TW2: Laser technology and optics design

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General optics design

Peter Loosen

- Survey
- paraxial optics
- Seidel aberrations
- diffraction
- combination of diffraction and aberration theory

TOS RWTH
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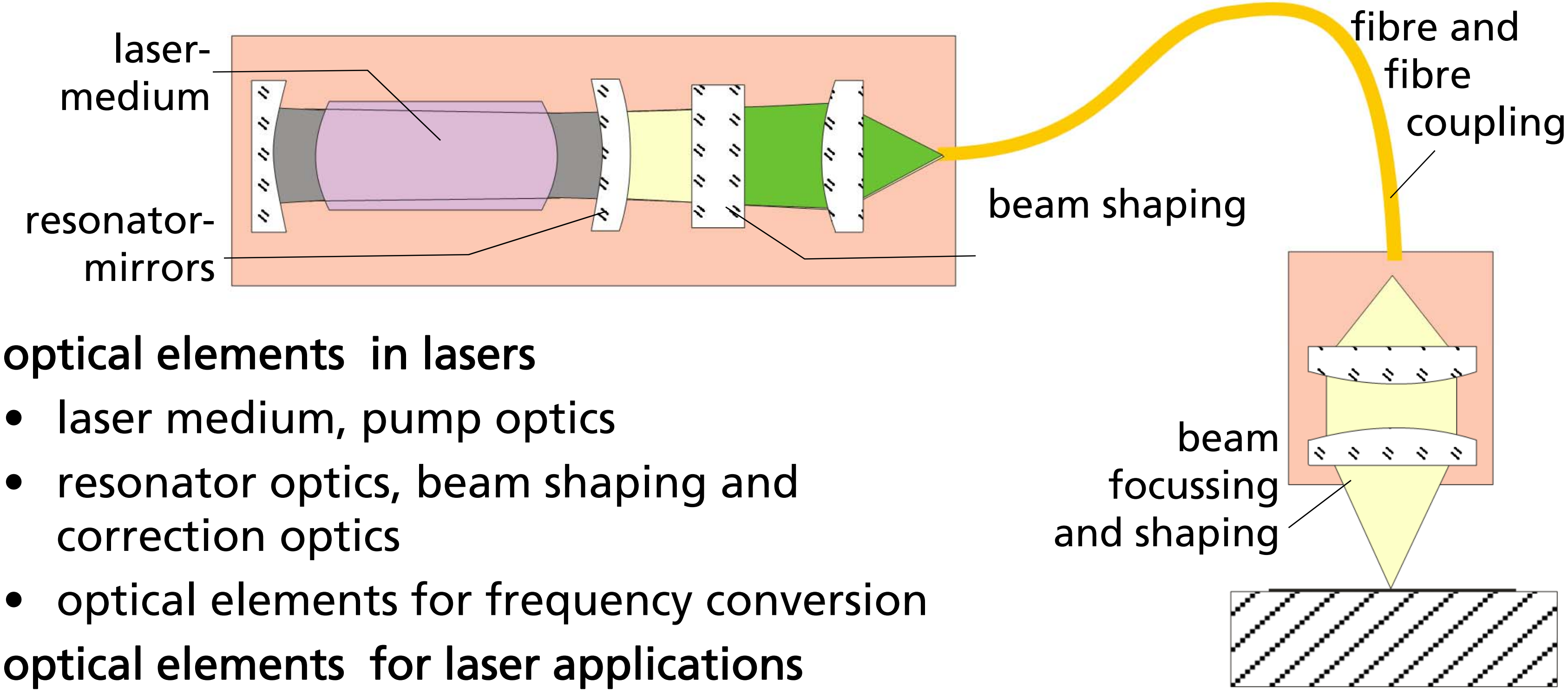
LEHRSTUHL FÜR TECHNOLOGIE OPTISCHER SYSTEME

Optics and optical systems with lasers



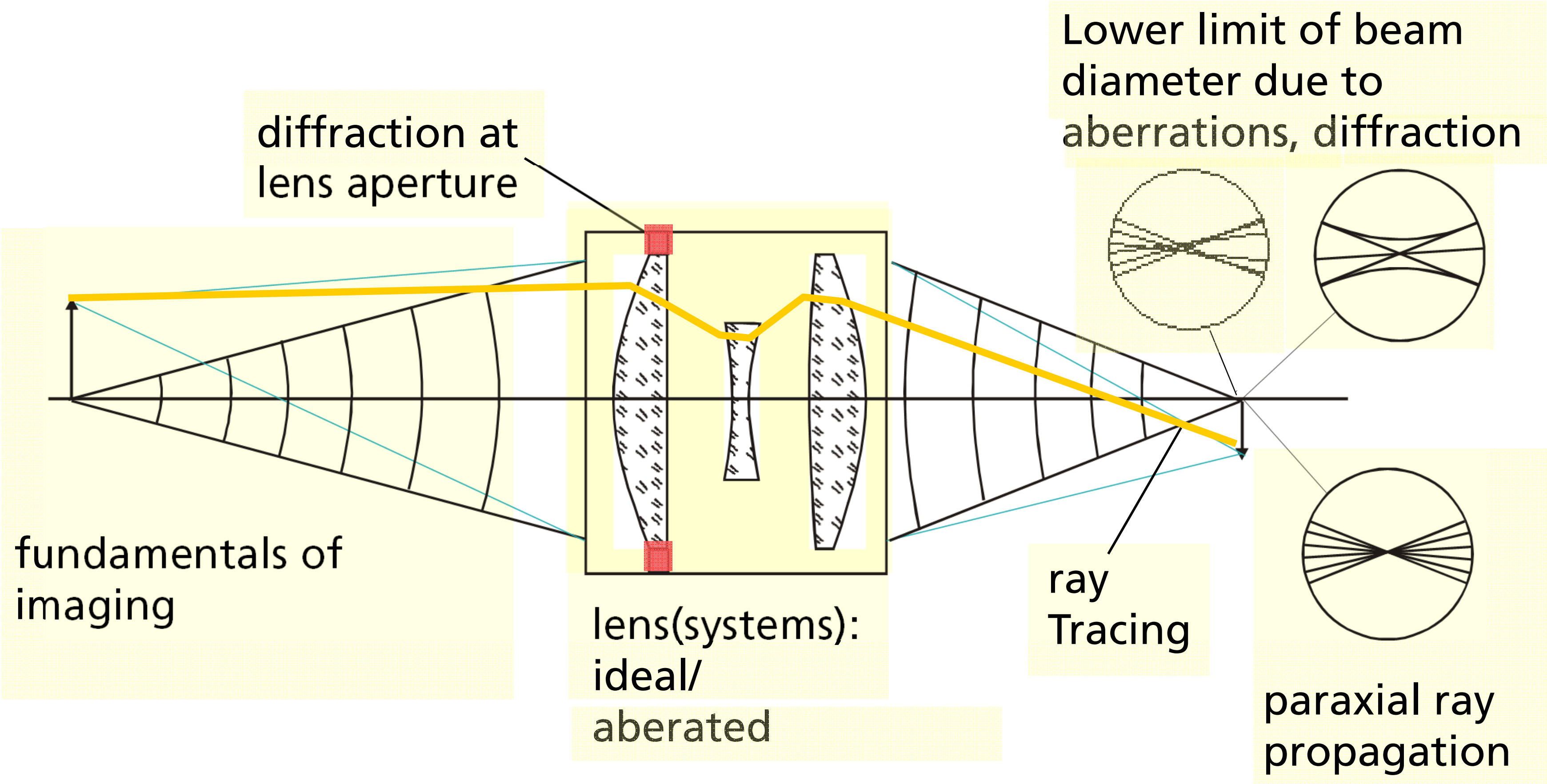
optical elements in lasers

- laser medium, pump optics
- resonator optics, beam shaping and correction optics
- optical elements for frequency conversion

optical elements for laser applications

- fiber coupling optics, beam propagation optics
- focusing optics, beam shaping systems

Survey, Topics



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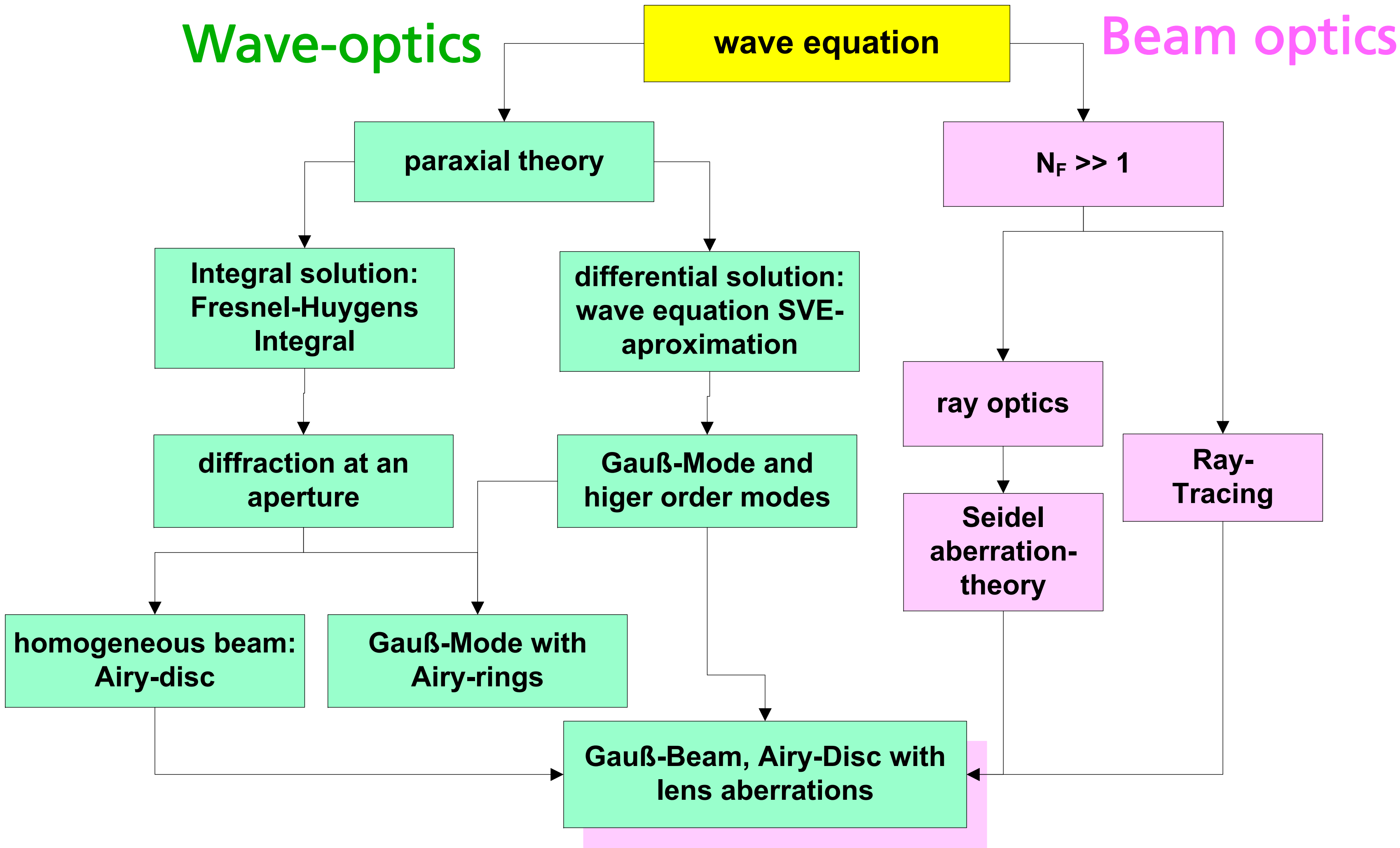
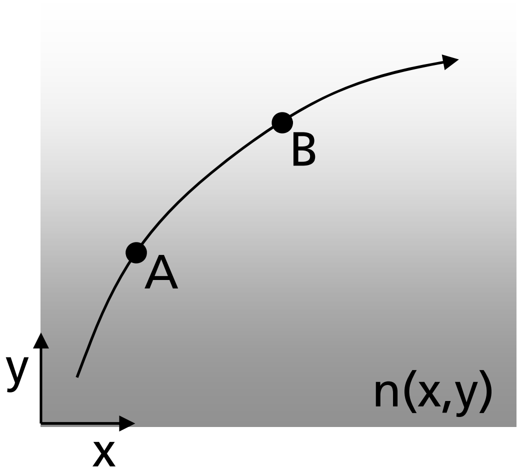
Wave optics, ray optics and combinations

