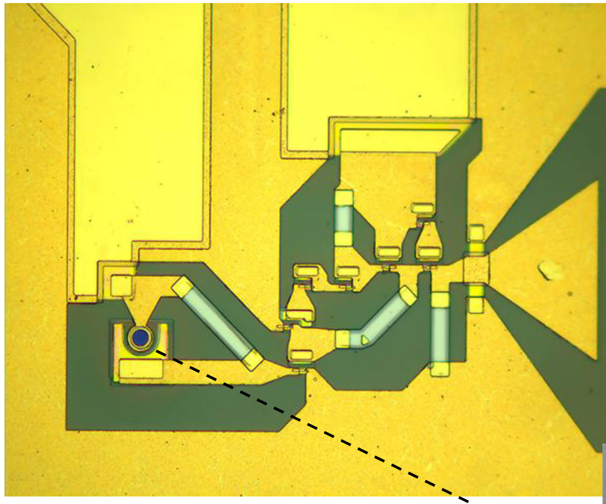
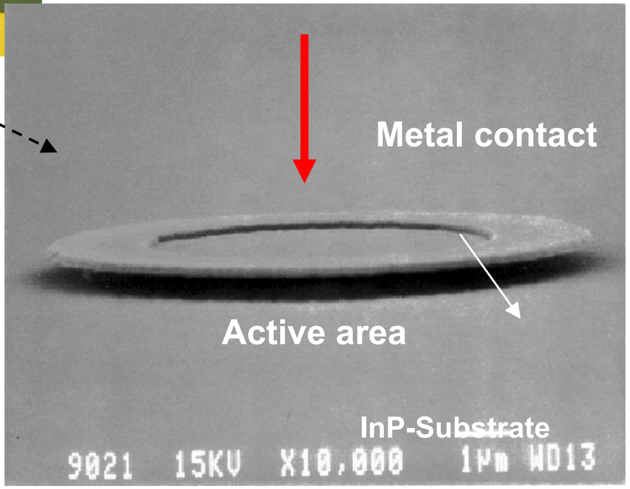
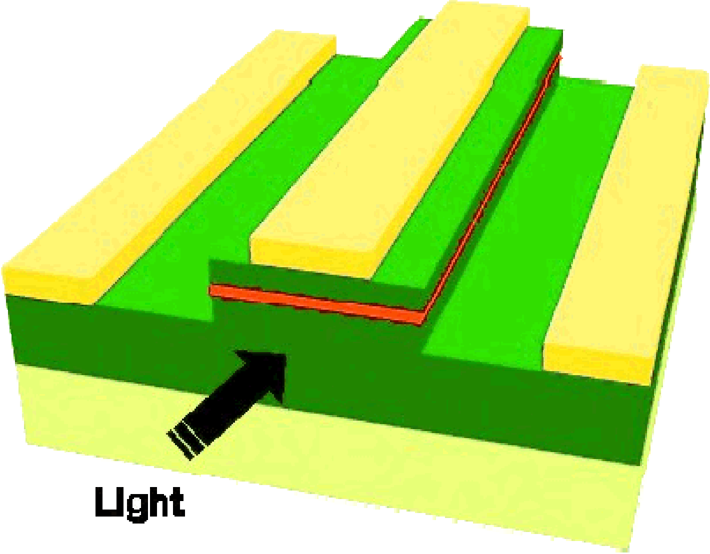
# 7 Optical Detectors and Receivers Circuits



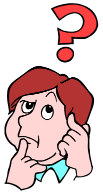
53 GHz InP/InGaAs-HBT Photoreceiver: Integrated Photodiode and Transimpedance-amplifier



Planar 30 GHz InP/InGaAs PD (integrated)



Schematic Traveling wave photodetector with side-illumination into a ridge-waveguide (bandwidth up to ~300 GHz with high efficiency)



## Goals of the chapter:

- Analysis of basic the process of conversion of light, resp. photons into electrical charge and current
- Mechanism of electron-hole pair generation in semiconductors by photon absorption
- Speed and absorption efficiency trade-offs in semiconductor pn-junction photodiodes
- Noise properties of the photodetection process in PiN-photodiodes
- Overview of Avalanche Photodiodes APD for internal current gain

## Methods for the Solution:

- Absorption and photocurrent generation in the depletion-layers of semiconductors
- Calculation of impulse response in vacuum- and PiN-photodiodes
- Analysis of rate fluctuations of the photocarriers as a model for the intrinsic shot-noise generation for photodetection
- Equivalent circuit for PDs
- Basics of photoreceivers design: PD- and receiver noise optimization

# 7 Optical Detectors and Receivers:

## **Introduction**: Detection principles

Primary goal of **photodetection**: **conversion of an optical signal into an electrical signal** (mainly current, voltage)

**Concept**:

The photons of a light field with the energy quantum $\underline{E = \hbar\omega > E_g}$ are absorbed (destroyed) and transfer the energy to a valence band electron, which makes a transition to the conduction-band generating a mobile a electron-hole pair.

$$\underline{\partial n / \partial t \approx R_{12,abs} = \alpha v_{gr} s_{ph} \sim \alpha v_{gr} \overline{|E|}^2}$$

Because the optical carrier frequency $\omega_{opt}$ is ~200 THz, it is not possible to observe the modulated carrier field in real-time

$E(x,t) = A(x,t)\cos\left(\omega_{opt}t - \beta(\omega)x\right)\Big|_{x=0} = \frac{A(x,t)}{2} e^{j(\omega t - \beta x)}\Big|_{x=0} + cc$ directly by carrier transport effects.

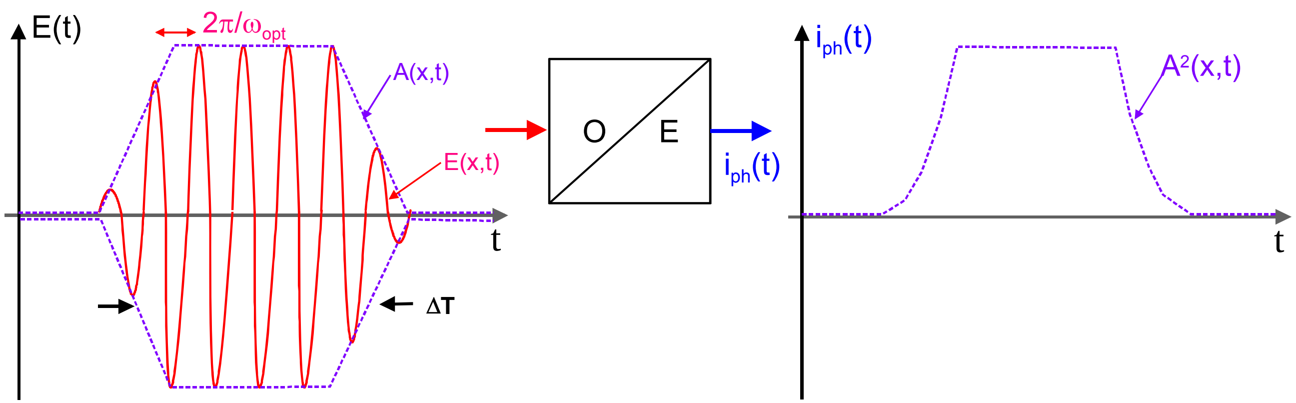
Today photodetectors (bandwidth DC-~300 GHz) measure the only **intensity-envelop** (averaged over many optical cycles).

PDs to detect the **average intensity or power $I_{opt}(t)$,** resp. **$P_{opt}(t) \sim A(x,t)^2$** by photo-generating a density $n(x,t)=p(x,t)$ of mobile e-h-pairs proportional to the photon density $s_{ph}$ in the EM-field.

$$I_{opt}(t) \cong A(t)A^*(t) = s_{ph}(t) v_{gr} \hbar\omega$$

*Photocurrent:*

$$i_{ph}(t) \sim R_{12,abs} \sim |A(t)|^2 \sim e\alpha s_{ph}(t) v_{gr}$$



# Functional principle of photodetectors based on the Internal and External Photoeffect:

Technical important photodetectors are based on the **absorption of photons combined with the generation and the transport of free or quasi-free charges**

Photon-Absorption → eh-pair generation → carrier drift / transit → induced external load current

**a)** Photogeneration of an **electron-hole pair in SC** , $\hbar\omega > E_g$ (internal photoeffect)

**b)** Generation of a **photoelectron** from metallic/SC photocathodes into vaccum $\hbar\omega > e\phi\left(\text{workfunction}\right)$ (external photoeffect)

The **absorptions rate $R_{12}$** is proportional to the Intensity $I_{opt}$ of the optical signal

**c)** **Charge transport** (mainly drift or diffusion) to the contacts and generation of a photocurrent in the external load circuit

Photogenerated charges are converted by the static (DC) external bias electrical field into a current $i_{ph}$

(current source character of the depletion layer in pn-junction diodes). ,

Remark: the generation (transition) time Δt of photocarriers is extremely short ~fs. The generation time Δt is roughly proportional to the inverse of the optical absorption bandwidth Δω of the optical transition.

# Functional Goals of Photodetectors for communication technology:

- **High efficiency**, resp. **responsivity R** [A/W] of the optical-electrical conversion $\bar{P}_{opt} R = i_{ph}$
- High **responsivity R** at the **communication wavelengths $\lambda$= (0.85), 1.30** and **1.55µm** requires a SC with a bandgap energy $E_g < 2\pi c/\lambda$ , resp. a suitable material composition of the SC for strong absorption $\alpha(\lambda)$.

  (Si, Ge, GaAs, InGaAs, ….)
- **High electrical bandwidth (low transit time + low electrical parasitics RC-time constants).**

  $f_{-3dB}$=0.8 x data rate (10 Gb/s → 8 GHz, 40 Gb/s → 32 GHz)
- **Minimal noise** of the detector (shot-noise limit of the photocurrent $i_{ph}(t)$) and **high Linearity** for analog signal detection
- **Detector-geometry** compatible to the fiber geometry (active area ~10 – 50µm , SM – MM fibers)

**Photodetectors which are important for fiberoptic communication are:**

- **PIN-Photodiodes** in the material system Si, Ge and InGaAs:
  in reverse-biased pn-photodiodes **e-h-pairs** are generated by photon absorption in the depletion layer.
  The internal bias-field separates the carrier pairs (to prevent recombination). e and h move in the bias field in opposite directions to the n and p-contacts.

- **Avalanche** (Lawinenverstärkungs)-**Photodiodes** (APD) from Si, Ge und InGaAs
  Special pn-photodiode structures, where the primary e-h-pairs are **multiplied M-times internally** by **impact-ionization in high field regions** of the APD.

- **Photoresistors** (resistance change by photogenerated carrier density Δn, Δp) and **Vacuum-Photodiodes** are of lower importance except for long wavelength detection and highest speed.

# 7.1 Transport dynamics of photogenerated carriers (pairs)

**Assumptions:**

- For technical applications the generation of a carrier-pair (e-h-pair) in SC occurs almost instantaneously, because the absorption transition time is in the fs-range.
- For the interaction of the optical field with the material of the photodetector we consider the particle character of the photon-field.

  The optical field is represented by a modulated **propagating photon-stream**, that is converted into a **electrical carrier stream by absorption**.

## 7.1.1 Concept of Photodetection (external photoeffect)

For the explanation of basic processes in photodetection we consider the **external photoeffect in a metal photocathode** in a vacuum-photodiode.

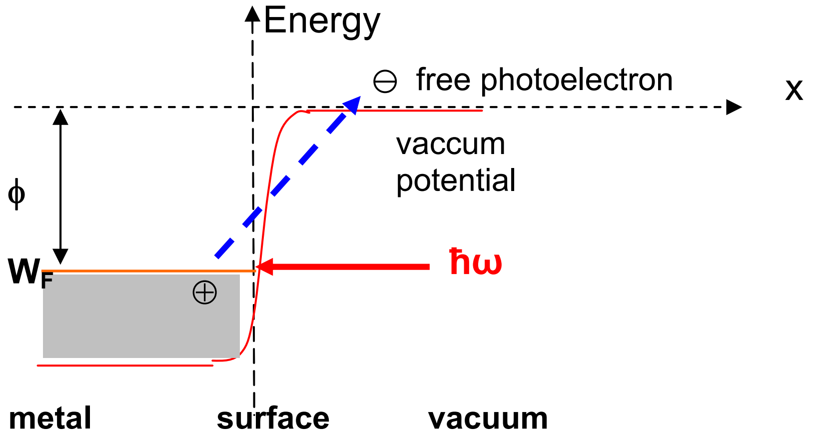
We use the following basic relations (Physik II):

If the Energy of Photons:

$$E_{ph} = \hbar\omega = hc/\lambda > e\phi \ ,$$

$$\phi = \text{work function (Austrittsarbeit) of a metal cathode}$$

then photoelectrons can move across the energy-barrier $\phi$ at the surface and leave the metal-photocathode (free photoelectron).



### Relation between photon density $s_{ph}$ and the light field intensity $I_{opt}$:

$$I_{opt}(x,t) = \partial P_{opt}(x,t) / \partial A = s_{ph}(x,t) \;\; \hbar\omega \;\; v_{gr} \rightarrow$$

$$s_{ph}(x,t) = I_{opt}(x,t) / (\hbar\omega \, v_{gr})$$

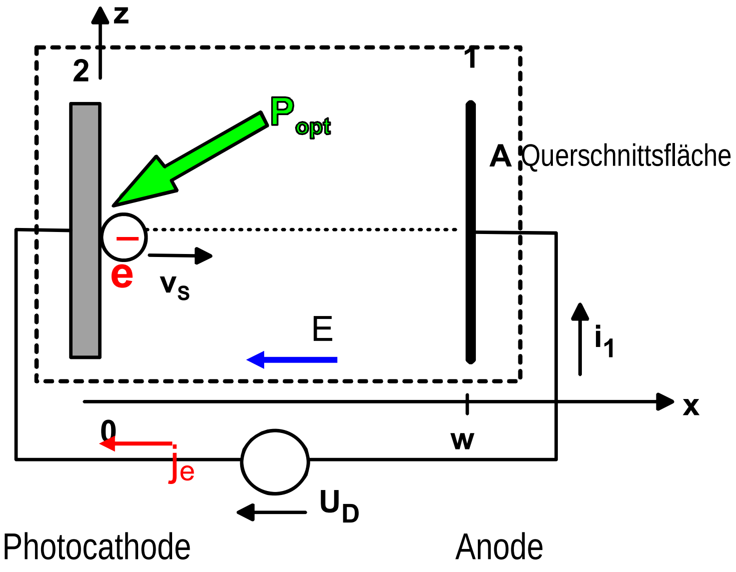
$I_{opt}$ = $P_{opt}$/A= intensity of the optical field (power/area) [W/m$^2$]

$s_{ph}$= photon density of the optical field (photons/unit volume) [m$^{-3}$]

$v_{gr}$ = group velocity

$P_{opt}$ = optical power [W]

### Schematic of Vacuum-Photodiode:



$I_e$ =$j_e$A = $i_1$ photocurrent, (Photoelectron-Emission)

$j_e$ = photocurrent density

R = responsivity [A/W]; is a function of ω and the material of the photocathode

$\eta_i$ = quantum efficiency, number of emitted photoelectrons per impinging photon $\eta_i$ <1

A = diode area, cross section pf the metal photocathode

$U_D$ = external DC-voltage source (differential (AC) resistance =0)

E = $U_D$/w = Bias-field strength (static)

w = length of drift region

The optical field of intensity $I_{opt}$ and frequency ω penetrates into the surface of the photocathode (metal) and generates an <u>average</u> electron current density $j_e$, if the photon energy ħω exceeds the workfunction ϕ of the metal: $\hbar\omega > W_{austritt} = e\phi$

For the **primary** photocurrent $i_e$=A $j_e$ we get due to the particle conservation:

$$I_e(t) = i_1 = e\frac{P_{opt}(t)}{\hbar\omega}\eta_i = P_{opt}(t)\frac{e\lambda}{hc}\eta_i \underbrace{=}_{Def.R} P_{opt}(t)R \quad ; \quad R = \frac{e}{\hbar\omega} = \frac{e\lambda}{hc}\eta_i \quad (responsivity, A/W)$$

division by area A →

$$j_e(t) = R\, I_{opt}(t) = e\eta_i \, s_{ph}(t) v_{gr}$$

# 7.1.1.1 Carrier transport and external photocurrent (OE-impulse response)

For the calculation of the external photocurrent $i_1(t)$ in the load circuit ($V_D$ = ideal voltage source) we consider a model (all relevant effects correctly represented), keeping the mathematics simple. These effects are also found in semiconductor PIN-diodes.

