frac{d}{dt}R(t)$$

- The trend of operating units with time is then

$$\int_0^t \lambda(\tau)\,d\tau = -\int_1^{R(t)} \frac{dR(\tau)}{R(\tau)}$$

Reliability Analysis (continued)

- Evaluation of the definite integrals gives

$$\int_0^t \lambda(\tau)\, d\tau = -\ln\left[R(t)\right]$$

- Assuming that λ is constant over the operating lifetime,

$$R(t) = \exp\left[-\int_0^t \lambda(\tau) d\tau\right] = e^{-\lambda t}$$

- At t=1/ λ, 1/e of the original units are still operating (defined as mean time between failures)

Reliability Analysis (continued)

- Frequently assess component reliability based on reciprocal of failure rate λ :

$$R(t) = e^{-\frac{t}{MTBF}}$$

  where MTBF=mean time between failures

- For a mission duration of N hours, estimate of component reliability becomes

$$R(mission) = e^{-\frac{N}{MTBF}}$$

Verifying a Reliability Estimate

- Given a unit reliability of R, what is the probability P of testing it 20 times without a failure?
- What is the probability Q that you will see one or more failures?
  - $R = 0.99 \implies P_{20\ successes} = 0.8179 \implies Q = 0.1821$
  - $R = 0.95 \implies P_{20\ successes} = 0.3584 \implies Q = 0.6416$
  - $R = 0.90 \implies P_{20\ successes} = 0.1216 \implies Q = 0.8784$

Confidence

- The confidence C in a test result is equal to the probability that you should have seen worse results than you did

P(observed *and all better outcomes*) + C =1

Example of Confidence - Saturn V

- 13 vehicle flights without a failure
- Assume a reliability value of R

$$R^{13} + C = 1$$

- Valador report (slide 7) listed 95% reliability

$$C = 1 - R^{13} = 1 - 0.95^{13} = 48.7\%$$

- What reliability could we cite with 80% confidence?

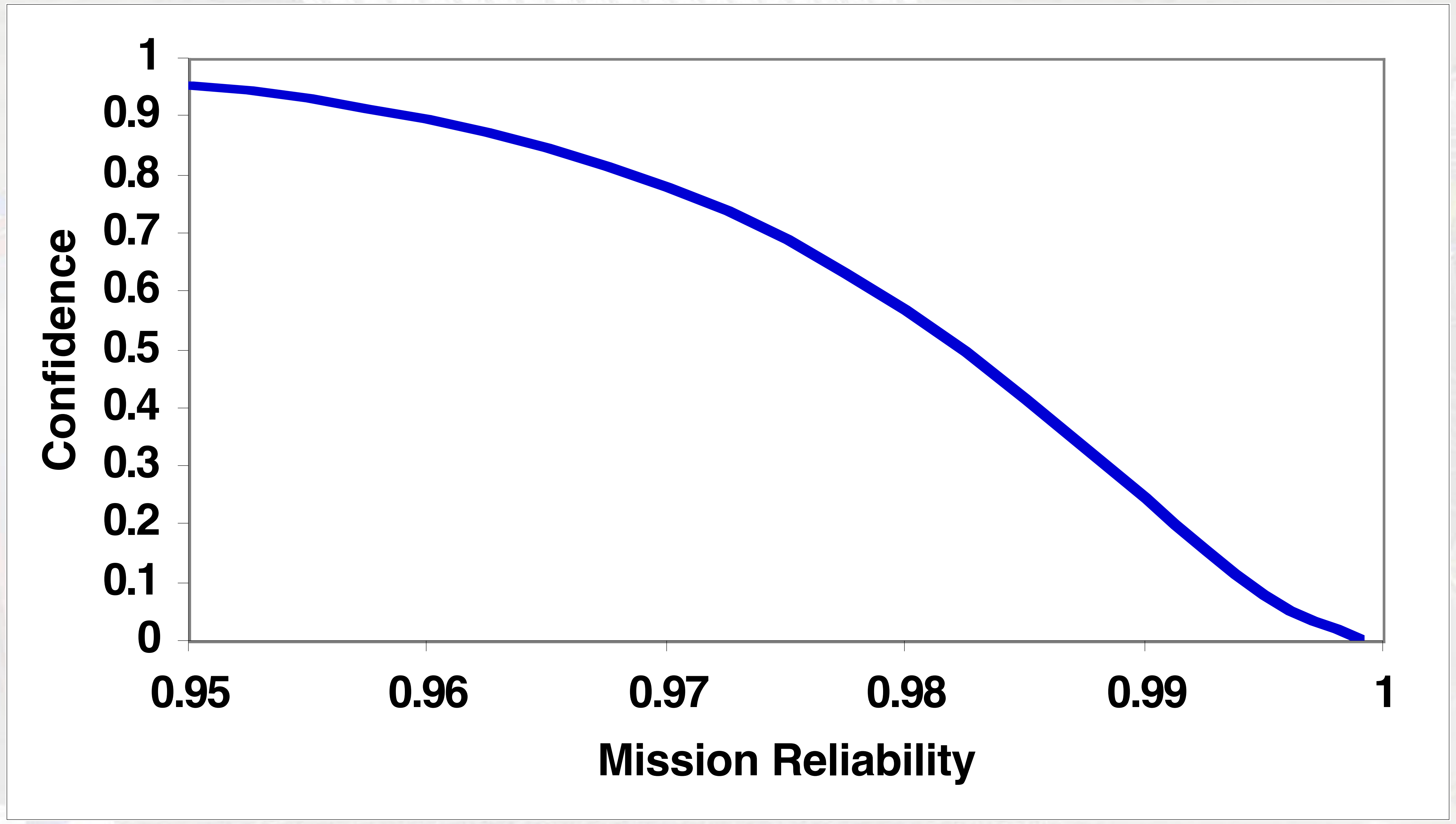
$$R = (1 - C)^{1/13} = 0.2^{0.07692} = 88.4\%$$

Example of Confidence

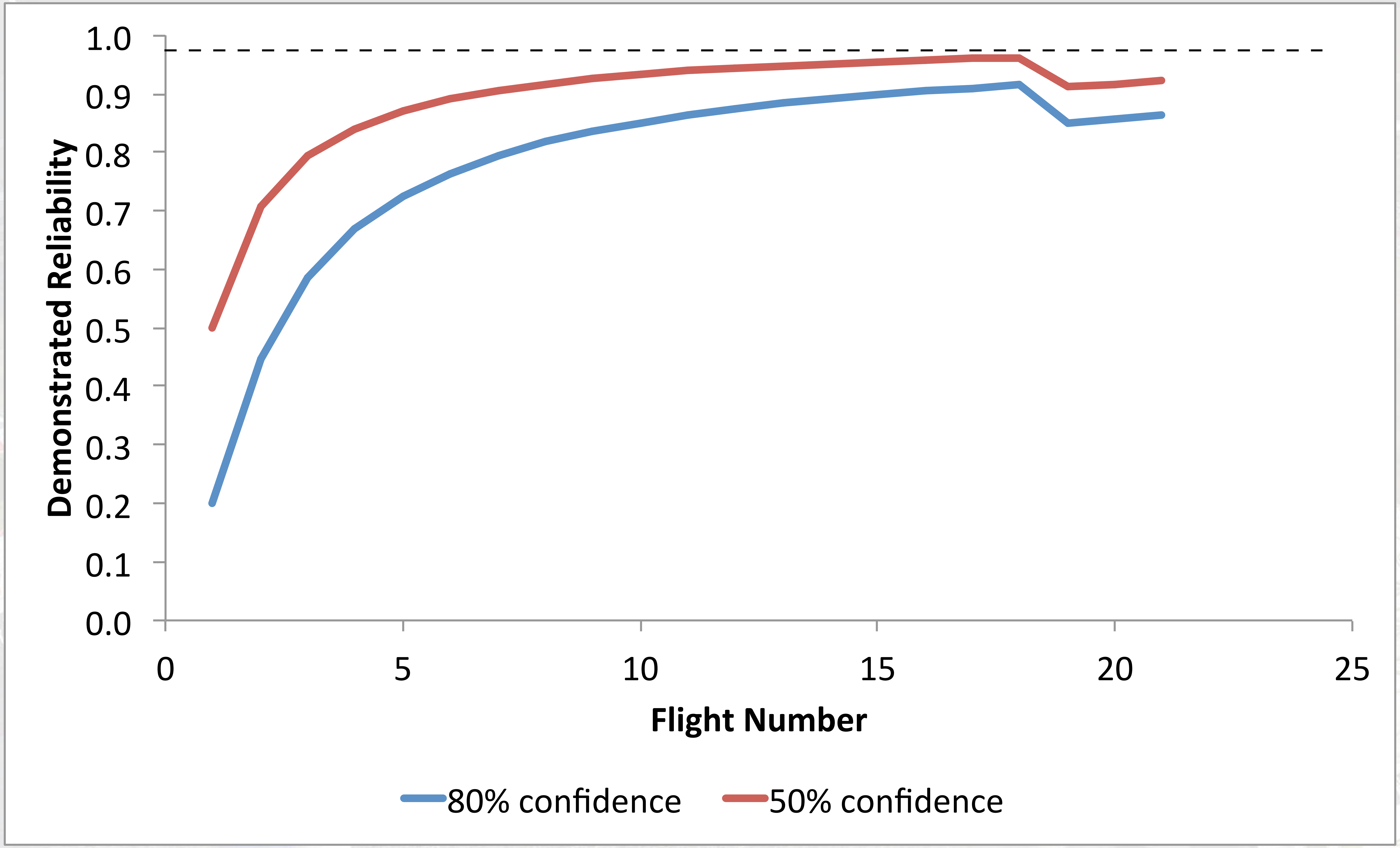
- 100 vehicle flights with 1 failure
- Assume a reliability value of R