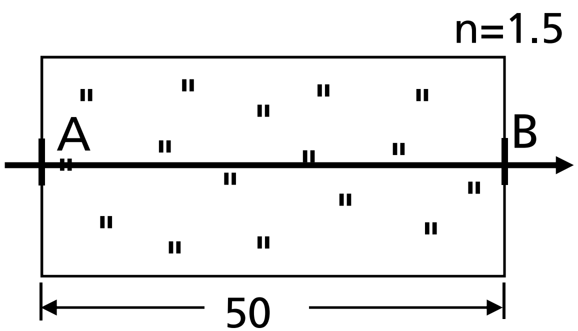


Optical path length L_{opt}



$$L_{opt} = c_0 \cdot \int_A^B dt = \int_A^B n(x,y)\,ds$$

$$n(x,y) = \frac{c_0}{v(x,y)}$$ Index of refraction



$$L_{opt} = \int_0^L n \cdot ds = n \cdot L$$

geometrical distance: 50mm
optical distance (@ n=1,5): 75mm

(pass-through time is 50% longer)

Fermat´s Principle

$$L_{opt} = \int_A^B n \cdot ds$$

$$= Min$$

$$T = \frac{1}{c_0} \int_A^B n \cdot ds$$

$$= Min$$

An optical ray between two point A and B always follwos a path, along which the optical path L_{opt} (resp. the travel time T) takes an minimum (compared to closely paths nearby)

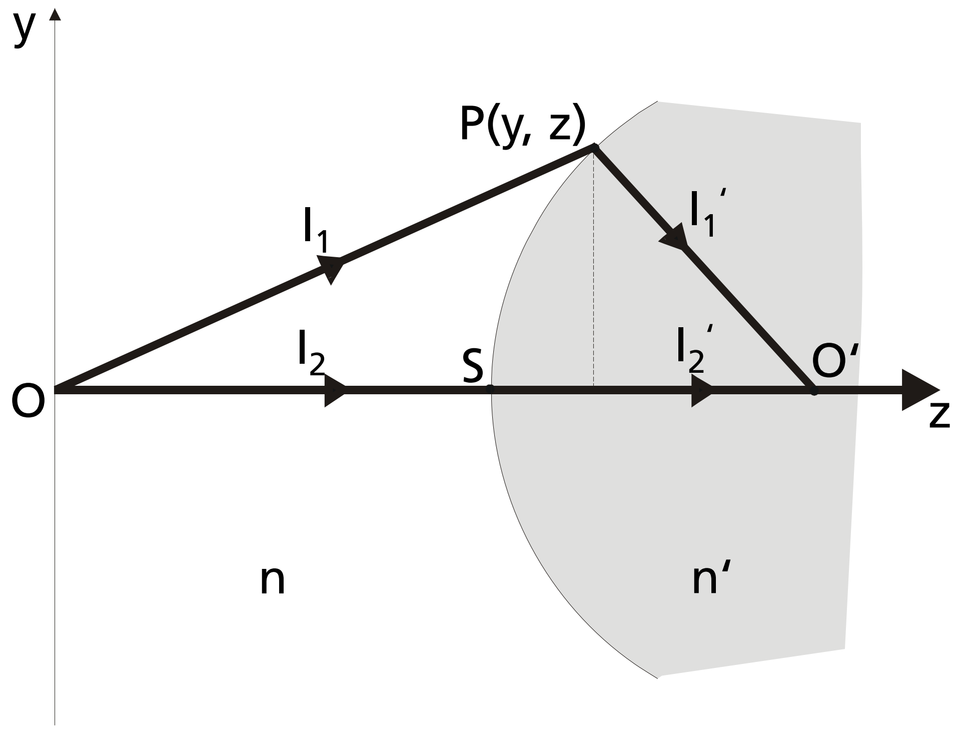
Refractive surface für ideal imaging

Fermat: all optical paths from 0 → 0‘ are equal:

$$n \cdot l_1 + n' \cdot l_1' = n \cdot l_2 + n' \cdot l_2' = const.$$

$$n \cdot \sqrt{y^2 + z^2} + n' \sqrt{y^2 + (l_2 + l_2' - z)^2} = const.$$

Equation of a cartesian rotation-oval



„Real“ imaging systems

„Disadvatages “ of ideal cartesian surfaces (Hyberboloide, ...)

- aberration-free imaging only for a single point, with extended surfaces strong aberrations
- surfaces are hard to manufacture
- precision requirements are high

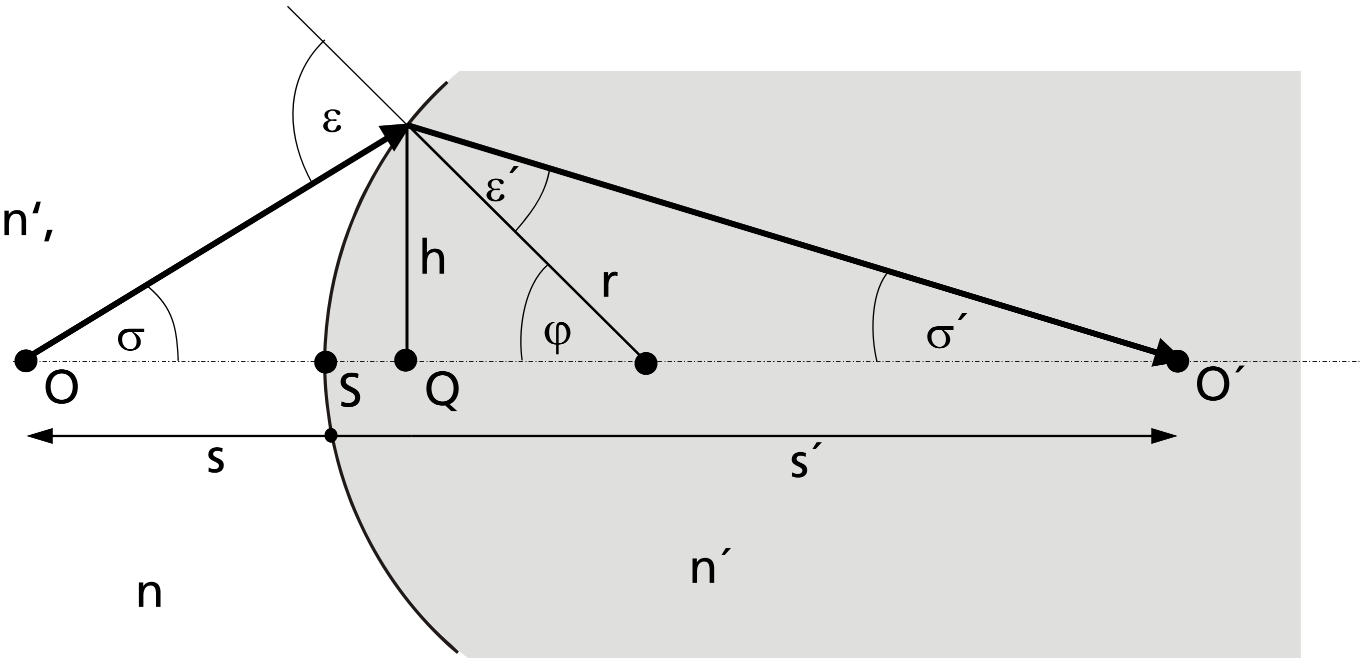
Systems in practical use

- spherical surfaces:
- manufacturing is simple and precise
- But no aberrations-free imaging, even not for single points
- aberration compensation by combination of surfaces and lens materials

Refraction at a spherically curved surface

One single spherically curved surface between media with n and n‘,

Small angle approximation



Relation between s and s`:

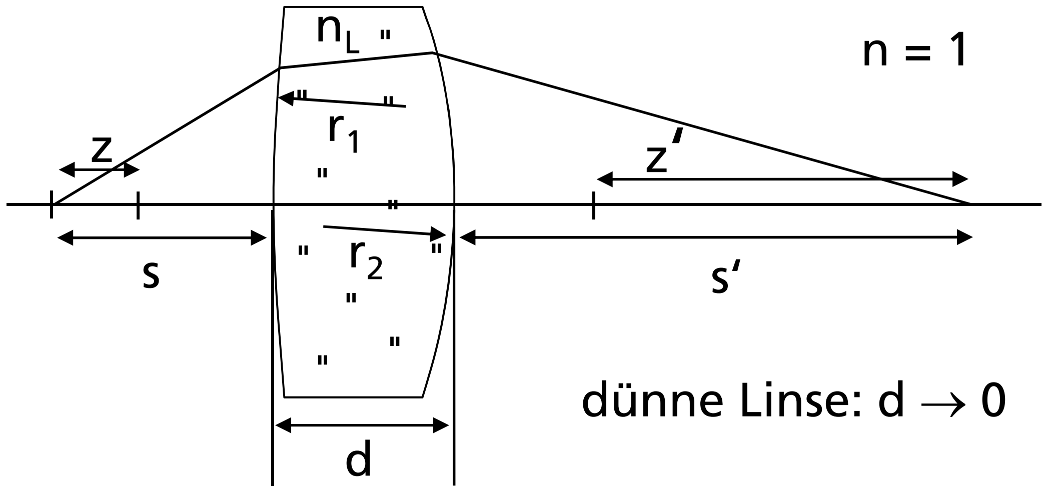
$$\frac{n'}{s'} - \frac{1}{s} = \frac{n'-1}{r}$$

