Chapter 6

Stress in Soil Mass

AAiT
Addis Ababa Institute of Technology
Addis Ababa University

Geotechnical Engineering Chair
School of Civil & Environmental Engineering

General Outline

- Introduction
- Geostatic Stress
- Additional Stress
- Stress Bulbs & Contact Pressure
- Stress States & Stress Paths

1. Introduction

- Definitions
- Fundamentals of Stress-Strain
- Material Responses to Normal Loading
- Stresses in Soil Mass

Introduction Definitions

Stress: or intensity of loading, is the load per unit area.

Strain: or intensity of deformation, is the ratio of the change in a dimension to the original dimension or the ratio of change in length to the original length.

Stiffness : resistance to deformation

Strength : ability to withstand an applied load without failure or plastic deformation

Poisson's ratio : ratio of transverse contraction strain to longitudinal extension strain in the direction of stretching force

Modulus of Elasticity: describes tendency of an object to deform along an axis when opposing forces are applied along that axis

Nominal stress = actual load/original cross-sectional area of specimen, i.e. no allowance is made for reduction in area, due to necking, as the load is increased.

Introduction Definitions

Stress **(*strain*)** ***state*** at a point is a set of stress (strain) vectors corresponding to all planes passing through that point.

Mohr's circle is used to graphically represent stress (strain) state for two-dimensional bodies.

Isotropic means the material properties are the same in all directions, and also the loadings are the same in all directions.

Anisotropic means the material properties are different in different directions, and also the loadings are different in different directions.

Elastic materials are materials that return to their original configuration on unloading and obey Hooke's law.

Plastic materials do not return to their original configuration on unloading.

Introduction cntd

Soils are not homogeneous, elastic, rigid bodies, so the determination of stresses and strains in soils is a particularly difficult task.

One may ask: “If soils are not elastic materials, then why do I have to study elastic methods of analysis?”

- An elastic analysis of an isotropic material involves only two constants—Young’s modulus and Poisson’s ratio—and thus if we assume that soils are isotropic elastic materials, then we have a powerful, but simple, analytical tool to predict a soil’s response under loading. We will have to determine only the two elastic constants from our laboratory or field tests.

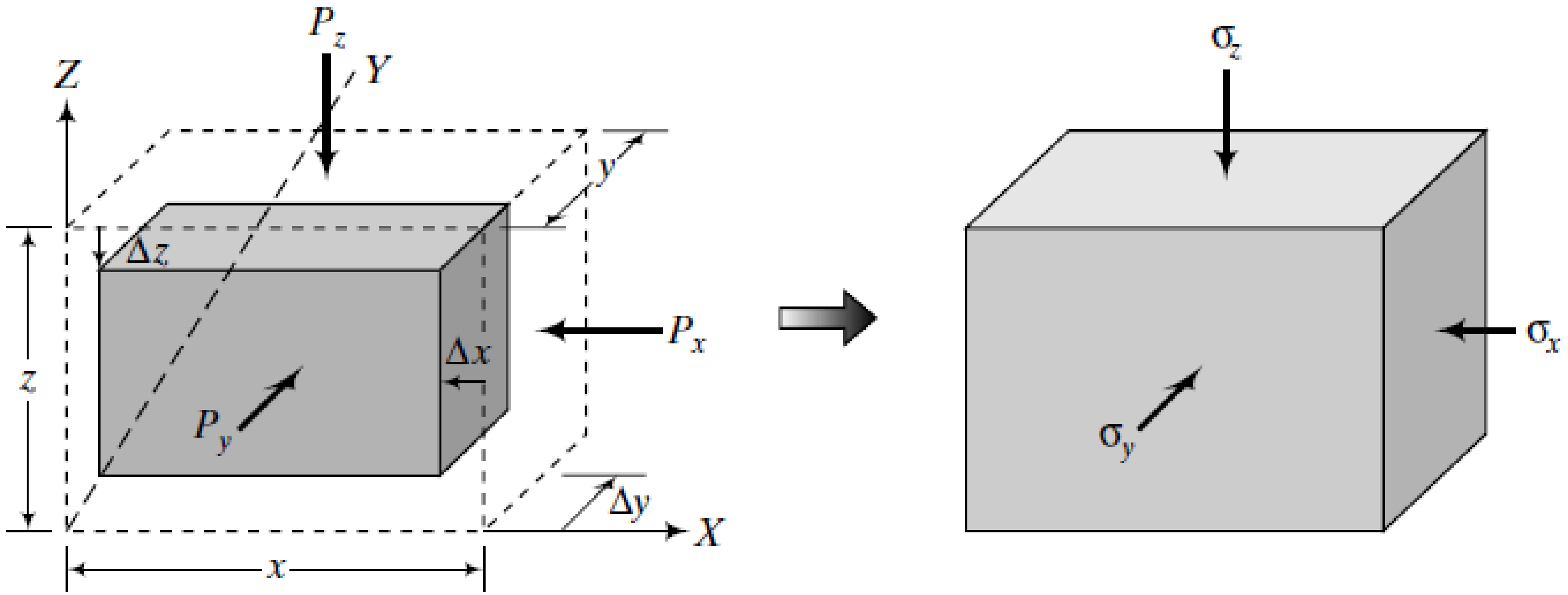
Introduction cntd

Stresses and Strains – A Recap of the Fundamentals

Normal Stresses and Strains

Consider a cube of dimensions $x = y = z$ that is subjected to forces

$P_x = P_y = P_z$, normal to three adjacent sides.



Introduction cntd

The normal stresses:

$$\sigma_z = \frac{P_z}{xy}, \sigma_x = \frac{P_x}{yz}, \sigma_y = \frac{P_y}{xz}$$

Consider a cube of dimensions $x = y = z$ that is subjected to forces $P_x = P_y = P_z$, normal to three adjacent sides.

Let us assume that under these forces the cube compressed by $\Delta x, \Delta y, \Delta z$ in the *X*, *Y*, and *Z* directions.

The strains in these directions, assuming they are small (infinitesimal):

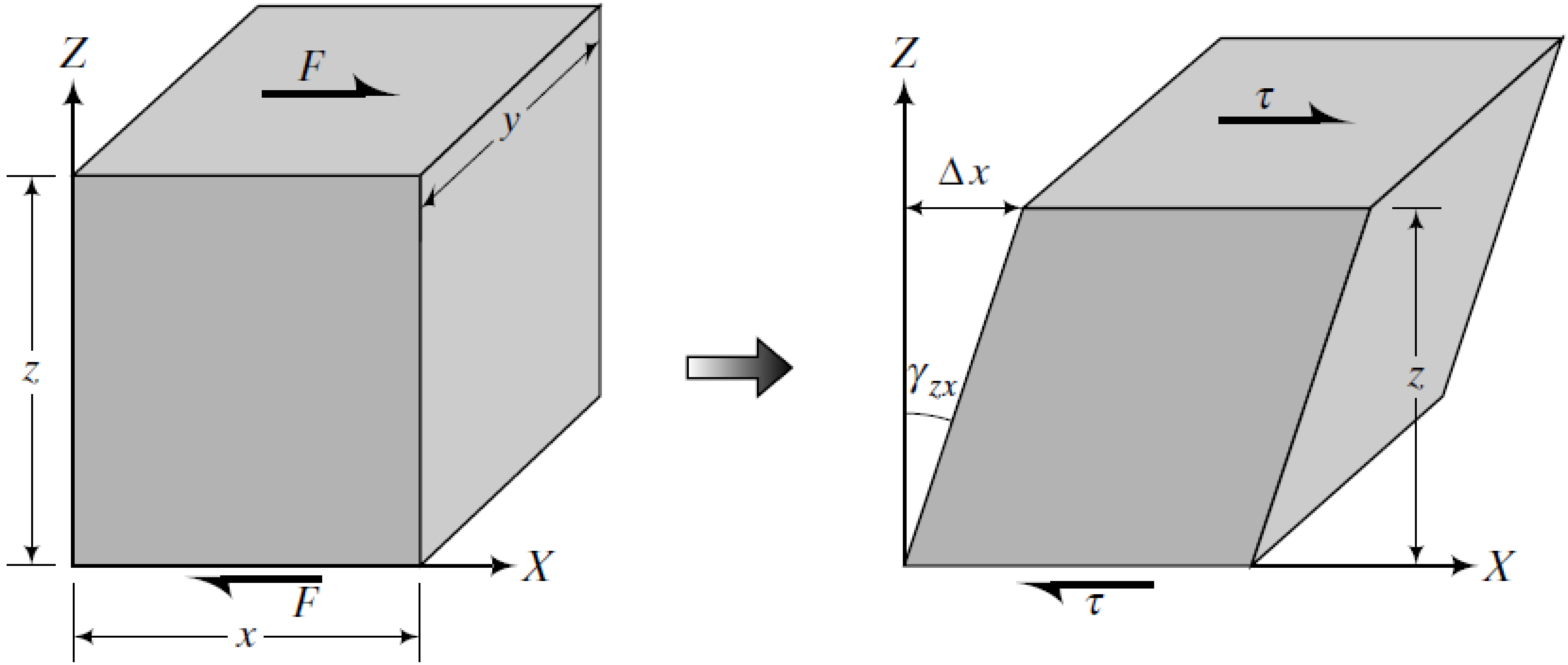
$$\varepsilon_z = \frac{\Delta z}{z}, \varepsilon_x = \frac{\Delta x}{x}, \varepsilon_y = \frac{\Delta y}{y}$$

Volumeteric strain:

$$\varepsilon_p = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Shear Stresses and Shear Strains

Let us consider, for simplicity, the *XZ* plane and apply a force *F* that causes the square to distort into a parallelogram, as shown below.



Introduction cntd

The force *F* is a shearing force, and the shear stress is

$$\tau = \frac{F}{xy}$$

Simple shear strain is a measure of the angular distortion of a body by shearing forces. If the horizontal displacement is $\Delta \boldsymbol{x}$, the shear strain or simple shear strain, $\boldsymbol{\gamma_{zx}}$, is

$$\boldsymbol{\gamma_{zx}} = \tan^{-\mathbf{1}} \frac{\Delta \boldsymbol{x}}{\boldsymbol{z}}$$

For small strains, $\tan \boldsymbol{\gamma_{zx}} = \boldsymbol{\gamma_{zx}}$, and therefore

$$\boldsymbol{\gamma_{zx}} = \frac{\Delta \boldsymbol{x}}{\boldsymbol{z}}$$

Introduction cntd

Material Responses to Normal Loading / Unloading

If we apply an incremental vertical load, ΔP, to a deformable cylinder of cross-sectional area A, the cylinder will compress by, say, Δz and the radius will increase by Δr.

The loading condition we apply here is called uniaxial loading.

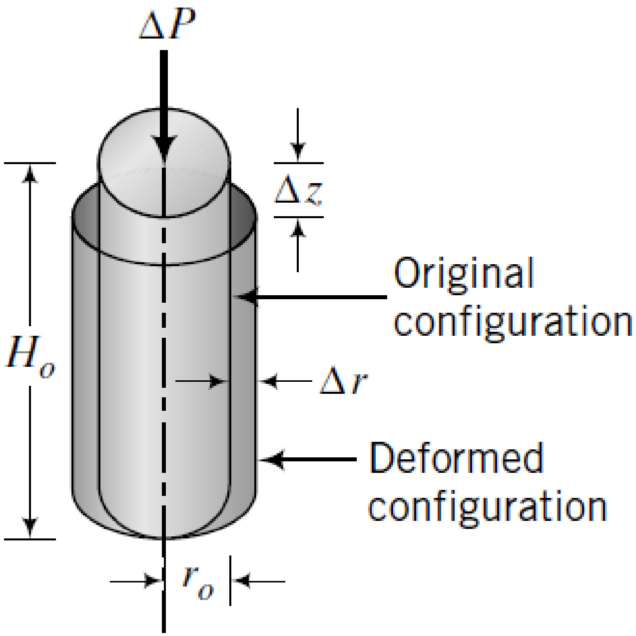
The change in vertical stress is

$$\Delta\sigma_z = \frac{\Delta P}{A}$$

The vertical and radial strains are, respectively,

$$\Delta\varepsilon_z = \frac{\Delta z}{H_o}$$

$$\Delta\varepsilon_r = \frac{\Delta r}{r_o}$$



Introduction cntd

Material Responses to Normal Loading / Unloading

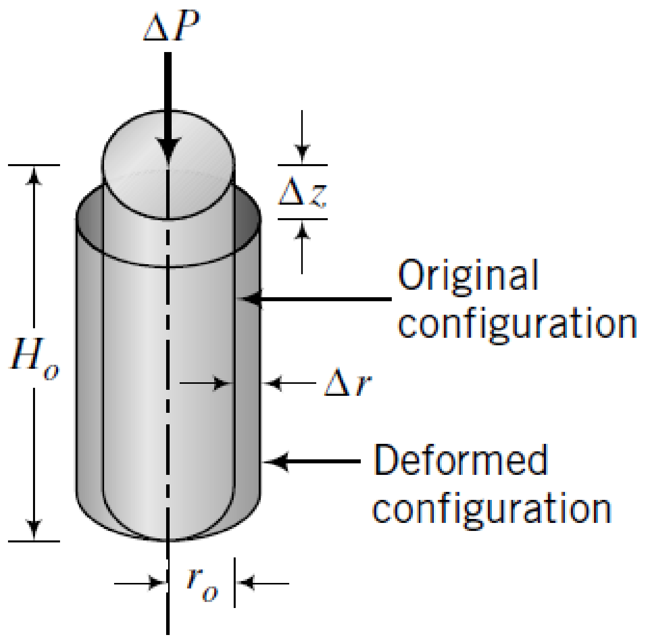
The ratio of the radial (or lateral) strain to the vertical strain is called Poisson's ratio, υ, defined as

$$\upsilon = \frac{-\Delta\varepsilon_r}{\Delta\varepsilon_z}$$

TABLE Typical Values of Poisson's Ratio

Soil type	Description	ν^a
Clay	Soft	0.35–0.40
	Medium	0.30–0.35
	Stiff	0.20–0.30
Sand	Loose	0.15–0.25
	Medium	0.25–0.30
	Dense	0.25–0.35

[a]These values are effective values, ν'.



Introduction cntd

Linear elastic materials

- For equal increments of ΔP, we get the same value of Δz.
- If at some stress point, say, at A , we unload the cylinder and it returns to its original configuration.

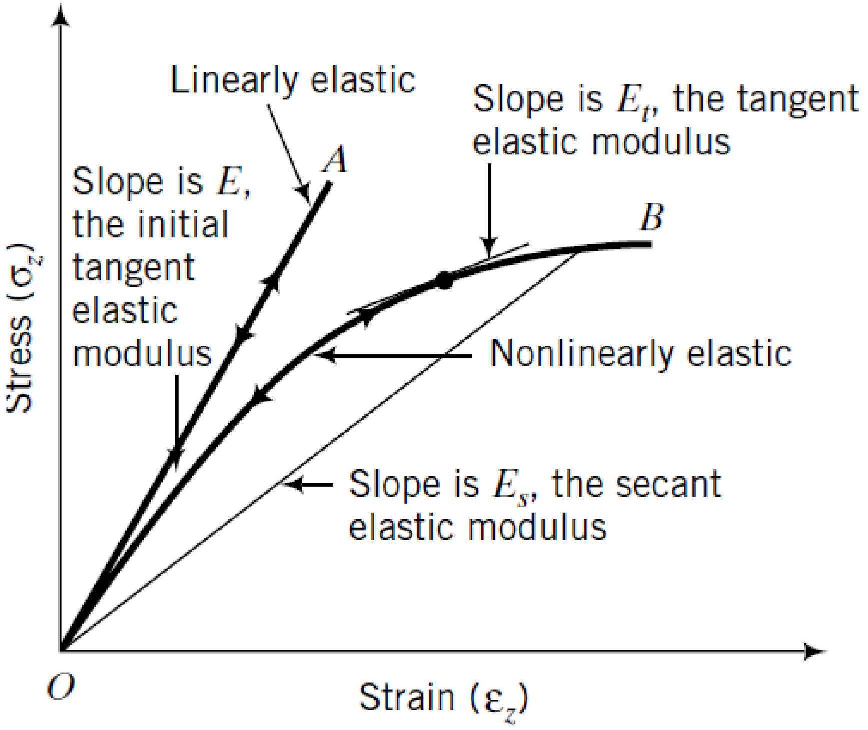
Non-linear elastic materials

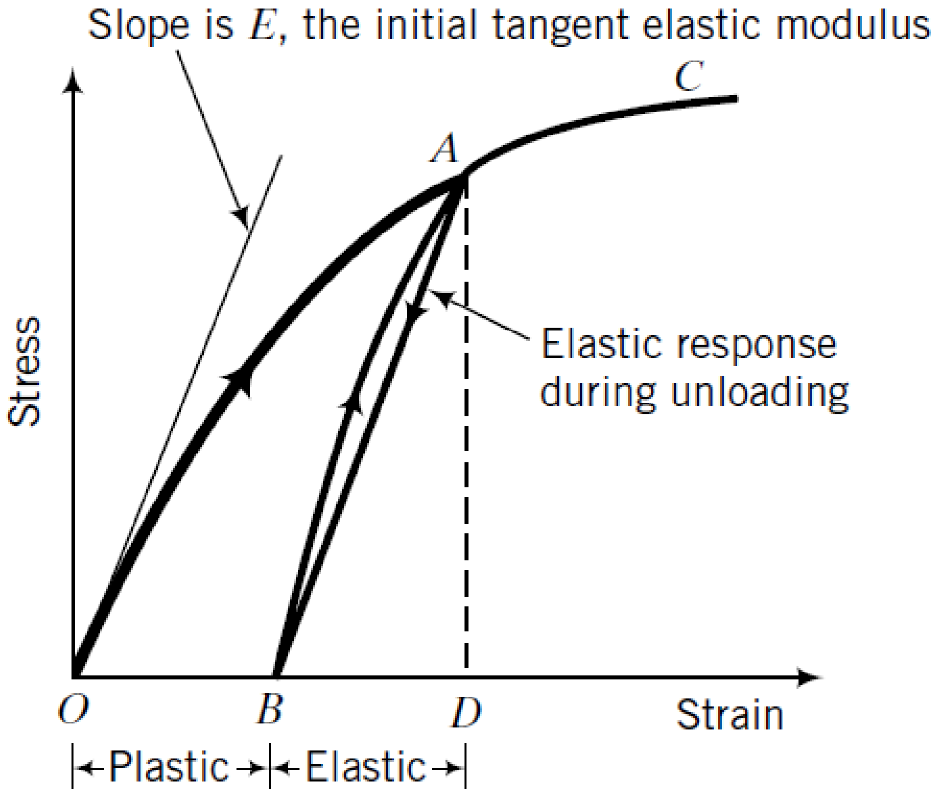
- For equal increments of ΔP, we get the different value of Δz.
- If at some stress point, say, at A , we unload the cylinder and it returns to its original configuration.

Elasto-plastic materials

- do not return to their original configurations after unloading.
- The strains that occur during loading consist an elastic or recoverable part and a plastic or unrecoverable part.

Introduction cntd





Introduction cntd

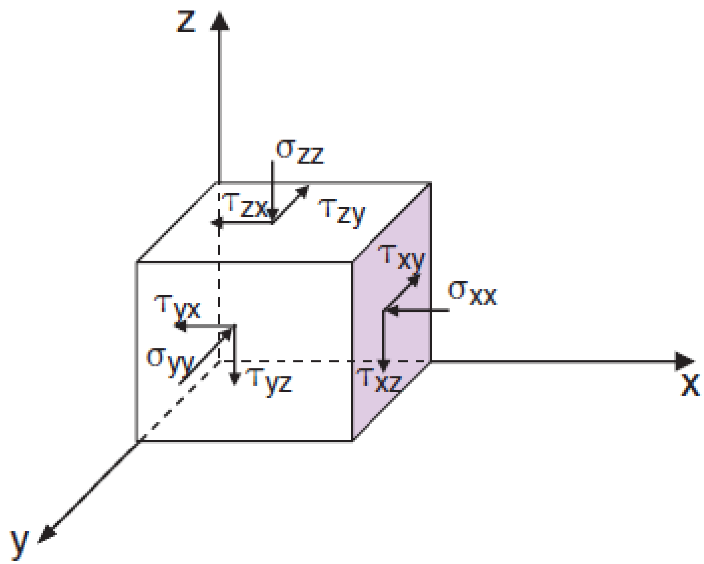
Moduli

- The elastic modulus or initial tangent elastic modulus (E) is the slope of the stress–strain line for linear isotropic material .
- The tangent elastic modulus (Et) is the slope of the tangent to the stress–strain point under consideration.
- The secant elastic modulus (Es) is the slope of the line joining the origin (0, 0) to some desired stress–strain point.

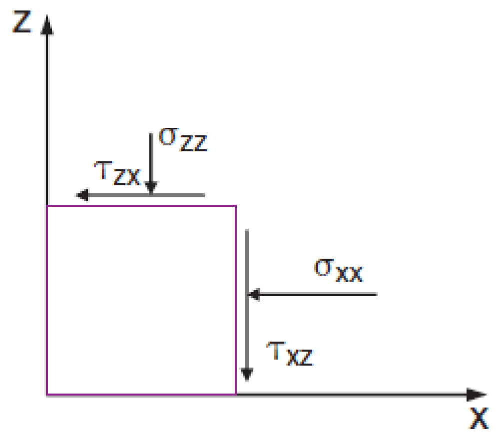
Introduction cntd

Stresses in Soil Mass

If we consider an elemental cube of soil at the point considered, then a solution by elastic theory is possible. Each plane of the cube is subjected to a stress, *σ*, acting normal to the plane, together with a shear stress, *τ*, acting parallel to the plane.



