

## Numerical Analysis of Dispersion Curves in Fluid-Filled Corrugated Pipes

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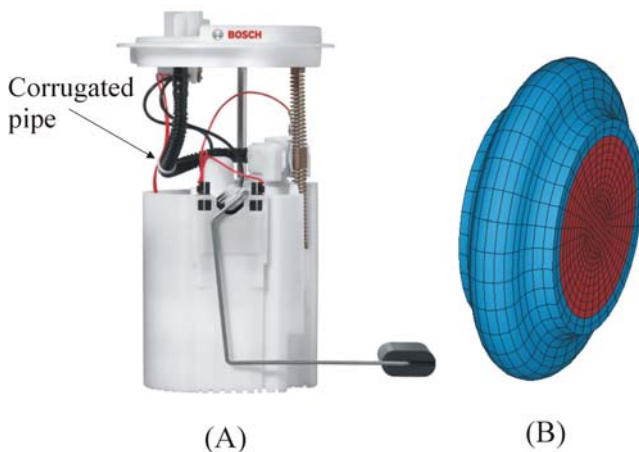
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### Introduction

The numerical analysis of harmonic wave propagation in corrugated pipes is studied in this paper. Such corrugated pipes are part of the engine fuel supply in automobiles and are located in the fuel supply module as depicted in Fig. 1, between the electric fuel pump and the fuel rail. The pipe is treated as a waveguide with periodic properties in direction of wave propagation. The analysis of these guided waves leads to dispersion curves showing the strong frequency-dependence of different wave modes. A complete understanding of the wave propagation in fuel-filled corrugated pipes is of major interest due to its potential of transmitting acoustic waves which may lead to undesired noise levels in the interior of vehicles. Mace et al. [1] have developed a method based on using standard FE-code in order to predict dispersion curves in solid waveguides called Waveguid-FE in the following. This technique is complemented by Maess et al. [2] for problems concerning acoustic-structure-interaction.

### Computation of Dispersion Curves

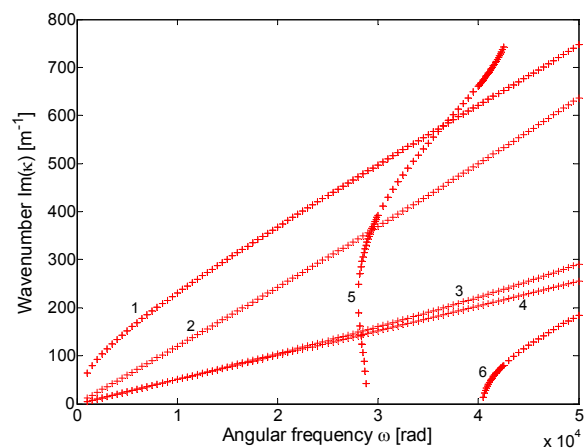
Details about FE-Modelling and the non-symmetric representation of the fully coupled equations are given in [3]. Following the idea of Mace et al. [1], a periodic segment model of the corrugated pipe is meshed using a standard FE-package as depicted in Fig. 1 (in this study ANSYS is used). The pipe segment consists of 10873 DOFs. Linear 8-node elements are used, both for the solid and fluid partition, and full FSI-coupling is applied. Equations are given in the references mentioned above [2],



**Figure 1:** Fuel supply module (courtesy of Robert Bosch GmbH) including fuel pump and corrugated pipes (A) and FE-model of a fuel-filled corrugated pipe segment (B).

this section only summarizes the main steps to solve for the dispersion curves. The assumption of harmonic wave propagation in the waveguide leads to the dynamic stiffness matrix. On the interface, continuity of the Dirichlet and Neumann data is imposed. The next critical step is the partition of the DOFs into a left and right portion with respect to the position on the pipe segment, whereas interior DOFs are condensed dynamically. A transfer matrix representation and an associated eigenproblem is obtained after applying periodicity conditions at successive cross-sections. From the eigenvalues, wave numbers are recovered, whereas the eigenvectors lead to wave mode shapes. Computations are performed for a corrugated pipe segment with a length of 4 mm. The material of the pipe shell is a Rilsan polymer ( $\rho_s = 1050 \text{ kg/m}^3$  and  $E = 330 \text{ MPa}$ ), whereas the fluid domain is a standard engine fuel ( $\rho_f = 800 \text{ kg/m}^3$  and  $K_f = 1250 \text{ MPa}$ ). The dispersion curves of the corrugated pipe are displayed in Fig. 2. Group velocities and power flows are obtained by postprocessing the different branches (Fig. 3 and Fig. 4) [2].

To avoid numerical errors, it is more convenient to compute group velocities as the ratio of the net power flow and the time-averaged energy density. Four different branches are visible in the complete observed frequency domain. The bending mode starts with zero velocity and is highly dispersive for low frequencies (branch 1). The torsional mode (branch 4) is characterized by the dispersion curve with the smallest slope (and therefore the highest velocity) and the fact that all the energy is located in the structure. The energy and phase velocity of the fluid-type mode



**Figure 2:** Dispersion curves: 1 bending mode; 2 long. mode; 3 fluid mode; 4 torsional mode; 5 1<sup>st</sup> cut-on mode; 6 2<sup>nd</sup> cut-on mode.

