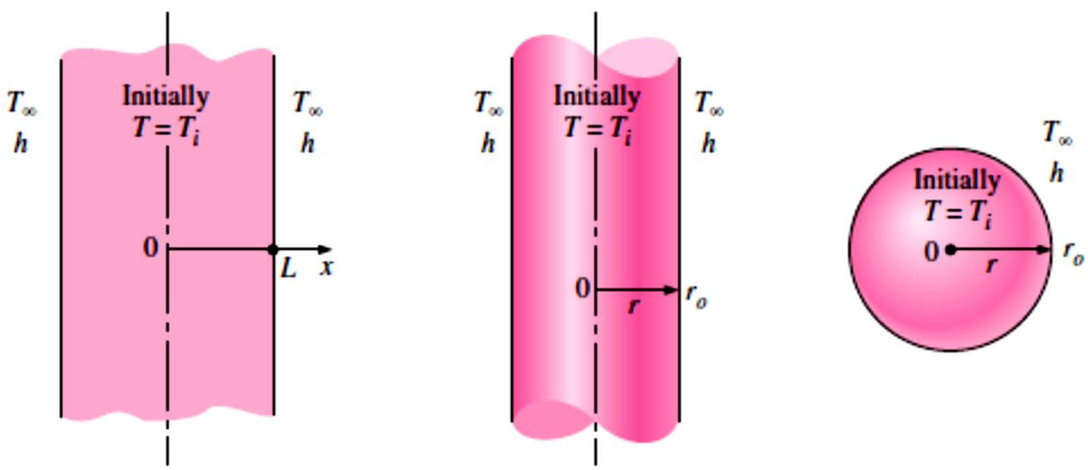
Heat Transfer I

Lecturer : Dr. Rafel Hekmat Hameed
Subject : Heat Transfer
Year : Third B.Sc.
University of Babylon
College of Engineering
Mechanical Engineering Dep.

Unsteady-State Conduction

CONVECTION BOUNDARY CONDITIONS

In most practical situations the transient heat-conduction problem is connected with a convection boundary condition at the surface of the solid. Naturally, the boundary conditions for the differential equation must be modified to take into account this convection heat transfer at the surface. In this section, we consider the variation of temperature with *time* and *position* in one-dimensional problems such as those associated with a large plane wall, a long cylinder, and a sphere. Consider a plane wall of thickness $2L$, a long cylinder of radius r_o, and a sphere of radius r_o initially at a *uniform temperature* T_i, as shown in figure below



(*a*) A large plane wall (*b*) A long cylinder (*c*) A sphere

At time $t = 0$, each geometry is placed in a large medium that is at a constant temperature T_∞ and kept in that medium for $t > 0$. Heat transfer takes place between these bodies and their environments by convection with a *uniform* and *constant* heat transfer coefficient h. Note that all three cases possess geometric and thermal symmetry: the plane wall is symmetric about its *center plane* ($x = 0$), the cylinder is symmetric about its *centerline* ($r = 0$), and the sphere is symmetric about its *center point* ($r = 0$). We neglect *radiation* heat transfer between these bodies and their surrounding surfaces.

In order to reduce the number of parameters, we nondimensionalize the problem by defining the following dimensionless quantities:

Dimensionless temperature:	$\theta(x, t) = \dfrac{T(x, t) - T_\infty}{T_i - T_\infty}$	
Dimensionless distance from the center:	$X = \dfrac{x}{L}$	
Dimensionless heat transfer coefficient:	$\text{Bi} = \dfrac{hL}{k}$	**(Biot number)**
Dimensionless time:	$\tau = \dfrac{\alpha t}{L^2}$	**(Fourier number)**

The nondimensionalization enables us to present the temperature in terms of three parameters only: X, Bi, and τ. This makes it practical to present the solution in graphical form. The dimensionless quantities defined above for a plane wall can also be used for a *cylinder* or *sphere* by replacing the space variable x by r and the half-thickness L by the outer radius r_o. Note that the characteristic length in the definition of the Biot number is taken to be the *half-thickness L* for the plane wall, and the *radius r_o* for the long cylinder and sphere instead of V/A used in lumped system analysis.

