PHYSICAL GEOGRAPHY

THEORY ON CENTRAL RECTILINEAR RECESSION OF SLOPES IV

BY

J. P. BAKKER AND J. W. N. LE HEUX

(Communicated by Prof. F. A. VENING MEINESZ at the meeting of Sept. 30, 1950)

6. *The ratio between the flat part and the curved part of the rocky nucleus*

The simplified construction of the isoclines by the drawing of some special points, however, is not our principal aim. The integral-curve has a flat part and a curved part. The unprotected flat part of the rocky nucleus being subject to further softening, we should like to know something about the ratio of these parts and about the influence of the data a, b and c upon this ratio.

This is one of the central problems of the interpretation of slopes in Nature. As the reader will be aware W. PENCK [18, p. 127—132] attempted to demonstrate that the slopes of our figures 18 and 19 [W. PENCK's figures 7 (p. 129) and 8 (p. 131)] are the results of accelerated vertical erosive action of streams.

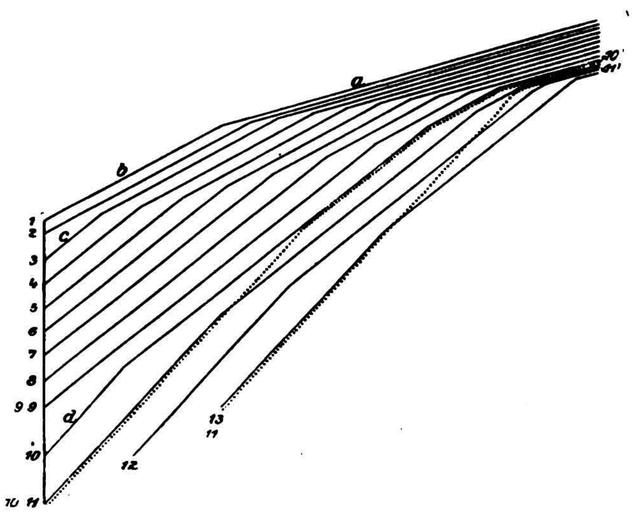

Fig. 18 (after W. PENCK)

"Das veranschaulicht fig. 7 (our figure 18) in der 13 aufeinanderfolgende Hanglagen konstruiert worden sind, die sich unter der Annahme *wachsender Erosionsintensität* ergeben" [W. PENCK, Morpholog. Analyse p. 128].

"Es ist ersichtlich (fig. 8, our figure 19), dasz sich die Hänge umso schärfer konvex gegen die Talkerben krümmen, je gröszer die Erosionsbeschleunigung ist, je rascher die Erosionsintensität wächst" [W. PENCK, Morphol. Analyse p. 131].

In this part of his theory W. PENCK jumped to the conclusion, that the convex "Gefällsbruch" between the steep part at the foot of his slopes and their flat upper parts could be formed exclusively by accelerated vertical fluvial erosion:

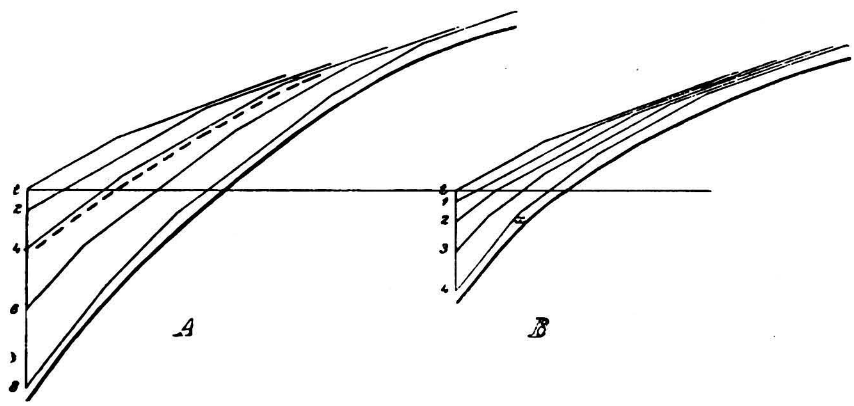

Fig. 19 (after W. PENCK)

"Der Gefällsbruch ist eine Diskontinuität der Abdachung; er wird aber verursacht durch eine kontinuierliche Zunahme der Erosionsintensität. Zum erstenmal begegnen wir dem Fall, dasz eine stetige Ursache morphologisch eine Unstetigkeit bewirkt" (Morphol. Analyse p. 131).

