IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

Quadripartitioned Single Valued Neutrosophic Refined Sets and Its Topological Spaces

[1] Arokia Pratheesha S V, [2] Mohana K

[1] Research Scholar, Nirmala College for Women, Coimbatore, (TN), India.
[2] Assistant Professor, Nirmala College for Women, Coimbatore, (TN),India.

Abstract: In this paper, we define quadripartitioned single valued neutrosophic refined sets and its properties. Also we examine some desired properties of quadripartitioned single valued neutrosophic refined sets. Further we introduce the concept of quadripartitioned single valued neutrosophic refined topological spaces and study the basic concepts with examples in detail.

Keywords: *Neutrosophic refined sets, Quadripartitioned single valued neutrosophic refined sets, Quadripartitioned single valued neutrosphic refined topology*

I. Introduction

The fuzzy set was introduced by Zadeh [21] in 1965 , where each element had a degree of membership. The intuitionstic fuzzy set (IFS for short) on a universe X was introduced by K.Atanassov [1] in 1986 as a generalization of fuzzy set , where besides the degree of membership and the degree of non – membership of each element.In 1998 , Smarandache [14] developed a new concept called neutrosophic set (NS) which is a generalization of fuzzy set and intuitionistic fuzzy set. In additionl to membership and non-membership function neutrosophic set has one extra component called indeterminacy membership function. Neutrosophic set theory deals with uncertainity factor i.e,indeterminacy factor which is independent of truth and falsity values . Since neutrosophic set handles indeterminate and inconsistent effectively , it is applied in many fields like decision support system , semantic web services , new economy 's growth , image processing , medical diagnosis etc.

Wang [18] (2010) introduced the concept of Single Valued Neutrosophic Set (SVNS) which is a generalization of classic set , fuzzy set , intuitionistic fuzzy set etc. Later Quadripartitioned Single Valued Neutrosophic set was introduced by Chatterjee [9] and it consist of four components namely truth,contradiction, unknown and falsity membership function in the real unit interval [0,1].

The notion of multisets was formulated first in [20] by Yager as generalization of the concept of set theory and then the multiset was developed by Blizard [2] and Calude et al [4] , as useful structures arising in many area of mathematics and computer science such as data base queries .Several authors from time to time made a number of generalization of set theory. Since then , Several researches[5,7,10,11,12,16,17] discussed more properties on fuzzy multisets .Shinoj and John [13] made an extension of the concept of fuzzy multisets by an intuitionstic fuzzy set , which called intuitionstic fuzzy multisets (IFMS).The cocept of FMS and IFMS fail to deal with indeterminancy. Recently, Deli et al. [6] used the concept of neutrosophic refined sets and studied some of their basic properties. The concept of neutrosophic refined set (NRS) is a generalization of fuzzy multisets and intuitionistic fuzzy multisets.

The purpose of this paper to construct a quadripartitioned single valued neutrosophic refined set(QSVNR) and quadripartitioned single valued neutrosophic refined topological space (QSNNRTS) which is a generalization of neutrosophic refined sets. This paper is arranged in the following manner. In section 2 contains basic definitions of quadripartitioned single valued neutrosophic set and neutrosophic refined set .In section 3 we define the concept of quadripartitioned single valued neutrosophic refined sets and its properties.In section 4 deals about the concept of quadripartitioned single valued neutrosophic refined topology, its interior and closure with examples are discussed. Further we introduce the concept of quadripartitioned sigle valued neutrosophic refined generalized closed sets and investigate their properties.

II. Preliminaries

2.1 Definition [1]

Let E be a universe. An intuitionistic fuzzy set I on E can be defined as follows:

$$I = \{ \langle x, \mu_1(x), \gamma_1(x) \rangle : x \in E \}$$

where, $\mu_1 : E \rightarrow [0,1]$ and $\gamma_1 : E \rightarrow [0,1]$ such that $0 \leq \mu_1(x) + \gamma_1(x) \leq 1$ for any $x \in E$.

2.2 Definition [14]

Let E be a space of points (objects),with a generic element in E denoted by x a neutrosophic set (N-set) A in E is characterized by a truth membership function T_A, an indeterminacy membership function I_A and a falsity membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of [0,1]. It can be written as

$$A = \{ \langle u, (T_A(x), I_A(x), F_A(x)) \rangle : x \in E, T_A(x), I_A(x), F_A(x) \in [0,1] \}$$

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

2.3 Definition [15,19]

Let E be a universe. A neutrosophic refined set (NRS) A on E can be defined as follows:

$$A = \{ \langle x, (T_A^1(x), T_A^2(x) \ldots\ldots, T_A^P(x)), (I_A^1(x), I_A^2(x) \ldots\ldots, I_A^P(x)), (F_A^1(x), F_A^2(x) \ldots\ldots, F_A^P(x)) \rangle : x \in E\}$$

where, $T_A^1(x), T_A^2(x) \ldots\ldots, T_A^P(x)$: $E \to [0,1]$, $I_A^1(x), I_A^2(x) \ldots\ldots, I_A^P(x)$:$E \to [0,1]$ and $F_A^1(x), F_A^2(x) \ldots\ldots, F_A^P(x)$:$E \to [0,1]$ such that $0 \le T_A^i(x) + I_A^i(x) + F_A^i(x) \le 3$ (i = 1,2,…..P) and $T_A^1(x) \le T_A^2(x) \le \ldots\ldots \le T_A^P(x)$ for any x∈E.

$(T_A^1(x), T_A^2(x) \ldots\ldots, T_A^P(x))$,$(I_A^1(x), I_A^2(x) \ldots\ldots, I_A^P(x))$ and $(F_A^1(x), F_A^2(x) \ldots\ldots, F_A^P(x))$ is the truth membership sequence, indeterminacy membership sequence and falsity membership sequence of the element x respectively. Also P is called the dimension of NRS A. We arrange the truth-membership sequence in decreasing order but the corresponding indeterminacy-membership and falsity- membership sequence may not be in decreasing or increasing order.

2.4 Definition [9]

Let X be a non-empty set. A quadripartitioned single valued neutrosophic set (QSVNS) A over X characterizes each element in X by a truth-membership function $T_A(x)$, a contradiction membership function $D_A(x)$, an unknown membership function $Y_A(x)$ and a falsity membership function $F_A(x)$ such that for each x∈X, $T_A(x), D_A(x), Y_A(x), F_A(x) \in [0, 1]$ and $0 \le T_A(x)+D_A(x)+Y_A(x)+F_A(x) \le 4$.

III. QUADRIPARTITIONED SINGLE VALUED NEUTROSOPHIC REFINED SETS

This section deals about the concept of quadripartitioned single valued neutrosophic refined sets and its properties.

3.1 Definition

Let X be a universe. A quadripartitioned single valued neutrosophic refined set (QSVNR) A on X can be defined as follows:

$$A = \{ \langle x, (T_A^1(x), T_A^2(x) \ldots\ldots, T_A^P(x)), (D_A^1(x), D_A^2(x) \ldots\ldots, D_A^P(x)),$$
$$(Y_A^1(x), Y_A^2(x) \ldots\ldots, Y_A^P(x)), (F_A^1(x), F_A^2(x) \ldots\ldots, F_A^P(x)) \rangle : x \in X\}$$

Where $T_A^1(x), T_A^2(x) \ldots\ldots, T_A^P(x)$: $X \to [0,1]$, $D_A^1(x), D_A^2(x) \ldots\ldots, D_A^P(x)$: $X \to [0,1]$,$Y_A^1(x), Y_A^2(x) \ldots\ldots, Y_A^P(x)$: $X \to [0,1]$ and $F_A^1(x), F_A^2(x) \ldots\ldots, F_A^P(x)$: $X \to [0,1]$ such that $0 \le T_A^i(x) + D_A^i(x) + Y_A^i(x) + F_A^i(x) \le 4$ (i=1,2,…..P) and $T_A^1(x), T_A^2(x) \ldots\ldots, T_A^P(x)$ for any x∈X. $(T_A^1(x), T_A^2(x) \ldots\ldots, T_A^P(x))$, $(D_A^1(x), D_A^2(x) \ldots\ldots, D_A^P(x))$, $(Y_A^1(x), Y_A^2(x) \ldots\ldots, Y_A^P(x))$, $(F_A^1(x), F_A^2(x) \ldots\ldots, F_A^P(x))$ is the truth membership sequence , a contradiction membership sequence, an unknown membership sequence and a falsity membership sequence of the element x, respectively . Also P is called the dimension of QSVNR (A).