Paraxial equation for the thin lens

$$\frac{1}{s'} - \frac{1}{s} = (n_L - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\frac{1}{f'} = (n_L - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

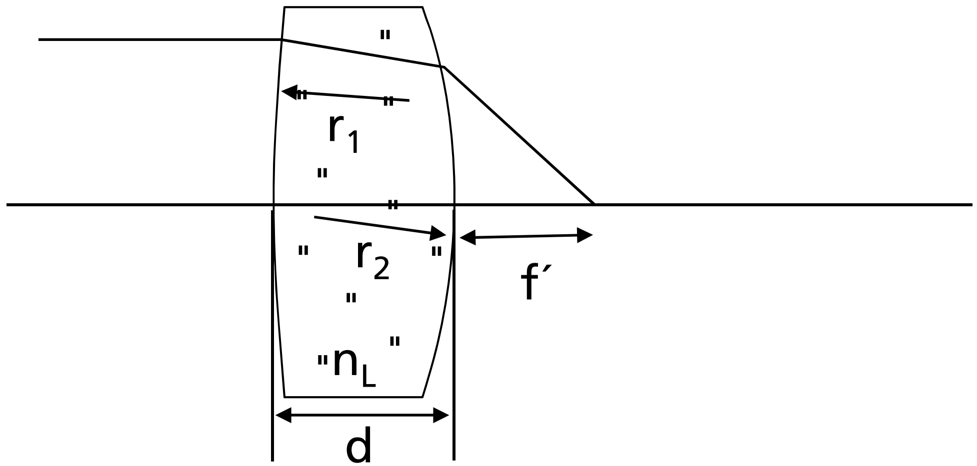
„Lens maker formula"



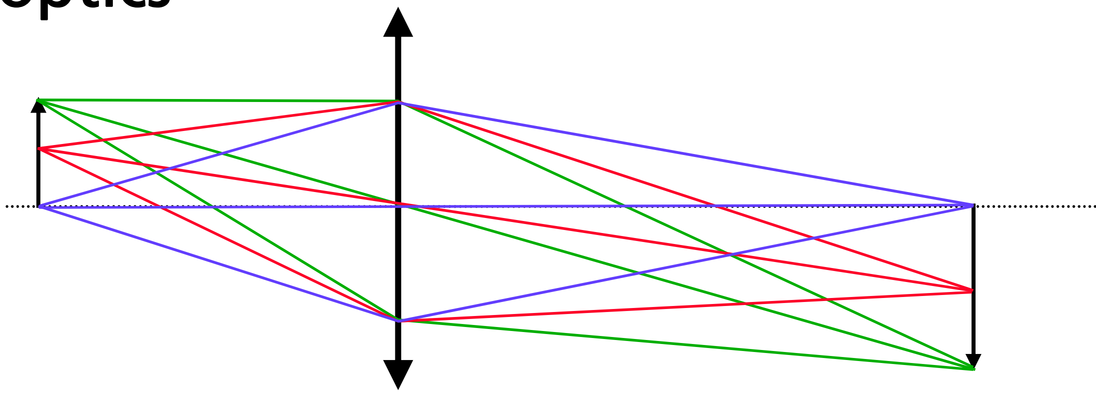
alternative formulation with curvature K of phase fronts :

$$K = \frac{1}{s} \qquad K' = \frac{1}{s'}$$

$$K - K' = \frac{1}{f}$$



Extension of paraxial optics



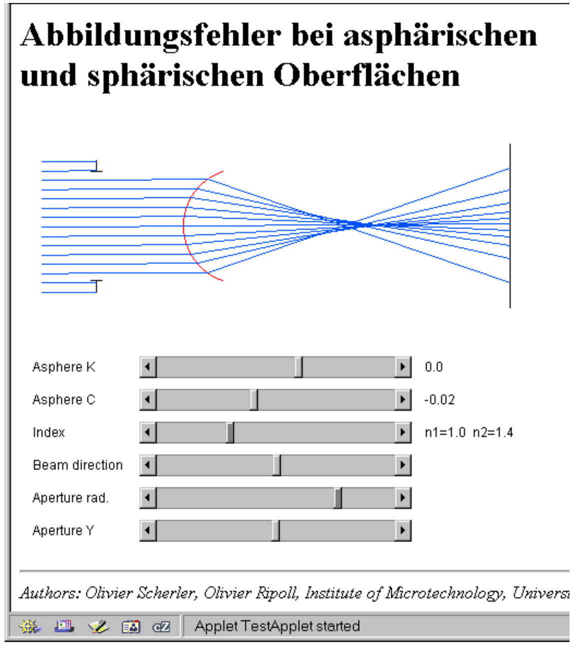
- paraxial optics („first order optics,“, „Gaußian optics“) describes the imaging by optical elements with ideal imaging properties
- all beams from every object point are striking exactly in the related image points
- extension to Seidel aberration optics and to ray-tracing

$$\sin(\varphi) = \varphi$$ Paraxial optics

$$-\frac{\varphi^3}{3!}$$ 3. order optics (Seidel optics)

$$+\frac{\varphi^5}{5!}-\frac{\varphi^7}{7!}+\ldots$$ numerical ray-tracing („all terms of the Taylor expansion“)

Aberrations of non-paraxial optics



Spherical surfaces:
K=0, C=1/R

Paraxial optics:

- all object beams are concentrated per definition in perfect image points
- For lenses with shperical surfaces only valid for paraxial beams ($\sin\theta \approx \theta$)

Real (non-paraxial) optics:

- beam direction has to be calculated at every surface with either the exact or a more accurate refractive law (ray-tracing, Seidel-theory)
- this procedure gives the aberrations of the optical system

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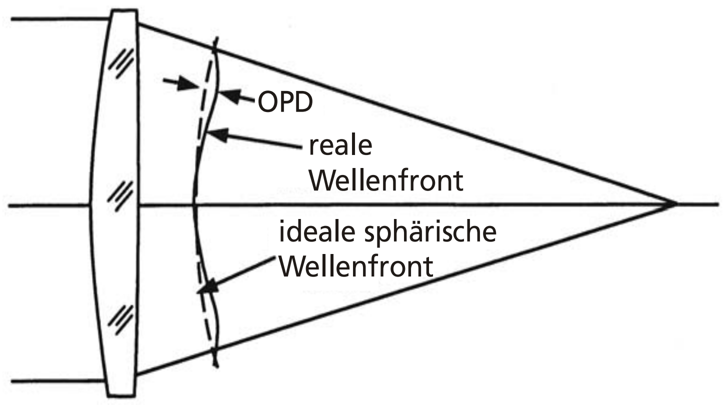
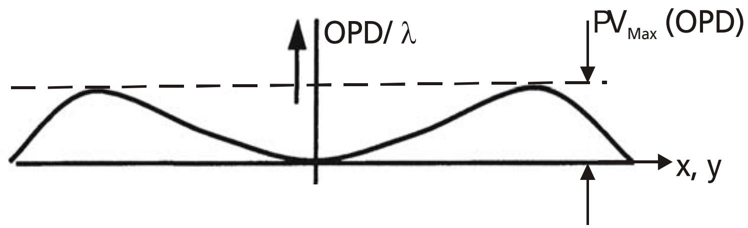
Optical path difference (OPD)

- an ideal paraxial system creates a perfect spherical wave front at the exit, which perfectly converges into a point
- a real optical system creates a distorted wave front with deviations from the iodeal spherical reference wave (OPD, wave front aberrations)

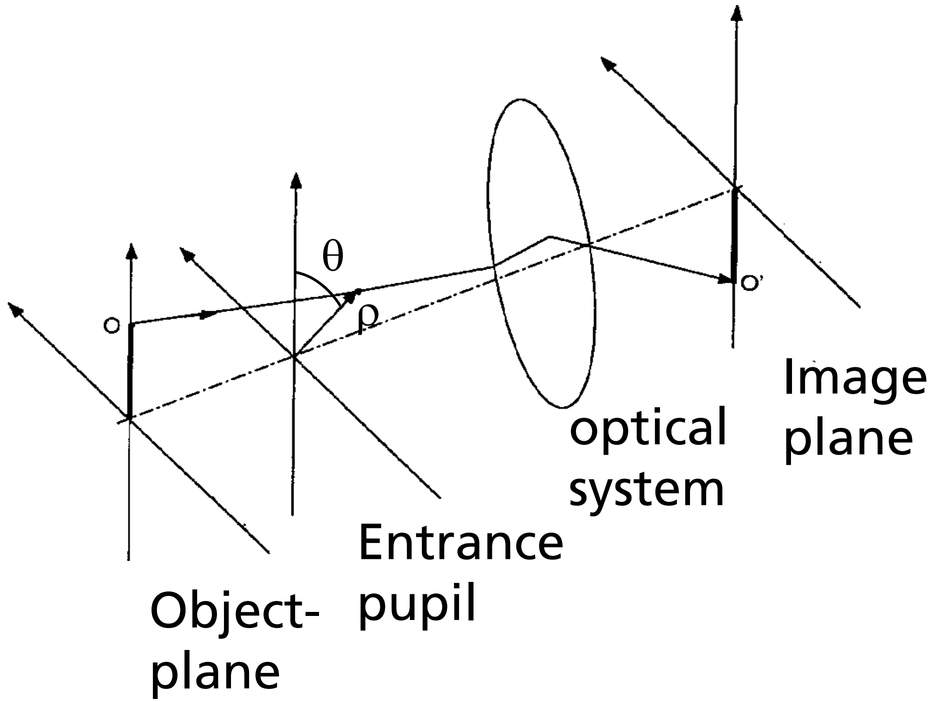


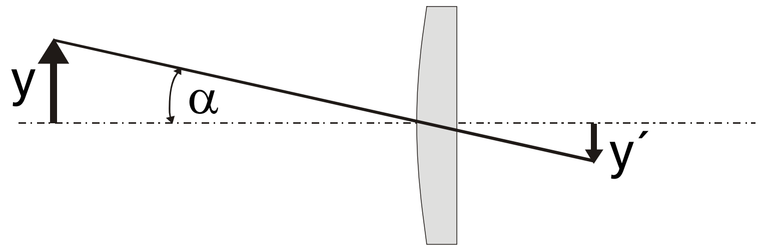


Rayleigh-criterion: a system is ray-optically perfect,
if PV_{Max} (OPD) $\leq \lambda/4$ (only limited by diffraction)

