

# The World of Quantum Matter

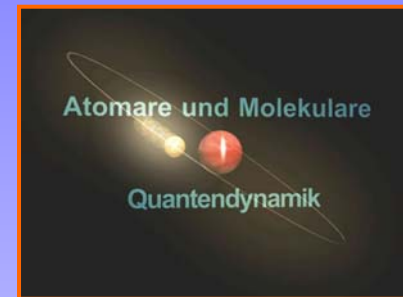


ALBERT-LUDWIGS-  
UNIVERSITÄT FREIBURG

Atomic and Molecular Quantum Dynamics

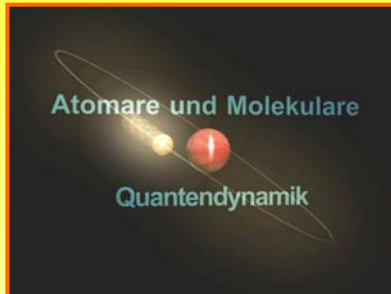
**founded 1457**

Matthias Weidemüller  
*Albert-Ludwigs-Universität Freiburg*



€€€ DFG (SPP 1116)  
EU (TMR Network „Cold Molecules“)  
Landesstiftung BW  
(Quanteninformation/Eliteförderung)

<http://quantendynamik.physik.uni-freiburg.de>



Matthias Weidemüller

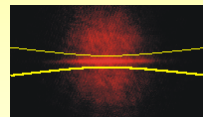
Roland Wester

Helga Müller (Sokr.)  
Ulrich Person (Ing.)  
Hartmut Götz (Ing.)



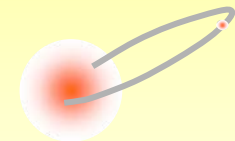
➤ Mixtures of ultracold atoms and cold molecules

Stephan Kraft (*Doct*)  
Jörg Lange (*Doct*)  
Peter Staantum (*PostDoc*)  
Benjamin Müller (*Dipl*)  
Christian Giese (*Dipl*)



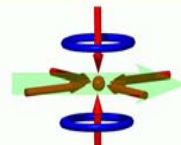
➤ Ultracold Rydberg gases and plasmas

Markus Reetz-Lamour (*Dokt*)  
Thomas Amthor (*Doct*)  
Johannes Deiglmayr (*Dipl*)  
Sebastian Westermann (*Dipl*)  
André de Oliveira (*GuestSci*)



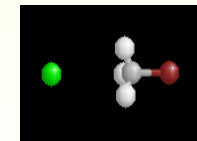
➤ Cold atom targets /coherent control with femtosecond pulses

Wenzel Salzmann (*Doct*)  
Ulrich Poschinger (*Dipl*)  
Judith Eng (*Dipl*)



➤ Quantum dynamics of ion-molecule reactions

Roland Wester (*PI*)  
Jochen Mikosch (*Doct*)  
Sebastian Trippel (*Doct*)  
Raphael Berhane (*Dipl*)



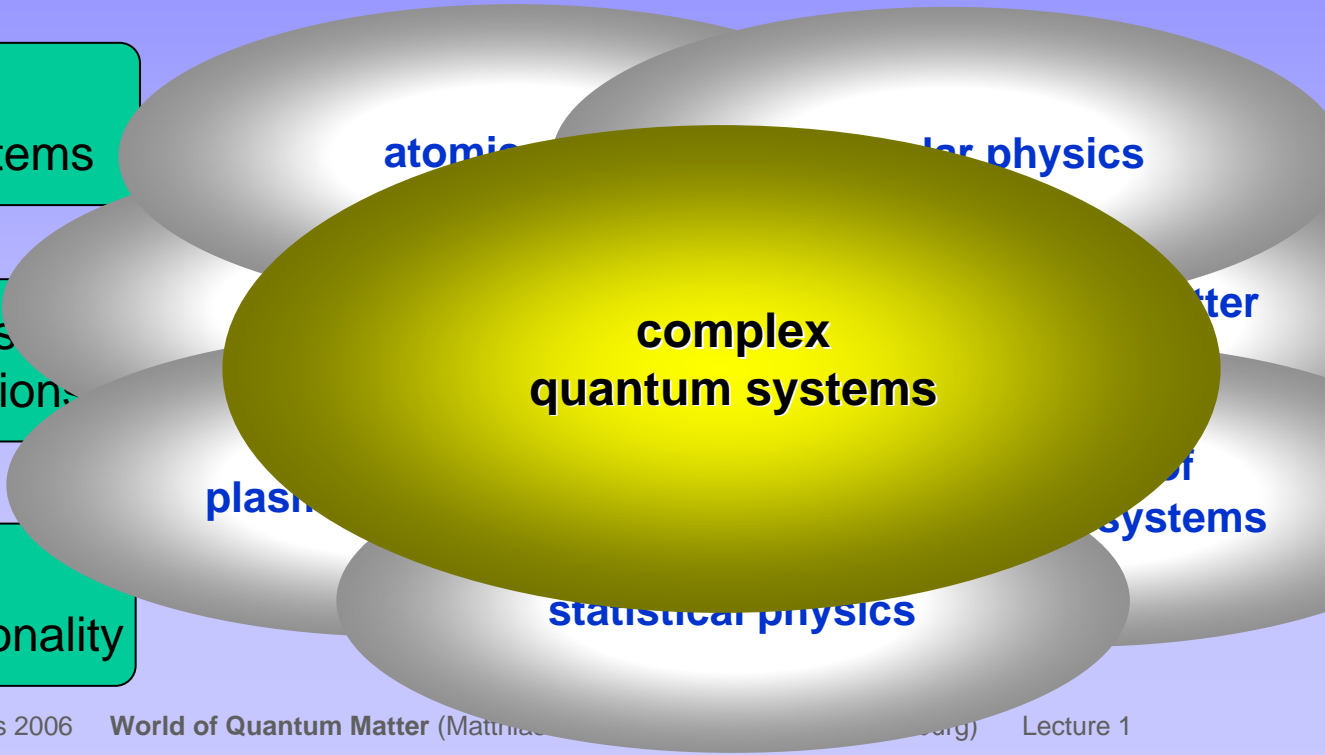
# The new era of quantum mechanics

Quantum Physics  
is undergoing the transition from  
the *Analytical Era*  
to  
the *Synthetic Era*

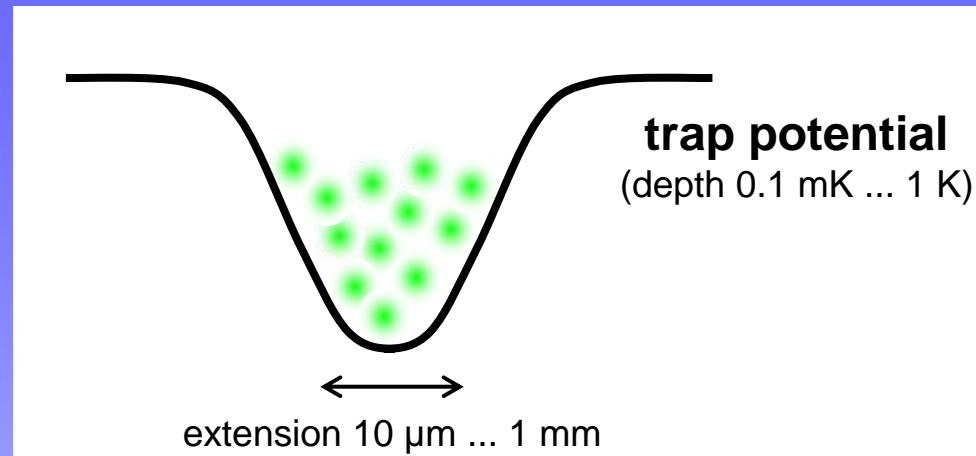
coherent control  
of simple quantum systems

many-body systems  
with controlled interactions

control of  
geometries and dimensionality



# Ultracold atoms in traps

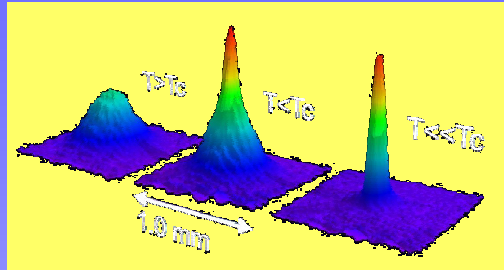


- **Negligible thermal energy** ( $k_B T \sim \text{neV}$ )  
dynamics determined by interactions, strong correlations
- **Large deBroglie wavelength** ( $\lambda_{\text{dB}} \sim \text{mm}$ )  
quantum degeneracy, scattering resonances
- **Very long observation times** ( $t_{\text{obs}} \sim \text{min}$ )  
ultrahigh resolution, observable effects of weak interactions