3.2 Definition

Consider two QSVNR set A and B over X. Then,

1. A is contained in B denoted by $A \tilde{\subseteq} B$ if $T_A^i(x) \le T_B^i(x)$, $D_A^i(x) \le D_B^i(x)$, $Y_A^i(x) \ge Y_B^i(x)$ and $F_A^i(x) \ge F_B^i(x)$ ∀ x∈X.
2. The Complement of A is denoted by $A^{\tilde{c}}$ and is defined by
$$A^{\tilde{c}} = \{ \langle x , (F_A^1(x), F_A^2(x) \ldots\ldots, F_A^P(x)) , (Y_A^1(x), Y_A^2(x) \ldots\ldots, Y_A^P(x)) ,$$
$$(D_A^1(x), D_A^2(x) \ldots\ldots, D_A^P(x)) , (T_A^1(x), T_A^2(x) \ldots\ldots, T_A^P(x)) \rangle : x \in X\}$$
i.e., $T_A^i(x) = F_A^i(x)$, $D_A^i(x) = Y_A^i(x)$, $Y_A^i(x) = D_A^i(x)$, $F_A^i(x) = T_A^i(x)$ for all x∈X and i =1,2,…P.

3.3 Definition

Let A, B ∈ QSVNR(X). Then,

1. If $T_A^i(x)=0$, $D_A^i(x) = 0$, $Y_A^i(x)= 1$ and $F_A^i(x)=1$ ∀ x∈X and i= 1,2,…..P then A is called null quadripartitioned single valued neutrosophic refined set and denoted by $\tilde{\phi}$.
2. If $T_A^i(x)=1$, $D_A^i(x) = 1$, $Y_A^i(x)= 0$ and $F_A^i(x)=0$ ∀ x∈X and i= 1,2,…..P then A is called universal quadripartitioned single valued neutrosophic refined set and denoted by $\tilde{X}$.

3.4 Definition

Let A, B ∈ QSVNR(X). Then,

1. The union of A and B is denoted by $A \tilde{\cup} B = C_1$ and is defined by

$$C_1 = \{\langle x, (T^1_{C_1}(x), T^2_{C_1}(x) \ldots\ldots, T^P_{C_1}(x)), (D^1_{C_1}(x), D^2_{C_1}(x) \ldots\ldots, D^P_{C_1}(x)), (Y^1_{C_1}(x), Y^2_{C_1}(x) \ldots\ldots Y^P_{C_1}(x)), (F^1_{C_1}(x), F^2_{C_1}(x) \ldots\ldots, F^P_{C_1}(x))\rangle : x \in X\}$$

where $T^i_{C_1}(x) = \max\{T^i_A(x), T^i_B(x)\}$, $D^i_{C_1}(x) = \max\{D^i_A(x), D^i_B(x)\}$,

$Y^i_{C_1}(x) = \min\{Y^i_A(x), Y^i_B(x)\}$, $F^i_{C_1}(x) = \min\{F^i_A(x), F^i_B(x)\}$ $\forall$ x∈X and i= 1,2,…..P

2. The intersection of A and B is denoted by $A \tilde{\cap} B = D_1$ and is defined by

$$D_1 = \{\langle x, (T^1_{D_1}(x), T^2_{D_1}(x) \ldots\ldots, T^P_{D_1}(x)), (D^1_{D_1}(x), D^2_{D_1}(x) \ldots\ldots, D^P_{D_1}(x)), (Y^1_{D_1}(x), Y^2_{D_1}(x) \ldots\ldots, Y^P_{D_1}(x)), (F^1_{D_1}(x), F^2_{D_1}(x) \ldots\ldots, F^P_{D_1}(x))\rangle : x \in X\}$$

where $T^i_{D_1}(x) = \min\{T^i_A(x), T^i_B(x)\}$, $D^i_{D_1}(x) = \min\{D^i_A(x), D^i_B(x)\}$,

$Y^i_{D_1}(x) = \max\{Y^i_A(x), Y^i_B(x)\}$, $F^i_{D_1}(x) = \max\{F^i_A(x), F^i_B(x)\}$ $\forall$ x∈X and i= 1,2,…..P

3.5 Proposition

Let A, B, C∈QSVNR set in X. Then,

1. $A \tilde{\cup} B = B \tilde{\cup} A$ and $A \tilde{\cap} B = B \tilde{\cap} A$
2. $(A \tilde{\cup} B) \tilde{\cup} C = A \tilde{\cup} (B \tilde{\cup} C)$ and $A \tilde{\cap} B) \tilde{\cap} C = A \tilde{\cap} (B \tilde{\cap} C)$

Proof: The proofs can be easily made.

3.6 Proposition

Let A, B, C∈QSVNR set in X. Then,

1. $A \tilde{\cap} \tilde{\phi} = \tilde{\phi}$ and $A \tilde{\cup} \tilde{\phi} = A$
2. $A \tilde{\cap} \tilde{X} = A$ and $A \tilde{\cup} \tilde{X} = \tilde{X}$
3. $(A \tilde{\cup} B) \tilde{\cap} C = (A \tilde{\cap} C) \tilde{\cup} (B \tilde{\cap} C)$
4. $(A \tilde{\cap} B) \tilde{\cup} C = (A \tilde{\cup} C) \tilde{\cap} (B \tilde{\cup} C)$

Proof: It is clear to prove the result from definition 3.3-3.4.

3.7 Theorem

Let A, B, C∈QSVNR set in X. Then,

1. $(A \tilde{\cup} B)^{\tilde{c}} = A^{\tilde{c}} \tilde{\cap} B^{\tilde{c}}$
2. $(A \tilde{\cap} B)^{\tilde{c}} = A^{\tilde{c}} \tilde{\cup} B^{\tilde{c}}$

Proof: Let A, B QSVNR(X) is given. From Definition 3.2 and Definition 3.4, we have

1. $(A \tilde{\cup} B)^{\tilde{c}} = \{\langle x, (\max\{T^1_A(x), T^1_B(x)\}, \max\{T^2_A(x), T^2_B(x)\}, \ldots, \max\{T^P_A(x), T^P_B(T^P_B(x)\}),$
$(\max\{D^1_A(x), D^1_B(x)\}, \max\{D^2_A(x), D^2_B(x)\}, \ldots\ldots, \max\{D^P_A(x), D^P_B(x)\}),$
$(\min\{Y^I_A(x), Y^1_B(x)\}, \min\{Y^2_A(x), Y^2_B(x)\} \ldots\ldots, \min\{Y^P_A(x), Y^P_B(x)\}),$
$(\min\{F^1_A(x), F^1_B(x)\}, \min\{F^2_A(x), F^2_B(x)\} \ldots\ldots, \min\{F^P_A(x), F^P_B(x)\})\rangle : x \in X\}^{\tilde{c}}$
$= \{\langle x, (\min\{F^1_A(x), F^1_B(x)\}, \min\{F^2_A(x), F^2_B(x)\} \ldots\ldots, \min\{F^P_A(x), F^P_B(x)\}),$
$(\min\{Y^I_A(x), Y^1_B(x)\}, \min\{Y^2_A(x), Y^2_B(x)\} \ldots\ldots, \min\{Y^P_A(x), Y^P_B(x)\}),$
$(\max\{D^1_A(x), D^1_B(x)\}, \max\{D^2_A(x), D^2_B(x)\}, \ldots\ldots, \max\{D^P_A(x), D^P_B(x),$
$(\max\{T^1_A(x), T^1_B(x)\}, \max\{T^2_A(x), T^2_B(x)\}, \ldots\ldots \max\{T^P_A(x), T^P_B(x)\})\rangle : x \in X\}$