$$R^{100} + 100R^{99}(1-R) + C = 1$$

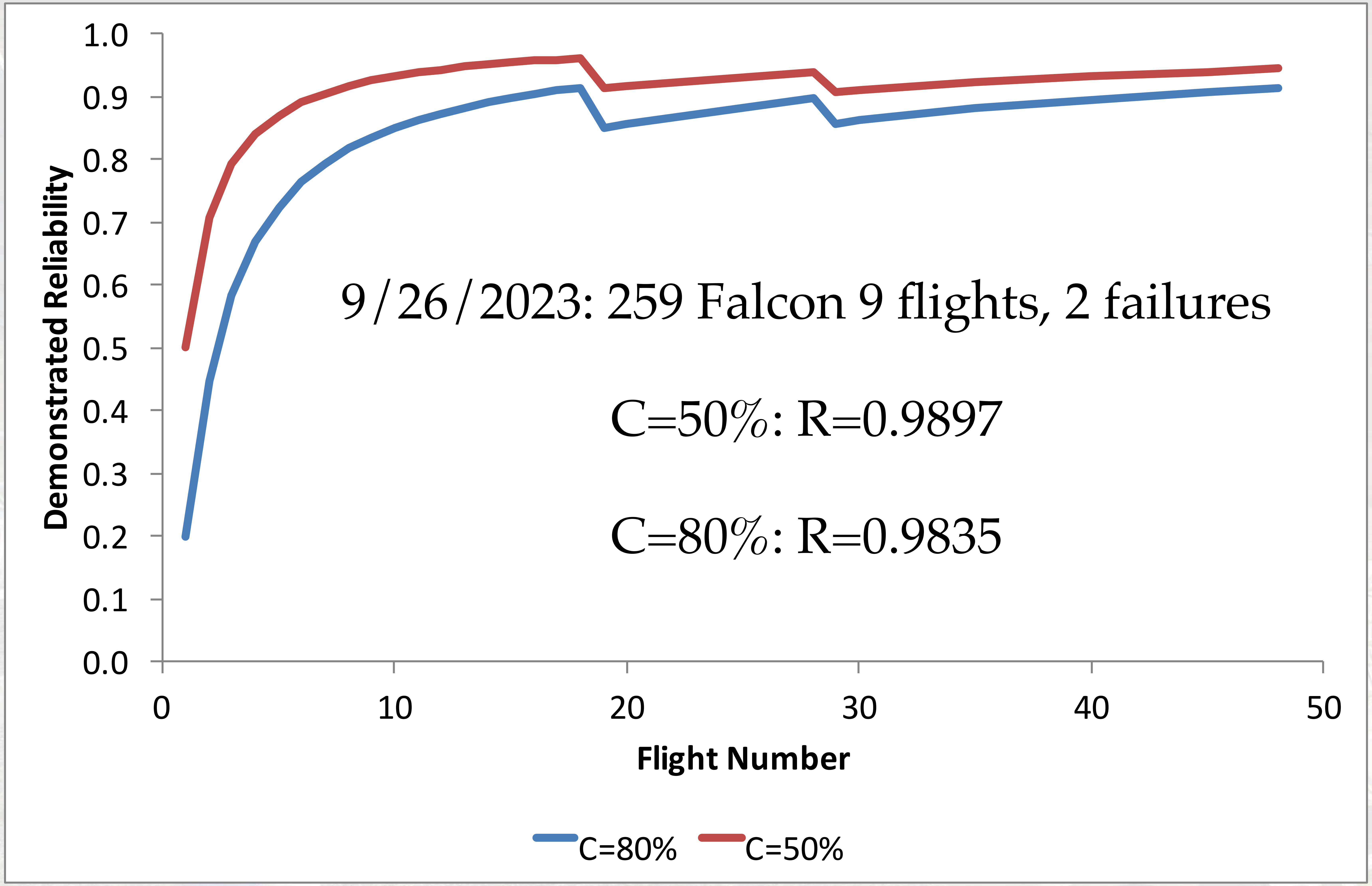
- Trade off reliability with confidence values



Falcon 9 Reliability Curves (2/28/16)



Falcon 9 Reliability Curves (2/27/18)



Definition of Redundancy

- Probability of k out of n units working = (number of combinations of k out of n) × P(k units work) × P(n-k units fail)

$$P(k|n) = \frac{n!}{k!(n-k)!} P^k (1-P)^{n-k}$$

- For the Falcon 9 example,

$$\boxed{\frac{n(n-1)}{2} R^{n-2}(1-R)^2} + \boxed{nR^{n-1}(1-R) + R^n} + C = 1$$

The results we saw

All better results

Redundancy Example

3 parallel computers, each has reliability of 95%:

- Probability all three work

$$P(3) = P^3 = (.95)^3 = .8574$$

- Probability exactly two work

$$P(2) = 3P^2(1 - P) = 3(.95)^2(.05) = .1354$$

- Probability exactly one works

$$P(1) = 3P(1 - P)^2 = 3(.95)(.05)^2 = .0071$$

- Probability that none work

$$P(0) = (1 - P)^3 = (.05)^3 = .0001$$

Redundancy Example

3 parallel computers, each has reliability of 95%:

- Probability all three work

$$P(3) = .8574$$

- Probability at least two work

$$P(3) + P(2) = .8574 + .1354 = .9928$$

- Probability at least one works

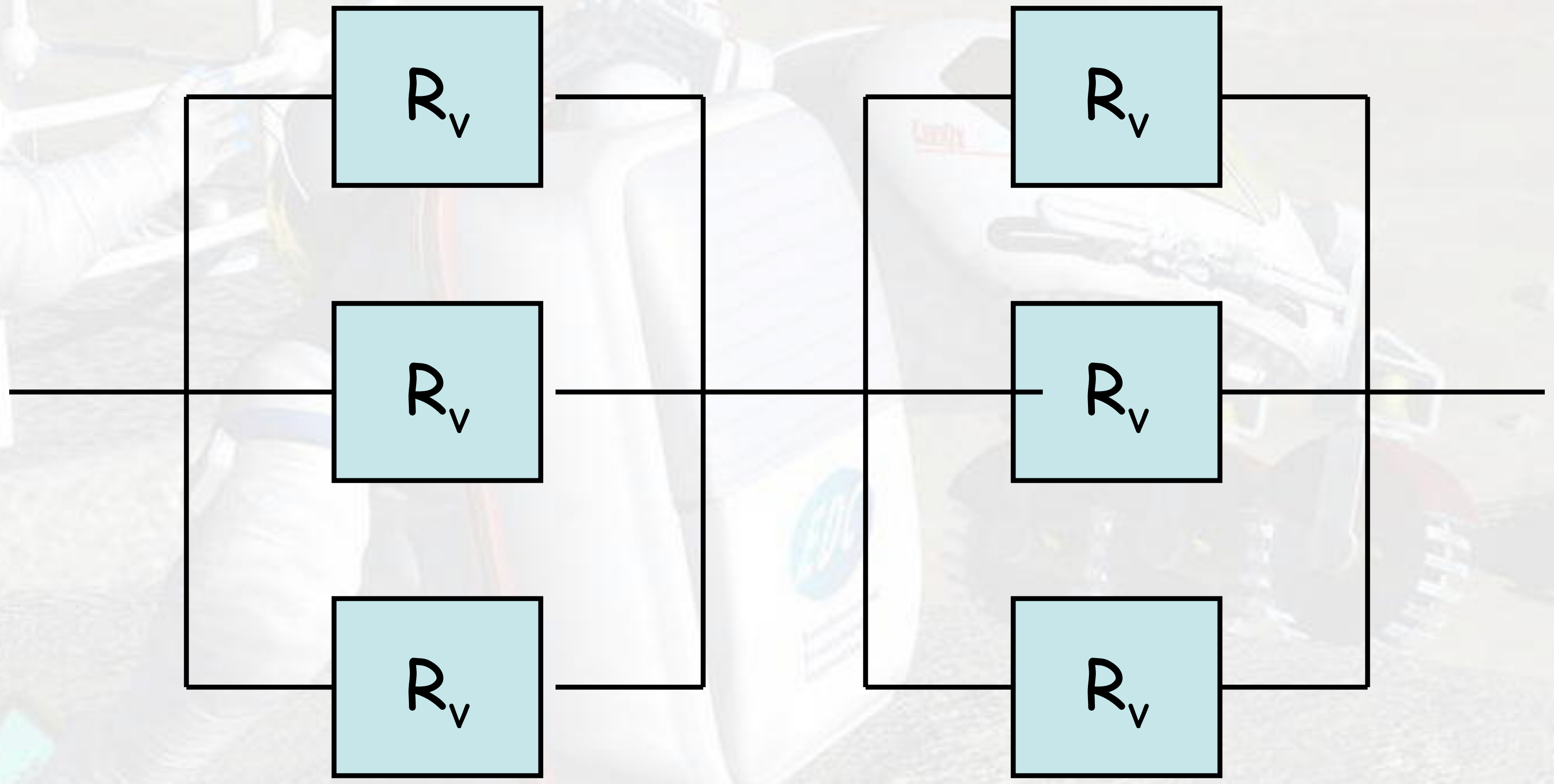
$$P(3) + P(2) + P(1) = .9928 + .0071 = .9999$$