(a) 3-d stress system



(b) 2-d stress system

Introduction cntd

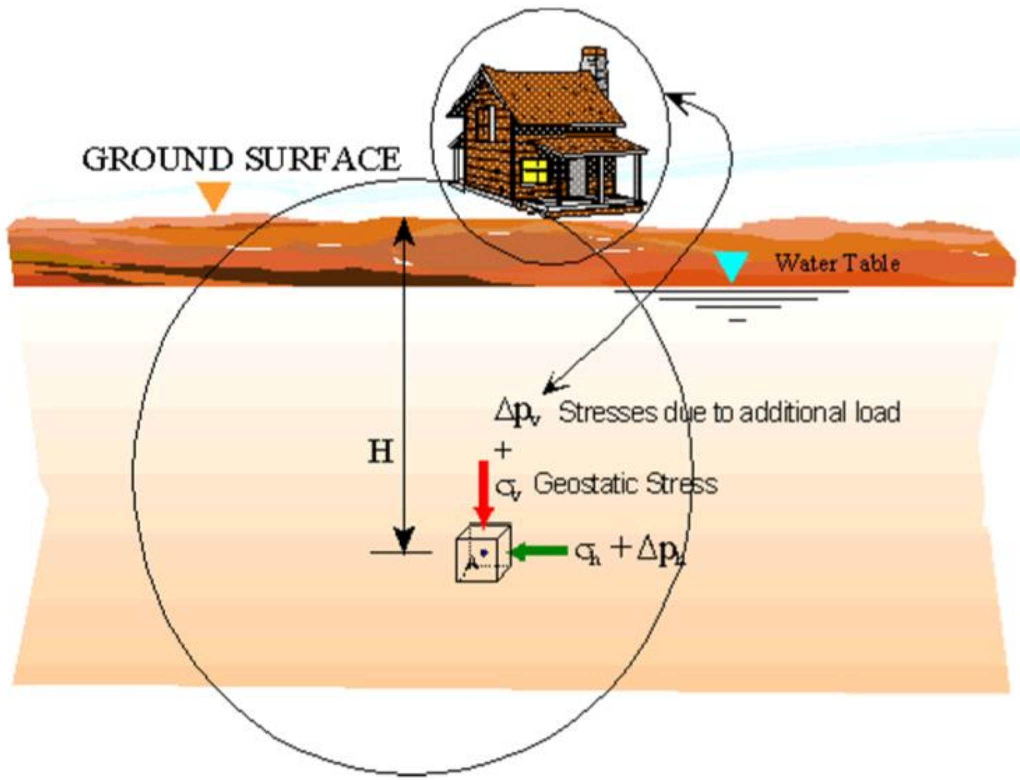
Stresses in Soil Mass

- There are therefore a total of six stress components acting on the cube.
- Once the values of these components are determined then they can be compounded to give the magnitudes and directions of the principal stresses acting at the point considered.
- Many geotechnical structures operate in a state of plane strain, i.e. one dimension of the structure is large enough for end effects to be ignored and the problem can be regarded as one of two dimensions.

Introduction cntd

Stresses in Soil Masses

- Geostatic stresses: from self weight of soil
- Additional stress: from imposed load



- Vertical stress
- Horizontal stress
- Shear stress

Introduction cntd

Vertical pressure at a point below ground surface is normally induced in four ways:

- **Geostatic Pressure/Overburden:** from self weight of soil; increases linearly with depth
  - **Hydrostatic Pressure:** from ground water; increases linearly with depth
- **Additional stress**: from an imposed structural load; decreases with depth in a non-linear manner
  - **Surcharge load**: a load of infinite extent on the surface; does not vary with depth.

2. Geostatic Stress

- Total Stress
- Pore Pressure
- Effective Stress
  - Effects of Capillarity
  - Effects of Seepage

Geostatic Stress

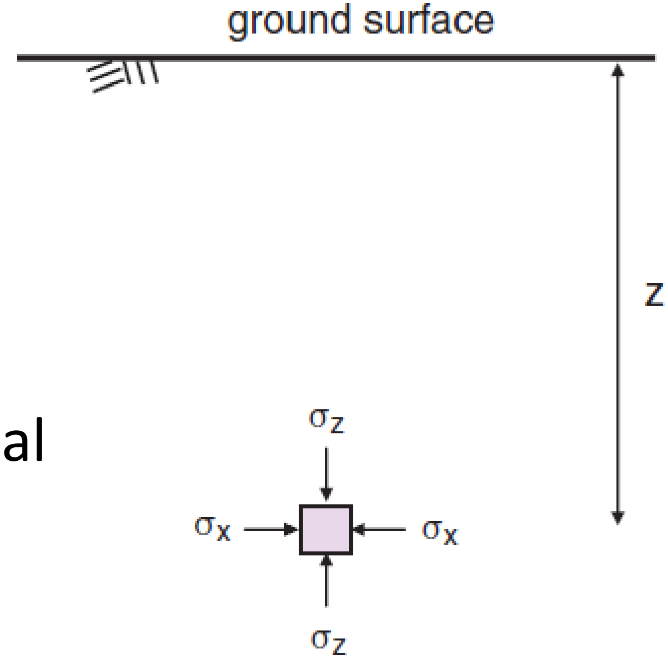
Total Stress

- is due to the weight of everything that lies above that point including soil, water and any load applied to the soil surface.

- increases with depth and with unit weight and the total vertical stress at depth z in the soil due to the weight of the soil acting above.

$$\sigma_z = \gamma z$$

where γ = unit weight of the soil.



If the soil is multi-layered, the total vertical stress is determined by summing the stresses induced by each layer of soil.

Geostatic Stress cntd

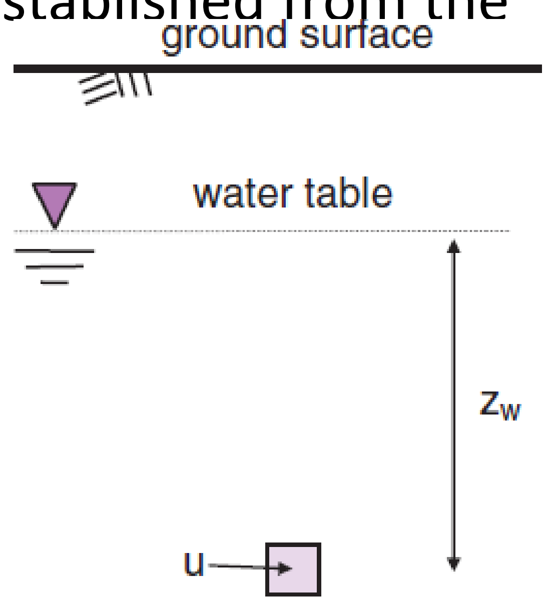
Pore Pressure

The water within the voids of a soil is known as pore water.

Pore water experiences pressure known as the pore pressure or pore water pressure, u.

In the case of a horizontal ground water table, we may be able to assume that no flow is taking place and the pore pressure at a point beneath the ground water table can be established from the hydrostatic pressure acting.

The magnitude of the pore pressure at the water table is zero.



$u = \gamma_w z_w$

where z_w = the depth below the water table.

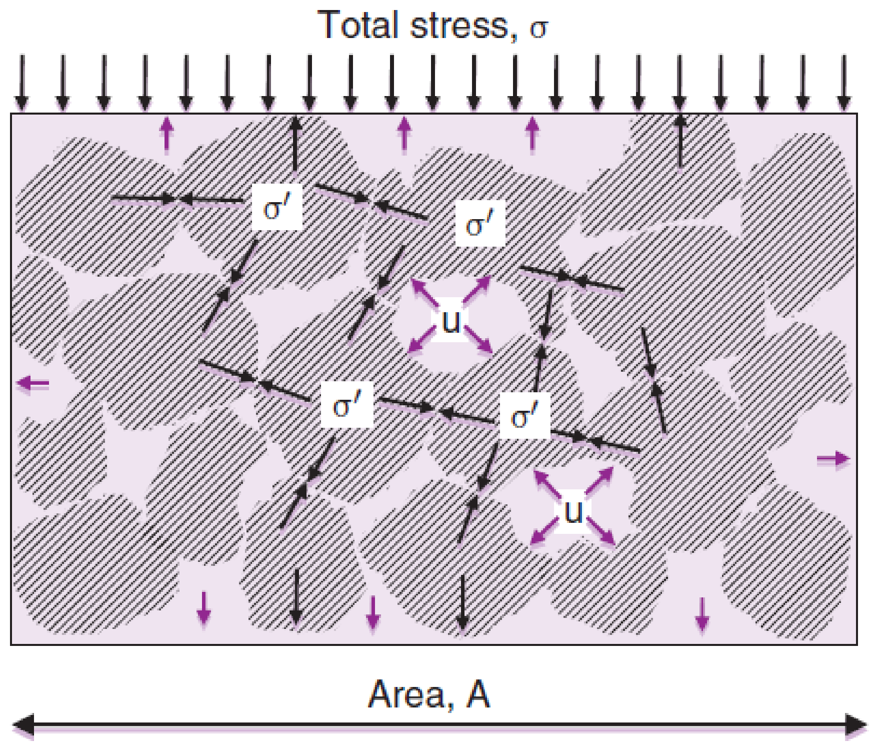
Geostatic Stress cntd

Effective Stress

The stress that controls changes in the volume and strength of a soil is known as the *effective stress.*

Consider saturated soils only.

When a load is applied to such a soil, it will be carried by the water in the soil voids (causing an increase in the pore water pressure) or by the soil skeleton (in the form of grain to grain contact stresses), or else it will be shared between the water and the soil skeleton.



The portion of the total stress carried by the soil particles is known as the effective stress, σ'.

Geostatic Stress cntd

Effective Stress

Terzaghi first presented the concept of effective stress in 1925.

He showed, from the results of many soil tests, that when an undrained saturated soil is subjected to an increase in applied normal stress, $\Delta\sigma$, the pore water pressure within the soil increases by Δu, and the value of Δu is equal to the value of $\Delta\sigma$. This increase in u caused no measurable changes in either the volumes or the strengths of the soils tested, and Terzaghi therefore used the term neutral stress to describe u, instead of the now more popular term pore water pressure.

$$\sigma' = \sigma - u$$

where the prime represents 'effective stress'.

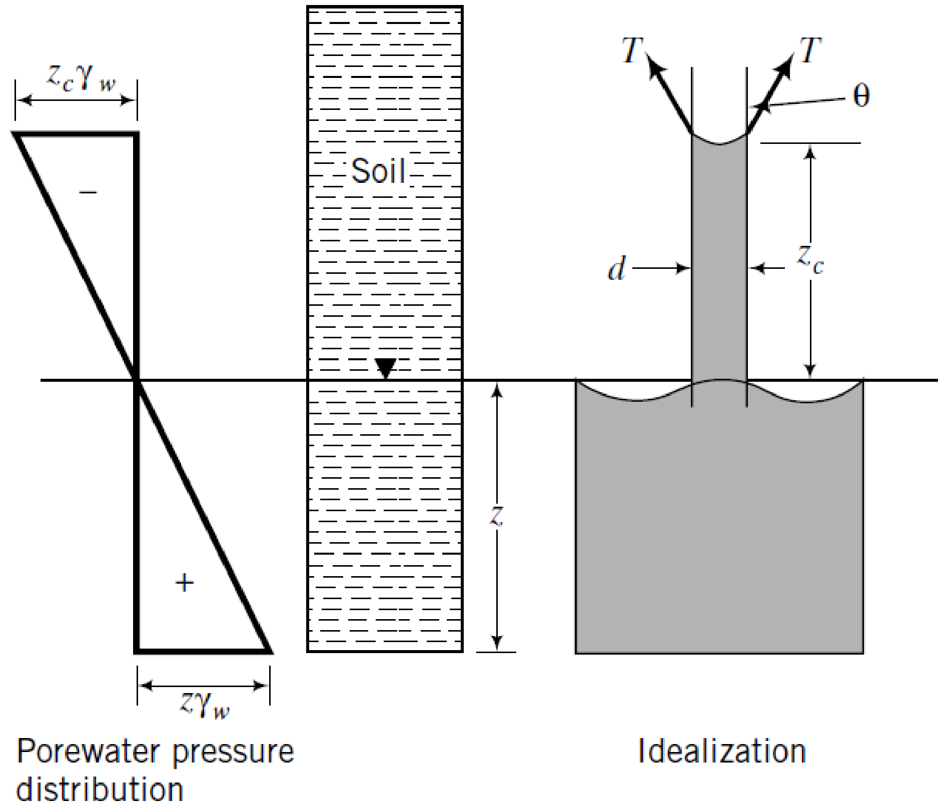
NB. This equation is applicable to all saturated soils.

Geostatic Stress cntd

Effect of Capillary

The porewater pressure due to capillarity is negative (suction), and is a function of the size of the soil pores and the water content.

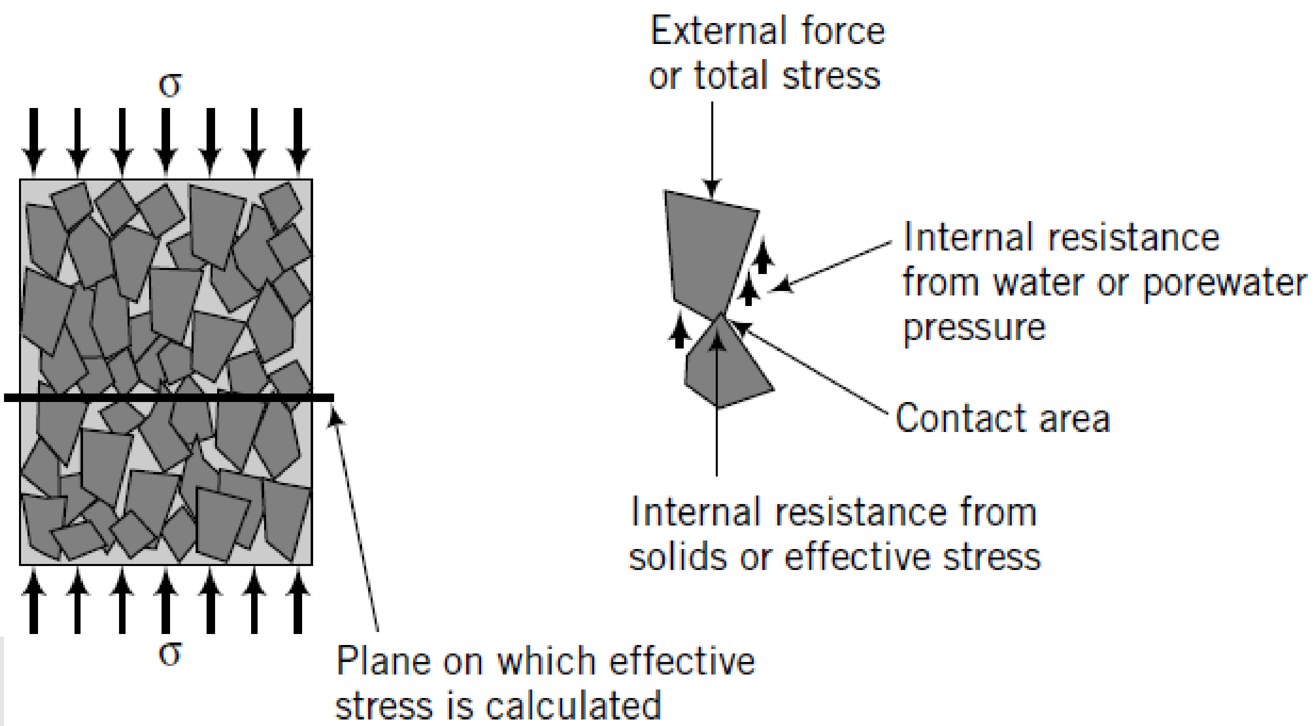
At the groundwater level, the porewater pressure is zero and decreases (becomes negative) as you move up the capillary zone.



Geostatic Stress cntd

The effective stress increases because the porewater pressure is negative.

For example, for the capillary zone, zc, the porewater pressure at the top is $-z_c\gamma_w$ and the effective stress is $\sigma' = \sigma - (-z_c\gamma_w) = \sigma + z_c\gamma_w$



Geostatic Stress cntd

Effect of Seepage

As water flows through soil it exerts a frictional drag on the soil particles, resulting in head losses.

The frictional drag is called seepage force in soil mechanics.

It is often convenient to define seepage as the seepage force per unit volume (it has units similar to unit weight), which we will denote by j_s.

If the head loss over a flow distance, L, is Dh, the seepage force is

$$j_s = \frac{\Delta h \gamma_w}{L} = i\gamma_w$$

Geostatic Stress cntd

If seepage occurs downward, then the seepage stresses are in the same direction as the gravitational effective stresses.

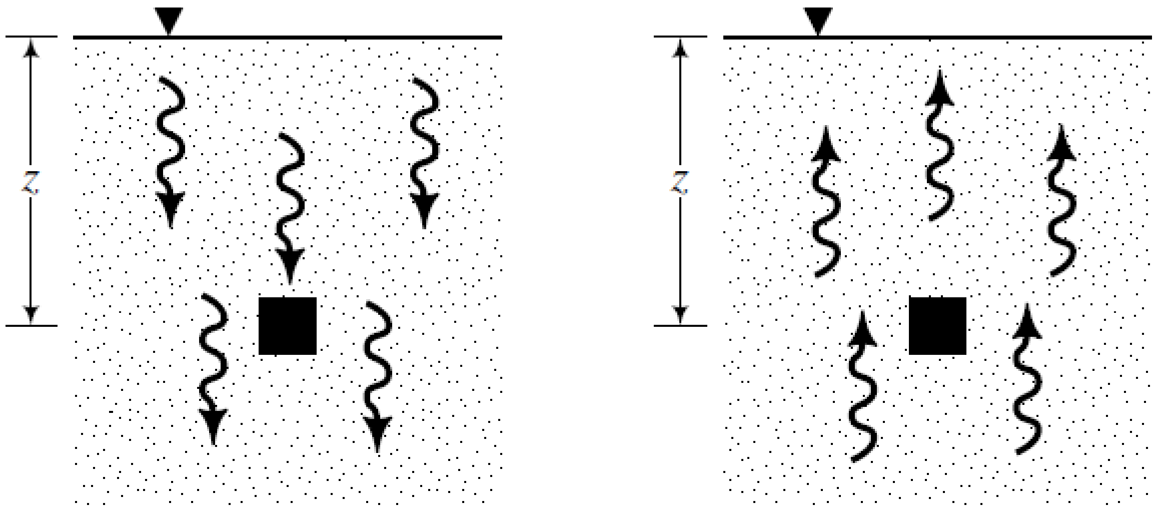
From static equilibrium, the resultant vertical effective stress is

$\sigma' = \gamma' z + iz\gamma_w = \gamma' z + j_s z$

If seepage occurs upward, then the seepage stresses are in the opposite direction to the gravitational effective stresses.

From static equilibrium, the resultant vertical effective stress is

$\sigma' = \gamma' z - iz\gamma_w = \gamma' z - j_s z$

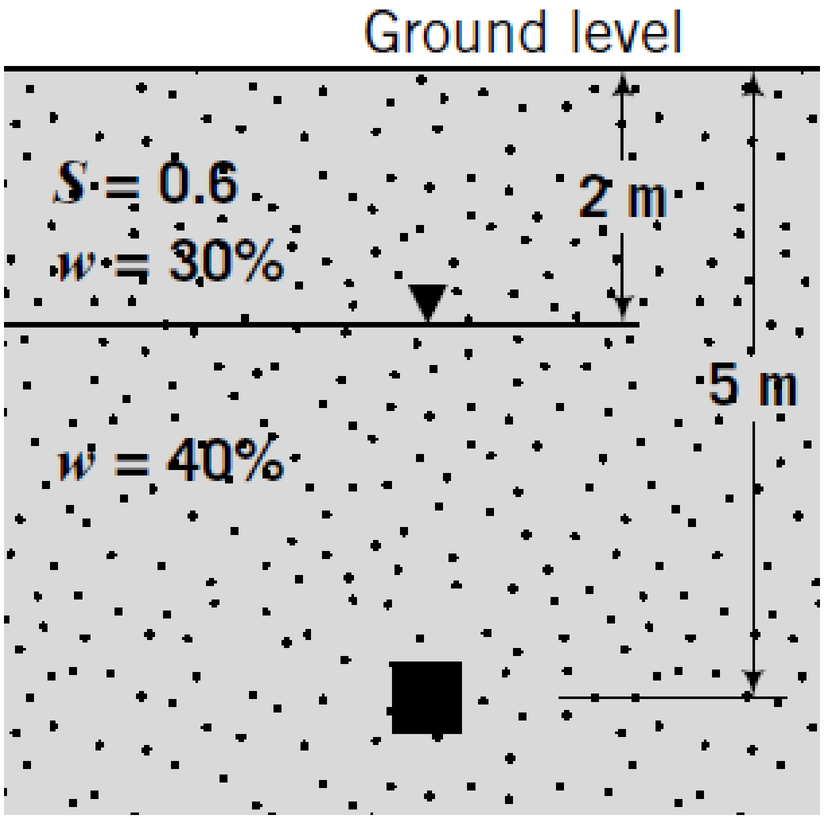


(a) Downward seepage (b) Upward seepage

Geostatic Stress cntd

EXERCISE 6.2.1 – EFFECTIVE STRESS

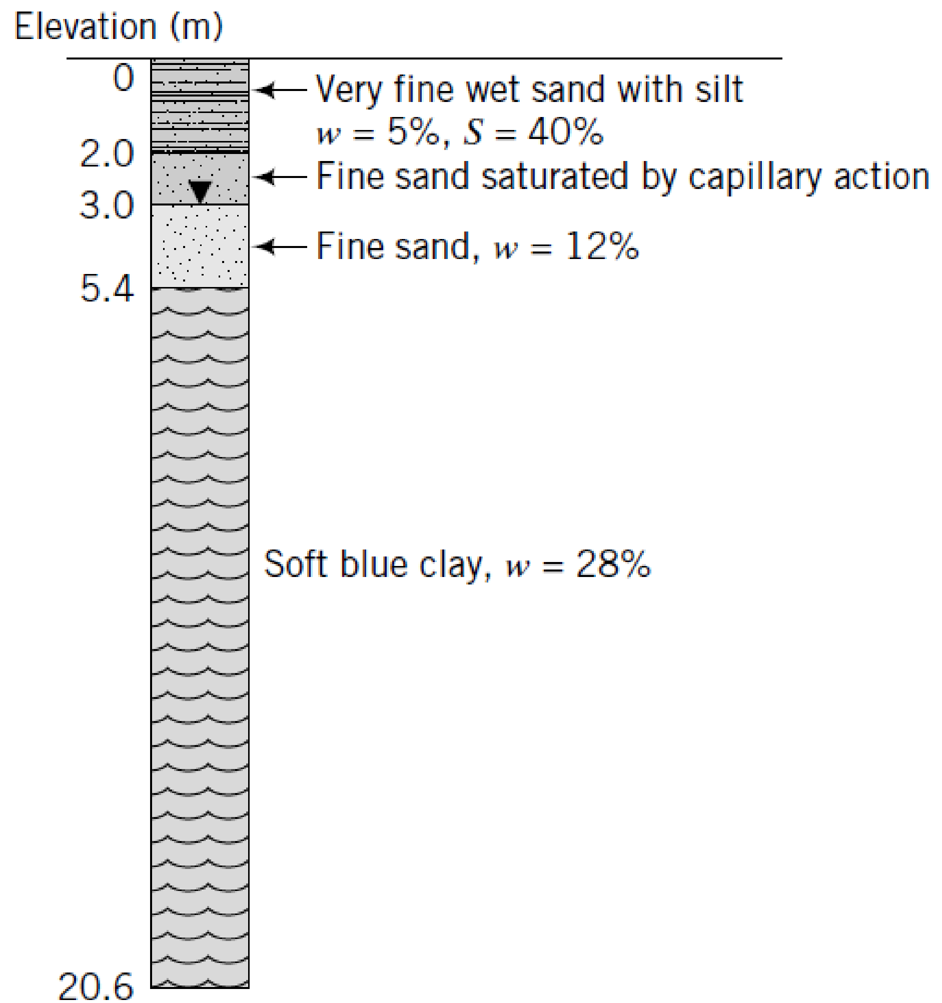
Calculate the effective stress for a soil element at depth 5 m in a uniform deposit of soil, as shown in figure below.