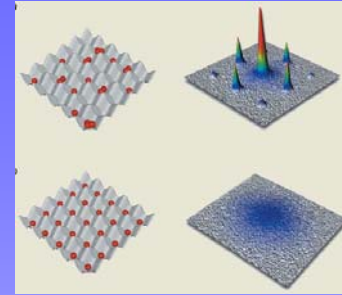
**FULL CONTROL over  
INTERNAL and EXTERNAL degrees of freedom**

# Systems under investigation (selection)

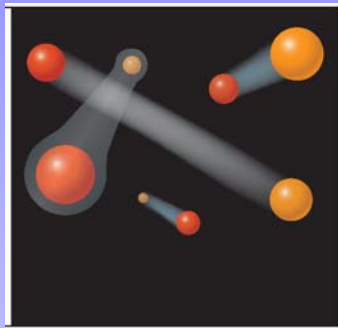
**Bose-Einstein condensates**



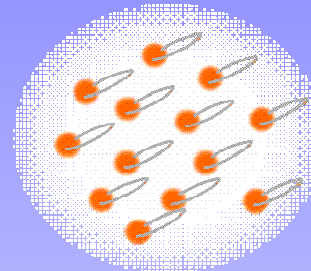
**ultracold gases in optical lattices**



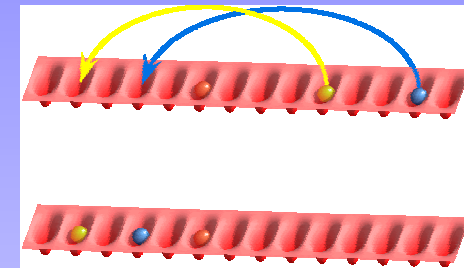
**quantum-degenerate Fermi gases**



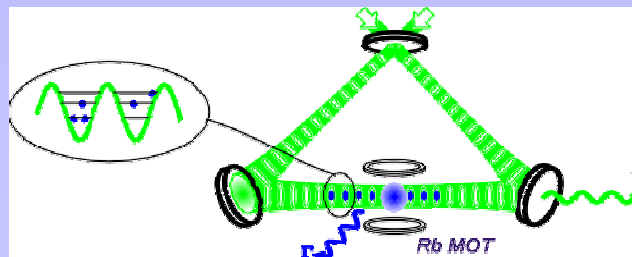
**Rydberg gases**



**single atoms**



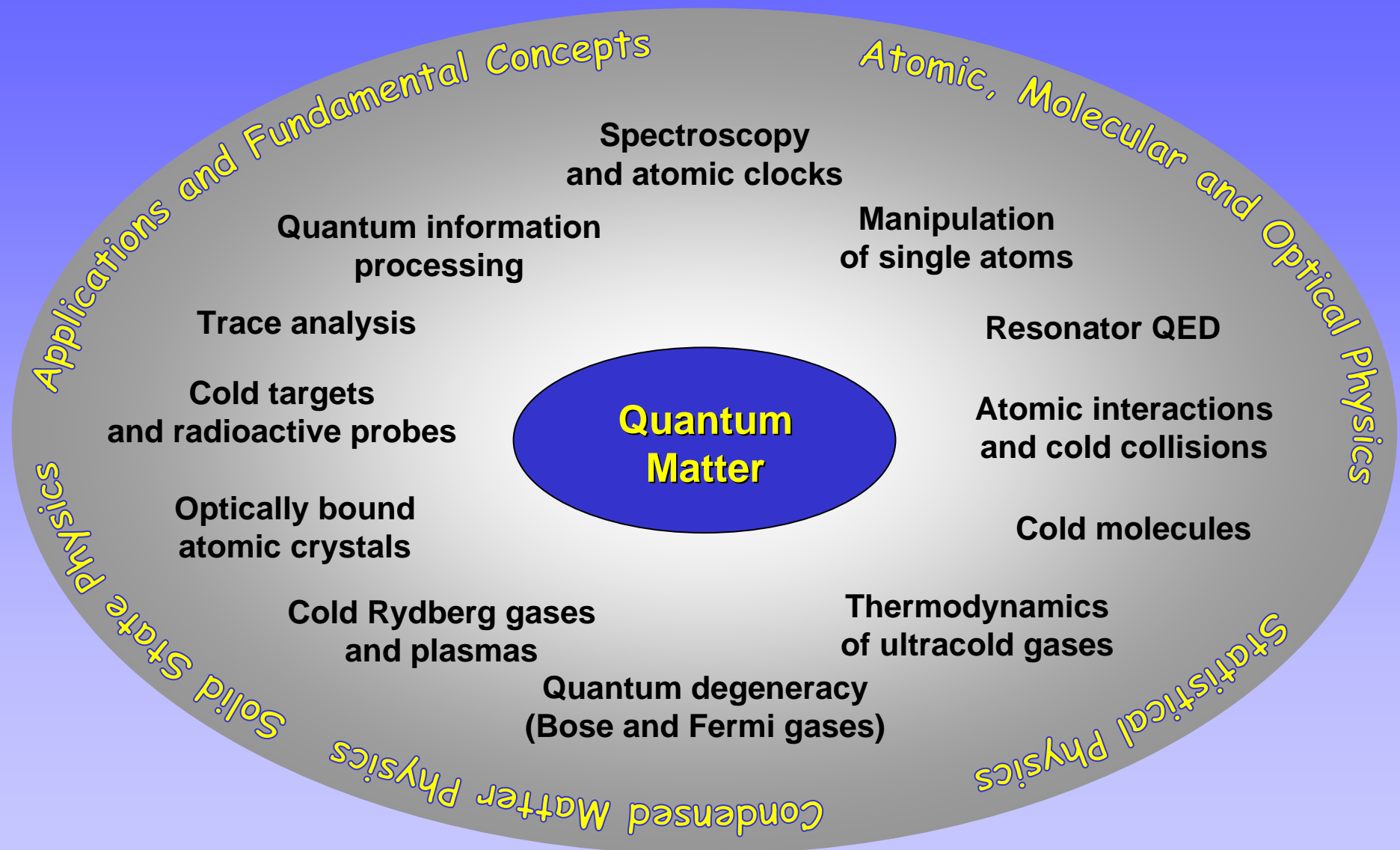
**atoms in high-finesse resonators**



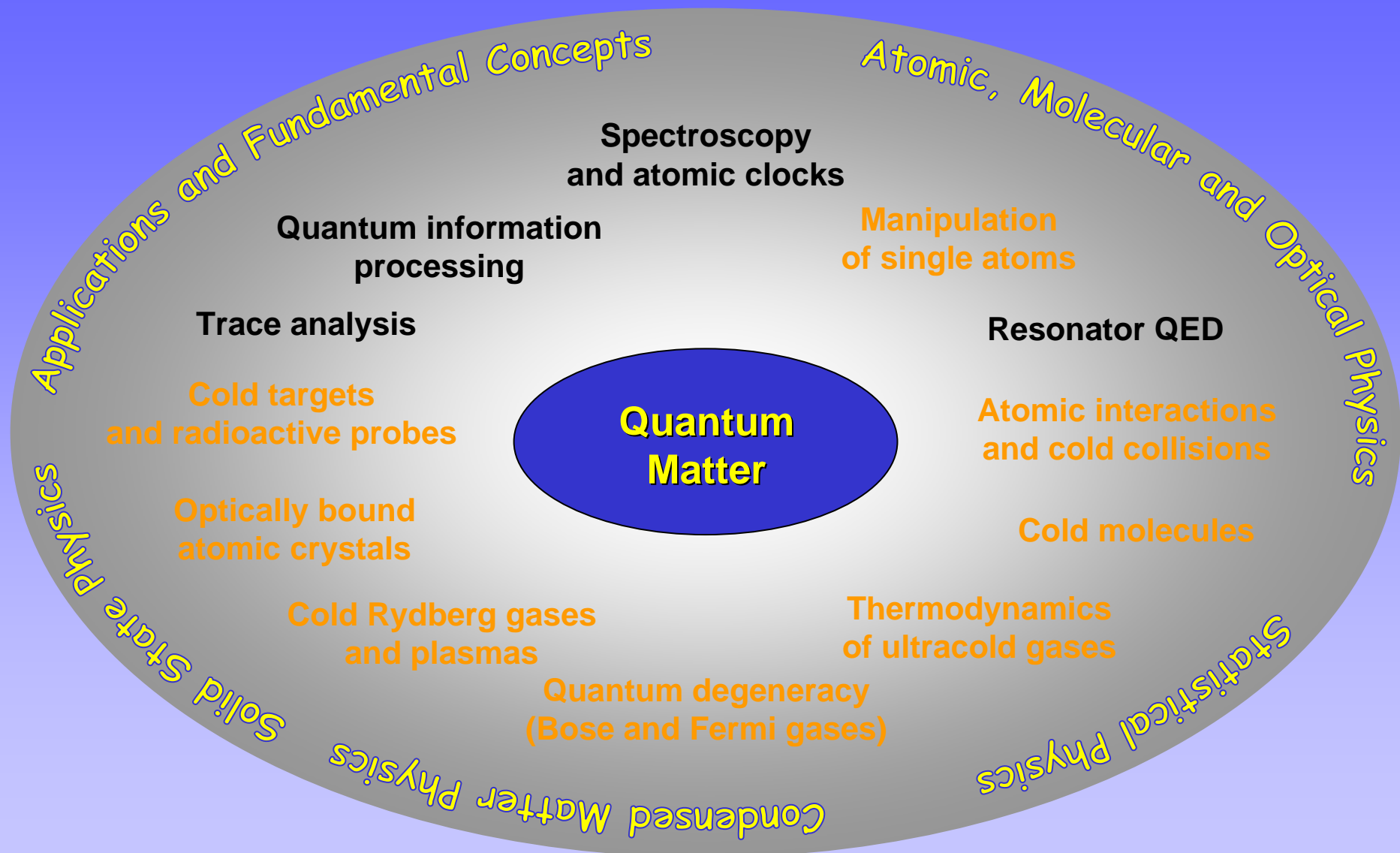
**trapped ions**



# The World of Quantum Matter



# The World of Quantum Matter



# Contents of the lectures

0. Primer on light-matter interactions
1. The way to absolute zero – cooling and trapping methods for atoms **Lecture 1**
2. Cold collisions
3. Bose-Einstein condensation **Lecture 2**
4. Degenerate Fermi gases
5. Cold Rydberg gases and plasmas **Lecture 3**
6. Ultracold molecules
7. Manipulation of single atoms **Lecture 4**
8. Cold atoms as targets for photon and particle beams



# Contents of the lectures

## 0. **Primer on light-matter interactions**

1. The way to absolute zero –  
cooling and trapping methods for atoms

**Lecture 1**

2. Cold collisions

3. Bose-Einstein condensation