$A^{\tilde{c}} = \{\langle x, (F^1_A(x), F^2_A(x) \ldots\ldots, F^P_A(x)), (Y^1_A(x), Y^2_A(x) \ldots\ldots, Y^P_A(x)),$
$(D^1_A(x), D^2_A(x) \ldots\ldots, D^P_A(x)), (T^1_A(x), T^2_A(x) \ldots\ldots, T^P_A(x))\rangle : x \in X\}$

$B^{\tilde{c}} = \{\langle x, (F^1_B(x), F^2_B(x) \ldots\ldots, F^P_B(x)), (Y^1_B(x), Y^2_B(x) \ldots\ldots, Y^P_B(x)),$
$(D^1_B(x), D^2_B(x) \ldots\ldots, D^P_B(x)), (T^1_B(x), T^2_B(x) \ldots\ldots, T^P_B(x))\rangle : x \in X\}$

$A^{\tilde{c}} \tilde{\cap} B^{\tilde{c}} = \{\langle x, (\min\{F^1_A(x), F^1_B(x)\}, \min\{F^2_A(x), F^2_B(x)\} \ldots\ldots, \min\{F^P_A(x), F^P_B(x)\}),$
$(\min\{Y^I_A(x), Y^1_B(x)\}, \min\{Y^2_A(x), Y^2_B(x)\} \ldots\ldots, \min\{Y^P_A(x), Y^P_B(x)\}),$
$(\max\{D^1_A(x), D^1_B(x)\}, \max\{D^2_A(x), D^2_B(x)\}, \ldots\ldots, \max\{D^P_A(x), D^P_B(x),$

(max{ T_A^1 (x), T_B^1(x)}, max{ T_A^2 (x), T_B^2 (x)}, ……. max{ T_A^P (x), T_B^P (x)}) ⟩ : x∈X}

$\Rightarrow (A \tilde{\cup} B)^{\tilde{c}} = A^{\tilde{c}} \tilde{\cap} B^{\tilde{c}}$.

2. Now consider,

$(A \tilde{\cap} B)^{\tilde{c}}$ = {⟨ x, (min{ T_A^1 (x), T_B^1(x)}, min{ T_A^2 (x), T_B^2 (x)}, …, min{ T_A^P (x), T_B^P (T_B^P (x)}),

(min{ D_A^1 (x) , D_B^1 (x)}, min{ D_A^2 (x) , D_B^2 (x)},….., min{ D_A^P (x) , D_B^P (x)}),

(max{ Y_A^I (x), Y_B^1(x)}, max{ Y_A^2 (x), Y_B^2 (x)}……, max{ Y_A^P (x), Y_B^P (x)}),

(max{ F_A^1 (x) , F_B^1 (x)},max{ F_A^2 (x) , F_B^2 (x)}……., max{ F_A^P (x) , F_B^P (x)}) ⟩ : x∈X}$^{\tilde{c}}$

= {⟨ x, (max{ F_A^1 (x) , F_B^1 (x)}, max{ F_A^2 (x) , F_B^2 (x)}……., max{ F_A^P (x) , F_B^P (x)}),

(max{ Y_A^I (x), Y_B^1(x)}, max{ Y_A^2 (x), Y_B^2 (x)}……, max{ Y_A^P (x), Y_B^P (x)}),

(min{ D_A^1 (x) , D_B^1 (x)}, min{ D_A^2 (x) , D_B^2 (x)},….., min{ D_A^P (x) , D_B^P (x),

(min{ T_A^1 (x), T_B^1(x)}, min{ T_A^2 (x), T_B^2 (x)}, ……. min{ T_A^P (x), T_B^P (x)}) ⟩ : x∈X}

$A^{\tilde{c}}$ = {⟨ x , (F_A^1(x), F_A^2 (x)….. , F_A^P (x)) , (Y_A^1(x), Y_A^2 (x)….. , Y_A^P (x)) ,

(D_A^1 (x), D_A^2 (x)……., D_A^P (x) , (T_A^1(x), T_A^2 (x)….. , T_A^P (x)) ⟩ : x∈X}

$B^{\tilde{c}}$ = {⟨ x , (F_B^1 (x), F_B^2 (x)….. , F_B^P (x)) , (Y_B^1(x), Y_B^2 (x)….. , Y_B^P (x)) ,

(D_B^1 (x), D_B^2 (x)……., D_B^P (x)), (T_B^1(x), T_B^2 (x)….. , T_B^P (x)) ⟩ : x∈X}

$A^{\tilde{c}} \tilde{\cup} B^{\tilde{c}}$ = {⟨ x, (max{ F_A^1 (x) , F_B^1 (x)}, max{ F_A^2 (x) , F_B^2 (x)}……., max{ F_A^P (x) , F_B^P (x)}),

(max{ Y_A^I (x), Y_B^1(x)}, max{ Y_A^2 (x), Y_B^2 (x)}……, max{ Y_A^P (x), Y_B^P (x)}),

(min{ D_A^1 (x) , D_B^1 (x)}, min{ D_A^2 (x) , D_B^2 (x)},….., min{ D_A^P (x) , D_B^P (x),

(min{ T_A^1 (x), T_B^1(x)}, min{ T_A^2 (x), T_B^2 (x)}, ……. min{ T_A^P (x), T_B^P (x)}) ⟩ : x∈X}

$\Rightarrow (A \tilde{\cap} B)^{\tilde{c}} = A^{\tilde{c}} \tilde{\cup} B^{\tilde{c}}$

Hence the proof.

IV. QUADRIPARTITIONED SINGLE VALUED NEUTROSOPHIC REFINED TOPOLOGY

In this section we define a topology quadripartitioned single valued neutrosophic refined set which is known as QSVNRT and also discuss its properties.

4.1 Definition

A quadripartitioned single valued neutrosophic refined topology(QSVNRT) on a non-empty set X is a family τ of QSVNR sets in X which satisfy the following conditions.

1. $\tilde{\phi}, \tilde{X} \in \tau$
2. $H_1 \tilde{\cap} H_2 \in \tau$ for any $H_1, H_2 \in \tau$
3. $\tilde{\cup} H_i \in \tau$ for every $\{H_i : i \in J\} \tilde{\subseteq} \tau$

Here the pair (X, τ) is called a quadripartitioned single valued neutrosophic refined topological space (QSVNRTS). All the elements of τ are called quadripartitioned single valued neutrosophic refined open set (QNROS) in X. A QSVNR set K is quadripartitioned single valued neutrosophic refined closed set (QNRCS) if and only if its complement of K is QNROS.

4.2 Example

Let X = {x,y} and $G_1, G_2, G_3, G_4 \in$ QSVNR(X)

G_1 = {⟨ x,{0.6,0.5,0.3,0.2},{0.5,0.4,0.2,0.4},{0.6,0.2,0.1,0.7} ⟩ ,
⟨ y,{0.6,0.5,0.6,0.2},{0.5,0.6,0.7,0.1},{0.4,0.5,0.2,0.3} ⟩ }

G_2= {⟨ x,{0.3,0.8,0.2,0.7},{0.2,0.7,0.1,0.6},{0.1,0.6,0.2,0.5} ⟩ ,
⟨ y,{0.4,0.5,0.5,0.7},{0.3,0.4,0.6,0.4},{0.4,0.5,0.2,0.1} ⟩ }

G_3={⟨ x,{0.6,0.8,0.2,0.2},{0.5,0.7,0.1,0.4},{0.6,0.6,0.1,0.5} ⟩ ,
⟨y,{0.6,0.5,0.5,0.2},{0.5,0.6,0.6,0.1},{0.4,0.5,0.2,0.1} ⟩ }

G_4 ={⟨ x,{0.3,0.5,0.3,0.7},{0.2,0.4,0.2,0.6},{0.1,0.2,0.2,0.7} ⟩ ,
⟨ y,{0.4,0.5,0.6,0.7},{0.3,0.4,0.7,0.4},{0.4,0.5,0.2,0.3} ⟩ }

Then the family τ = { $\tilde{\phi}$, G_1, G_2, G_3, G_4, $\tilde{X}$ } is called a QSVNRT on X.