Geostatic Stress cntd

EXERCISE 6.2.2 – EFFECTIVE STRESS DISTRIBUTION

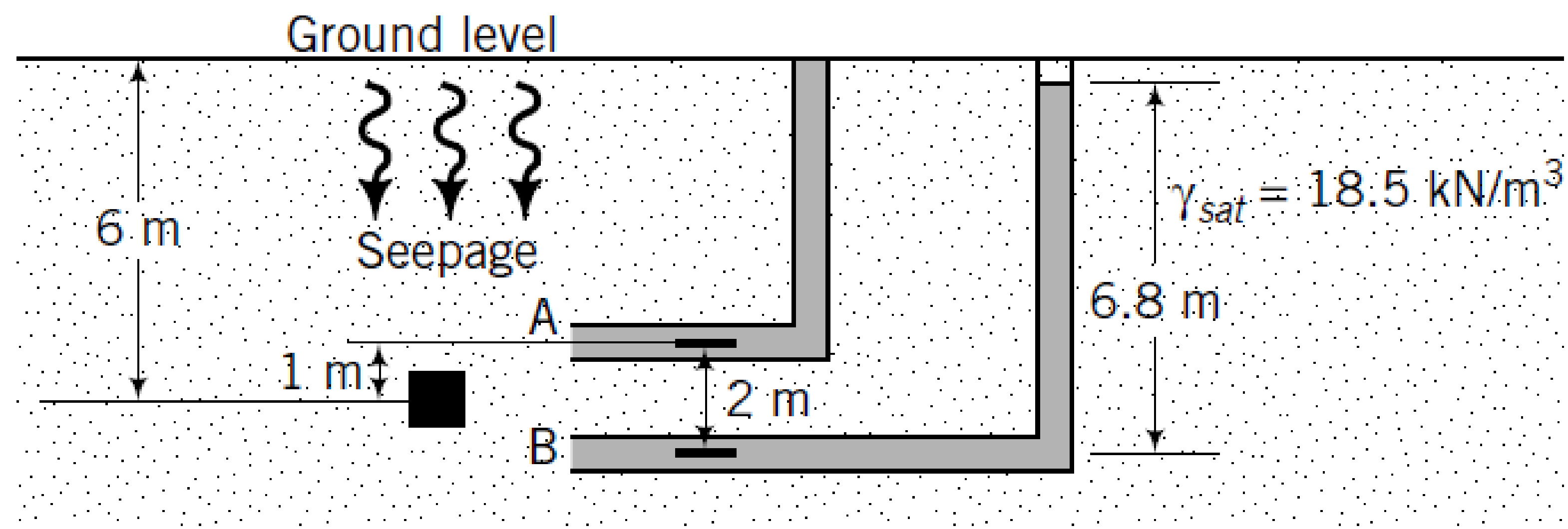
A borehole at a site reveals the soil profile shown in figure.
Plot the distribution of vertical total and effective stresses with depth.



EXERCISE 6.2.3 – EFFECTS OF SEEPAGE

Water is seeping downward through a soil layer, as shown in figure. Two piezometers (A and B) located 2 m apart (vertically) showed a head loss of 0.2 m.

Calculate the resultant vertical effective stress for a soil element at a depth of 6 m.



Geostatic Stress cntd

Horizontal Components of Geostatic Stresses

The ratio of the horizontal principal effective stress to the vertical principal effective stress is called the lateral earth pressure coefficient at rest (Ko), that is,

$$K_o = \frac{\sigma_3'}{\sigma_1'}$$

K_o applies only to effective principal, not total principal, stresses.

To find the lateral total stress, one must add the porewater pressure.

NB. The porewater pressure is hydrostatic and, at any given depth, the porewater pressures in all directions are equal.

Geostatic Stress cntd

EXERCISE 6.2.4 – Horizontal Geostatic Stresses

Calculate the horizontal effective stress and the horizontal total stress for the soil element at 5 m in Exercise 6.2.1 if $K_o = 0.5$.

Geostatic Stress cntd

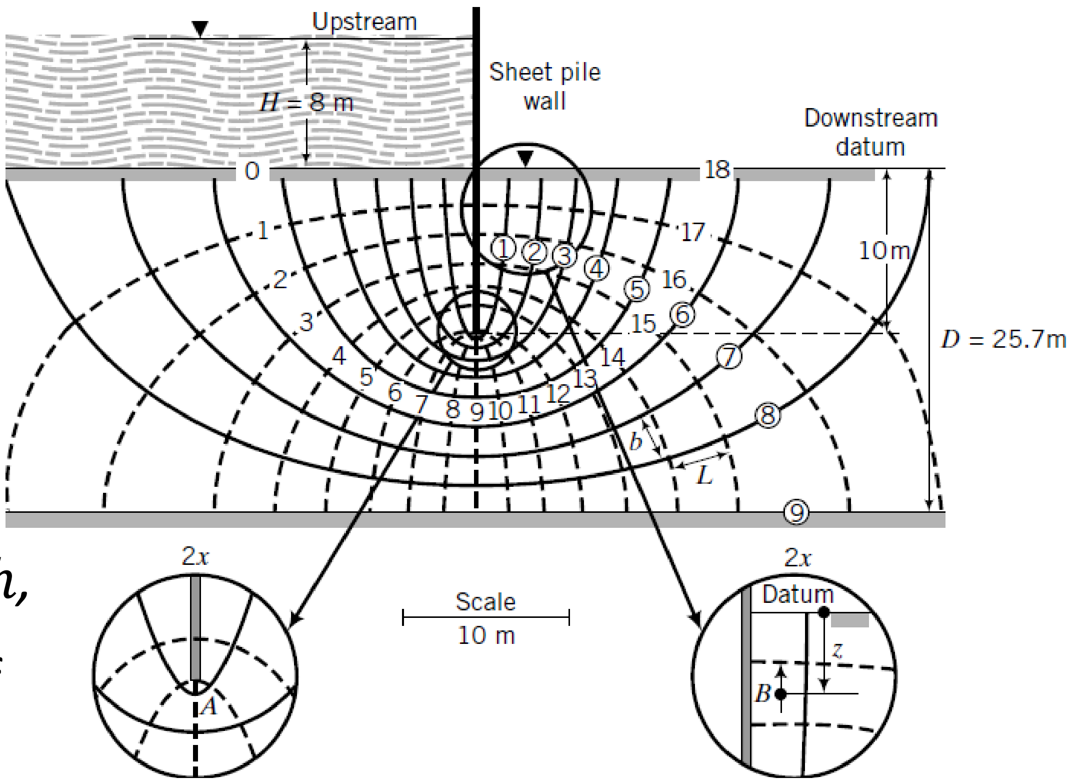
Porewater Pressure Distribution in Flow Regime

The porewater pressure at any point *j* within the flow domain (flownet) is calculated as follows:

1. Select a datum.

[Downstream water level is chosen as the datum in the figure here]

2. Determine the total head at *j*: $H_j = \Delta H - (N_d)_j \Delta h$, where $(N_d)_j$ is the number of equipotential drops at point *j*; $(N_d)_j$ can be fractional. For example, at *B*, $H_B = \Delta H - 16.5\Delta h$



Geostatic Stress cntd

3. Subtract the elevation head at point *j* from the total head H_j to get the pressure head.

For point *B* , the elevation head h_z is -*z* (point *B* is below the datum). The pressure head is then $\left(h_p\right)_j = \Delta H - (N_d)_j \Delta h - h_z$

4. The porewater pressure is $u_j = \left(h_p\right)_j \gamma_w$

The uplift force per unit length (length is normal to the *XZ* plane) is found by calculating the porewater pressure at discrete points along the base and then finding the area under the porewater pressure distribution diagram, that is,

$$P_w = \sum_{j=1}^{n} u_j \, \Delta x_j$$

Geostatic Stress cntd

Uplift Forces under Dams

The uplift force per unit length (length is normal to the *XZ* plane) is found by calculating the porewater pressure at discrete points along the base and then finding the area under the porewater pressure distribution diagram, that is,

$$P_w = \sum_{j=1}^{n} u_j \, \Delta x_j$$

where P_w is the uplift force per unit length, u_j is the average porewater pressure over an interval Δx_j, and *n* is the number of intervals.

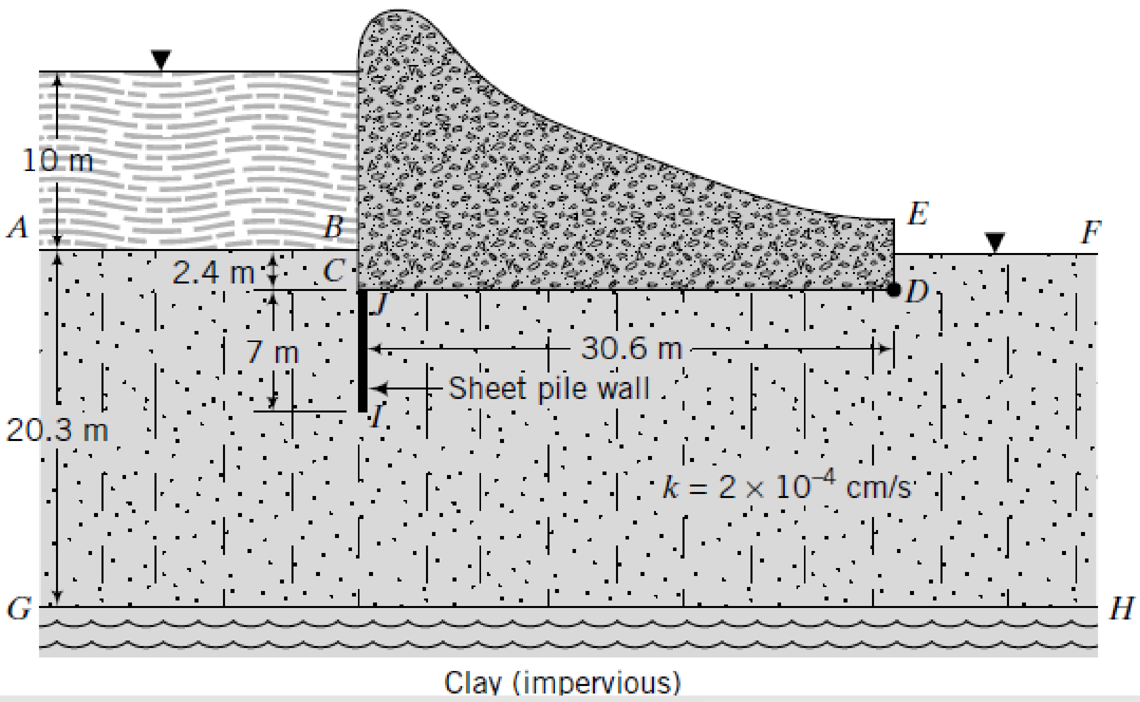
It is convenient to use Simpson's rule to calculate P_w :

$$P_w = \frac{\Delta x}{3}\left(u_1 + u_n + 2\sum_{i=3}^{n} u_i + 4\sum_{i=2}^{n} u_i\right)$$

Geostatic Stress cntd

EXERCISE 6.2.5 – *Pore Pressure Distribution & Uplift Forces*

A dam, shown in figure below, retains 10 m of water. A sheet pile wall (cutoff curtain) on the upstream side, which is used to reduce seepage under the dam, penetrates 7 m into a 20.3-m-thick silty sand stratum. Below the silty sand is a thick deposit of practically impervious clay. The average hydraulic conductivity of the silty sand is 2.0×10^{-4}cm/s. Assume that the silty sand is homogeneous and isotropic.



Geostatic Stress cntd

(a) Draw the flownet under the dam.

(b) Calculate the flow rate, *q*.

(c) Calculate and draw the porewater pressure distribution at the base of the dam.

(d) Determine the uplift force.

(e) Determine and draw the porewater pressure distribution on the upstream and downstream faces of the sheet pile wall.

(f) Determine the resultant lateral force on the sheet pile wall due to the porewater.

(g) Determine the maximum hydraulic gradient.

(h) Will piping occur if the void ratio of the silty sand is 0.8?

(i) What is the effect of reducing the depth of penetration of the sheet pile wall?

3. Additional Stress

- Infinite Loads
  - Surcharge Load
- Finite Loads
  - Point Load
  - Strip Load
    - Embankment Loads
  - Circular Load
  - Rectangular Load
    - Approximate Methods
  - Irregularly Shaped Areas

Additional Stress

Introduction

- The determination of the stress distributions created by various applied loads has occupied researchers for many years.
- The basic assumption used in all their analyses is that the soil mass acts as a continuous, homogeneous and elastic medium.
- The assumption of elasticity obviously introduces errors but it leads to stress values that are of the right order and are suitable for most routine design work.

Additional Stress

- In most foundation problems, however, it is only necessary to be acquainted with the increase in vertical stresses (for settlement analysis) and the increase in shear stresses (for shear strength analysis)
- Surface loads are divided into two general classes, finite and infinite. However, these are qualitative classes and are subject to interpretation.
- Examples of finite loads are point loads, circular loads, and rectangular loads. Examples of infinite loads are fills and surcharges.

Additional Stress

- The increases in soil stresses from surface loads are total stresses. These increases in stresses are resisted initially by both the porewater and the soil particles.

- Most soils exist in layers with finite thicknesses. The solution based on a semi-infinite soil mass will not be accurate for these layered soils.

Additional Stress

- ***Elastic Half Space***: the space is bounded by a horizontal plane and consists of a homogeneous, isotropic, and elastic material extending endlessly downwards and horizontally below the horizontal plane.
- ***Elastic Full Space***: consists of a homogeneous, isotropic, and elastic material but the application of the load is isnside the space; typically used for structures buried in the ground (eg. Tunnels)
- ***Elastic Quarter Space***: has an important application in determining stresses at the back of retaining wall due to surface loads.

Additional Stress

- The distribution of stresses within a soil from applied surface loads or stresses is determined by assuming that the soil is a semi-infinite, homogeneous, linear, isotropic, elastic material.
- A semi-infinite mass is bounded on one side and extends infinitely in all other directions; this is also called an “elastic half-space.”
- Because of the assumption of a linear elastic soil mass, we can use the principle of superposition. That is, the stress increase at a given point in a soil mass in a certain direction from different loads can be added together.

Stresses induced by uniform surface surcharge

- Road, railway, fill, ice, etc
- In the case of a uniform surcharge spread over a large area it can be assumed that the increase in vertical stress resulting from the surcharge is constant throughout the soil.
- Here, the vertical total stress at depth z, is given by

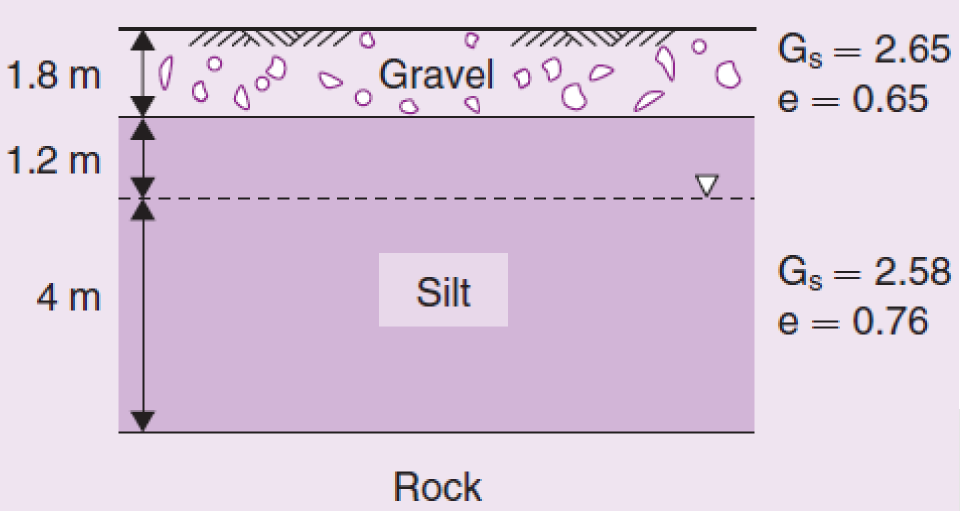
$$\sigma_z = \gamma z + q$$

where q is the magnitude of the surcharge (kPa).

EXERCISE 6.3.1 – SURCHARGE LOAD ON EFFECTIVE STRESS

Details of the subsoil conditions at a site are shown in figure below together with details of the soil properties. The ground surface is subjected to a uniform loading of 60 kPa and the groundwater level is 1.2 m below the upper surface of the silt. It can be assumed that the gravel has a degree of saturation of 50% and that the silt layer is fully saturated.

Determine the vertical effective stress acting at a point 1 m above the silt/rock interface.



Stresses induced by point load

- Electric line pole, light stand, column, etc
- Boussinesq (1885) evolved equations that can be used to determine the six stress components that act at a point in a semi-infinite elastic medium due to the action of a vertical point load applied on the horizontal surface of the medium.
- His expression for the increase in vertical stress is:

$$\Delta\sigma_z = \frac{3Q}{2\pi z^2\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}}$$