- Probability that none work

$$P(0) = (1 - P)^3 = (.05)^3 = .0001$$

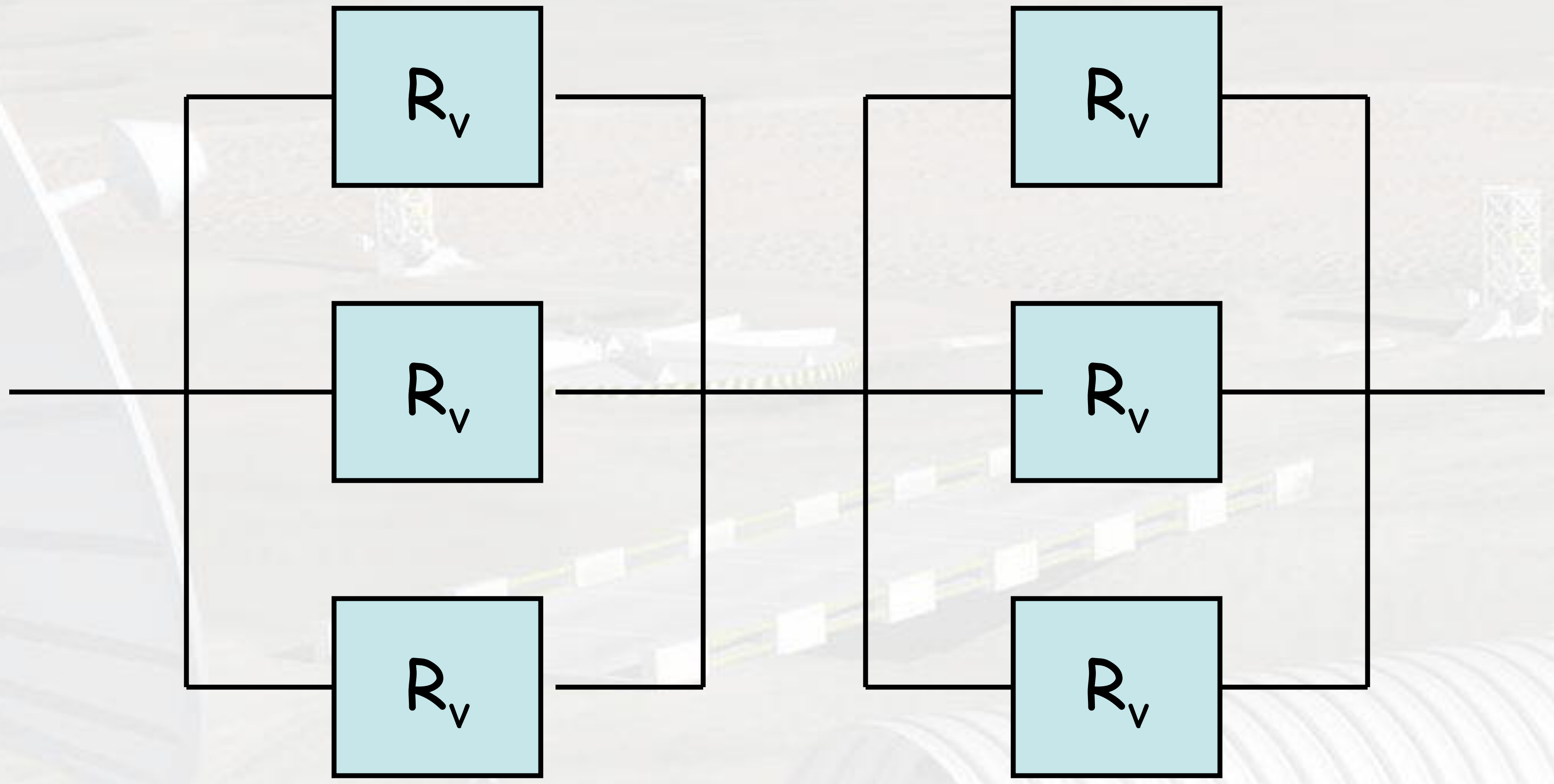
Reliability Diagrams

- Example of Apollo Lunar Module ascent engine
- Three valves in each of oxidizer and fuel lines
- One in each set of three must work
- R_v=0.9 --> R_{system}=.998



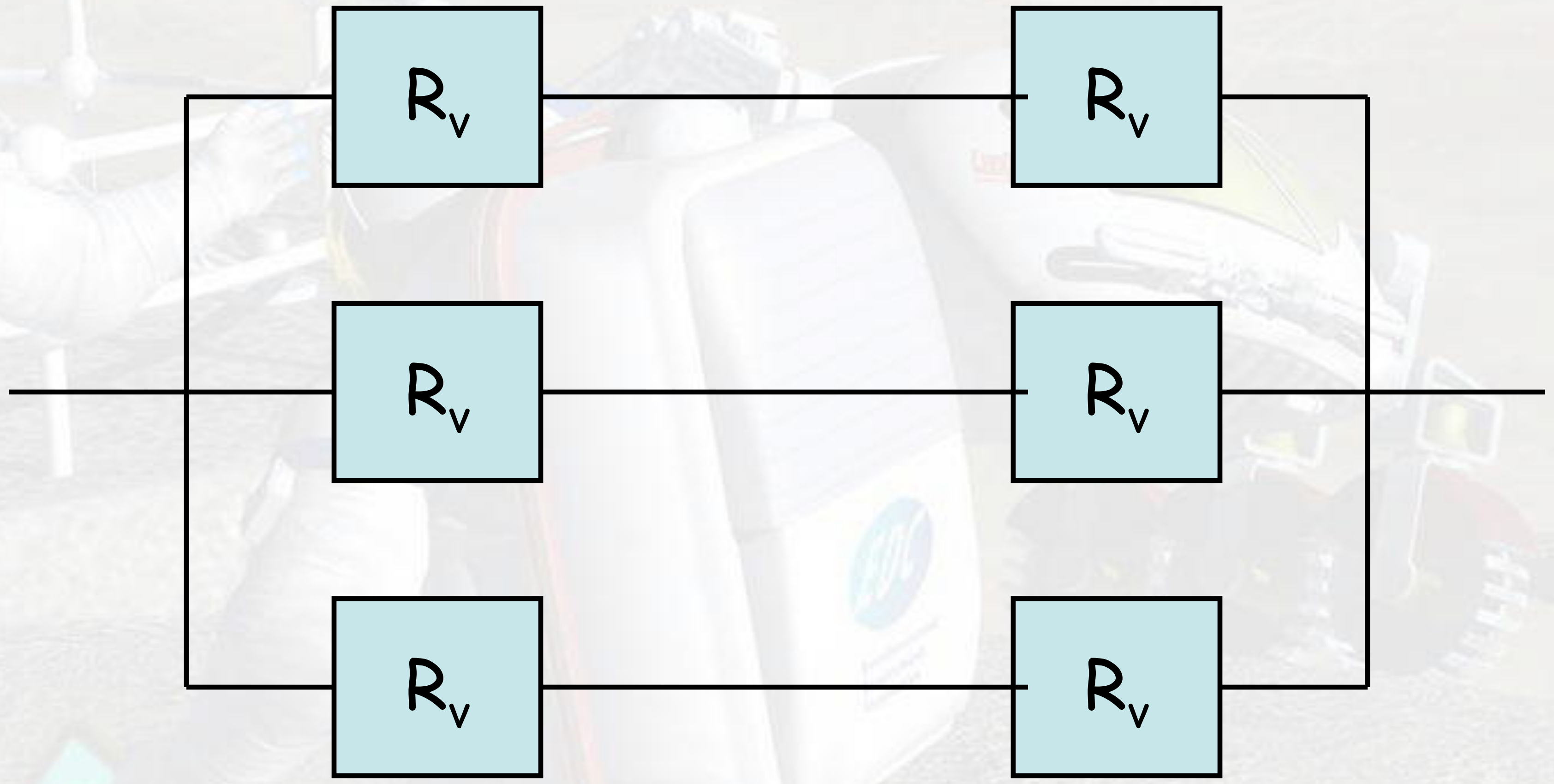
$$R_{system} = \left[1-(1-R_v)^3\right]^2$$

