ECE 307 - Homework 7 Solutions

First we provide the solutions of the problems from Clemen's 2nd edition.
At the end of the document you may find the solutions to the exercises from Clemen's 3rd edition.

Problem 7.1

7.1. We often have to make decisions in the face of uncertainty. Probability is a formal way to cope with and model that uncertainty.

Problem 7.3

$P(A \text{ and } B) = 0.12$

$P(A \text{ and } \overline{B}) = 0.29$

$P(A) = 0.41$

$P(B) = 0.65$

$P(\overline{B}) = 0.35$

$P(B \mid A) = \frac{0.12}{0.41} = 0.293$

$P(A \mid B) = \frac{0.12}{0.65} = 0.185$

$P(\overline{A} \mid \overline{B}) = \frac{0.06}{0.35} = 0.171$

Problem 7.4

$$P(A \text{ or } B) = P(A \text{ and } B) + P(A \text{ and } \overline{B}) + P(\overline{A} \text{ and } B) = 0.12 + 0.53 + 0.29 = 0.94$$

$$\text{or } P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.41 + 0.65 - 0.12 = 0.94$$

$$\text{or } P(A \text{ or } B) = 1 - P(\overline{A} \text{ and } \overline{B}) = 1 - 0.06 = 0.94$$

Problem 7.6

7.6.a.	Joint.	P(left-handed and red-haired) = 0.08
b.	Conditional	P(red-haired | left-handed) = 0.20
c.	Conditional	P(Cubs win | Orioles lose) = 0.90
d.	Conditional	P(Disease | positive) = 0.59
e.	Joint	P(success and no cancer) = 0.78
f.	Conditional	P(cancer | success)
g.	Conditional	P(food prices up | drought)
h.	Conditional	P(bankrupt | lose crop) = 0.50
i.	Conditional, but with a joint condition: P(lose crop | temperature high and no rain)	
j.	Conditional	P(arrest | trading on insider information)
k.	Joint	P(trade on insider information and get caught)

Problem 7.7

For Product B, EMV = \$8M(0.38) + \$4M(0.12) + 0(0.50) = \$3.52M

Var(B) $= 0.38(8 - 3.52)^2 + 0.12(4 - 3.52)^2 + 0.50(0 - 3.52)^2$

= 13.8496 "Millions-of-dollars squared"

Standard Deviation for B = $\sigma_B = \sqrt{13.8496}$ = \$3.72M.

For Product C, there is no variation. Thus, Var(C) = 0 and $\sigma_C = 0$.

Problem 7.9

$P(\bar{A}) = 1 - P(A) = 1 - 0.10) = 0.90$

$P(\overline{B} \mid A) = 1 - P(B \mid A) = 1 - 0.39 = 0.61$

$P(\overline{B} \mid \bar{A}) = 1 - P(B \mid \bar{A}) = 1 - 0.39 = 0.61$

$P(B) = P(B \mid A)\, P(A) + P(B \mid \bar{A})\, P(\bar{A}) = 0.39(0.10) + 0.39(0.90) = 0.39$

$P(\overline{B}) = 1 - P(B) = 1 - 0.39 = 0.61$

At this point, it should be clear that A and B are independent because $P(B) = P(B \mid A) = P(B \mid \bar{A}$))) = 0.39. Thus, $P(A) = P(A \mid B) = P(A \mid \overline{B}) = 0.10$, and $P(\bar{A}) = P(\bar{A} \mid B) = P(\bar{A} \mid \overline{B}) = 0.90$. (Actually, the fact that A and B are independent can be seen in the statement of the problem.)

Problem 7.11

7.11. a. The basic setup for this problem is the same as it is for the previous problem. We already have the joint probabilities, so we can start by calculating the expected values of X and Y:

$$E(X) = \frac{(-2) + (-1) + 0 + 1 + 2}{5} = 0$$

$$E(Y) = 0.2\ (0) + 0.4\ (1) + 0.4\ (2) = 1.2$$

Thus, we have the following table:

X	Y	X - E(X)	Y - E(Y)	(X-E(X))(Y-E(Y))
-2	2	-2	0.8	-1.6
-1	1	-1	-0.2	0.2
0	0	0	-1.2	0
1	1	1	-0.2	-0.2
2	2	2	0.8	1.6

The covariance is the expected value of the numbers in the last column, each of which can occur with probability 1/5. Calculating this expected value gives a covariance of zero. Likewise, the correlation equals zero.

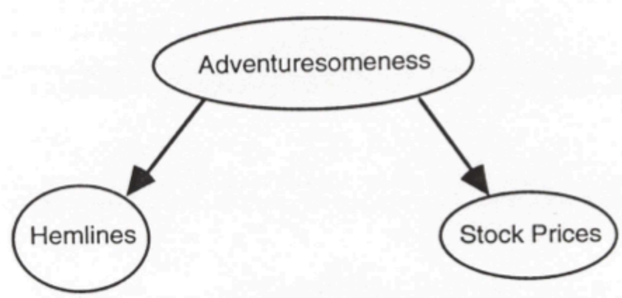
b. $P(Y = 2) = 0.4$, but $P(Y = 2 \mid X = -2) = P(Y = 2 \mid X = 2) = 1.0$ and $P(Y = 2 \mid X = 0) = 0$

c. $P(X= -2) = 0.2$, but $P(X = -2 \mid Y = 2) = 0.5$ and $P(X = -2 \mid Y = 0) = 0$

d. Clearly, X and Y are dependent. In fact, $Y = |X|$. But it is not a linear relationship, and the covariance relationship does not capture this nonlinear relationship.

