This article was downloaded by: [CDL Journals Account]

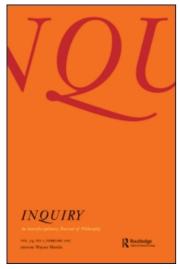
On: 12 May 2011

Access details: *Access Details:* [subscription number 922973516]

Publisher Routledge

Informa Ltd Registered in England and Wales Registered Number: 1072954 Registered office: Mortimer House, 37-

41 Mortimer Street, London W1T 3JH, UK



Inquiry

Publication details, including instructions for authors and subscription information: http://www.informaworld.com/smpp/title~content=t713393858

Frege's Elucidatory Holism

Clinton Tolleya

^a University of California, San Diego, USA

Online publication date: 12 May 2011

To cite this Article Tolley, Clinton(2011) 'Frege's Elucidatory Holism', Inquiry, 54:3,226-251

To link to this Article: DOI: 10.1080/0020174X.2011.574999 URL: http://dx.doi.org/10.1080/0020174X.2011.574999

PLEASE SCROLL DOWN FOR ARTICLE

Full terms and conditions of use: http://www.informaworld.com/terms-and-conditions-of-access.pdf

This article may be used for research, teaching and private study purposes. Any substantial or systematic reproduction, re-distribution, re-selling, loan or sub-licensing, systematic supply or distribution in any form to anyone is expressly forbidden.

The publisher does not give any warranty express or implied or make any representation that the contents will be complete or accurate or up to date. The accuracy of any instructions, formulae and drug doses should be independently verified with primary sources. The publisher shall not be liable for any loss, actions, claims, proceedings, demand or costs or damages whatsoever or howsoever caused arising directly or indirectly in connection with or arising out of the use of this material.



Frege's Elucidatory Holism

CLINTON TOLLEY

University of California, San Diego, USA

(Received 2 March 2010)

I argue against the two most influential readings of Frege's methodology in the philosophy of logic. Dummett's "semanticist" reading sees Frege as taking notions associated with semantical content—and in particular, the semantical notion of truth—as primitive and as intelligible independently of their connection to the activity of judgment, inference, and assertion. Against this, the "pragmaticist" reading proposed by Brandom and Ricketts sees Frege as beginning instead from the independent and intuitive grasp that we allegedly have on the latter activity and only then moving on to explain semantical notions in terms of the nature of such acts. Against both readings, I argue, first, that Frege gives clear indication that he takes semantical and pragmatical notions to be equally primitive, such that he would reject the idea that either sort of notion could function as the base for a non-circular explanation of the other. I argue, secondly, that Frege's own method for conveying the significance of these primitive notions—an activity that Frege calls "elucidation"—is, in fact, explicitly circular in nature. Because of this, I conclude that Frege should be read instead as conceiving of our grasp of the semantical and pragmatical dimensions of logic as far more of a holistic enterprise than either reading suggests.

I. Frege's alleged orders of explanation

In what follows I argue against the two currently most influential readings of Frege's methodology in the philosophy of logic, what I call the "semanticist" and the "pragmaticist" readings, respectively.

The semanticist reading has been the most common and influential of the two interpretations, due in large part to its being championed by Michael Dummett. In Dummett's view, Frege takes notions associated with semantical content—and in particular, the semantical notion of truth—as primitive and as intelligible independently of their connection to the notions of the mental activity of judgment and inference or the linguistic practice of assertion,

Correspondence Address: Clinton Tolley, Department of Philosophy, University of California, San Diego, 9500 Gilman Drive, La Jolla, CA, 92093-0119, USA. Email: ctolley@ucsd.edu

0020-174X Print/1502-3923 Online/11/030226-26 © 2011 Taylor & Francis

DOI: 10.1080/0020174X.2011.574999

such that the former semantical notions can then be used to explain what judgment, inference, and assertion are.¹ Dummett himself views this feature of Frege's method as a "retrograde step on Frege's part", part and parcel with Frege's eventual conception of the True as an object to which certain sentences refer in the same way that other complex terms stand for individuals.² This, in turn, is what motivates one of Dummett's most well-known criticisms of Frege—namely, that Frege fails to recognize that "it is part of the concept of truth that *we aim* at making true statements" (emphasis added).³

Yet despite its considerable influence, Dummett's interpretation faces a serious and quite straightforward obstacle. This is the simple fact that in both of Frege's two most sustained presentations of the foundations of his logic—his 1879 Begriffsschrift and the first (1893) volume of his Grundgesetze der Arithmetik⁴—Frege explicitly includes among his "primitive signs [Urzeichen]" notation not just for the expression of semantic content, but also for the expression of the acts of judgment, assertion, inference, and definition. In fact, Frege makes use of the very same signs for the expression of the very same acts in both the earlier and the later works. This would seem to imply, then, that Frege means to accord a much more fundamental status to these pragmatical notions than Dummett's reading suggests.

Dummett's own treatment of this simple fact is unsatisfying. Though he acknowledges that "Frege indeed tried to bring in" pragmatical considerations through his "theory of assertion", Dummett claims that these are brought in only "afterwards" and so "too late", which Dummett takes to show that Frege fails to recognize that the very notion of semantic content "is not given in advance of our going in for the activity of asserting". A closer look at the *Begriffsschrift* and the *Grundgesetze*, however, gives precisely the opposite impression, as the notions of acts of judgment and assertion are introduced at one and the same time—indeed, in the very same sections—as the notion of judgeable content itself.

It is with good reason, therefore, that Dummett's semantics-first, or "semanticist", interpretation of Frege, as I will call it, has come in for criticism of late, precisely for its failure to take the measure of the intimate link that Frege appears to recognize between the notions of semantical content and certain kinds of mental activity. Two of the more sustained criticisms along these lines have been provided by Robert Brandom and Thomas Ricketts. Brandom, for example, rightly points out that Frege's increasingly explicit discussion of assertoric force in his later writings at the very least "immunizes Frege somewhat from Dummett's scandalized response" to Frege's introduction of the doctrine of truth-values as *Bedeutungen*, since, as Brandom also (rightly) points out, the latter aspect of Frege's views "need not be the *whole* story about sentences" from the logical point of view (emphasis added). Ricketts, too, criticizes Dummett for "read[ing] a contemporary view back into Frege" concerning the priority of semantics to pragmatics, one that is "antithetical to Frege's conception of judgment". 10

Yet while there is clearly something right in these efforts to correct the blind spots of the semanticist reading, there is also reason to think that Brandom and Ricketts have gone too far in the opposite direction. This is because their proposed replacement for Dummett's reading is one that, in effect, simply turns the semanticist interpretation on its head, arguing that we ought to see Frege as instead embracing a pragmatics-first, or "pragmaticist", approach to the foundations of logic.

For his part, Brandom concedes that Dummett is right to think that semantical considerations come to play a uniquely foundational role for Frege after 1890, once Frege conceives of a truth-value as the *Bedeutung* of a sentence. Prior to this stage in Frege's development, however, Brandom argues that Dummett gets things exactly backwards, insofar as Dummett fails to appreciate the extent to which "the young Frege" in fact means to give priority to pragmatical notions to such an extent that (like Brandom himself) Frege intends to provide a reductive explanation of the very notion of semantical content in terms of the activity or practice of assertion and inference. In other words, on Brandom's reading "the young Frege" ends up holding views consonant with Brandom's own "pragmatist" attempt "to explain what is asserted by appeal to features of assertings", and so explain "in general, the content by the act, rather than the other way around". 12

Ricketts, by contrast, enjoins an even more thoroughgoing rejection of Dummett's semanticist interpretation in favor of a pragmaticist reading, ¹³ insofar as Ricketts argues that throughout Frege's entire career (and so even after the introduction of truth-values as *Bedeutungen* of sentences) Frege's conception of the foundations of logic is always rooted first and foremost in his conception of the activity of judgment. For Ricketts, Frege is more accurately thought of as putting forward a "*metaphysics* of judgment" (emphasis added), with the intent of replacing the metaphysics of objects and concepts that would otherwise have allowed for Frege's approach to stand closer to contemporary formal semantics, as Dummett's reading implies. ¹⁴

Yet precisely because it is ultimately simply an inversion of Dummett's position, the very same sort of problem that stands in the way of Dummett's semanticist interpretation would seem to stand in the way of Brandom's and Ricketts' pragmaticist one as well. For it is hard to see how the pragmaticist reading will be able to do justice to the other half of the simple fact we noted above—namely, the fact that both the younger and the older Frege include what he himself calls "primitive signs" for *not just* pragmatical *but also* semantical notions at the beginning of both of his mature works on logic.

Indeed, this simple fact suggests that Frege does not share the assumption that the pragmaticist and the semanticist interpretations both have in common—namely, that Frege must begin his work in logic by taking either one or the other of these notions to be intelligible *independently* of the other.¹⁵ As we have already seen, Dummett assumes that Frege takes as his explanatory starting point an independently intelligible notion of semantical content

that Frege then intends to use "afterwards" (or so Dummett's Frege hopes) to explain activity like assertion and aiming at the true. By contrast, Brandom and especially Ricketts assume instead that Frege should be seen as beginning his explanations in logic from the independent and intuitive grasp that we have on certain practices, on our own acts of communicating, judging, asserting, and inferring, and only then moving on to explain semantical content and its articulation in terms of these more primitive notions.