4.3 Definition

Let (X, τ) be a QSVNRTS and A = { ⟨x, T_A^i(x), D_A^i(x), Y_A^i(x), F_A^i(x) ⟩ :x∈X}for i= 1,2,…..,P be QSVNR set in X.Then quadripartitioned single valued neutrosophic refined closure (QNRCl(A)) and quadripartitioned single valued neutrosophic refined interior (QNRInt(A)) of A are defined by,

QNRCl(A) = $\tilde{\cap}$ { K: K is a QNRCS in X and A $\tilde{\subseteq}$ K}

QNRInt(A) = $\tilde{\cup}$ { L: L is a QNROS in X and L $\tilde{\subseteq}$ A}

Here QNRCl(A), QNRInt(A) are QNRCS and QNROS respectively in X. Further,

A is QNRCS if and only if A=QNRCl(A)

A is QNROS if and onlyif A=QNRInt(A)

4.4 Example

Consider an Example 4.2 and its QSVNRT τ and if

N = { ⟨ u,{0.6,0.9,0.2,0.1},{0.6,0.8,0.1,0.2},{0.4,0.8,0.1,0.2} ⟩

⟨ v,{0.7,0.5,0.3,0.1},{0.7,0.8,0.4,0.1},{0.6,0.7,0.1,0.1} ⟩ }

QNRInt(N) = { ⟨ u,{0.6,0.8,0.2,0.2},{0.5,0.7,0.1,0.4},{0.6,0.6,0.1,0.5} ⟩ ,

⟨v,{0.6,0.5,0.5,0.2},{0.5,0.6,0.6,0.1},{0.4,0.5,0.2,0.1} ⟩ }

QNRCl(N) = $\tilde{X}$

4.5 Proposition

For any QSVNR set A in (X, τ) we have,

a) QNRCl($A^{\tilde{c}}$)= (QNRInt(A))$^{\tilde{c}}$

b) QNRInt($A^{\tilde{c}}$) = (QNRCl(A))$^{\tilde{c}}$

Proof: Consider a QSVNR set A denoted by

A = {⟨x, T_A^i(x), D_A^i(x), Y_A^i(x), F_A^i(x) ⟩ : x∈X} for i= 1,2,…..,P

and denote the family of QSVNR subsets contained in S are indexed by the family,

A = {⟨x, $T_{L_j}^i$ (x), $D_{L_j}^i$ (x), $Y_{L_j}^i$ (x), $F_{L_j}^i$ (x) ⟩ : x∈X} for i= 1,2,…..,P

Then we get,

QNRInt(A) = { ⟨x, max $T_{L_j}^i$ (x), max $D_{L_j}^i$ (x), min $Y_{L_j}^i$ (x), min $F_{L_j}^i$ (x) ⟩ : x∈X} for i= 1,2,…..,P

and hence (QNRInt(A))$^{\tilde{c}}$ = {⟨x, min $F_{L_j}^i$ (x), min $Y_{L_j}^i$ (x),max $D_{L_j}^i$ (x),max $T_{L_j}^i$ (x) ⟩ : x∈X} for i=1,2,…..,P

Since, $T_{L_j}^i \leq T_A^i$, $D_{L_j}^i \leq D_A^i$, $Y_{L_j}^i \geq Y_A^i$ and $F_{L_j}^i \leq F_A^i$ for each j∈J and the complement of family of QSVNR sets A denoted by

$A^{\tilde{c}}$ = {⟨x,, $F_{L_j}^i$ (x) , $Y_{L_j}^i$ (x) , $D_{L_j}^i$ (x), $T_{L_j}^i$ (x) ⟩ : x∈X} for i= 1,2,…..P

we find that,

QNRCl($A^{\tilde{c}}$)= {⟨x, min $F_{L_j}^i$ (x) ,min $Y_{L_j}^i$ (x),max $D_{L_j}^i$ (x),max $T_{L_j}^i$ (x) ⟩ : x∈X} for i=1,2,…..,P

Therefore we get,

QNRCl($A^{\tilde{c}}$)= (QNRInt(A))$^{\tilde{c}}$

b) It is similar to the proof of (a).

4.6 Definition

A QSVNR set A in a QSVNRTS (X, τ) is called,

i) Quadripartitioned single valued neutrosophic refined semi open set (QNRSOS) if
A $\tilde{\subseteq}$ QNRCl(QNRInt(S))

ii) Quadripartitioned single valued neutrosophic refined semi closed set (QNRSCS) if
QNRInt(QNRCl(A)) $\tilde{\subseteq}$ A

iii) Quadripartitioned single valued neutrosophic refined pre-open set (QNRPOS) if
A $\tilde{\subseteq}$ QNRInt(QNRCl(A))

iv) Quadripartitioned single valued neutrosophic refined pre-closed set (QNRPCS) if
QNRCl(QNRInt(A)) $\tilde{\subseteq}$ A

v) Quadripartitioned single valued neutrosophic refined α-open set (QNRαOS) if
A $\tilde{\subseteq}$ QNRInt(QNRCl(QNRInt(A)))

vi) Quadripartitioned single valued neutrosophic refined α-closed set (QNRαCS) if
QNRCl(QNRInt(QNRCl(A))) $\tilde{\subseteq}$ A

vii) Quadripartitioned single valued neutrosophic refined β-open set (QNRβOS) if
A $\tilde{\subseteq}$ QNRCl(QNRInt(QNRCl(A)))

viii) Quadripartitioned single valued neutrosophic refined β-closed set (QNRβCS) if
QNRInt(QNRCl(QNRIntA))) $\tilde{\subseteq}$ A

ix) Quadripartitioned single valued neutrosophic refined regular open if and only if
A =QNRInt(QNRCl(A))

x) Quadripartitioned single valued neutrosophic refined regular closed if and only if
A = QNRCl(QNRInt(A))

4.7 Definition

Let (X, τ) be a QSVNRT and A= {⟨x, T_A^i(x), D_A^i(x), Y_A^i(x), F_A^i(x)⟩ : x∈X} for i= 1,2,…..,P be QSVNR set in X. Then quadripartitioned single valued neutrosophic refined semi closure(QNRSCl) and quadripartitioned single valued neutrosophic refined semi interior(QNRSInt) of A are defined by,

$$\text{QNRSCl}(A) = \tilde{\cap}\{K : K \text{ is a QNRSCS in X and } A \tilde{\subseteq} K\}$$

$$\text{QNRSInt}(A) = \tilde{\cup}\{L : L \text{ is a QNRSOS in X and } L \tilde{\subseteq} A\}$$

4.8 Result

Let A be a QSVNRS in (X, τ), then

1) QNRSCl(A) = A $\tilde{\cup}$ QNRInt(QNRCl(A))
2) QNRSInt(A) = A $\tilde{\cap}$ QNRCl(QNRInt(A))