Additional Stress cntd

Stresses induced by point load

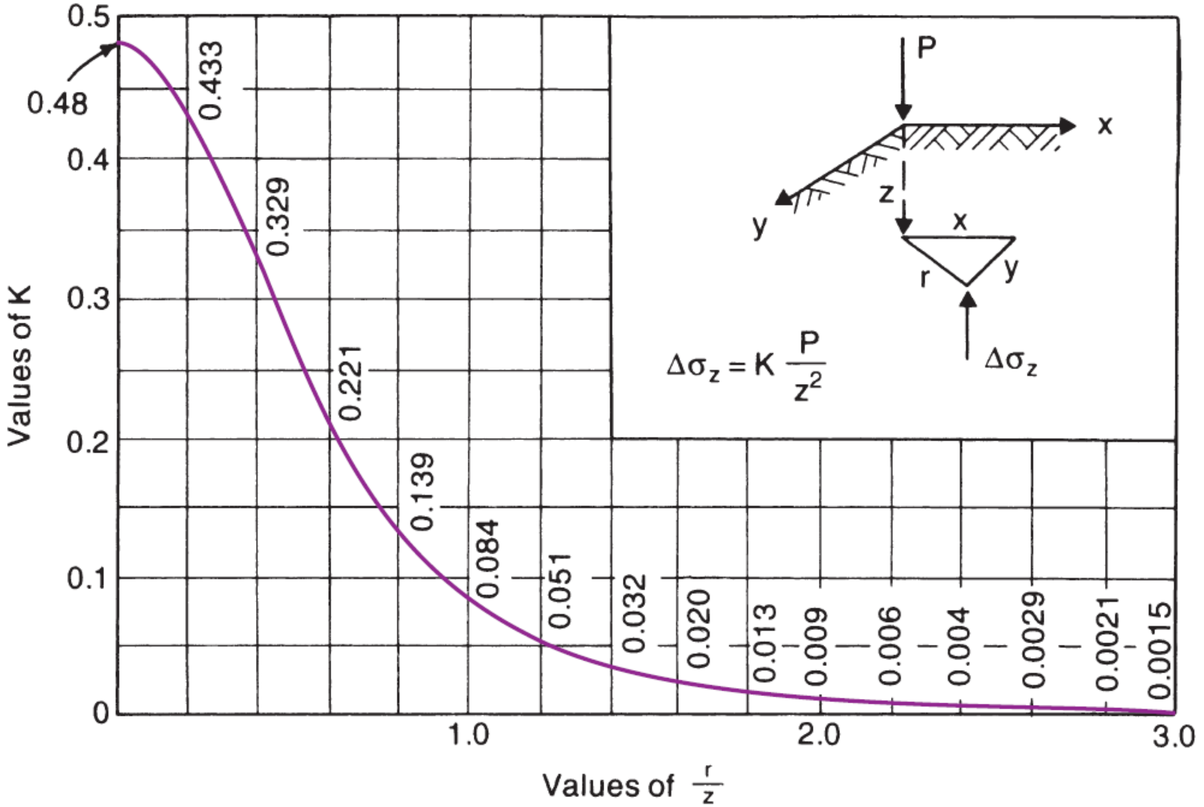
The expression has been simplified to:

$$\Delta\sigma_z = \frac{Q}{z^2}K$$

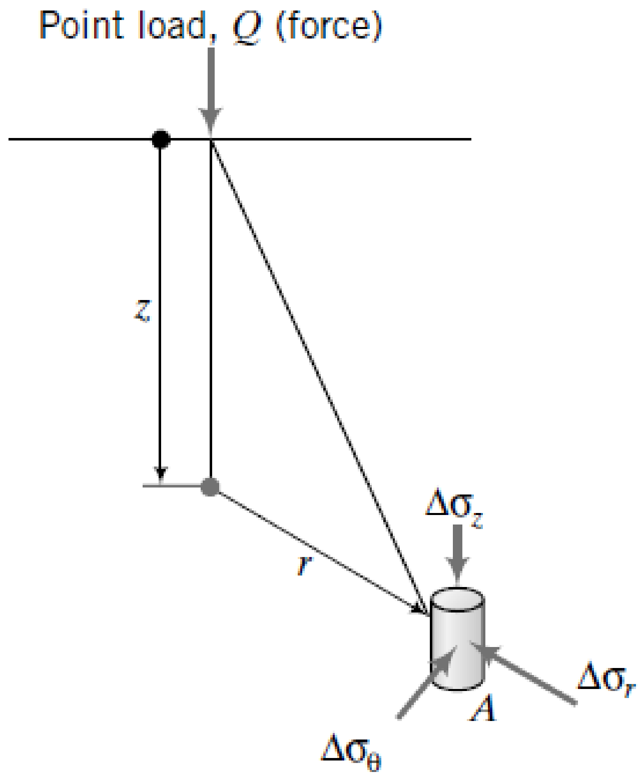
where K is an

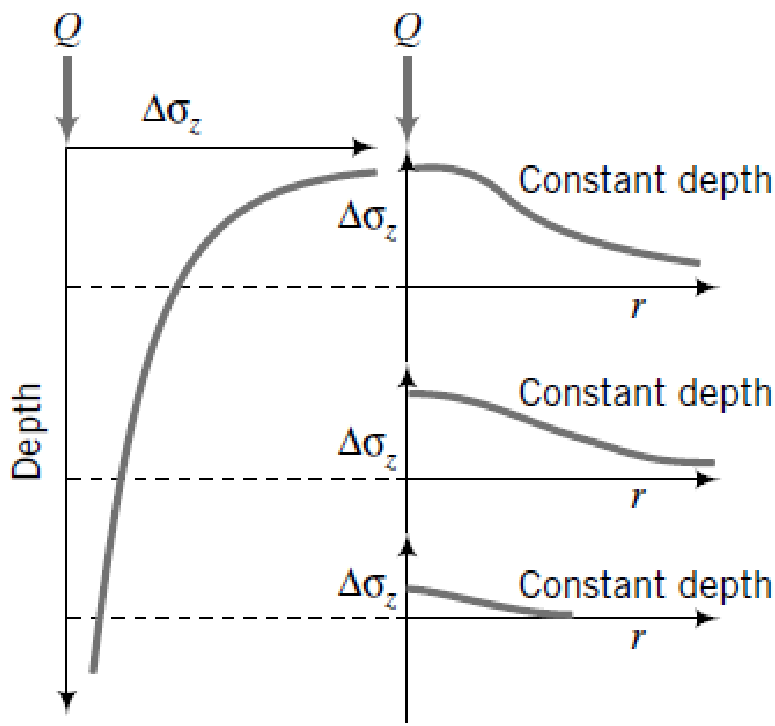
influence factor.

$$K = \frac{3}{2\pi}\frac{1}{\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}}$$



Additional Stress cntd





$$\Delta\sigma_r = \frac{Q}{2\pi}\left(\frac{3r^2 z}{(r^2+z^2)^{5/2}} - \frac{1-2\nu}{r^2+z^2+z(r^2+z^2)^{1/2}}\right)$$

$$\Delta\sigma_\theta = \frac{Q}{2\pi}(1-2\nu)\left(\frac{z}{(r^2+z^2)^{3/2}} - \frac{1}{r^2+z^2+z(r^2+z^2)^{1/2}}\right)$$

$$\Delta\tau_{rz} = \frac{3Q}{2\pi}\left[\frac{rz^2}{(r^2+z^2)^{5/2}}\right]$$

EXERCISE 6.3.2 - Point load

a) A concentrated load of 400 kN acts on the surface of a soil.

Determine the vertical stress increments at points directly beneath the load to a depth of 10 m.

b) A pole carries a vertical load of 200 kN. Determine the vertical total stress increase at a depth 5 m (a) directly below the pole and (b) at a radial distance of 2 m.

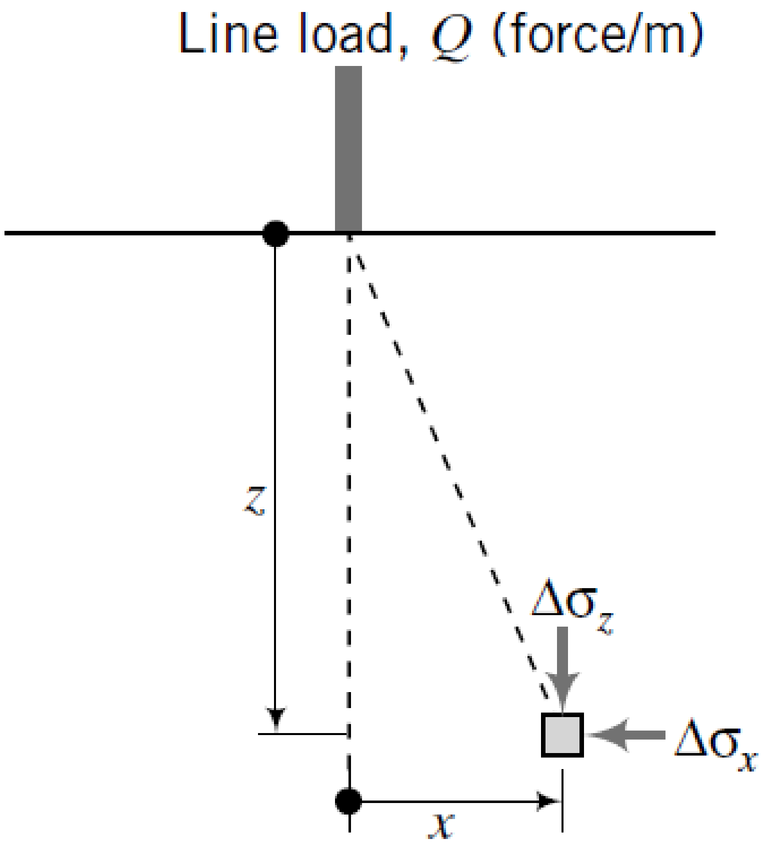
Stresses induced by uniform line load

- Rack or rail loading, load from a long brick wall.

$$\Delta\sigma_z = \frac{2Qz^3}{\pi(x^2+z^2)^2}$$

$$\Delta\sigma_x = \frac{2Qx^2z}{\pi(x^2+z^2)^2}$$

$$\Delta\tau_{zx} = \frac{2Qxz^2}{\pi(x^2+z^2)^2}$$

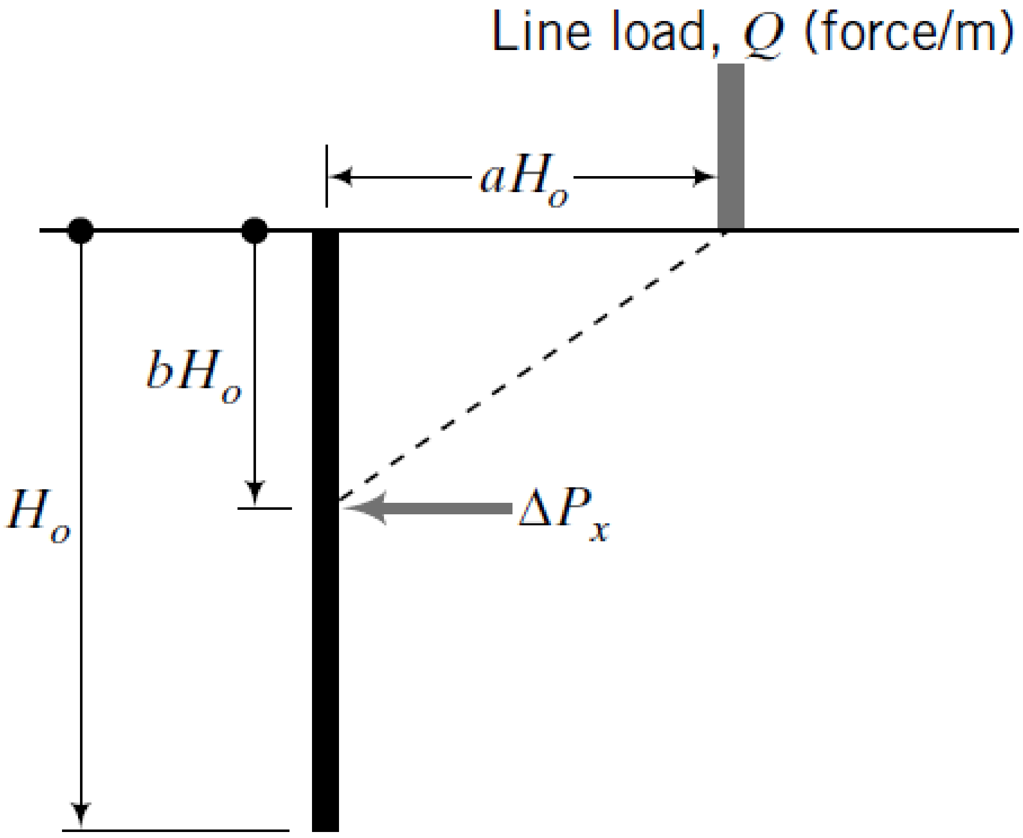


Line Load Near a Buried Earth-Retaining Structure

$$\Delta\sigma_x = \frac{2Qa^2b}{\pi H_o(a^2+b^2)^2}$$

The increase in lateral force is

$$\Delta P_x = \frac{2Q}{\pi\,(a^2+1)^2}$$



Stresses induced by strip load

- Strip foundation - a structure of finite width and infinite length on a soil surface.
- Two types of strip loads are common in geotechnical engineering.
- One is a load that imposes a uniform stress on the soil, for example, the middle section of a long embankment.
- The other is a load that induces a triangular stress distribution over an area of width B . For example the stress under the side of an embankment.

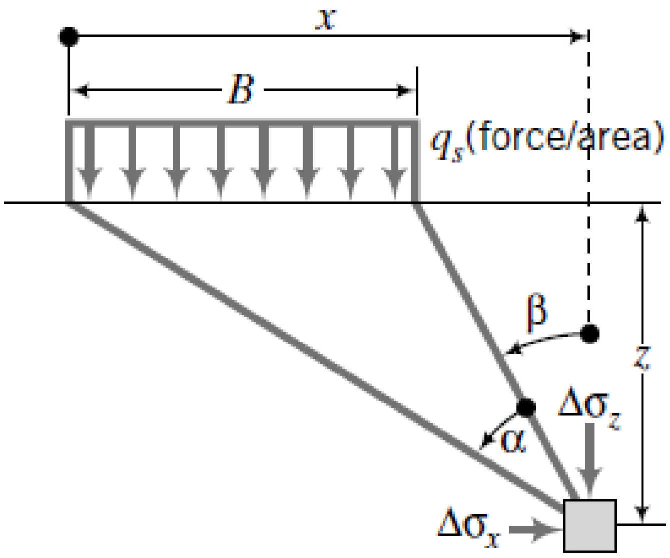
Additional Stress cntd

Uniform surface stress

$$\Delta\sigma_z = \frac{q_s}{\pi}[\alpha + \sin\alpha\cos(\alpha + 2\beta)]$$

$$\Delta\sigma_x = \frac{q_s}{\pi}[\alpha - \sin\alpha\cos(\alpha + 2\beta)]$$

$$\Delta\tau_{zx} = \frac{q_s}{\pi}[\sin\alpha\sin(\alpha + 2\beta)]$$

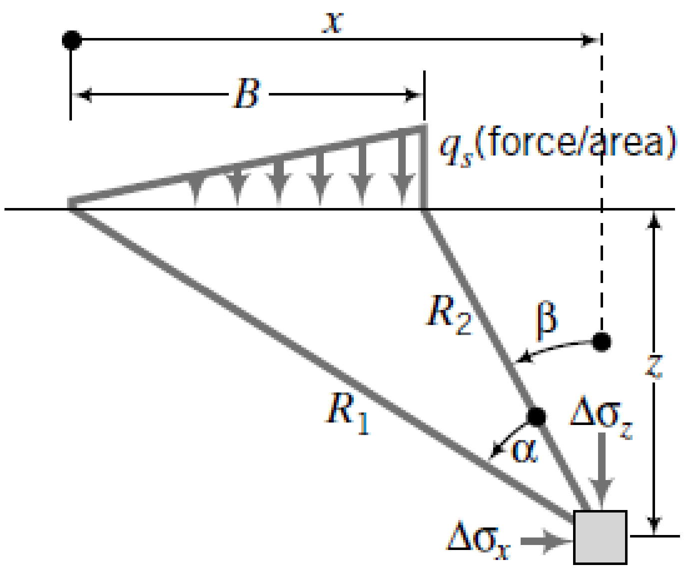


Linear varying surface stress

$$\Delta\sigma_z = \frac{q_s}{\pi}\left[\frac{x}{B}\alpha - \frac{1}{2}\sin 2\beta\right]$$

$$\Delta\sigma_x = \frac{q_s}{\pi}\left[\frac{x}{B}\alpha - \frac{z}{B}\ln\frac{R_1^2}{R_2^2} + \frac{1}{2}\sin 2\beta\right]$$

$$\Delta\tau_{zx} = \frac{q_s}{2\pi}\left[1 + \cos 2\beta - 2\frac{z}{B}\alpha\right]$$



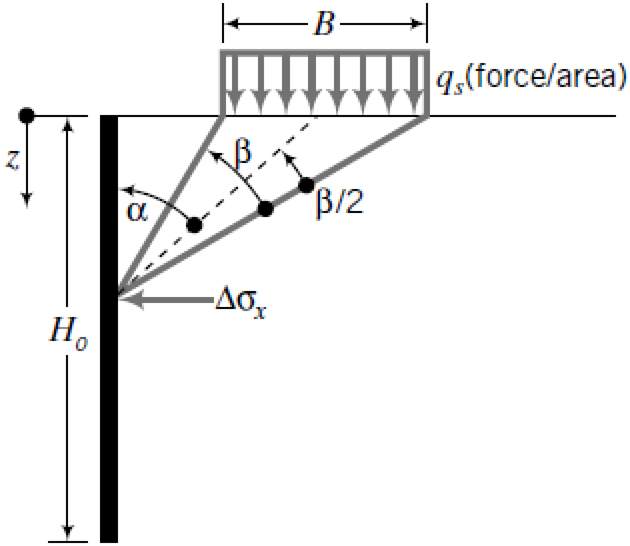
Uniform surface stress near retaining wall

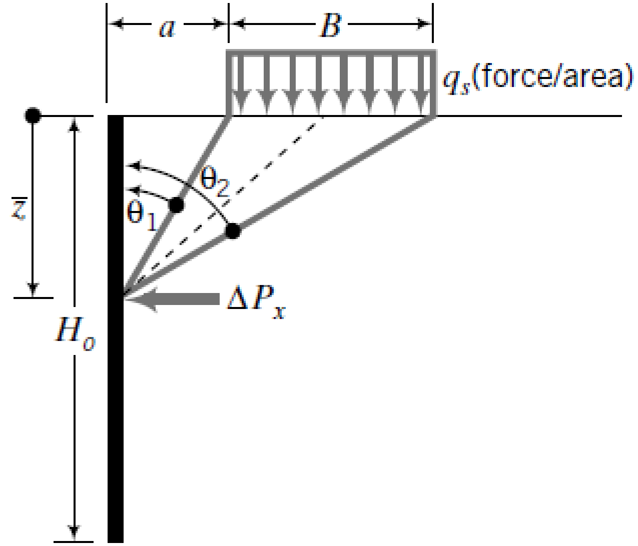
$$\Delta\sigma_x = \frac{2q_s}{\pi}[\beta - \sin\beta\cos 2\alpha] \;\&\; \Delta P_x = \frac{q_s}{90}[H_o(\theta_2 - \theta_1)]$$

$$\bar{z} = \frac{H_o^2(\theta_2 - \theta_1) - (R_2 - R_1) + 57.3BH_o}{2H_o(\theta_2 - \theta_1)}$$

where $\theta_1 = \tan^{-1}\left(\frac{a}{H_o}\right), \theta_2 = \tan^{-1}\left(\frac{a+B}{H_o}\right)$

$R_1 = (a+B)^2(90 - \theta_2), and\ R_2 = a^2(90 - \theta_1)$



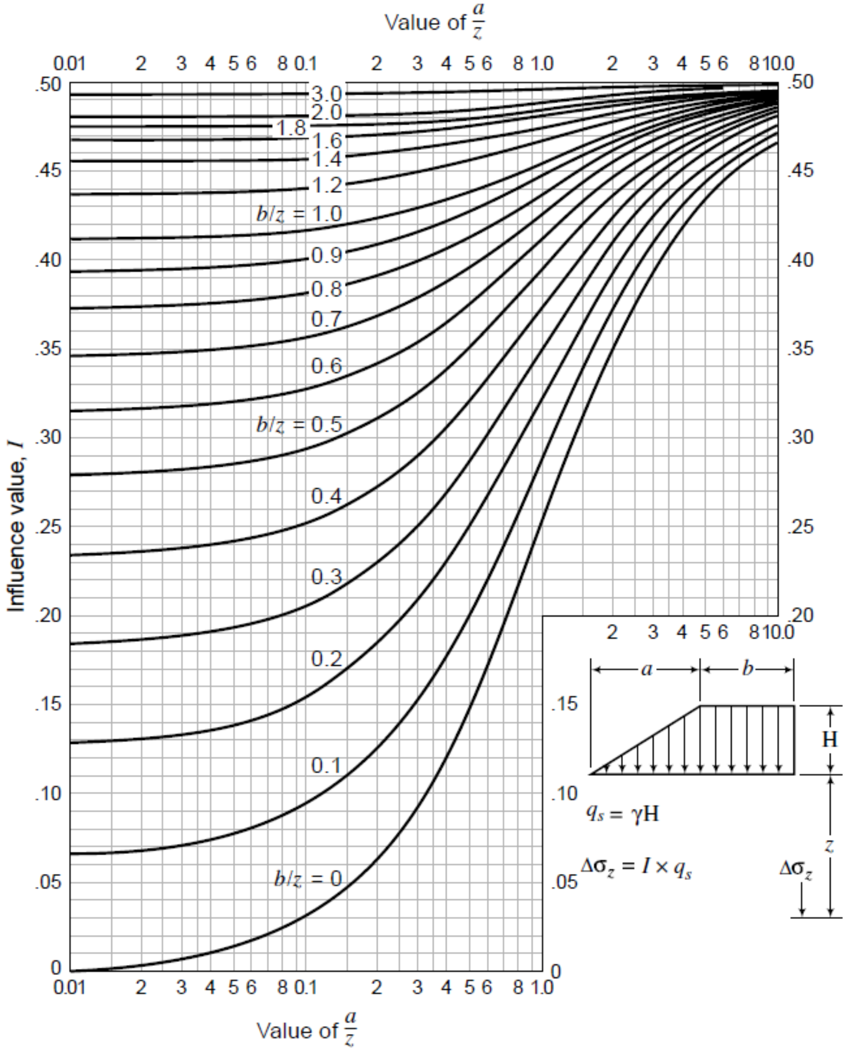


Additional Stress cntd

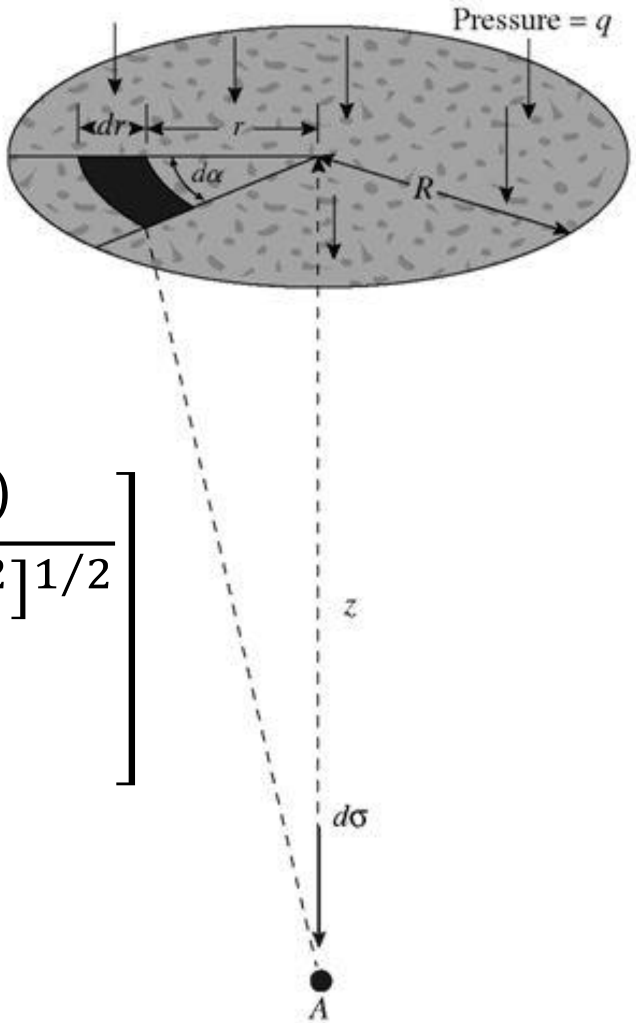
Stresses induced by embankment load

Loads from an embankment can be considered as a combination of a rectangle and two triangular strip loads.