My main goal here will be to show that this shared interpretive assumption is, in fact, mistaken. A closer look at Frege's account of these notions makes it quite clear that Frege simply does not embrace either the semanticist or the pragmaticist orders of explanation. For rather than intending for *either* the act-dimension *or* the content-dimension of the subject-matter of logic to have explanatory priority over the other, Frege instead takes *both* to be equally primitive. It is only once we free ourselves of the assumption that Frege intends his discussion of either semantics or pragmatics to proceed in a reductive fashion that we can hope to make sense of the straightforwardly coordinate treatment that Frege gives to these two dimensions, not just in each of his official presentations of his logical system, but throughout his writings on logic.

I will develop my alternative non-reductive interpretation of Frege's methodology as follows. I begin in Section II by doing a key bit of stagesetting to put us in a position to properly appreciate the implications of Frege's inclusion of signs for both semantical notions and pragmatical notions on the list of the *Urzeichen* of his newly fashioned language for logic itself, his Begriffsschrift. The key point that we will build toward here is Frege's claim that the significance of any genuinely primitive sign is indefinable. To this end I first introduce Frege's systematic distinction between the definition or reductive explanation of the significance of a sign, on the one hand, and the "elucidation [Erläuterung]" of the significance of a sign on the other. As I show in Section II, Frege classifies a sentence as an elucidation if, like a definition, it is meant to convey the significance of certain words or signs, but if, unlike a definition, it is forced to rely solely on what Frege calls "hints [Winke]" or "figurative expressions [bildliche Ausdrücke]" to accomplish this task, due to the primitive, unanalyzable nature of the significance at issue.

In Section III, I turn to the introductory sections of Frege's mature exposition of his system of logic, his 1893 *Grundgesetze*, in order to show that Frege is quite clear about his belief that the significance of the *Urzeichen* of logic in particular cannot be defined but can only be elucidated. Yet since Frege includes signs for both semantical and pragmatical notions among the "primitive signs" of logic, it follows that neither sort of notion (as the significance of these signs) could be taken as providing the basis for the definition of the other, since, by Frege's lights, neither sort of notion can be defined *at all*. In

Section IV, I broaden our view to show that this commitment to the primitive standing of both sorts of notions is not peculiar to the *Grundgesetze*, providing evidence from throughout Frege's mature writings that he thinks both sorts of notions cannot be defined but can only be elucidated.¹⁶

Yet even if at this point we will have shown that Frege cannot be saddled with any attempt at an explanatory ordering via *definitions*, the possibility might remain that Frege nevertheless takes his *elucidations* of the primitive signs to stand in either one or the other asymmetric order of explanation, and so in this way might still yet signal his embrace of either a semanticist or pragmaticist methodology. What is more, something along these latter lines might seem to fit better with the tenor of the semanticist and pragmaticist interpretations in the first place, insofar as the kind of dependence that Frege is being said to uphold is typically described in weaker, or at least less precise, terms than that of explicitly definitional dependence.

In Section V, I show that this remaining possibility is excluded by the elucidations that Frege actually gives of his primitive signs. Rather than being ordered in the asymmetric way that these interpretations would require, Frege's elucidations are readily seen to move more or less directly in a *circle*. This in turn strongly suggests that, far from being reductive in nature, the type of intelligibility-relation that Frege actually thinks obtains at the level of foundational notions in logic is decidedly *holistic*, such that our grasp on any one of these notions can only be achieved by a sense of its place among the rest of the primitives as well. But if Frege countenances neither definitional nor elucidatory asymmetry, then there would seem to be little ground remaining for ascribing to him a reductive–explanatory methodology of either sort.

In Section VI, I conclude by addressing a worry that will have arisen for many readers (especially those under the influence of the semanticist interpretation) concerning the consistency of my non-reductive interpretation with Frege's well-known antipathy toward psychologism. For it might seem difficult to square the emphasis I place throughout on Frege's persistent inclusion of the notions of certain mental acts among the list of primitive logical elements, on the one hand, with Frege's well-known insistence, on the other, "always to keep sharply separated what is psychological from what is logical", as he puts things at the outset of his Grundlagen der Arithmetik (Gl, p. x). My response will focus on the fact that, on the interpretation I will develop, we need only admit that Frege means to include such acts within logic, but do not need to follow the pragmaticist in making the stronger claims either that Frege means to ground all of logic in such acts, or that logic is even concerned with all that there is to say about such acts. It will be open for us instead to fully recognize the extent to which Frege takes mental acts like judgment and inference to be, in effect, Janus-faced, and so to require treatment by both logic and psychology in order to be completely understood.

II. Definitions, elucidations, and the *Urelemente* of logic

Though even in his early writings Frege uses the term "elucidation [Erläuterung]" in a way that anticipates its use in later texts, 17 it is only in his mature period that he spends any time elaborating what he means by this term, and only later, in particular, that Frege specifies how an elucidation differs from a genuine or proper "definition [Definition]". The two most sustained discussions of elucidations are found in Frege's second of two essays entitled "Über die Grundlagen der Geometrie" from 1906, and in his unpublished manuscript from 1914, "Logik in der Mathematik". In the present section, I will abbreviate these two works as "UGG" and "LiM", respectively.

In "LiM" Frege explains what a definition is in the following manner:

When a simple sign is introduced to replace a group of signs, such a stipulation is a definition. . . . We shall call the simple sign the definiendum, and the complex group of signs which it replaces the definiens. The definiendum acquires its sense only from the definiens. This sense is built up [aufgebaut] out of the sense of the parts of the definiens. (PW, p. 208; NS, p. 224)

One might, of course, go on to ask after how the signs used in the definiens have acquired their sense. This would be to focus on each of the signs in the given definition as a potential definiendum itself, one whose sense might be conveyed by giving a further definition through which its sense is displayed by being "built up" out of still simpler elements. In "UGG", Frege makes it clear, however, that he thinks that the process of further defining the parts of any given definiens must come to a halt: "We must recognize primitive logical elements [logische Urelemente] that are not definable" (p. 301; CP, p. 300; KS, p. 288). These elements count as primitive because they will function as the undefined definers, as it were, in the sense that all other words will gain their senses by being "built up" out of these elements. This points to what Frege calls "genuine significance [eigentliche Bedeutsamkeit] of definition"—namely, that it provides "the logical construction [Aufbau]" of a sense "out of primitive elements [*Urelemente*]" ("UGG", p. 303; CP, p. 302; KS, p. 289). 18

Now, Frege takes it to be a condition for communication that we all mean the same things by the symbols we deploy.¹⁹ Frege thinks that the securing of such semantical consensus can be achieved by definition.²⁰ Yet for definitions to be effective, consensus concerning the significance of the signs for the primitive elements that are contained in the definiens must already have been achieved, since it is ultimately only in virtue of such a shared grasp of the meanings of these signs that the definiens can confer a commonly understood meaning on the definiendum in the first place, such that the definition itself can achieve its "genuine significance". Frege recognizes, however, that with respect to the words (signs) that signify the primitive indefinable elements themselves—what Frege will call "primitive signs [Urzeichen]" in the Grundgesetze²¹—definitions cannot possibly play the role of securing such consensus, since the relevant sense at issue is simple and so cannot be "built up", definition-wise, at all. For these reasons, then, Frege insists that, with respect to achieving consensus on the sense of the primitive signs, "here something else must enter in" besides definition, in order to "serve the purposes of researchers understanding one another and the communication of science"; it is this "something else" that Frege calls "elucidation" ("UGG", p. 301; CP, p. 300; KS, p. 288).²²

In "LiM", Frege argues for the necessity of elucidations from a slightly different angle, by asking us to consider what must take place "when we begin science":

When we begin science, we cannot avoid using words from ordinary language. But these words are for the most part not really appropriate for scientific purposes, because they are not sufficiently determinate and are fluctuating in their use. Science needs technical expressions that have entirely determinate and fixed references, and in order to make these references understood and to exclude possible misunderstandings, one gives *elucidations*. (PW, p. 207; NS, p. 224; emphasis added)

Frege therefore takes elucidations to be required for science even to get started, to prepare the way for the execution of the "construction [Aufbau]" of the science proper.

In fact, in "UGG", Frege goes so far as to place elucidations outside of the science proper: elucidations themselves, Frege writes, "should not appear in the system, but rather must precede it", such that "within the system we must simply presuppose that the word [for the primitive] is known" (p. 306; CP, pp. 304-05; KS, p. 292; emphasis added).²³ Indeed, Frege takes this to be of a piece with the fact that, when one begins science, "one cannot in general circumvent the necessity of presupposing words as known" (ibid.; emphasis added). Definitions, by contrast, do belong to the "construction" of the science in a quite straightforward manner. As we have already seen, in a definition, the definiendum acquires its sense by this sense being "built up"—or, as we might also render "aufgebaut", "constructed"—out of the sense of the component parts of the definiens.

Unlike a definition, then, an elucidation "does not construct the sense of a sign in this way, out of simpler pieces, but rather treats it [i.e., the sense] as simple" (PW, p. 208; NS, p. 224). What is it, though, to convey the significance of a sign in such a way that treats it as the simple entity that it is? In certain cases, perhaps in empirical science, we might be able to convey the significance of a primitive sign through pointing, through ostensive definition. This route, however, is not open in the case of logical primitives, insofar as the significance of the basic signs in logic will not consist in something that can be presented to the senses in the first place.²⁴ In fact, in "LiM", Frege implies that elucidations of primitive signs *must* make use of other bits of language: "Of course, for [elucidation] one can *only* use further words of the language"; "we cannot *avoid* using the words from our ordinary language" (PW, p. 207; NS, p. 224; my italics).²⁵ This will be something that elucidations share with ordinary (non-ostensive) definitions. Yet since we cannot be using the other signs in an elucidatory sentence to "build up" or "construct" the sense of what we mean by the primitive sign at issue, Frege recognizes that elucidations must be doing something else with words.