## 1) Assumptions:

- all photoelectrons are moving in a constant, external **bias field** $E=V_D/w$ with a **constant velocity $v_S$** from the **photo-cathode to the anode** (which is not very realistic for a vacuum photodiode with accelerated charges, but a good approximation for SCs)
- the density of the photoelectrons n(x,t) is low ➨ **no space charge effects and no coulomb interaction** between the moving charge
- the motion x(t) of all photoelectron is identical (only 1-dimensional motion)
- for the **impulse response** we assume an **optical $P_{opt}(t)=W_{opt}\delta(t)$-pulse** exciting a homogeneous **charge sheet** at the photocathode instantaneously ($W_{opt}$=energy of the optical pulse=$N_{ph}\hbar\omega$, $N_{ph}$=number of photons in the δ-pulse)

$$I_{opt}(t)=I_{opt}'\delta(t) \quad ; \quad \left[\delta(t)=1/s\right] \qquad \text{integrating} \quad A\int I_{opt}dt \quad with \quad \int\delta(t)dt=1 \rightarrow$$

$$W_{opt}=AI_{opt}'=N_{ph}\hbar\omega \quad \rightarrow \quad I_{opt}(t)=I_{opt}'\delta(t)=W_{opt}/A\delta(t)=N_{ph}\hbar\omega/A\delta(t) \quad ; \quad P_{opt}(t)=W_{opt}\delta(t)=N_{ph}\hbar\omega\delta(t)$$

Absorption of this optical intensity δ-pulse leads to an “instantaneous” generation of a free **charge sheet** with the charge $Q_e$:

$$Q_e=e\eta_i N_{ph}=W_{opt}/(\hbar\omega)e\eta_i=W_{opt}R \quad \text{using}: R=e\eta_i/(\hbar\omega)$$

## 2) Procedure: Determination of the displacement current at the electrodes

- depending on the actual position $x_e(t)$ a **single photoelectron e** influences a **time-dependent charge $Q_1(x_e(t))$** on the photocathode and a charge **$Q_2(x_e(t))$** on the anode with **$Q_1(t)+Q_2(t)=e$**.
- for calculating the elementary **induced current $i_1(t)$** in the load circuit (a short for an ideal bias voltage source $V_D$) we need to determine the influenced **time-dependent electrode charges $Q_1(t)$ and $Q_2(t)$**:

$$i_1(t) = \frac{\partial Q_1(t)}{\partial t} = -\frac{\partial Q_2(t)}{\partial t} \qquad with \quad Q_1(t) + Q_2(t) = -e \quad (charge\ conservation\ for\ a\ single\ electron)$$

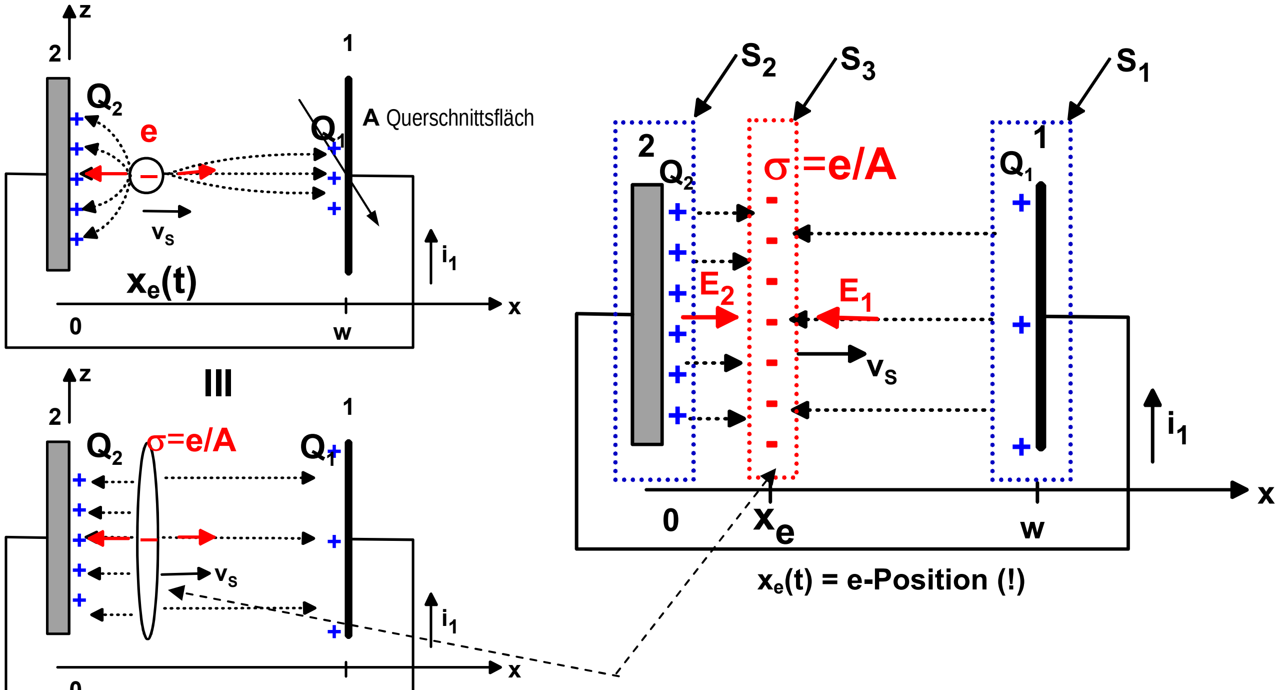
**We are looking for the impulse response of photocurrent $i_{ph}(t)=i_1(t)$ in the load short circuit ($R_L=0$)**

## Concept of solution: capacitor containing a “moving charge sheet”

Solving the problem for an individual electron (moving point charge) would require the determination a 3-dimensional electric field distribution $E(x,y,z, x_e)$ between two plates with a constant potential difference.

**Electrostatic equivalence:**

S= surface of integration for the Gauss relation



$x_e(t)$ = e-Position (!)

equivalent “smeared out” electron charge

Remark:
only dynamic charges are shown, not the static charges of the bias-voltage.

**“Gedanken-Experiment”**:

To reduce the 3D- to a 1D- field problem we make the „Gedanken”-experiment:

because the field equation are linear and as all electrons in the moving charge sheet behave identical, we can say that the photocurrent response of the lateral charge sheet is just the sum of the elementary responses of an individual point charge (superposition). This means that we get the same elementary response as if we **“think” of an** **electron “smeared out” homogenously over the whole cross-section A.**
This leads to a 1D potential calculation of an **elementary charge sheet** at the **electron position $x_e(t)$** with a charge density

$$\sigma(x_e) = \eta_i W_{opt} e / (\hbar\omega) / A = N_{ph} e / A \eta_i$$

With the charge-sheet density per photoelectron **$\sigma(x_e(t))=e/A$** the homogeneous electric fields $E_1(t)=E_1(x_e(t))$, $E_2(t)=E_2(x_e(t))$ can be determined from the **"Gausschen Satz"** in the x-dimension $\oiint_A \varepsilon_o \varepsilon_r \vec{E}\, d\vec{A} = \iiint_V \rho dV = Q$ .

The problem of calculating the homogeneous case with the moving charge sheet $\sigma(x_e) = e/A$ at $x_e(t)$ is 1-dimensional and provides $Q_1(x_e(t))$, $Q_2(x_e(t))$, resp. $i_1(t)$, $i_2(t)$.

**Remark:** the static, time-independent charges on the plates caused by the constant bias-voltage $V_D$ are not considered, because the corresponding constant charges do not generate an external current in the load.

Under the convention of considering the directed field vectors $E_{1,2}$ as positive (see figure), we get:

$$i_{ph}(t) = i_1 = \frac{\partial}{\partial t} Q_1(t) = -\frac{\partial}{\partial t} Q_2(t) \qquad \textit{charge conservation: Kirchhoff current law}$$

$$\textit{with :} \qquad Q_1(t) + Q_2(t) = e \qquad \textit{charge neutrality}$$

As we assumed an ideal bias-voltage source $V_D$ (AC-resistance =0, short circuit) there is no AC-voltage v(t) between the plates.

Observe that $E_1(x_e(t))$ and $E_2(x_e(t))$ are constant in x in the time-dependent intervals $[0, x_e(t)]$ and $[x_e(t), w]$

*Because the dynamic load voltage* $v(t) = 0$ *we obtain :*

$$v(t) = \int_0^w E(x)dx = x_e(t)E_2(t) - \left[w - x_e(t)\right]E_1(t) = 0 \qquad \textit{Kirchhoff Voltage Law} \quad (eq.1)$$

*and with the assumption of a constant carrier velocity* $v_s \rightarrow \quad x_e(t) = v_S\, t$

*formulating the Gauss-Relation (resp. with the Maxwell equation:* $\nabla D = \rho$ , $\varepsilon = \varepsilon_0 \varepsilon_r$ *)* *we obtain for the surface S with area A* $\rightarrow$

$$\varepsilon E_1 A = Q_1 \,, \quad resp.\ D_1(w) = \varepsilon E_1 = Q_1 / A = \sigma_1(w) \qquad (surface\ S_1)$$

$$\varepsilon E_2 A = Q_2 \,, \quad resp.\ D_2(0) = \varepsilon E_2 = Q_2 / A = \sigma_2(0) \qquad (surface\ S_2)$$

$$\varepsilon E_2 A + \varepsilon E_1 A = e \,, \ resp.\ D_2 + D_1 = e / A = -\sigma_3(x_e) \qquad (surface\ S_3) \quad (eq.2)$$

**Remark:**

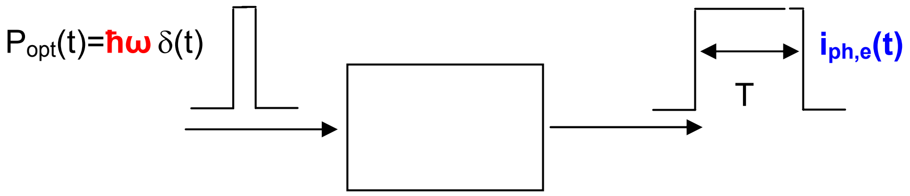
$E_1$ and $E_2$ are spatially, but not temporally constant (independent of x , but dependent on the $x_e$-position of the electrons).
A = cross-section of the diode A, with $\sqrt{A} >> w$ (no fringing effects considered)

By elimination of $E_1$ and $E_2$ with eq.1 and 2 we get for the plate charges

$$Q_2(t) = e\frac{w - x_e(t)}{w} \quad and \quad Q_1(t) = e\frac{x_e(t)}{w} \quad \rightarrow$$

$$\frac{\partial}{\partial t}Q_1(t) = i_1(t) = \underline{i_{ph,e}(t) = ev_S / w = e\frac{1}{T}} \qquad 0 \le t \le T = w / v_S$$

**Elementary impulse response: photon → photo-current**



**General: $I_{ph}(t) = P_{opt}(t)\,(R/e) * i_{ph,e}(t)$**

**Time-domain:**

The electron that moves with constant velocity $v_S$ produces a **rectangular elementary current pulse $i_D(t)=i_{ph,e}(t)$** with the **amplitude e/T** and the **duration $T=w/v_s$** (transit time)

(**Elementary event of a photoelectron** moving from the cathode to the anode)

**remark**: during the transit of the electron there is only a displacement current at the contacts – only when the electrons reach the anode, there is a conduction (particle) current.

For a $N_{ph}$-photon pulse we have the generalization: $I_{ph}(t) = \eta_i N_{ph} i_{ph,e}(t) = e\eta_i N_{ph} v_s / w \quad for \quad 0 < t < w / v_s$

## Photodetector Bandwidth and frequency transfer function

Because all electrons produce the same elementary current impulse response $i_{ph,e}(t)$, we obtain the frequency response $i_{ph,e}(\omega)$ by a Fourier-transform. The normalized single-photo-electron frequency response is identical to the total normalized response of the PD.

The **transport current** of the photoelectron **$i_{ph,e}(t)$** is the response to the **single photon impulse** $\underline{P_{opt,e}(t) = \hbar\omega_{opt}\delta(t)}$ under the assumption that the emission of the photoelectron occurs instantaneously:

*Photocurrent per photo – e: (photocurrent impulse response)*

$$i_{ph,e}(t)=\frac{e}{T}\left[\sigma(t)-\sigma(t-T)\right],\quad \sigma(t)=Step-funktion,\quad \delta(t)=Dirac-Funktion$$

$\downarrow$ *Fourier – Transform* $\left(e^{-j\omega t}\right)$

$$i_{ph,e}(\omega)=F\left(i_{ph,e}(t)\right)=\int_0^\infty i_{ph,e}(t)e^{-j\omega t}dt \qquad \textit{single photoelectron current density spectrum}$$

*Because we consider all photoelectrons as identical we can apply the superpostion of the impulse responses:*

$$I_{ph}(t)=\sum_{N_i} i_{ph,e}(t)=N_i i_{ph,e}(t) \qquad \textit{with the Fourier-transform } F$$

$$\textit{and with the pulse-energy:}\quad W_{opt}=\int P_{opt}(t)dt=N_i\hbar\omega_{opt}\int\delta(t)dt \quad \textit{and} \quad F(\delta(t))=1$$

$$I_{ph}(\omega)=F\left[\frac{\eta_i}{\hbar\omega_{opt}}W_{opt}\delta(t)*i_{ph,e}(t)\right]=\frac{\eta_i}{\hbar\omega_{opt}}W_{opt}\cdot i_{ph,e}(\omega)=$$

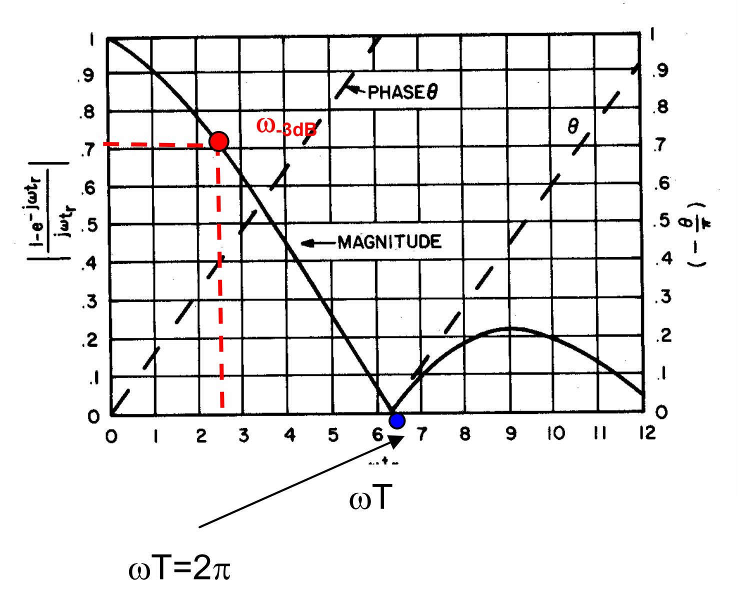
$$=\frac{\eta_i e}{\hbar\omega_{opt}}W_{opt}\int_0^T\frac{1}{T}\;e^{-j\omega t}dt=$$

$$\underline{I_{ph}(\omega)=W_{opt}R\,\frac{e^{-j\omega T}-1}{-j\omega T}}\quad ;\quad T=w/v_s$$

$$\boxed{\left|I_{ph}(\omega)\right|=W_{opt}R\;\sqrt{2}\frac{\sqrt{1-\cos(\omega T)}}{\omega T}=W_{opt}(\omega)R\;\frac{\sin(\omega T/2)}{\omega T/2}}$$

**Frequency response of a photo detector with constant carrier drift velocity**

**Transit time dominated frequency response of a photodiode:** $\sin(\omega T/2)/(\omega T/2)$



**The first zero of the frequency response occurring at $\omega_o = 2\pi / T$ and the -3dB-bandwidth $\omega_{-3dB} \cong 1/2\omega_0$ is determined by the transit time T=w/$v_s$ and the carrier velocity $v_s$. Fast photodetectors require therefore short transit times T.**

### General response:

The time-domain response of the photodetector for general optical input waveform $P_{opt}(t)$ is obtained by a convolution with the impulse-response:

$$\boxed{P_{opt}(t) R/e * i_{ph,e}(t) \quad \bullet\!-\!-\!\circ \quad P_{opt}(\omega) R/e \cdot i_{ph,e}(\omega)}$$

## 7.1.1.2 Noise processes in Photodetectors (shot noise)

**Introduction and goal:**

The photocathode is illuminated by light of constant intensity $I_{opt} = P_{opt} / A$ producing a constant **average photocurrent**

$$\overline{I_{ph}} = R\, I_{opt}\, A = e \lim_{T\to\infty} \frac{N_T}{T}$$

with $N_T$=number of generated photo electron in the time interval T

- The resulting photocurrent $i_{ph}(t)$ consists of a large number of $N_T$ elementary current pulses $i_{ph,e}(t-t_k)$ occurring at the **random emission time $t_k$** in the measurement interval T.
- The number $N_T(t)$ fluctuates around an average value $\overline{N_T}$ with $\Delta N_T(t)$ because the **times $t_k$ of photo-electron emission is not exactly defined but fluctuate statistically** (all quantum mechanical calculations result just in average values [expectation values] for the transition rates).