It is clear that especially the undermost curves of W. PENCK's figures 8*A* and 8*B* (our figures 19*A* and 19*B*) have approximately the same shapes as the curves of central rectilinear recession for $a = 25°$ and $22°$, $\beta = 52°$ and c about 1/3. If it is assumed, that the screes were removed afterwards it is possible to explain W. PENCK's slopes in quite an other way. These slopes may be the results of central rectilinear recession by weathering removal of a wall with an initial slope angle of 52°, without any direct effect on the rocky nucleus of accelerated vertical fluvial erosion. Moreover we demonstrated in the second part of our theory (16, p. 1158—1159) that in special conditions knickpoints in convex slopes can be formed exclusively by the free play of weathering removal recession, without having any direct effect of vertical fluvial erosion.

It is our conviction, that W. PENCK's theory is not justified from a quantitatively exact standpoint, a fact which has been pointed out before by JOH. SÖLCH (19) and J. P. BAKKER (6*a*, 20 and 21). In our opinion the theoretical knowledge of slope development based upon a mathematical treatment of the phenomena in Nature has not yet proceeded sufficiently to jump to W. PENCK's morphotectonic conclusions. It should not be overlooked, that without a rigorous mathematical treatment such series of two-dimensial drawings have not any deductive value. In the most favourable case series of two-dimensial figures or blockdiagrams etc., drawn with the purpose to demonstrate a morphogenetic succession — for instance of cyclic landscape development in arid or humid climates — are nothing but series of co-existing types of present-day landscapes. Reasoning inductively — induction is methodical guesswork — scientists such as DAVIS and W. PENCK assume, that there must be a logical morphogenetic relation between the landscapetypes of their series, but a deductive evidence for the exactness of this assumption is completely lacking. It is clear that from a logical standpoint there is a great danger in these pseudo-deductive treatments of physionomic geomorphologic problems. There are two lacunae in their logical construction: 1. an external one, 2. an internal one:

1. The conditions on which the pseudo-deductions are based, are not established at all or quite insufficiently. For instance in W. PENCK's treatise on slope development over an unchanging baselevel (18, p. 109—115), the author started from one or a small number of constant a-values and a high negative c-value, but overlooked, that halfway he changed his constant a-values into completely unknown variable ones.
2. As we already mentioned the internal lacuna in the logical construction of several present day theories on slope development is the most dangerous one. It is the lacuna of exactly justified evidence of the reality of a special landscape succession. Is a landscape B really the successor of A and C of B etc.?

If a scientist is not able to establish the physico-mathematical relation between C and B, B and A etc. his opinion has only the value of a supposition or an apriorism — we shall call it in future the special morphogenetic apriorism — and the demonstration in his pseudo-deductions has only the value of a parable. From most textbooks and other publications on geomorphology the impression might be gathered, that we know already fairly much of cyclic development of mountain forms. In our opinion, however, our actual knowledge of several parts of physionomic geomorphology is extremely poor, especially as regards such regions, where it is impossible to find datable sediments and fossil soiltypes on the different plateau-like forms, terraces etc. For our knowledge of slope development and several other physionomic geomorphologic problems a decision pro or con mathematical methods is at the same time a choice between science and parable, a question of to be or not to be.