**Lecture 2**

4. Degenerate Fermi gases

5. Cold Rydberg gases and plasmas

**Lecture 3**

6. Ultracold molecules

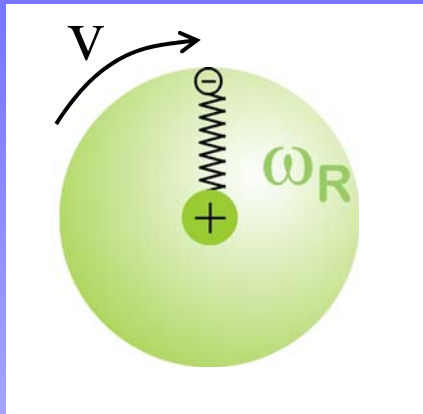
7. Manipulation of single atoms

**Lecture 4**

8. Cold atoms as targets for photon and particle beams

# Different look at light-matter interactions

## Lorentz atom



- magnetic dipole moment through ring current
- no electric dipole moment through symmetry

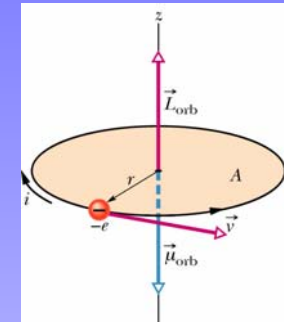
## Interaction with external fields

### magnetic fields

$$E_{\text{mag}} = -\vec{\mu} \cdot \vec{B}$$

### magnetic moment

$$\vec{\mu} = -\frac{e}{2m_e} \vec{L}_{\text{orbit}}$$



### electric fields

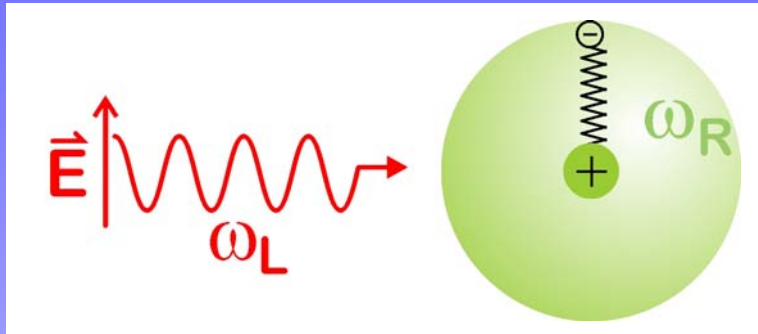
$$E_{\text{el}} = -\vec{\phi}_{\text{ind}} \cdot \vec{\mathcal{E}}$$

### induced dipole moment

$$\vec{\phi}_{\text{ind}} = \underline{\underline{\alpha}} \vec{\mathcal{E}}$$

# Different look at light-matter interactions

## Electric polarizability



### Equation of motion

$$\ddot{\varphi} + \Gamma_{\omega} \dot{\varphi} + \omega_0 \varphi = -\frac{e^2}{m_e} \mathcal{E}(t)$$

classical damping rate

$$\Gamma_{\omega} = \frac{e^2 \omega^2}{6\pi\epsilon_0 m_e c^3}$$

### Polarizability

$$\alpha = 6\pi\epsilon_0 c^3 \frac{\Gamma/\omega_0^2}{\omega_0^2 - \omega^2 - i(\omega^3/\omega_0^2)\Gamma}$$