Reliability Diagrams (how not to…)



$$R_{system} = \left[1-(1-R_v)^3\right]^2$$

R_v=0.9 --> R_{system}=.998

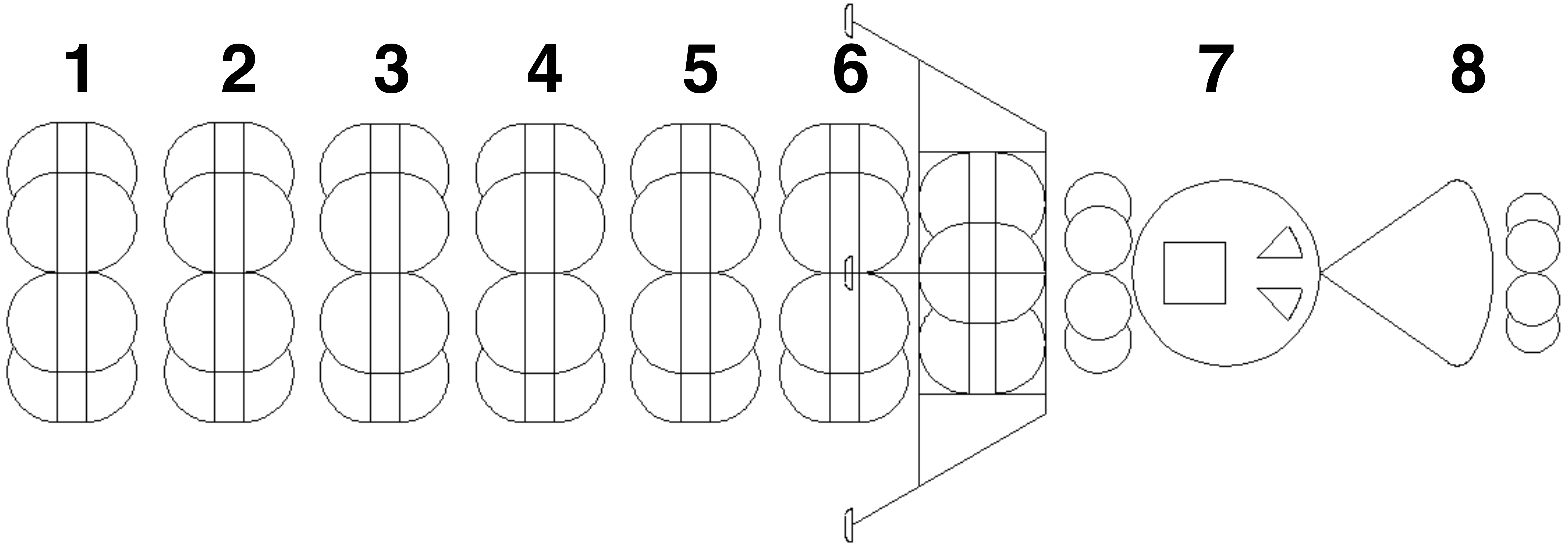


$$R_{system} = \left[1-(1-R_v^{\,2})^3\right]$$

R_v=0.9 --> R_{system}=.993

Earth Departure Configuration

8 launches and 7 dockings required to start mission



Assume P_{launch}=0.97 and P_{dock}=0.99

$P_{no\ failures} = P_{launch}^8\ P_{dock}^7 = 0.73$

$P_{all\ boost\ modules} = P_{launch}^6\ P_{dock}^5 = 0.792$

$P_{all\ boost\ modules} = P_{no\ failures} + P_{1\ failure} =$

$$0.792+6(1-P_{launch})P_{launch}^6\ P_{dock}^5 = 0.792+0.143 = 0.935$$

Spares - The Big Picture

- **Have to get 6 functional boost modules for each of 10 missions**
- **Have to get functional lunar vehicle and crew module for each mission**
- **Assume composite reliability =0.97(0.99)=0.96**

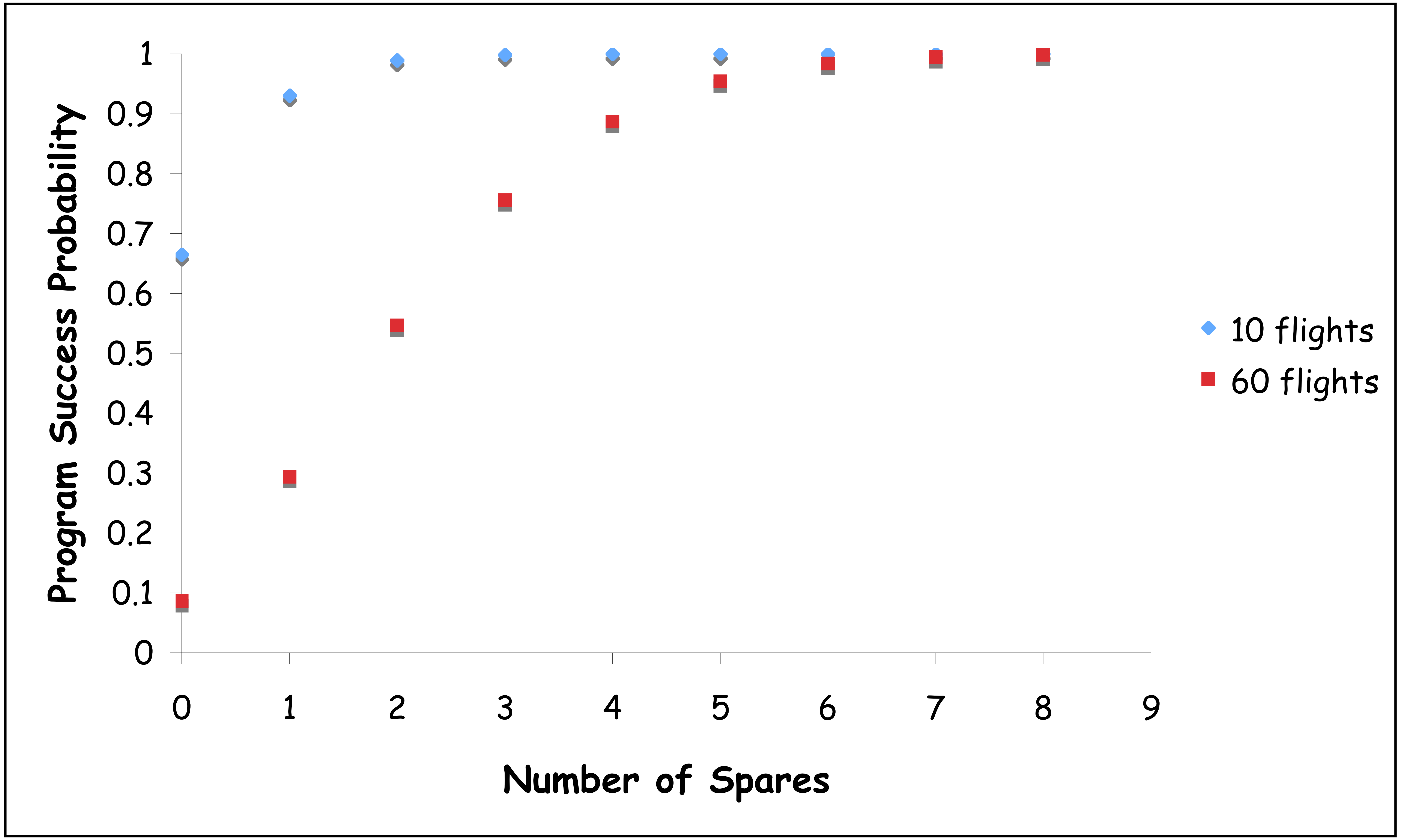
$$P(n \mid n) = p^n$$

$$P(n \mid n+1) = n(p^{n-1})(1-p)(p)$$

$$P(n \mid n+2) = \frac{n(n-1)}{2}(p^{n-2})(1-p)^2(p)$$

$$P(n \mid n+m) = \frac{n!}{(n-m)!m!}(p^{n-m})(1-p)^m(p)$$

Effect of Fleet Spares on Program



Spares Strategy Selection

- **VSE approach:**
  - 2 launches and 1 dock: $P=(0.97)^2(0.99)=0.931$
  - Program reliability over 10 missions: $0.931^{10}=0.492$
- **Goal: meet VSE program reliability**
  - 1 lander and 1 CEV spare - $p=0.9308$ each
  - 2 boost module spares - $p=0.5464$
  - Program reliability: $(0.9308)^2(0.5464)=0.473$
- **Alternate goal: 85% program reliability**
  - 2 lander, 2 CEV, 4 BM spares: $(0.9893)^2(0.8871)=0.868$
  - 1 lander, 1 CEV, 6 BM spares: $(0.9308)^2(0.9838)=0.852$