Apart from W. PENCK's ideas the problem of knickpoints in slopes is a very important one for the bicyclic or monocyclic interpretation of rounded summits, ridges and plateau-like forms. Already 30 years ago SÖLCH discussed this problem in his very interesting publication: "Eine Frage der Talbildung" (22, p. 67—68). For the special case of "Eckflurs" the author's conclusion is, that a bicyclic interpretation as well as a monocyclic one is possible, but he believes that the former interpretation is more probable. "Allein es wird doch wohl nur in Ausnahmen aus einem Grat oder einem gewölbten Rücken ebenflurige, scharfrandige Platte mit steilen Gehängen hervorgehen".

Indeed, if we follow the current opinions a priori a bicyclic interpretation seems to be the most probable one, but we must not forget, that up to this day the knickpointproblem and his various possibilities have never been investigated from a completely exact standpoint; not even for the simplest conditions (see our theory p. 1158—1159 and some aspects of this problem in BAKKER, 23, p. 10—29).

In the following lines we shall give a mathematical method, which is necessary as introduction for an exact treatment of the knickpointproblem.

In the first place, a mathematical definition of the expression "flat part" is wanted. We will call "flat part" of the integral-curve the part limited by the isoclines $p = \tan \alpha$ and $p = \tan \alpha'$, if the difference $\tan \alpha' - \tan \alpha$ is 0,1. Of course, another small value may be taken. So, $\alpha' - \alpha$ is the angle between the directions of the tangents to the integral-curve in the extremities of the nearly straight part. In fig. 14 and 15, the flat part lies between the isoclines 0,4 and 0,5; $\alpha' - \alpha$ is nearly 5°.

The ratio between the flat part and the curved part depends upon the intersection-point of isocline $\tan \alpha + 0,1$ with the integral-curve. Observing the course of the isoclines in fig. 14 and 15 (part III of our theory), we see that the scale of their meeting-points with the curve shows some resemblance to the scale of their meeting-points with the slope.

Therefore, our following examination will be based on the definition:

> *The ratio between the flat part and the curved part of the integral-curve depends upon the intersection-point S of isocline tan α + 0,1 with the slope in the case of a plateau as well as in the case of a crest.*

Construction of S from given values of a, b and c

As we have seen sub 5 (part III of our theory) the simplified construction of S requires the construction of Q (fig. 20). Now Q is the meeting-point of HA and NB. Choosing OK as unit, $KH = h = \tan \beta = 1/b$. So, the coordinates of H are $(1, 1/b)$. Since $AD = KG = \tan \alpha = 1/a$ and $OA = AD \cot \beta = b/a$, the coordinates of A are $(b/a, o)$. Thus, the equation of HA is

$$b\,y = \frac{ax - b}{a - b}.$$

Since $BE = KF = \tan \gamma = p$ and $OB = BE \cot \beta = pb$, the coordinates of B are (pb, o). The coordinates of C being $(m, 1/b)$, the equation of NB is

$$b\,y = \frac{x - pb}{m - pb}.$$

From $\frac{ax-b}{a-b}=\frac{x-pb}{m-pb}$ we derive $x=\frac{b\,(m-ap)}{a\,(m-1)-b\,(ap-1)}$.

Now again $m = 2(1-c)\,(ap-1)+1$ so $m-1 = 2\,(1-c)\,(ap-1)$ and $m-ap = (1-2c)\,(ap-1)$. Therefore

$$x=\frac{b\,(1-2c)\,(ap-1)}{(ap-1)\,\{2a\,(1-c)-b\}}=\frac{b\,(1-2c)}{2a\,(1-c)-b}$$