What is this "something else"? In "UGG", Frege claims that the kind of understanding achieved through elucidation "would not come about without a figurative use of an expression [eine Bildlichkeit des Ausdrucks]" (p. 301; CP, p. 301; KS, p. 288). What is more, Frege holds that we are forced to produce an elucidatory sentence only when "there is nothing else to do but to lead the reader or listener by means of hints [Winke] to what is meant by the word", as he puts the point in "Über Begriff und Gegenstand" (p. 193; CP, p. 183; KS, p. 168). Hence, because it involves an imaginative use of language, Frege thinks that "we have to count on a thoughtful cooperation [ein verständnisvolles Entgegenkommen], a guessing [Erraten] of that which one has in mind", for elucidation to be successful ("LiM", PW, p. 207; NS, p. 224). 28

So described, the procedure of elucidations might seem like a very precarious way to begin anything that might hope to become rigorous science.²⁹ In any case, what is crucial for our purposes is, first, that Frege recognizes certain "elements" within science as primitive, and, second, that he takes the signs for such elements to have a significance which is indefinable. However it is that we do manage to grasp these senses, in such grasping Frege thinks we are confronted by "a logically primitive phenomenon [logische Urerscheinung] which must simply be recognized [anerkannt] and cannot be reduced [zurückgeführt] to something simpler", as Frege puts it in an earlier essay of 1903.³⁰ As a consequence, wherever we find Frege (a) identifying something as to be symbolized by a primitive sign of a science, or (b) identifying a sign in a science as one whose significance cannot be defined but only elucidated, we can be confident that we are dealing with one of the things that Frege takes to be an "Urelement" of the science at issue.

III. Frege's list of logical Urelemente in Grundgesetze I

With these points in mind, let us now turn to Frege's major work of his mature period, the first volume of his *Grundgesetze der Arithmetik*. ³¹ At the outset of this work, Frege begins by identifying the "primitive signs [*Urzeichen*]" for the science of logic itself—i.e., the primitive signs of the "*Begriffsschrift*" (Gg I, p. 5; BLA, p. 33). Immediately before his presentation of these signs, however, Frege makes what is for us a now-familiar qualification:

It will not always be possible to define everything in a strict manner [regelrecht], precisely because our endeavor must be to trace our way back to what is logically simple [das logisch Einfache], which as such is not genuinely [eigentlich] definable. I must be satisfied, therefore, with indicating [hinzuweisen] what I intend by means of hints [Winke]. (Gg I, p. 4; BLA, p. 32)

Hence, Frege recognizes that in his own presentation of the science of logic, he will have to rely upon signs whose significance cannot be defined but only elucidated, because their significance cannot be "built up" out of anything more simple.

It is significant that these remarks come on the final page of the prefatory material that precedes the first main Part of the work, a Part that is entitled "Primitive signs [Urzeichen]". As might now be suspected, in this First Part Frege aims to convey the significance of his notation in ordinary language. Frege even already allows himself here to make use of basic arithmetical examples and even uses arithmetical expressions to help bring out the significance of the primitive signs of his logic, despite the fact that the final goal of the rest of the work itself is nothing other than to provide a rigorous definition ("construction [Aufbau]") of the significance of arithmetical expressions out of these more basic logical expressions.³² Frege recognizes that his use of arithmetical expressions already in this First Part might seem problematic, as he himself has not yet "defined" the significance of arithmetical signs. Nevertheless, Frege excuses himself on the grounds that he is using such expressions simply in order "to form examples more easily and to facilitate understanding by means of hints [durch Winke das Verständnis zu erleichtern]" what his own basic "primitive signs [Urzeichen]" mean (Gg I, §5, p. 9 fn. 1; BLA, p. 38 fn. 13). In other words, the sentences in this First Part that introduce the primitive signs are not—and indeed, cannot be—meant as definitions of these signs, but rather are meant as elucidatory of their significance. In fact, Frege himself points up this fact by reserving the title of "Definitions" for the next (Second) Part of the work. This is, of course, in perfect accord with the results of our earlier discussion—namely, that the exposition of the significance of the primitive signs must be undertaken prior to giving any "genuine definitions" within the system of a science.³³

If we now turn to the actual list of *Urzeichen* that Frege gives in the First Part of *Grundgesetze* I, what do we find? If Dummett's semanticist interpretation of Frege were correct, then we might expect to find only signs associated with semantic content. If, by contrast, the pragmaticist line of Brandom and Ricketts were correct, then we might expect only signs for pragmatical notions to appear on this list. Yet, as we have already anticipated at the outset, what we find instead is that Frege includes signs for *both* notions among the primitive signs of logic. More specifically, we find, on the one hand, signs for "referring [bedeuten]" or "designating [bezeichnen]" objects, concepts, relations,

truth-values, course-of-values, extensions,³⁴ and we are told that all such signs also serve to "express [ausdrücken]" a "sense [Sinn]" (Gg I, §2, p. 7; BLA, p. 35). Yet we also find, on the other hand, primitive signs for several mental acts. The first and most well-known of these is a sign for the act of judgment, "\(\tilde{-}\)"—i.e., the sign he calls the "judgment-stroke" (Gg I, §5).³⁵ Shortly thereafter Frege introduces signs for two further mental acts: first, signs for "inference" (Gg I, §14)—signs he will call "transition-signs" in later sections (Gg I, §26 and §48)—and second, a sign for "definition" (Gg I, §27).³⁶

IV. The discussion of logical *Urelemente* in other contemporaneous writings

At this point, one might be worried that I am placing undue emphasis on Frege's designation of certain signs as primitive within the notation developed in *Grundgesetze*. Perhaps this designation is peculiar to this work, or simply the result of the constraints of writing a textbook? In order to allay such concerns, let me now provide further confirmation of the foregoing interpretation of Frege's views by furnishing passages from other writings in Frege's mature period in which he claims explicitly that each one of the notions designated by a primitive sign in the *Grundgesetze* is in fact a primitive, indefinable element in logic and therefore only able to be elucidated.

Let me begin with the notions belonging to semantics and, in particular, the notions pertaining to reference, such as that of object, concept, and truth. In his 1903 "Über die Grundlagen der Geometrie I", Frege introduces the notions of object and concept by pointing to a correlative distinction among the parts of a sentence (sign), e.g., "two is a prime number":

The first constituent, "two", is a proper name of a certain number; it designates an object, a whole that no longer requires completion. The predicative constituent, "is a prime number", on the other hand, does require completion and does not designate an object. I also call the first constituent saturated, the second, unsaturated. To this difference in signs there of course corresponds an analogous one in the realm of references: to the proper name there corresponds an object; to the predicative part, something I call a concept. (p. 371; CP, p. 281; KS, p. 269)

After this preliminary indication of what he means by "concept" and "object", however, Frege immediately goes on to make the following qualification:

this is not supposed to be a definition; for the decomposition [Zerfallen] into a saturated and an unsaturated part must be considered as a primitive logical phenomenon [logische Urerscheinung] which must simply be

recognized, but which cannot be reduced [zurückgeführt] to something simpler. (ibid.)

What is more, Frege acknowledges that he is "well aware that expressions like 'saturated' and 'unsaturated' are figurative [bildlich] and only serve to indicate [hinzuweisen] what is meant—whereby one must always count on the co-operative understanding [entgegenkommende Verständnis] of the reader" (p. 372; CP, p. 281; KS, pp. 269–70). Hence, just as with their correlates in the official notation of the Begriffsschrift, Frege shows here that he also takes the ordinary language terms "object" and "concept" to have a significance that cannot be defined, but only elucidated.

Frege gives a similar treatment to the notion of an object in his 1891 "Funktion und Begriff". After claiming that concepts can be understood as kinds of functions (namely, as functions whose values are always truth-values (cf. p. 16; CP, p. 146; KS, p. 133)), and claiming that "objects without restriction" can be admitted "as arguments and values of functions", Frege admits that "the question arises what it is that we are here calling an object" (p. 18; CP, p. 147; KS, p. 134). In response, Frege again insists that the answer to such a question cannot take the form of a definition:

I regard a regular [schulgemäβe] definition as impossible, since we have here something that, due to its simplicity [Einfachheit], does not allow for logical analysis [Zerlegung]. It is only possible to indicate [hinzudeuten] what is meant. (p. 18; CP, p. 147; KS, p. 134)

The primitiveness of the notion of a concept is likewise reiterated in Frege's 1892 "Über Begriff und Gegenstand". There Frege insists that the "explanation [Erklärung]" that he has given of the word "concept" in his earlier writings (such as the Grundlagen) "is not meant as a genuine [eigentliche] definition" (193; CP 182; KS 167), precisely for the following now-familiar reasons:

One cannot require that everything shall be defined, any more than one can require that a chemist shall decompose [zerlege] every substance. What is simple cannot be decomposed, and what is logically simple cannot have a genuine definition. . . For the introduction of a name for something logically simple, a definition is not possible. There is nothing for it but to lead the reader or hearer, by means of hints [Winke], to understand what is intended in connection with the word (p. 192; CP, pp. 182–83; KS, pp. 167–68)