Seidel aberrations: coordinate system

- Expression of the OPD (or "q") in the coordinates for a rotational symmetric optical system with spherical surfaces
- power series expansion of the OPD
- use of the Seidel approximations





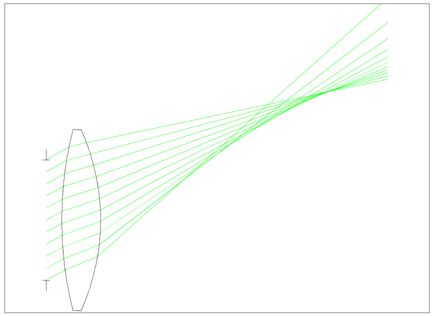
Seidel aberration terms

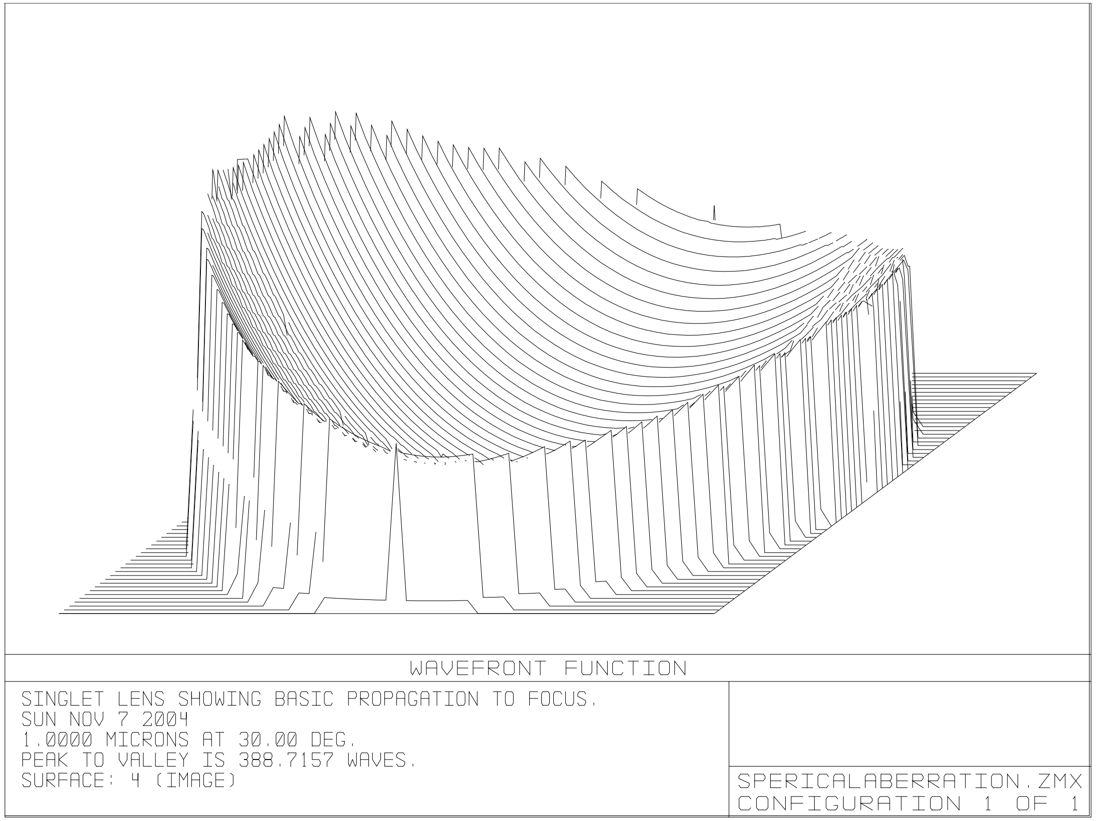
$q(y',\rho,\theta)$	=	$C_{020} \cdot \rho^2$	defokussing
	+	$C_{111} \cdot y' \cdot \rho \cdot \cos\theta$	tilt
	+	$C_{040} \cdot \rho^4$	spherical aberration
	+	$C_{131} \cdot y' \cdot \rho^3 \cdot \cos\theta$	coma
	+	$C_{222} \cdot y'^2 \cdot \rho^2 \cdot \cos^2\theta$	astigmatism
	+	$C_{220} \cdot y'^2 \cdot \rho^2$	image field curvature
	+	$C_{311} \cdot y'^3 \cdot \rho \cdot \cos\theta$	distortion
	+	… (higher terms)	

- all expansion terms are depending from object size and aperture radius with different powers
- Reduction of aberrations at small object sizes and small apertures

Example of an OPD

- non-optimized single lens
- entrance: oblique parallel ray bunch (30°)
- OPD behind the last refracting surface
- PV = 388 Waves





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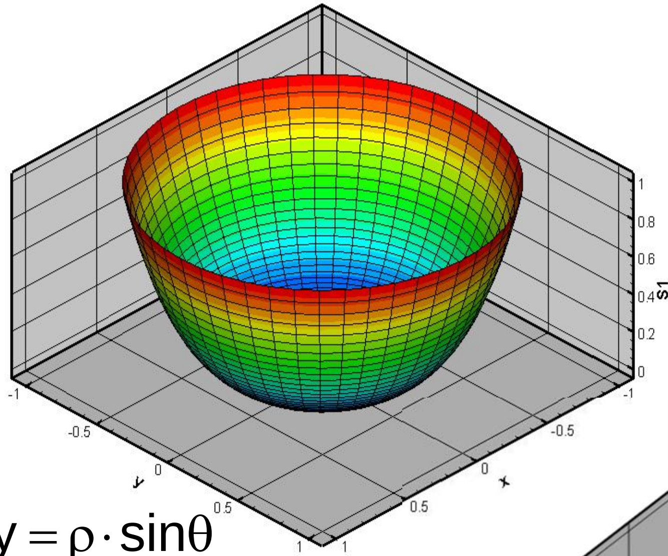
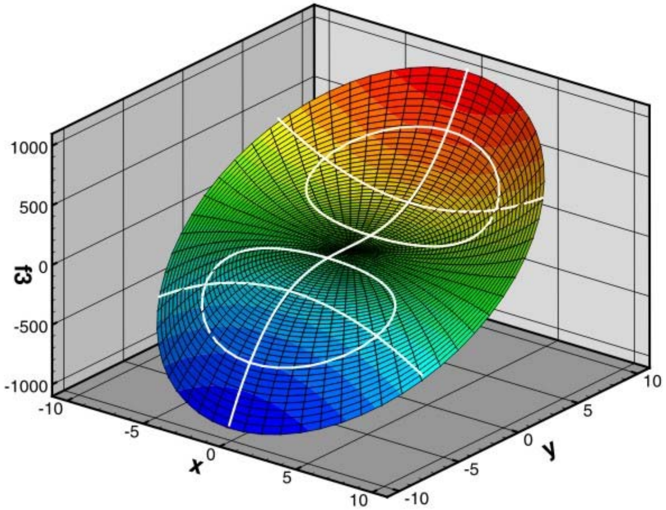
Examples of pure Seidel-terms (1/2)



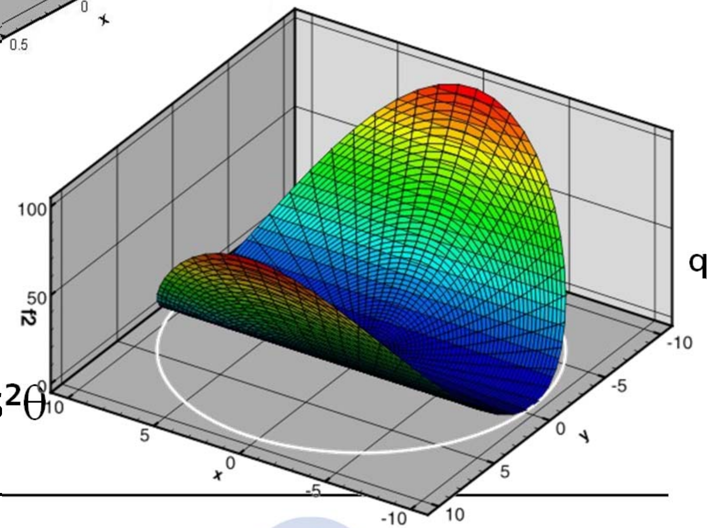
$C_{040} \cdot \rho^4$
(spherical aberration)



$y = \rho \cdot \sin\theta$

$\rho^2 = x^2 + y^2$

$C_{131} \cdot y' \cdot \rho^3 \cdot \cos\theta$
(coma)

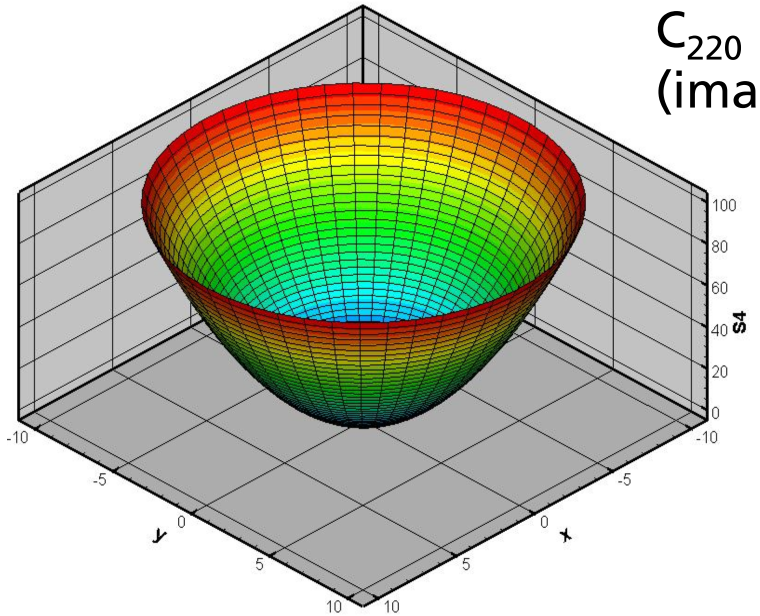


$C_{222} \cdot y'^2 \cdot \rho^2 \cdot \cos^2\theta$
(astigmatism)