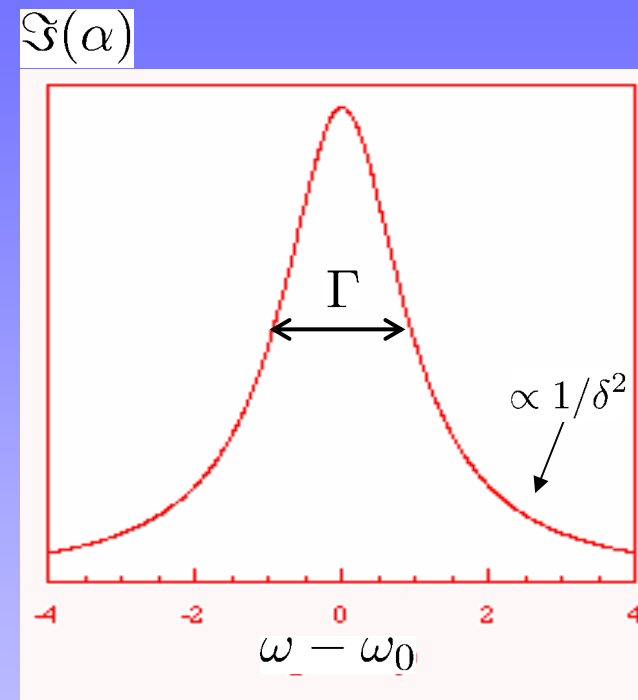
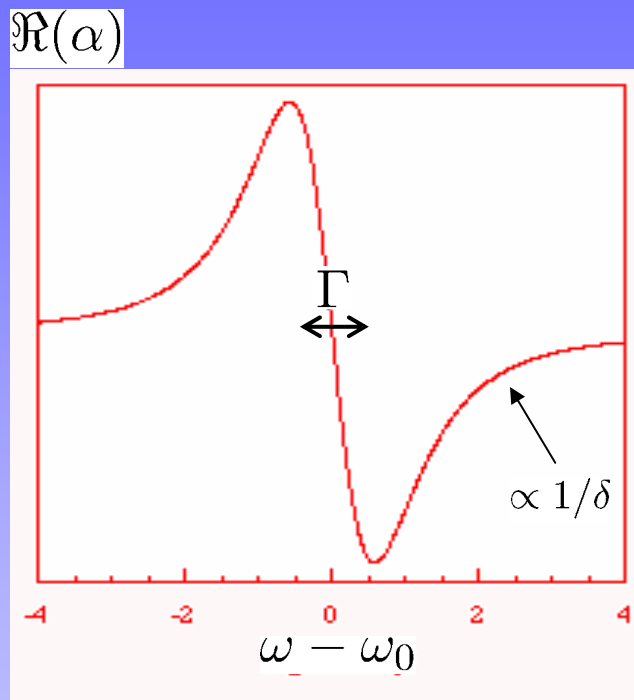
with  $\Gamma \equiv \Gamma_{\omega_0}$

### Susceptibility

$$\chi_{el} = \mathcal{N} \alpha / \epsilon_0$$

# Different look at light-matter interactions

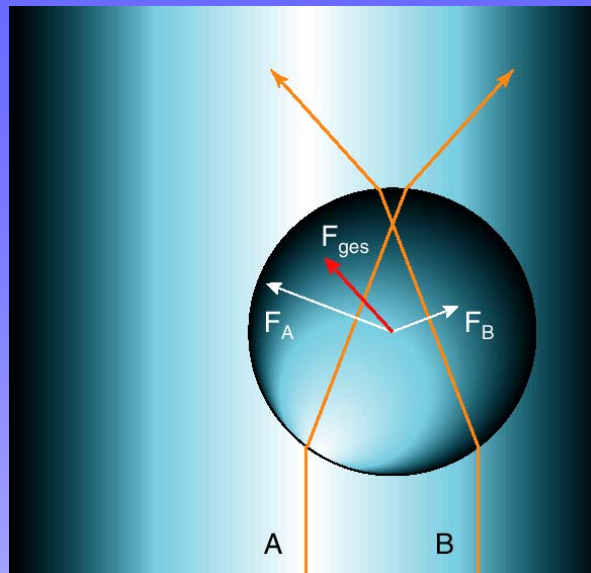
## Electric polarizability (cont'd)



## Refractive index (small $\chi$ )

$$n \simeq 1 + \frac{1}{2}\chi_{\text{el}} = 1 + \frac{1}{2}\mathcal{N}\alpha/\epsilon_0$$

# Different look at light-matter interactions



## Dipole force

Interaction potential

$$U_{\text{dip}} = -\frac{1}{2} \langle \vec{\phi}_{\text{ind}} \cdot \vec{\mathcal{E}} \rangle = -\frac{1}{2\epsilon_0 c} \Re(\alpha) I$$

Dipole force

$$\vec{F}_{\text{dip}} = -\vec{\nabla} U_{\text{dip}} = \frac{1}{2\epsilon_0 c} \Re(\alpha) \vec{\nabla} I$$

## Scattering force (light pressure)

Scattering rate

$$\Gamma_{\text{sc}} = P_{\text{abs}} / \hbar\omega = -\frac{1}{\epsilon_0 \hbar c} \Im(\alpha) I$$

Scattering force

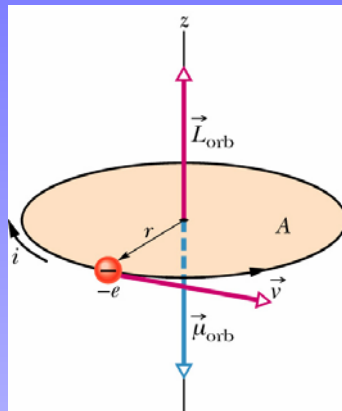
$$\vec{F}_{\text{sc}} = \hbar \vec{k} \Gamma_{\text{sc}} = \frac{1}{\epsilon_0 c} \Im(\alpha) I \vec{k}$$

# Different look at atom-field interactions

## Why does the classical picture work so well?

### Magnetic dipole moment

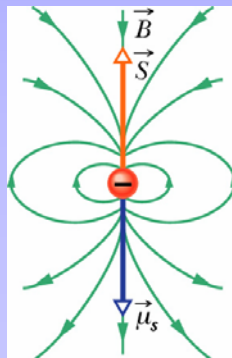
#### Orbital momentum



$$\vec{\mu}_{\text{Bahn}} = -\frac{e}{2m_e} \vec{L}_{\text{Bahn}}$$

$$|\vec{\mu}_{\text{Bahn}}| = -\mu_{\text{Bohr}} \frac{|\vec{L}|}{\hbar}$$

#### Electron spin



$$\vec{\mu}_S = -\frac{e}{m_e} \vec{S}$$

$$|\vec{\mu}_S| = \pm \frac{1}{2} \hbar \frac{e}{m_e} \equiv \pm \mu_{\text{Bohr}}$$

$$\mu_{\text{Bohr}} = 9.72 \times 10^{-24} \text{ J/T}$$

#### Current operator

$$\vec{J}(\vec{r}) = \frac{\hbar}{2\mu i} \psi^*(\vec{r}) \vec{\nabla} \psi(\vec{r}) + \text{c.c.}$$

For wavefunction  $n, l, m$ :

$$\vec{J}_{nlm}(\vec{r}) = \frac{\hbar}{\mu} m \frac{\rho_{nlm}(\vec{r})}{r \sin \vartheta} \vec{e}_\varphi(\vec{r})$$