In fact, we find Frege making this very same qualification almost every time he uses such phrases ("incomplete" vs. "complete", "in need of supplementation" vs. "whole", "unsaturated" vs. "saturated") to "indicate" what he means by what he takes to be the more general distinction between function and argument.³⁷ It is unsurprising, then, that we find Frege insisting as well that the very term "function" itself is not something that can be "defined" either, but can only be "elucidated" through "figurative expressions"—again, precisely because its meaning, too, belongs among what is simple, primitive, and unanalyzable.³⁸

Frege displays a very similar attitude toward the two truth-values themselves, truth and falsity. In the outline of the contents that Frege places at the outset of his unpublished 1897 "Logik", Frege notes that he will discuss both the fact that "true cannot be defined", and also the status of "true" as "original [ursprünglich] and simple" (PW, p. 126; NS, p. 137). In the body of the manuscript itself, Frege claims, first, that "truth is obviously something so original and simple that a reduction [Zurückführung] to something even simpler is not possible" (PW, p. 129; NS, p. 140). Frege then adds to this that though he is able to "elucidate the sense in which I use the word 'false", this word is "as little susceptible to a genuine [eigentliche] definition as the word 'true" (PW, p. 130; NS, p. 141; emphasis added). In the late (1918) essay "Der Gedanke", Frege reiterates this point, claiming again that the "content [Inhalt]" or "reference [Bedeutung]" of the word "true" is "entirely sui generis [einzigartig] and indefinable" (pp. 60–61; CP, pp. 353–54; KS, pp. 344–45).³⁹

Turning from reference to sense, we find Frege giving the same qualifications concerning the sentences by which he tries to convey what he understands to be the sense of an assertoric sentence, or what he calls a "thought [Gedanke]". In his late essay, "Der Gedanke", for example, just before Frege gives the definition-like statement that "by 'thought' I mean something for which, as such, the question of truth can arise", Frege claims explicitly that he does not actually intend this statement to offer a definition of "thought" (p. 60; CP, p. 353; KS, p. 344). It is presumably, therefore, instead meant to be elucidatory of "thought". Frege is even more explicit about the primitive standing of the notion of thought in the late essay "Die Verneinung". In a footnote, Frege again makes the following definition-like statement about judging: "judging [urteilen], one can say, is the acknowledging of something as true" (p. 151n; CP, p. 381 fn. 13; KS, p. 370 fn. 10). And immediately after this, Frege makes the following definition-like statement about thought itself: "that which is acknowledged as true can only be a thought [Gedanke]" (ibid.). At this point, Frege claims that the "original kernel" of the notion of judgment "now seems to have cracked in two": "one part lies in the word 'thought', the other in the word 'true" (ibid.). Yet Frege also insists that "here we must stop", reminding us yet again that "the impossibility of an infinite regress in definition is something we must be prepared for in advance" (ibid.). Since we already know that Frege thinks that "true" cannot itself be defined, the implication here is surely that the same thing must be true of "thought" as well.

Let me turn, finally, to further evidence beyond the *Grundgesetze* of Frege's commitment to the equally primitive, indefinable standing of the notions belonging to pragmatics, and, in particular, the primitive status of the notion of the act of judging. In "Über Sinn und Bedeutung", for example, after claiming that "judgments can be regarded as advances from a thought to its truth-value", Frege immediately qualifies the status of this claim by insisting that "naturally this cannot be a definition" because "judging [das Urteilen] is something that is entirely *sui generis* [etwas ganz Einzigartiges] and incomparable" (p. 35; CP, p. 165; KS, p. 150). And in his later "Die Verneinung", after again reminding us that "not everything can be defined", Frege indicates that he takes "what a judgment is" to be one of the things that "according to its essence is not definable" (p. 150; CP, p. 381; KS, p. 370). 40 This is, of course, exactly what we would expect, given the status that we have already seen Frege accord to judgment and its corresponding primitive sign within the official presentation of Frege's logical system itself in *Grundgesetze*.

V. The holism in Frege's elucidations

The evidence from the two previous sections demonstrates that, throughout his mature writings, Frege gives every indication that he takes both semantical as well as pragmatic notions to be primitive, simple, and indefinable as such, which implies (a fortiori) that neither is definable in terms of the other. In fact, as I have already noted above (cf. Section I), the classification of the signs for acts like that of judgment as well as signs for contents as both among the logical primitives is something that is present from the very outset of Frege's career. 41 For both the younger and the older Frege, each stands on par with the other, as Urelemente of logic, and hence each sort of notion must be represented (signified) within any notation for the science of logic via its own distinct Urzeichen. 42 In fact, it is for this very reason that Frege is critical of alternative notations for logic, not only for failing to mark all of the distinctions at the level of semantic content that are relevant for logic (such as the distinction between an individual object and a concept, as Frege emphasizes to Anton Marty),⁴³ but also for failing to include signs that mark the performing of distinctly logical mental acts, as is evident from Frege's exchanges with Peano and in his correspondence with Philip Jourdain. 44 No notation can be counted as sufficient for logic if it were to exclude either sort of sign.

Now, the fact that Frege thinks there must be primitive *signs* for both semantic content as well as pragmatical features implies that Frege takes both kinds of things to be among the primitive *elements* of the science of logic. Hence if the main thesis of either the semanticist or the pragmaticist interpretation were that Frege intends to offer genuine definitions of either sort of notion, then the foregoing would be sufficient to show that these interpretations are clearly on the wrong track. From the early pages of the *Begriffsschrift*, through the introductory Part of the *Grundgesetze*, as well as

throughout the rest of Frege's mature writings, Frege give every indication that he thinks that neither pragmatic notions like those of judgment and inference, nor semantical notions like those of object and truth, are notions that can be defined or reductively explained in terms of the other, since neither is the sort of thing that can be defined at all.

Ultimately, however, it is not clear that Dummett, Brandom, or Ricketts intend their claims about Frege's alleged orders of explanation to require that Frege provide explicit or genuine definitions of the one sort of notion in terms of the other. Rather, they pose their claims in terms that are not exactly Frege's own, speaking of our ability to "grasp" certain notions prior to or "independently" of others, or of one notion's "supervening" on another. For this reason, these interpretations might still be salvaged if it turned out that Frege were committed to some other non-definitional form of asymmetric conceptual dependence obtaining between semantical and pragmatical notions. 46

Now, as we have seen, Frege does insist that these primitive notions do participate in non-definitional relations of intelligibility. Even though each primitive notion is *sui generis* and indefinable per se, Frege believes that its significance can be grasped or understood through elucidations. Hence, there remains the possibility that Frege *does* think that a kind of asymmetric conceptual dependence obtains among these notions, albeit one that is not of the sort that can be captured in "genuine" or "regular" *definitions*, but instead one that can only be manifest in the order of the *elucidations* one should give of these primitives.

Once we take a closer look, however, at the actual series of elucidations that Frege gives of his primitive elements, we can see that no such asymmetric ordering is manifest. In fact, following out the course of Frege's actual elucidations would seem to suggest exactly the opposite conclusion. This is because, as I will show in a moment, Frege's own elucidations of the primitives form a *circle*: each of the logical Ur-elements is elucidated by Frege precisely in terms of the other Ur-elements. It would seem much closer to the truth, therefore, to say that Frege's own non-definitional "explanation" of his primitive terms makes evident his presumption that we are capable of only a decidedly *non*-reductive understanding of how these notions all hang together, such that the understanding of any one term is reciprocally, and not asymmetrically, dependent on our understanding of the others.

We can see this sort of circular, reciprocally dependent, interconnection obtaining even among the elucidations that we have cited in the previous sections. Consider again, for example, Frege's elucidations from "Über die Grundlagen der Geometrie I" and "Funktion und Begriff". In these, the notions of object, concept, and truth-value are all elucidated in conjunction with one another, even while Frege continues to resist the idea that these "figurative" interlinkings could in any way be taken to form a definition. ⁴⁷ In turn, the significance of "truth" itself is something Frege gestures at by linking

it with "judgment" and "thought", as we saw from the passage from "Die Verneinung". 48 This mirrors Frege's method in the *Grundgesetze* itself, where Frege introduces the notion that the truth-values are what sentences "refer to [bedeuten]" by focusing on the contrast between what a logician expresses by merely "writing out [hinschreiben]" a sentence rather than intending this inscription to "assert [behaupten]" the thought so expressed (Gg I, §2, p. 7; BLA, p. 35). Yet as assertion is elucidated in terms of the sensible manifestation of judgment (cf. "Der Gedanke", p. 62; CP, pp. 355–56; KS, p. 346), and since, as we saw above, judgment itself is elucidated in terms of the act of advancing from a thought to a truth-value, 49 the path that Frege lays out for understanding the significance of his primitive signs is one that closes back on itself.

What is especially striking about this, when viewed from the perspective of the two prevailing interpretations, is that Frege shows no sign of concern whatsoever with the circularity inherent in his elucidations. This provides further confirmation of the fact that Frege cannot possibly intend his elucidatory use of language to effect a reduction of something to something else. In fact, Frege's task in elucidations would seem to consist instead in introducing his readers to something on the order of a "hermeneutical" circle—what might well be called an "elucidatory circle", as Erich Reck has happily put a similar point in a recent essay. For the circularity of Frege's elucidations strongly suggests that what is ultimately primitive could only be our grasp of the set of logical primitives as a whole, insofar as it is only a tour of the whole landscape of das Logische that Frege thinks will make possible the genuine recognition and appreciation of any of its parts. The provided that the provided recognition and appreciation of any of its parts.