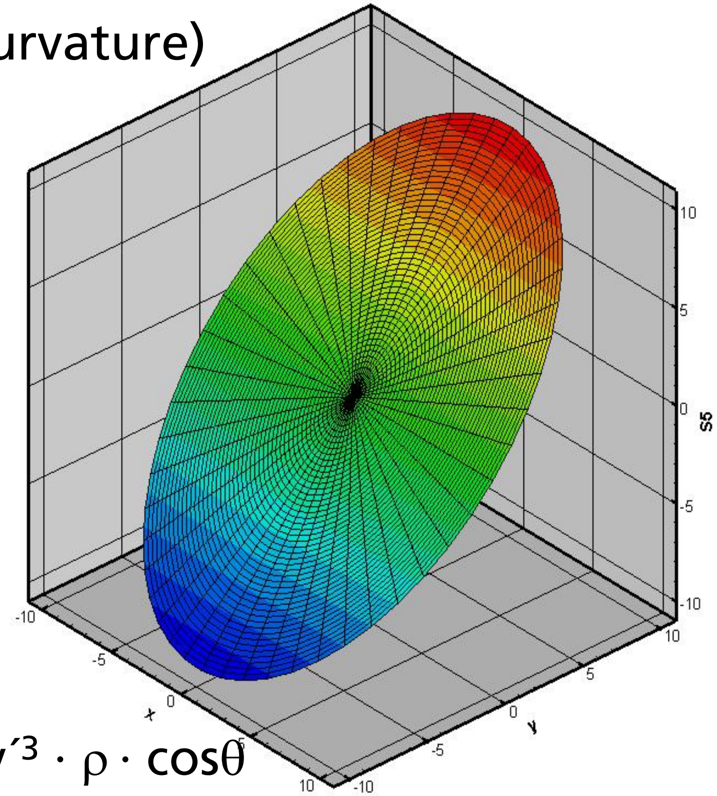
$q(y',\rho,\theta) =$

$C_{040} \cdot \rho^4$	sph. Aberration
$+ \ C_{131} \cdot y' \cdot \rho^3 \cdot \cos\theta$	Koma
$+ \ C_{222} \cdot y'^2 \cdot \rho^2 \cdot \cos^2\theta$	Astigmatismus
$+ \ C_{220} \cdot y'^2 \cdot \rho^2$	Bildfeldwölbung
$+ \ C_{311} \cdot y'^3 \cdot \rho \cdot \cos\theta$	Verzeichnung

Examples of pure Seidel-terms (2/2)



$C_{220} \cdot y'^2 \cdot \rho^2$
(image field curvature)



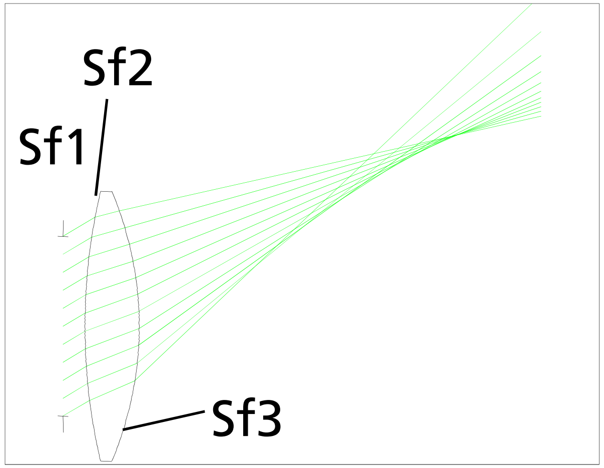
$C_{311} \cdot y'^3 \cdot \rho \cdot \cos\theta$
(distortion)

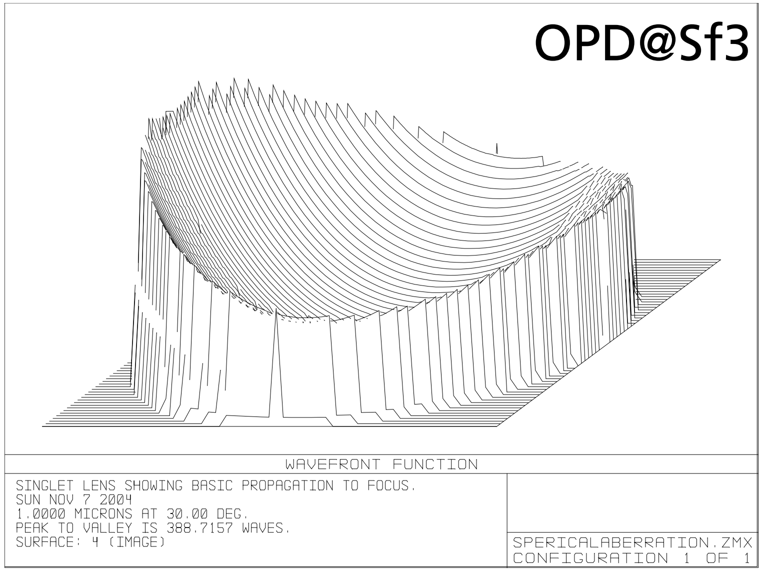
$q(y',\rho,\theta) =$

$C_{040} \cdot \rho^4$	sph. Aberration
$+ \; C_{131} \cdot y' \cdot \rho^3 \cdot \cos\theta$	Koma
$+ \; C_{222} \cdot y'^2 \cdot \rho^2 \cdot \cos^2\theta$	Astigmatismus
$+ \; C_{220} \cdot y'^2 \cdot \rho^2$	Bildfeldwölbung
$+ \; C_{311} \cdot y'^3 \cdot \rho \cdot \cos\theta$	Verzeichnung

Examples of Seidel expansion coefficients

Singlet with 30° field angle





Listing of Aberration Coefficient Data

Seidel Aberration Coefficients:

Surf	SPHA C040	COMA C131	ASTI C222	FCUR C220	DIST C311
STO	0.000000	0.000000	0.000000	0.000000	0.000000
2	0.028782	0.088388	0.271442	0.282752	1.701932
3	0.952040	-0.008754	0.184380	0.471253	-0.004334

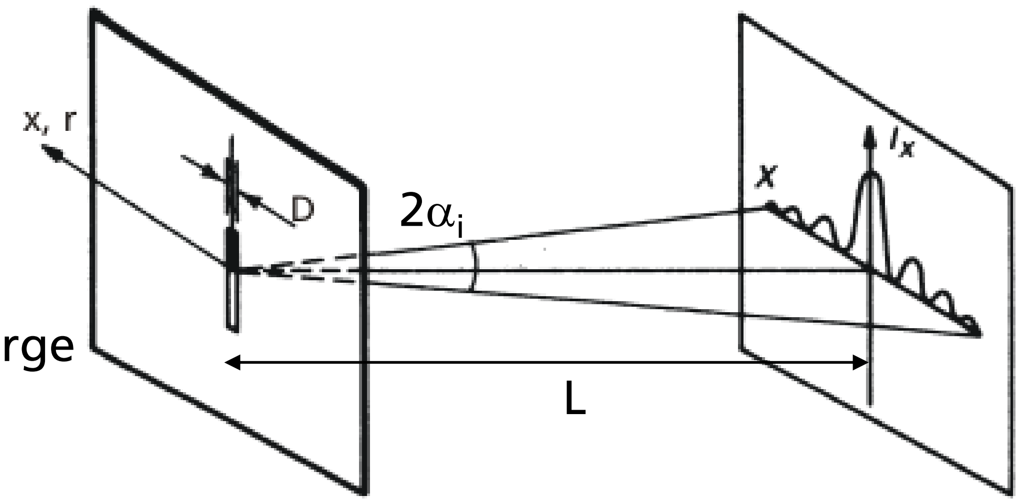
Further discussion of Seidel theory

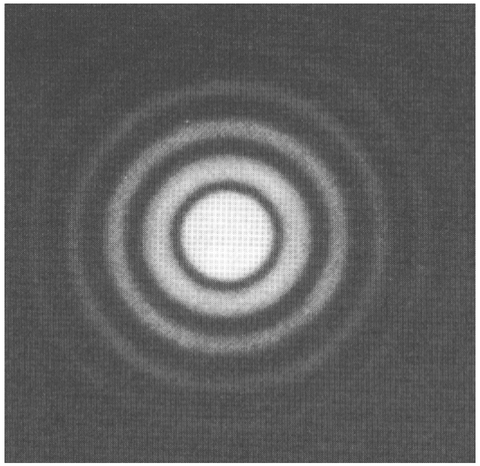
… and aberration coefficients
and methods to compensate aberrations within the Seidel theory
and ray-tracing calculations
are discussed in the next presentation, given by Martin Traub

In the following we will discuss diffraction and the physical interpretation of the Rayleigh-criterion, which defines the limit for ray-optical optimizations

Diffraction: slit and circular aperture

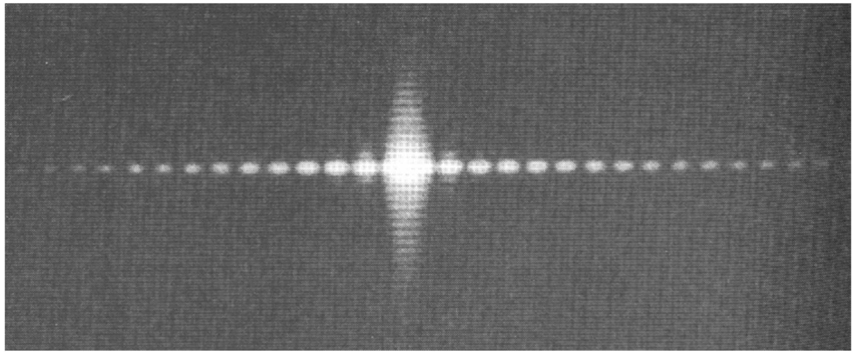
- plane Wave illuminates the screen from the left side
- screen openings: slit with width D or circular aperture with diameter D
- pictures below show the intensity distribution at large distances L$\rightarrow\infty$ (N_F<<1)





