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VALUATION OF COMPANY MERGER FROM THE SHAREHOLDERS' POINT OF VIEW

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Abstract
By means of a company merger formerly legally and economically independent companies are tied up to an economic entity. To order the financial state of affairs after the merger, the current shareholders must revalue their stake in the merged company. The interest is focused on the valuation of shares and, consequently, on the allocation of the future economic benefits of the merged company to each owner. Despite the apparent relevance of company mergers in practice, the scientific literature deals with this topic only in an unsatisfying manner. After some early simple model-oriented approaches with the aim to define an ideal exchange ratio, the valuation problem of a merger was taken up again not earlier than in Hering (2004). Based on his considerations, the aim of our paper is to extend and generalize the valuation methods for a company merger and foremost to set the algebra for the computation of the critical share by using maximization of wealth as target function on a firm foundation.
We assign a certain marginal quota to the shareholders representing the minimum share in the merged company which puts them in a financial position no worse than compared to the going concern basis. For this reason, we introduce the state marginal quota model as an innovative valuation approach that considers both existing market imperfections and individual expectations of a specific shareholder. To pinpoint our key finding: If private financial redistributions are available, our extended and generalized model shows that the marginal quota α^* in question cannot be "trivially" obtained as a ratio of utilities. Instead, it is essential to consider the private decision field of a shareholder to allow a restructuring of the dividend payout stream offered by the merged company in order to reach at least a level of utility which is comparable to the state before the merger.

Keywords: business valuation, company merger, decision function, shareholder, marginal quota, maximization of wealth.

JEL Classification: C61, G31, G34

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Introduction

In contrast to an acquisition or a sale of an entire company, a *merger* of a company does not necessarily imply a transfer of ownership (Gerold, 1989, pp.361-362; Matschke, Brösel and Matschke, 2010, p.7; Brösel, Matschke and Olbrich, 2012, p.243; Brösel, Toll and Zimmermann, 2012). Rather, the old respectively current owners have to revalue their stake in the newly established larger company. For a definition of a merger we distinguish between an (economically) broad and a (legally) narrow interpretation. According to the interpretation of a merger in a broad sense, some formerly legally and economically independent firms merge and are consolidated into a new entity regardless of legal aspects (Dodd, 1980, p.105; Matschke, Brösel and Matschke, 2010; Brösel, Toll and Zimmermann, 2012, p.92). If it is the case that at least one of the companies loses its legal independence after the merger, we speak of a merger in a narrow sense since now a legally and economically completely new entity arises. Both interpretations have in common that all shareholders of the formerly independent companies gain property in the merged company.

Subject to the legally-based interpretation of a merger (merger in a narrow sense), either one of the companies is at the head of the remaining ones or all companies form a new legal entity on an equal basis. The economically-based interpretation of a merger (merger in a broad sense) includes all afore-mentioned cases and it is also possible that all parties retain their legal personalities. This rather broad definition includes as well the special cases in which a holding is installed with legally independent subsidiaries or where new shareholders are integrated who contribute a new business operations unit to a company or provide merely venture capital, for instance.

In general, a merger is *undertaken* to satisfy certain enterpreneurial goals (Milgrom and Roberts, 1992; Malekzadeh and Nahavandi, 2007, p.79; Haleblian et al., 2009). Apart from financial motives like a better supply of equity or debt capital (Scott, 1977, p.1235) or the reap of tax benefits (Lewellen, 1971; Lam and Chiu, 2005), there are also personal motives like aspirations for power or prestige (Jensen and Meckling, 1976; Grossman and Hart, 1983; Jensen, 1986; Eisenhardt, 1989) as well as a deliberate or undeliberate sort of hubris (Roll, 1986; Hayward and Hambrick, 1997; Kroll, Toombs and Wright, 2000; Seth, Song and Pettit, 2000; Malmendier and Tate, 2005, 2008; Chatterjee and Hambrick, 2007; Homberg and Osterloh, 2010). Strategic motives are concerned with the optimization of the production processes of a company. Some managers want to beef up productivity by means of consolidating companies of the same line of business, which have identical value-added activities (horizontal mergers), by realizing economies of scale, by reducing production costs per unit or by combining supply chains which may lead to better sourcing conditions (Wright, 1936; Perry and Porter, 1985; Amihud, Dodd and Weinstein, 1986, p.401; Goold and Campbell, 1998). Due to sweeping market forces, they want to achieve a higher market share or a greater market power (Ansoff, 1965). If companies of the same line of business are consolidated which operate on different levels of value-added activities (vertical mergers), managers aim particularly at reducing costs for the coordination of business activities along a certain line of business as well as at reducing the reliance on certain stakeholders by interconnecting the corresponding input and output channels with the ultimate goal to gain less output-related risks (Amihud, Dodd and Weinstein, 1986, p.401; Eschen and Bresser, 2005; Grill and Bresser, 2013). Motives for a merger of companies of different lines of business (conglomerate mergers) are the enlargement of the product portfolio (Markowitz, 1952) and a jointly use of formerly not completely used machines or assets (Lam and Chiu, 2005; Eschen and Bresser, 2005; Grill and Bresser, 2013) as well as cost reductions within existing input or output channels.