If this is right, however, then Frege's own conception of the relation between semantical and pragmatical notions in logic would seem to be at *two* steps removed from either the semanticist or the pragmaticist interpretation. As we saw in the previous sections, Frege clearly thinks we cannot build up or construct the sense of either pragmatical or semantical primitives in anything like a definition. And as we have just seen, Frege does not seem to think that we can grasp any one of these senses independently of grasping them all.

VI. Concluding remarks and a note about Frege's anti-psychologism

I began by signaling my sympathy with a key motivation for the pragmaticist interpretation—namely, the insufficiency of Dummett's claim that Frege intends to use an allegedly independent grasp we have of semantic content in order to explain what sorts of mental acts are possible or appropriate in relation to that content. I hope by now to have brought to light considerable grounds for rejecting Dummett's proposal. The very same kinds of reasons, however, push against the rest of the pragmaticist counter-proposal—i.e., the proposal that for Frege, logic must instead take its start from the pragmatic

notions as opposed to the notions that pertain to semantic content. It is just as wrong to construe Frege as thinking that either the notions pertaining to reference (like that of an object, concept, and truth), or those pertaining to sense (such as thought) can be explained or defined in terms of the activity of judgment and inference, as it is wrong to construe Frege as thinking of judgment and assertion as an afterthought.⁵² Frege takes both kinds of notions to be not only primitive in themselves but also takes our understanding of each to be equally essential to our very grip on the sphere of das Logische itself. But then, far from attempting a reduction in either direction, Frege's methodology is better seen as embracing a decidedly non-reductive, holistic conception of the order of intelligibility among the semantical and pragmatical notions associated with logic.⁵³

Why has this aspect of Frege's methodology failed to be sufficiently appreciated? One factor, I think, may be the fact that Frege clearly does intend to put forward explicitly reductive definitions (explanations) of what many before him have thought to be primitive notions, as a means to realizing his logicist program. Indeed, one of the most central, well-known, and welladvertised goals of Frege's lifelong work is nothing other than the definition of the basic concepts and objects in arithmetic (number, the numbers, etc.) in terms of more primitive, purely logical concepts and objects (non-self-identity, extensions, etc.).⁵⁴ Even so, there is no reason to infer from Frege's reductive approach to arithmetic to his commitment to a reductive-explanatory methodology within *logic* itself. In fact, careful attention to Frege's construal of his logicist program itself actually provides further evidence of the equally primitive standing of semantics and pragmatics within logic, insofar as Frege indicates that a genuine reduction of arithmetic will require not only that the semantic content of arithmetical expressions be given an analysis in terms of purely logical semantic content, but also a demonstration that what appears to be distinctly arithmetical activity is in fact a species of purely logical activity, such that "calculating [Rechnen] becomes drawing inferences [Schlussfolgern]", as Frege puts it in Grundlagen §87 (Gl, p. 99).

There is, however, another even more formidable obstacle that stands in the way of the appreciation of the co-primitive standing that we have seen Frege giving to both semantics and pragmatics within logic. This is an obstacle that threatens, in particular, the plausibility of the idea that Frege could accord an equally primitive status within logic to notions of mental acts—namely, the long-standing emphasis that Frege's readers have given to Frege's opposition to what he took to be the increasing and misguided intrusion of psychology within logic that he saw as common in the logic textbooks of his day.⁵⁵ For it might seem that any interpretation on which a fundamental, ineliminable role is assigned to notions like the act of judgment and of inference within logic would simply be ruled out by Frege's intentions "always to keep sharply separate what is psychological from what is logical", to again quote from the early pages of Frege's *Grundlagen* (Gl, p. x).

Yet even if Frege's anti-psychologism cannot be denied, there is also no way around the simple fact that we have focused on throughout the foregoing—namely, that Frege actually does include signs for these mental acts of judgment, inference, and definition on his list of *Urzeichen* for the *Begriffsschrift*. There is, therefore, also no way around the fact that he clearly intends, not only to include such notions among *das Logische*, but to place them at its very foundations.

Can these two threads of Frege's thought be made to co-exist without inconsistency? Many of Frege's readers have concluded that they cannot. In fact, from their initial publication, Frege's persistent inclusion of notation for these pragmatical elements has remained something of a mystery among Frege's interpreters. Among Frege's earliest readers, Russell, Philip Jourdain, and Wittgenstein all took its inclusion to be a mistake on Frege's part, insofar as it appeared fit *only* to capture something "psychological", since a concern with this sort of notion seemed to cut directly against Frege's opposition to the intrusion of psychology within logic. ⁵⁶

We can, however, begin to relieve the sense of tension here, by recognizing two key points. The first is that Frege's main target in the anti-psychologistic passages is primarily those who would not only import the findings of *empir*ical psychological research into logic, but allow these findings to provide the basis for the identification of logical laws—those, that is, who would identify certain laws as logical despite the fact that they had been formed from empirical generalization from observations of individual mental states.⁵⁷ Yet being against intrusion of this sort, and being against the grounding of logic upon this sort of psychological data, is perfectly compatible with incorporating the notions of mental acts of judgment and inference as such in the specification of the subject-matter of logic. Or at least it would be compatible, so long as we, first, do not invoke these notions as providing the (sole) ground for all other ostensibly logical objects, concepts, and laws, and, secondly and more importantly, so long as we have reason to believe that the significance of these notions is not exhausted by the data provided in empirical ("inner") observation.

How do things stand with Frege in these respects? Concerning the first, we have already argued in general against attributing to Frege any order of explanation that would purport to provide logic with a pragmaticist foundation. What about the second? Frege accepts, of course, that the acts of judgment and inference are, *in part*, psychological; he says this explicitly in his 1919 "Aufzeichnungen für Ludwig Darmstaedter": "both grasping a thought and making a judgment is an act [*Tat*] of a knowing subject, and are to be assigned to psychology" (PW, p. 253; NS, p. 273). Yet in this same passage Frege implies that he does not take this to *exhaust* the nature of judgment, writing that "both acts, however, relate to [*erstrecken sich auf*; literally 'extend themselves toward'] something that does not belong to psychology" (ibid.). Frege makes this feature of his views even more explicit in his 1897 "Logik", claiming that

with distinctly logical acts like judgment, "something comes into view that is essentially [wesentlich] no longer psychical in the proper sense [im eigentlichen Sinne seelisch]" (PW, p. 145; NS, p. 157). For this reason, Frege is here moved to claim that a distinctly logical act is "a process which lies on the very limit of the mental [an der Grenze der Seelischen] and which for that reason cannot be completely understood from a purely psychological standpoint" (ibid.; emphasis added).

This leads us to a final, crucial point, one that nicely complements our previous discussion: far from excluding them from treatment in logic, Frege shows here that he believes that the *complete* understanding of such acts like judgment and inference can *only* be achieved through the complementary treatment that such acts receive *within logic*. To be sure, Frege also makes it clear that he thinks that there will be *more* to say about judgment *qua* mental act than logic will disclose; Frege's point here is simply that there is something that logic, too, has to say about such acts as well.⁵⁸ In any case, the main upshot of the foregoing has been that Frege thinks that this is something that logic *must* say, not just if the nature of acts like judgment and inference is to be genuinely understood, but if *any* of the rest of the primitive notions of logic—the notions of object, concept, thought, truth, and so on—are to be genuinely understood themselves.⁵⁹

Notes

- 1. See especially Frege: Philosophy of Language (Harvard, 1973), pp. 432–33.
- 2. Op.cit., p. 7.
- Dummett, M. (1959) "Truth", Proceedings of the Aristotelian Society, 59, pp. 141–62; here,
 p. 143. See as well Dummett, M. (1993), The Origins of Analytic Philosophy (Cambridge, MA: Harvard University Press), pp. 12–13.
- 4. I will use the following abbreviations for Frege's works:
 - BLA: Basic Laws of Arithmetic, M. Furth (Ed., trans.) (Berkeley, CA: University of California Press, 1964);
 - BS: Begriffsschrift und andere Aufsätze, I. Angelelli (Ed.) second edn (Hildesheim: Olms, 1964);
 - CN: Conceptual Notation and related articles, T. Bynum (Ed., trans.) (Oxford: Clarendon, 1972);
 - CP: Collected Papers on Mathematics, Logic, and Philosophy, B. McGuinness (Ed.), trans. M. Black et al. (Oxford: Blackwell, 1984);
 - Gg I: Grundgesetze der Arithmetik, I. Band (Jena: H. Pohle, 1893);
 - Gl: Grundlagen der Arithmetik (Breslau: W. Koelbner, 1884);
 - KS: Kleine Schriften, I. Angelleli (Ed.) (Hildesheim: Olms, 1967);
 - NS: Nachgelassene Schriften, H. Hermes et al. (Eds.) (Hamburg: F. Meiner, 1969);
 - PMC: Philosophical and Mathematical Correspondence, B. McGuinness (Ed.), trans., H. Kaal (Oxford: Blackwell, 1980);
 - PW: Posthumous Writings, H. Hermes et al. (Eds.), trans., P. Long and R. White (Oxford: Blackwell, 1979);
 - WB: Wissenschaftlicher Briefwechsel, Gabriel et al. (Eds.) (Hamburg: F. Meiner, 1976). I will cite Frege's works according to the pagination from the original publication (where possible) first, then the most common English translation (where available), and