4.9 Definition

Let (X, τ) be a QSVNRT and A={⟨x, T_A^i(x), D_A^i(x), Y_A^i(x), F_A^i(x)⟩ : x∈X} for i= 1,2,…..,P be QSVNR set in X. Then quadripartitioned single valued neutrosophic refined α closure(QNR α Cl) and quadripartitioned single valued neutrosophic refined α interior(QNR α Int) of A are defined by,

$$\text{QNR}\alpha\text{Cl}(A) = \tilde{\cap}\{K : K \text{ is a QNR}\alpha\text{CS in X and } A \tilde{\subseteq} K\}$$

$$\text{QNR}\alpha\text{Int}(A) = \tilde{\cup}\{L : L \text{ is a QNR}\alpha\text{OS in X and } L \tilde{\subseteq} A\}$$

4.10 Definition

Let (X, τ) be a QSVNRTS and A = { ⟨x, T_A^i(x), D_A^i(x), Y_A^i(x), F_A^i(x) ⟩ :x∈X} for i= 1,2,…..,P be QSVNR set in X. Then quadripartitioned single valued neutrosophic refined pre closure (QNRPCl) and quadripartitioned single valued neutrosophic refined pre interior (QNRPInt) of A are defined by,

$$\text{QNRPCl}(A) = \tilde{\cap}\{K : K \text{ is a QNRPCS in X and } A \tilde{\subseteq} K\}$$

$$\text{QNRPInt}(A) = \tilde{\cup}\{L : L \text{ is a QNRPOS in X and } L \tilde{\subseteq} A\}$$

4.11 Result

Let A be a QSVNR set in (X, τ) then

1) QNRαCl(S) = A $\tilde{\cup}$ QNRCl(QNRInt(QNRCl(A)))
2) QNRαInt(S) = A $\tilde{\cap}$ QNRInt(QNRCl(QNRInt(A)))

4.12 Proposition

Let (X, τ) be a QSVNRTS and B, C be a QSVNR sets in X. Then the following properties hold:

a) QNRInt(B) $\tilde{\subseteq}$ B
b) B $\tilde{\subseteq}$ QNRCl(B)
c) B $\tilde{\subseteq}$ C ⇒ QNRInt(B) $\tilde{\subseteq}$ QNRInt(C)
d) B $\tilde{\subseteq}$ C ⇒ QNRCl(B) $\tilde{\subseteq}$ QNRCl(C)
e) QNRInt(QNRInt(B)) = QNRInt(B)
f) QNRCl(QNRCl(B)) = QNRCl(B)
g) QNRInt(B $\tilde{\cap}$ C) = QNRInt(B) $\tilde{\cap}$ QNRInt(C)
h) QNRCl(B $\tilde{\cup}$ C) = QNRCl(B) $\tilde{\cup}$ QNRCl(C)
i) QNRInt($\tilde{X}$) = $\tilde{X}$
j) QNRCl($\tilde{\phi}$) = $\tilde{\phi}$

Proof: The proof of (a), (b) and (i) are straightforward. It is easy to prove the result (d) from (a) and Definition 3.3
(g) From QNRInt(B $\tilde{\cap}$ C) $\tilde{\subseteq}$ QNRInt(B)and QNRInt(B $\tilde{\cap}$ C) $\tilde{\subseteq}$ QNRInt(C) we get,
QNRInt(B $\tilde{\cap}$ C) $\tilde{\subseteq}$ QNRInt(B) $\tilde{\cap}$ QNRInt(C) by the result of B $\tilde{\subseteq}$ C, B $\tilde{\subseteq}$ A ⇒ B $\tilde{\subseteq}$ C $\tilde{\cap}$ A
whereA,B,C are QSVNR sets in E. Now from the fact of QNRInt(B) $\tilde{\subseteq}$ B and
QNRInt(C) $\tilde{\subseteq}$ C we see that, QNRInt(B) $\tilde{\cap}$ QNRInt(C) $\tilde{\subseteq}$ B $\tilde{\cap}$ C and also

QNRInt(B) $\tilde{\cap}$ QNRInt(C) $\in \tau$ we get, QNRInt(B) $\tilde{\cap}$ QNRInt(C) $\tilde{\subseteq}$ QNRInt(B $\tilde{\cap}$ C) which shows the required proof.

The rest can be proved easily from the previous results and the Proposition 4.6.

4.13 Definition

Let (X, τ) be a quadripartitioned single valued neutrosophic refined topological space. A subset A of a space (X, τ) is called

i) generalized closed set (QNRg-closed) if QNRCl(S) $\tilde{\subseteq}$ L whenever A $\tilde{\subseteq}$ L and L is a quadripartitioned single valued neutrosophic refined open set in X.

ii) generalized pre-closed (QNRgP-closed) set if QNRPCl(A) $\tilde{\subseteq}$ L whenever A $\tilde{\subseteq}$ L and L is a quadripartitioned single valued neutrosophic open set in X.

iii) generalized semi closed(QNRgS-closed) set if QNRSCl(A) $\tilde{\subseteq}$ L whenever A $\tilde{\subseteq}$ L and L is a quadripartitioned single valued neutrosophic refined open set in X .

iv) α generalized closed set (QNαg-closed) if QNRαCl(A) $\tilde{\subseteq}$ L whenever A $\tilde{\subseteq}$ L and L is a quadripartitioned single valued neutrosophic refined open set in X.

4.14 Example

Let X = {x,y} and $\tau = \{\tilde{\phi}, G_1, G_2, G_3, G_4, \tilde{X}\}$ where

G_1 = { ⟨ x,{0.6,0.5,0.3,0.2},{0.5,0.4,0.2,0.4},{0.6,0.2,0.1,0.7} ⟩,
⟨ y,{0.6,0.5,0.6,0.2},{0.5,0.6,0.7,0.1},{0.4,0.5,0.2,0.3} ⟩ }

G_2 = { ⟨ x,{0.3,0.8,0.2,0.7},{0.2,0.7,0.1,0.6},{0.1,0.6,0.2,0.5} ⟩,
⟨ y,{0.4,0.5,0.5,0.7},{0.3,0.4,0.6,0.4},{0.4,0.5,0.2,0.1} ⟩ }

G_3 = { ⟨ x,{0.6,0.8,0.2,0.2},{0.5,0.7,0.1,0.4},{0.6,0.6,0.1,0.5} ⟩,
⟨ y,{0.6,0.5,0.5,0.2},{0.5,0.6,0.6,0.1},{0.4,0.5,0.2,0.1} ⟩ }

G_4 = { ⟨ x,{0.3,0.5,0.3,0.7},{0.2,0.4,0.2,0.6},{0.1,0.2,0.2,0.7} ⟩,
⟨ y,{0.4,0.5,0.6,0.7},{0.3,0.4,0.7,0.4},{0.4,0.5,0.2,0.3} ⟩ }

Then (X, τ) is a QSVNRTS. Consider a QSVNR set are

S = { ⟨ u,{0.1,0.2,0.8,0.7},{0.2,0.1,0.8,0.7},{0.1,0.1,0.7,0.6} ⟩,
⟨ v,{0.2,0.3,0.6,0.6},{0.1,0.4,0.7,0.6},{0.1,0.1,0.7,0.6} ⟩ } is a QNRg-closed in X.

4.15 Theorem

Every QNRCS is a QNRg-closed set in (X, τ).

Proof: Let A be a QNRCS and A $\tilde{\subseteq}$ L where L be QNROS in (X, τ). Since A is QNRCS, QNRCl(A) $\tilde{\subseteq}$ A[since A= QNRCl(A)]. Therefore QNRCl(A) $\tilde{\subseteq}$ A $\tilde{\subseteq}$ L. Hence A is a QNRg-closed set in (X, τ).

4.16 Remark

The converse of the above theorem need not be true. In Example 4.14 S is QNRg-closed set but not QNRCS.

4.17 Theorem

Let B and C be QNRg-closed sets in (X, τ) then B $\tilde{\cup}$ C is also QNRg- closed set in (X, τ).