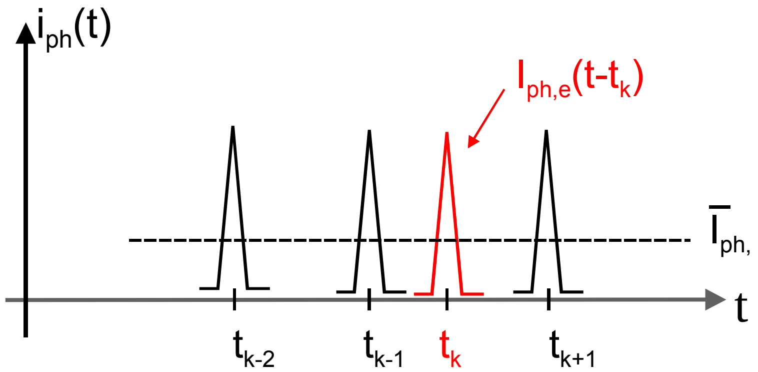
These number fluctuations result in **random current fluctuations $i_{ph,n}(t)$** (noise current) in the photo current $i_{ph}(t)$.

**The statistical fluctuation $i_{ph,n}(t)$ around the average photocurrent $\overline{I_{ph}}$ is the inherent noise of the photodetection process:**

$$i_{ph}(t) = \sum_{k=1}^{N_e} i_{ph,e}(t - t_k) = \overline{I_{ph}} + i_{ph,n}(t)$$

(Definition)

**The goal is to calculate from a simple model the power spectrum $S_T(\omega)$ of the noise process $i_{ph,n}(t)$.**



- The current noise is an inherent, irreversible property of the absorption and carrier emission process of the photodetection and can not be eliminated. It represents the **absolute detection minimum**.
- In case that the optical power $P_{opt}(t)$, resp. the intensity $I_{opt}(t)$ is modulated by a signal $p_s(t)$, resp. $i_s(t)$, $P_{opt}(t)=\overline{P}+p_s(t)$, resp. $I_{opt}(t)=\overline{I}+i_s(t)$, then the photocurrent shows also the related signal component $i_S(t)=R\, p_S(t)$.

**Signal photocurrent $i_{ph,S}(t)$**: $i_{ph,S}(t)=R\ p_S(t)$

For the total current we write:

$$i_{ph}(t)=\overline{I_{ph}}+i_{ph,s}(t)+i_{ph,n}(t)=R\,\overline{P}_{opt}+R\,p_s(t)+i_{ph,n}(t)$$

?

In order to detect the signal current with a low error probability, we require intuitively (see chap.9) that:

$i_{ph,s}(t)>>i_{ph,n}(t)$ (dependent on the desired signal error criterion, eg. SNR, BER)

Because the noise photocurrent $i_{ph,n}(t)$ is a time-unlimited, statistical signal no Fourier-transform exist and we can only describe it by its autocorrelation function $\langle i_{ph,n}(t)i_{ph,n}(t-\tau)\rangle$, resp. by its

**power density spectrum** $i^2_{ph,n}(\omega)=\lim\limits_{T\to\infty} S_T(\omega)=F\left(\langle i_{ph,n}(t)i_{ph,n}(t-\tau)\rangle\right)$

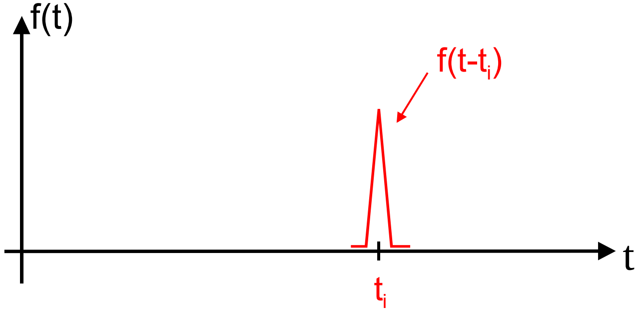
# Calculation of the noise power density spectrum $i_{ph,n}^{2}(\omega)$ of the photocurrent noise:

## Power density spectrum S(ω) of a sequence of random events f(t-t$_i$):

- In electronic and optoelectronic devices the electrical or photon current results from the flow of particles (drift, diffusion, generation, recombination, etc.), resulting from a **large number of statistical elementary events** (eg. single electron transport between photocathode and anode → current pulses in the load circuit).

  These so called **rate processes** consist in the simplest case of a collection of identical random events with a defined **average event rate** $\overline{N}$ in the time interval T.

- The **elementary event f(t-t$_i$)** [eg. $i_{ph,e}(t)$] takes place at **random time t$_i$** (assumption: all $t_i$ are independent of each other and uncorrelated)



**Identical events** (transport of a charge quantum e)**:**

**Definition:** $x_T(t)$ is a time function of the event sequence in a limited measurement interval T [-T/2,T/2] containing $N_T$ events

$$x_T(t) = \sum_{i=1}^{N_T} f(t-t_i) \qquad for \;\; -T/2 < t < +T/2$$

$$x_T(t) = 0 \qquad\qquad for \;\; |t| > T/2$$

$N_T / T = \overline{N}$ is the average event number in the interval T

Because $x_T(t)$ is a function of finite energy its Fourier-transform **does exist** !

The Fourier-transform of $x_T(t)$ gives by making use of the **delay-operator** $e^{-j\omega t_i}$ for the time $t_i$ of event f(t-t$_i$) :

$$X_T(\omega) = F\left(x_T(t)\right) = \sum_{i=1}^{N_T} F_i(\omega) = \sum_{i=1}^{N_T} e^{-j\omega t_i} F_0(\omega)$$

The Fourier-spectrum $F_i(\omega)$ of the event $f(t-t_i)$ is

$$F_i(\omega) = \int_{-\infty}^{+\infty} f(t-t_i) e^{-j\omega t} d\,t = e^{-j\omega t_i} F_0(\omega)$$

using the $F_0(\omega) = F[f(t)]$ of the elementary event $f(t)$ at $t=0$

$$F_0(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} d\,t \quad and \quad f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F_0(\omega) e^{+j\omega t} d\,\omega \quad (2-sided\ spectrum)$$

From the **amplitude density spectrum $X_T(\omega)$** we obtain the related **power density spectrum $S_T(\omega)$** as:

**Remark:** the amplitude density spectrum $X_T(\omega)$ of the time-limited process $x_T(t)$ must exist, because its energy is finite.

$$X_T(\omega) = \sum_{i=1}^{N_T} F_i(\omega) = \sum_{i=1}^{N_T} e^{-j\omega t_i} F_0(\omega) = \sum_{i=1}^{N_T} e^{-j\omega t_i} F\left(f(t)\right)$$

*The "energy"* $E_{x,T}$ *in the interval* $T$ *is:*

$$E_{x,T} = \int_{-T/2}^{+T/2} x_T^2\, dt = \int_{-T/2}^{+T/2} \underbrace{F^{-1}\left(X_T(\omega)\right)}_{x_T(t)} \underbrace{F^{-1}\left(X_T(\omega)\right)^*}_{x_T{}^*(t)} dt$$

*The average power* $S$ *of the event-sequence during the symmetric interval* $\left[-\frac{T}{2}, +\frac{T}{2}\right]$ *becomes usin the PSD* $S_T(\omega)$:

$$S = \frac{E_{x,T}}{T} \underbrace{=}_{\substack{Definition\\ von\ S_T}} \int_{-\infty}^{+\infty} S_T(\omega) d\omega = \frac{1}{T} \int_{-T/2}^{+T/2} \underbrace{\frac{1}{2\pi} \int_{-\infty}^{+\infty} X_T^*(\omega) e^{-j\omega t} d\omega}_{x_T{}^*(t)} \; \underbrace{F^{-1}\left(X_T(\omega)\right)}_{x_T(t) \neq f(\omega)} dt$$

$S_T =$ *power spektrale density (PSD) of event – sequence* $x_T(t)$

*Using the technique of the exchange of the integrations:*

$$S = \frac{1}{T}\frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} X_T^*(\omega)e^{-j\omega t}\,F^{-1}\left(X_T(\omega)\right)dt\,d\omega = \frac{1}{T}\frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-j\omega t}F^{-1}\left(X_T(\omega)\right)dt\,X_T^*(\omega)\,d\omega =$$

$$S = \frac{1}{T}\frac{1}{2\pi}\int_{-\infty}^{+\infty} F\left\{F^{-1}\left(X_T(\omega)\right)\right\}X_T^*(\omega)\,d\omega \quad \rightarrow$$

$$S_T(\omega) = \frac{1}{T}\frac{1}{2\pi}X_T(\omega)X_T(\omega)^* = \frac{1}{T}\frac{1}{2\pi}\left|X_T(\omega)\right|^2$$

*If we insert* $X_T(\omega) = \sum_{i=1}^{N_T} e^{-j\omega t_i}F_0(\omega)$ *in the above equation for* $S_T(\omega)$ *or using Pasevals-Theorem* $\int x_T(t)^2\,dt = \frac{1}{2\pi}\int S_T(\omega)\,d\omega$*:*

$$S_T(\omega) = \frac{1}{T}\,\frac{1}{2\pi}\left\{\left|F_0(\omega)\right|^2 N_T + \left|F_0(\omega)\right|^2 \underbrace{\sum_{i=1}^{N_T}\sum_{i\neq k}^{N_T} e^{-j\omega(t_i-t_k)}}_{=0\ \text{to be proven}}\right\}$$

The double sum is zero

$$\sum_{i=1}^{N_T}\sum_{i\neq k}^{N_T} e^{-j\omega(t_i-t_k)} = 0$$

because the $t_i$ are per definition uncorrelated random variable and the double sum only contains unit-vectors with random phase angles (not a mathematically rigorous proof).

Making use of $\overline{N} = N_T / T$ ($\overline{N}$ =average rate of events) →

## Power density spectrum of the time-limited random rate process:

$$\boxed{S(\omega) = \lim_{T\to\infty} S_T(\omega) = \lim_{T\to\infty}\frac{\left|X_T(\omega)\right|^2}{2\pi T} = \lim_{T\to\infty}\frac{X_T(\omega)X_T(\omega)^*}{2\pi T} = \left|F_0(\omega)\right|^2\frac{\overline{N}}{2\pi}}$$

**(2-sided power spectrum,** per unit ω**)**

**➡ The power density spectrum of a random rate process is proportional the average event rate**

## Power density spectrum S(ω) of a random sequence of identical elementary events:

$$S(\omega)= \left|F_0(\omega)\right|^2 \frac{\bar{N}}{2\pi} \qquad (densit\ related\ to\ \ \omega)$$

$$S(f)= \left|F_0(f)\right|^2 \bar{N} \qquad 2-sided\ spectrum \qquad (density\ related\ to\ \ f)$$ with $f=\omega/2\pi$

$$S(f)= \left|F_0(f)\right|^2 2\bar{N} \qquad 1-sided\ spectrum \qquad f>0$$

make use of $S(f)=2\pi S(\omega)$

If we apply the above derived shot-noise relations to the process of photodetection in a vacuum photodiode we have for the elementary event f(t) a rectangular current pulse of duration T:

$$i_{ph,e}(t)=ev_S/w \qquad 0<t<w/v_s=T$$

$$i_{ph,e}(t)=0 \qquad t<0;\ w/v_s<t$$

$$F_0(\omega)=e\frac{e^{-j\omega T}-1}{-j\omega T} \quad ; \quad (2-sided)\rightarrow$$

$$\left|F_0(\omega)\right|^2=e^2\left[2\frac{1-\cos(\omega T)}{(\omega T)^2}\right] \quad \rightarrow \quad S(\omega)= \left|F_0(\omega)\right|^2\frac{\bar{N}}{2\pi}$$

## Shot-Noise of the photocurrent

$$S_{i_{ph}}(\omega)= \frac{1}{\pi}\bar{N}e^2\left[2\ \frac{1-\cos(\omega T)}{(\omega T)^2}\right]=\frac{1}{\pi}e\bar{I}_{ph}\left[2\ \frac{1-\cos(\omega T)}{(\omega T)^2}\right]$$

$$S_{i_{ph}}(f)= 2e\bar{I}_{ph}\left[2\frac{1-\cos(2\pi Tf)}{(2\pi Tf)^2}\right]$$

$$falls\ \ \omega<<2\pi/T \quad \rightarrow \quad S_{i_{ph}}(f)=2e\bar{I}_{ph} \qquad \left(using\ \cos(\varepsilon)\approx 1-\varepsilon^2/2\ ;\ \ \varepsilon<<1\right)$$

This noise 1-sided spectrum is also called 2eI-noise

## Summary:

- The goal of photodetectors is

  1) conversion of optical power into electrical charge,

  2) charge separation and transport and

  3) the generation of electrical signal in external load circuits

- The noise power density spectrum S($\omega$) of rate processes of discrete, uncorrelated random events is equal to the event rate.

  The frequency dependence of S($\omega$) of a rate process is the magnitude squared of the F-transform of the elementary event.

- Because the spectrum of shot-noise is independent of frequency at low frequency, this noise is also called „white noise“

- **Shot noise is a fundamental limit of the photodetection determining the smallest signal that can be detected**

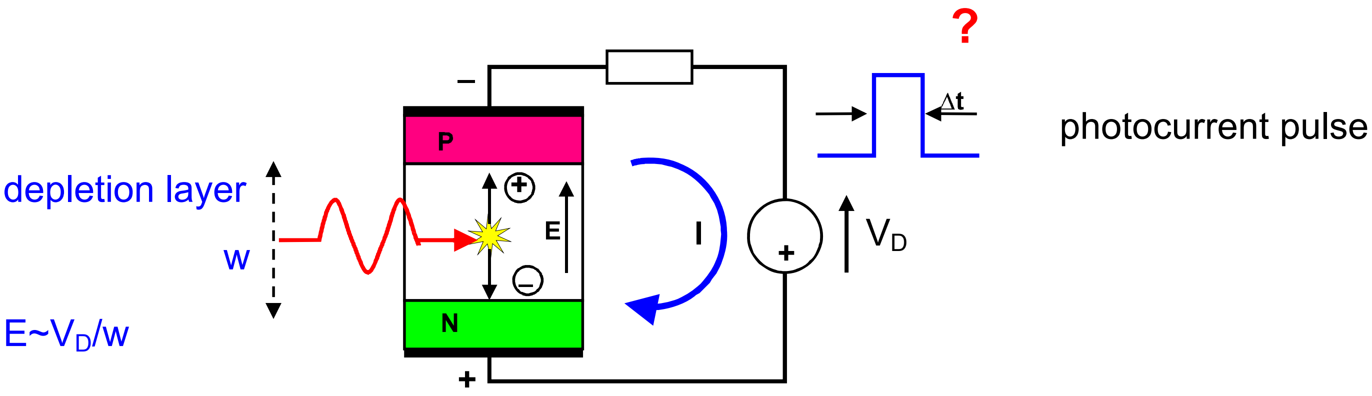
- Many noise processes in SC-devices such as photogeneration, carrier injection, diffusion, photoemission in pn-diodes and bipolar transistors exhibit rate proportional noise.