- then the standard German editions (e.g., "Über Sinn und Bedeutung' (p. 27; CP, p. 158; KS, p. 144)"). In the case of the *Grundlagen*, the 1980 printing of the second revised edition of J.L. Austin's translation, lightly edited by Michael Dummett and Brian McGuinness (Oxford: Blackwell, 1980), has the same pagination as the original 1884 edition. All translations throughout (of both Frege's works and others) are my own, though I have in each case consulted the cited English rendering (where available).
- 5. For the "judgment-stroke", see Begriffsschrift §2 and Grundgesetze §5; in the latter text, Frege says that the judgment-stroke "contains assertion" (Gg I, §5, p. 10; BLA, p. 39). For the sign for inference (what Frege calls "transition-signs" in the Grundgesetze), see Begriffsschrift §6 and Grundgesetze §14 and §26. For the definition-"stroke", see Begriffsschrift §24 and Grundgesetze §27.
- 6. This contrasts with Frege's views on the nature of semantic content, which, of course, develop significantly throughout his career. In the Begriffsschrift, for example, the semantic content is referred to only as "content [Inhalt]", whereas the Grundgesetze incorporates the distinction Frege has made in the meantime between "sense [Sinn]" and "reference [Bedeutung]". Compare Begriffsschrift §2 and Grundgesetze §2. For Frege's explanation of the development of his views on "content [Inhalt]", see the "Introduction" to the Grundgesetze (Gg I, p. x; BLA, p. 6).
- 7. "Truth", p. 143.
- 8. Again, see Begriffsschrift §2 and Grundgesetze, §5.
- 9. Brandom, Articulating Reasons (Harvard, 2000), p. 214 fn. 12.
- "Objectivity and objecthood: Frege's metaphysics of judgment", in: L. Haaparanta and J. Hintikka (Eds.), Frege Synthesized, pp. 65–95 (Dordrecht: D. Reidel, 1986); here, p. 76; Ricketts links this criticism to Dummett in an endnote to this passage (cf., p. 94 fn. 18).
- See Articulating Reasons, pp. 49f; 59f; and 125; compare as well Brandom's Making It Explicit (Harvard, 1994), pp. 94–97, and also his earlier "Asserting", Noûs 17.4 (November 1983), pp. 637–50.
- 12. Articulating Reasons, p. 4. Brandom recognizes that, especially in light of the widespread acceptance of Dummett's reading, his own reading of Frege will seem to some to be an "odd photograph", with its subject "portrayed from unusual vantage points" (Making It Explicit, p. xii). Note that the kind of reductive explanation that is at issue here is one that is connected, though not identical, with Brandom's additional interpretive claim that the early Frege embraces an "inferentialist semantics", according to which the subpropositional contents of a given proposition are to be identified by way of the inferential (deductive) consequences that the proposition itself has. This second claim amounts only to saddling Frege with a commitment to a kind of reductive explanation within semantics, and not necessarily one for the explanation of semantics as a whole in terms of something else (i.e., pragmatics).
- 13. Let me emphasize here that I do not in any way mean to claim that Ricketts himself (in his own voice) means to endorse the most ambitious features of Brandom's own reductive project. This is one of the reasons I will refer to what is common in Brandom's and Ricketts' readings of Frege as an ascription of "pragmaticism" rather than "pragmatism", allowing the latter to be reserved for Brandom's own label for his own reductive program.
- 14. Here I allude to the sub-title of Ricketts' essay: "Frege's Metaphysics of Judgment" (cf. p. 65); for Ricketts rejection of Frege as first and foremost a semanticist in Dummett's sense, see pp. 65–66. Ricketts even goes so far as to claim that, properly understood, in Frege's logic "talk of meaning is to drop away" altogether (p. 91). In a later (1996) essay, "Logic and truth in Frege", *Proceedings of the Aristotelian Society*, Supp. 70, pp. 121–40, Ricketts makes a similar claim about the eliminability of talk of thoughts (as species of Fregean "Sinn") in Frege's logic: "Frege's notion of a thought does not figure directly in his views about the subject-matter of logic" (p. 137).

- 15. Brandom especially has emphasized the fact that, by his lights, Frege's intention, both early and late, is one of placing certain notions in a "reductive order of explanation", though Brandom thinks Frege switches his opinion about which order is appropriate; cf. *Articulating Reasons*, p. 11f. Ricketts puts the point in terms of the "supervenience" of semantical notions on pragmatical ones; cf., "Objectivity and objecthood", p. 66.
- 16. By Frege's "mature" period, I mean the period after the early 1890s, after his drawing of the famous distinction between Sinn and Bedeutung. In fact, I think there is good reason to think that Frege held a version of this non-reductive view throughout his entire career; in several footnotes below I will point to passages from Frege's earlier writings that suggest as much.
- 17. See, for example, the use of "elucidation" in his early 1880–1 essay, "Booles rechnende Logik und die Begriffsschrift" (PW, p. 27; NS, p. 30 (quoted below)). This use, along with many others, is entirely covered over by the English translation, which (in this case) renders "Erläuterungen" as "prose glosses"; at other times—for example, in the passages from "Logik in der Mathematik" that I discuss below—"Erläuterung" is translated as "illustrative example" (PW, p. 207), "explanation" (PW, p. 216), and "illustration" (PW, p. 235). Throughout I have regularized the translations.
- 18. This line of thought remains constant throughout Frege's writings. It is reiterated, for example, in one of Frege's last writings, his 1924–5 "Erkenntnisquellen": "not everything can be defined; only what has been conceptually analyzed [zerlegt] can be reconstituted [wieder zusammengesetzt] out of the parts yielded by analysis [Zerlegung]. But what is simple cannot be analyzed and hence not defined" (PW, p. 271; NS, p. 290).
- Cf. "UGG": if communication in science is to be possible at all, then there is clearly "a need to make sure that we designate the same thing by the same sign (word)" (p. 301; CP, p. 300; KS, p. 288).
- Cf. "UGG": "Once the investigators have come to an understanding about the primitive elements and their designations, an agreement about what is logically composite [das logisch Zusammengesetze] is easily achieved through definition" (p. 301; CP, p. 300; KS, p. 288).
- 21. See below, Section III. Frege deploys this terminology as well in two unpublished essays from 1880–82 that compare his own notation (*Begriffsschrift*) with that of Boole's; cf. PW, pp. 36f and 48f; NS, pp. 40–41 and 55–57.
- 22. The link to communication might appear to be essential; indeed, in "UGG", Frege indicates that he thinks elucidation would not be necessary for the solitary scientist: "Someone who pursued research only for themselves would have no use for it" (p. 301; CP, pp. 300–01; KS, pp. 288). On the other hand, Frege appears to think that, at least for humans, all thinking—even uncommunicated thinking—takes place in language; compare Frege's early 1882 essay "Über die wissenschaftliche Berechtigung einer Begriffsschrift" (p. 48; BS, p. 106; CN, p. 83), the 1897 "Logik", (PW, p. 142; NS, p. 154), and especially the late 1924–25 "Erkenntnisquellen der Mathematik und der mathematischen Naturwissenschaften" (PW, p. 269; NS, p. 288).
- 23. Frege makes this very point in the above passage from "LiM" as well: elucidation "precedes the construction [Aufbau] of the system and does not belong in the system" (my italics), with the result being that "in the construction itself, it must be presupposed that the words have determinate and familiar references" (PW, p. 207; NS, p. 224; emphasis added). As Frege puts it in "UGG", elucidations belong to the "propaedeutic" to science (p. 300; CP, p. 300; KS, p. 288).
- 24. Frege argues for this point at length with respect to the subject-matter of arithmetic and logic in the *Grundlagen*, especially in §§26–27, and it sets the agenda for his discussion of the special sort of "recognition–judgments" that constitute mathematical equations later in the same work (cf. §§55 et seq.).

- 25. This contrasts with the later use of "Erläuterung" in the tradition that flows out of Frege, through the Tractatus, and into logical positivism; compare Schlick's use of "Erläuterung" in his essay, "Die Wende in der Philosophie" (in Erkenntnis, 1930), according to which they appear to consist in literal pointings.
- For a similar use of "bildlich", see "Über die Grundlagen der Geometrie I" (p. 372;
 CP, p. 281; KS, pp. 269–70) and "Über Begriff und Gegenstand" (p. 205; CP, p. 194;
 KS, pp. 167–8).
- Compare the similar use of "Winke" later in the same essay (p. 205; CP, p. 194; KS, p. 178).
 I quote this passage, as well as the passages from the previous note, below in Section IV.
- 28. Compare "Erkenntnisquellen": "it is not possible without a mutual understanding [entgegenkommendes Verständnis] to make designations of a logically unanalyzable [nicht zerlegbare] content intelligible to others" (PW, p. 271; NS, p. 290). Many of the foregoing points are encapsulated nicely in the following passage from Frege's December 27 1899 letter to Hilbert, though here he casts things in terms of "reference", rather than "sense": "I would not want to count elucidatory sentences [Erläuterungssätze] as part of mathematics itself but refer them to the antechamber [Vorhof], the propaedeutics. They are similar to definitions in that they too are concerned with laying down the reference [Bedeutung] of a sign (or word)... [But] in such a case the reference to be assigned is logically simple, then one cannot give a proper definition but must confine oneself to warding off the unwanted references among those that occur in linguistic usage and to indicating [hinzuweisen] to the wanted one, and here one must of course always rely on a co-operative understanding with guessing [ein entgegenkommendes, erratendes Verständnis]. Unlike definitions, such elucidations cannot be used in proofs because they lack the necessary precision, which is why I should like to refer them to the antechamber, as I said above" (PMC, pp. 36-37; WB, p. 63).
- 29. Frege himself recognizes that the role of "guessing" in elucidations leaves open the possibility that "again and again new elucidations will be necessary", such that "in theory, one will never genuinely [eigentlich] achieve one's goal", of mutual understanding ("LiM", PW, p. 207; NS, p. 224). Indeed, one might well wonder about whether the achievement of such mutual understanding could ever be shown in principle. "In practice", however, Frege is confident that we are (somehow) able "to come to understand the significance of the words" in this way (ibid.; cf. "UGG", p. 301, CP, p. 301; KS, p. 288). At one point, Frege suggests that this is because "we can put trust in the fact that other people understand words, inflexions, and sentence-formations in what is essentially the same way that we do ourselves" ("Über Begriff und Gegenstand", p. 195, CP, p. 285; KS, p. 169).