Proof: Since P and R are QNRg-closed sets in (X, τ) we get QNRCl(B) $\tilde{\subseteq}$ L and QNRCl(C) $\tilde{\subseteq}$ L whenever B,C $\tilde{\subseteq}$ L where L is QNROS in (X, τ). This implies B $\tilde{\cup}$ C is also a subset ofL where L is QNROS in X. Then QNRCl(B $\tilde{\cup}$ C) = QNRCl(B) $\tilde{\cup}$ QNRCl(C). i.e., QNRCl(B $\tilde{\cup}$ C) $\tilde{\subseteq}$ L. Therefore B $\tilde{\cup}$ C is QNRg-closed set in (X, τ).

4.18 Theorem

Let B and C are QNRg-closed sets in (X,τ) then, QNRCl(B $\tilde{\cap}$ C) $\tilde{\subseteq}$ QNRCl(B) $\tilde{\cap}$ QNRCl(C).

Proof: Since B and C are QNRg-closed sets in (X, τ) we get QNRCl(B) $\tilde{\subseteq}$ L and QNRCl(C) $\tilde{\subseteq}$ L whenever B,C $\tilde{\subseteq}$ L where L is QNROS in (X, τ). This implies that B $\tilde{\cap}$ C is also a subset of L where L is QNROS. Since B $\tilde{\cap}$ C $\tilde{\subseteq}$ B and B $\tilde{\cap}$ C $\tilde{\subseteq}$ C and also we know that if B $\tilde{\subseteq}$ C then QNRCl(B) $\tilde{\subseteq}$ QNRCl(C).Therefore QNRCl(B $\tilde{\cap}$ C) $\tilde{\subseteq}$ QNRCl(B) and QNRCl(B $\tilde{\cap}$ C) $\tilde{\subseteq}$ QNRCl(C) which implies that QNRCl(B $\tilde{\cap}$ C) $\tilde{\subseteq}$ QNRCl(B) $\tilde{\cap}$ QNRCl(C).Hence proved.

4.19 Remark

The intersection of two QNRg-closed sets need not be a QNRg-closed set which is shown in the following example.

4.20 Example

Let X= {x,y} and $\tau = \{\tilde{\phi}, G_1, G_2, G_3, G_4, \tilde{X}\}$where

$G_1 = \{\langle x,\{0.6,0.5,0.3,0.2\},\{0.5,0.4,0.2,0.4\},\{0.6,0.2,0.1,0.7\}\rangle, \langle y,\{0.6,0.5,0.6,0.2\},\{0.5,0.6,0.7,0.1\},\{0.4,0.5,0.2,0.3\}\rangle\}$

$G_2 = \{\langle x,\{0.3,0.8,0.2,0.7\},\{0.2,0.7,0.1,0.6\},\{0.1,0.6,0.2,0.5\}\rangle, \langle y,\{0.4,0.5,0.5,0.7\},\{0.3,0.4,0.6,0.4\},\{0.4,0.5,0.2,0.1\}\rangle\}$

$G_3 = \{\langle x,\{0.6,0.8,0.2,0.2\},\{0.5,0.7,0.1,0.4\},\{0.6,0.6,0.1,0.5\}\rangle, \langle y,\{0.6,0.5,0.5,0.2\},\{0.5,0.6,0.6,0.1\},\{0.4,0.5,0.2,0.1\}\rangle\}$

$G_4 = \{\langle x,\{0.3,0.5,0.3,0.7\},\{0.2,0.4,0.2,0.6\},\{0.1,0.2,0.2,0.7\}\rangle, \langle y,\{0.4,0.5,0.6,0.7\},\{0.3,0.4,0.7,0.4\},\{0.4,0.5,0.2,0.3\}\rangle\}$

Then (X, τ) is a QSVNRTS.Consider a QNRg-closed sets

$S = \{\langle x,\{0.1,0.2,0.8,0.7\},\{0.2,0.1,0.8,0.7\},\{0.1,0.1,0.7,0.6\}\rangle, \langle y,\{0.2,0.3,0.6,0.6\},\{0.1,0.4,0.7,0.6\},\{0.1,0.1,0.7,0.6\}\rangle\}$

$T = \{\langle x,\{0.1,0.1,0.9,0.8\},\{0.2,0.1,0.8,0.7\},\{0.1,0.1,0.6,0.7\}\rangle, \langle y,\{0.1,0.2,0.6,0.7\},\{0.1,0.3,0.7,0.6\},\{0.1,0.2,0.6,0.7\}\rangle\}$

$S \tilde{\cap} T = \{\langle x,\{0.1,0.1,0.9,0.8\},\{0.2,0.1,0.8,0.7\},\{0.1,0.1,0.7,0.7\}\rangle, \langle y,\{0.1,0.2,0.6,0.7\},\{0.1,0.3,0.7,0.6\},\{0.1,0.1,0.7,0.7\}\rangle\}$is not a QNRg-closed set.

4.21 Theorem

Let S be QNRg-closed set in (X, τ) and $S \tilde{\subseteq} T \tilde{\subseteq}$ QNRCl(S) then T is QNRg-closed set in (X, τ).

Proof: Let $T \tilde{\subseteq} L$ where L is QNROS in (X, τ). Then $S \tilde{\subseteq} T$ implies $S \tilde{\subseteq} F$. Since S is QNRg-closed, we get QNRCl(S) $\tilde{\subseteq}$ L whenever $S \tilde{\subseteq} L$. And also $S \tilde{\subseteq}$ QNRCl(T) implies QNRCl(T) $\tilde{\subseteq}$ QNRCl(S). Thus QNRCl(T) $\tilde{\subseteq}$ L and so T is QNRg-closed set in (X, τ).

4.22 Theorem

A QNRg-closed set S is ONRCS if and only if QNRCl(S)-S is QNRCS.

Proof: First assume that S is QNRCS then we get QNRCl(S) = S and so QNRCl(S) – S = $\tilde{\phi}$ which is QNRCS. Conversely assume that QNRCl(S)–Sis QNRCS. Then QNRCl(S)-S = $\tilde{\phi}$ that is QNRCl(S) = S. This implies that S is QNRCS. Hence proved.

4.23 Result

Let A be a QSVNR set in (X, τ), then

1)QNRPCl(A) = A $\tilde{\cup}$ QNRCl(QNRInt(A))

4.24 Example

Let X ={u,v} and $\tau = \{\tilde{\phi}, G_1, G_2, G_3, G_4, \tilde{X}\}$where

$G_1 = \{\langle x,\{0.2,0.3,0.4,0.5\},\{0.4,0.3,0.6,0.7\},\{0.6,0.4,0.6,0.7\}\rangle, \langle y,\{0.4,0.2,0.1,0.3\},\{0.2,0.4,0.1,0.5\},\{0.4,0.3,0.2,0.6\}\rangle\}$

$G_2 = \{\langle x,\{0.1,0.2,0.5,0.3\},\{0.6,0.5,0.4,0.2\},\{0.4,0.5,0.3,0.2\}\rangle, \langle y,\{0.3,0.1,0.4,0.5\},\{0.3,0.2,0.7,0.5\},0.5,0.6,0.5,0.4\}\rangle\}$

$G_3 = \{\langle x,\{0.2,0.3,0.4,0.3\},\{0.6,0.5,0.4,0.2\},\{0.6,0.5,0.3,0.2\}\rangle, \langle y,\{0.4,0.2,0.1,0.3\},\{0.3,0.4,0.1,0.5\},\{0.5,0.6,0.2,0.4\}\rangle\}$

$G_4 = \{\langle x,\{0.1,0.2,0.5,0.5\},\{0.4,0.3,0.6,0.7\},\{0.4,0.5,0.6,0.7\}\rangle, \langle y,\{0.3,0.1,0.4,0.5\},\{0.2,0.2,0.7,0.5\},\{0.4,0.3,0.5,0.6\}\rangle\}$

Then a QSVNR set,

$A = \{\langle x,\{0.2,0.3,0.5,0.4\},\{0.5,0.4,0.3,0.6\},\{0.6,0.7,0.5,0.3\}\rangle, \langle y,\{0.5,0.1,0.3,0.4\},\{0.4,0.2,0.6,0.3\},\{0.5,0.4,0.3,0.2\}\rangle\}$is a QNRgP-closed in X.