$$\rho(\vec{r}) = |\psi(\vec{r})|^2$$

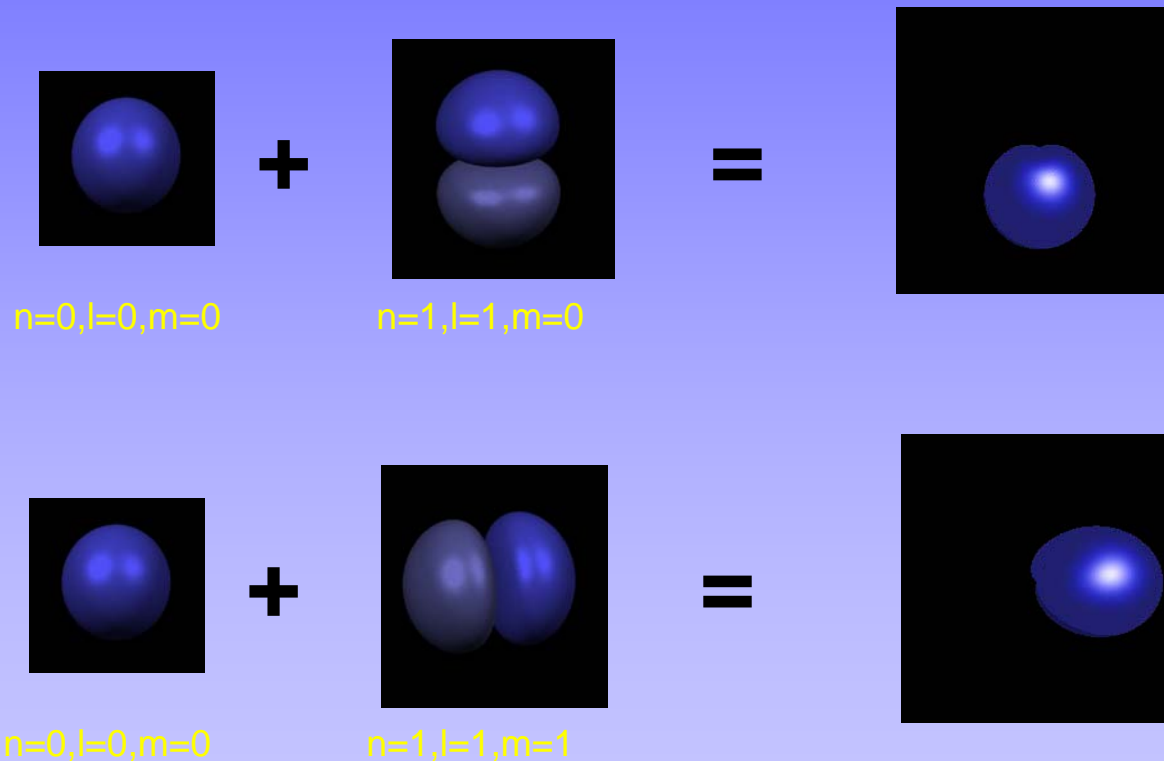
$$\langle \vec{\mu}_{\text{mag}} \rangle_{nlm} = -m \frac{e\hbar}{2\mu} \vec{e}_z$$

# Different look at light-matter interactions

## Why does the classical picture work so well?

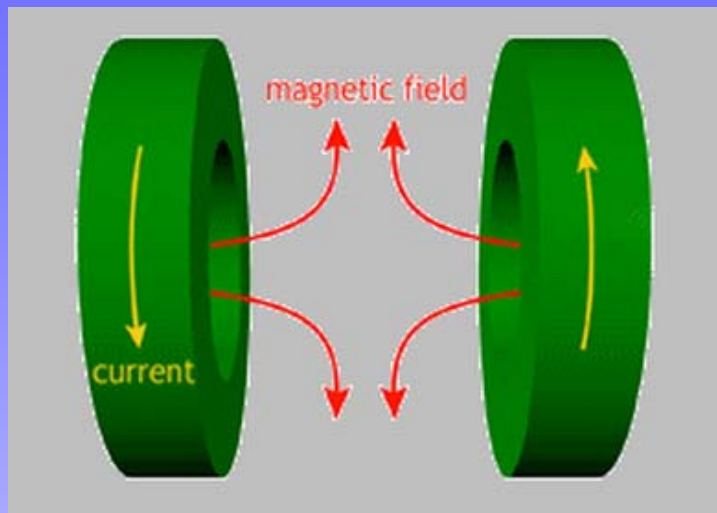
### Electric dipole moment

Superposition of eigenfunctions (ex. s- and p-orbitals)

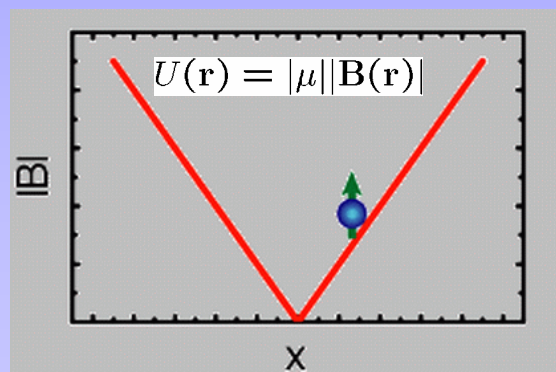


# The trap zoo: magnetic traps

## Quadrupole trap



### Potential



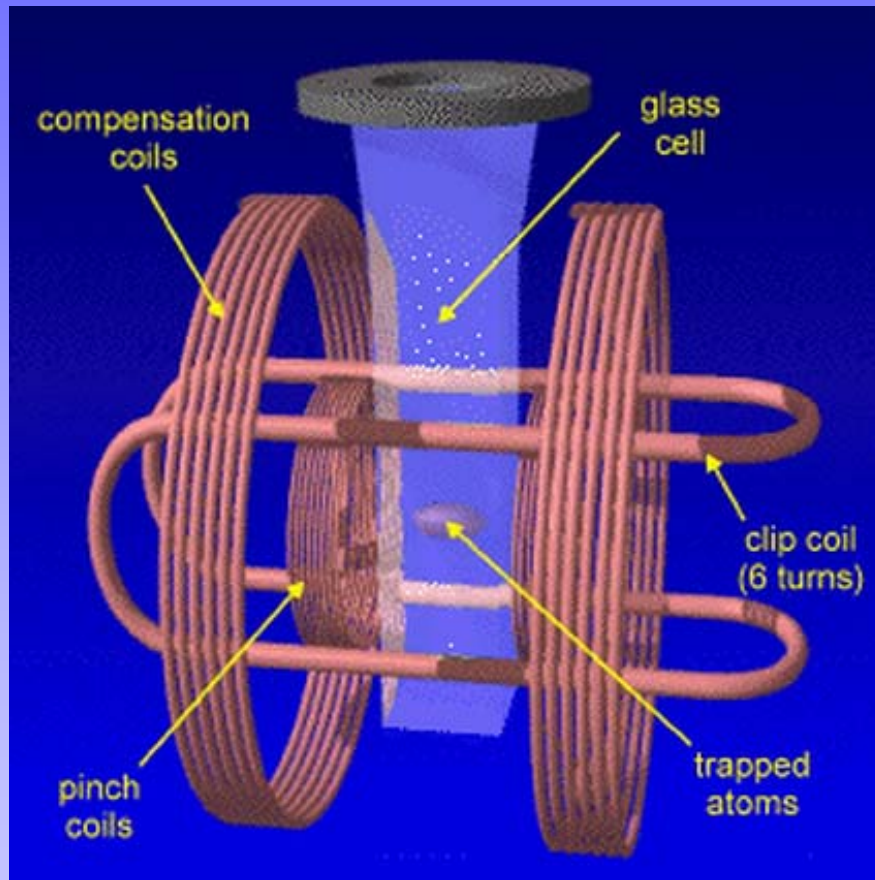
### Problem:

Spin flips close to zero field  
(Majorana flip)