and

$$y=\frac{1}{b}\cdot\frac{ax-b}{a-b}=\frac{-1}{2a\,(1-c)-b}.$$

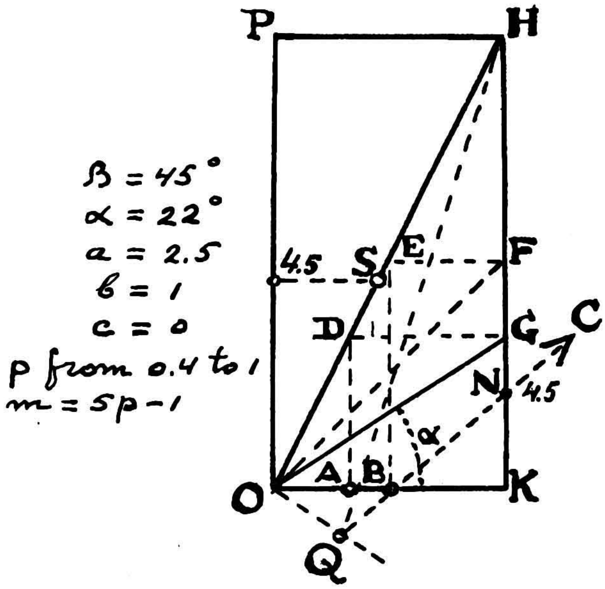

Fig. 20

Consequently: Q lies on the straight line

(20) $$x = -\,b(1-2c)y$$

To get some idea of the ratio between the flat part and the curved part of the rocky nucleus, without drawing any curve, we now proceed as follows.

In the right-angled triangle of fig. 20, OH is the slope and $\angle\ HOK = \beta$ the initial slope-angle of the wall of a plateau or crest.

Draw OP parallel to KH with a regular scale from 0 to 10. KH gets a quadratic scale from 0 to 10. Draw OG so that $\angle\ GOK = \alpha$, the slope-angle of the screes. Cot $\alpha = a$. Draw OF to define the "flat part". $\angle\ FOG$ is the angle between the directions of the tangents in the extremities of the nearly straight part of the integral-curve. FG may be taken 0,1. Transpose the points G and F to OK by drawing GDA and FEB. Draw HA and OQ [cot $KOQ = -\,b(1-2c)$] with the point of intersection Q. Finally, join Q to B and produce this line till the quadratic scale is intersected in N. The number of N gives the value of the ordinate of S. Construct S by means of the regular scale upon OP, then $HS : SO$ is approximately the ratio between the flat part and the curved part (in horizontal sense for a plateau and in vertical sense for a crest).

7. *Discussion of the influence of a, b and c upon the ratio*

I. *Influence of a*

To discuss the influence of a upon the place of the point N (fig. 20), we will calculate the ordinate KN of this point.

For the coordinate of Q we have found

$$x = \frac{b\,(1-2c)}{2a\,(1-c)-b} \quad , \quad y = \frac{-1}{2a\,(1-c)-b}.$$

The coordinates of B are $OB = x = BE \cot \beta = KF$. $b = \tan a'$. $b = b/a'$ and $y = o$ $(\tan a' = \tan a + 0.1 = (10 + a)/10a)$.

The equation of QB is:

$$\frac{x - \dfrac{b}{a'}}{\dfrac{b\,(1-2c)}{2a\,(1-c)-b} - \dfrac{b}{a'}} = \frac{y}{\dfrac{-1}{2a\,(1-c)-b}}.$$

For $x = 1$, $y = KN = \dfrac{b - a'}{b\,\{(a-a')\,(2c-1) - (a-b)\}}$

y becomes zero, if $b = a'$ or $\tan \beta - \tan a = 0.1$.

Conclusion:

In the case of a small difference between β and a, the rocky nucleus may be regarded as a "flat part" over its whole length.

Introducing the value of $a' = \dfrac{10a}{10+a}$ we find

$$y = \frac{1}{b}\,\frac{10\,(a-b) - ab}{2a^2\,(1-c) + 10\,(a-b) - ab}.$$

The ratio: $\dfrac{\text{flat part}}{\text{curved part}}$ becomes

$$\frac{1-y}{y} = \frac{2a^2}{10\,(a-b) - ab} \cdot (1-c) = (1-c)\,f\,(a).$$

The function $f(a)$ becomes minimum for $a = \dfrac{20\,b}{10-b}$, independent of c and then, the ratio has the value

$$\frac{80\,b}{(10-b)^2} \cdot (1-c) \text{ dependent upon } c.$$

Since $a = \dfrac{1}{\tan a}$ and $b = \dfrac{1}{\tan \beta}$, the condition $a = \dfrac{20\,b}{10-b}$ becomes $\tan \beta - 0.1 = 2 \tan a$.