- $\Delta\sigma_z = 2I' q_o$



Stress induced due to a uniformly loaded circular area



- circular foundation of an oil or water tank.

$$\Delta\sigma_z = q_s\left[1-\left(\frac{1}{1+(r_o/z)^2}\right)^{3/2}\right] = q_s I_c$$

$$\Delta\sigma_r = \Delta\sigma_\theta = \frac{q_s}{2}\left[(1+2v)-\frac{4(1+v)}{[1+(r_o/z)^2]^{1/2}} + \frac{1}{[1+(r_o/z)^2]^{3/2}}\right]$$

EXERCISE 6.3.3 - Uniform Circular Load

A circular foundation of diameter 100 m exerts a uniform pressure on the soil of 450 kPa.

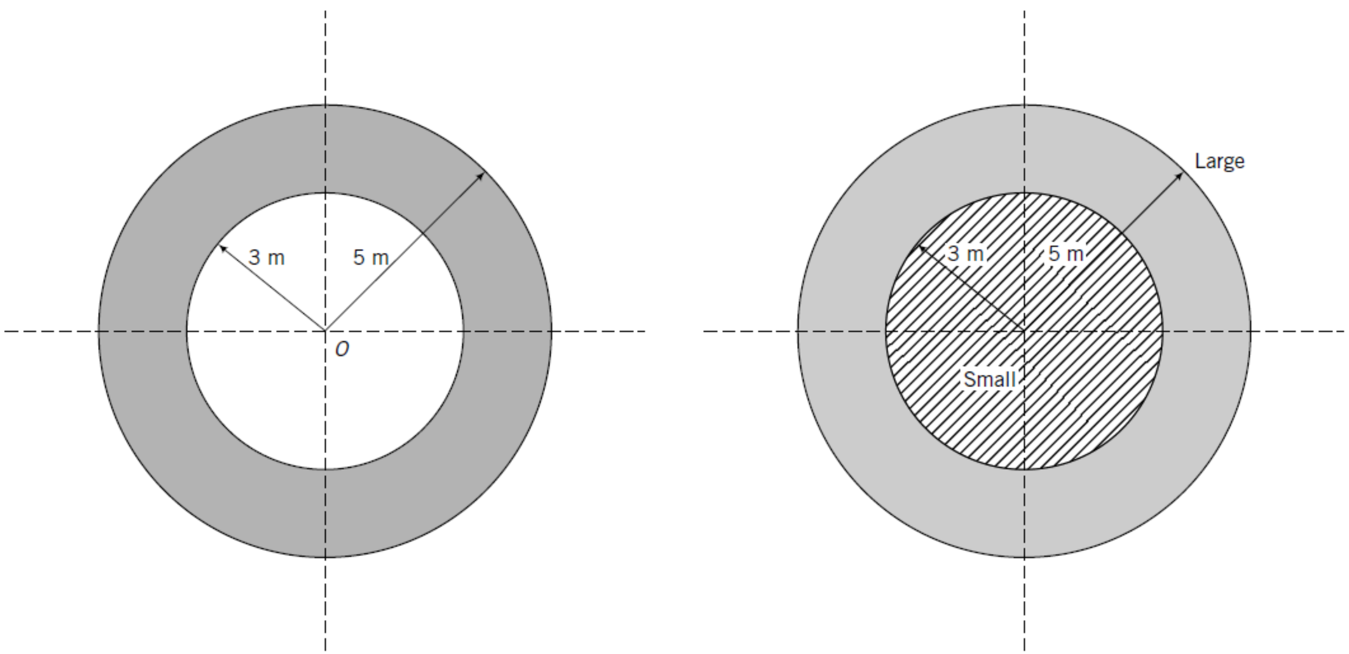
Determine the vertical stress increments for depths up to 200 m below its centre.

EXERCISE 6.3.4 ***Vertical Stress Increase Due to a Ring Load***

A silo is supported on a ring foundation, as shown in the figure. The total vertical load is 4 MN.

(a) Plot the vertical stress increase with depth up to 8 m under the center of the ring.

(b) Determine the maximum vertical stress increase and its location.



Additional Stress cntd

Stresses induced by uniform rectangular load

- Raft or rectangular footing
- The increases in stresses below the *corner* of a rectangular area of width *B* and length *L* are

$$\Delta\sigma_z = \frac{q_s}{2\pi}\left[\tan^{-1}\frac{LB}{zR_3} + \frac{LBZ}{R_3}\left(\frac{1}{R_1^2} + \frac{1}{R_2^2}\right)\right] = q_s I_z$$

$$\Delta\sigma_x = \frac{q_s}{2\pi}\left[\tan^{-1}\frac{LB}{zR_3} - \frac{LBZ}{R_1^2 R_3}\right] = q_s I_x$$

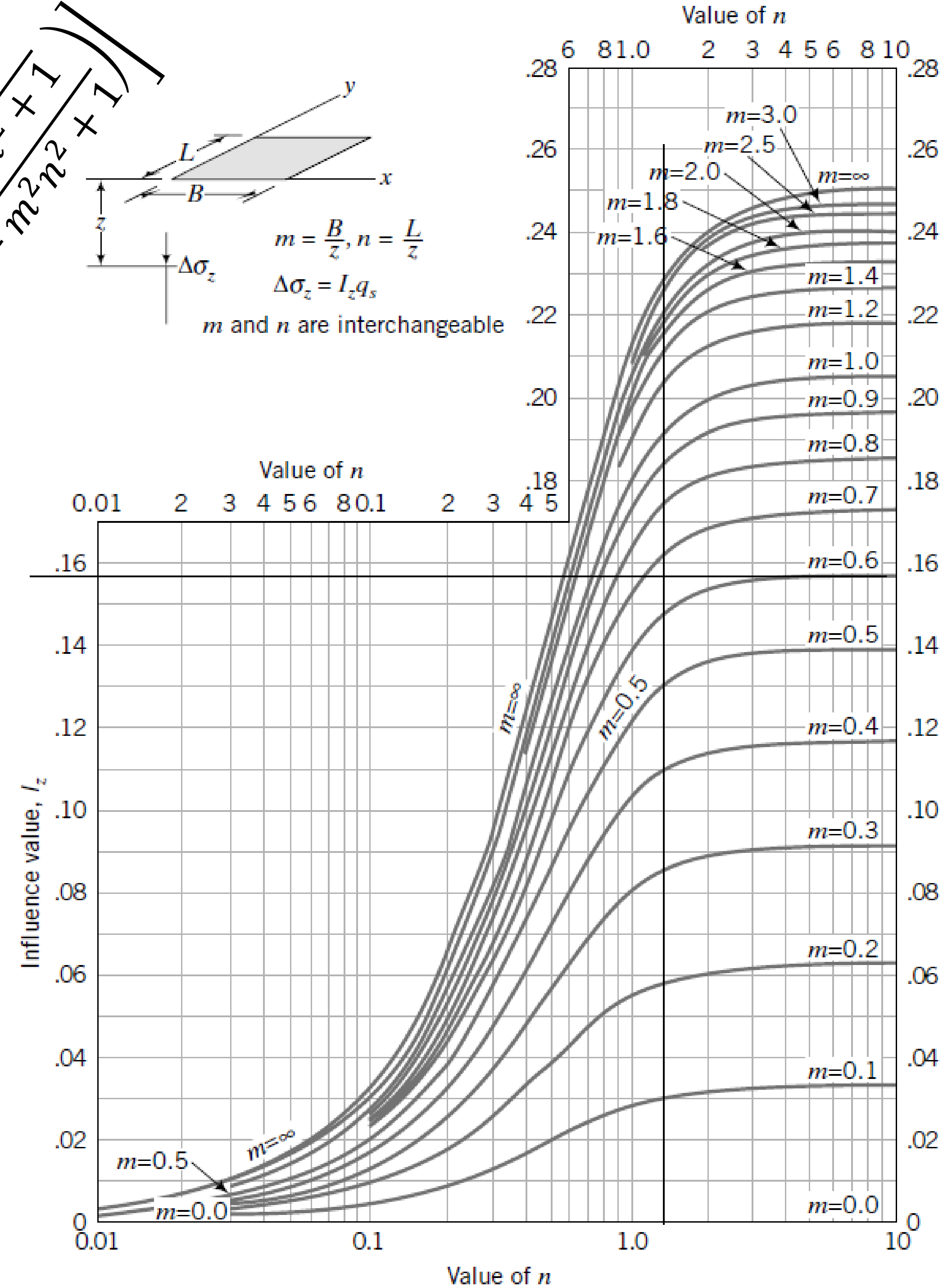
$$\Delta\sigma_y = \frac{q_s}{2\pi}\left[\tan^{-1}\frac{LB}{zR_3} - \frac{LBZ}{R_2^2 R_3}\right] = q_s I_y$$

$$\Delta\tau_{zx} = \frac{q_s}{2\pi}\left[\frac{B}{R_2} - \frac{z^2 B}{R_1^2 R_3}\right] = q_s I_\tau$$

where $R_1 = (L^2 + z^2)^{1/2}, R_2 = (B^2 + z^2)^{1/2}, and\ R_3 = (L^2 + B^2 + z^2)^{1/2}$

$$I_z = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+-m^2n^2+1}\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right)+\tan^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2-m^2n^2+1}\right)\right]$$

After Fadum (1948)

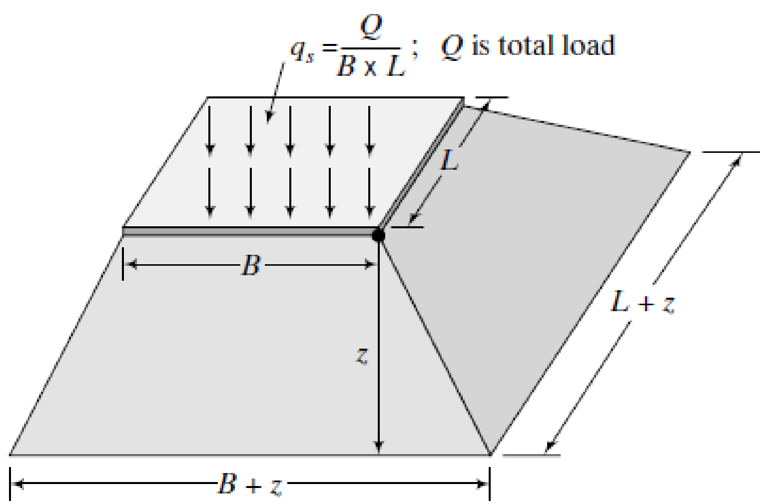


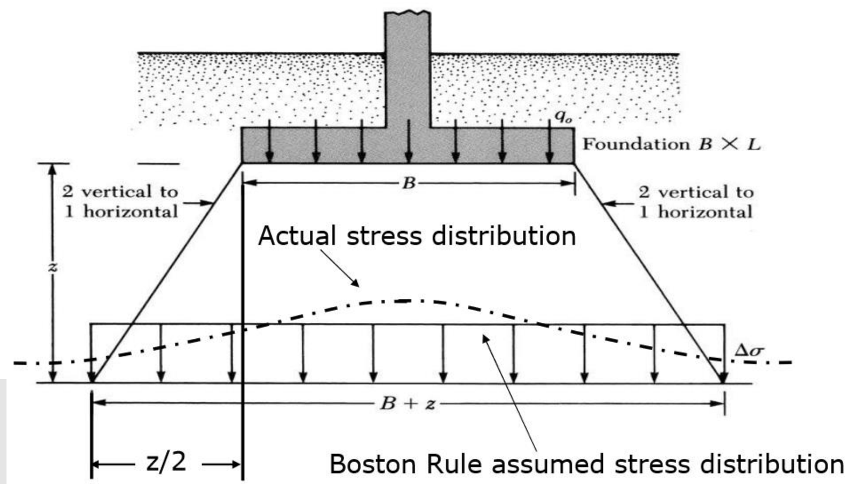
Additional Stress cntd

Approximate Methods for Rectangular Load

- Used in preliminary analyses of vertical stress increases under the center of rectangular loads.
- reasonably accurate (compared with Boussinesq's elastic solution) when z > B.
- The vertical stress increase under the center of the load is

$$\Delta\sigma_z = \frac{Q}{(B+z)(L+z)} = \frac{q_s BL}{(B+z)(L+z)}$$





EXERCISE 6.3.5 - Rectangular Load

A 4.5 m square foundation exerts a uniform pressure of 200 kPa on a soil.

Determine

(i) the vertical stress increments due to the foundation load to a depth of 10 m below its centre and

(ii) the vertical stress increment at a point 3 m below the foundation and 4 m from its centre along one of the axes of symmetry.

EXERCISE 6.3.6 - Rectangular Load

A rectangular concrete slab, 3 m X 4.5 m, rests on the surface of a soil mass. The load on the slab is 2025 kN.

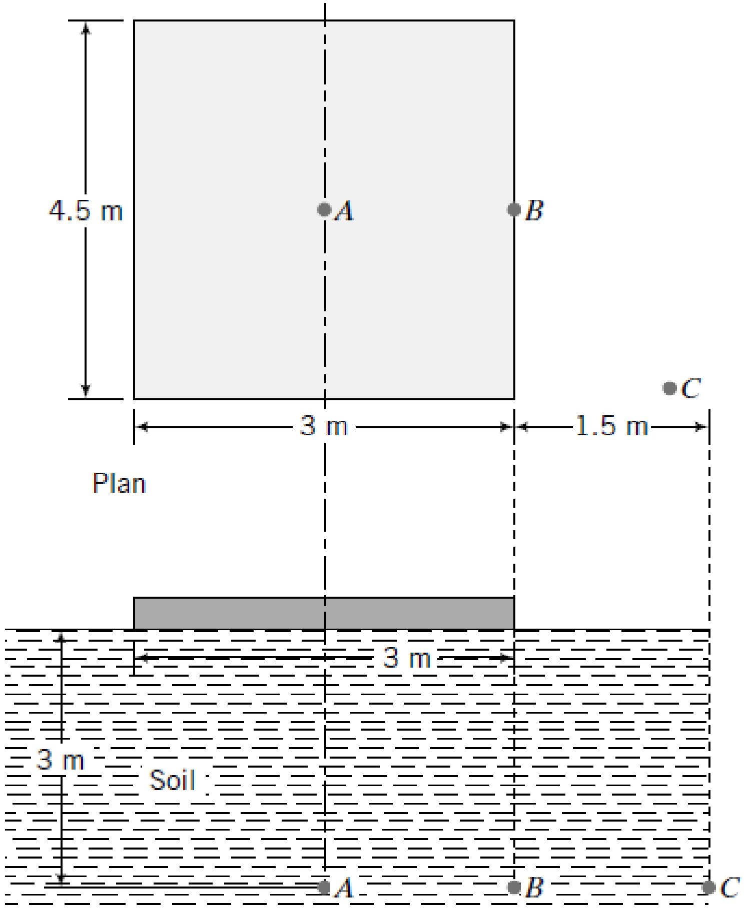
Determine the vertical stress increase

at a depth of 3 m

(a) under the center of the slab, point *A;*

(b) under point *B*; and

(c) at a distance of 1.5 m from a corner, point *C*.



Stresses induced by uniform irregular load

Newmark's chart: -constructed based on Boussinesque's solution

-enables the vertical stress to be determined at any point below an area of any shape carrying a uniform pressure q.

- Newmark (1942) developed a chart to determine the increase in vertical stress due to a uniformly loaded area of any shape.
- The chart consists of concentric circles divided by radial lines . The area of each segment represents an equal proportion of the applied surface stress at a depth *z* below the surface.

Additional Stress cntd

If there are 10 concentric circles and 20 radial lines, the stress on each circle is $q_s/10$ and on each segment is $q_s/(10 * 20)$

The radius-to-depth ratio of the first (inner) circle is found by setting $\Delta\sigma_z = 0.1q_s$, that is,

$$0.1q_s = q_s \left[1 - \left\{\frac{1}{1 + (r_o/z)^2}\right\}^{3/2}\right]$$

from which $r/z = 0.27$.

For the other circles, substitute the appropriate value for $\Delta\sigma_z$;

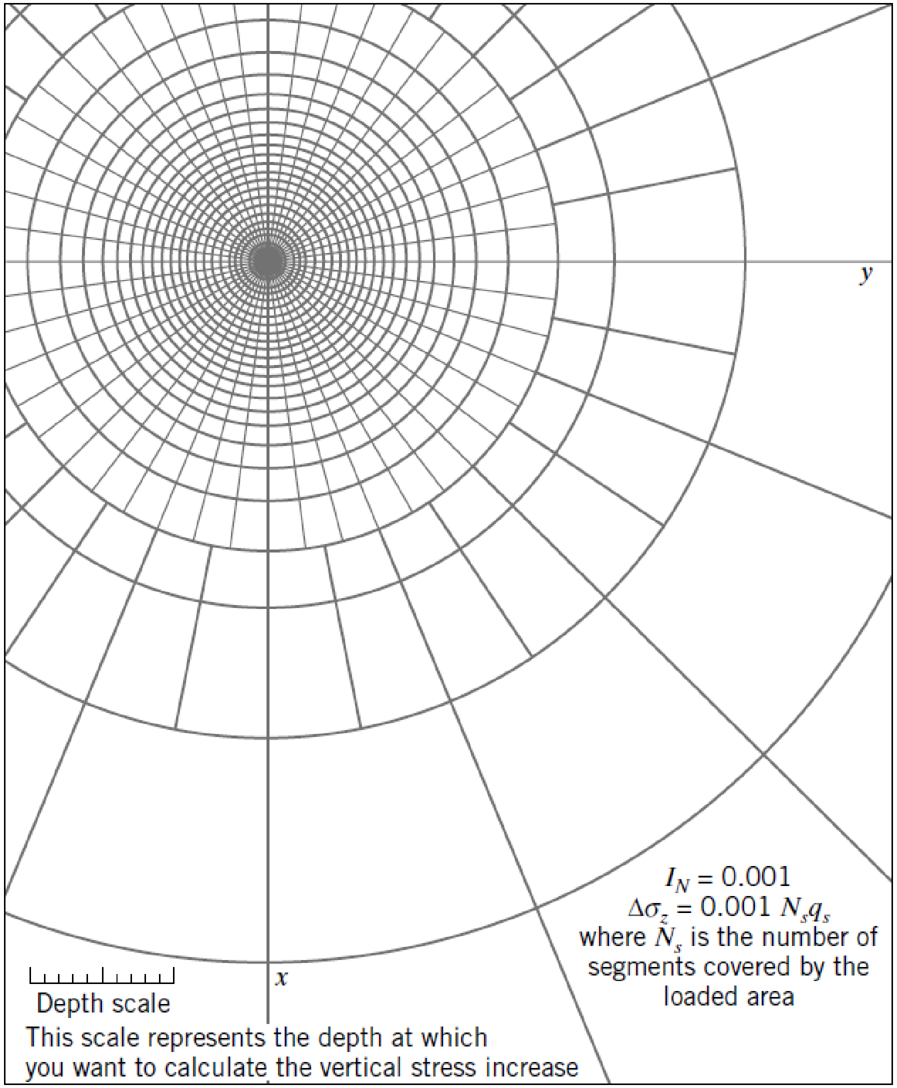
for example, for the second circle $\Delta\sigma_z = q_s$, and find *r/z*.

Additional Stress cntd

Newmark's chart:

The chart is normalized to the depth; that is, all dimensions are scaled by a factor initially determined for the depth.

Every chart should show a scale and an influence factor I_N



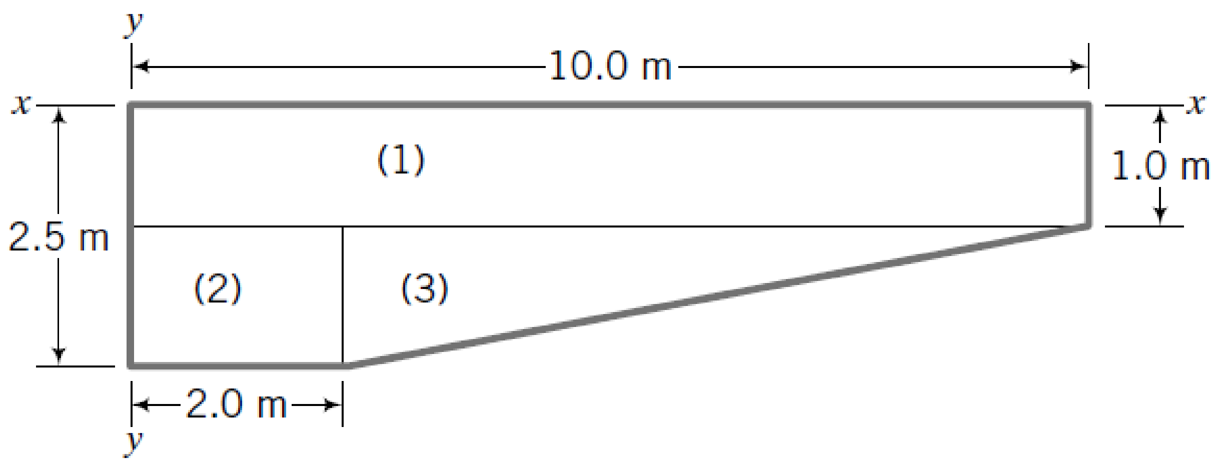
Additional Stress cntd

The procedure for using Newmark's chart is as follows:

1. Set the scale, shown on the chart, equal to the depth at which the increase in vertical stress is required. We call this the depth scale.
2. Identify the point below the loaded area where the stress is required. Let us say this point is A.
3. Plot the loaded area, scaling its plan dimension using the depth scale with point A at the center of the chart.
4. Count the number of segments (Ns) covered by the scaled loaded area. If certain segments are not fully covered, one can estimate what fraction is covered.
5. Calculate the increase in vertical stress as $\Delta\sigma_z = q_s I_N N_s$.

EXERCISE 6.3.7 ***Vertical Stress Increase Due to an Irregular Loaded Area***

The plan of a foundation of uniform thickness for a building is shown in figure. Determine the vertical stress increase at a depth of 4 m below the centroid. The foundation applies a vertical stress of 200 kPa on the soil surface.