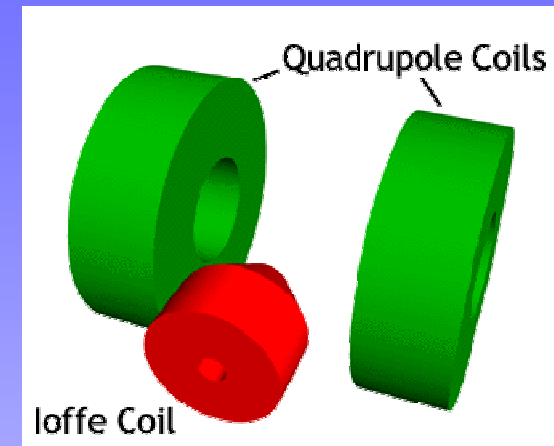


# The trap zoo: magnetic traps

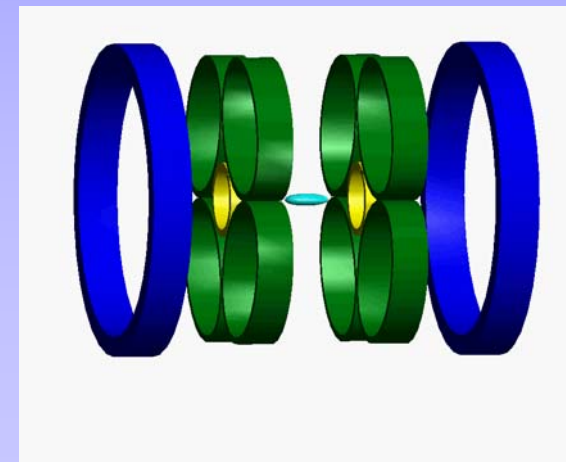
## Joffe-Pritchard trap



## QUadrupole Ioffe Configuration

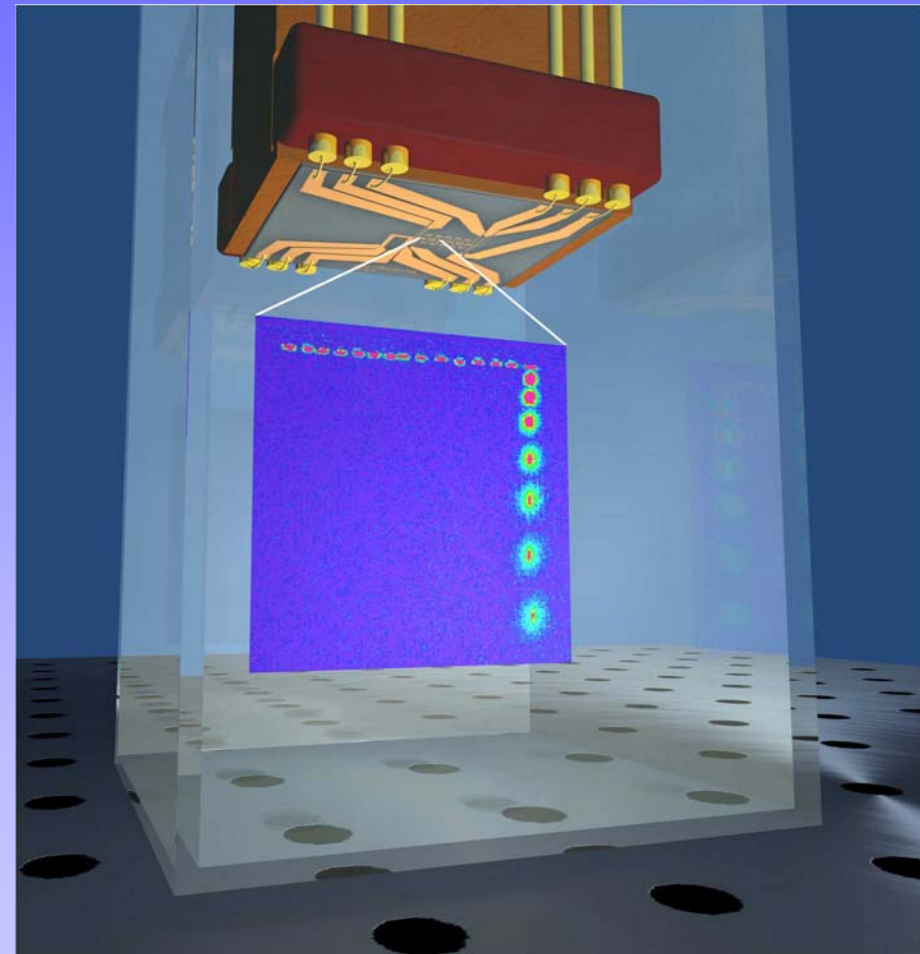
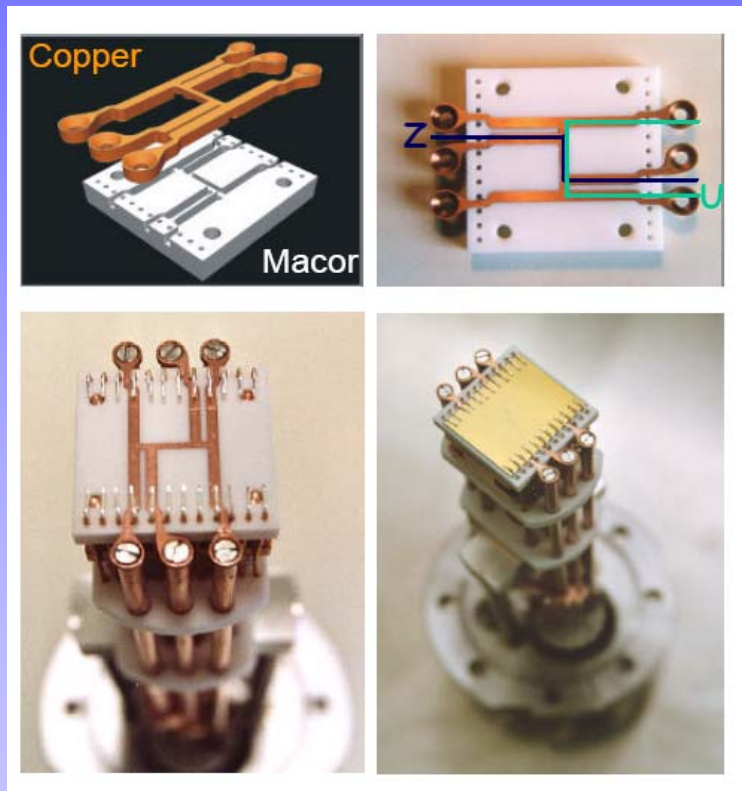


## Cloverleaf configuration



# The trap zoo: magnetic traps

## Chip traps

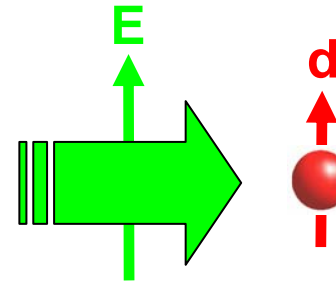


courtesy J. Reichel (ENS Paris)

# The trap zoo: optical dipole traps

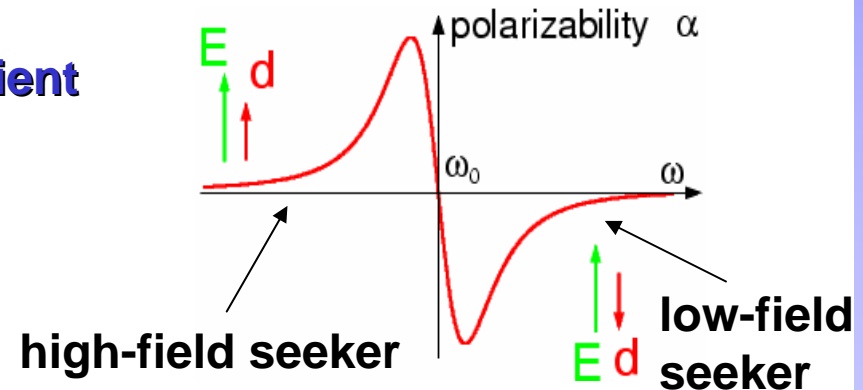
**Laser light** *far-detuned from resonance*  
induces an oscillating **atomic dipole**

$$\mathbf{d} = \underline{\underline{\alpha}} \mathbf{E}$$



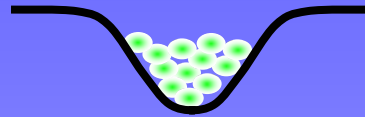
Dipole experiences force prop. to the  
**polarizability** and the **intensity gradient**

$$\mathbf{F}_{\text{dip}} = -\nabla(\mathbf{d} \cdot \mathbf{E})$$
$$\propto -\underline{\underline{\alpha}} \nabla I \equiv -\nabla U_{\text{dip}}$$