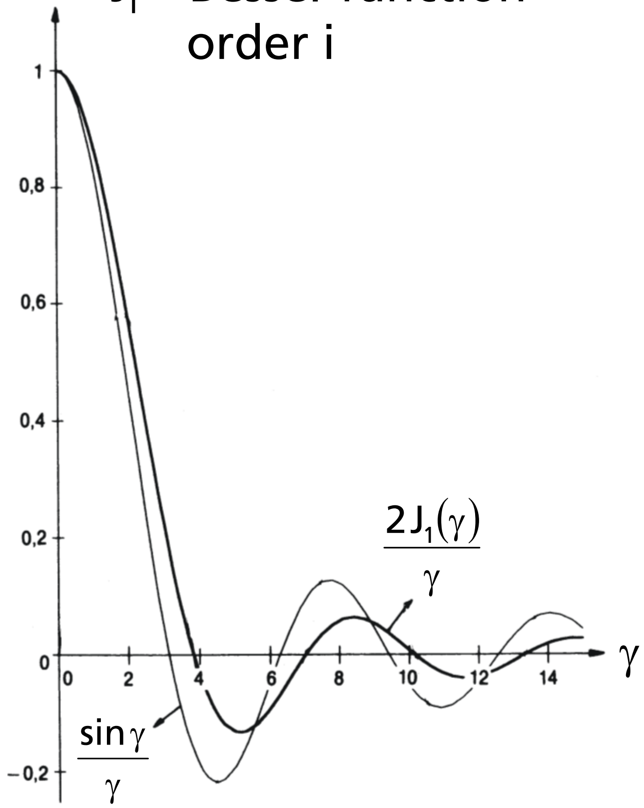
circular aperture

slit aperture



Diffraction at slit and circular aperture: far-field I(x,y)

J_i – Bessel-function order i



far-field intensity distribution:

circle (diameter D) : $I_S(\gamma) = I_0 \cdot \left(\frac{2 J_1(\gamma)}{\gamma} \right)^2$

slit (width D) : $I_K(\gamma) = I_0 \cdot \left(\frac{\sin \gamma}{\gamma} \right)^2$

$$\gamma = \frac{1}{2} \cdot k \cdot D \cdot \sin\alpha$$

first null of I_S (circular aperture) at:

$$\gamma = \frac{1}{2} \cdot k \cdot D \cdot \sin\alpha = 3{,}83 \qquad (k = 2\pi / \lambda)$$

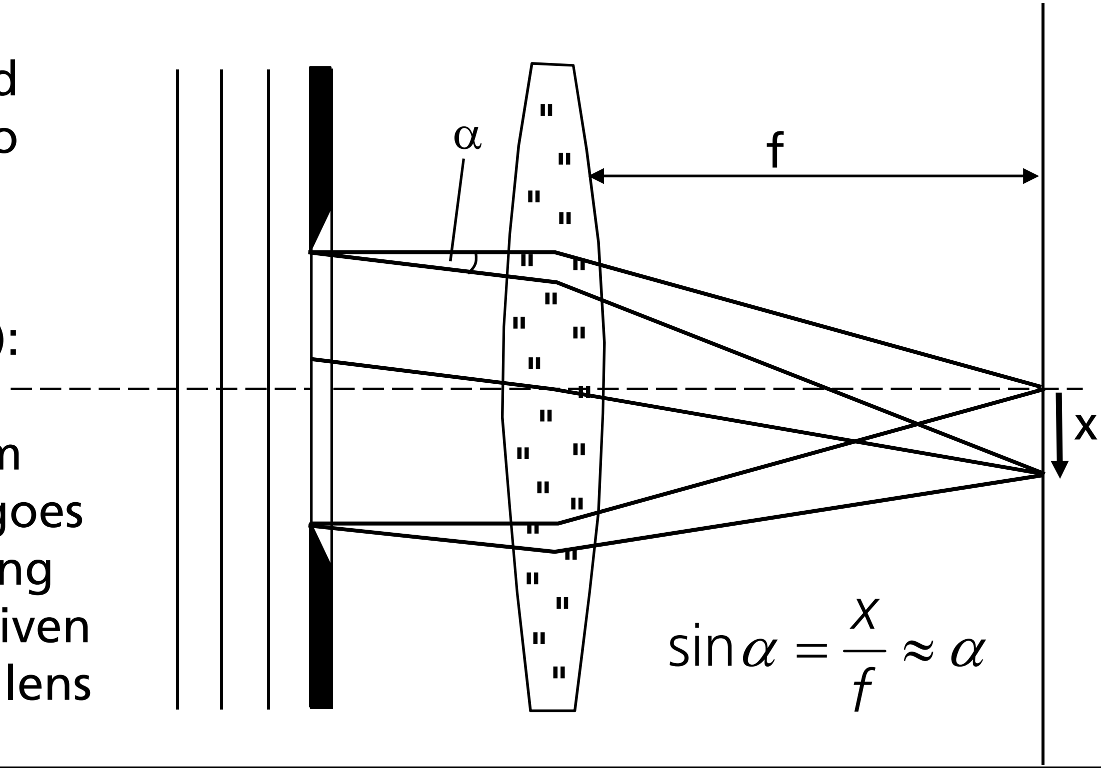
$$\sin\alpha_{1.Min} \approx \alpha_{1.Min} \approx 1{,}22 \cdot \frac{\lambda}{D}$$

Far-field at the focus of a lens

- for the description of diffraction processes at optical systems it is necessary to discuss, which diffraction pattern (near-field, far-field) is generated by optical systems

- a convergent lens transforms the far-field diffraction pattern into the focal plane of the lens:

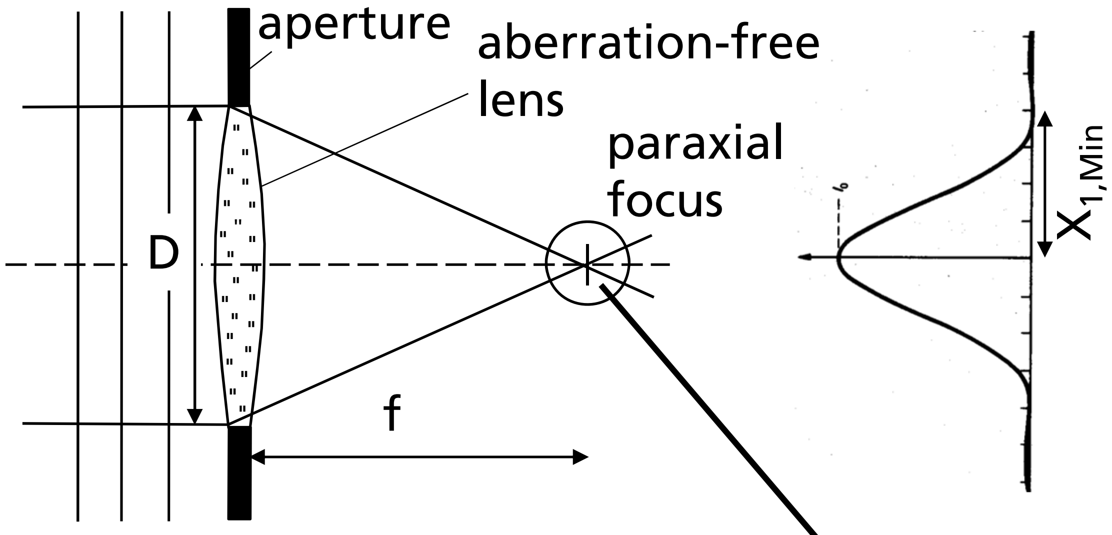
- far-field ($L \to \infty$, $N_F \to 0$): phase differences for elementary waves from boundary and center goes to zero, that is according to Fermat´s principle given in the focal plane of a lens

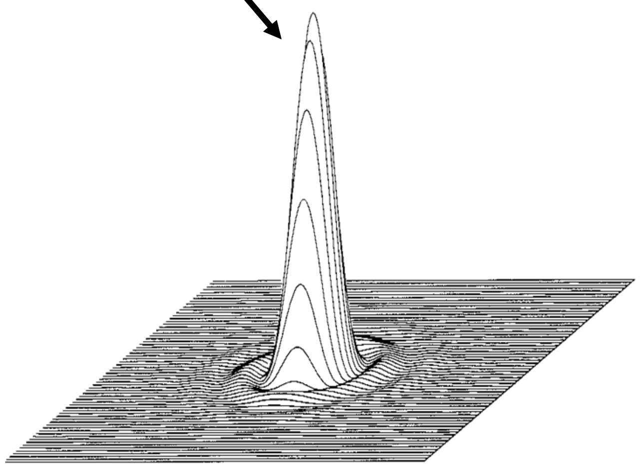


$$\sin\alpha = \frac{x}{f} \approx \alpha$$

Minimum focal radius

- focusing of a plane wave with a circular symmetric ideal paraxial lens
- superposition of focusing and diffraction at lens aperture
- due to diffraction the plane wave is not focused to a point, but to a diffraction pattern (Airy-disc)





$$\alpha_{1.Min} = \frac{x_{1.Min}}{f} = 1{,}22 \cdot \frac{\lambda}{D}$$

$$x_{1.Min} = 1{,}22 \cdot \frac{\lambda}{D} \cdot f \approx \lambda \cdot f/\#$$

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Transition to beams with Gaußian intensity profiles

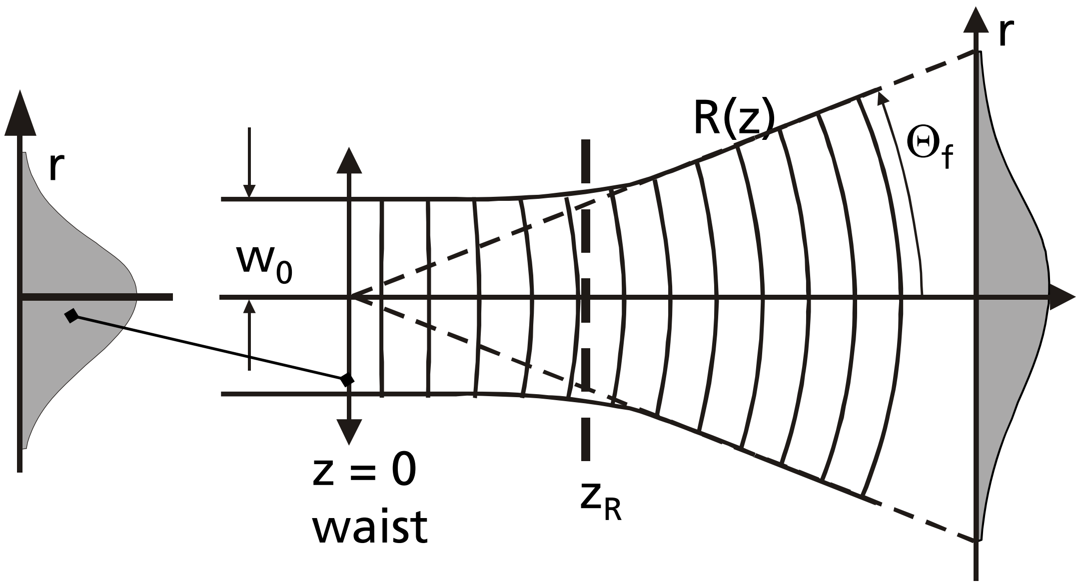
- in classical optics an homogenously filled aperture is a usefull approximation to many practical setups
- in theses cases the discussed minimum focal radius is a valid approximation
- when using laser beams however, it is more appropriate to calculate the achievable focal radius on the basis of the laser modes, which are typically used
- for the simplest case, a fundamental Gaussian beam is discussed in the following