4.25 Theorem

Every QNRCS is a QNRgP-closed but not conversely.

Proof: Let A be a QNRCS in X and A $\tilde{\subseteq}$ L where L be QNROS in (X, τ).Since QNRPCl(A) $\tilde{\subseteq}$ QNRCl(A) and A is a QNRCS in X , QNRPCl(A) $\tilde{\subseteq}$ QNRCl(A) = A $\tilde{\subseteq}$ L . Hence A is a QNRgP-closed set in (X, τ).

4.26 Example

In Example 4.24 A is QNRgP-closed set but not QNRCS.

4.27 Theorem

Every QNRαCSis a QNRgP-closed set but not conversely.

Proof: Let A be a QNRαCS in X and A $\tilde{\subseteq}$ L where L be QNROS in (X, τ). By hypothesis, QNRCl(QNRInt(QNRCl(A))) $\tilde{\subseteq}$ A and since A $\tilde{\subseteq}$ QNRCl(A), QNRCl(QNRInt(A)) $\tilde{\subseteq}$ QNRCl(QNRInt(QNRCl(A))) $\tilde{\subseteq}$ A. Here QNRCl(A) $\tilde{\subseteq}$ A $\tilde{\subseteq}$ L. Therefore A is a QNRgP-closed set in X.

4.28 Example

In Example 4.24 A is QNRgP-closed set but not QNR α CS.

4.29 Theorem

Every QNRg-closed set is a QNRgP-closed set but not conversely.

Proof: Let A be a QNRg-closed set in X and A $\tilde{\subseteq}$ L where L be QNROS in (X, τ). Since QNRPCl(A) $\tilde{\subseteq}$ QNRCl(A) and by hypothesis, QNRPCl(A) $\tilde{\subseteq}$ L. Therefore A is a QNRgP-closed set in X.

4.30 Example

Let X={x,y} and τ = { $\tilde{\phi}$, G_1, G_2, G_3, G_4, $\tilde{X}$ }where

G_1 = { ⟨ x,{0.2,0.3,0.4,0.5},{0.4,0.3,0.6,0.7},{0.6,0.4,0.6,0.7} ⟩ ,
⟨ y,{0.4,0.2,0.1,0.3},{0.2,0.4,0.1,0.5},{0.4,0.3,0.2,0.6} ⟩ }

G_2 = { ⟨ x,{0.1,0.2,0.5,0.3},{0.6,0.5,0.4,0.2},{0.4,0.5,0.3,0.2} ⟩ ,
⟨ y,{0.3,0.1,0.4,0.5},{0.3,0.2,0.7,0.5},0.5,0.6,0.5,0.4} ⟩ }

G_3 = { ⟨ x,{0.2,0.3,0.4,0.3},{0.6,0.5,0.4,0.2},{0.6,0.5,0.3,0.2} ⟩ ,
⟨ y,{0.4,0.2,0.1,0.3},{0.3,0.4,0.1,0.5},{0.5,0.6,0.2,0.4} ⟩ }

G_4 = { ⟨ x,{0.1,0.2,0.5,0.5},{0.4,0.3,0.6,0.7},{0.4,0.5,0.6,0.7} ⟩ ,
⟨ y,{0.3,0.1,0.4,0.5},{0.2,0.2,0.7,0.5},{0.4,0.3,0.5,0.6} ⟩ }

Then a QSVNR set,

A = { ⟨ x,{0.1,0.2,0.5,0.6},{0.3,0.1,0.7,0.6},{0.2,0.3,0.7,0.8} ⟩ ,
⟨ y,{0.2,0.1,0.6,0.5},{0.1,0.2,0.8,0.6},{0.4,0.3,0.6,0.7} ⟩ }is a QNRgP-closed in X. But it is not a QNRg-closed in X.

4.31 Theorem

Every QNRRCS set is a QNRgP-closed set but not conversely.

Proof: Let A be a QNRRCS in X and hence by Definition 4.6, A = QNRCl(QNRInt(A)) which implies QNRCl(A)=QNRCl(QNRInt(A)). Therefore QNRCl(A) = A i.e., A is a QNRCS in X .By Theorem4.26, A is a QNRgP-closed in X.

4.32 Example

In Example 4.30 A is QNRgP-closed set but not QNRRCS.

4.33 Theorem

Every QNRPCS set is a QNRgP-closed set but not conversely.

Proof: Let A be a QNRPCS in X and A $\tilde{\subseteq}$ L where L be a QNROS in (X, τ). By Definition 4.6, QNRCl(QNRInt(A)) $\tilde{\subseteq}$ A which implies QNRPCl(A) = A $\tilde{\cup}$ QNRCl(QNRInt(A)) $\tilde{\subseteq}$ A. Therefore QNRPCl(A) $\tilde{\subseteq}$ L. Hence A is a QNRgP-closed set in X.

4.34 Example

In Example 4.24 A is QNRgP-closed set but not QNRPCS.

4.35 Theorem

Every QNRαg-closed set is a QNRgP-closed set but not conversely.

Proof: Let A be a QNRαg-closed in X and A $\tilde{\subseteq}$ L where L be a QNROS in (X, τ). By Definition 4.13, S $\tilde{\cup}$ QNRCl(QNRInt(QNRCl(S))) $\tilde{\subseteq}$ L which implies QNRCl(QNRInt(QNRCl(A))) $\tilde{\subseteq}$ L and QNRCl(QNRInt(A)) $\tilde{\subseteq}$ F . Therefore QNRPCl(A) = S $\tilde{\cup}$ QNRCl(QNRInt(A)) $\tilde{\subseteq}$ L . Hence A is a QNRgP-closed in X.

4.36 Example

In Example 4.30 A is QNRgP-closed set but not QNRα g-closed.

4.37 Example

Let X={x,y} and τ = { $\tilde{\phi}$, G_1 , $\tilde{X}$ }where

G_1= {⟨ x,{0.5,0.3,0.7,0.6},{0.4,0.2,0.3,0.5},{0.3,0.1,0.5,0.4} ⟩,
⟨ y,{0.4,0.2,0.6,0.5},{0.3,0.1,0.5,0.4},{0.2,0.1,0.4,0.3} ⟩ }

$G_1^{'}$ = {⟨ x,{0.6,0.7,0.3,0.5},{0.5,0.3,0.2,0.4},{0.4,0.5,0.1,0.3} ⟩,
⟨ y,{0.5,0.6,0.2,0.4},{0.4,0.5,0.1,0.3},{0.3,0.4,0.1,0.2} ⟩ }

Then a QSVNR set G_1=A is a QNRSCS but not a QNRgP-closed set in X.

4.38 Example

In Example 4.24 A is a QNRgP-closed set but not QNRSCS.

4.39 Proposition

1. QNRgS-closed set and QNRgP-closed sets are independent to each other.
2. QNRSCS and QNRgP-closed set are independent to each other.

4.40 Example

Let X={x,y} and τ = { $\tilde{\phi}$, G_1 , $\tilde{X}$ }where

G_1= {⟨ x,{0.5,0.4,0.7,0.6},{0.4,0.3,0.6,0.5},{0.3,0.2,0.5,0.4} ⟩,
⟨ y,{0.3,0.2,0.5,0.4},{0.2,0.1,0.5,0.3},{0.1,0.2,0.3,0.4} ⟩ }

$G_1^{'}$ = {⟨ x,{0.6,0.7,0.4,0.5},{0.5,0.6,0.3,0.4},{0.4,0.5,0.2,0.3} ⟩,
⟨ y,{0.4,0.5,0.2,0.3},{0.3,0.5,0.1,0.2},{0.4,0.3,0.2,0.1} ⟩ }

Then a QSVNR set G_1=A is a QNRgS-closed set but not a QNRgP-closed set in X.