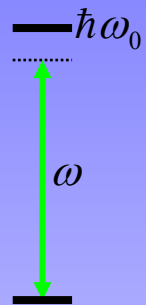


# The trap zoo: optical dipole traps

optical dipole force  
 $F_{\text{dip}} = -\nabla U_{\text{dip}}$   
 optical dipole potential

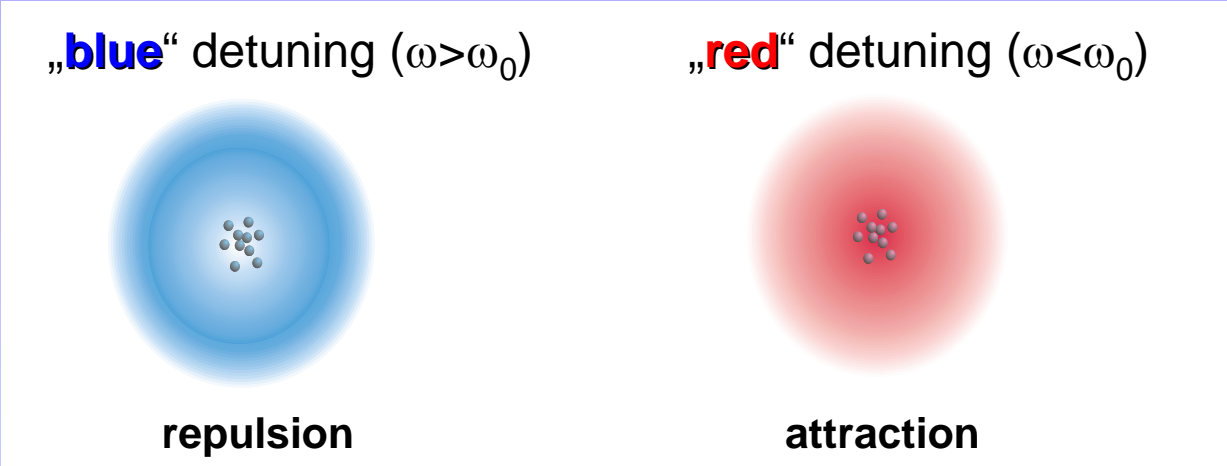


R. Grimm, M. Weidemüller, Yu.B. Ovchinnikov,  
 Adv. At. Mol. Opt. Phys. **42**, 95 (2000)



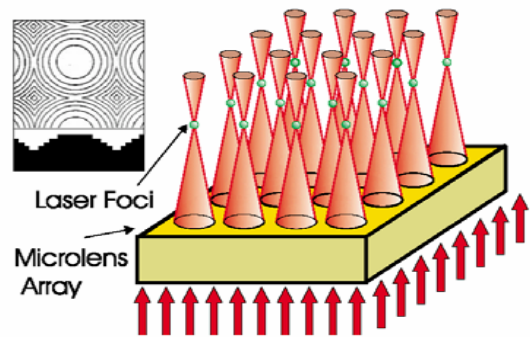
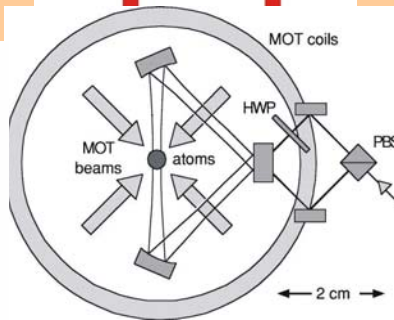
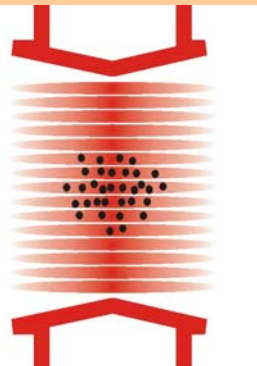
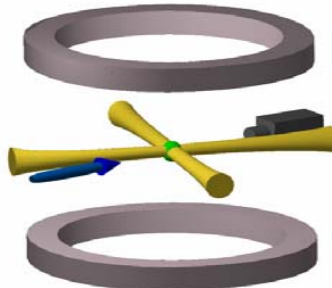
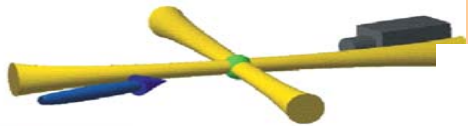
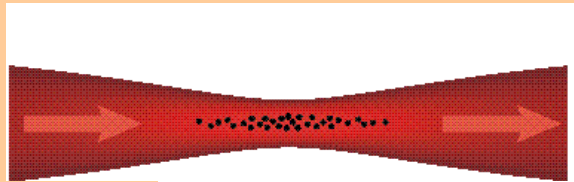
$$U_{\text{dip}}(\mathbf{r}) = -\frac{3\pi c^2}{2\omega_0^3} \left( \frac{\Gamma}{\omega_0 - \omega} + \frac{\Gamma}{\omega_0 + \omega} \right) I(\mathbf{r}) \quad \text{dipole potential}$$

$$\Gamma_{\text{sc}}(\mathbf{r}) = -\frac{3\pi c^2}{2\hbar\omega_0^3} \left( \frac{\omega}{\omega_0} \right)^3 \left( \frac{\Gamma}{\omega_0 - \omega} + \frac{\Gamma}{\omega_0 + \omega} \right)^2 I(\mathbf{r}) \quad \text{scattering rate}$$

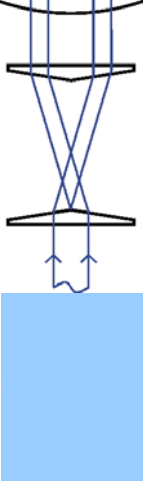
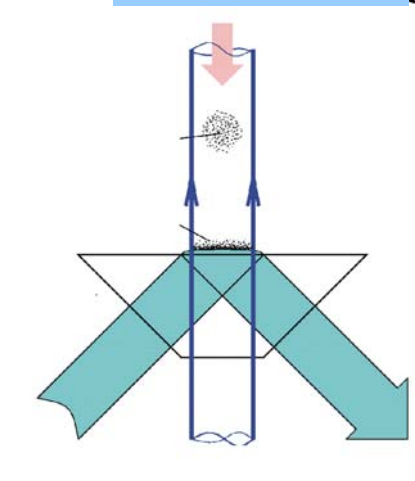
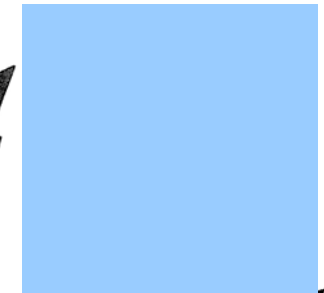
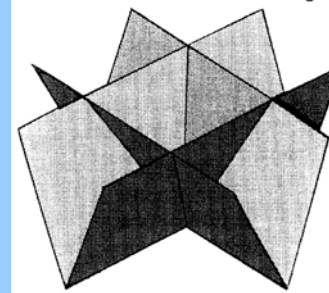
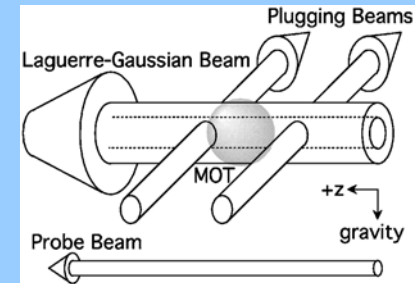