Gaussian beam

$$\frac{E(r,z,t)}{E_0} = \frac{w_0}{w}\exp\left(-\left(\frac{r}{w}\right)^2\right)\cdot\exp(i\cdot(\omega t-\phi_T-\phi_L))$$

amplitude-factor

phase-factor



beam radius:

$$w = w_0\cdot\sqrt{1+(z/z_R)^2}$$

transvers. phase term:

$\phi_T = kr^2 / 2R$

longitudinal phase term:

$\phi_L = kz - \text{arctg}\,(z/z_R)$

radius of curvature of the phase fronts:

$$R(z) = z\cdot\left(1+(z_R/z)^2\right)$$

Rayleigh-length:

$$z_R = \pi w_0^2/\lambda$$

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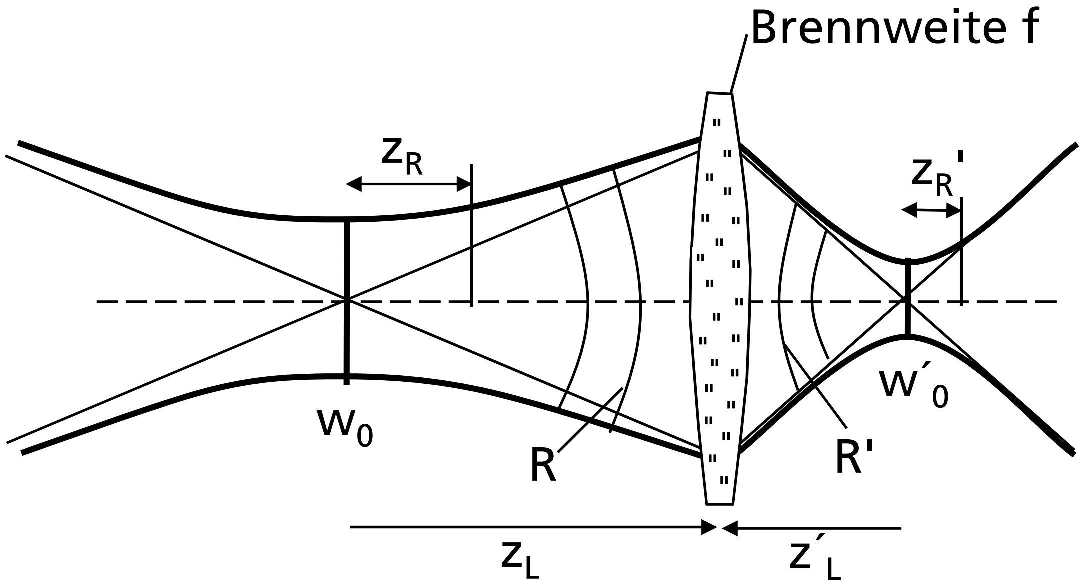
Transformation of a Gaussian beam with a paraxial lens

$$K = \frac{1}{R(z_L)} \qquad K' = \frac{1}{R'(z_L')}$$

$$K - K' = \frac{1}{f}$$

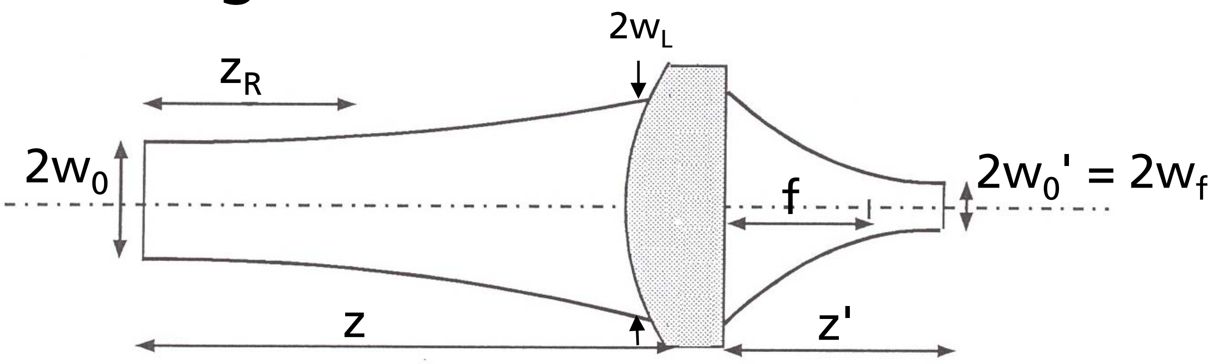
(paraxial lens equation)



a Gaussion beam (w_0, z_R, R) is transformed with a paraxial lens into an other Gaussion beam (w_0', z_R', R'):

… by transformation of the phase-front curvature K according to the paraxial lens fromula

Focussing of a Gaussian beam

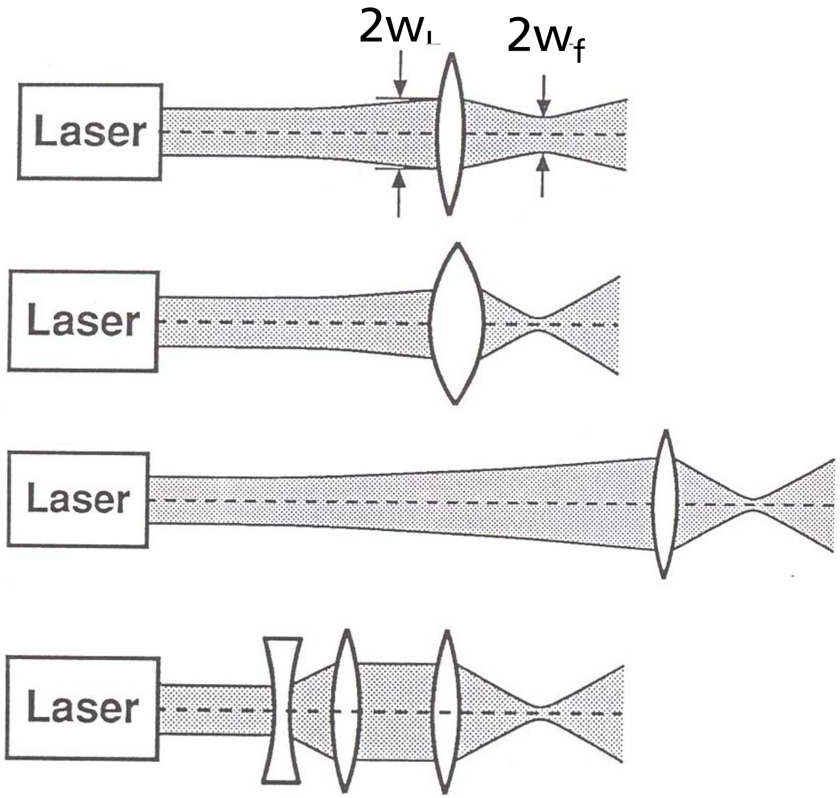


$$w_L = w_0 \sqrt{1 + (z/z_R)^2} \qquad z_R = \frac{\pi w_0^2}{\lambda} \qquad z >> f$$

$$w_{f,Gauss} = \frac{2}{\pi} \lambda \cdot \frac{f}{2 w_L} \qquad w_f^{Min} \approx \lambda$$

$$w_{f,Gauss} = 0{,}64 \cdot \lambda \cdot f / 2 w_L \qquad w_{f,Airy} = 1{,}22 \cdot \lambda \cdot f / D$$

Focussing of a Gaussain beam



small beam diameter at lens
large focal length
⇒ large focal radius

small focal length
⇒ small focal radius

large beam diameter at lens due to large object distance
⇒ small focal radius

large beam diameter at lens due to lens system
⇒ small focal radius

$$w_f = \frac{2}{\pi}\lambda \cdot \frac{f}{2w_L}$$

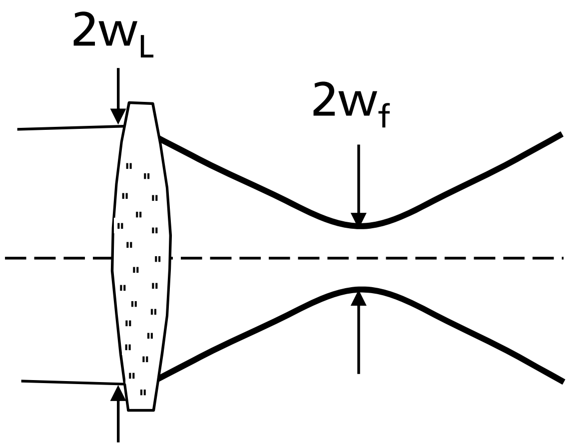
w_L beam radius at focal lens
w_f focal radius

Combination of beam- and diffraction optics

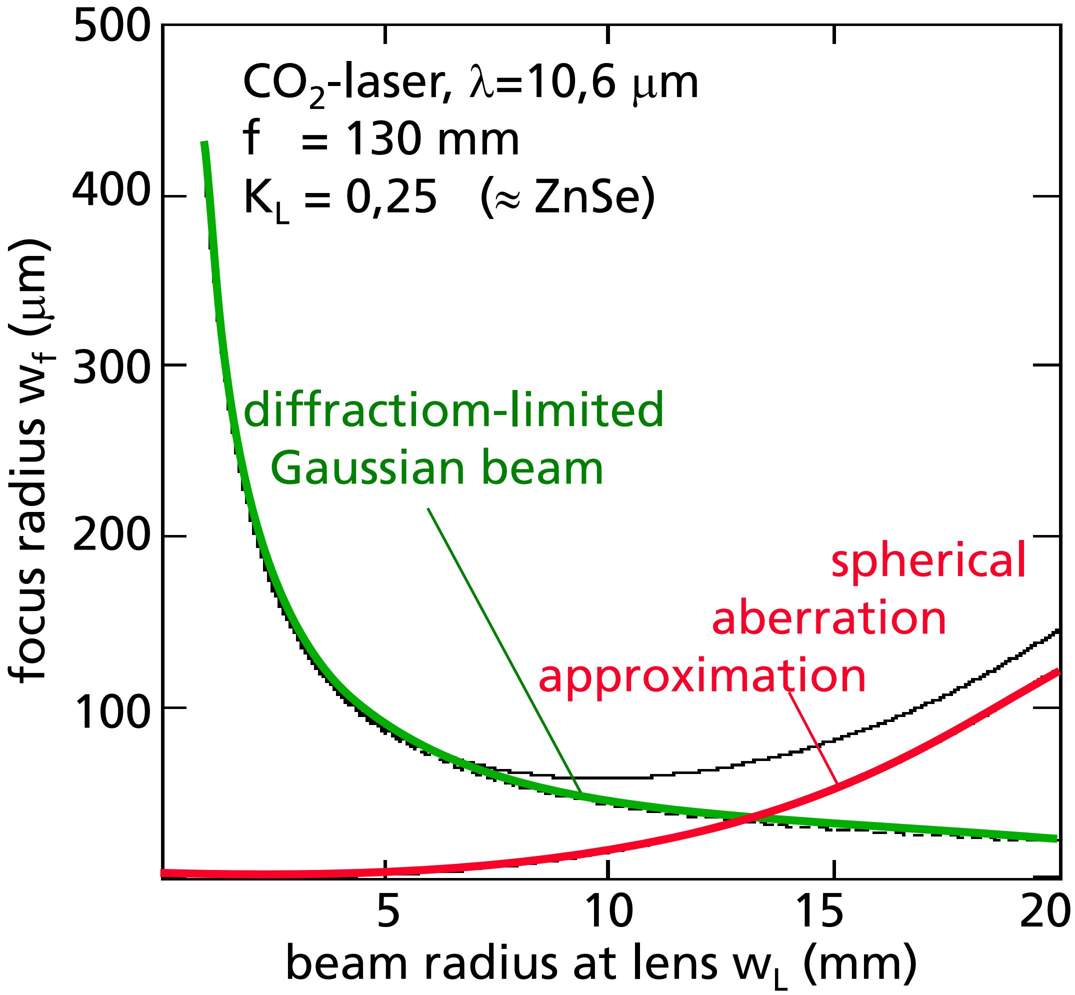
Mathematical complexity is increased from 1. – 4.