Intercorrelated Failures

- Some failures in redundant systems are common to all units
  - Software failures
  - "Daisy-chain" failures
  - Design defects
- Following a failure, there is a probability f that the failure causes a total system failure

Intercorrelated Failure Example

3 parallel computers, each has reliability of 95%, and a 30% intercorrelated failure rate:

- Probability all three work

$$P(3) = P^3 = (.95)^3 = .8574$$

- Probability exactly two work (one failure)

$$P(2) = 3P^2(1 - P) = 3(.95)^2(.05) = .1354$$

  – Probability the failure is benign (system works)

$$P\left(2_{safely}\right) = .7(.1354) = .0948$$

  – Probability of intercorrelated failure (system dies)

$$P\left(2_{system\ failure}\right) = .3(.1354) = .0406$$

Intercorrelated Failure Example

(continued from previous slide)

- Probability exactly one works (2 failures)

$$P(1) = 3P(1-P)^2 = 3(.95)(.05)^2 = .0071$$

  - Probability that both failures are benign

$$P\left(1_{safely}\right) = .7^2(.0071) = .0035$$

  - Probability that a failure is intercorrelated

$$P\left(1_{system\ failure}\right) = \left(1-.7^2\right)(.0071) = .0036$$

Redundancy Example with Intercorrelation

3 parallel computers, each has reliability of 95%, and a 30% intercorrelated failure rate:

- Probability all three work

$$P(3) = .8574$$

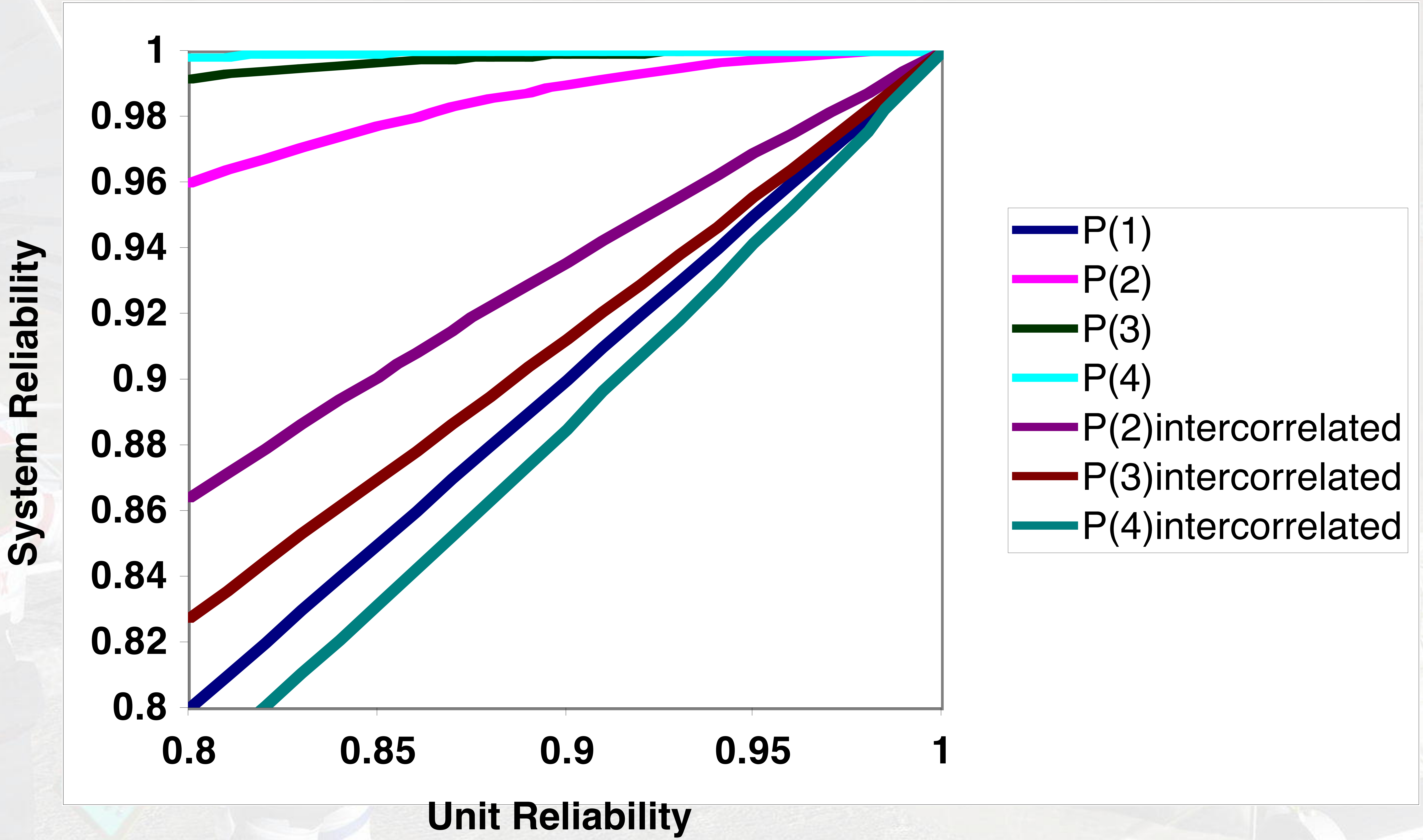
- Probability at least two work

$$= .8574 + .0948 = .9522 \quad (was\ .9928)$$

- Probability at least one works

$$= .9522 + .0035 = .9557 \quad (was\ .9999)$$

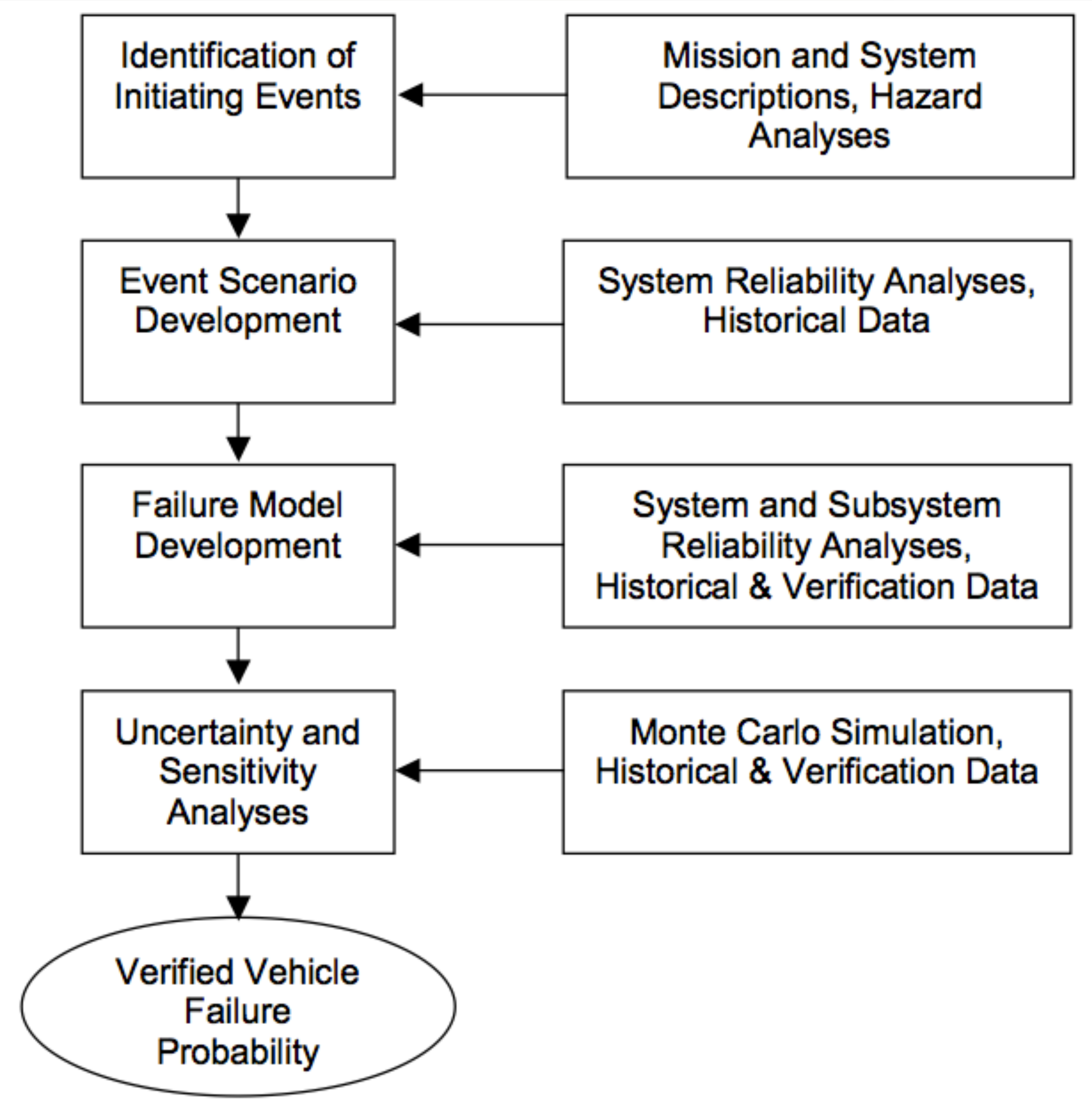
System Reliability with 30% Intercorrelation