Conclusion:

If there exists the relation tan β — 0.1 = 2 *tan a between the initial slope-angle of the wall of a plateau or crest and the slope-angle of the screes the ratio between flat part and curved part of the rocky nucleus has a minimum value, dependent upon the ratio between rock-volume and screes-volume.*

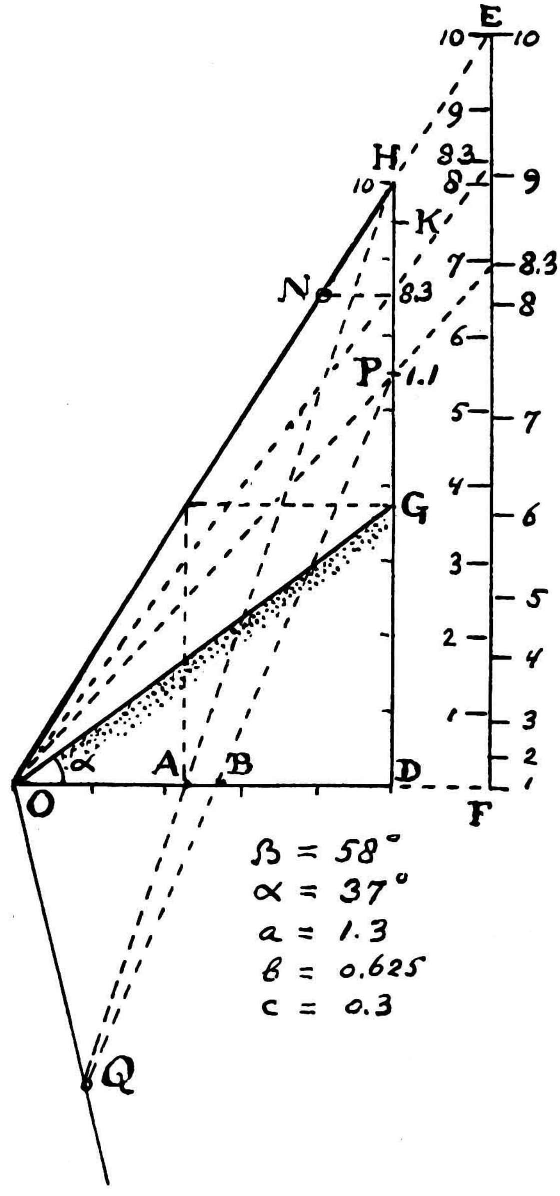

Fig. 21

Example:

In fig. 21 we have chosen $\angle\ HOD = \beta = 58°, OD = 1, DH = \tan\beta = 1.6$ The angle a can easily be constructed by taking $HK = 0.1$ and bisecting KD in G. It follows, that $\tan a = \frac{3}{4}$.

G transposed to OD gives A and H joined to A meets OQ in Q ($\tan DOQ =$ $= \frac{-1}{b(1-2c)} = -4$, if we choose $c = 0.3$). Q joined to B ($AB = 0.1$) meets DH in P. The value of $PD = y$ is obtained by putting $a = \frac{20b}{10-b}$ in the formula $y = \frac{1}{b}\frac{10(a-b) - ab}{2a^2(1-c) + 10(a-b) - ab}$ which gives

$$y = \frac{1}{b}\frac{(10-b)^2}{(10+b)^2 + 40b(1-2c)}$$

and then putting $b = \frac{5}{8}$, $c = 0.3$. We find $y = 1.1$ measured with OD as unit. The number of P on the (auxiliary) quadratic scale is 8.3 (right side of EF). Transposed to the regular scale (left side of EF) and brought back to HD, we find the point N on the slope.