Though I cannot address this here, a further problem arises concerning elucidations, insofar as Frege appears to admit that certain elucidatory sentences are ultimately nonsensical. This comes to the fore most famously in "Über Begriff und Gegenstand", where Frege attempts to elucidate the distinction between objects and concepts by "asserting" that "the concept horse is not a concept", which he recognizes is "an awkwardness of language", but which he insists "cannot be avoided", if the senses at issue are to be conveyed (pp. 196-97; CP, pp. 185-86; KS, p. 170). For further discussion of this point, and for an argument that Frege's idea that this sort of nonsensical use of language could somehow nevertheless be elucidatory is something that is inherited and radicalized in Wittgenstein's conception of "elucidation" in the Tractatus, see Conant, J. (2000) "Elucidation and nonsense in Frege and the early Wittgenstein", in: A. Crary and R. Read (Eds.), The New Wittgenstein, pp. 174–217 (Abingdon: Routledge); see as well Jolley, K. (2007) The Concept "Horse" Paradox and Wittgensteinian Conceptual Investigations (London: Ashgate). For a general interpretation of Frege's work that places substantial emphasis on keeping track of the peculiar kind of expression that elucidations constitute, see Weiner, J. (1990) Frege in Perspective (Cornell, NY: Cornell University Press), especially Ch. 6.

- 30. From Frege's first (1903) essay entitled "Über die Grundlagen der Geometrie", p. 371; CP, p. 281; KS, p. 269; emphasis added. In the later "UGG" Frege repeats this point: in cases where "a reduction [Zurückführung] to what is known is not possible", "a definition is not possible" either, and instead here "one must recognize [anerkennen] a primitive element and so must be satisfied with an elucidation" (p. 305; CP, p. 304; KS, p. 292).
- Volume One was published in 1893; the second Volume didn't appear until a decade later (1903).
- 32. Compare, among other places, the Introduction to the *Grundgesetze*, where Frege tells us that in what follows he "defines [definiere] Number [Anzahl] as an extension of a concept" (p. x; my italics.). Compare as well the "definition" of Number given in *Grundlagen* §68 and also in §107.
- 33. Frege does not devote any sections of the *Grundgesetze* itself to explicitly thematizing the distinction between definitions and elucidations in these very terms. Nevertheless, the distinction itself is clearly at work, and even the terminology surfaces at several points (cf. §1 fn. 2, Gg I, p. 6 fn. 1; BLA, p. 34 fn. 10), including one point in the Second Part, where Frege again explicitly draws a sharp distinction between what is meant to be accomplished by an elucidation and what is the task instead of a definition. In §34, Frege provides a "definition" of the notation he means to use to express the function represented by "x belongs to the extension of the concept F". In §35, Frege admits that his exposition has contained more than the mere definition of this notation, as it also contains an "elucidation" of it. Frege emphasizes, however, that in the context of proofs, he will appeal *only* to the definition, and admits, moreover, that "our elucidation could be wrong in other respects without placing the correctness of those proofs in question; for only the definition itself is the foundation [*Grundlage*] for this construction [*Aufbau*]" (Gg I, p. 54; BLA, p. 94).
- 34. Signs for functions, concepts, arguments are introduced in Gg I, §1; those for objects and truth-values in Gg I §2; those for course-of-values and extensions in Gg I, §3; those for relations in Gg I, §4.
- 35. Frege makes clear throughout his writings that he means to use "judgment" in the sense of an "act [*Tat*]" on the part of a subject. Frege claims in *Grundgesetze* §5, for example, that the function of this sign is to "contain assertion [*Behauptung*]" (Gg I, p. 10; BLA, p. 39). This contrasts with signs that refer to or designate any concept (function) or object, as Frege makes clear in his contemporary essay, "Funktion und Begriff": "the judgment-stroke cannot be used to construct a functional expression; for it does not serve, in conjunction with the other signs, to designate an object", such that "\(\dagger \) 2 + 3 = 5' does not designate *anything* [*bezeichnet nichts*] but instead *asserts* something [*behauptet etwas*]" (p. 22n; CP, p. 149 fn. 7; KS, p. 137 fn. 7; emphasis added). Cf. as well "Die Verneinung" (p. 151n; CP, p. 381 fn. 13; KS, p. 370 fn. 10).
- 36. As with "judgment", evidence from other writings makes clear that Frege means to be using "inference" and "definition" so as to pick out acts on the part of the logician or mathematician. In the opening paragraph of his 1914 "Logik in der Mathematik", for example, Frege not only explicitly characterizes both inference and definition as acts, and so on a par with judgment, but also claims that they are acts that are especially central to both logic and mathematics. Frege argues that "mathematics has closer ties with logic than any other discipline" precisely because "almost the entire activity [Tätigkeit] of the mathematician consists in inferring [Schliessen]" with the other main "part of the mathematician's activity" consisting in "defining [Definieren]" (PW, p. 203; NS, p. 213).

It is worth noting as well that Frege's description of the act of defining here is continuous with the description of definition that we met with above in section II: defining is the act "through which we introduce a new name, in which we determine that it should have the same sense and the same reference as a phrase made up of already known signs" (Gg I, p. 45; BLA, p. 82).

- 37. In "Über Begriff und Gegenstand", for example, Frege again admits that such phrases are "of course only figurative [bildlich] expressions", claiming that "all that I wish or am able to do here is to give hints [Winke]" (p. 205; CP, p. 194; KS, p. 178).
- 38. Compare Frege's 1904 "Was ist eine Funktion?": "The peculiarity of functional signs, which we here call 'unsaturatedness', naturally has something answering to it in the functions themselves. They too may be called 'unsaturated', and in this way we mark them out as fundamentally different from numbers. Of course, this is no definition; but likewise none here is possible. I must confine myself to indicating [hinzuweisen] what I have in mind through figurative [bildlich] expression, and here I rely on the co-operative understanding [das entgegenkommende Verständnis] of the reader" (p. 667; CP, p. 292; KS, pp. 279–80). Compare as well "Logik in der Mathematik": "It is not possible to give a definition of what a function is, because we have here to do with something simple [Einfaches] and unanalyzable [Unzerlegbares]. It is only possible to lead one to [hinzuführen] what is meant, and to make it more distinct through connecting it to what is known. Here an elucidation must take the place of a definition, one that must, of course, count on a co-operative understanding" (PW, p. 235; NS, p. 254). Compare, finally, the late "Erkenntnisquellen" (PW, p. 271; NS, p. 290).
- 39. In fact, in *Grundgesetze* I, Frege makes the more general point that there can be no "explanation [*Erklärung*]" of "phrases like 'to have a reference' or 'to refer to something", because "their application [i.e., the application of these phrases] always presupposes that one has already known [*erkannt*] some names as referring [*bedeutungsvoll*]" (Gg I, §30, p. 46; BLA, p. 85).
- 40. This, I hope, should remove any impression that what I called "the definition-like statement" about judgment quoted above from this same page of "Die Verneinung" is in fact meant by Frege to function as a definition.
- 41. Compare especially Begriffsschrift §2 and "Booles rechnende Logik" (PW, p. 11n; NS, p. 11n), as well as the passages cited in note 5. In his earlier period, of course, Frege only officially recognizes one primitive semantical notion —namely, an undifferentiated notion of "content [Inhalt]"—though (as many have noted) anticipations of a more finegrained semantical distinction can be found already in Frege's discussion of identity in Begriffsschrift §8.
- 42. Recall the passage cited above from "Funktion und Begriff", in which Frege is explicit about the fact that that the signs for semantical content and the signs for pragmatical elements do not even signify what they do in the same way, as the former sort of sign "designates" an object or a concept, while the latter "does not designate anything", but instead "asserts something" (p. 22n; CP, p. 149 fn. 7; KS, p. 137 fn. 7; emphasis added).
- 43. Compare Frege's 1882 letter to Marty (PMC, p. 100; WB, p. 164).
- Compare Frege's 1897 essay "Über die Begriffsschrift des Herrn Peano und meine eigene" (p. 377; CP, p. 247; KS, p. 232), and the draft of a letter to Jourdain (PMC, pp. 78–79; WB, pp. 126–27).
- 45. Dummett, for example, criticizes Frege for ultimately failing to put enough into place for us "to gain any grip on what it is to express a sense, or more particularly, a thought, if we try to view the activity of expressing thoughts in isolation from the various kinds of linguistic acts which may be performed in relation to them", and also failing to recognize that "until a connection is made between the truth-value of a sentence and the linguistic activities of asserting, questioning, etc., which can be accomplished by its utterance, we are in the dark as to what truth and falsity are" (Frege: Philosophy of Language, p. 415; emphasis added). Brandom, for his part, speaks of the later Frege's shift to "understanding truth in terms of prior primitive reference relations", whereas the earlier Frege did not propose such an "understanding" (Articulating Reasons, p. 52). Ricketts describes the kind of intelligibility-relation at issue as consisting in the fact that certain "categories" are "wholly supervenient" on others ("Objectivity and objecthood", p. 66); he also construes