# 7.1.2 Optical carrier generation in semiconductor photodetectors

## Internal Photoeffect in SC:

Photons with an energy $\hbar\omega \geq E_g$ produce valence-to-conduction band transitions by **stimulated absorption $R_{12}$** and generate **mobile electron-hole pairs** in the depletion layer of a reverse biased PIN-diode.

→ **Electron-hole pair generation $R_{12}=\alpha v_{gr} s_{ph}$**



- If an electric field E is present (eg. in the depletion layer w of a reverse biased pn-junction) it separates the weakly bound e-h-pair (coulomb attraction, excitons) and prevents them from recombining again $R_{spont} \sim 0$.

  Electron and hole start drifting in <u>opposite</u> directions thus generating induced photocurrents in the load circuit.

- If the eh-pair (excitons) would not be separated, then the eh-pair could recombine (**inverse process, eh-pair-annihilation**) without producing a contact current.

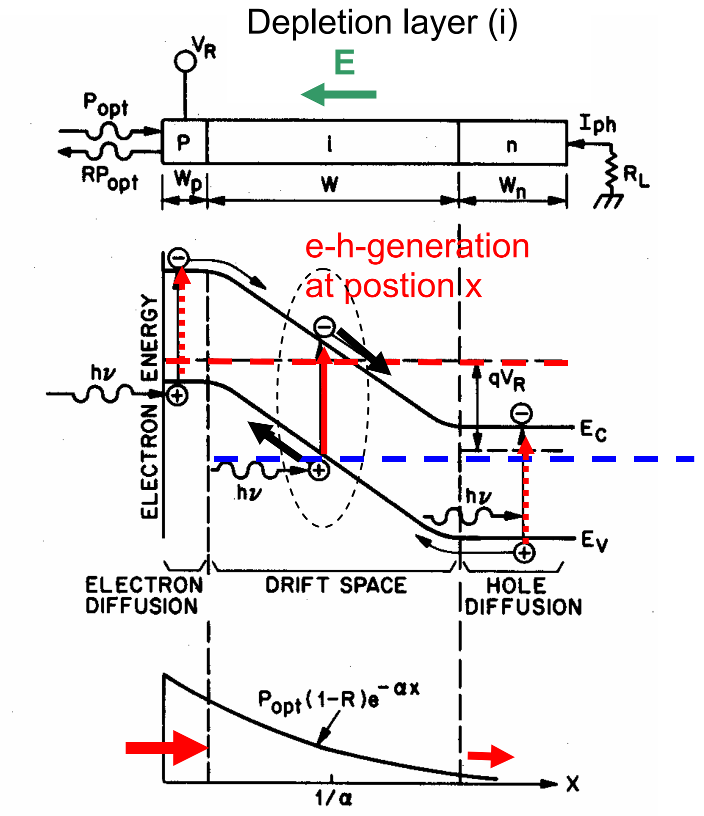
# Band diagram of elementary electron-hole pair generation in pn-Photodiode:

An efficient implementation for e-h-pair generation and pair separation is the **depletion layer w of a pn-junction diode**.

The reverse voltage across the depletion layer w is the sum of the of the

1) **external voltage $V_D$** and 2) the internal **diffusion potential** $\phi_{bi} = (kT/e) ln\left(N_A N_D / n_i^2\right)$ :

$$E \cong \left(U_D + \phi_{bi}\right)/w \quad for\ w_p < x < w_p + w$$



e-Diffusion- E=0  e,h-Drift- E>0  h-Diffusions-Transport E=0

$N_A$=Acceptor doping of the p-area

$N_D$= Donor doping of the n-area

- the PD must be constructed in such a way that the light is mainly absorbed in the depletion layer w>1/α.
- avoid absorption in the neutral (E=0) contact n- and p-layers ($w_n$, $w_p$) where e-h-pairs recombine or diffuse slowly (thin contacts, transparent contacts)

**Band diagram of reverse biased PIN-PD for $V_D$<0**

**Intensity distribution** (exponential decay by light-absorption)

At high electric fields the electrons and holes transit the depletion width w (~μm) at ~saturation velocity $v_{sat}$ (~$10^7$cm/s) very quick so that e-h-recombination is negligible.

# Calculation of the total terminal current: Static carrier transport

**Continuity equation of e and h in** an illuminated <u>incremental</u> volume element (dxA) at position x of the depletion layer w (diffusion is very small → drift dominated transport):

*Determine the electron current at w from the continuity equation at x:*

$$\frac{\partial}{\partial t}n = -\frac{\partial}{\partial t}p = R_{12}\left(x, I_{opt}\right) - \frac{1}{e}\frac{\partial}{\partial x} j_{drift,e} - R_{spont}\left(x\right)$$

*absorption* *transport*

*absorbtion and drift dominated SC* $\left|j_{diff,e}\right| << \left|j_{drift,e}\right|$ : $R_{12} >> R_{21}, R_{spont}$

*and static condition* $\frac{\partial}{\partial t} = 0$ *and the boundary condition* $j_{drift,e}\left(0\right) = 0$

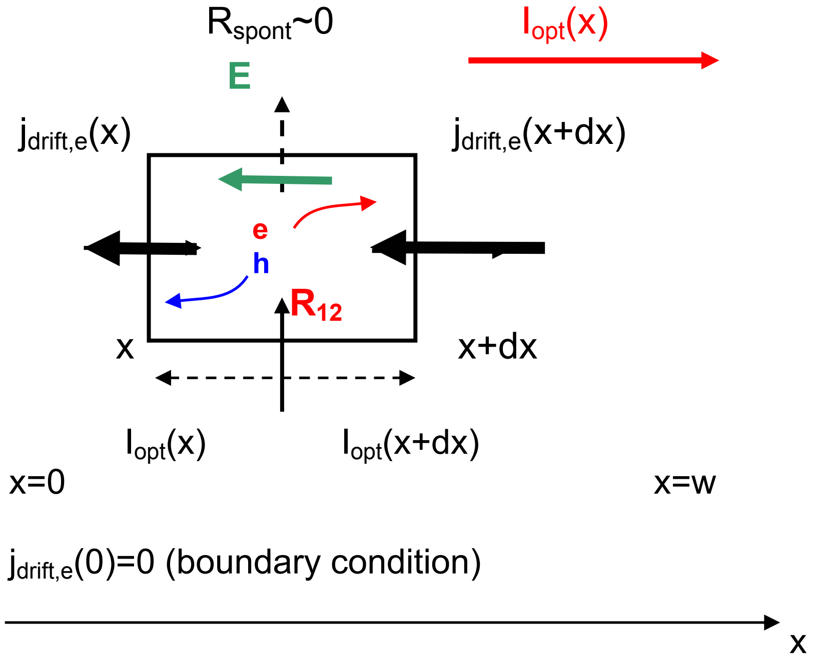
$$\frac{\partial j_{drift,e}}{\partial x} = eR_{12}\left(x, I_{opt}\left(x\right)\right) \rightarrow \int_0^w dx$$

$$I_{ph,e}\left(w\right) = A\left(j_{drift,e}\left(w\right) - \underbrace{j_{drift,e}\left(0\right)}_{=0\,(boundary)}\right) = A\, j_{drift,e}\left(w\right) = eA\int_0^w R_{12}\left(I_{opt}\right)dx =$$

$$I_{ph,e}\left(w\right) \underbrace{=}_{R_{12}=\alpha n_{ph} v_{gr}} eA\int_0^w \alpha\, n_{ph}\left(x\right) v_{gr}\, dx \underbrace{=}_{\alpha=-\frac{\partial I_{opt}}{\partial x}\frac{1}{I_{opt}}} -\frac{eA}{\hbar\omega}\int_{I_{opt}(0)}^{I_{opt}(w)} \partial I_{opt} =$$

$$I_{ph,e}\left(w\right) = \frac{e}{\hbar\omega}\left(P_{opt}\left(0\right) - P_{opt}\left(w\right)\right) = \frac{e}{\hbar\omega}P_{opt}\left(0\right)\underbrace{\frac{\left(P_{opt}\left(0\right) - P_{opt}\left(w\right)\right)}{P_{opt}\left(0\right)}}_{\eta} = \frac{e\eta}{\hbar\omega}P_{opt}\left(0\right)$$

*hole current at 0 :* $\left(analog\ calculation\right) \rightarrow I_{ph,h}\left(0\right) = eA\int_0^w \alpha\, n_{ph}\left(x\right) v_{gr}\, dx$



$j_{drift,e}(0)=0$ (boundary condition)

current convention: + ←

# Material properties and wavelength dependence of $\alpha(\omega)$ in SC:

Chap.5 showed that photons with energy $\hbar\omega \geq E_g$ in SC are absorbed (absorption coefficient $\alpha(\omega)$) such that the intensity $I_{opt}(x)$ decreases exponentially with increasing penetration depth x:

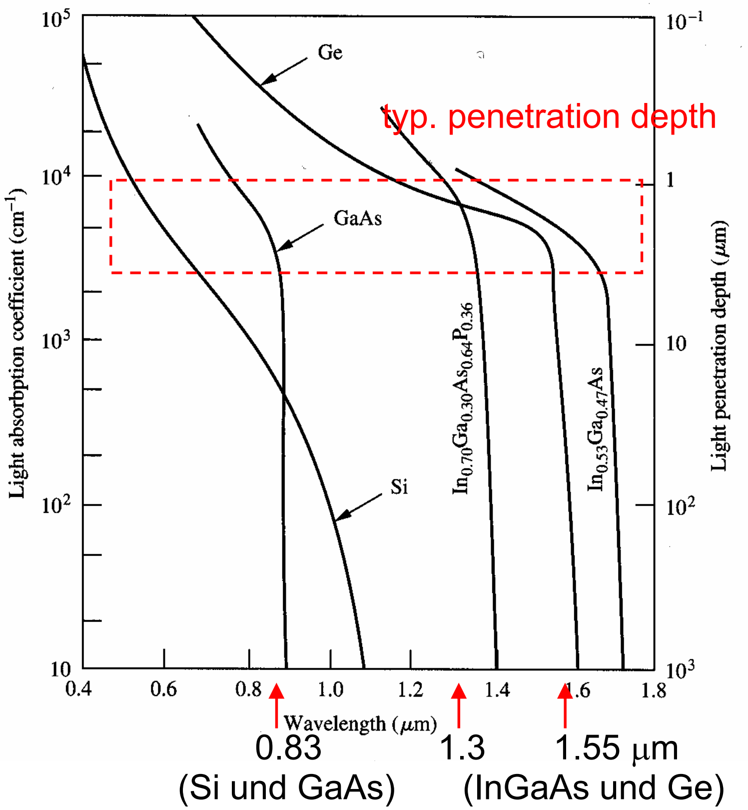
$$I_{opt}(x) = I_{opt}(0)e^{-\alpha(\omega)\,x}$$

$\alpha(\omega)$ is dependent on

1) SC material
2) Bandgap $E_g$,
3) Density of states $\rho_n$, $\rho_p$, resp. reduced density $\rho_r$
4) Matrixelement $\langle u_c | x | u_v \rangle$ and
5) Position of the Quasi-Fermi-levels $E_{FQ,n}$ and $E_{FQ,p}$.

Suitable materials are direct (high $\alpha$) and indirect SC (rel. small $\alpha$).

**Absorption $\alpha(\lambda)$ of different semiconductors**



Suitable material systems for the communication wavelengths are:

$\lambda$=0.85µm : Si, Ge, GaAs, InP ; $\lambda$=1.30µm : Ge, InGaAs ; $\lambda$=1.55µm : Ge, InGaAsP

Typical penetration depth range from 1-10µm, resp. $\alpha=10^4 - 10^3$ cm$^{-1}$.

# 7.1.3 Carrier drift transport in depletion layers

## Drift-transport of e-h-pairs in the depletion layer

e-h-pairs generated at a position $x_e$ in depletion layer of reverse biased pn-junctions are transported by drift by the E-field in the depletion layer. Diffusion is negligible because of the small carrier densities n and p. Electrons and holes are

1) separated in the E-field, then
2) accelerated by the E-field and
3) reaching the saturation velocity $v_{sat,n}$ , resp. $v_{sat,p}$ after a short time $\Delta t$~0.1ps or a short distance $\Delta x$~0.1$\mu$m t:

**Estimate of $\Delta$t and $\Delta$x to reach the saturation velocity $v_{sat,n}$ for electrons:** (ballistic motion without scattering)

$$m_e \frac{\partial^2}{\partial t^2} x = -eE \quad \textit{Newton equation of motion} \rightarrow$$

$$m_e v(t) = -eEt \quad \rightarrow \quad v(t) = -\frac{eE}{m_e} t \quad \rightarrow \quad v(\Delta t) = v_{sat,n} = -\frac{eE}{m_e}\Delta t \quad \rightarrow \quad \textit{time to reach } v = v_{sat}: \quad \Delta t = \frac{v_{sat,n} m_e}{eE}$$

$$x(t) = -\frac{e\ E}{2m_e} t^2 \quad \rightarrow \quad \textit{distance to reach } v{=}v_{sat}: \quad \Delta x(\Delta t) = -\frac{1}{2}\ \frac{v_{sat,n}{}^2 m_e}{eE}$$

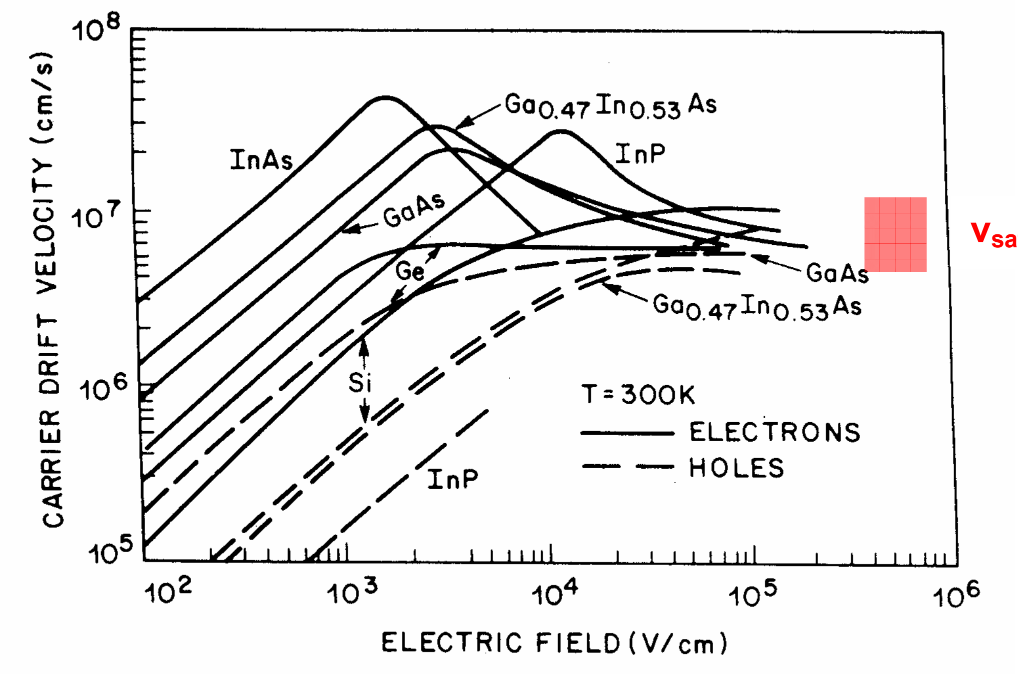
**Example for electrons in GaAs:**

$$v_{sat,n} \cong 10^5 m/s \quad ; \quad E = 10kV/cm$$

$$m_e = 0.07 m_0 \cong 0.7\ \ 10^{-31} kg \qquad \rightarrow \Delta t \cong 50 fs\ ,\ \Delta x(\Delta t) \cong 0.03 \mu m$$

For practical photodetector devices we can assume that the electrons reach the saturation velocity almost instantaneously.

# **Static carrier velocity v(E) vers. electric field E for typical SC:** (scattering limited motion)



**$v_{sat}$: typ. 0.6 – 2 $10^7$cm/s**

- Low field velocity:

  $v=\mu E$, $\mu$=carrier mobility

- High field velocity:

  $v=v_{sat} \neq f(E)$

  The static saturation velocity $v_{sat}$ of many SC is in the range of 6-9 $10^6$cm/s.

- III-V-SC reach their saturation velocity at a much lower field E, resp. operation voltages than eg. Si.