4. Stress Bulbs & Contact Pressure

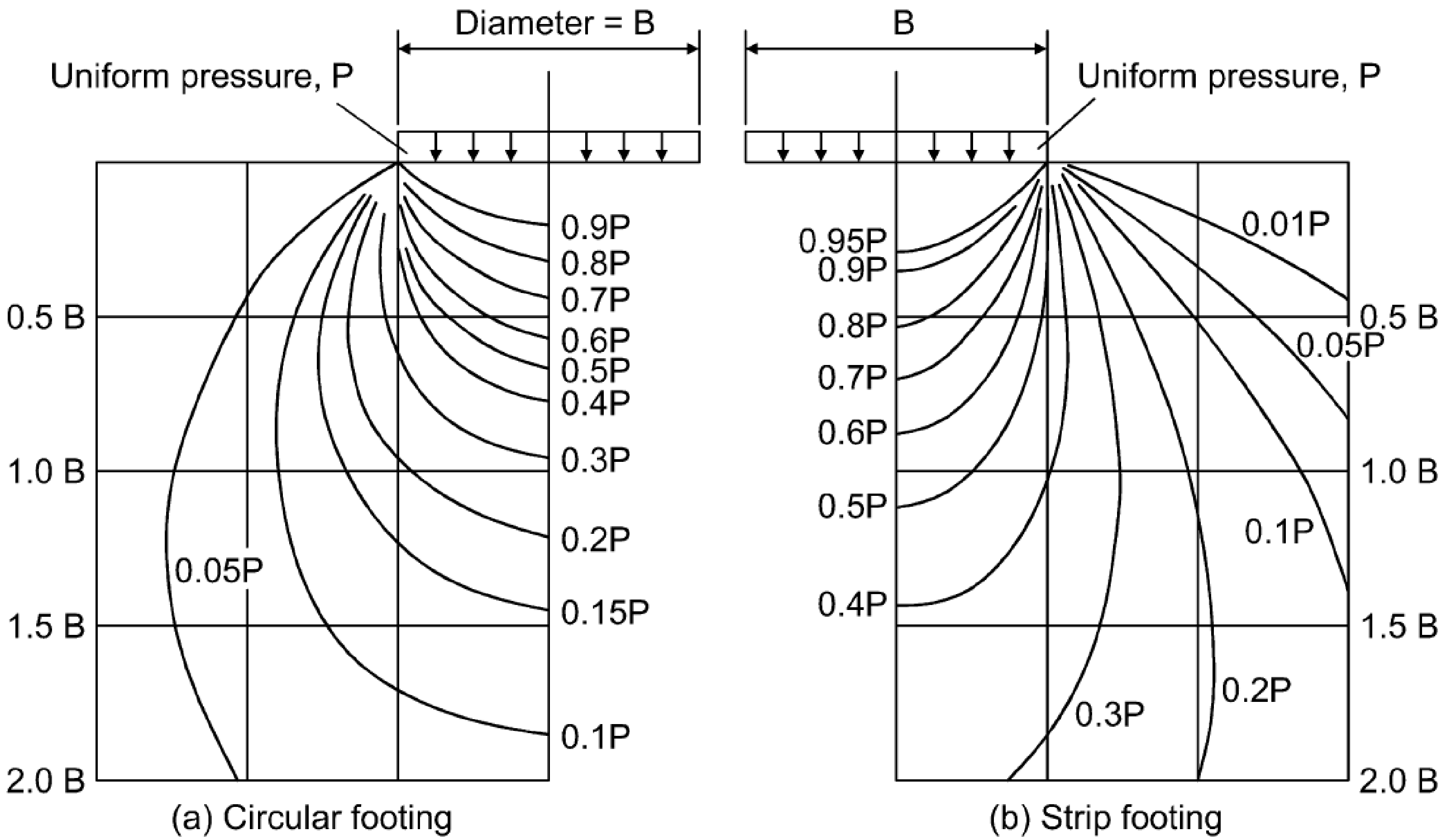
- Stress Bulbs
- Contact Stress

Stress Bulbs & Contact Pressure

Stress bulbs / Bulbs of Pressure / Isobars

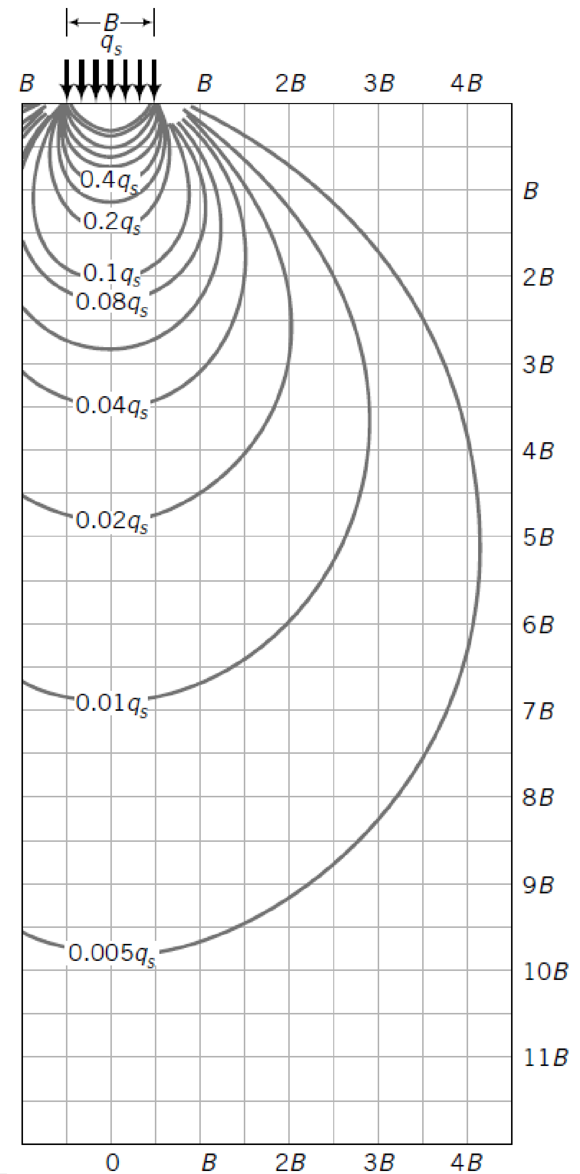
- If points of equal vertical pressure are plotted on a cross-section through the foundation, diagrams found are called stress bulbs.
- constitute another method of determining vertical stresses at points below a foundation that is of regular shape.
- From a bulb of pressure one has some idea of the depth of soil affected by a foundation.
- Significant stress values go down roughly to 2.0 times the width of the foundation,

Stress Bulbs & Contact Pressure



Stress Bulbs & Contact Pressure

- The distribution of vertical stress below a uniformly loaded square foundation is shown.
- The increase in vertical stress is about 10% below a depth of 2*B*; *B* is the diameter of the foundation.
- The vertical stress decreases from the center of the foundation outward, reaching a value of about 10% at a horizontal distance of *B*/2 from the edge at a depth of *B*.

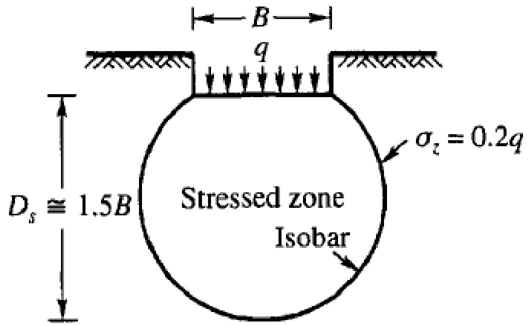


Stress Bulbs & Contact Pressure

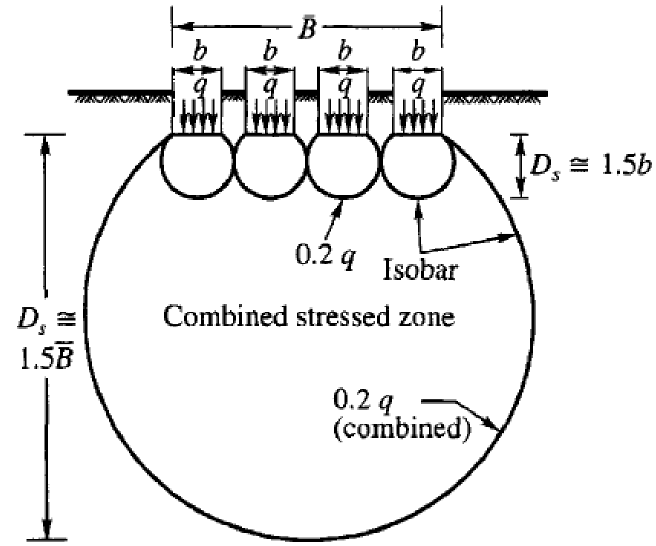
- This chart gives the increase in stress below a square foundation and below a strip foundation for a uniform elastic soil.
- By using this chart, one can get the increase in stress at any location in the soil mass in the vicinity of the foundation.
- It is particularly useful for obtaining the increase in stress at the edge and at the center of the footing because this difference can affect the distortion of the foundation.

Stress Bulbs & Contact Pressure

- For all practical purposes, one can take a stress contour which represents 20% of the contact pressure q (i.e equal to 0.2q) the depth of which is defined as ***significant depth D_s***.
- An isobar constructed in such manner is assumed to enclose a soil mass which is responsible for the settlement of the structure.



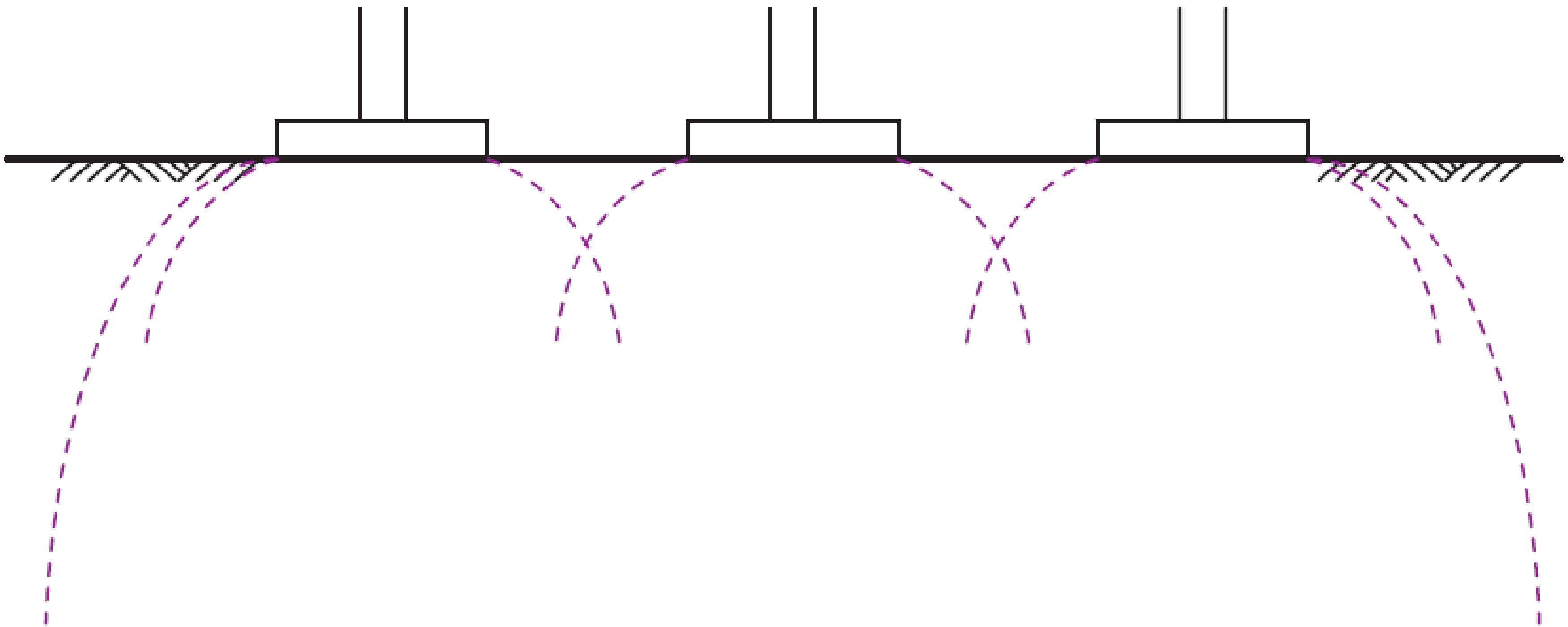
(a) Significant depth of stressed zone for single footing



(b) Effect of closely placed footings

Stress Bulbs & Contact Pressure

- Small foundations will act together as one large foundation unless the foundations are at a greater distance apart (c/c) than five times their width, which is not usual.



Stress Bulbs & Contact Pressure

EXERCISE 6.4.1 – Stress Bulb for Point Load

A single concentrated load of 1000 kN acts at the ground surface.

Construct an isobar for $\Delta\sigma_z = 40\ kN/m^2$ by making use of the

Boussinesq equation given as

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

Stress Bulbs & Contact Pressure

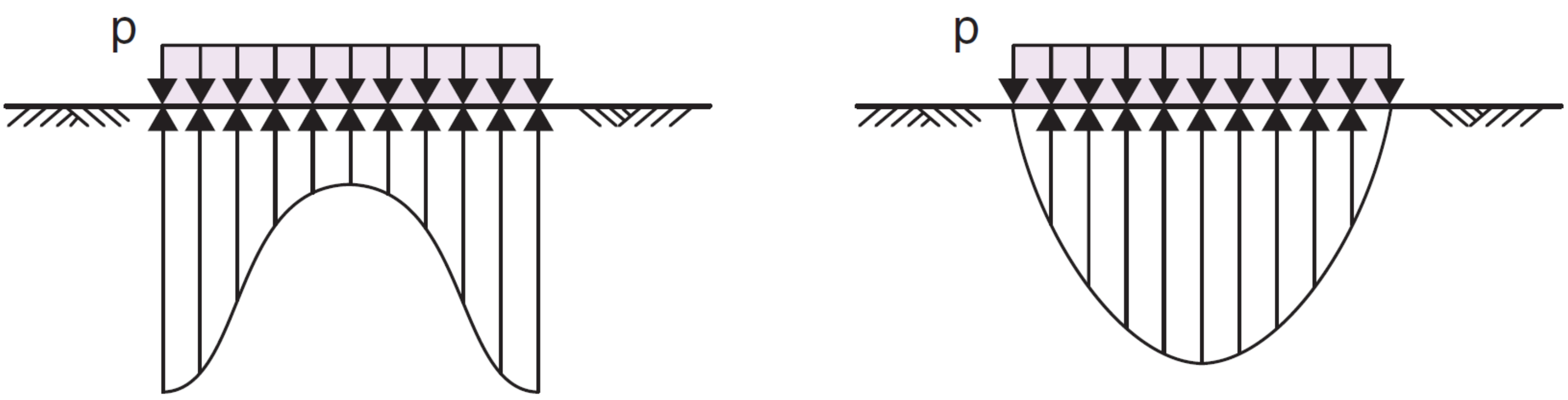
Contact Pressure: is the actual pressure transmitted from the foundation to the soil.

- A uniformly loaded foundation will not necessarily transmit a uniform contact pressure to the soil.
- This is only possible if the foundation is perfectly flexible. The contact pressure distribution of a rigid foundation depends upon the type of soil beneath it.
- On the assumption that the vertical settlement of the foundation is uniform, it is found from the elastic theory that the stress intensity at the edges of a foundation on cohesive soils is infinite.
- For a rigid surface footing sitting on sand stress at the edges is zero and the pressure distribution is roughly parabolic.

Stress Bulbs & Contact Pressure

- The relative rigidity of the foundation (a system that transfers the load to the soil) to the soil mass influences the stress distribution within the soil.

Fig. Contact pressure distribution under a rigid foundation loaded with a uniform pressure, p.



(a) Cohesive soil

(b) Cohesionless soil

Stress Bulbs & Contact Pressure

- The elastic solutions presented so far are for flexible loads and do not account for the relative rigidity of the soil foundation system.
- If the foundation is rigid, the stress increases are generally lower (15% to 30% less for clays and 20% to 30% less for sands) than those calculated from the elastic solutions.
- Traditionally, the stress increases from the elastic solutions are not adjusted because soil behavior is nonlinear and it is better to err on the conservative side.

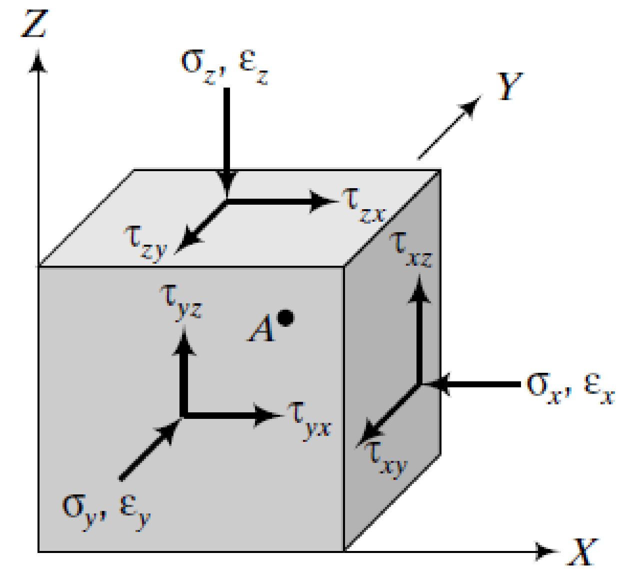
Stress Bulbs & Contact Pressure

- Under flexible foundation the pressure is uniform at the foundation level.
- Although foundations can be rigid, using the flexible solution in all cases is recommended for the following reason.
- Full-scale measurements indicate that the initially uneven pressure distribution under relatively rigid foundations redistributes itself and becomes close to the constant pressure under a flexible footing.
- This is attributed to the inability of the soil to sustain a large stress gradient for a long period of time.

5. Stress States & Stress Paths

- General State of Stress
- Conditions of Stress and Strain
- Stress and Strain States
- Stress & Strain Invariants
- Stress Paths

Stress States & Stress Paths



General State of Stress

Stresses and strains for a linear, isotropic, elastic soil are related through Hooke's law.

For a general state of stress , Hooke's law (elastic equation or elastic stress–strain constitutive equation) is given as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu) \end{bmatrix}\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}$$

where *E* is the elastic (or Young's) modulus and ν is Poisson's ratio.

Stress States & Stress Paths cntd

From the elastic stress–strain constitutive equation

$$\gamma_{zx} = \frac{2(1+v)}{E}\tau_{zx} = \frac{\tau_{zx}}{G}$$

where $G = \frac{E}{2(1+v)}$ is shear modulus.

$E, G \text{ and } v \text{ are called the elastic parameters}$.

Only two of these parameters— either $E \text{ or } G \text{ and } v$ — are required to solve problems dealing with isotropic, elastic materials.

Poisson’s ratio for soils is not easy to determine, and a direct way to obtain G is to subject the material to shearing forces.

Stress States & Stress Paths cntd

For nonlinear elastic materials, the tangent modulus or the secant modulus is used in the elastic equation and the calculations are done incrementally for small increments of stress.

The elastic and shear moduli for soils depend on the stress history, the direction of loading, and the magnitude of the applied strains.

Soil type	Description	E^a (MPa)	G^a (MPa)
Clay	Soft	1–15	0.5–5
	Medium	15–30	5–15
	Stiff	30–100	15–40
Sand	Loose	10–20	5–10
	Medium	20–40	10–15
	Dense	40–80	15–35

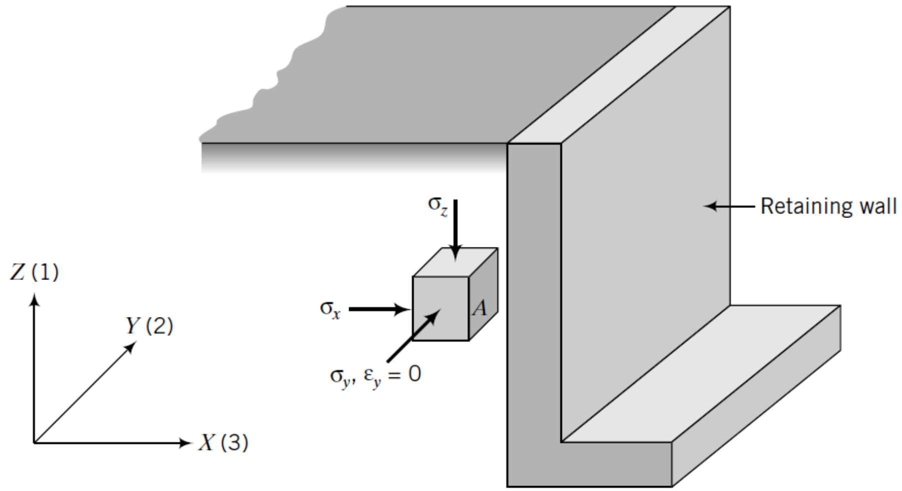
[a]These are average secant elastic moduli for drained condition

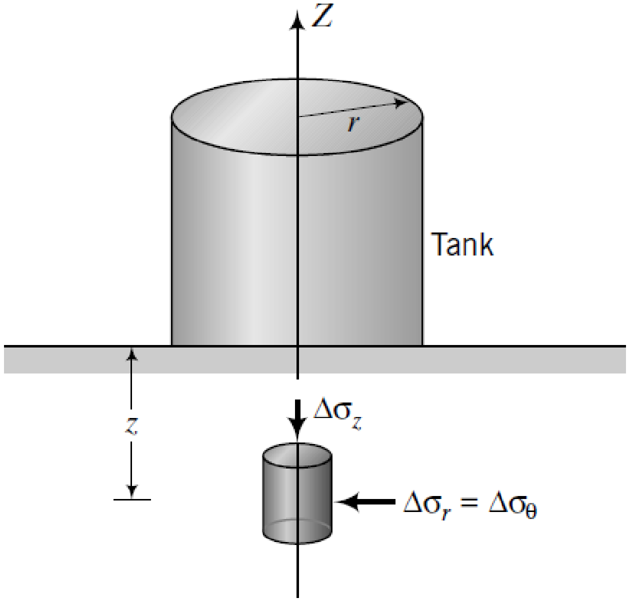
Stress States & Stress Paths cntd

Conditions of Stress and Strain

There are two conditions of stresses and strains that are common in geotechnical engineering.

- Plane strain condition : the strain in one direction is zero.
- Axial symmetry (the axisymmetric condition) : where two stresses are equal.