If after an assessment of all pros and cons the decision for a merger is in a certain sense positive, the distribution of the ownership rights (connected to the shares) and the allocation of the future economic benefits of the merged company to all parties is the *chief problem* which has to be solved (Matschke, Brösel and Matschke, 2010, p.7). The related main objective of a valuation is to find the minimum quota or proportion of shares in the merged company. This quota will also be called marginal quota because it inflicts no economic losses on a valuation subject, which consists of a single shareholder or a group of owners. In the current publication it is demonstrated how this valuation problem can be formulated from the shareholders' viewpoint applying a so-called state marginal quota model. To identify the contributions of the current publication, it is worthwhile to discuss first of all the current state of art concerning the valuation of corporate mergers which is done in the second section. In section three, a general formulation and exemplary considerations of the state marginal quota model are given. Finally, the results of the present contribution are critically reviewed and a short outlook is given in section four.

1. Review of the scientific literature

In spite of a high practical relevance of mergers, there are only quite a few scientific publications which address the valuation problem of a company merger by means of a model-oriented approach (Silberman, 1968; Larson and Gonedes, 1969; Ramanathan and Rappaport, 1971; Matschke, 1975, pp.327-336; Kipping, 1982; Nonnenmacher, 1982; Yagil, 1987; Hering, 2004; Cigola and Modesti, 2008; Giacomello, 2008; Kürsten, 2008; Moretto and Rossi, 2008; Tagliavini, 2008; Toll, 2011, pp.110-137). The valuation problem of a company merger is treated scarcely in textbooks which deal with corporate valuations in general. This is even more astounding if we understand that the classical valuation models for an acquisition or a sale of a company cannot be applied to the given valuation problem of a company merger at once without a number of modifications. In fact, the valuation problem of a merger needs a special attention.

On balance, the valuation problem of a company merger has been treated only in an unsatisfying manner by the scientific community despite its relatively high practical relevance. Furthermore, there had been only a few models for the computation of decision values for quite a long time which were based solely on discounted cash flow considerations (Silberman, 1968) with the aim to define an ideal exchange ratio (Larson and Gonedes, 1969; Ramanathan and Rappaport, 1971; Nonnenmacher, 1982; Yagil, 1987; Cigola and Modesti, 2008; Giacomello, 2008; Kürsten, 2008; Moretto and Rossi, 2008; Tagliavini, 2008). In, 2004, Hering (2004) proposed a *state marginal quota model* as investment-oriented method. Thereby, he addressed a company merger by assuming that formerly independent companies were consolidated into a larger economic entity. Hence, the shareholders of the companies in question have only two disparate alternatives for actions at their disposal. Either they decide in favor of the merger or their companies stay independent. From the shareholders' viewpoint, the valuation of a merger should happen in the following way, identifying *three distinct steps*: In a first step, the possible withdrawals (also referred to as "dividend payouts") of the party of old shareholders are computed in a so-called *base approach* where the original state is assessed before the merger takes place. The base program defines an optimal individual investment and financing program.

Clearly, the base program reveals the optimal target value attainable for the shareholders which should be at least reached again after the merger. In a second step, the attention is focused on the consolidated company for which a so-called *merger approach* is formulated. The resulting merger program defines the maximum dividend payouts to the unified group of shareholders of the merged company. In a third and final step, by considering the optimal target values of the base and merger programs, the *marginal quota* α^* can be computed as the minimum share to be assigned to the concerned party of shareholders of the formerly independent companies satisfying the additional constraint not to cause any economic harm to them.

In Hering (2004), maximization of income was tacitly assumed as a target for all shareholders. A similar analysis for maximizing the total asset value as target was only confined to a special case (Hering, 2014, pp.96-97) in which the marginal quota was "trivially" determined as a ratio of utilities making direct use of the given weighting factors w_t referring to the preferred structure of dividend payouts. However, such a ratio of utilities leads only to the optimal quota if the redistributions within the financing objects (increase or decrease in private debt obligations (borrowing) or in financing investments (lending)) are neglected. If such *private financial redistributions* are possible and necessary, the marginal quota α^* in question cannot be "trivially" obtained as a ratio of utilities. Instead, the private decision fields of the concerned shareholders have to be taken into account to determine the unique marginal quota which guarantees a comparable utility by considering the future cash flows which are available before and after the merger. At this critical point, the *research objective* of the present contribution can be formulated: Based on the proposals in Hering (2004; 2014, pp.96-97), it shall be shown how the marginal quota can be determined in a more general case in which private financial redistributions are allowed by assuming maximization of wealth as target.