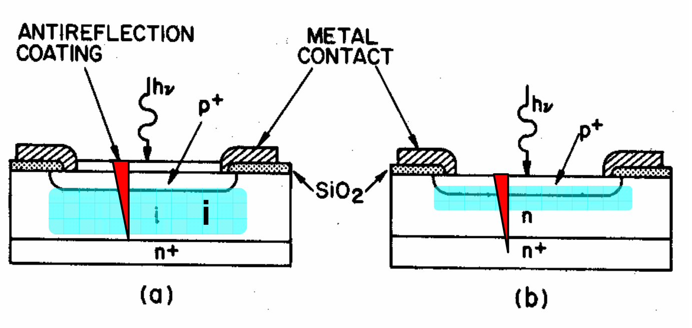
# 7.2 PIN-Photodiodes

## 7.2.1 Planar PIN-Diodes

**Functional goals:**

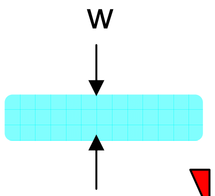
- Depletion layer must be close to the surface to keep the absorption in the n- and p-layers small because light penetration occurs only over $1/\alpha \sim 1\text{-}10\mu m$.
- Depletion layer thickness $w > 1/\alpha$ for high absorption efficiency
- w must also be a trade-off between short transit time $T = w/v_{sat}$ , a high electrical bandwidth $\omega_{-3dB} \cong 1/T = v_{sat}/w$ and low space charge capacitance $C_j=\varepsilon_0\varepsilon_r A/w$ and a high sensitivity R.

**Typical structure for planar photodiodes with vertical illumination:**



pin-Photodiode pn-Photodiode

Photodiodes are coated on their surface by anti-reflection coatings made from multilayer-stacks of $SiO_2$ and $SiN_4$ in order to reduce the large air/SC-reflection of 30% to a few %.



Depletion layer

▼ Intensity distribution vertical to the surface

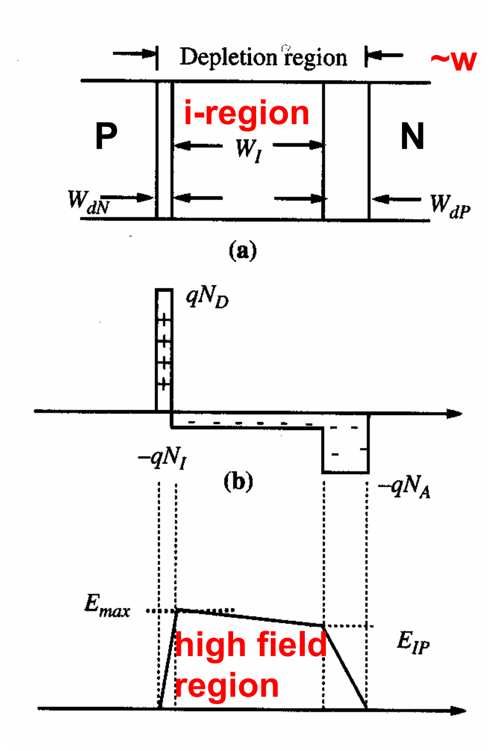
➡ large depletion width requires low doping concentrations (undoped I-layer)

### Layer sequence and doping of PIN-PD

There are conflicting requirements:

the depletion layer should be as close as possible to the surface for high absorption efficiency, on the other hand the contact p+-layer should not be too thin in order to keep the contact resistance $R_s$ and the time constant $R_sC_j$ low.

# Field distribution in a reverse biased PiN-Photodiode:



Space charge distribution $\rho(x)$

Fieldstrength E

Depending on the desired **electrical bandwidth $\omega_{-3dB}$**:

- The diameter A should be close to the fiber diameter of ~15- 50µm if butt-coupling without optics is anticipated
- The maximum depletion layer capacitance $C_j$ ~A ~1/$w_i$, should be a small as possible
- the depletion layer width $w_i$ is a compromise between efficiency (~$w_i$) and transit bandwidth (~1/$w_i$).

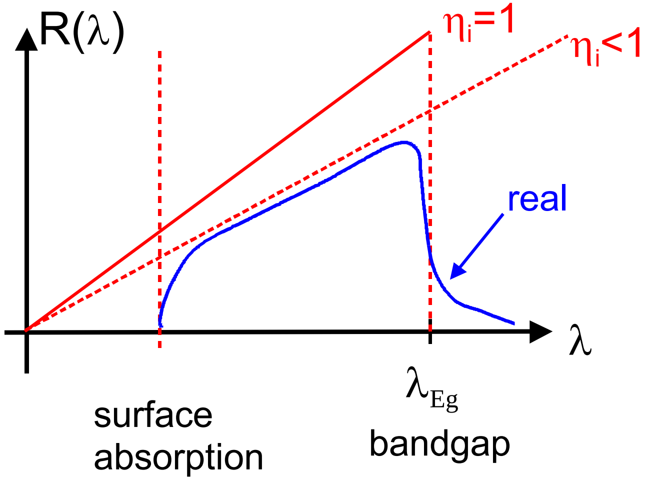
(To avoid this trade-off one can use **side-illuminated planer wave-guide PD** reaching bandwidth >300 GHz)

The **optical operation wavelength range of PIN-PD** (usable wavelength range)

$$R(\lambda) = \frac{e\lambda\eta_i}{hc_0} \sim \lambda$$ is limited at:

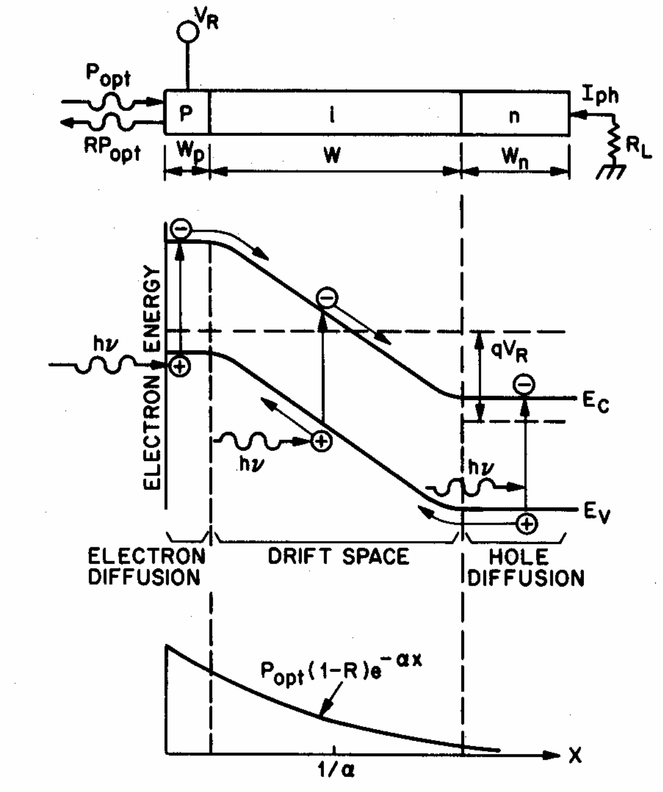
<u>long wavelengths by</u>:
- Bandgap $E_g$
- Large penetration depth and low efficiency

<u>Short wavelengths by</u>:
- Short penetration depth to reach the depletion layer, absorption in the thick top contact layer

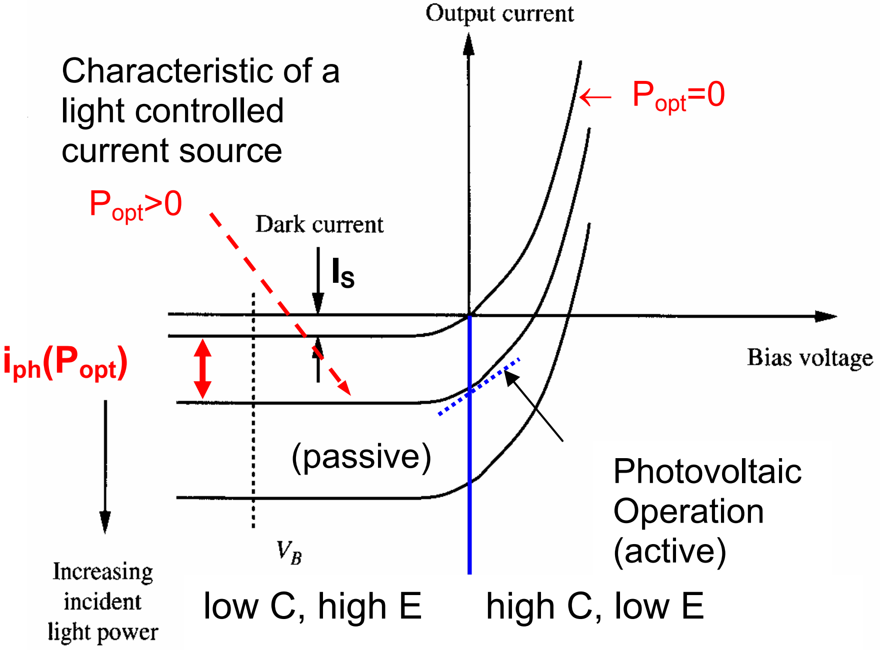


# 7.2.1.1 Band diagram of reverse-biased PIN-Diodes

PD are operated at reverse bias -$V_D$ for **current source characteristic, low depletion capacitance and high bandwidth**.



**Typical I-V-Characteristic of a Photodiode:**



Without illumination the PD only exhibits a **thermal and surface leakage current (dark current) $I_S$**, which must however be as small as possible, because they cause an inherent **2el-noise contribution**.

$$i_D(t) \cong \underbrace{I_S}_{\substack{\text{Dark}\\ \text{current}}} + \underbrace{i_{s,n}(t)}_{\substack{\text{Shot-noise of}\\ \text{dark current } I_S}} + \underbrace{i_{ph,n}(t)}_{\substack{\text{Shot-noise of}\\ \text{average Photocurrent}}} + \underbrace{RI_{opt}(t)A}_{\substack{\text{Signal}\\ \text{current}}} \qquad with\ \ V_D < 0$$

A = active Diode area
R = sensitivity of the PD [A/W]

**A PIN-PD can be operated without bias-voltage $V_D$=0, but its output resistance $r_0$ is low, the depletion width w small and its depletion capacitance $C_j(0)$ high (operation as active solar cells).**

# 7.2.1.2 Drift-Transport in i-depletion region

In contrast to the vacuum-PD the **inner photo effect in SC** produces distributed **e-h-pairs by a generation rate $R_{12}(x)$ proportional to $I_{opt}(x)$** which cause an **electron-** and **hole current**.

**Goals:**

As the intensity $I_{opt}(x)$ and the e-h-generation in PD are position dependent we integrate the incremental photocurrents $\partial j_p(x)$, $\partial j_n(x)$ over the depletion layer width 0<x<w to get the total terminal currents (holes and electrons) of the device $i_{ph}(t)$.

We are making the following assumptions and simplifications for the carrier transport in the i-region of the PD:

1) the carrier drift current is much higher than the diffusion current (small carrier density in the depletion layer, small $P_{opt}$)

$$j_{drift,n} = env_n >> j_{diff,n} = eD_n \frac{\partial}{\partial x} n$$

$$j_{drift,p} = epv_p >> j_{diff,p} = eD_p \frac{\partial}{\partial x} p$$

2) the electric field is $E \cong (-V_D + \phi_{bi})/w$ for $V_D<0$, meaning that we have neglected the carrier space charge of the drifting carriers (small photocurrents $i_{ph}$ )

3) e and h move with their saturation velocities $v_{s,n}$ and $v_{s,p}$

$$v_n(x) = v_{s,n} \quad , \quad v_p(x) = v_{s,p}$$

4) the rate of the stimulated absorption $R_{12}(x,t)$ is proportional to the intensity $I_{opt}(x,t)$.

$$R_{12}(x,t) = \alpha v_{gr} s_{ph} = +\alpha I_{out}(x,t)/(\hbar\omega)$$

5) carrier recombination by spontaneous recombination or other recombination processes (defects, traps etc.) is neglected

$R_{spont} \approx n/\tau << G_n = \text{thermal generation rate}$ because the transit time $T<<\tau$

6) 1-dimensional problem (x). The cross-section is A.

**Rate- or Continuity-equations for an incremental volume element Adx:**

With these assumptions we obtain the following continuity equations for electrons (and holes):

$$\frac{\partial}{\partial t}n = +R_{12}(x) - R_{spont}(x) - \frac{1}{e}\frac{\partial}{\partial x} j_{drift,n}(x) \cong +R_{12}(x) - \frac{1}{e}\frac{\partial}{\partial x} j_{drift,n}(x)$$

*if we assume* $R_{12}(x) >> R_{spont}(x)$ *and velocity saturation* $j_{drift} = e\,n(x)\,v_{sat,n}$

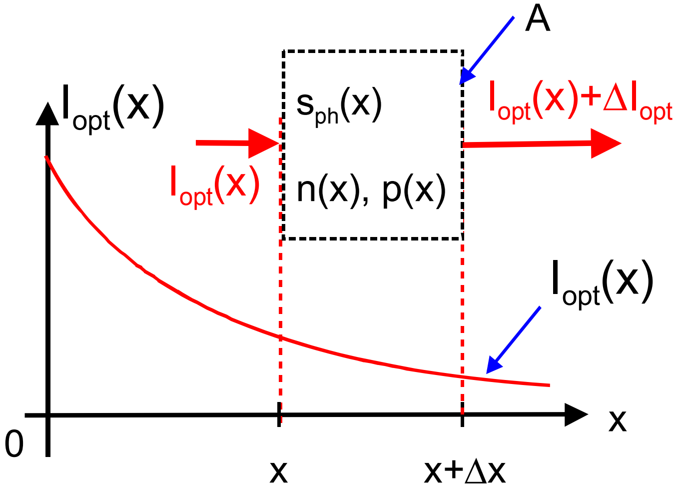
$$\frac{\partial}{\partial t}n = +R_{12}(x) - v_{sat,n}\frac{\partial}{\partial x} n(x)$$

Calculation of the relation between $I_{opt}$ and $R_{12,\,abs}$: (see chap.5)

*by the definition of* $\alpha$ *and using* $\partial x = v_{gr}\,\partial t \rightarrow$

$$\alpha = -\frac{\partial I_{opt}}{I_{opt}\,\partial x} \rightarrow \alpha I_{opt} = -\frac{\partial I_{opt}}{\partial x} = \frac{\partial s_{ph}\ \hbar\omega\ v_{gr}}{\partial x} = \frac{\partial s_{ph}\,\hbar\omega}{\partial t} = R_{12}\hbar\omega$$

Bild



$$R_{12}(x,t) = \frac{\alpha(\omega)}{\hbar\omega} I_{opt}(x,t) = R' I_{opt}(x,t) \quad , \quad R' = \frac{\alpha(\omega)}{\hbar\omega}$$

$$\frac{\partial}{\partial t}n \cong +R' I_{opt}(x) - \frac{1}{e}\frac{\partial}{\partial x} j_{drift,n}(x) \qquad static: \frac{\partial}{\partial t} = 0 \rightarrow$$

$$\frac{\partial}{\partial x} j_{drift,n}(x) = e v_{sat,n}\frac{\partial}{\partial x} n(x) = eR' I_{opt}(x) \qquad integrating\ \ across\ the\ deplation\ layer\ with\ x = 0 \rightarrow w$$

$$j_{drift,n}(w) - j_{drift,n}(0) = eR'\int_0^w I_{opt}(x)\,dx$$

*similar for holes:*

$$-j_{drift,p}(w) + j_{drift,p}(0) = eR'\int_0^w I_{opt}(x)\,dx$$

Under the assumption of

1) no diffusive current injection from the n- and p-areas and
2) current continuity through the depletion layer (no recombination or thermal generation)

we get:

$$1) \rightarrow j_n(0) = 0 \quad , \quad j_p(w) = 0 \qquad (\text{not very realistic assumption})$$