We can now discuss the influence of a on the shape of the integral-curve. In fig. 21, for given values of b and c, the flat part of this curve cannot be smaller than HN, in horizontal parallel projection for a plateau and in vertical parallel projection for a crest. The flat part increases, if $\tan a$ decreases from GD to zero and if $\tan a$ increases from DG to DK (maximum flat part for $\tan \beta - a = 0.1$).

In general terms:

If a decreases from ∞ *to* $\frac{20b}{10-b}$, *the flat part decreases.*

If a decreases from $\frac{20b}{10-b}$ *to b, the flat part increases.*

II. *Influence of b.*

It is a remarkable fact, that the value b does not appear in the equation of the isoclines

$$px = y\frac{h^2 - my^2}{h^2 - y^2} \text{ and } px = y\frac{k^2 - mx^2}{k^2 - x^2} \qquad [m = 2(1-c)(ap-1) + 1].$$

That means: shape and position of the isoclines are independent of the initial slope-angle of the wall of a plateau or a crest. Calculating the point of intersection of an isocline and the slope with equation $x = by$, we find:

$$pb = \frac{h^2 - my^2}{h^2 - y^2} \text{ (plateau) and } pb = \frac{k^2 - mx^2}{k^2 - x^2} \text{ (crest)}$$

($x = 0$, $y = 0$ are the coordinates of the second point of intersection in the origin).

If we seek the locus of the meeting-point, when b is variable, the value b must be eliminated between the two last equations and $x = by$. The result is:

$$px = y\frac{h^2 - my^2}{h^2 - y^2} \text{ and } px = y\frac{k^2 - mx^2}{k^2 - x^2}$$

the equations of the isoclines themselves.

We have seen that the angle a of the screes being given, the ratio between the flat part and the curved part of the integral curve is determined by the meeting-point of the isocline $p = \tan a + 0.1$ with the slope. *Thus, the locus of this meeting-point, when* β *varies from* 90° *to* a, *is the isocline* $p = \tan a + 0.1$ *in the case of a plateau as well as in the case of a crest.*