Problem 7.12

7.12. The influence diagram would show conditional independence between hemlines and stock prices, given adventuresomeness:



Thus (blatantly ignoring the clarity test), the probability statements would be P(Adventuresomeness), P(Hemlines | Adventuresomeness), and P(Stock prices | Adventuresomeness).

Problem 7.14

7.14.

	B	$\overline{B}$	
A	0.204	0.476	0.68
$\overline{A}$	0.006	0.314	0.32
	0.21	0.79	1

P(B and A) = P(B | A) P(A) = 0.30 (0.68) = 0.204

P(B and $\overline{A}$) = P(B | $\overline{A}$) P($\overline{A}$) = 0.02 (0.32) = 0.006

$$P(\overline{B} \mid A) = \frac{P(\overline{B} \text{ and } A)}{P(A)} = \frac{0.476}{0.68} = 0.70$$

$$P(\overline{B} \mid \overline{A}) = \frac{P(\overline{B} \text{ and } \overline{A})}{P(\overline{A})} = \frac{0.314}{0.32} = 0.98$$

P(B) = P(B and A) + P(B and $\overline{A}$) = 0.204 + 0.006 = 0.21

P($\overline{B}$) = 1 - P(B) = 1 - 0.21 = 0.79

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{0.204}{0.21} = 0.970$$

P($\overline{A}$ | B) = 1 - P(A | B) = 1 - 0.970 = 0.030

$$P(A \mid \overline{B}) = \frac{P(A \text{ and } \overline{B})}{P(\overline{B})} = \frac{0.476}{0.79} = 0.603$$

P($\overline{A}$ | $\overline{B}$) = 1 - P(A | $\overline{B}$) = 1 - 0.603 = 0.397

Problem 7.16

7.16. a. $E(X) = 0.05\,(1) + 0.45\,(2) + 0.30(3) + 0.20(4)$
$= 0.05 + 0.90 + 0.90 + 0.80$
$= 2.65$

$Var(X) = 0.05\,(1\text{-}2.65)^2 + 0.45\,(2\text{-}2.65)^2 + 0.30(3\text{-}2.65)^2 + 0.20(4\text{-}2.65)^2$
$= 0.05\,(2.72) + 0.45\,(0.42) + 0.30(0.12) + 0.20(1.82)$
$= 0.728$

$\sigma_X = \sqrt{0.728} = 0.853$

b. $E(X) = 0.13\,(-20) + 0.58\,(0) + 0.29(100)$
$= -2.60 + 0 + 29$
$= 26.40$

$Var(X) = 0.13\,(-20 - 26.40)^2 + 0.58\,(0 - 26.40)^2 + 0.29(100 - 26.40)^2$
$= 0.13\,(2152.96) + 0.58\,(696.96) + 0.29(5416.96)$
$= 2255.04$

$\sigma_X = \sqrt{2255.04} = 47.49$

c. $E(X) = 0.368\,(0) + 0.632\,(1) = 0.632$

$Var(X) = 0.368\,(0 - 0.632)^2 + 0.632\,(1 - 0.632)^2$
$= 0.368\,(0.632)^2 + 0.632\,(0.368)^2$
$= 0.368\,(0.632)\,[0.632 + 0.368]$
$= 0.368\,(0.632)$
$= 0.233$

$\sigma_X = \sqrt{0.233} = 0.482$

For those who solved the problems from Clemen's 3rd edition the solutions for the exercises that are different from those of the 2nd edition are given below:

Problem 7.11 (3rd edition)

This is problem 7.12 in the 2nd edition and the solution is given above.

Problem 7.12 (3rd edition)

In many cases, it is not feasible to use a discrete model because of the large number of possible outcomes. The continuous model is a "convenient fiction" that allows us to construct a model and analyze it.

Problem 7.14 (3rd edition)

P(offer) = 0.50
P(good interview | offer) = 0.95
P(good interview | no offer) = 0.75

$P(\text{offer} \mid \text{good interview}) = P(\text{offer} \mid \text{good})$

$$= \frac{P(\text{good} \mid \text{offer})\, P(\text{offer})}{P(\text{good} \mid \text{offer})\, P(\text{offer}) + P(\text{good} \mid \text{no offer})\, P(\text{no offer})}$$

$$= \frac{0.95\ (0.50)}{0.95\ (0.50) + 0.75\ (0.50)}$$

$$= 0.5588$$

Problem 7.16 (3rd edition)

$E(X) = (1 - p)\ (0) + p\ (1) = p$

$$\begin{aligned} \text{Var}(X) &= (1 - p)\ (0 - p)^2 + p\ (1 - p)^2 \\ &= (1 - p)\ p^2 + p\ (1 - p)^2 \\ &= (1 - p)\ p[p + (1 - p)] \\ &= (1 - p)\ p \end{aligned}$$