2. Research methodology

We consider a company participating in a merger. The shareholders of the company are faced with two mutually excluding alternatives for actions. Either they decide in favor of the merger or their companies will stay independent. For the related decision-making the *state marginal quota model* according to Hering (2004) lends itself to be used as a suitable method.

The first step evaluates the *baseline*, i.e. the original state before the merger (Hering, Olbrich and Steinrücke, 2006, p.410; Brösel, Matschke and Olbrich, 2012, pp.249-250; Lerm, Rollberg and Kurz, 2012, pp.263-265). For the owners of the merging companies we postulate maximizing of wealth as target such that we have to maximize the weighted sum GW of all dividend payouts G_t at the points of time t weighted with the weighting factors w_t which mirror the unique consumption preferences of the owners (Hering, 2015, pp.150-153). Already known fixed-dividend payouts to the owners as well as all so far predetermined cash flows (due to existing operating activities or given debt obligations, for instance) are considered in the fixed cash flow stream **b** balancing the liquidity conditions. To ensure the company's existence beyond the planning horizon n, a fixed cash flow balance b_n can be integrated as a sufficiently high fictive dividend payout at the end of period n that guarantees the continuation of the desired level of fixed-dividend payouts beyond the planning horizon.

Furthermore, we make the following assumptions (Hering, Olbrich and Steinrücke, 2006; Hering and Toll, 2015; Hering, Toll and Kirilova, 2016): The planning period extends n years, whereby t = 0 is the reference moment of decision-making. Initially, we have j = 1, 2, ... , m investment and financing objects at our disposal. These may include given opportunities for borrowing, investing in interest-bearing financial assets as well as unlimited cash holdings. The cash flow stream of any object j is given as follows: $\mathbf{g}_j := (g_{j0}, g_{j1}, \ldots, g_{jt}, \ldots, g_{jn})$. Thereby, g_{jt} describes the cash surplus at the point in time t. How often a certain object j can be realized at most is indicated by a constant x_j^{max}, which bounds the decision variable x_j from above. For all x_j with $x_j^{max} = \infty$ there are no upper bounds. The variables G_t and x_j are confined to non-negative quantities. The liquidity conditions ensure that at any time t, the sum of all cash outflows is never greater than the sum of all cash inflows. To determine the optimal investment and financing program for a given decision field before the merger, we have to solve the following *pre-merger approach* (base approach) which defines the baseline (Weingartner, 1963; Hax, 1964; Hering, Olbrich and Steinrücke, 2006; Brösel, Matschke and Olbrich, 2012; Hering, 2015):

$$\max. GW; \; GW := \sum_{t=0}^{n} w_t \cdot G_t$$

$$-\sum_{j=1}^{m} g_{jt} \cdot x_j + G_t \le b_t \;\forall\; t \in \{0, 1, 2, \ldots, n\}; \; x_j \le x_j^{max} \;\; \forall\; j \in \{1, 2, \ldots, m\}$$

$$x_j \ge 0 \;\forall\; j \in \{1, 2, \ldots, m\}; \; G_t \ge 0 \;\forall\; t \in \{0, 1, 2, \ldots, n\}$$

The optimal solution (*base program*) of this optimization approach delivers the dividend payouts G_t*, as well as the maximal target value $GW^{max} = GW^*$.

In a second step, we consider the *merged company*. By means of the merger approach, we have to determine the total amount of payouts distributable to all shareholders of the consolidated company. Thereby, we have to take into account that due to different voting or ownership rights the merged company may follow a different target function (e.g. maximization of income or some other structured form of wealth maximization) in comparison to the merging companies. In the following, we assume once more maximization of wealth and use a superscript F to indicate quantities related to the merged company:

$$\max. GW^F; \; GW^F := \sum_{t=0}^{n} w_t^F \cdot G_t^F$$

$$-\sum_{j=1}^{m^F} g_{jt}^F \cdot x_j^F + G_t^F \le b_t^F \;\forall\; t \in \{0, 1, 2, \ldots, n\}; \; x_j^F \le x_j^{F\,max} \;\forall\; j \in \{1, 2, \ldots, m^F\}$$

$$x_j^F \ge 0 \;\forall\; j \in \{1, 2, \ldots, m^F\}; \; G_t^F \ge 0 \;\forall\; t \in \{0, 1, 2, \ldots, n\}$$

The *merger program* delivers information about the optimal dividend payouts G_t^{F*} at times t which are obtainable by all shareholders of the consolidated company and the related actions j that must be taken. Thereby, it is irrelevant if G_t^{F*} results from maximization of wealth or may represent merely a dummy variable for the optimal dividend payout at the point of time t for any other kind of target function – e.g. maximization of income. The model just implies that the party of old shareholders of the unmerged company pursues some kind of wealth maximization and that the computed utility of the pre-merger program GW* is the minimal requirement for the party of old shareholders after the merger.