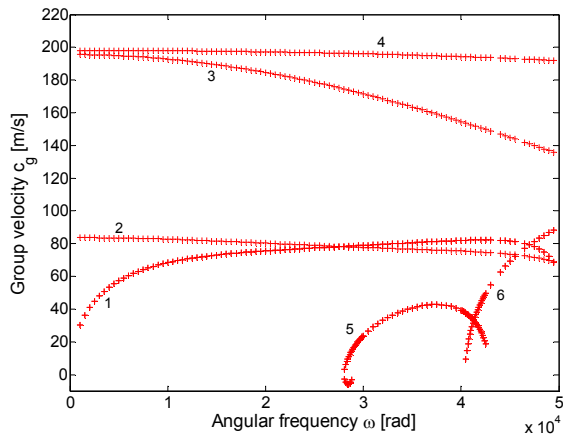


Figure 3: Group velocities.

(branch 3) decreases for higher frequencies. Moreover, two cut-on modes are visible (branch 5 and 6). Note that the 1<sup>st</sup> cut-on mode starts with a negative group velocity. Fig. 4 shows power flows in the fluid domain relative to the energy located in the structure. Partially, negative power transmissions are obtained. This indicates that the energy propagates against the direction of phase.

An animation tool (Structural Dynamic Toolbox) is used to visualize the different wave modes. Eleven periodic corrugated pipe segments are assembled and the wave forms of free wave modes are displayed by continuing the segment results periodically. As representative examples, the fluid-type mode and the bending mode are shown in Fig. 5. The animation of the fluid mode reveals that a positive pressure field leads to an expansion of the pipe shell and bending of the segment radii, whereas a negative pressure of the acoustic fluid results in a contraction of the structure. In addition, this animation tool helps to interpret the wave form of the cut-on modes. The 1<sup>st</sup> cut-on mode turns out to be a bending-type mode, whereas the 2<sup>nd</sup> cut-on mode is characterized by an elliptical deformation of the cross-section.

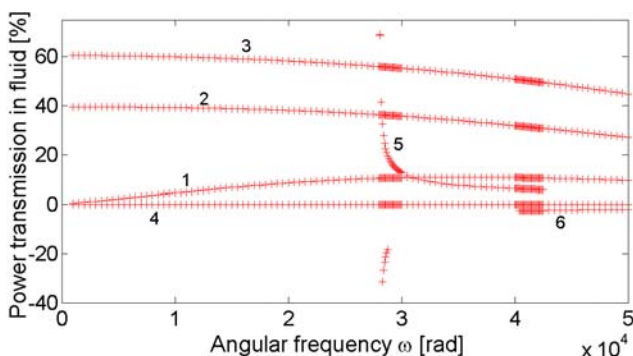


Figure 4: Energy distribution in the fluid in %.

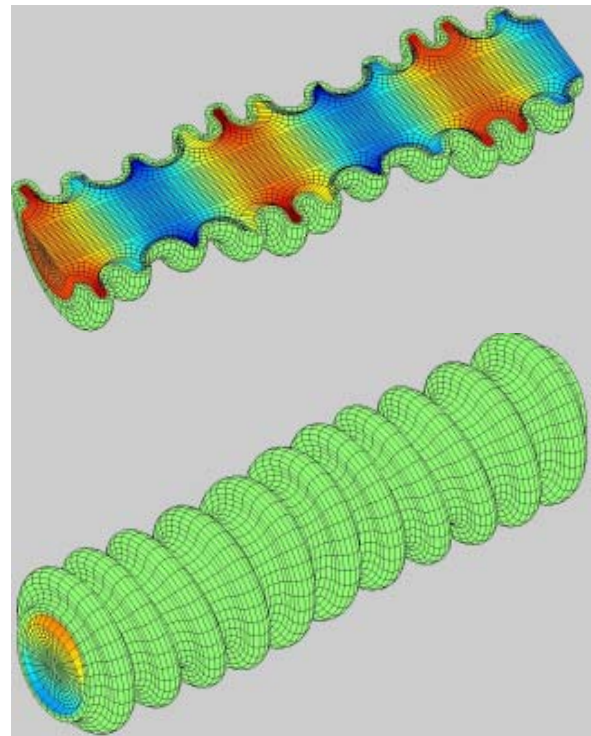


Figure 5: Illustration of the fluid mode (top, longitudinal section) and bending mode (bottom).

## Conclusion

It is shown that the Waveguide-FE technique from Mace et al. [1] and Maess et al. [2] is useful for practical examples of waveguides with more complicated geometries where analytical solutions of dispersion curves are not available. Wave numbers, group velocities and power transmissions are computed for a corrugated pipe. The use of standard FE-packages simplifies the computation of dispersion curves and allows the consideration of a large class of fluid-filled pipe problems.

## Acknowledgements

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## References

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