Probabilistic Risk Assessment

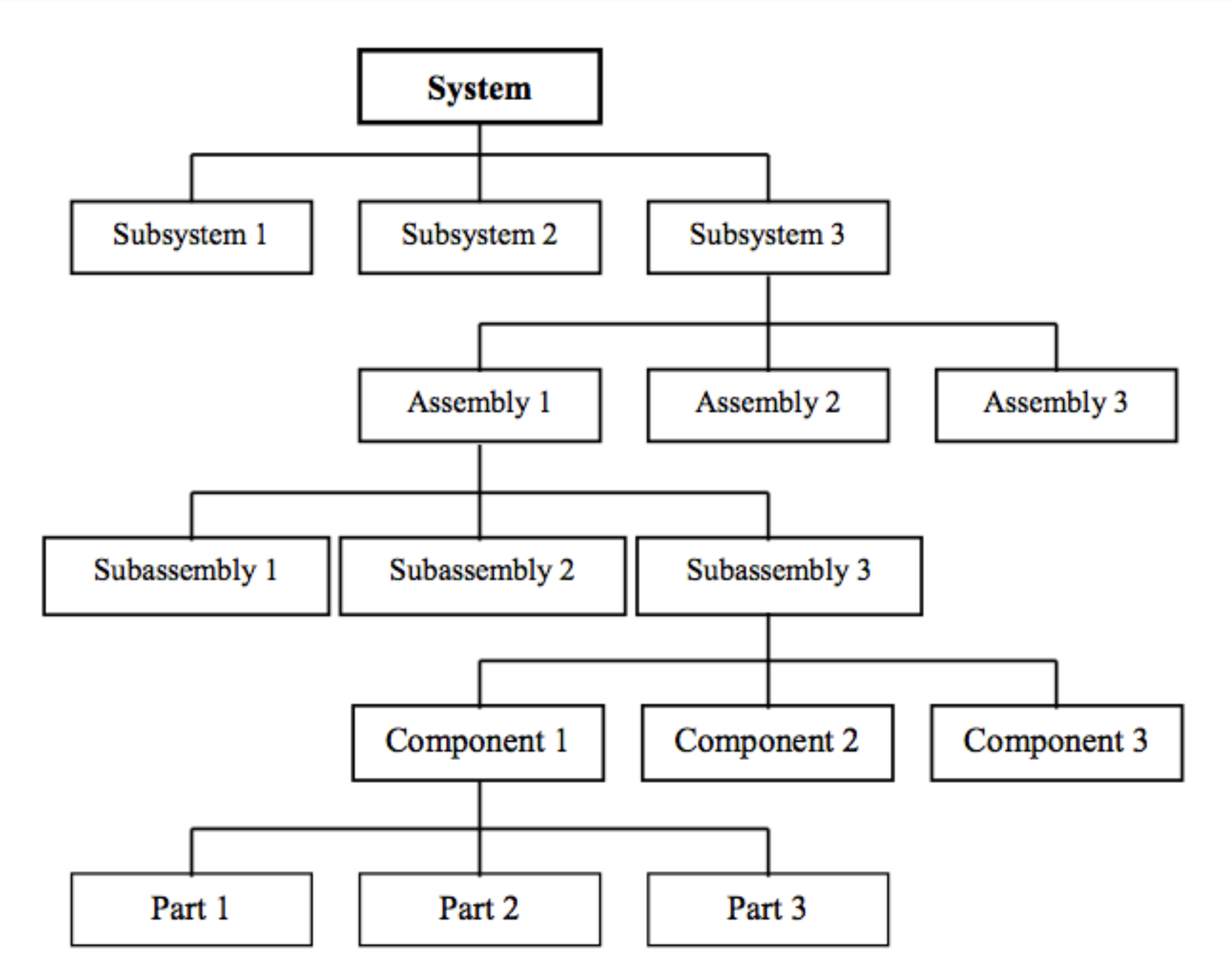
- Identification and delineation of the combinations of events that, if they occur, could lead to an accident (or other undesired event)
- Estimation of the chance of occurrence for each combination
- Estimation of the consequences associated with each combination.

PRA Process Flowchart



FAA, “Guide to Reusable Launch and Reentry Vehicle Reliability Analysis” April 2005

System Breakdown Chart



FAA, “Guide to Reusable Launch and Reentry Vehicle Reliability Analysis” April 2005