After this preliminary considerations, we are now able to compute the unique *marginal quota* α^*, which satisfies the party of old shareholders after the merger. The methodical *complexity* of the calculation of the marginal quota depends upon whether or not private financial redistributions are possible and necessary to reconfigure the dividend payout stream of the merged company with the goal to reach at least a level of utility which is comparable to the state before the merger (Hering and Toll, 2017, p.298).

Only if *private financial redistributions are not possible* or are *possible* but *not necessary* to satisfy the level of utility of the pre-merger program, the minimum demandable quota in the merged company can be computed by the following so-called "trivial valuation formula":

$$\alpha^* = \frac{\overbrace{GW^*}^{\text{Utility before the merger}}}{\underbrace{\sum_{t=0}^{n} w_t \cdot G_t^F*}_{\text{Utility after the merger}}}. \quad (1)$$

It may be the case that a dividend payout is drawn out at the same time before and after the merger (by maximizing the terminal value, for instance). Hence, there is no need for private financial redistributions to level the utilities of the optimal dividend payout streams of the pre-merger and merger programs. In contrast, if *private financial redistributions* are *possible* and *necessary*, the required marginal quota α^* cannot be "trivially" determined as a ratio of utilities. Rather, we have to consider the private decision field of a shareholder to allow a redistribution of the dividend payout stream of the merged company in order to obtain comparable utilities.

If it is possible for the old owners to restructure the new dividend payout stream of the merged company to obtain the same utility as before the merger, the merger can be indeed a viable alternative. This can be accomplished by means of supplementary private financial redistributions. In a *valuation approach* we denote a variation of the j-th private financial object by the variable Δx_j^P belonging to the cash flows g_{jt}^P at various points of times t. In total, there are j = 1, 2, ... , m^P different private financing and cash investment activities. There may be of course limits to an increase or decrease of private financing or cash investment activities. For example, a private financial object can be maximally reduced to a level as could be realized before the merger. Moreover, in the private area, there is often an upper limit to additional borrowing. According to Hering (2004), we will assume in the sequel that only one single valuation approach with the target function "min AQ" has to be solved representative for the party of old owners, which leads to a huge reduction in complexity (Toll, 2011, pp.125-126; Hering, 2014, p.98; Hering and Toll, 2017, p.299).

Regarding maximization of wealth as target, we formulate:

$$\text{min. } AQ;\ AQ := \alpha;$$

$$\sum_{j=1}^{m^P} g_{jt}^P \cdot \Delta x_j^P + \alpha \cdot G_t^F{*} - G_t \geq 0\ \forall\ t \in \{0, 1, 2, \ldots, n\};\ \sum_{t=0}^{n} w_t \cdot G_t \geq GW^*;$$

$$-\Delta x_j^P \geq -\Delta x_j^{P\max}\ \forall\ j \in \{1, 2, \ldots, m^P\};$$

$$\Delta x_j^P \geq 0\ \forall\ j \in \{1, 2, \ldots, m^P\};\ G_t \geq 0\ \forall\ t \in \{0, 1, 2, \ldots, n\};\ \alpha \geq 0$$

As afore-mentioned, a similar valuation problem for maximization of income had been analyzed in Hering (2004) which is different to some extent to the present case in which the dividend payout stream of the valuation program has to satisfy at least the target value GW*, but where the dividend payout stream of the valuation program with the distributions G_t needs not to be identical in structure compared to their counterparts in the base program (whose dividend payouts are here denoted by G_t^*). Hence, we postulate solely that

$$\underbrace{\underbrace{\sum_{j=1}^{m^P} g_{jt}^P \cdot \Delta x_j^P}_{\substack{\text{redistributions within}\\ \text{private borrowing}\\ \text{or cash investment}}} + \underbrace{\alpha \cdot G_t^F{*}}_{\substack{\text{dividend payouts for the share } \alpha\\ \text{in the optimal dividend payout}\\ \text{stream of the merged company}}}}_{\substack{\text{cash outflow to the shareholders}\\ \text{(after the merger)}}} \geq \underbrace{G_t}_{\substack{\text{desired dividends by}\\ \text{the shareholders}\\ \text{under the constraint}\\ \sum_{t=0}^{n} w_t \cdot G_t \geq GW^*}}. \tag{2}$$