The transient temperature charts for a large plane wall, long cylinder, and sphere were presented by M. P. Heisler in 1947 and are called **Heisler charts**. There are *three* charts associated with each
geometry: the first chart is to determine the temperature ***T_o* at the center** of the geometry at a given time t. The second chart is to determine the ***temperature at other locations*** at the same time in terms of T_o. The third chart is to determine the total amount of ***heat transfer*** up to the time t. See figures (4-5) to (4-16) in Holman book.

§ Assumptions in using Heisler charts:

- Constant T_i and thermal properties over the body
- Constant boundary fluid T_∞ by step change
- Simple geometry: slab, cylinder or sphere

§ Limitations:

- Far from edges
- No heat generation (Q=0)
- Relatively long after initial times (F_o or $\tau > 0.2$)

The temperature of the body changes from the initial temperature T_i to the temperature of the surroundings T_∞ at the end of the transient heat conduction process. Thus, the *maximum* amount of heat that a body can gain (or lose if $T_i > T_\infty$) is simply the *change* in the *energy content* of the body. That is,

$$Q_{max} = mC_p(T_\infty - T_i) = \rho V C_p(T_\infty - T_i) \qquad \text{(kJ)}$$

where m is the mass, V is the volume, ρ is the density, and C_p is the specific heat of the body. Thus, Qmax represents the amount of heat transfer for $t \to \infty$. The amount of heat transfer Q at a finite time t will obviously be less than this maximum. A *negative* sign for Q_{max} indicates that heat is *leaving* the body.

SOLVED EXAMPLES

See EX. 4-6 and 4-7 in Holman book

A long 20-cm-diameter cylindrical shaft made of stainless steel 304 comes out of an oven at a uniform temperature of 600°C . The shaft is then allowed to cool slowly in an environment chamber at 200°C with an average heat transfer coefficient of h =80 W/m^2 · °C. Determine the temperature at the center of the shaft 45 min after the start of the cooling process. Also, determine the heat transfer per unit length of the shaft during this time period.

The properties of stainless steel 304 at room temperature are
k= 14.9 W/m · °C, ρ=7900 kg/m^3, C_p = 477 J/kg · °C, and $\alpha = 3.95 \times 10^{-6}$ m^2/s

Noting that the radius of the shaft is r_o = 0.1 m, from Fig. 4–8 we have

$$\left.\begin{aligned} \frac{1}{\text{Bi}} &= \frac{k}{hr_o} = \frac{14.9\ \text{W/m}\cdot{}^\circ\text{C}}{(80\ \text{W/m}^2\cdot{}^\circ\text{C})(0.1\ \text{m})} = 1.86 \\ \tau &= \frac{\alpha t}{r_o^2} = \frac{(3.95\times10^{-6}\ \text{m}^2/\text{s})(45\times60\ \text{s})}{(0.1\ \text{m})^2} = 1.07 \end{aligned}\right\} \frac{T_o - T_\infty}{T_i - T_\infty} = 0.40$$

And

$$T_o = T_\infty + 0.4(T_i - T_\infty) = 200 + 0.4(600 - 200) = 360^\circ\text{C}$$

To determine the actual heat transfer, we first need to calculate the maximum heat that can be transferred from the cylinder, which is the sensible energy of the cylinder relative to its environment. Taking L = 1 m,

$$\begin{aligned} m &= \rho V = \rho \pi r_o^2 L = (7900\ \text{kg/m}^3)\pi(0.1\ \text{m})^2(1\ \text{m}) = 248.2\ \text{kg} \\ Q_{max} &= mC_p(T_\infty - T_i) = (248.2\ \text{kg})(0.477\ \text{kJ/kg}\cdot{}^\circ\text{C})(600-200)^\circ\text{C} \\ &= 47{,}354\ \text{kJ} \end{aligned}$$

The dimensionless heat transfer ratio is determined from Fig. 4–15 for a long cylinder to be

$$\left.\begin{aligned} \text{Bi} &= \frac{1}{1/\text{Bi}} = \frac{1}{1.86} = 0.537 \\ \frac{h^2\alpha t}{k^2} &= \text{Bi}^2\tau = (0.537)^2(1.07) = 0.309 \end{aligned}\right\} \frac{Q}{Q_{max}} = 0.62$$

Therefore,

$$Q = 0.62Q_{max} = 0.62 \times (47{,}354 \text{ kJ}) = 29{,}360 \text{ kJ}$$

which is the total heat transfer from the shaft during the first 45 min of the cooling.