Stress States & Stress Paths cntd

Plane Strain Condition

Consider an element of soil, A, behind a retaining wall.

Because the displacement that is likely to occur in the Y direction (Δy) is small compared with the length in this direction, the strain tends to zero; that is, $\varepsilon_y = \Delta y / y \cong 0$.

One can then assume that soil element A is under a plane strain condition.

Since we are considering principal stresses, we will map the X, Y, and Z directions as 3, 2, and 1 directions.

In the case of the retaining wall, the Y direction (2 direction) is the zero strain direction, and therefore $\varepsilon_2 = 0$

Stress States & Stress Paths cntd

Plane Strain Condition

Hooke's law for a plane strain condition is

$$\varepsilon_1 = \frac{1+v}{E}[(1-v)\sigma_1 - v\sigma_3]$$

$$\varepsilon_3 = \frac{1+v}{E}[(1-v)\sigma_3 - v\sigma_1]$$

$$\sigma_2 = v(\sigma_1 + \sigma_3)$$

EXERCISE 6.5.1 Plane Strain Condition

A retaining wall moves outward, causing a lateral strain of 0.1% and a vertical strain of 0.05% on a soil element located 3 m below ground level.

Assuming the soil is a linear, isotropic, elastic material with E=5000 kPa and $\upsilon = 0.3$, calculate the increase in stresses imposed.

If the retaining wall is 6 m high and the stresses you calculate are the average stresses, determine the lateral force increase per unit length of wall.

Stress States & Stress Paths cntd

Axisymmetric Condition

Consider a water tank or an oil tank founded on a soil mass.

The radial stresses (σ_r) and circumferential stresses (σ_θ) on a cylindrical element of soil directly under the center of the tank are equal because of axial symmetry.

The oil tank will apply a uniform vertical (axial) stress at the soil surface and the soil element will be subjected to an increase in axial stress, $\Delta\sigma_z = \Delta\sigma_1$, and an increase in radial stress, $\Delta\sigma_r = \Delta\sigma_\theta = \Delta\sigma_3$.

Will a soil element under the edge of the tank be under an axisymmetric condition? The answer is no, since the stresses at the edge of the tank are all different; there is no symmetry.

Stress States & Stress Paths cntd

Axisymmetric Condition

Hooke's law for the axisymmetric condition is

$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - 2v\sigma_3]$$

$$\varepsilon_3 = \frac{1}{E}[(1 - v)\sigma_3 - v\sigma_1]$$

NB. Plane strain and axisymmetric stress conditions are ideal conditions. In reality, the stress conditions imposed on soils are much more complicated.

EXERCISE 6.5.2 Axisymmetric Condition

An oil tank is founded on a layer of medium sand 5 m thick underlain by a deep deposit of dense sand. The geotechnical engineer assumed, based on experience, that the settlement of the tank would occur from settlement in the medium sand.

The vertical and lateral stresses at the middle of the medium sand directly under the center of the tank are 50 kPa and 20 kPa, respectively.

The values of E and v are 20 MPa and 0.3, respectively.

Assuming a linear, isotropic, elastic material behavior, calculate the strains imposed in the medium sand.

Stress States & Stress Paths cntd

Anisotropic, Elastic States

Anisotropic materials have different elastic parameters in different directions.

Anisotropy in soils results from essentially two causes.

1. The manner in which the soil is deposited. This is called structural anisotropy and it is the result of the kind of soil fabric that is formed during deposition. The soil fabric produced is related to the history of the environment in which the soil is formed. A special form of structural anisotropy occurs when the horizontal plane is a plane of isotropy. And it is called transverse anisotropy.

2. The difference in stresses in the different directions. This is known as stress-induced anisotropy.

Stress States & Stress Paths cntd

Transverse anisotropy, also called cross anisotropy, is the most prevalent type of anisotropy in soils.

- If we were to load the soil in the vertical direction (Z direction) and repeat the same loading in the horizontal direction, say, the X direction, the soil would respond differently; its stress–strain characteristics and strength would be different in these directions.
- However, if we were to load the soil in the Y direction, the soil's response would be similar to the response obtained in the X direction.

Stress States & Stress Paths cntd

Anisotropic, Elastic States

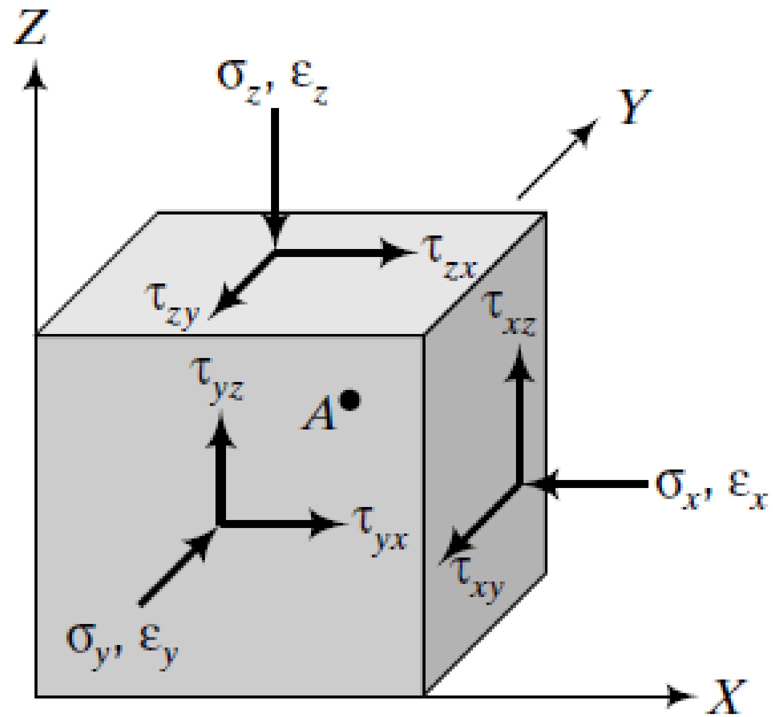
- The implication is that a soil mass will, in general, respond differently depending on the direction of the load.
- For transverse anisotropy, the elastic parameters are the same in the lateral directions (X and Y directions) but are different from the vertical direction.
- In the laboratory, the direction of loading of soil samples taken from the field is invariably vertical.
- Consequently, we cannot determine the five desired elastic parameters from conventional laboratory tests.

Stress States & Stress Paths cntd

Stress and Strain States

Stress state: (at a point) is the set of stress vectors corresponding to all planes passing through that point.

Strain state: (at a point)



Mohr's Circle for Stress States

Sign convention:

-Compressive stresses are positive for soils.

-Counterclockwise shear is positive and $\sigma_z > \sigma_x$.

The two coordinates of the circle are (σ_z, τ_{zx}) and (σ_x, τ_{xz}).

For equilibrium, $\tau_{xz} = -\tau_{zx}$; these are called complementary shear stresses and are orthogonal to each other.

Stress States & Stress Paths cntd

Mohr's Circle for Stress States

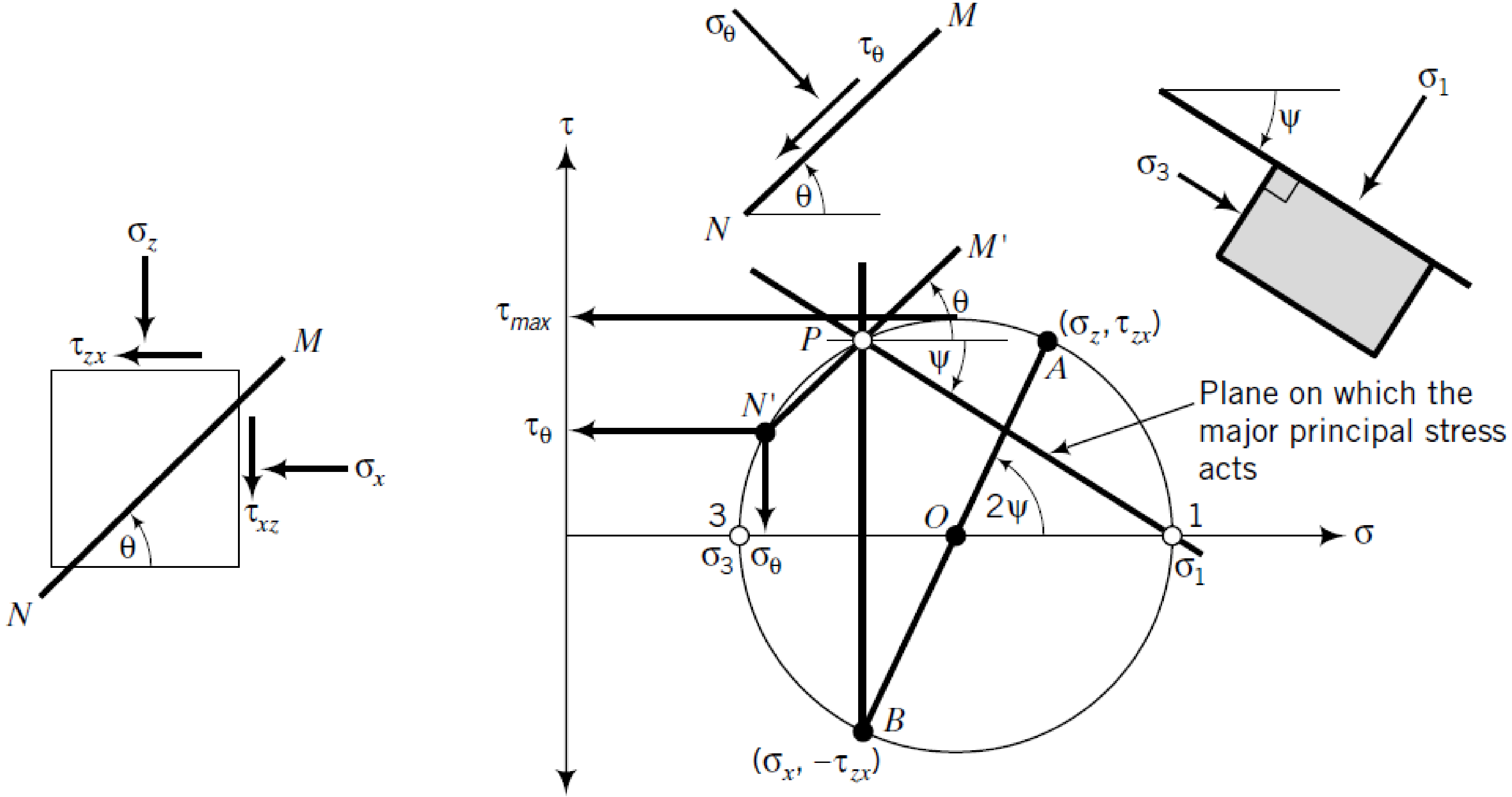
Plot these two coordinates on a graph of shear stress (ordinate) and normal stress (abscissa) .

Draw a circle with AB as the diameter. The circle crosses the normal stress axis at 1 and 3.

The stresses at these points are the major principal stress, σ_1, and the minor principal stress, σ_3.

Stress States & Stress Paths cntd

Mohr's Circle for Stress States



Stress States & Stress Paths cntd

Mohr's Circle for Stress States

$$\sigma_1 = \frac{\sigma_z + \sigma_x}{2} + \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{zx}^2}$$

$$\sigma_3 = \frac{\sigma_z + \sigma_x}{2} - \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{zx}^2}$$

The angle between the major principal stress plane and the horizontal plane (ψ) is

$$\tan \psi = \frac{\tau_{zx}}{\sigma_1 - \sigma_x}$$

Stress States & Stress Paths cntd

The stresses on a plane oriented at an angle $\boldsymbol{\theta}$ from the major principal stress plane are

$$\boldsymbol{\sigma_\theta = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\theta}$$

$$\boldsymbol{\tau_\theta = \frac{\sigma_1 - \sigma_3}{2}\sin 2\theta}$$

The stresses on a plane oriented at an angle $\boldsymbol{\theta}$ from the horizontal plane are

$$\boldsymbol{\sigma_\theta = \frac{\sigma_z + \sigma_x}{2} + \frac{\sigma_z - \sigma_x}{2}\cos 2\theta + \tau_{zx}\sin 2\theta}$$

$$\boldsymbol{\tau_\theta = \tau_{zx}\cos 2\theta - \frac{\sigma_z - \sigma_x}{2}\sin 2\theta}$$

Stress States & Stress Paths cntd

The maximum (principal) shear stress is at the top of the circle with magnitude

$$\boldsymbol{\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}}$$

NB. The stress $\boldsymbol{\sigma_z}$ acts on the horizontal plane and the stress $\boldsymbol{\sigma_x}$ acts on the vertical plane for our case.

If one draws these planes in Mohr's circle, they intersect at a point, P, called the pole of the stress circle.

It is a special point because any line passing through the pole will intersect Mohr's circle at a point that represents the stresses on a plane parallel to the line.

Stress States & Stress Paths cntd

Mohr's Circle for Strain States

Major principal strain:

$$\varepsilon_1 = \frac{\varepsilon_z + \varepsilon_x}{2} + \sqrt{\left(\frac{\varepsilon_z - \varepsilon_x}{2}\right)^2 + \left(\frac{\gamma_{zx}}{2}\right)^2}$$

Major principal strain:

$$\varepsilon_3 = \frac{\varepsilon_z + \varepsilon_x}{2} - \sqrt{\left(\frac{\varepsilon_z - \varepsilon_x}{2}\right)^2 + \left(\frac{\gamma_{zx}}{2}\right)^2}$$

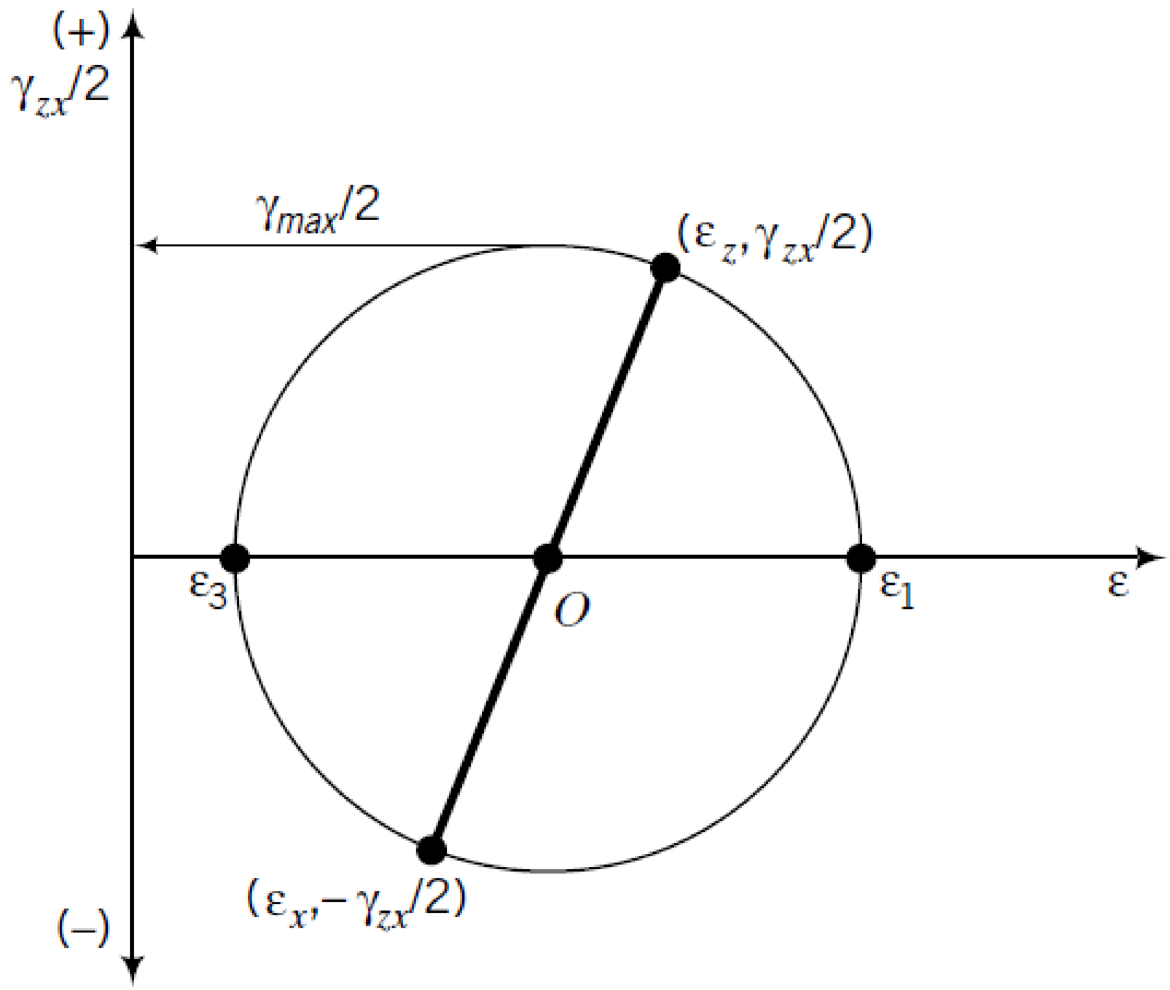
where γ_{zx} is called the engineering shear strain or simple shear strain.

$$\gamma_{max} = \varepsilon_1 - \varepsilon_3$$

NB. In soils, strains can be compressive or tensile. There is no absolute reference strain.

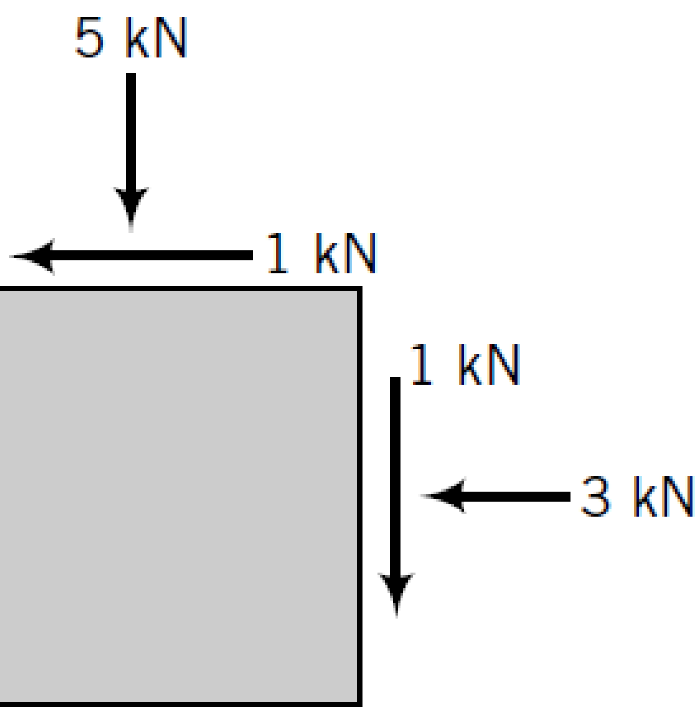
Stress States & Stress Paths cntd

Mohr's Circle for Strain States



EXERCISE 6.5.3 Mohr's Circle for Stress States

A sample of soil (0.1 m X 0.1 m) is subjected to the forces shown in the figure.



Determine

(a) $\boldsymbol{\sigma_1}$, $\boldsymbol{\sigma_3}$, $\boldsymbol{\psi}$

(b) the maximum shear stress; and

(c) the stresses on a plane oriented at 30° counterclockwise from the major principal stress plane.

Stress States & Stress Paths cntd

- The stresses and strains discussed in previous slides are all dependent on the axis system chosen.
- We have arbitrarily chosen the Cartesian coordinate and the cylindrical coordinate systems. We could, however, define a set of stresses and strains that are independent of the axis system.
- Such a system will allow us to use generalized stress and strain parameters to analyze and interpret soil behavior.
- In particular, we will be able to represent a three-dimensional system of stresses and strains by a two-dimensional system.

Stress States & Stress Paths cntd

Definitions

Mean stress, **p,** is the average stress on a body or the average of the orthogonal stresses in three dimensions.

Deviatoric stress, **q,** is the shear or distortional stress or stress difference on a body.

Stress path is a graphical representation of the locus of stresses on a body.

Stress States & Stress Paths cntd

Stress and Strain Invariants

-measures that are independent of the axis system.

Stress invariants provide measures of

- mean stress
- deviatoric or distortional or shear stress.

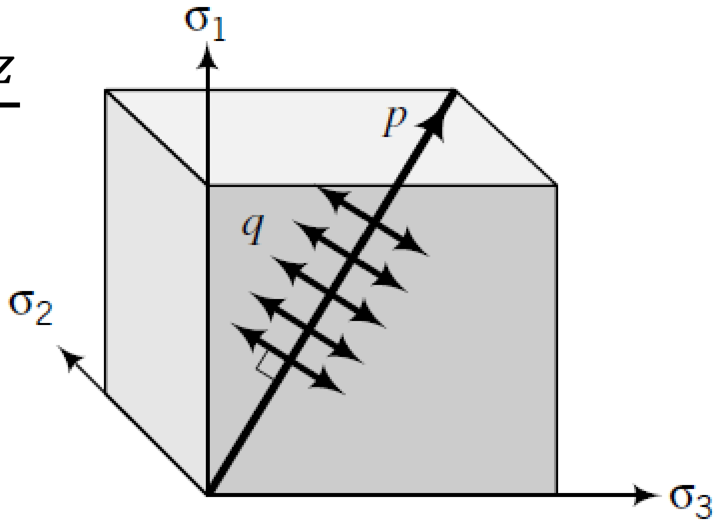
Strain invariants provide measures of

- volumetric strains
- deviatoric or distortional or shear strains.