Our next task is the derivation of the so-called valuation formulas for determining α^*. For this purpose, we will at first formulate the dual optimization problem of the given primal optimization problem in which we denote the dual structure variables of the liquidity conditions by d_t^P, the dual structure variables referring to the upper limits by u_j^P and, finally, the dual structure variable for the utility constraint by δ:

$$\text{max. } Y;\ Y := \delta \cdot GW^* - \sum_{j=1}^{m^P} \Delta x_j^{P\max} \cdot u_j^P;$$

$$\sum_{t=0}^{n} g_{jt}^P \cdot d_t^P - u_j^P \leq 0\ \forall\ j \in \{1, 2, \ldots, m^P\};\ \sum_{t=0}^{n} G_t^{F*} \cdot d_t^P \leq 1;$$

$$-d_t^P + w_t \cdot \delta \leq 0\ \forall\ t \in \{0, 1, 2, \ldots, n\};$$

$$d_t^P \geq 0\ \forall\ t \in \{0, 1, 2, \ldots, n\};\ u_j^P \geq 0 \forall\ j \in \{1, 2, \ldots, m^P\};\ \delta \geq 0$$

We assume that the primal valuation approach has an optimal solution with $\alpha^* > 0$. According to the theorem of complementary slackness, the constraint dual to α has to be fulfilled in the optimal solution of the dual valuation approach as an identity:

$$\sum_{t=0}^{n} G_t^F * \cdot d_t^P = 1. \tag{3}$$

Furthermore, we postulate only well-posed optimization problems and, hence, each dual structure variable d_t^P for every liquidity condition is positive and the relationships $d_{t-1}^P \geq d_t^P$ $\forall$ $t \in \{1, 2, ... , n\}$ are satisfied since rate-free cash holdings are always available in the private area, which put a floor under the future rates. The last equation implies that at least one $d_t^P > 0$ is positive which leads us in combination with $d_{t-1}^P \geq d_t^P$ $\forall$ $t \in \{1, 2, ... , n\}$ to $d_0^P > 0$. Hence, the endogenous discount factors can be introduced by the relations $\rho_t^P = d_t^P/d_0^P$ $\forall$ t to allow a reformulation of the above equation as:

$$\underbrace{\sum_{t=0}^{n} G_t^F * \cdot \rho_t^P}_{\text{net present value after the merger}} = \frac{1}{d_0^P}. \tag{4}$$

The left-hand side of this equation corresponds to the net present value of the dividend payout stream of the merged company ("net present value after the merger").

We are able to formulate the dual constraint belonging to any object j in a canonical form by introducing the slack variable μ_j and by considering the relations $d_t^P = \rho_t^P \cdot d_0^P$ with $d_0^P > 0$:

$$\sum_{t=0}^{n} g_{jt}^P \cdot \rho_t^P + \frac{\mu_j^P}{d_0^P} = \frac{u_j^P}{d_0^P} \quad \Leftrightarrow \quad C_j^P + \frac{\mu_j^P}{d_0^P} = \frac{u_j^P}{d_0^P}. \tag{5}$$

The first term on the left-hand side represents the net present value of object j. If financial redistributions in the private area are enforced with $\Delta x_j^P > 0$ and $\Delta x_j^P < \Delta x_j^{Pmax}$, which means that the corresponding primal structure variables are below their upper limits, we can infer from the theorem of complementary slackness that the related dual structure variables u_j^P and the dual slack variables μ_j^P vanish. However, if one of the objects reaches its upper ceiling after the private redistributions and, consequently, is fully realized ($\Delta x_j^P > 0$ and $\Delta x_j^P = \Delta x_j^{Pmax}$), the dual structure variable u_j^P assumes a non-negative value. Hence, a financial redistribution in the private area always implies a non-negative net present value:

$$\Delta x_j^P > 0 \quad \Rightarrow \quad C_j^P = \frac{u_j^P}{d_0^P} \geq 0. \tag{6}$$

By the same token, we can derive the identity $d_t^P = w_t \cdot \delta$ from $G_t > 0$ for any point in time t which is associated with a certain dividend payout. Furthermore, for an economically reasonable target function the dual structure variables d_t^P of the liquidity conditions will all be positive, which ensures that $\delta > 0$ and, by the theorem of complementary slackness, enforces the identity:

$$\sum_{t=0}^{n} w_t \cdot G_t = GW^*. \tag{7}$$

Even if the dividends G_t are different compared to the base program, the sum of the weighted distributions will always be equal to the target value of the primal base program in the respective optimal solution. Therefore, the term $\delta \cdot GW^*$ appearing in the target function of the dual valuation approach can be interpreted as:

$$\delta \cdot GW^* = \delta \cdot \sum_{t=0}^{n} w_t \cdot G_t = \sum_{G_t > 0} d_t^P \cdot G_t \tag{8}$$