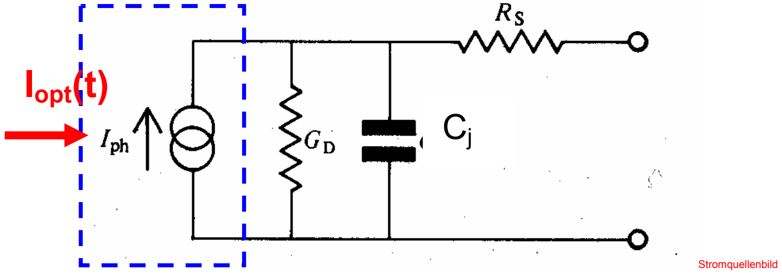
$$j_{drift,n}(w) = eR' \int_0^w I_{opt}(x)\,dx = j_{drift,p}(0) = eR' \int_0^w I_{opt}(x)\,dx$$

$$\underline{i_{ph} = AeR' \int_0^w I_{opt}(x)\,dx = i_{ph}(I_{opt}) \neq f(U_D)} \qquad \text{Current source characteristic, because } v_{sat} \text{ does not depend on voltage}$$

$$\underline{i_{ph} = AeR' \frac{1}{\alpha} \int_0^{I_{opt}(0)} dI_{opt} = \frac{e}{\hbar\omega}\left(P_{opt}(0) - P_{opt}(w)\right)}$$

**Intensity controlled current source** (Absorption in the neutral n- and p-areas and dark-current neglected)

## Electrical equivalent model of the pn-photodiode:



Transit-time limited response F(ω)

RC-Low Pass-response

# 7.2.2 Quantum efficiency of photodiodes

## Intensity distribution and responsivity R

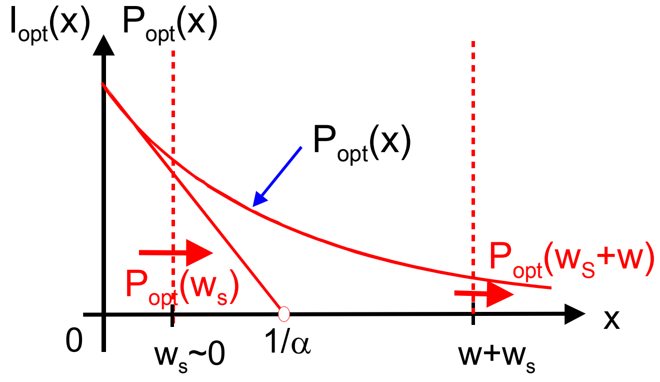
**Goal:**

Determination of how much optical power is converted into photocurrent as the light transits the **depletion layer**, where complete conversion is assumed

In the highly doped and non-depleted n- and p-contact areas [0,$w_s$] and [$w_s$+w, ∞], which are field free (E=0) there is undesired absorption (loss) of light. The e-h-pairs generated in these areas would have to **diffuse** (slow transport) to the depletion layer. If these e-h-pairs are generated away from the depletion layer by more than one **diffusion length $L_D$** they mostly recombine before reaching the high field area ($w_s<L_D$)

Therefore these parts of the absorbed optical power in the n- and p-contact layers do not contribute to the signal in the external load circuit and are lost.

To simplify the estimate of the absorption efficiency η in the depletion layer we assume that the thickness of the surface-contact layer is negligible compared to the thickness of the depletion layer w: $w_{surface,n,p} = w_s$ , $w_{surface,n,p} << w$.



$$I_{opt}(x) \cong I_{opt}(0)\, e^{-\alpha x}$$

$$P_{opt}(x) = A I_{opt}(0)\, e^{-\alpha x} = P_{opt,in}\, e^{-\alpha x}$$

The absorption efficiency or Quantum-efficiency $\eta$ of a PD is defined as:

$$\eta = \frac{P_{opt,absorbiert}}{P_{opt,in}} = \frac{P_{opt,in} - P_{opt,in}(w)}{P_{opt,in}} = \left[1 - e^{-\alpha w}\right]$$

*limiting ideal case:*
$w \to \infty \quad \to \eta = 1$

➡ thick intrinsic i-layers maximize the efficiency $\eta \to 1$ at the expense of transit time T and bandwidth B

The photocurrent $i_{ph}$ at the terminals becomes:

$$i_{ph} = P_{opt,absorbiert}\frac{e}{\hbar\omega} = P_{opt,in}\frac{e}{\hbar\omega}\left[1 - e^{-\alpha w}\right] = P_{opt,in}\underbrace{\frac{e}{\hbar\omega}\eta}_{Responsvity\ R}$$

($w_s$ of the contacting layer is neglected)

The total diode current of the reverse biased PIN-PD including **dark- or leakage current $I_S$** and the two **noise currents** from 1) the dark current and 2) the average signal current, **$i_{n,phot}$** and the **signal current $i_{ph}$**:

$$\boxed{i_D(t) \cong \frac{\eta e}{\hbar\omega}P_{opt,in}(t) + I_S + \underbrace{i_{n,phot}(t)}_{\sqrt{2eR\bar{P}_{opt}\Delta f}} + \underbrace{i_{n,s}(t)}_{\sqrt{2eI_s\Delta f}}}$$

# 7.2.3 Impulse and frequency response of PIN-Photodiodes (PD)

**Goal:**

**Determination of the intrinsic (**transit time limited, no electric parasitics**) impulse response of the i-layer of PiN-PD. The Fourier transform of the the δ-impulse response provides the frequency transfer function.**

For the calculation of the elementary, local **current impulse responses $i_{ph,e}(t,x_g)$** and **$i_{ph,p}(t,x_g)$** of a photogenerated **e-h-pair** with a **generation location $x_g$** within the depletion layer ($0< x_g <w$), we proceed in the same way as for the vacuum-PD, but have to consider the presence of two types of mobile carriers:

To get the **total** (whole depletion layer) **current impulse response $i_{ph}(t)$** we have to integrate over all **elementary** current impulse $i_{ph}(t,x_g)$ responses from all generation locations $0< x_g <w$ in the depletion layer.

We have to consider:

- Carrier pair-generation of two sort of charge particles, negative e and positive h traveling in opposite directions
- The particles move immediately with their saturation velocity $v_s$. The saturation velocity of electrons is often much higher than the saturation velocity of the slower holes: $v_{s,n} >> v_{s,p}$
- The depletion layer with the absorption $w<<1/\alpha$ (weak absorption) is assumed to be illuminated instantaneously (no propagation effects of the optical pulse through the depletion layer) by the optical δ-pulse:

$$I_{opt}(x,t)=I_{opt}(0)\delta(t)e^{-\alpha x} \quad with \quad A\int I_{opt}(0)\delta(t)dt=W_{opt}=\text{optical pulse energy}$$

$$I_{opt}(x,t)=W_{opt}(0)/A\,\delta(t)e^{-\alpha x}\xrightarrow[\alpha<<1/w]{} I_{opt}(x,t)\simeq W_{opt}(0)/A\,\delta(t)\neq f(x)$$

- The local absorption, resp. generation rate $R_{12}(x,t)=\alpha\left(n_{ph}v_{gr}\right)=\alpha I_{opt}(x,t)/(\hbar\omega)$ of photocarriers is distributed over the depletion layer width w. The elementary impulse response of individual e-h-pairs depends on the position $0<x_g<w$.
- The e-h carrier density is low ➡ no field distortion and no Coulomb-interaction between the carriers.

# Optical $\delta(t)$-pulse excitation and charge sheet Q' formation at $x_g$:

The spatially distributed optical $\delta$-pulse $I_{opt}(t,w_g)= W_{opt}(0)/A\delta(t)$ generates a two charge-sheets $\Delta Q_e=Q'_e\Delta x$ and $\Delta Q_h=Q'_h\Delta x$ in the incremental volume $A\Delta x$ (A=diode cross-section):

$$Q'_e = \frac{\partial Q_e}{\partial x} = -enA = \rho_e A \quad ; \quad Q'_h = \frac{\partial Q_h}{\partial x} = epA = \rho_h A \quad ; \quad Q' = \textit{charge per length} \quad \textit{n, p=electron-, hole density}$$

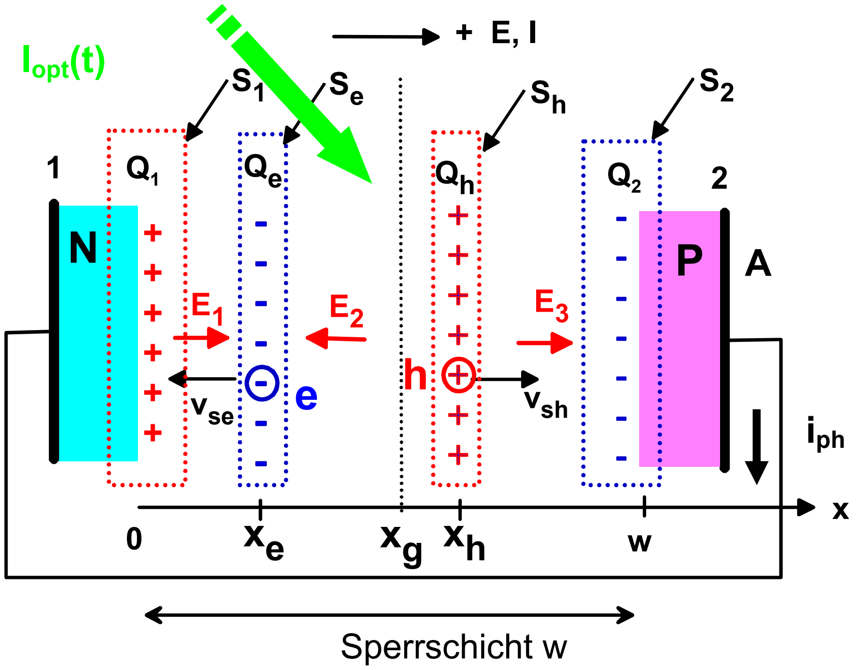
From the definition of $R_{12}(t,x_g)$ we get for the charges in the two charge sheets of length $\Delta x$:

$$\frac{\partial}{\partial t}\Delta Q_e = \frac{\partial}{\partial t}Q_e'\Delta x = -eR_{12}\left(t,x_g\right)A\Delta x = -e\alpha I_{opt}/(\hbar\omega)A\Delta x = -e\alpha W_{opt}\delta(t)/(\hbar\omega)\Delta x \xrightarrow{\int dt}$$

$$\Delta Q_e = Q_e'\Delta x = -e\alpha W_{opt}\int\delta(t)dt/(\hbar\omega)\Delta x = -e\alpha W_{opt}/(\hbar\omega)\Delta x \rightarrow \underline{Q_e' = -e\alpha W_{opt}/(\hbar\omega)} \quad ana\log: \quad \underline{Q_h' = e\alpha W_{opt}/(\hbar\omega)}$$

## Elementary-Event: Generation of an e-h-pair at the position $x_g$,

**contact n-area generation $x_g$ p-area contact**



After generation at t=0 at $x_g$ the e and h move immediately with the saturation velocities $\mathbf{v_{s,n}}$ und $\mathbf{v_{s,p}}$ apart in opposite directions.

$x_g$ = generation location of eh-pair at t=0 !

Position of the carrier sheets from $x_g(t=0)$ at time t.

$\mathbf{x_e=x_g-v_{se}t}$ $\mathbf{x_h=x_g+v_{sh}t}$

We use the same „Gedanken“-experiment of „charge smearing out“ for both electrons and holes as for the vacuum diode For the detailed derivation see Appendix 7B.

Applying the Gaussian relation for the closed surfaces $S_1$, $S_2$, $S_e$, $S_h$ and Kirchhoffs voltage law V(t)=0 we obtain for the time dependence of the charges $\Delta Q_1(t,x_g)$ and $\Delta Q_2(t,x_g)$ resp. by differentiation the induced photocurrent $\Delta I_{ph}(t,x_g)$ in the load.

$$Example: \quad current\ on\ the \quad p-side$$

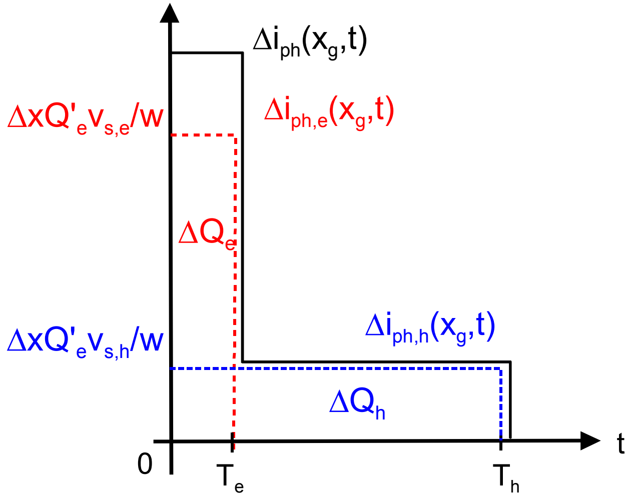
$$\Delta i_{ph}(t) = -\frac{\partial}{\partial t}\Delta Q_2(t) = \frac{\partial}{\partial t}\Delta Q_1(t)$$

$Q_{e,h}'$= e- or h-charge generated at $x_g$

$$\Delta i_{ph}(t,x_g) = \underbrace{\Delta x Q_h' \frac{v_{s,h}}{w}}_{\substack{holes\\ intransit}} - \underbrace{\Delta x Q_e' \frac{v_{s,e}}{w}}_{\substack{electrons\\ intransit}} = \Delta x Q_h' \frac{(v_{s,h}+v_{s,e})}{w} = photocurrent\,/\,unit\ length\ dx$$

This current equation demonstrates that the elementary currents of e and h add up (superposition of both currents). The current amplitudes are independent of $x_g$ (but this equation is valid only as long as the particles are present resp. moving, before being absorbed in the neutral contact region).

**Elementary e- and h-current pulses of the e-h-pair generated at $x_g$ :**



**Duration of the h-current pulse:**

$$T_h(x_g) = \frac{w-x_g}{v_{s,h}}$$

**Duration of the e-current pulse:**

$$T_e(x) = \frac{x_g}{v_{s,e}}$$

It is obvious that the durations of two elementary current pulses depend on the generation location $x_g$.

Bild des Treppenpulses selbst

# Impulse response of the photocurrent for a constant intensity distribution in a PIN-photodiode:

We calculate now for the case of a spatial constant optical intensity $I_{opt}(t,x)=I_0\delta(t)$ in the photodiode (weak absorption, $\alpha<<1$) the total current pulse $I_{ph}(t)$ from all local elementary current pulses $\Delta i_{ph}(t, x_g)$:

To get the total e- and h-current pulse we have to integrate ($\int dx_g$) over all rectangular elementary current pulses generated at $x_g$ in the depletion layer $0<x_g<w$ .

The integration of all elementary impulse responses gives for the total e-current pulse a **triangular pulse** using $Q'_h=Q'$:

$$I_{ph,e}(t)=-\frac{\partial}{\partial t}Q_{2,e,tot}(t)=Q'\frac{1}{w}\left[wv_{s,e}-v_{s,e}^2t\right]=Q'\left[v_{s,e}-\frac{v_{s,e}^2}{w}t\right] \qquad \textit{(left moving electrons: only displacement current)}$$

$$\textit{Triangular pulse with amplitude:}\quad Q'v_{s,e} \quad \textit{and duration:}\quad T_e=w/v_{s,e}$$

$$\textit{Tranported charge:}\quad Q'w/2 \quad (\textit{observe the definition of the positive current direction of } I_{ph})$$

Observe that we calculated only the displacement current induced by the left-moving e at electrode 2.

The determination for the h-current pulse at x=w proceeds in the same way, but there is for the holes an addition convection current at the electrode 2 (holes flowing into the electrode 2):

$$I_{ph,h}(t)=\underbrace{I_{ph,h}(t)}_{\substack{\textit{displacement}\\ \textit{current}}}+\underbrace{I_{ph,h,conv}(t)}_{\substack{\textit{convection}\\ \textit{current}}}=-Q'v_{s,h}^2t+Q'v_{s,h} \qquad (\textit{right moving holes: displacement and convection currents})$$

$$\textit{Triangular pulse with amplitude:}\quad Q'v_{s,h} \quad \textit{and duration:}\quad T_h=w/v_{s,h}$$

$$\textit{Transported Charge:}\quad Q'w/2 \quad \rightarrow \textit{total transported charge: } Q_{tot}=Q'w$$

We get from the integration [see appendix 7B] the total photocurrent impulse response (response to an optical δ-pulse $W_{opt}\delta(t)$):

$$I_{ph}(t)=I_{ph,e}(t)+I_{ph,h}(t)=$$

$$\underline{I_{ph}(t)=Q'w\left[\frac{v_{s,e}}{w}-\frac{v_{s,e}^2}{w^2}t\right]+Q'w\left[\frac{v_{s,h}}{w}-\frac{v_{s,h}^2}{w^2}t\right]}$$

Observe that we calculated the displacement- and particle current of the right-moving h at electrode 2.

