## Ch. 5 Logarithmic, Exponential, and Other Transcendental (*nonalgebraic*) Functions

# 5.10 Hyperbolic Functions

**Note:**

**The Gateway Arch in St. Louis, Missouri was constructed using the hyperbolic cosine function. The equation used to construct the arch was**

$$y = 693.8597 - 68.7672 \cosh 0.0100333x ,$$

$-299.2239 \le x \le 299.2239$, **where** $x$ **and** $y$ **are measured in feet.**

## Trigonometric Functions:

sin $x$ *and* cos $x$ ⇨ point $(x, y)$ *on* *the* unit circle, $x^2 + y^2 = 1$

## Hyperbolic Functions:

sinh $x$ *and* cosh $x$ ⇨ point $(x, y)$ *on* *the* unit hyperbola, $y^2 - x^2 = 1$

sinh $x$ *read as* "the hyperbolic sine of $x$" : "*cinch x*"

cosh $x$ *read as* "the hyperbolic cosine of $x$" : *rhymes w/* "*gosh x*"

## Definition of the Hyperbolic Functions: (p 395)

## Graphs of the Hyperbolic Functions: (p 396)

## Hyperbolic Identities: (p 397)

## Derivatives and Integrals of the Hyperbolic Functions:

(p 397)

Examples: **Find the derivative of each function.**

1. (p403 #18) $g(x) = ln(\cosh x)$

2. (p403 #20) $y = x \cosh x - \sinh x$

Example: **Find the integral.**

1. (p403 #46) $\int \operatorname{sech}^3 x \tanh x \, dx$

Examples: **Find the derivative of each function.**

1. (p403 #18) $g(x) = ln(\cosh x)$

$$g'(x) = \frac{1}{\cosh x} \cdot \frac{d(\cosh x)}{dx} = \frac{1}{\cosh x} \cdot \sinh x \ (1)$$

$$= \frac{\sinh x}{\cosh x} = \tanh x$$

2. (p403 #20) $y = x \cosh x - \sinh x$

$$y' = x \cdot \frac{d(\cosh x)}{dx} + \cosh x \cdot \frac{d(x)}{dx} - \frac{d(\sinh x)}{dx}$$

$$y' = x \cdot \sinh x \ (1) + \cosh x \cdot (1) - \cosh x \ (1)$$

$$y' = x \cdot \sinh x$$

Example: **Find the integral.**

1. (p403 #46) $\int \operatorname{sech}^3 x \ \tanh x \ dx$

let $u = \operatorname{sech} x$

$du = -\operatorname{sech} x \tanh x \ dx$

$$\int \operatorname{sech}^3 x \tanh x \, dx = -\int (\operatorname{sech} x)^2 \left(-\operatorname{sech} x \tanh x \, dx\right)$$

$$= -\int u^2 \, du$$

$$= -\frac{u^3}{3} + C$$

$$= -\frac{1}{3} \operatorname{sech}^3 x + C$$

# Videotape 12 [31 minutes]

## Inverse Hyperbolic Functions: (p 399)

## Derivatives and Integrals Involving Inverse Hyperbolic Functions: (p 401)

Example: **Find the integral.**

1. (p403 #52) $$\int \frac{2}{x\sqrt{1+4x^2}}\,dx$$

$$\int \frac{2}{x\sqrt{1+4x^2}}\,dx = 2\int \frac{1}{x\sqrt{1+(2x)^2}}\,dx$$

let $u = 2x$
$du = 2\,dx$

$$= 2\int \frac{1 \cdot 2\,dx}{2x\sqrt{1+(2x)^2}}$$

$$= 2\int \frac{du}{u\sqrt{1+u^2}} \qquad u = 2x,\quad a = 1$$

use!

$$\int \frac{du}{u\sqrt{a^2 \pm u^2}} = -\frac{1}{a}\ln \frac{a+\sqrt{a^2 \pm u^2}}{|u|} + C$$

$$= 2\left(-\frac{1}{1}\ln \frac{1+\sqrt{1+u^2}}{|u|}\right) + C$$

$$= -2\ln\left(\frac{1+\sqrt{1+4x^2}}{|2x|}\right) + C$$

**Texas Instruments**

***The hyperbolic functions are available only in the*** **CATALOG.**

# Assignment

p403 #1, 3, 5, 7 (identities)
#13,
#15-27 odd (derivatives)
#37 (power cable)
#39-53 odd (integrals - omit #49)
#79, 81 (Area)