Failure Modes and Effects Analysis

FAILURE MODES, EFFECTS, AND CRITICALITY ANALYSIS WORKSHEET

System: Upper Stage Propulsion System

Mission: Satellite Delivery to GEO

Phase: Orbital Insertion

Ref. Drawing: GTYD-1002B008

Sheet 1 of 20

Prepared by: John Smith

Reviewed by: Janet Jones

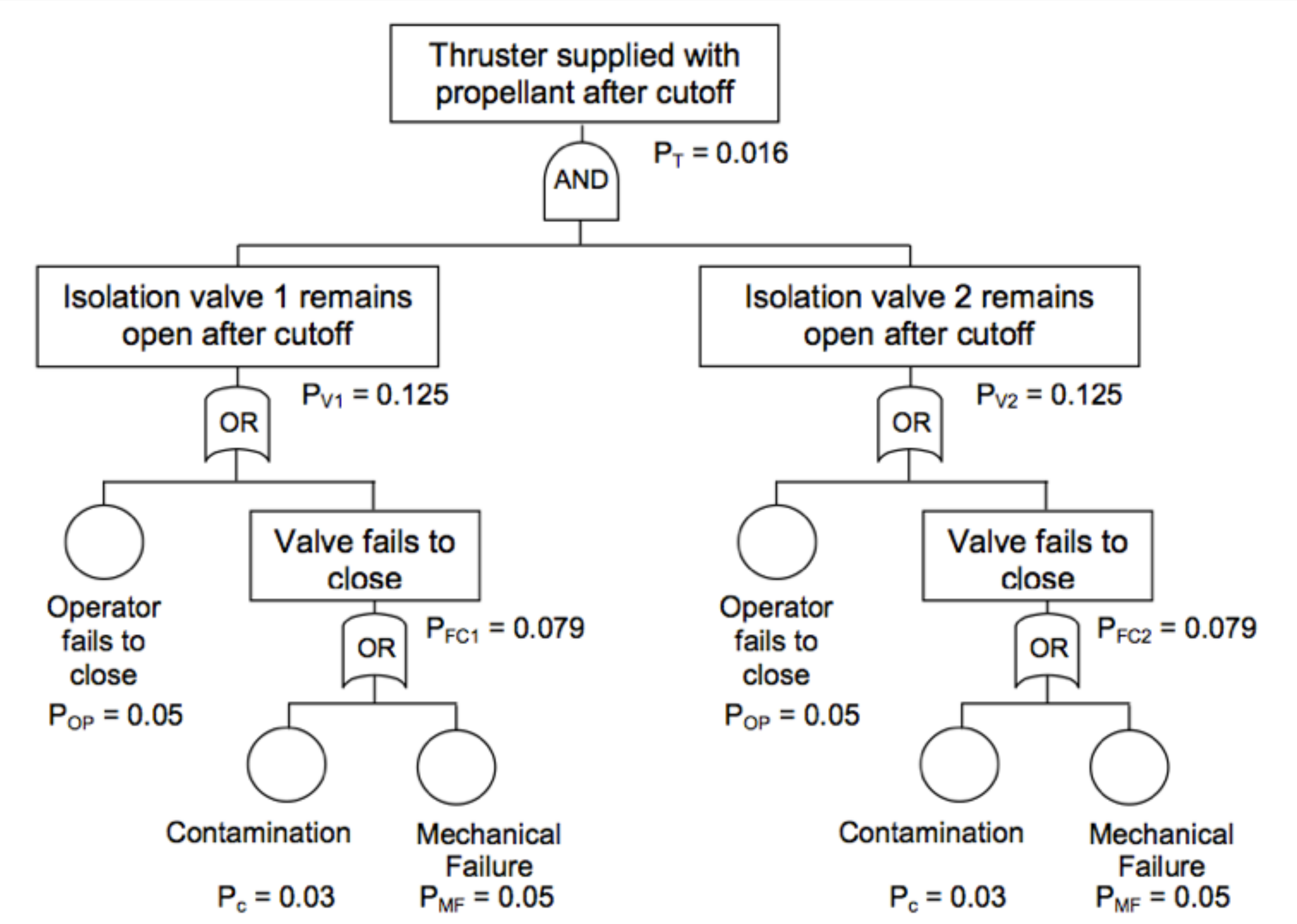
Approved by: Sharon Jackson

Date: January 2, 2004

ID	Item	Failure Modes	Failure Causes	Failure Effects	Risk Assessment			Detection Methods and Controls
					Sev.	Prob.	Risk	
2.0	Combustion Chamber	a. Coolant loss b. Seal failure	a. Manufact. process problem b. Cyclic fatigue	a. Reduced performance, burn-through, possible crash and injury to involved public b. Reduced performance	a.II b.III	a.C b.D	a.6 b.14	a. Inspect welds b. Seal redundancy

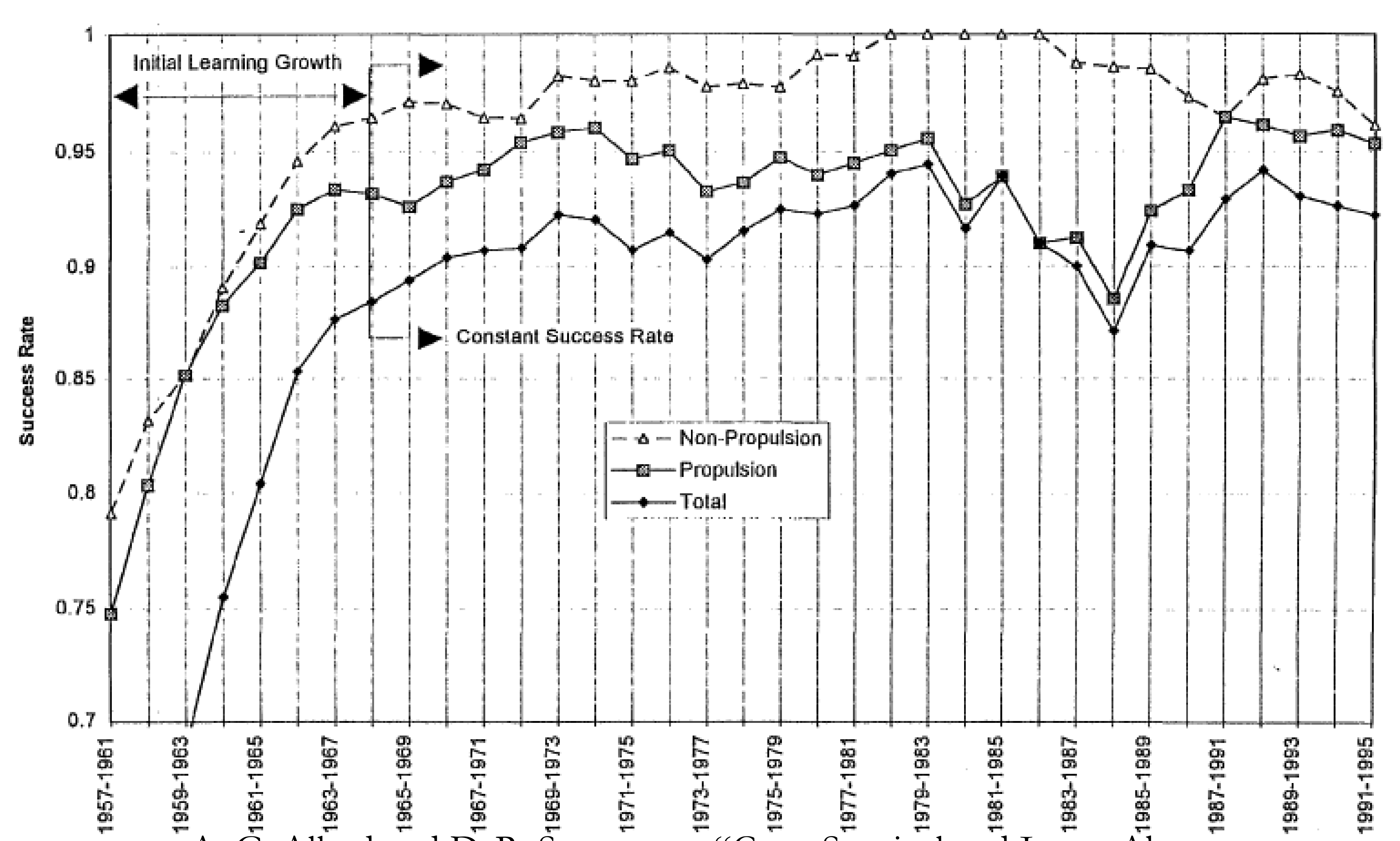
FAA, "Guide to Reusable Launch and Reentry Vehicle Reliability Analysis" April 2005

Fault Tree Analysis



FAA, “Guide to Reusable Launch and Reentry Vehicle Reliability Analysis” April 2005

U.S. Launch Reliability - 5 yr. rolling avgs.