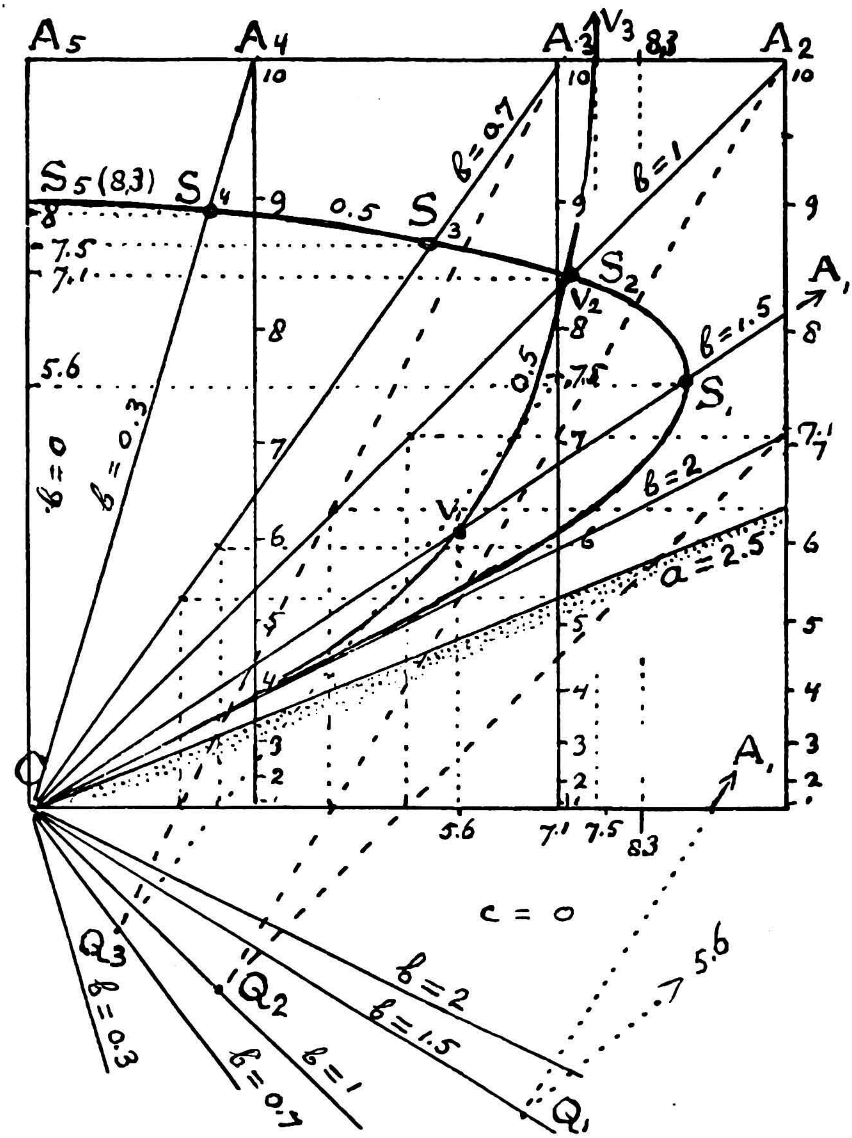

Fig. 22

Example: In fig. 22, $a = 2.5$; $c = 0$. Since $\tan a = 0.4$, we have constructed isocline 0.5 for a plateau and for a crest. Both isoclines meet on the slope OA_2 in S_2, because $k = h$. The ordinate of S_2, constructed by means of Q_2 ($b = 1$) is 7.1. Now the ratio between the flat part and the curved part of the integral curve in S_2A_2: S_2O (horizontally measured for a plateau, vertically measured for a crest). In this manner, the ratio can easily be read off if b varies from 0 to 2. For a plateau the points of division are S_5—S_1 for a crest V_5—V_1. For $b = 2$, the slope is tangent to isocline 0.5, the curved part disappears and we find again: *In the case of a small difference between β and α ($\tan\beta - \tan\alpha = 0.1$), the integral curve has no curved part.*