4.41 Example

Let X={x,y} and τ = { $\tilde{\phi}$, G_1 , $\tilde{X}$ }where

G_1= {⟨ x,{0.6,0.7,0.2,0.3},{0.7,0.8,0.4,0.5},{0.9,0.8,0.3,0.5} ⟩,
⟨ y,{0.8,0.6,0.3,0.2},{0.6,0.7,0.1,0.2},{0.5,0.6,0.2,0.3} ⟩ }

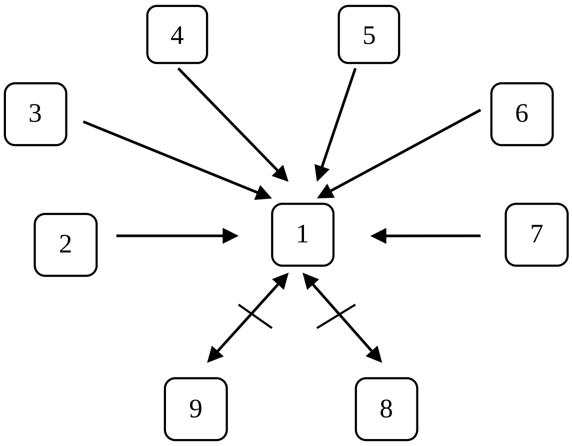
$G_1^{'}$ = {⟨ x,{0.3,0.2,0.7,0.6},{0.5,0.4,0.8,0.7},{0.5,0.3,0.8,0.9} ⟩,
⟨ y,{0.2,0.3,0.6,0.8},{0.2,0.1,0.7,0.6},{0.3,0.2,0.6,0.5} ⟩ }

Then a QSVNR set

A = {⟨ u,{0.5,0.6,0.3,0.4},{0.6,0.7,0.5,0.6},{0.8,0.7,0.4,0.6} ⟩,
⟨ v,{0.7,0.5,0.4,0.3},{0.5,0.6,0.2,0.3},{0.6,0.5,0.3,0.4} ⟩ }is QNRgP-closed set but not a QNRgS-closed set in X.

The following implications are true.

1.QNRg-closed set 2.QNRCS 3.QNRg-closed set 4.QNRPCS 5.QNRRCS
6. QNR α CS 7.QNR α g-closed set 8.QNRgS-closed set 9.QNRSCS



Here A $\rightarrow$ B denotes A implies B but not conversely and A $\not\leftrightarrow$ B means A and B are independent of each other and none of them is reversible.

4.42 Remark

The union of any two QNRgP-closed sets is n ot a QNRgP-closed setwhich is shown in the following example.

4.43 Example

Let X={x,y} and τ = { $\tilde{\phi}$, G_1 , $\tilde{X}$ }where

G_1= {⟨ x,{0.5,0.3,0.7,0.6},{0.4,0.2,0.3,0.5},{0.3,0.1,0.5,0.4}⟩,
⟨ y,{0.4,0.2,0.6,0.5},{0.3,0.1,0.5,0.4},{0.3,0.2,0.4,0.3}⟩}

Considser two QSVNR sets

A_1 = {⟨ x,{0.5,0.2,0.7,0.6},{0.4,0.1,0.4,0.6},{0.3,0.1,0.6,0.5}⟩,
⟨ y,{0.4,0.1,0.7,0.6},{0.3,0.2,0.6,0.5},{0.2,0.1,0.5,0.4}⟩}

A_2 = {⟨ x,{0.4,0.3,0.7,0.5},{0.3,0.1,0.7,0.5},{0.2,0.1,0.6,0.7}⟩,
⟨ y,{0.3,0.2,0.7,0.8},{0.2,0.1,0.7,0.6},{0.1,0.2,0.5,0.4}⟩}

which are QNRgP-closed sets but $A_1 \tilde{\cup} A_2$ is not a QNRgP-closed set in X.

V. CONCLUSION

In this paper, we defined on quadripartitioned single valued neutrosophic refined sets and its properties.Further we introduced the concept of quadripartitioned single valued neutrosophic refined topological space and studied the basic concepts with examples in detail.

REFERENCES

[1] Atanasov K.T, Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems20 (1986), 87-96.
[2] W.D. Blizard , Multiset theory,Notre Dame Journal of Formal Logic 30(1)(1989) 36-66.
[3] S.Broumi,I.Deli,and F.Smarandache On Neutrosophic Refined Sets and their Applications in Medical Diagnosis,Journal of NewTheory,6(2015) 88-98.
[4] C.S. Calude, G. Paun, G.Rozenberg, A.Salomaa,Lecture notes in computer science: Multiset Processing Mathematical,Computer Science, and Molecular Computing Points of View, 2235(2001) Springer-New York.
[5] S.Das,M.B.Kar and S.Kar, Group multi-criteria decision making using intuitionistic multi-fuzzy sets,Journa; Of Uncertainity Analysis and Applications 10(1) (2013) 1-16.
[6] I.Deli and S.Broumi,Neutrosophic multisets and its application in medical diagnosis(2014)
[7] R.Muthuraj and S.Balamurugan ,Multi-Fuzzy Group and its Level Subgroups,Gen.Math.Notes17(1) (2013) 74-81.
[8] Mohanasundari M,Mohana K,Quadripartitioned Neutrosophic Mappings with its relation and Quadripartitioned Neutrosophic Topology,International Journal of Mathematics and its applications,9,1,(2021),83-93.
[9] Rajashi Chatterjee, P.Majumdar and S.K.Samanta, On some similarity measures and entropy on quadripartitioned single valued neutrosophic sets, Journal of Intel-ligent Fuzzy Systems 30(2016) 2475-2485

[10] S.Sebastian and T.V. Ramakrishnan, Multi-fuzzy extension of crisp functionsusing fridge functions,Annals of Fuzzy Mathematics and Informations2(1) (2011)1-8.

[11] S.Sebastian and T.V. Ramakrishnan, Multi-fuzzy Subgroups,Int.J.Contemp.Math.Sciences 6(8) (2011)365-372.

[12] S.Sebastian and T.V. Ramakrishnan, Multi-fuzzy Sets, International Mathematical Forum 5(50) (2010) 2471-2476.

[13] T.K.Shinoj and S.J. John, Intuitionistic fuzzy multisets and its application in medical diagnosis,World Academy of Science,Engineering and Technology 6 (2012) 01-28.

[14]F.Smarandache, A unifying Field in Logics.Neutrosophy: NeutrosophicProbability,Setand Logic,Rehoboth:American Research press (1998).

[15] F.Smarandache,n-valued Refined Neutrosophic Logic and its Applications in Physics,Progress in Physics, 143-146,Vol.4,2013

[16] A.Syropoulos, On generalized fuzzy multisets and their use in computation,Iranian Journal of Fuzzy Systems 9(2) (2012) 113-125.

[17] A.S. Thomas and S.J.John, Multi-fuzzy rough sets and relations,Annals of Fuzzy Mathematics and informatics x/x (2013) xx-xxx.

[18] Wang H, Smarandache F, Zhang YQ, Sunderraman R, Single valued neutrosophic sets, Multispace Multistruct 4.,(2010), 410-413.

[19] S.Ye,J.Ye, Dice similarity measure between single valued neutrosophic multisets and its application in medical diagnosis,Neutrosophic Sets and Systems 6(2014) 48-52.

[20] R. R. Yager, On the theory of bags (Multisets),Int.J.of General System,13(1986) 23-27.

[21] L.A. Zadeh,Fuzzy Sets , Inform. And Control,8 (1965) 338-353.