Due to the concepts of duality and symmetry, we can infer $Y^* = AQ^* = \alpha^*$. By means of the above relations, we are now able to derive from the target function Y of the dual valuation approach the general marginal quota formula (Toll, 2011, pp.126-130; Hering, 2014, pp.99-102; Hering and Toll, 2017, pp.300-301):

$$\alpha^* = \delta \cdot GW^* - \sum_{j=1}^{m^P} \Delta x_j^{P\,max} \cdot u_j^P = \sum_{G_t > 0} d_t^P \cdot G_t - \sum_{j=1}^{m^P} \Delta x_j^{P\,max} \cdot u_j^P$$

$$\Leftrightarrow \frac{\alpha^*}{d_0^P} = \underbrace{\underbrace{\sum_{G_t > 0} G_t \cdot \rho_t^P}_{\substack{\text{net present value of the}\\ \text{dividends comparable}\\ \text{to the base program}}} - \underbrace{\sum_{C_j^P > 0} \Delta x_j^{P\,max} \cdot C_j^P}_{\substack{\text{net present value of the private}\\ \text{financial redistributions according}\\ \text{to the valuation program}}}}_{\text{net present value before the merger}}. \tag{9}$$

From this, we are able to derive different kinds of valuation formulas with certain degrees of complexity. The "*complex valuation formula*" for the determination of the marginal quota at maximization of wealth reads as:

$$\alpha^* = \frac{\dfrac{\alpha^*}{d_0^P}}{\dfrac{1}{d_0^P}} = \frac{\sum_{G_t > 0} G_t \cdot \rho_t^P - \sum_{C_j^P > 0} \Delta x_j^{P\,max} \cdot C_j^P}{\sum_{t=0}^{n} G_t^F{}^* \cdot \rho_t^P}$$

$$= \frac{\text{net present value before the merger}}{\text{net present value after the merger}}. \tag{10}$$

If none of the objects reaches its ceiling after private financial redistributions ($C_j^P = 0$), the "complex valuation formula" reduces to a "*simplified valuation formula*":

$$\alpha^* = \frac{\sum\limits_{G_t > 0} G_t \cdot \rho_t^P}{\sum\limits_{t=0}^{n} G_t^F * \cdot \rho_t^P}. \tag{11}$$

If the dividends G_t of the valuation program are proportional to the distributions G_t^F* of the merger program by a factor of b, which means that $G_t = b \cdot G_t^F* \; \forall \; t \in \{0, 1, 2, \ldots, n\}$ with $0 < b < 1$, this formula can be even further simplified to:

$$\alpha^* = \frac{\sum\limits_{t=0}^{n} b \cdot G_t^F * \cdot \rho_t^P}{\sum\limits_{t=0}^{n} G_t^F * \cdot \rho_t^P} = b \text{ and, hence, by definition } \alpha^* = \frac{G_t}{G_t^F *}, \tag{12}$$

which is the "*trivial valuation formula*" for a maximization of wealth. In the special case that maximization of the terminal values is intended in both approaches, this equation boils down to:

$$\alpha^* = \frac{G_n}{G_n^F *} = \frac{EW^*}{EW^F *}, \tag{13}$$

which finally closes the circle to the modeling approach proposed by Hering (2014, pp.96-97): As recommended therein, the marginal quota can be "trivially" computed by means of a ratio of utilities which is based exclusively and directly on the given weighting factors w_t referring to the desired structure of payouts. However, a determination which is based on a ratio of utilities, as suggested in the mentioned contribution, leads only in the *special case* to an optimal solution in which we can ignore private financial redistributions (an increase or a decrease in private borrowing or cash investment) altogether. As an example, for a maximization of the terminal value the above-mentioned trivial procedure is fully justified since private financial restructurings are not required to make the optimal dividend payout streams of the base and merger programs comparable in terms of utility. Hence, in this special case the marginal quota corresponds simply to the ratio of the terminal values before and after the merger (Hering, 2014, p.97). Concluding, it can be stated that the *availability of private financial redistributions* always favors a consideration of the private decision field of each shareholder instead of using the "trivial" ratio of utilities to determine the marginal quota α^* since without restructuring the dividend payout stream of the merged company it may not be possible to reach a comparable utility (Hering and Toll, 2017).

3. Exemplary considerations, results and discussion

To elucidate the general approach presented above, we will create an *example* with a fictitious data base to illustrate the determination of the minimum demandable quota in the merged company from the viewpoint of an affected shareholder (Hering and Toll, 2017). Under the assumption of (quasi-)certainty we will consider a planning period with 3 periods (n = 3), whereby each period extends over one year and delimits the points in time t – 1 and t.

Company A to be involved in a merger pursues maximization of the terminal value. The pre-merger approach (base approach), which describes the *baseline*, delivers the optimal target value EW* = 30. Hence, according to the *base program* (without the merger in question), the owners of company A can draw on the dividend payout stream (0, 0, 0, 30), defined between the points in time t = 0 and t = 3. After the merger, company A will be a part of the larger company X. In the *negotiations*, the controversial point is the allocation of property rights or shares of the merged company X. Hence, for the concerned party of old owners of company A the question arises which marginal quota α^* of the larger company X they should receive to be no worse off than before.