1. pure ray-optical description of the optical system and optimization according to the Rayleigh-criterion:
   $PV(OPD) < \lambda/4$
2. separate ray-optical calculation of the spherical aberration Δy and diffractive optical calculation of the radius of the Gaussian beam w_0; „total beam“ diameter: $R = \Delta y + w_0$
3. ray-optical calculation of the OPD of the optical system, transformation into the exit pupil; diffraction of this amplitued and phase distribution at the exit pupil, calculation of the focal spot
4. calculation of the full passage of the beam through the optical system by means of the diffraction integral: diffraction between two diffracting/reflecting surfaces, phase-front transformation at the surfaces

Method #2: Gaussian beam with spherical aberration

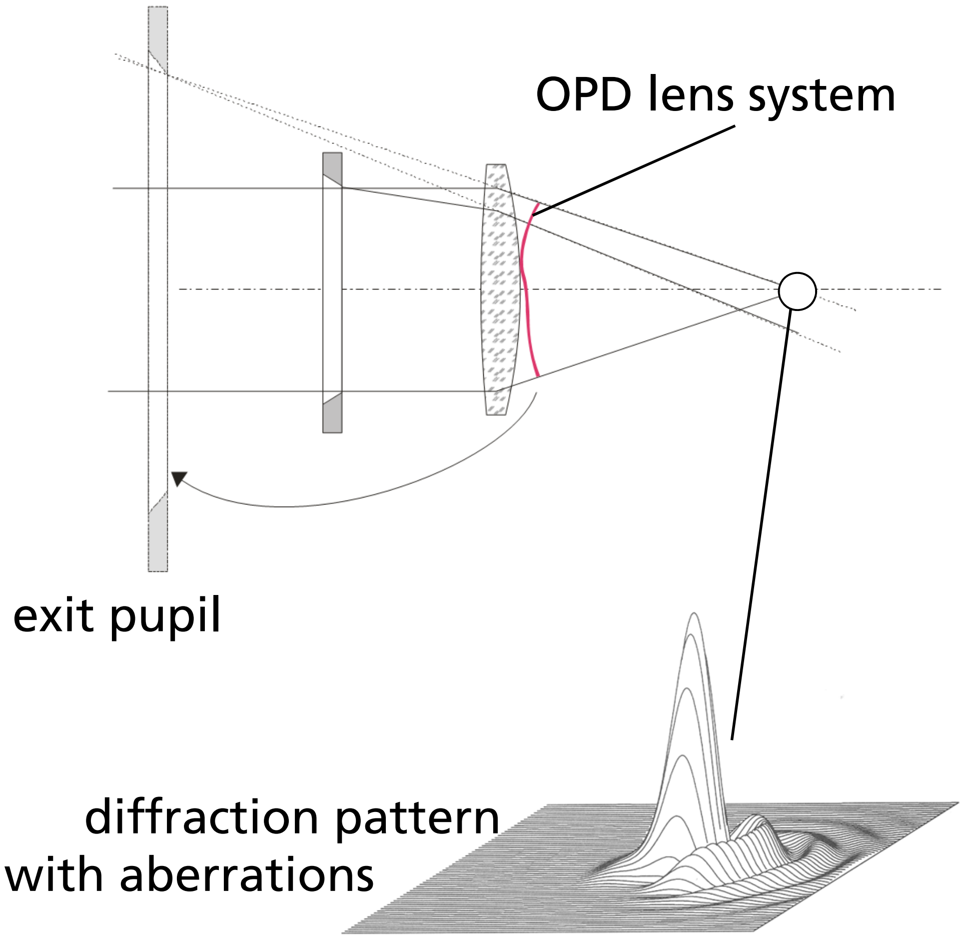


$$w_f = \frac{\lambda \cdot f}{\pi \cdot w_L} + K_L \cdot \frac{w_L^3}{f^2}$$



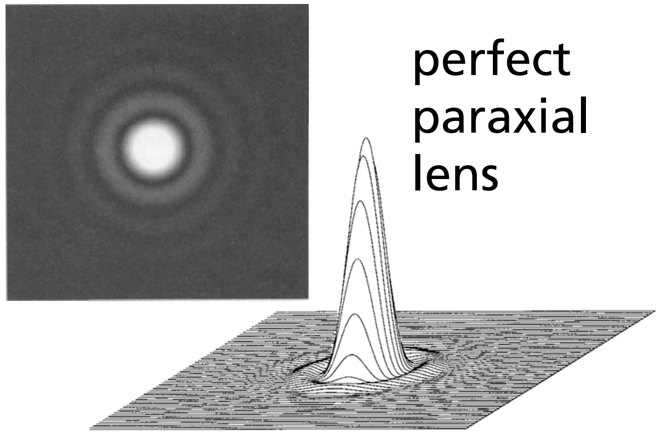
Method #3: diffraction with ray-optical OPD

- ray-optical calculation of the OPD of the full optical system at the exit (red)
- transformation into the exit pupil
- diffraction of the intensity-/phase-distribution at the exit pupil
- calculation of the intensity-/phase-distribution in the focal plane



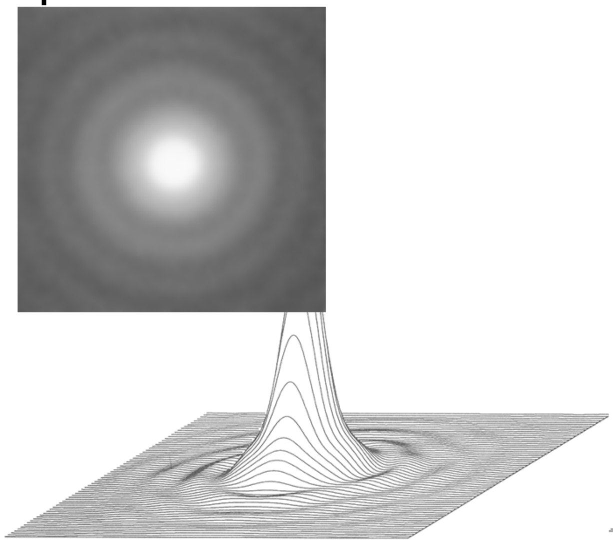
Airy-disk with aberrations

- lens with aberrations causes enlargement of the airy-disc
- below $PV_{OPD} < \lambda/4$ (Rayleigh-criterion) the airy-disc is nearly unchanged: „aberration-free imaging"

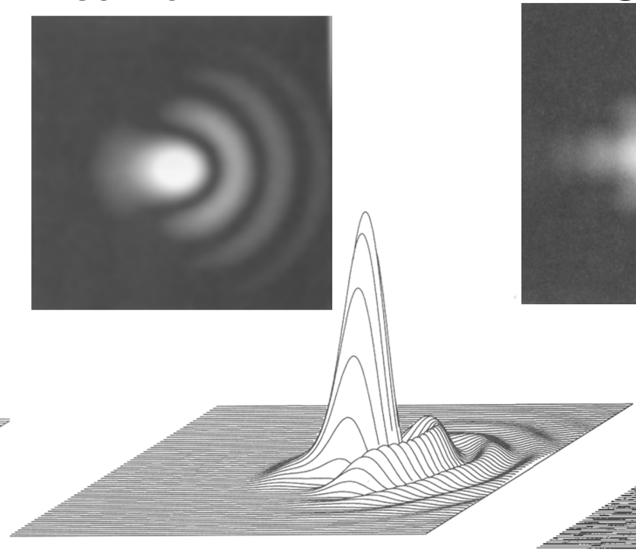


perfect paraxial lens

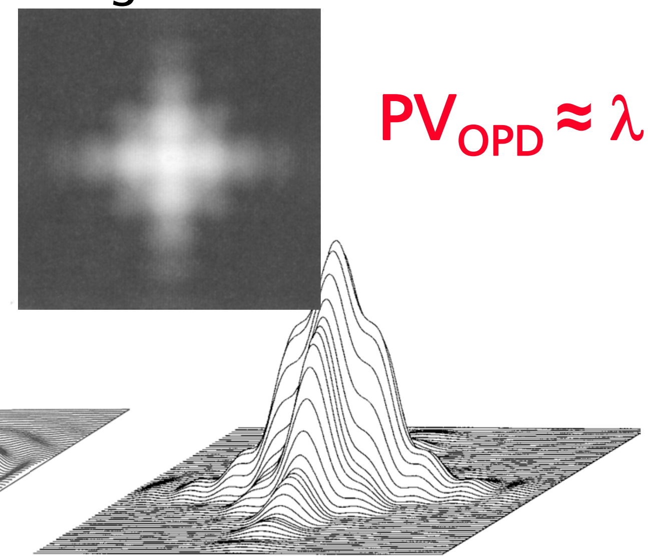
+ spherical aberration



+ coma



+ astigmatism



$PV_{OPD} \approx \lambda$

TOS RWTH
RHEINISCH-WESTFÄLISCHE TECHNISCHE HOCHSCHULE AACHEN
LEHRSTUHL FÜR TECHNOLOGIE OPTISCHER SYSTEME