- the situation as one in which Frege takes "our grasp" of one "notion" to be "exhausted by" our "apprehension" of something else (op. cit., p. 89).
- 46. Here I am indebted to an anonymous reviewer for raising this as a possible (and quite compelling) rejoinder to the foregoing line of criticism.
- Frege gives a similar elucidation of "object", "concept", and "truth-value" in Grundgesetze I, \$\$2-3.
- 48. Frege often puts this point in terms of the link between truth and the "assertoric force" with which a certain sentence is uttered; compare Frege's 1897 "Logik" (PW, p. 129; NS, p. 140); also "Der Gedanke" (p. 63; CP, p. 356; KS, p. 347); and "Über Sinn und Bedeutung" (p. 34; CP, p. 164; KS, p. 149).
- In addition to the passage quoted above from "Über Sinn und Bedeutung" (cf. p. 35; CP, p. 165; KS, p. 150), see as well *Grundgesetze* I §5 (Gg I, p. 9; BLA, p. 38).
- 50. Compare Rech, E. (2007) "Frege on truth, judgment, and objectivity", Grazer Philosophische Studien, 75, pp. 149–73: Frege's set of elucidations "involves a kind of circle—not a vicious circle, but a hermeneutical one—which distinguishes it from more linear explanations, reductions, or definitions" (p. 157; cf. p. 163). Here Reck focuses primarily upon the circular connection between Frege's accounts of judgment, thought, and truth—with the result being, in his view, that these accounts "form an elucidatory circle, or here, a triangle" (p. 157). As I have indicated above, however, the point readily extends to Frege's account of object, concept, and function as well.

On this and other several other points, my own views have perhaps the closest affinities with the main lines of the position sketched in Reck's essay, a position also suggested, though not elaborated, by certain remarks in Kremer, Michael (2000) "Judgment and Truth in Frege", Journal of the History of Philosophy, 38. (As I note below, I also am in agreement with Tyler Burge's criticisms of Dummett and Ricketts, though Burge does not link these criticisms with his own positive thesis concerning the questions of methodology at issue in the present essay.) At times, however, Reck casts his own interpretation in such a way that draws it closer instead to the version of pragmaticism endorsed by Ricketts. Compare, for example, Reck's explicit endorsement of Ricketts's proposal that, instead of reading Frege as being committed to a reductive account of the notion of objectivity in terms of an "object-based metaphysics", Frege is actually committed to an explanation of objectivity in terms of a "judgment-based metaphysics" (p. 156), such that Frege takes "the notion of judgment" rather than "the notion of object" to be "central and primary" and so to have ultimate explanatory priority (ibid). Nevertheless, despite this endorsement, Reck's sense of closeness to Ricketts seems ultimately to be due in large part to one claim made in Ricketts' more recent article ("Logic and truth in Frege") that suggests that Ricketts himself is moving toward a position that is perhaps closer to the non-reductive interpretation that I am proposing here. This is Ricketts' claim that Frege's remarks about judgment "are equally elucidatory of judgment and truth" such that "we have no grasp on the one apart from a grasp of the other" (op. cit., p. 131; emphasis added). Reck cites this with approval (cf. p. 157 fn. 10), as does Kremer (op. cit., p. 580).

51. The very notion of "parts" here needs to be treated with care, since the pragmatic and semantical dimensions would now each be counted as "parts" only in the sense of non-independent aspects or moments of *das Logische*, each able to be isolated out only by abstraction from what is ultimately a unitary phenomenon, and not in the sense of each being self-standingly intelligible pieces. For an account of parts and wholes which draws out the nature and importance of the distinction between these two sorts of "parts" (self-standing "pieces" vs. non-independent "moments"), along with the distinctions between the two sorts of wholes that they each comprise, see the Third of Husserl's *Logische Untersuchungen* (Max Niemeyer, 1901), II. Band, p. 222f. Though I cannot develop the

- point here, this seems to provide us with yet another, till now unnoticed, but quite significant, point of concord between these two thinkers on the nature of logic and logical investigation.
- 52. This places me in accord with Tyler Burge on both fronts. I am in complete agreement with Burge's criticisms of Dummett in Burge's "Frege on truth" (1986, reprinted in *Truth*, *Thought*, *and Reason* (Oxford, 2005)): "Frege nowhere, to my knowledge, writes or implicates that the sense of a sentence is given 'in advance' of our going in for the activity of assertion. Denotation [i.e., *Bedeutung*] is motivated and justified by Frege in terms of our 'striving after truth'" (p. 130). I also agree with Burge's criticisms of Ricketts (cf. his "Frege on knowing the third realm" (1992), also reprinted in *Truth*, *Thought*, *and Reason*), that Ricketts is wrong to think that Frege intends to explain "ontological" notions by way of "logical" ones, where the latter is understood along Ricketts' lines as the doctrine of judgment, because Frege thinks instead that "logic and ontology are mutually entangled" and "interrelated", and that "a *full* understanding of logic involves appeal to notions like logical object, function, thought content" and so on (op. cit., p. 311 fn. 15; emphasis added).
- 53. In fact, Frege's approach would seem to be one that Brandom himself identifies but then puts to one side, for not being "bold" enough: an approach according to which "one instead simply explore[s] and unpack[s] the relations among the different aspects" of a subject-matter (*Articulating Reasons*, p. 205 fn. 6). To his credit, Brandom has more recently scaled back his earlier interpretive claims about Frege, admitting in his *Tales of the Mighty Dead* (Harvard, 2002), for example, that though "Frege opens the doorway to that pragmatism about content that consists in a kind of social practical functionalism or use theory of meaning", Frege "clearly is not tempted to pass through it himself" (p. 75). Instead, Brandom now appears to acknowledge that Frege is better read as committed to a "reciprocal sense-dependence" between the notions of inference and truth (p. 60).
- 54. This ambition is announced already in the Preface to the *Begriffsschrift*, and its achievability is, in Frege's mind, made "very probable" by the arguments of the *Grundlagen* (cf. §87 and §109), though Frege does not think that it has been conclusively realized until the constructions provided in the *Grundgesetze*.
- For an especially polemical display of this sentiment toward "the corrupting incursion of psychology into logic", see the opening pages of *Grundgesetze* I (Gg I, pp. xiv–f; BLA, p. 12f).
- 56. Jourdain asks Frege in a 1914 letter "whether [Frege] now regard[s] assertion (⊢) as merely psychological" (PMC, p. 78; WB, p. 126). Russell's early worries about the judgment-stroke can be found in *Principles of Mathematics* §478, though he and Whitehead eventually decide to include the notation for assertion in their 1910 *Principia Mathematica* I (cf. pp. 8–9). For Wittgenstein's explicit exclusion of Frege's (and eventually Russell's) concern with acts from logic, compare *Tractatus* §4.442 as well as the earlier remarks in his 1913 "Notes on logic" (*Notebooks*, pp. 96 and 103). For a very thoughtful discussion of the tensions that might arise due to Frege's twin commitments to anti-psychologism and to the centrality of judgment, see Ch. 3 of Wayne Martin's *Theories of Judgment* (Cambridge, 2006), especially p. 84f; compare as well David Bell, *Frege's Theory of Judgment* (Oxford, 1979).
- 57. See, for example, Grundlagen §27.
- 58. Hence there is a sense in which Frege is not as anti-psychologistic as, say, the Wittgenstein of the *Tractatus* appears to be, since Frege does not take the main point of the *Begriffsschrift* to be that of providing a map of the crystalline structure of logical space (tautologies), and does not endorse a (quasi-)picture theory of the significance of logical notation itself. (I say "quasi-" as Wittgenstein himself ultimately does not mean for the propositions of logic to "picture" anything in the strict sense of this term, since he does not think that the propositions of logic "say" anything. Rather they merely "show [zeigen]" or "display [aufweisen]" something about what can be said—namely, the "form"

or "scaffolding" of what is sayable, which is at the same time the form of reality or the world itself; cf. *Tractatus* §4.121 and §6.124.) For Wittgenstein, a logical notation does not represent or depict anything that "takes place" or "happens": "In logic, process and result are equivalent. (Which is why there are no surprises.)" (*Tractatus* §6.1261). Frege, by contrast, makes clear from his earliest writings that he views the language of logic to be something that will both express thoughts (a *lingua*) but also be useful for the performing of inferences (and so a *calculus*); cf. Frege's 1880–1 "Booles rechnende Logik und die Begriffsschrift" (PW, p. 12f; NS, p. 14f). To this extent, Frege takes logic to be about something we do, about our activity.

59. I would like to thank Jeremy Heis, Karolina Huebner, Samantha Matherne, Sasha Newton, Erich Reck, and a very perceptive and thoughtful anonymous referee for their helpful comments on earlier drafts. I would also like to thank Erich Reck, Jim Conant, Michael Kremer, Robert Pippin, and the graduate students in my Frege seminar at UCSD in Fall 2009 for many fruitful discussions of these (and many other) themes in Frege's thought.