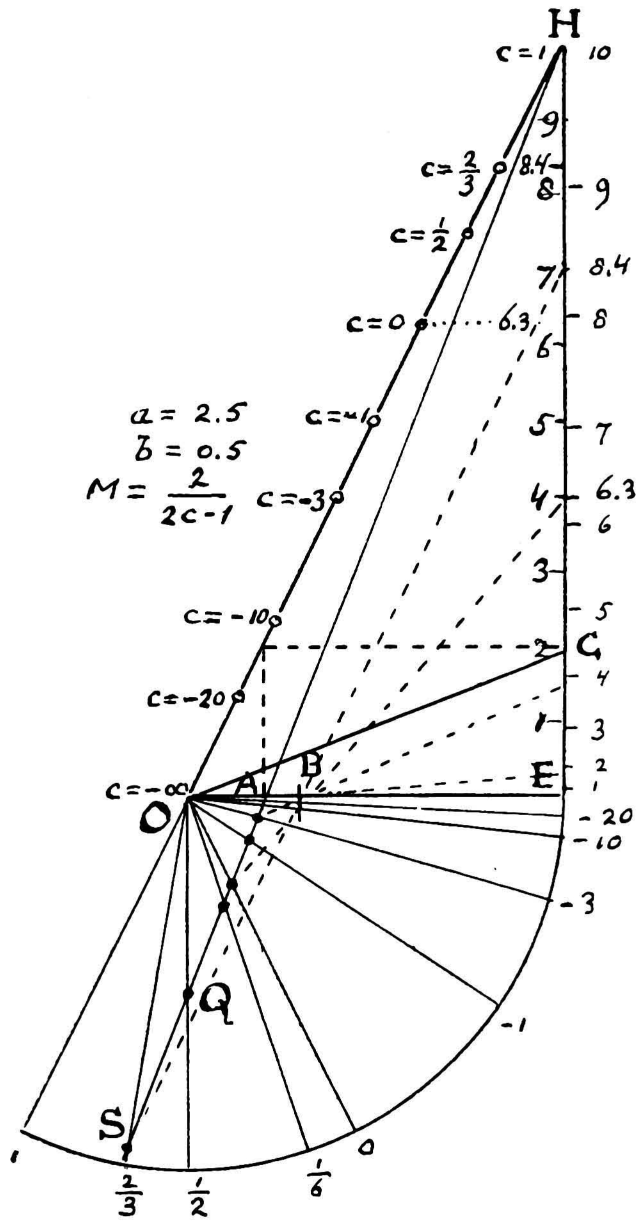

Fig. 23

Reversely, the isoclines may be constructed by seeking their meetingpoints upon the lines OA_1, OA_2 etc.

In fig. 22, the flat part in both cases cannot be smaller than 1.7 $(OS_5 = \frac{h}{\sqrt{m}} = \frac{10}{\sqrt{1{,}5}} = 8.3$ and the asymptote of isocline 0.5, $x = \frac{k}{\sqrt{m}})$.

Assuming that the lines AS are measured up to their points of intersection with $A_5\,A_2$, we can say about the influence of b upon the shape of the integral curve: *If, for given values of a and c, the value of b increases, the flat part of the integral curve increases also, till the maximum is reached for* $b = \frac{10a}{10+a}$

III. *Influence of c*

In fig. 23, $a = 2.5$; $b = 0.5$. The fragment $AB = 0.1$ remains invariable in position when c varies from 1 to $-\infty$ (for the c-values in Nature, see part II of our theory).

We have drawn the direction of OQ $(M = \frac{2}{2c-1})$ for $c = 1, \frac{2}{3}, \frac{1}{2}, \frac{1}{6}, 0, -1, -3, -10$ and -20.

The invariable line HA meets these directions in the points Q, which are joined to B. The meetingpoints with the quadratic scale are transposed to the slope OH by means of the double-scale EH. About the influence of c upon the shape of the integral curve, we observe: *Changes in the positive values of c between 0 and ½ have little influence: the flat part increases if c decreases. The curved part disappears for high negative values of c* (practically beginning with $c = -25$. $M = -\frac{1}{51} \tan \beta$).

Finally, we wish to express our gratitude to Mrs A. v. d. Zee-de Bruijne for her help in the correction of the text and to Miss C. Vervoort and Mr W. Koldewijn, assistants for physical geography at the University of Amsterdam for their kindness in preparing text and figures for the press.

LITERATURE

16. Bakker, J. P. and le Heux, J. W. N., Theory on central rectilinear recession of slopes, part. I and II. Proc. Kon. Ned. Akad. v. Wetensch. Amsterdam **50**, no. 8 and 9, 959–966 and 1154–1162 (1947).
17. Gau, E., Calculs numériques et graphiques. Coll. A. Colin, no. 60, Paris.
18. Penck, W., Die Morphologische Analyse. Stuttgart (1924).
19. Sölch, Joh., W. Pencks Morphologische Analyse. Geograph. Zeitschrift, 31–36 (1926).
20. Bakker, J. P., Cyclustheorie en morphologische analyse, part II. Tijdschrift Koninkl. Nederlands Aardrijkskundig Genootschap **49**, 3–20 (1932).
21. Bakker, J. P., Over Piedmonttrappen. Tijdschr. Kon. Ned. Aardr. Gen. **51**, 303–329 (1934).
22. Sölch, Joh., Eine Frage der Talbildung. Festband Albrecht Penck, 66–92. Bibl. Geograph Handbücher. (Stuttgart 1918).
23. Bakker, J. P., Over tectogene en morfogene gelijktijdigheid bij de jongere gebergtevorming in West- en Midden-Europa in het kader van denudatieve altiplanatie. Natuurwetenschappelijk Tijdschrift (Gent) **30**, 3–53 (1948).