First of all, we postulate that even for the *merged company X maximization of the terminal value is pursued*. Company X can draw on a larger asset base and because of this advantage the higher optimal target value $EW^{F*} = 100$ is computed by solving the *merger approach*. Therefore, the dividend payout stream (0, 0, 0, 100) can be distributed to all shareholders of the merged company. The concerned party of old shareholders of company A must now determine the *marginal quota* α^* which satisfies their desire to be no worse off than referred to the state before the merger. Since *no private financial redistributions are necessary* to restructure the dividend payout stream of the merged company to make both utilities comparable, the marginal quota α^* can be computed by means of the "*trivial valuation formula*":

$$\alpha^* = \frac{GW^*}{\sum_{t=0}^{n} w_t \cdot G_t^{F*}} = \frac{30}{0 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 + 1 \cdot 100} = 0.3 = 30\%. \quad (14)$$

Concluding, the party of old owners of company A must demand at least a share of 30% in the merged company X. According to Table no. 1, no private redistributions are required to make the dividend payout streams of the base and merger programs comparable if maximization of the terminal value is assumed before and after the merger.

Table no. 1: Valuation program in the trivial case

Point in time	t = 0	t = 1	t = 2	t = 3
Dividend payout stream of the merged company	0	0	0	100
Share α^* = 30%	0	0	0	30
Private redistributions	0	0	0	0
Dividend payout stream desired by the party of old owners	0	0	0	30

To be able to apply the "simplified" and "complex valuation formula", the example has to be slightly changed: The merged company X now pursues *another target different to company A* and can now draw on a maximally distributable initial capital of $G_0^F* = 80$. Hence, the dividend payout stream of the merged company distributable to all of its owners is now (80, 0, 0, 0). Because of the different payout schemes, the marginal quota $\alpha*$ cannot be trivially determined anymore. Rather, the concerned party of old owners of company A has to use *private financial redistributions* to make the dividend payout stream of the merged company comparable in terms of utility.

For the transformation of the new dividend payout stream into its desired structure, we postulate: Apart from an unlimited opportunity to invest money in financial assets in the private portfolio, the concerned party of old owners of company A has sufficiently high deposits to satisfy a demand of cash by reducing current financial assets at an interest rate of 5% p.a. In this case, no object within the financial redistributions touches its ceiling so that only marginal objects with a net present value of $C_j^P = 0$ are used in restructuring. The marginal object is always the financial asset in the private portfolio whose endogenous marginal interest rate is already known in each year as 5%. The application of the "*simplified valuation formula*" delivers:

$$\alpha* = \frac{\sum_{G_t>0} G_t \cdot \rho_t^P}{\sum_{G_t^F*>0} G_t^F* \cdot \rho_t^P} = \frac{30 \cdot 1.05^{-3}}{80} = \frac{25.91512796}{80} = 0.32393 \approx 32.3939\%. \quad (15)$$

Under consideration of the private financial redistributions as characterized above, the marginal quota of the merged company is now 32.3939%. The respective financial redistributions to the planning horizon (t = n) required to restructure the initial capital of the merged company are presented in Table no. 2 whereby single-period financial assets are employed which have to be refinanced from year to year by repaying and investing the corresponding cash as appropriate.

Table no. 2: Valuation program in the simplified case

Point in time	t = 0	t = 1	t = 2	t = 3
Dividend payout stream of the merged company	80	0	0	0
Share $\alpha*$ = 32.3939%	25.9151	0	0	0
Increase in the financial asset at 5%	–25.9151	–27.2109	–28.5714	0
Repayment from private means		27.2109	28.5714	30
Dividend payout stream desired by the party of old owners	0	0	0	30

To treat the *complex case* for the valuation of a merger exemplarily, we will assume that the party of old owners can invest money in financial assets only up to a limit of 10 MU at 5% p.a. and is moreover able to invest additional cash only at an interest rate of 3% p.a. Due to this *restriction*, the financial asset at 5% p.a. reaches its upper ceiling in every year and has consequently a positive net present value. For the determination of the marginal quota α*, it is therefore necessary to apply the "*complex valuation formula*":

$$\alpha^* = \frac{\sum_{G_t>0} G_t \cdot \rho_t^P - \sum_{C_j^P>0} \Delta x_j^{P\max} \cdot C_j^P}{\sum_{G_t^F*>0} G_t^F* \cdot \rho_t^P} = \frac{\frac{30}{1.03^3} - \left(-10 + \frac{0.5}{1.03} + \frac{0.5}{1.03^2} + \frac{10.5}{1.03^3}\right)}{80}$$

$$= \frac{27.45424978 - 0.565722271}{80} = 0.336106593 \approx 33.6107\%. \quad (16)$$

The party of old owners of company A should demand a minimum share of 33.6107% in the consolidated company to avoid any economic losses. The initial capital distributed by the merged company is redistributed to the planning horizon (t = n) by means of the financial assets at 5% p.a. and 3% p.a. (Table no. 3).