Stress States & Stress Paths cntd

Mean Stress – the space diagonal on a graph with orthogonal principal stress axes σ_1 , σ_2 , σ_3

$$p = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{\sigma_x + \sigma_y + \sigma_z}{3}$$



Deviatoric or Shear Stress

- line normal to the mean stress on a graph with orthogonal principal stress axes σ_1 , σ_2 , σ_3

$$q = \frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$$

Stress States & Stress Paths cntd

Volumetric Strain

$$\varepsilon_p = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Deviatoric or Distortional or Shear Strain

$$\varepsilon_q = \frac{\sqrt{2}}{3}\left[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2\right]^{1/2}$$

Stress States & Stress Paths cntd

Axisymmetric Condition, $\sigma_2' = \sigma_3'$ or $\sigma_2 = \sigma_3$; $\varepsilon_2 = \varepsilon_3$

$$p' = \frac{\sigma_1' + 2\sigma_3'}{3} \; and \; p = \frac{\sigma_1 + 2\sigma_3}{3}$$

$$p' = p - u$$

$$q = \sigma_1 - \sigma_3$$

$$q' = \sigma_1' - \sigma_3' = (\sigma_1 - \Delta u) - (\sigma_3 - \Delta u) = \sigma_1 - \sigma_3$$

$\therefore q = q'; shear\ is\ unaffected\ by\ porewater\ pressure.$

$$\varepsilon_p = \varepsilon_1 + 2\varepsilon_3 \qquad \qquad \varepsilon_q = \frac{2}{3}(\varepsilon_1 - \varepsilon_3)$$

Stress States & Stress Paths cntd

Plane Strain Condition, $\varepsilon_2 = 0$

$$p' = \frac{\sigma_1' + \sigma_1' + \sigma_3'}{3} \; and \; p = \frac{\sigma_1 + \sigma_1 + \sigma_3}{3}$$

$$p' = p - u$$

$$q' = q = \frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$$

$$\varepsilon_p = \varepsilon_1 + \varepsilon_3 \qquad \varepsilon_q = \frac{2}{3}(\varepsilon_1^2 + \varepsilon_3^2 + \varepsilon_1\varepsilon_3)^{1/2}$$

Stress States & Stress Paths cntd

Hooke's Law using Stress Invariants

The stress and strain invariants for an elastic material are related as:

$$\varepsilon_p^e = \frac{1}{K'} p' \; where \; K' = \frac{p'}{\varepsilon_p^e} = \frac{E'}{3(1-2v')}$$

is the effective bulk modulus and the superscript *e* denotes elastic.

$$\varepsilon_p^e = \frac{1}{3G} q \; where \; G = G' = \frac{E'}{2(1+2v')} \; is \; called \; the \; shear \; modulus.$$

Generalized Poisson's ratio

$$v' = \frac{3K' - 2G}{2G + 6K'}$$

EXERCISE 6.5.4 Stress & Strain Invariants

A cylindrical sample of soil 50 mm in diameter and 100 mm long is subjected to an axial effective principal stress of 400 kPa and a radial effective principal stress of 100 kPa. The axial and radial displacements are 0.5 mm and 20.04 mm, respectively.

Assuming the soil is an isotropic, elastic material, calculate

(a) the mean and deviatoric stresses,

(b) The volumetric and shear (distortional) strains, and

(c) the shear, bulk, and elastic moduli.

Stress States & Stress Paths cntd

Stress Paths : curves or graphs illustrating the changing stress states during loading or unloading.

- used to show results of laboratory tests, results from numerical simulations or results from measurements in the field.

- Stress paths are presented in a plot showing the relationship between stress parameters and provide a convenient way to allow a geotechnical engineer to study the changes in stresses in a soil caused by loading conditions.

Stress States & Stress Paths cntd

Plotting Stress Paths using Stress Invariants

Consider a cylindrical soil sample.

Apply equal increments of axial and radial stresses ($\Delta\sigma_z = \Delta\sigma_r = \Delta\sigma$) to an initially stress-free sample.

Since no shearing stresses is being applied on the horizontal and vertical boundaries, the axial and radial stresses are principal stresses; that is, $\Delta\sigma_z = \Delta\sigma_1$ and $\Delta\sigma_r = \Delta\sigma_3$

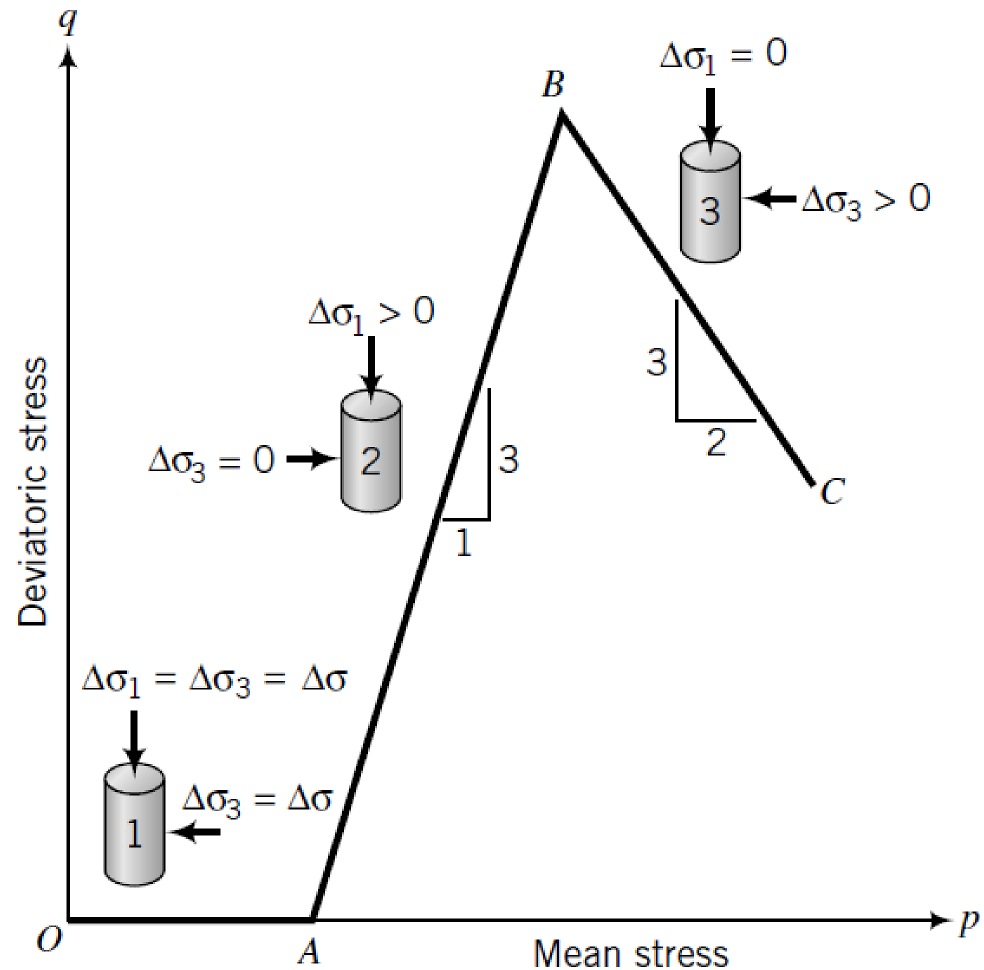
This kind of loading condition is called isotropic compression; that is, the stresses in all directions are equal ($\Delta\sigma_1 = \Delta\sigma_2 = \Delta\sigma_3$).

Stress States & Stress Paths cntd

OA → stress path for isotropic compression.

$$\Delta p_1 = \frac{\Delta\sigma_1 + 2\Delta\sigma_3}{3} = \frac{\Delta\sigma_1 + 2\Delta\sigma_1}{3} = \Delta\sigma_1$$

$$\Delta p_1 = \Delta\sigma_1 - \Delta\sigma_3 = \Delta\sigma_1 - \Delta\sigma_1 = 0$$



AB → $\Delta\sigma_3 = 0$, but continue

to increase σ_1, that is,
$\Delta\sigma_1 > 0$

BC → keep σ_1 constant

$(\Delta\sigma_3 = 0)$ but increase σ_3
$\Delta\sigma_3 > 0$

NB. note that *q* decreases

but *p* increases for stress path BC

Stress States & Stress Paths cntd

Total Stress Path and Effective Stress Path

The soil solids and the porewater must carry the applied increase in stresses in a saturated soil.

If the soil porewater is allowed to drain from the soil sample, the increase in stress carried by the porewater, called excess porewater pressure (Δu), will continuously decrease to zero and the soil solids will have to support all of the increase in applied stresses.

Stress States & Stress Paths cntd

Stress Path OA

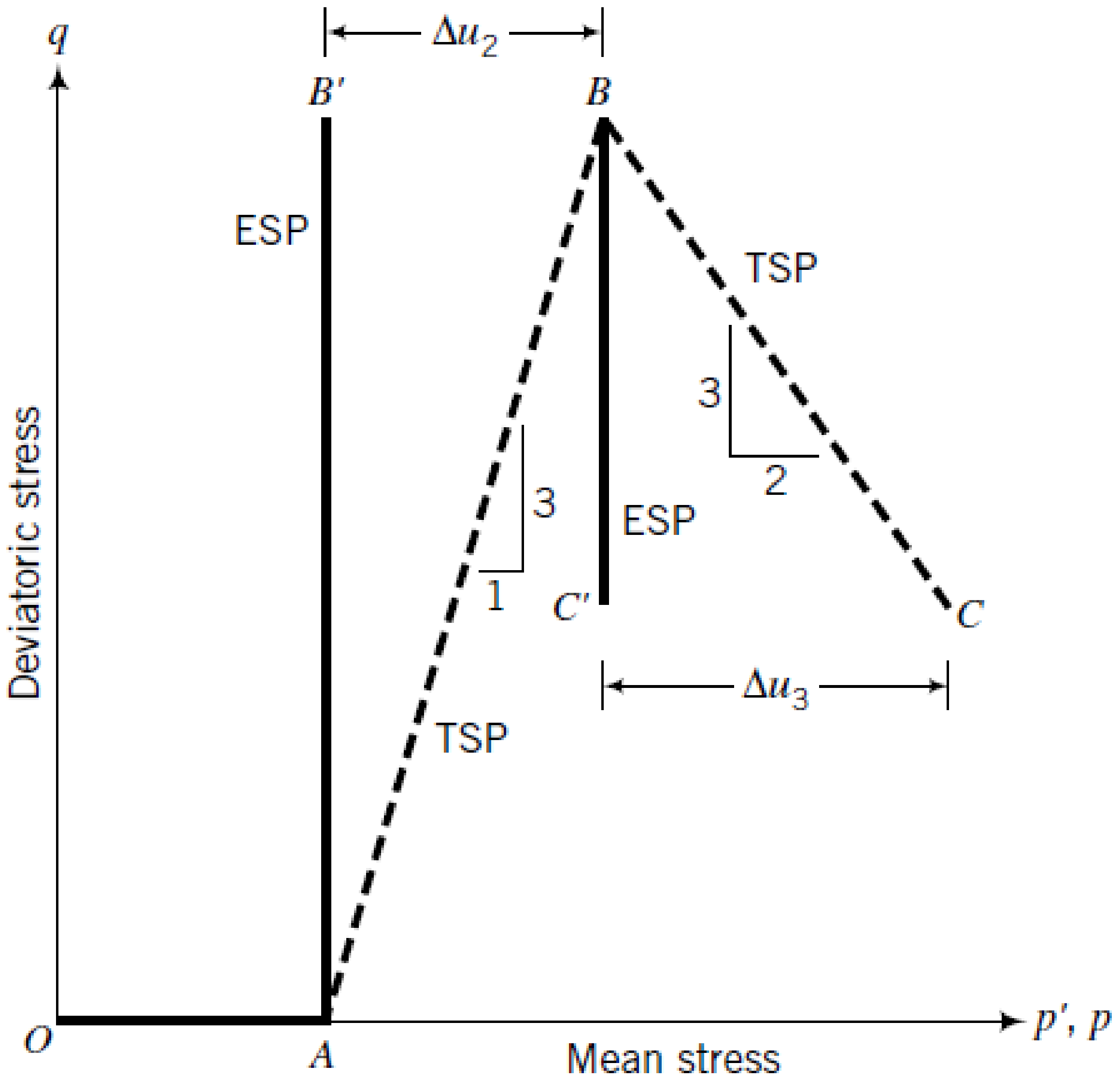
When the excess porewater is allowed to drain, it is called the drained condition in geotechnical engineering.

Since the excess porewater pressure (Δu_1) dissipates as water drains from the soil, the mean effective stress at the end of each increment is equal to the mean total stress; that is,

$$\Delta p_1' = \Delta p_1 - \Delta u_1 = \Delta p_1 - 0 = \Delta p$$

NB. The effective stress path (ESP) and the total stress path (TSP) are the same.

Stress States & Stress Paths cntd



Stress States & Stress Paths cntd

Stress Paths AB and BC

In geotechnical engineering, the term undrained is used to denote a loading situation in which the excess porewater cannot drain from the soil. The implication is that the volume of our soil sample remains constant.

If our soil were an isotropic, elastic material, then

$$\Delta\varepsilon_p^e = \frac{\Delta p'}{K'} = 0 \qquad \Leftrightarrow \qquad \Delta p' = 0 \; or K' = \infty$$

There is no reason why K' should be ∞ . The act of preventing the drainage of the excess porewater cannott change the (effective) bulk modulus of the soil solids.

The only tenable solution is $\Delta p' = 0$

Stress States & Stress Paths cntd

In terms of total stress, $\Delta\varepsilon_p^e = \frac{\Delta p}{K'} = 0$

where $K = E_u/3(1 - v_u)$ and the subscript u denotes undrained condition.

In this case, Δp cannot be zero since this is the change in mean total stress from the applied loading.

Therefore, the only tenable solution is $K = K_u = \infty$, which leads to $v_u = 0.5$

The implications of these equations for a linear, isotropic, elastic soil under undrained conditions are:

1. The change in mean effective stress is zero and, consequently, the effective stress path is vertical.

2. The undrained bulk modulus is ∞ and $v_u = 0.5$

Stress States & Stress Paths cntd

The deviatoric stress is unaffected by porewater pressure changes.

$$G = G_u = \frac{E_u}{2(1 + 2v_u)}$$

Since $G = G_u = G'$, then

$$\frac{E_u}{2(1 + 2v_u)} = \frac{E'}{2(1 + v')}$$

And, by substituting $v_u = 0.5$

$$E_u = \frac{1.5E'}{(1 + v')}$$

NB. For many soils, $v' \approx \frac{1}{3}$ and, as a result, $E_u \approx 1.1E'$; that is, the undrained elastic modulus is about 10% greater than the effective elastic modulus.

Stress States & Stress Paths cntd

Stress Paths using 2D Stress Parameters

For two-dimensional stresses, we can use an alternative stress path presentation based on Mohr's circle.

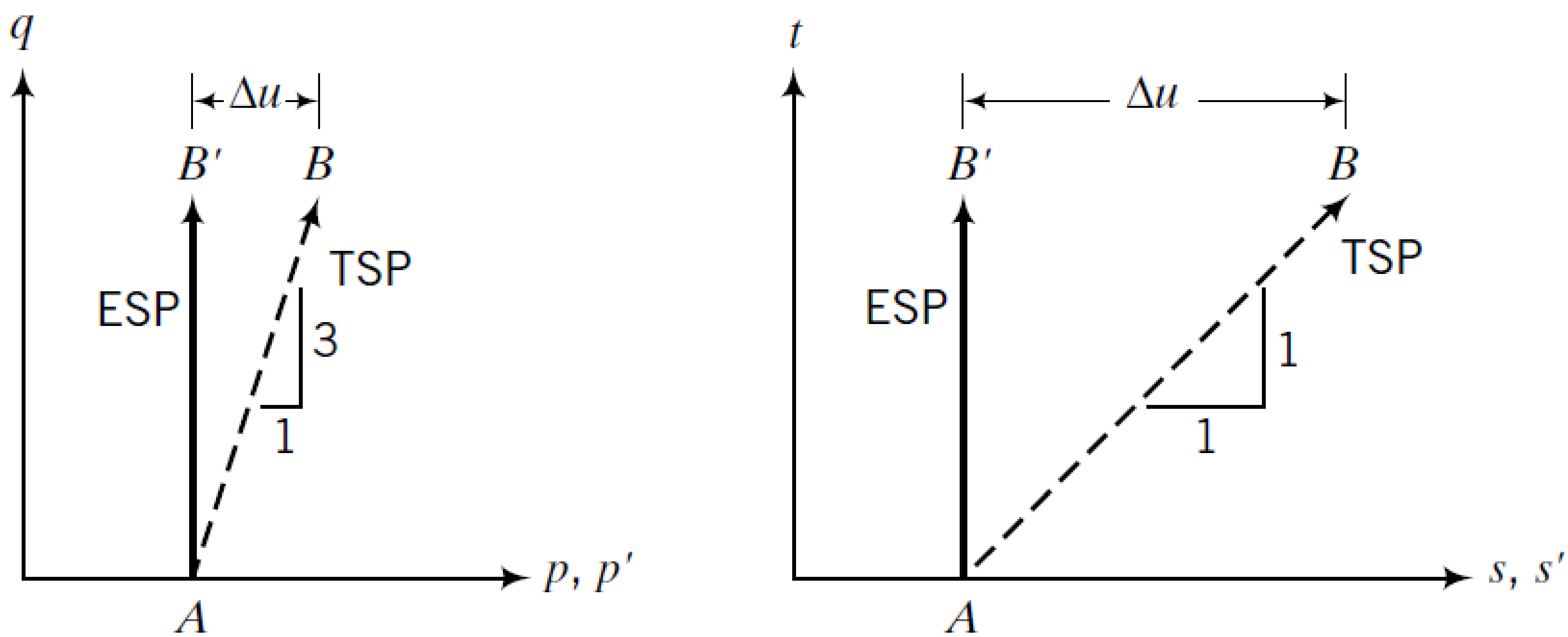
$$t = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_1' - \sigma_3'}{2}$$

$$s = \frac{\sigma_1 + \sigma_3}{2};\ s' = \frac{\sigma_1' + \sigma_3'}{2}$$

where t and s are the radius and center of Mohr's circle, respectively, and represent the maximum shear stress and mean stress, respectively.

Stress States & Stress Paths cntd

Stress Paths using 2D Stress Parameters



Stress States & Stress Paths cntd

Summary of Procedures to Plot Stress Paths

1. Determine the loading conditions drained or undrained, or both.

2. Calculate the initial loading values of $p'_o, p_o\ and\ q_o$.

3. Set up a graph of p'(and *p*, if you are going to also plot the total stress path) as the abscissa and *q* as the ordinate. Plot the initial values of (p'_o, q_o) and (p_o, q_o).

4. Determine the increase in stresses $\Delta\sigma_1, \Delta\sigma_2\ and \Delta\sigma_3$. These stresses can be negative.

5. Calculate the increase in stress invariants $\Delta p', \Delta p\ and\ \Delta q$. These stress invariants can be negative.

Stress States & Stress Paths cntd

6. Calculate the current stress invariants as $p' = p'_o + \Delta p', p = p_o + \Delta p \; and \; q = q_o + \Delta q$. The current value of p' cannot be negative, but q can be negative.

7. Plot the current stress invariants (p', q) and (p, q).

8. Connect the points identifying effective stresses, and do the same for total stresses.

9. Repeat items 4 to 8 for the next loading condition.

10. The excess porewater pressure at a desired level of deviatoric stress is the mean stress difference between the total stress path and the effective stress path.

Stress States & Stress Paths cntd

EXERCISE 6.5.5 ***Stress Paths Due to Axisymmetric Loading***

Two cylindrical specimens of a soil, A and B, were loaded as follows. Both specimens were isotropically loaded by a stress of 200 kPa under drained conditions. Subsequently, the radial stress applied on specimen A was held constant and the axial stress was incrementally increased to 440 kPa under undrained conditions. The axial stress on specimen B was held constant and the radial stress incrementally reduced to 50 kPa under drained conditions.

Plot the total and effective stress paths for each specimen, assuming the soil is a linear, isotropic, elastic material.

Calculate the maximum excess porewater pressure in specimen A.

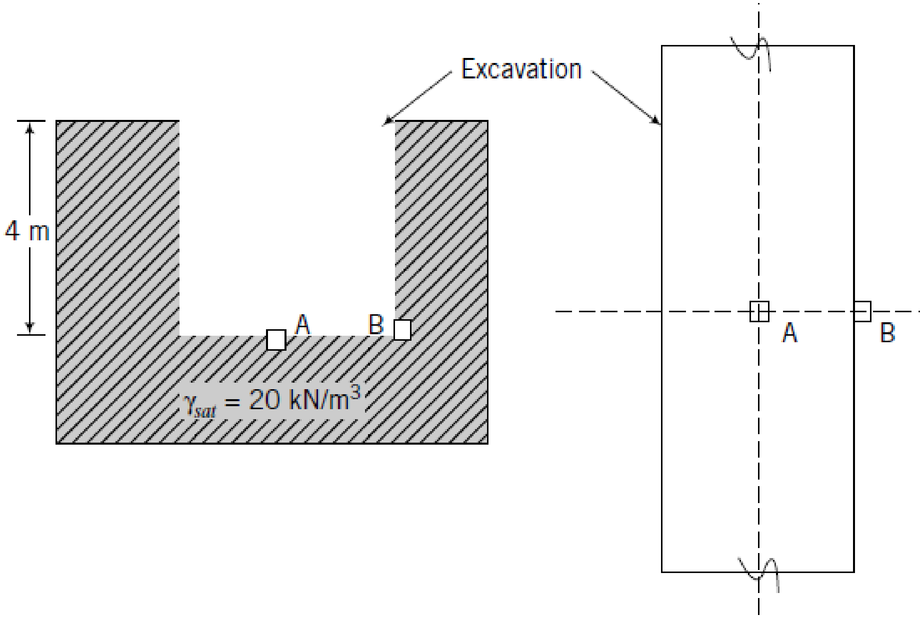
Stress States & Stress Paths cntd

EXERCISE 6.5.6 ***Stress Paths in (p, q) and (s, t) Spaces***

A long excavation is required in a stiff saturated soil for the construction of a building. Consider two soil elements. One, element A, is directly at the bottom of the excavation along the center line and the other, element B, is at the open face.

(a) Plot the stress paths in (p, q) & (s, t) spaces for elements A & B.

(b) If the soil is an isotropic, linear elastic material, predict the excess porewater pressures.



አመሰግናለሁ!

Galatoma!