A. G. Allred and D. R. Sauvageau, "Crew Survival and Intact Abort using Solid Rocket Boosters" AIAA 96-3156, July, 1996

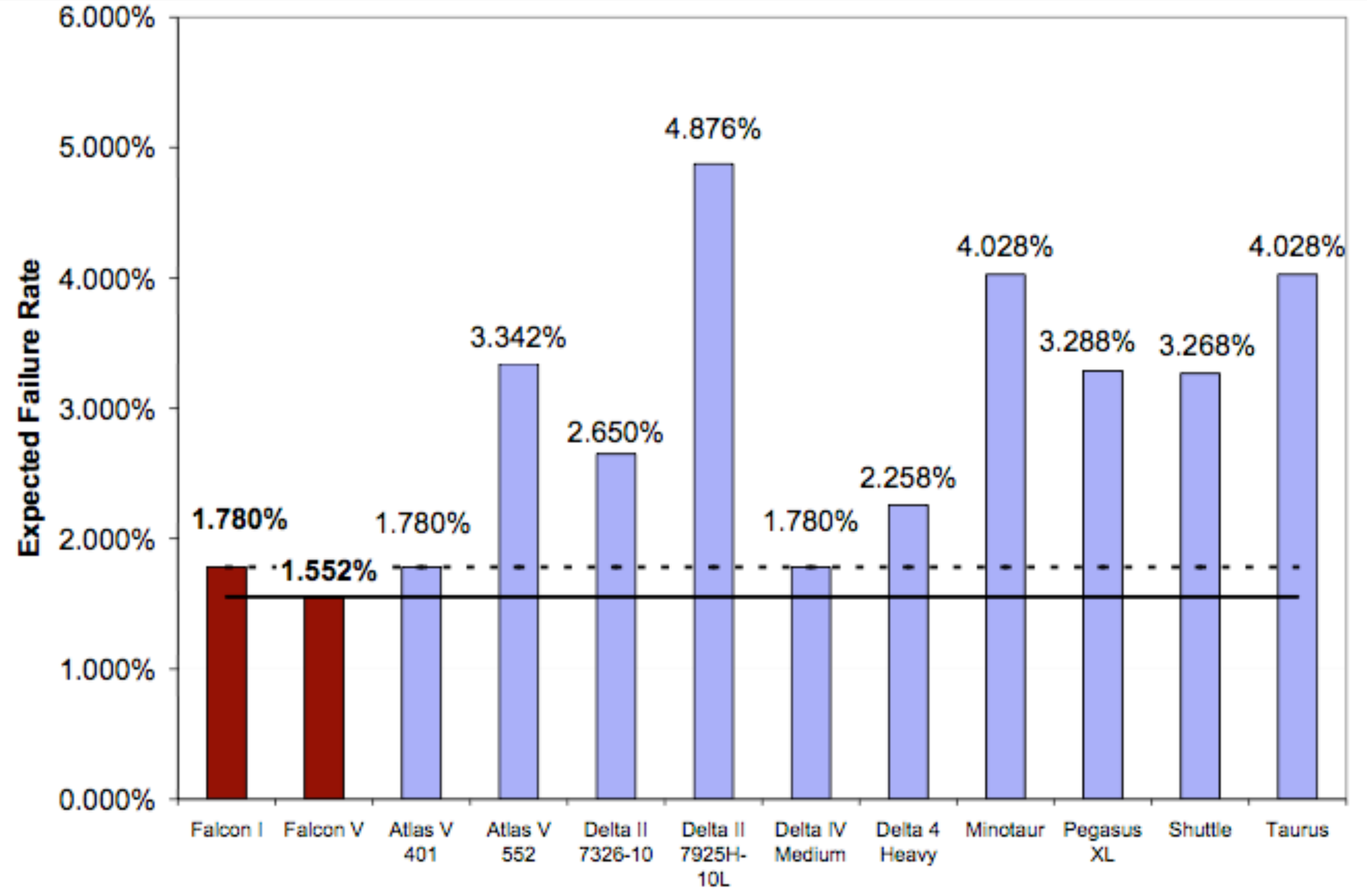
LV Subsystem Failures 1984-2004

Failure Type	Failures	Total Events	Individual Percent Failure Rate
Liquid Propulsion (Start)	3	1255	0.239%
Liquid Propulsion (In-flight)	3	1255	0.239%
Total Liquid Failure	**6**	**1255**	**0.478%**
Solid Propulsion (Shell)	4	1831 (all solids)	0.218%
Solid Propulsion (TVC)	3	571 (TVC only)	0.525%
Solid Propulsion with TVC (TVC and Shell Failure Modes)	**--**	**--**	**0.743%**
Stage, Booster, and Payload Separations	6	2577	0.233%
Fairing Separation	1	357	0.280%
Small Solid Booster Separations	1*	1165	0.086%
Electrical	2	470	0.426%
Avionics	2	470	0.426%
Other	1	470	0.213%

*Did not result in total mission loss.

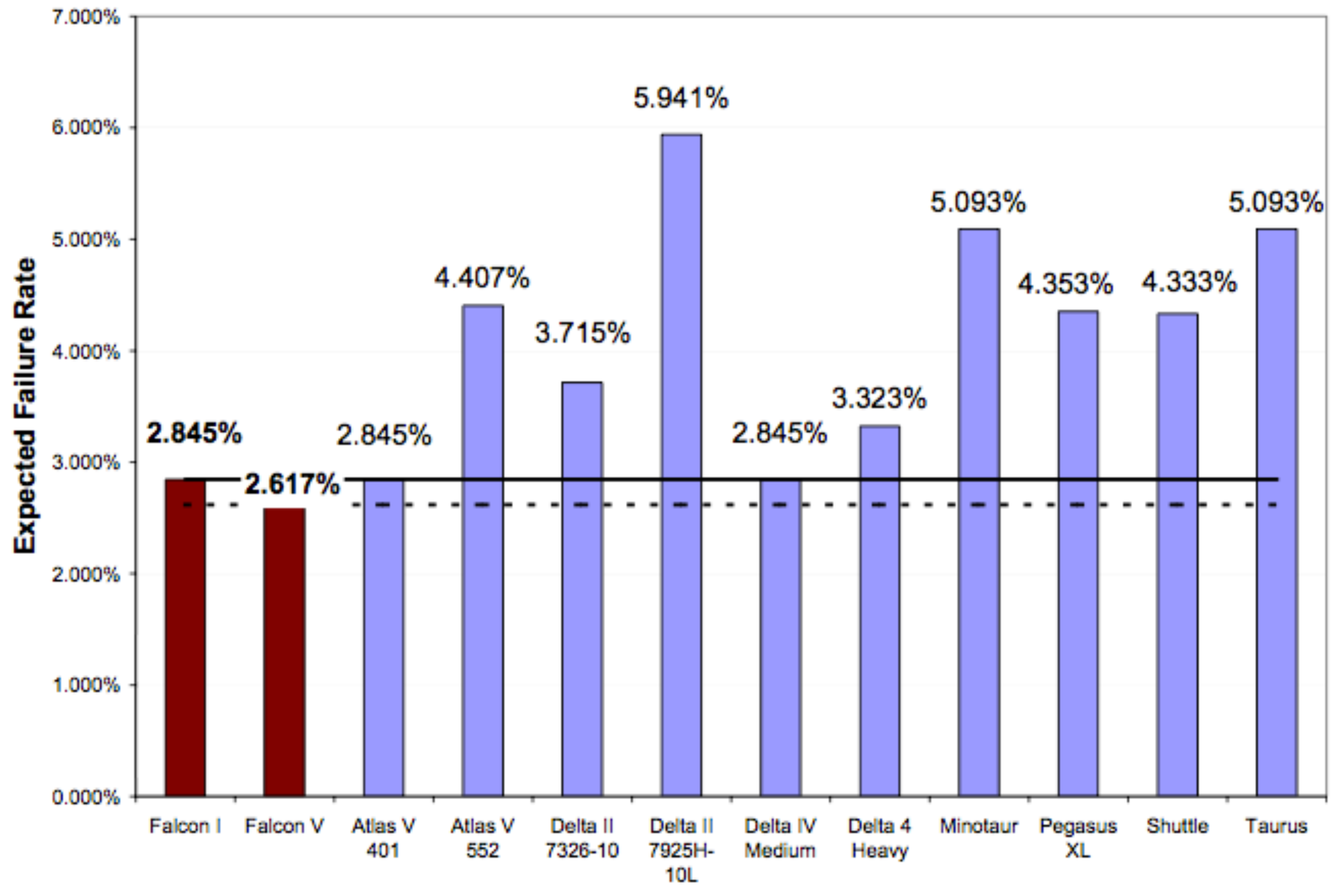
Futron Corporation, "Design Reliability Comparison for SpaceX Falcon Vehicles" Nov. 2004

Expected Failure Rates from Prop/Sep



Futron Corporation, “Design Reliability Comparison for SpaceX Falcon Vehicles” Nov. 2004

Failure Rates from All Causes



Futron Corporation, "Design Reliability Comparison for SpaceX Falcon Vehicles" Nov. 2004

Concept of System Resiliency

- Initial flight schedule

✈ ✈ ✈ ✈ ✈ ✈ ✈ ✈ ✈ ✈ ✈

- Hiatus period following a failure

✈ ✠ ✈ ✈ ✈ ✈ ✈

- Backlog of payloads not flown in hiatus

✈ ✈ ✈ ✈

- Surge to fly off backlog

✈ ✠ ✈✈ ✈✈ ✈✈ ✈✈✈

- Resilient if backlog is cleared before next failure occurs (on average)

Resiliency Variables

r - nominal flight rate, flts/yr

d - down time following failure (yrs)

k - fraction of flights in backlog retained

S - surge flight rate/nominal flight rate

m - average/expected flights between failures

rd - number of missed flights

krd - number of flights in backlog

(S-1)r - backlog flight rate

Definition of Resiliency

- Example for Delta launch vehicle
- r = 12 flts/yr
- d = 0.5 yrs
- k = 0.8
- S = 1.5
- m = 30
- Srkd/(S-1) = 14.4 < 30 - system is resilient!

$$\frac{Srkd}{S-1} \leq m$$

Shuttle Resiliency (post-*Challenger*)

r = 9 flts/yr

d = 2.5 yrs

k = 0.8

S = .67 (6 flts/yr)

m = 25

✓ System has negative surge capacity due to reduction in fleet size - cannot *ever* recover from hiatus without more extreme measures

Modified Resiliency

k′ - retention rate of all future payloads
(k′≤S for S<1)

- New governing equation for resiliency:

$$\frac{Srk'd}{S-k'} \leq m$$

- Implication for shuttle case:

✓k<.417 to achieve modified resiliency

Shuttle Resiliency (post-*Columbia*)

- r = 5 flts/yr
- d = 2 yrs
- S = .8 (4 flts/yr)
- m = 56 (average missions/failure)
- Modified resiliency requires k'≤0.7 for all future payloads

Today's Tools

- Calculation of probabilities
- Expected value and utility theory
- Failure rate and MTBF
- Redundancy and intercorrelated failures
- Resiliency calculations