# e- and h-current impulse response in a PIN-photodiode with constant intensity in the depletion layer:

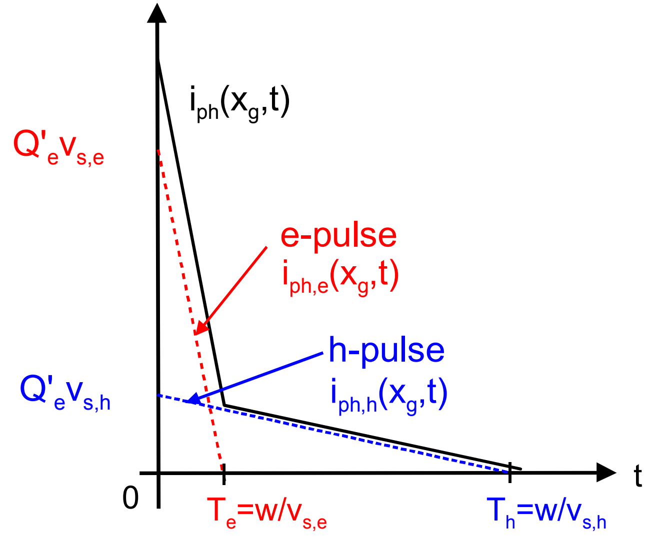


Bild der Stossantwort selbst

Remark:
the above current equations are only valid as long as the resp. particles are present and moving.
Otherwise the currents are =0.

- holes are often much slower $v_{s,h} << v_{s,e}$, thus they produce a long tail in the impulse response, whereas fast electrons form a short spike. The tail leads to an undesired increase of the frequency response at low frequencies $\omega < v_{s,h}/w$.

- for the carrier type, which flows **into** the contact we have to determine 2 current contributions
  1) **displacement current**, and
  2) **conductions- (particle) current**

Because $Q_2$ on the electrodes 2 is used for the current calculation, it is the holes, that generate an additional particle current at x=w (p-area).

The frequency response of the normalized $H(\omega)/H(0)$ can be obtained from the impulse response $i_{ph,eleemnt}(t)$ at least in principle by a Fourier-transformation F:

$$\frac{H(\omega)}{H(0)} = \frac{I_{ph}(\omega)}{I_{ph}(0)} = \frac{F\left(I_{ph}(t)\right)}{I_{ph}(0)}$$

For real diodes we would have to carry out the same calculation with a more realistic, exponential intensity distribution.

# 7.2.4 Electrical equivalent model of the PiN-photodiode

## Small Signal equivalent model of PIN-PD

Beside the frequency response of the primary photocurrent $i_{ph}(t)$ due to the transit time T of the i-absorption layer, there are additional parasitic elements, which modify the modulation transferfunction:

- Depletion layer capacitance $C_j$
- Contact / access resistance $R_S$
- Load resistance $R_L$ ; load capacitance $C_L$

$C_j$ and $R_S$ and $R_L$ form an additional RC-low pass filter, reducing the intrinsic bandwidth (transit time limited) of the PiN-diode further.

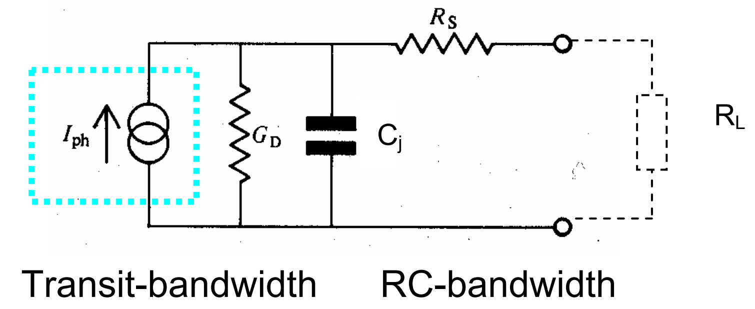
The calculation of the depletion layer capacitance is simple, because in the PiN-structure the diode voltage drops over the completely depleted i-layer:

$$C_j \cong \varepsilon_0 \varepsilon_r A / w$$

The calculation of $R_S$ is not trivial because of the 2-dimensional current flow and the contact ring geometry.

- for reaching small RC-time-constants $\tau = (R_S + R_L) C_j$ ~A, the area A of fast PIN-PD must be small.
- an optimization of the I-layer thickness w for bandwidth and efficiency is necessary.

## Electrical small-signal equivalent circuit of the PiN-Photodiode:



$G_D$ is mostly very small and can be neglected compared to $R_S$ and $R_L$.

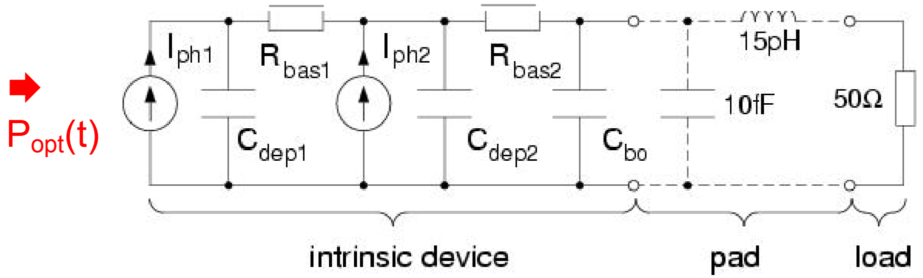
## Speed Limitation of the Small Signal Properties

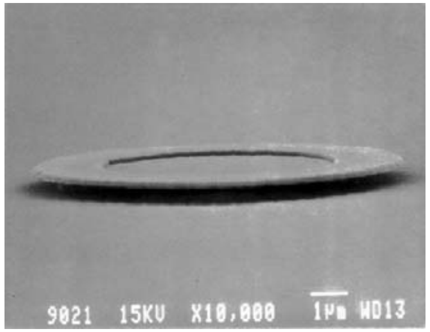
Planar Photodiodes are limited with respect to bandwidth due to:

- Trade-off for the depletion layer thickness w
- Drift of slow holes
- Minimal diode area for still efficient fiber-PD coupling

Typical PIN-PD for fast communication systems reach easily a bandwidth between 30-60 GHz. For research prototypes record values of >300 GHz have been reached  (side-illuminated traveling waveguide structure PD).

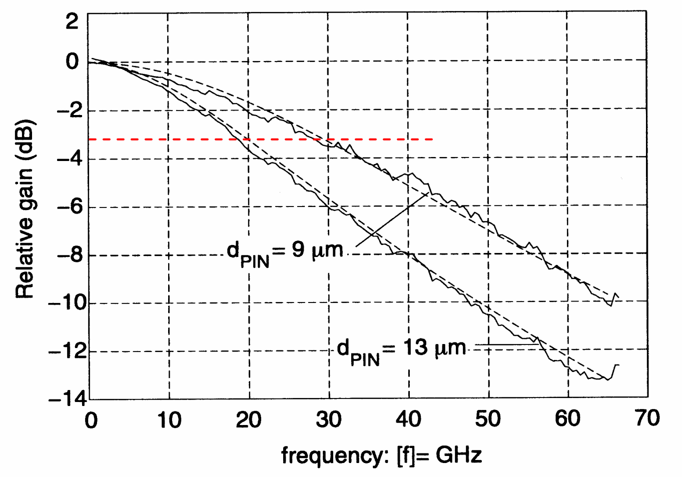
## Example of circuit model, SEM-micrograph and measurement of an InP/InGaAs-PD with 15µm diameter)





∅ ~9µm

Simulated and measured frequency response @$R_L$=50Ω:



# 7.2.5 Noise Model of the PIN-Photodiodes

As already mentioned the **dark current $I_s$** (reverse current) and also the **average signal-photocurrent** $\overline{i}_{ph}$ are both random rate processes producing 2el-noise.

The series resistance $R_S$ in addition produces thermal $4kTR_s$-noise.

The power density spectrum of these noise sources are:

- **Dark current noise** (signal independent)

$$i_{n,s}^2(\omega) = 2ei_S |F(\omega)|^2$$

- **Signal-photo current noise** (signal dependent)

$$i_{n,ph}^2(\omega) = 2e\overline{i}_{ph} |F(\omega)|^2$$

- **Thermal noise in series resistor $R_s$** (signal independent)

$$u_{n,r}^2(\omega) = 4kTR_S$$

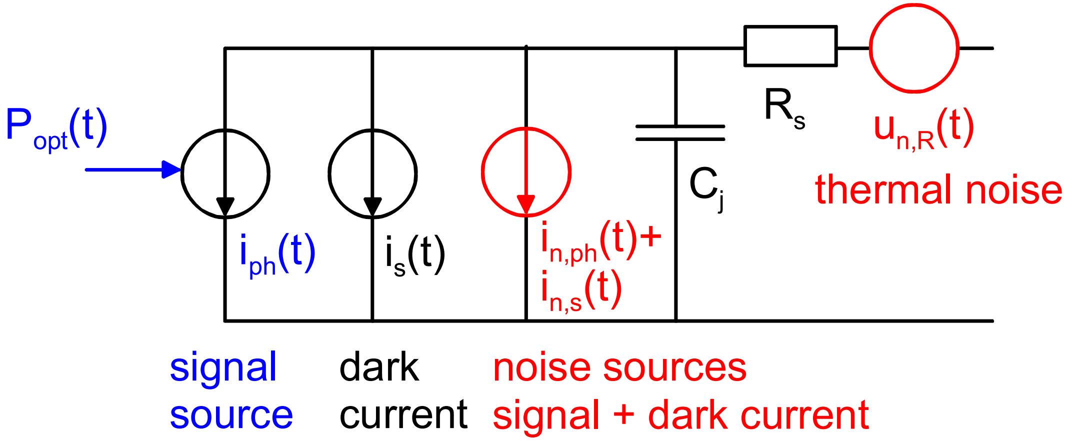
- **Additional noise from the optical source (RIN)**

$$i_{RIN}^2(\omega) = RIN(\omega)\, \overline{i}_{ph}^{\,2}$$ (see RIN-definition in chap.6)

**Noise-equivalent circuit model**

To include the noise effects we include 3 additional AC-current sources representing the noise currents. The noise current sources are characterized by their power spectral densities:

$$.i_{n,S}^2(\omega),\ i_{n,ph}^2(\omega) \text{ and } u_{n.r}^2(\omega)$$



Because the noise signal are random, nothing can be said about the time dependence of $i_n(t)$. Beside the information about the spectral power density only $\overline{i_n(t)}=0\,;\,\overline{i_n^{\,2}(t)}=2e\overline{I_{ph}}B$ the assumed Gaussian **amplitude probability density function P($i_n$)** can be related to the spectral power density $i_n^2(\omega)$ of the noise signal (see chap.9).

# 7.3 Avalanche-Photodiode **(APD)** (Lawinendurchbruch-Dioden, qualitativ)

Avalanche (Lawinenmultiplikations)-Photodiodes (APD) are a modified PIN-photodiode, which contains an **internal current amplification mechanism** for the primary (carriers generated by direct light absorption) photocurrent.
The carrier multiplication results from **impact ionization by "hot" e and/or holes in a very high electrical fields**.

## Principle of operation: Carrier multiplication by impact-ionization

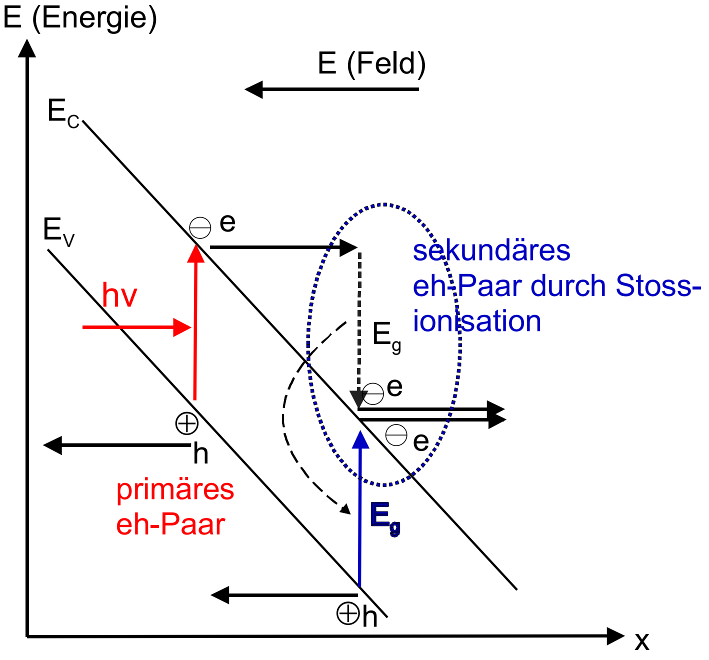
The electric E-field, close to the **breakdown** of the semiconductor, accelerates carriers so much, that they get enough kinetic energy between collision for exciting a valenceband-electron into the conductionband $E_{kin}\sim eE_g$.

By this **elementary e-h-pair generation due to impact-ionization** the primary particle has increased the total particle number by 1.

Repetition of the process leads to an **M-fold ionization avalanche or carrier number increase by M during transit**.

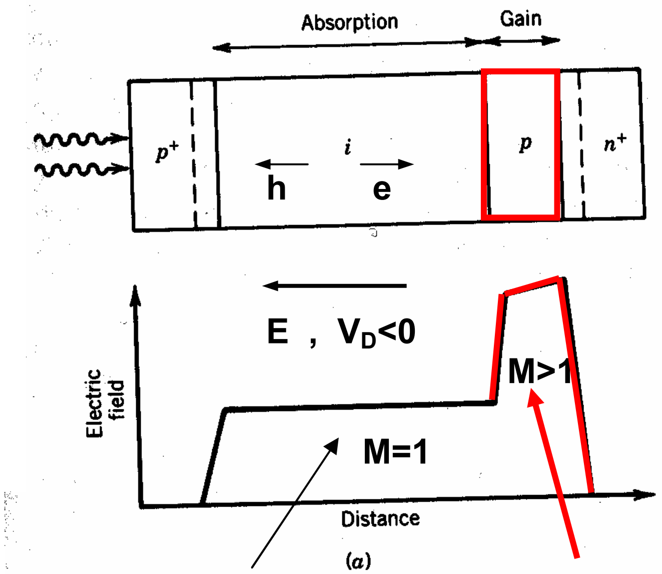
→ **primary e-h-Generation by photon absorption**

→ **secondary e-h-Generation by impact ionization**

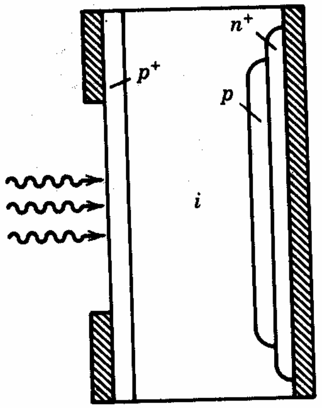


**primary** **secondary eh-pair**

## Schematic structure of a $p^+ipn^+$-AP



## Planar $p^+ipn^+$-APD:



(b)

long Drift - and Absorptions-region, low field

short Impact-Ionization-, Multiplication-region, high field, low absorption

The advantage of the APD is a very compact mechanism for **current-multiplication** $M = I_{ph,tot} / i_{ph,prim}$ of the primary photocurrent generated by absorption in the I-region of the APD.

$$\boxed{i_{ph}(t) = i_{ph,prim}(t)M = i_{ph,prim}(t) + i_{ph,prim}(t)(M-1)}$$

$i_{ph,prim}$ = primary photocurrent by photon absorption (for M=1, resp. $V_D \rightarrow 0$)
$i_{ph}$ = total photocurrent = primary photocurrent by absorption + secondary current by impact-ionization

## Excess-Noise in APDs: (Multiplication Noise)

However not only the primary photocurrent of the signal $i_{ph,prim}$ is amplified by M (but also the noise of the primary current):

The **primary shot-noise** is amplified, but because the ionization (multiplication by M) is also a statistical process with $M = \bar{M} + \Delta M$. This leads to an **excess current noise** (dependent on M) contribution of the APD-gain added to the signal.