A large slab of aluminum has a thickness of 10 cm and is initially uniform in temperature at 400◦C. It is then suddenly exposed to a convection environment at 90°C with h=1400 W/m^2.°C. How long does it take the center to cool to 180°C?

$$L = 5 \text{ cm} \qquad T_i = 400°\text{C} \qquad T_\infty = 90°\text{C} \qquad h = 1400 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$k = 204 \ \frac{\text{W}}{\text{m} \cdot °\text{C}} \qquad \alpha = 8.4 \times 10^{-5} \ \text{m}^2/\text{s} \qquad \frac{\theta_0}{\theta_i} = \frac{180-90}{400-90} = 0.29$$

$$\frac{k}{hL} = \frac{204}{(1400)(0.05)} = 2.91 \qquad \frac{\alpha\tau}{L^2} = 4.2 \qquad \tau = \frac{(4.2)(0.05)^2}{8.4 \times 10^{-5}} = 125 \text{ sec}$$

A plate of stainless steel (18% Cr, 8% Ni) has a thickness of 3.0 cm and is initially uniform in temperature at 500°C. The plate is suddenly exposed to a convection environment on both sides at 40°C with h=150W/m^2 .°C. Calculate the times for the center and face temperatures to reach 120°C.

$$L = 0.015 \text{ m} \qquad T_i = 500°\text{C} \qquad T_\infty = 40°\text{C} \qquad h = 150 \ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$$

$$k = 16.3 \ \frac{\text{W}}{\text{m} \cdot °\text{C}} \qquad \alpha = 0.44 \times 10^{-5} \ \text{m}^2/\text{s} \qquad \frac{k}{hL} = \frac{6.3}{(150)(0.015)} = 7.24$$

$$\text{at } \frac{x}{L} = 1.0 \qquad \frac{\theta}{\theta_0} = 0.93$$

$$\text{For } \frac{\theta_0}{\theta_i} = \frac{120-40}{500-40} = 0.174 \qquad \frac{\alpha\tau}{L^2} = 13.9$$

$$\tau = \frac{(13.9)(0.015)^2}{0.44 \times 10^{-5}} = 711 \text{ sec}$$

At the face $\frac{\theta}{\theta_i} = \frac{\theta_o}{\theta_i}$ at the center

$$\frac{\theta_o}{\theta_i} = \frac{\theta_o}{\theta}\,\frac{\theta}{\theta_i}$$

$$\text{For } \frac{\theta}{\theta_i} = 0.174 \qquad \frac{\theta_0}{\theta_i} = \frac{0.174}{0.93} = 0.187 \qquad \frac{\alpha\tau}{L^2} = 13$$

$$\tau = \frac{(13)(0.015)^2}{0.44 \times 10^{-5}} = 665 \text{ sec}$$

A fused-quartz sphere has a thermal diffusivity of 9.5×10^{-7} m^2/s, a diameter of 2.5 cm, and a thermal conductivity of 1.52 W/m.°C. The sphere is initially at a uniform temperature of 25°C and is suddenly subjected to a convection environment at 200°C. The convection heat-transfer coefficient is 110 W/m^2 .°C. Calculate the temperatures at the center and at a radius of 6.4 mm after a time of 3 min.

$\alpha = 9.5\times10^{-7}\ \text{m}^2/\text{s}$ $\quad r_0 = 1.25\ \text{cm}$ $\quad k = 1.52\ \dfrac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$

$T_i = 25^\circ\text{C}$ $\quad T_\infty = 200^\circ\text{C}$ $\quad h = 110\ \dfrac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$ $\quad \tau = 3\ \text{min} = 180\ \text{sec}$

$$\frac{k}{hr_0} = \frac{1.52}{(110)(0.0125)} = 1.105 \qquad \frac{r}{r_0} = \frac{0.64}{1.25} = 0.51$$

$$\frac{\alpha\tau}{r_0^2} = \frac{(9.5\times10^{-7})(180)}{(0.0125)^2} = 1.094 \qquad \frac{\theta_0}{\theta_i} = 0.12 \qquad \frac{\theta_r}{\theta_0} = 0.89$$

center $T = (25-200)(0.12)+200 = 179^\circ\text{C}$ $\quad r = 6.4\ \text{mm}$

$T = (25-200)(0.12)(0.89)+200 = 181.3^\circ\text{C}$

H.W

4-51 4-53 4-56 4-59