Table no. 3: Valuation program in the complex case

Point in time	t = 0	t = 1	t = 2	t = 3
Dividend payout stream of the merged company	80	0	0	0
Share α* = 33.6107%	26.8885	0	0	0
Increase in the financial asset at 5%	–10	–10	–10	0
Increase in the financial asset at 3%	–16.8885	–17.8952	–18.9320	0
Repayment from private means	0	27.8952	28.9320	30
Dividend payout stream desired by the party of old owners	0	0	0	30

The repercussions of a limitation in the 5%-financial asset emphasize the importance of private financial redistributions. If they can be executed without bounds, the minimum demandable share is smaller compared to a case with a restricted setting. To sum up, we can say that the marginal quota is the larger the more severe the restrictions in the private area are, as this example might suggest.

Conclusions

Despite the apparent relevance of company mergers in practice, the scientific literature deals with this issue only in an unsatisfying manner and gives no real clues for model-oriented solutions for the determination of the marginal quota. After some early simple discounted cash flow considerations (Silberman, 1968) for a required exchange ratio (Larson and Gonedes, 1969; Ramanathan and Rappaport, 1971; Nonnenmacher, 1982; Yagil, 1987), the valuation problem of a merger was taken up again not earlier than in Hering (2004). Thereby, the *state marginal quota model* as investment-theoretical method was proposed. Based on the considerations in Hering (2004; 2014, pp.96-97), the *aim of the* *present contribution* is to extend and generalize the valuation methods for a company merger and foremost to set the algebra for the computation of the marginal quota by using maximization of wealth as target function on a firm foundation.

As proposed in Hering (2014, pp.96-97), the marginal quota can be "trivially" computed as a ratio of the respective utilities before and after the merger which are based exclusively and directly on the given weighting factors w_t of withdrawals. However, the solution approach as developed in the present contribution shows that the marginal quota α^* in question cannot be "trivially" obtained as a ratio of utilities if private financial redistributions are available. Instead, for the determination of the critical share it is required to consider the private decision field of a shareholder to allow a restructuring of the dividend payout stream offered by the merged company in order to reach at least a level of utility which is comparable to the state before the merger.

While the necessity to model real-life imperfections argues in favour of the state marginal quota model, the marginal quota determination using the general model certainly has its *drawbacks*. Since all investment and financing objects enter directly into a large-scale optimization model, this requires high efforts for information-gathering and processing. Therefore, since a central simultaneous planning by means of a general model is often ruled out due to its inherent complexities, we have introduced partial-analytical valuation formulas as an implication of the state marginal quota model which are almost indispensable for practical decision-making situations. Knowing the endogenous marginal interest rates, the single objects can be valued in an isolated form whereby all governing interdependencies in the related decision field are properly considered. Unfortunately, applying a partial model presumes that the model-endogenous quantities are known. As a consequence, the valuation formulas suffer from the dilemma that they rely on information that can only be deduced from the solution of the total formulation of the state marginal quota model which we actually try to avoid (dilemma of the theory of endogenous prices). One way to evade this *dilemma* in large-scale enterprises is the approximate decomposition which combines an approximate decentralized control via endogenous shadow prices with the instruments of sensitivity and risk analyses, as well as a rolling planning (Hering, 2014, pp.174-200; 2015). Thereby, we were able to integrate the problem of uncertainty as well, which has been dismissed in the present contribution, in a heuristic manner.

Future research should be addressed to refining the state marginal quota model. For example, it is possible to give up the complexity-reducing linear structure, which would require a more general nonlinear framework (Pfaff, Pfeiffer and Gathge, 2002). Nonlinear synergy effects are particularly interesting. In addition, the model could be expanded to take into account market imperfections and cash flow ambiguities by applying simultaneous planning approaches in a heuristic combination with a simulative risk analysis (Hurd, 1954; Hertz, 1964; Salazar and Sen, 1966). Further, it remained unclear how an uninfluential shareholder should act in a situation in which he is not willing to vote for a merger, but where he cannot avoid it because of a lack of influence. According to Toll (2010, p.101), this situation may be relevant if the uninfluential shareholder has no opportunities in his private area to restructure an undesired dividend payout stream of the merged company, whereas the payout stream of the unmerged company were indeed compatible with his own preferences. In this situation, it is imaginable that the marginal quota may become very large and, thus, unrealizable from the viewpoint of an uninfluential shareholder. He may be able to satisfy his consumption needs only by a sale of shares before or after the merger. For this valuation problem, models should be applied for a share deal which were not in the focus of the present contribution. Furthermore, future research should address the opposite of a company merger, a split of a parent company, by applying our current research findings to this challenging field.

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