# 7.3.1 Concept of Avalanche build-up in APDs

**Absorption- and Multiplication Areas**

APDs mostly consist from 2 functional regions:

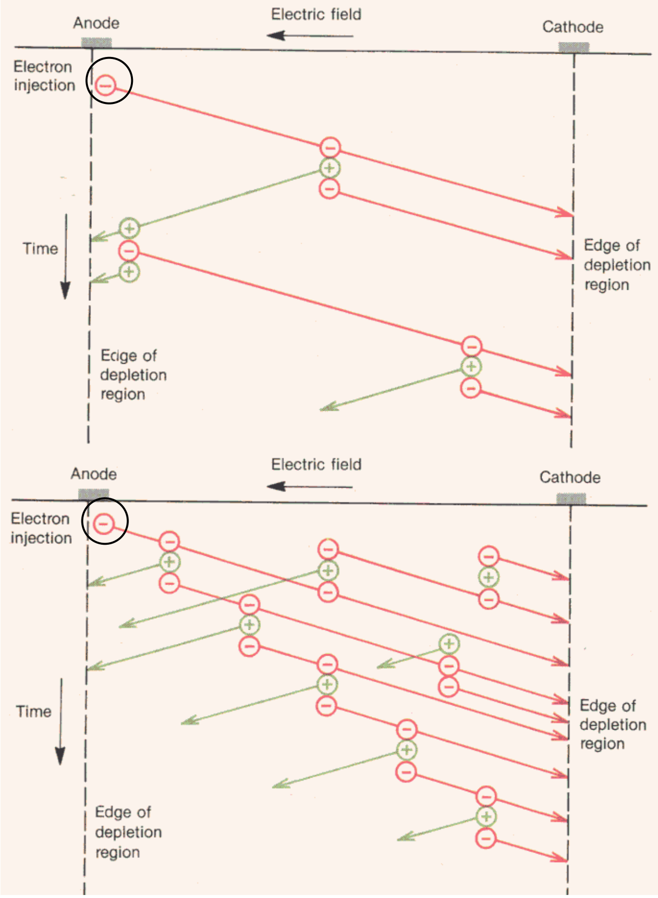
1. **Absorption region (i)**,
   where the primary e-h-pair generation occurs by photon absorption, but the electric field is too low for impact ionization.
   Wide layer for high quantum efficiency.

2. **Multiplication region ($pn^+$)**,
   little absorption occurs in the thin layers. The field E in a narrow region ($pn^+$-depletion region) is so high that carrier ionization takes leading to a multiplication M of the primary injected current from the absorption region.

**Transport- und Multiplications-Process:**

- generation of primary eh-pairs in the i-region by photon-absorption.
- Holes drift to the left to the $p^+$-contact region where they are majority-carriers
- electron drift to the right entering the multiplication $pn^+$-region, where they are accelerated by the high E-field until their kinetic energy $>eE_g$ and impact ionization occurs
- The ionization process continues (M-1)-times, leading to a total multiplication of $M(V_D)$.

**Schematic representation of avalanche-multiplication:**



**With e- and h-Ionization**

○ Avalanche start

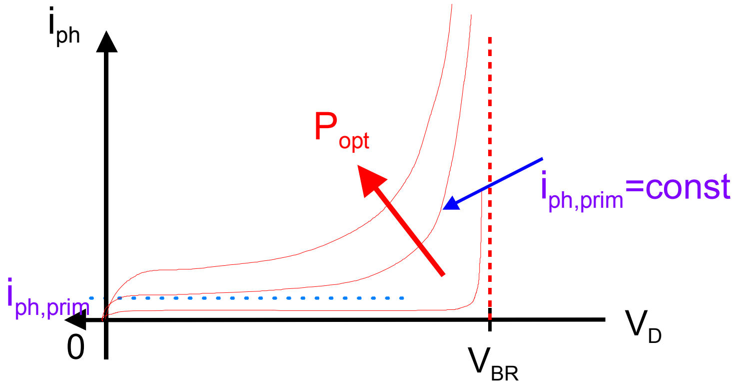
**With only e-Ionization**

Because of the statistical nature of impact ionization M is a fluctuating value leading to an **internal multiplication-noise** (excess-noise) generation.

As the APD is operated close to breakdown, the multiplication factor M is very dependent on reverse diode voltage. M reach practical values of 3 – 1000, however for reasons of **minimizing excess-noise M~20-40 are practical values**.

**Remark:** the carrier avalanche resulting from the initial e-h-photopair can, depending on the details of the APD structure and the materials involved, consist of 2 carrier avalanches for e and h.

# 7.3.2 I-V-characteristic and electrical equivalent APD-models



## V-I-Characteristic (without proof)

- An increase of $V_D$ lead to an increase in E, the ionization-rates and the multiplication $M(V_D)$
- The multiplication factor has a singularity at the breakdown voltage $U_{BR}$, making the voltage stabilization and thermal control of the APD difficult. Still practical are M~1000.

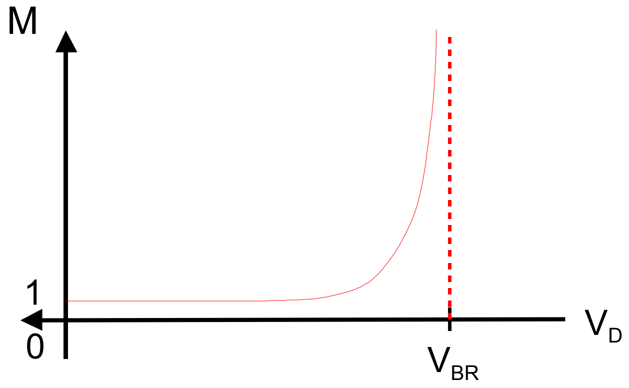
$$i_{ph}(t)=i_{ph,prim}\left(I_{opt}(t)\right)M\left(V_D\right)$$

### Multiplication-Characteristic $M(V_D)$ of APDs

From the $i_{ph}(V_D)$-characteristic we obtain the multiplication factor $M(V_D)$:

$$M\left(U_D\right)=\frac{i_{ph}}{i_{ph,prim}}\cong\frac{1}{\left[1-\left(\frac{V_D}{U_{BR}}\right)\right]}$$

for $V_D$ , $U_{BR}<0$



# 7.3.3 Multiplication-Noise of APDs

**Signal-Amplification**

$i_{ph}(\omega) = i_{ph,prim}(\omega)\ M$ *current gain*

$P_{ph}(\omega) = R_L\ i^2_{ph,prim}(\omega)\ M^2$ *power gain*

$R_L$ = load resistance

## Excess multiplication-noise and noise figure F(M)

The primary photocurrent $i_{ph,prim}$ contains at least shot-noise $i^2_{ph,n,prim} = 2e\,\overline{i}_{ph,prim}$ for an optical source with a RIN=0. Fluctuation in M lead to additional noise such that the noise power increases with $M^X$, x>2 rather than $M^2$ as for the ideal, multiplication-noise-free case.

**Amplification of the primary noise power:**

$$i^2_{ph,n} = i^2_{ph,n,prim}(\omega) M^x B = i^2_{ph,n,prim}(\omega) M^2 F(M) B \quad ; \quad x > 2$$

B=noise bandwidth

$$P_{ph,n} = i^2_{ph,n,prim}(\omega) M^x B\, R_L$$

**F is the Noise-Figure of the APD** depending on $M(V_D)$ is defined as:

$$F(M) = M^{x-2}$$

**Carrier multiplication inherently increases the noise of the photodetection process by the noise factor F.**

As a **figure of merit** for the quality of a noisy signal or for the error-probability in signal dectection one defines the **signal-to-noise ratio** SNR:

$$SNR = \frac{P_{ph}}{P_{ph,n}} = \frac{i^2_{ph,prim} M^2}{i^2_{ph,n,prim} M^x B} = \frac{i^2_{ph,prim}}{2ei_{ph,prim} B} \quad M^{2-x} = \frac{i_{ph,prim}}{2eB} M^{2-x}$$

**SNR decreases with increasing M and BER increases**

The APD is a relative low noise, broadband (GHz) amplifier for modest gain M~10-20 (larger M reduces the bandwidth of the APD)

- **For the case that the noise is dominated by the noise of the photodetector or by the optical source (RIN) then and APD does not improve the SNR as compared to a simple PIN-PD.**

## Optimal Gain $M_{opt}$ for APD and preamplifier:

In general the gain M of the APD is still too low to amplify a small optical signal into the Volt-range. Therefore additional gain from a following electronic amplifier (eg. current-voltage transimpedance amplifier) is required.

This amplifier with a transimpedance R adds also noise, which may or may not dominate at high frequencies the photodetection noise.

### Amplifier noise:

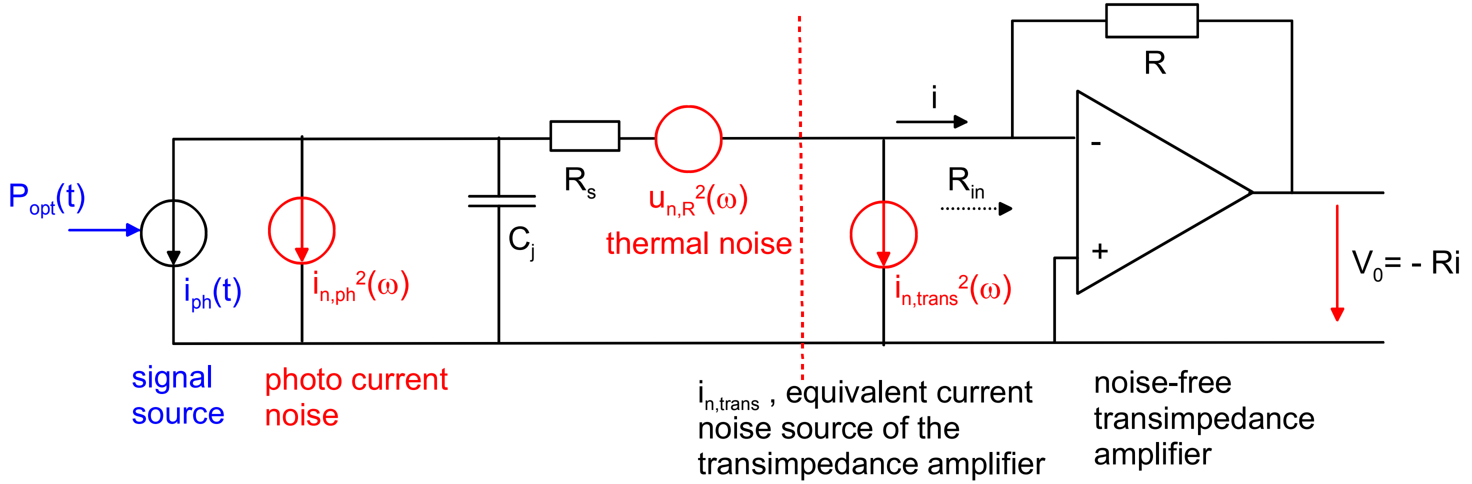
The noise of the amplifier is represented by an **equivalent input noise current source $i_{n,trans}(t)$ at its input**.

**This input noise source $i_{n,trans}(t)$ is characterized by its spectral noise power density $i_{n,trans}^2(\omega)$.**

The APD with gain M produces the signal photocurrent $i_{ph}(t)=M\, i_{ph,prim}(t)$, characterized by its amplitude spectral density $i_{ph}(\omega)$.
In addition the signal current is superposed by a noise current generated in the APD, with a spectral power density $i_{ph,n}^2(\omega)$.

For the calculation of the SNR and the noise figure F of the PD and the amplifier, we evaluate the noise power produced by all noise sources at the input impedance $R_{in}$ of the transimpedance-amplifier.

# Equivalent circuit of the APD and the transimpedance amplifier:



$R_{in}$ = input impedance of the Trans-impedance-amplifier

R = $v_0/i$ = transimpedance (ideal)

Calculation of SNR at the input impedance $R_{in}$ of the “noise-free” transimpedance amplifier and neglected thermal noise $u_{n,r}$.

*input signal power at frequency* $\omega$:

$$P_{S,in} = R_{in} i^2_{ph,prim}(\omega) M^2(\omega) \le R_{in} \bar{i}^2_{ph,prim} M^2(\omega) \quad if \quad i_{ph,prim}(\omega) < \bar{i}_{ph,prim} \quad \left( for\ 100\% - modulation\ P_{opt} = \bar{P}_{opt} + \bar{P}_{opt} cos(\omega t) \right)$$

*noise power at the input* $R_{in}$ :

$$P_{n,in} = R_{in} 2e\bar{i}_{ph,prim} M^x(\omega) B + R_{in} i^2_{n,trans}(\omega) B$$

$$= \underbrace{R_{in} 2e\bar{i}_{ph,prim} F(\omega) M^2(\omega) B}_{APD-noise} + \underbrace{R_{in} i^2_{n,trans}(\omega) B}_{equivalent\ Amplifier-noise}$$

*APD- or amplifier noise may dominate*

$$using \quad F(M) = M^{x-2} , \quad x > 2 \quad ; \quad B = bandwidth$$

Now we evaluate the SNR of APD **and** transimpedance-amplifier versus APD-Gain M for 100% signal-modulation:

$$SNR(M) = \frac{P_{S,in}}{P_{n,in}} = \frac{\bar{i}^2_{ph,prim}(\omega) M^2(\omega)}{2e\bar{i}_{ph,prim} F(M) M^2(\omega) B \;+\; i^2_{n,trans}(\omega) B}$$

$$a)\; M = klein \quad \rightarrow 2e\bar{i}_{ph,prim} \quad F(M) M^2(\omega) B << i^2_{n,trans}(\omega) B \qquad \textit{dominated by amplifier-noise}$$

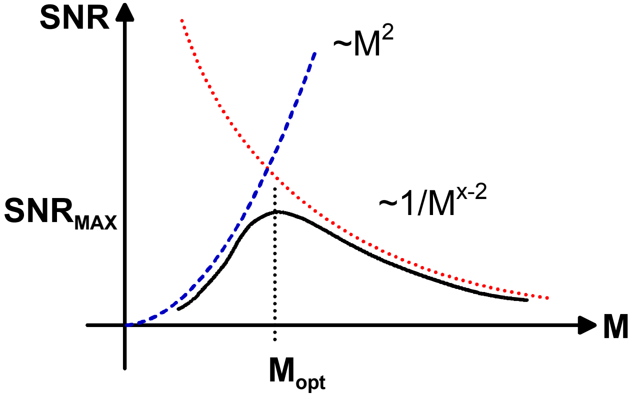
$$SNR = \frac{\bar{i}^2_{ph,prim}(\omega) M^2(\omega)}{i^2_{n,trans}(\omega) B} \approx M^2$$

$$b)\; M \; gross \quad \rightarrow 2e\bar{i}_{ph,prim} \quad F(M) M^2(\omega) >> i^2_{n,trans}(\omega) \quad dominated \;\; by \;\; APD - noise$$

$$SNR = \frac{\bar{i}_{ph,prim}(\omega)}{2e\, F(M) B} \approx M^{-(x-2)} \;, \qquad x > 2$$

**→ It is obvious that an optimum multiplication $M_{opt}$ must exist, where the SNR reaches its maximum:**

**SNR of a APD vers. M:**



Amplifier dominated noise APD-dominated noise

## Conclusions:

- Semiconductor photodiodes (PD) offer a very efficient optical-electrical conversion ($\eta_i$~50 – 90%) combined with high bandwidth (40 → 300 GHz) and shot-noise limited detection
- Photodiodes fabricated from different semiconductors cover a wast range of optical wave lengths from UV → FIR, but in particular the communication wavelengths at 0.8, 1.3 und 1.55µm.
- Photodiodes are compact, low cost and mass produced. Fiber-coupling and high speed packaging may be a non-negligible cost factor
- A drawback is that fast photodiodes require small detection areas and precise fiber alignment
- At high data rates the noise of electronic preamplifiers is often dominant, therefore optical pre-amplifiers (SOAs, EDFAs) can be used if cost considerations are not dominant.
- The depletion width w in PiN-photodiodes determines (trade-off) the transit time $\tau$, the responsivity R and also influences the external bandwidth by defining the depletion capacitance $C_j$. Therefore w has to be carefully optimized in photodiode design.
- The bandgap $E_g$ and saturation velocity $v_s$ determine as material parameters essentially the range of the optical sensitivity and the bandwidth of the PD.