# The "zoo" of dipole traps

- Red-detuned dipole traps

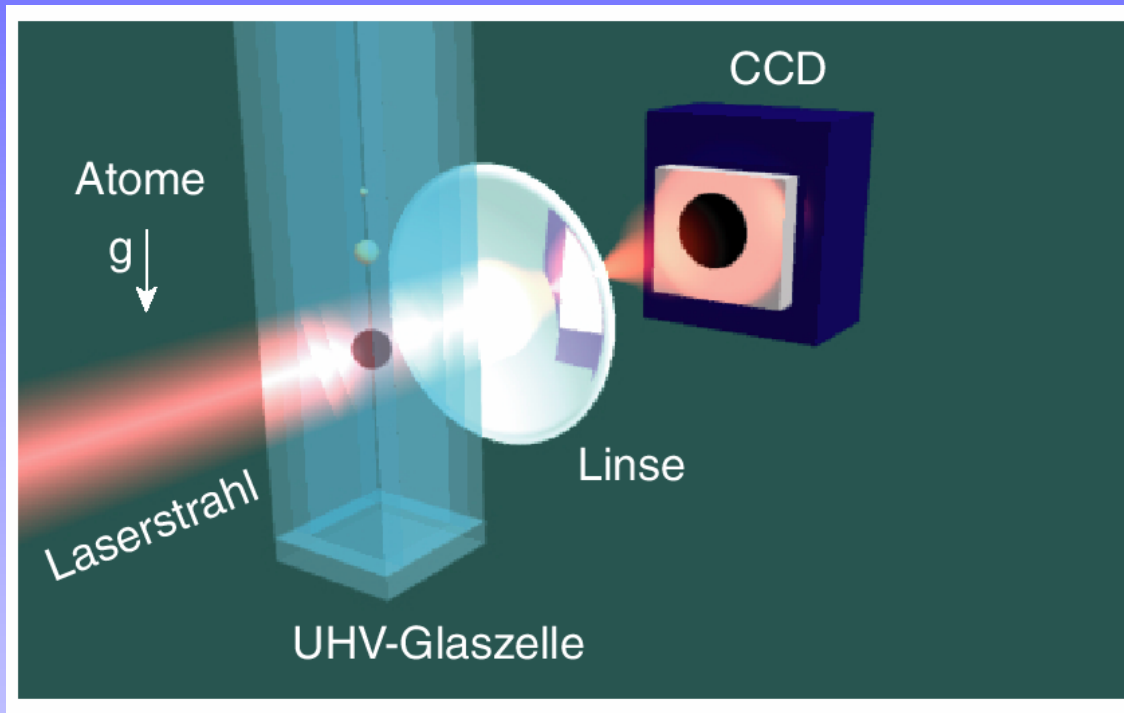


- Blue-detuned dipole traps

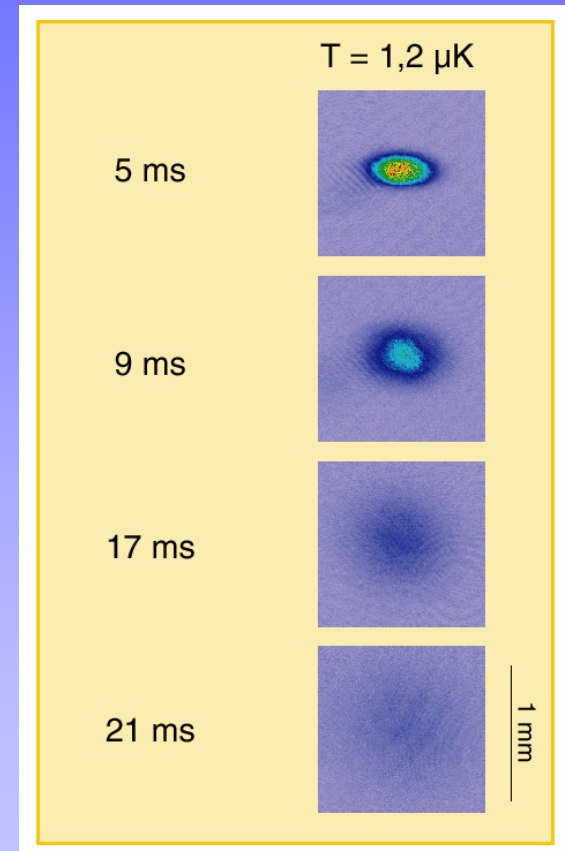


# Diagnositics

Absorption imaging of the atom cloud



CCD camera pictures

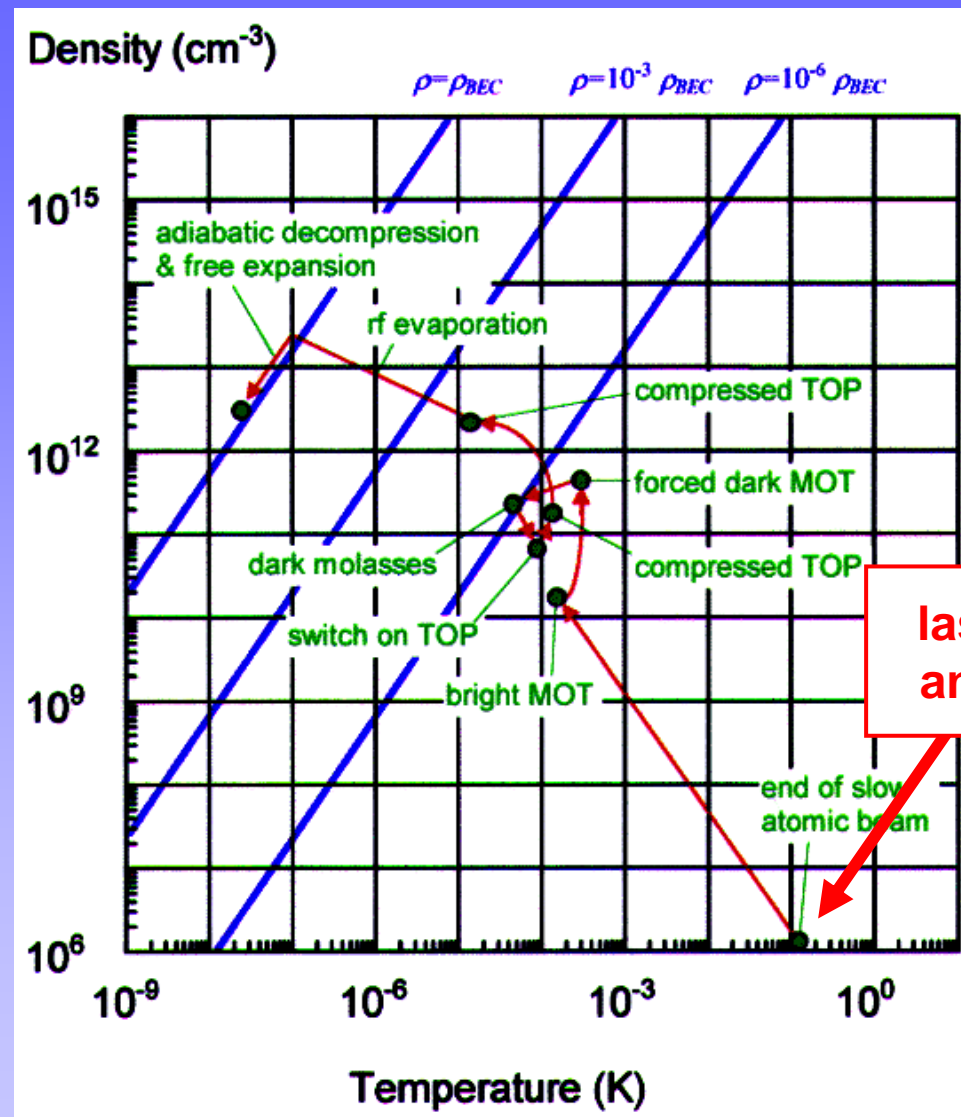


I. Bloch, T.W. Hänsch *et al.* @ Universität München

# Contents of the lectures

0. Primer on light-matter interactions
- 1. The way to absolute zero – cooling and trapping methods for atoms** **Lecture 1**
2. Cold collisions
3. Bose-Einstein condensation **Lecture 2**
4. Degenerate Fermi gases
5. Cold Rydberg gases and plasmas **Lecture 3**
6. Ultracold molecules
7. Manipulation of single atoms **Lecture 4**
8. Cold atoms as targets for photon and particle beams

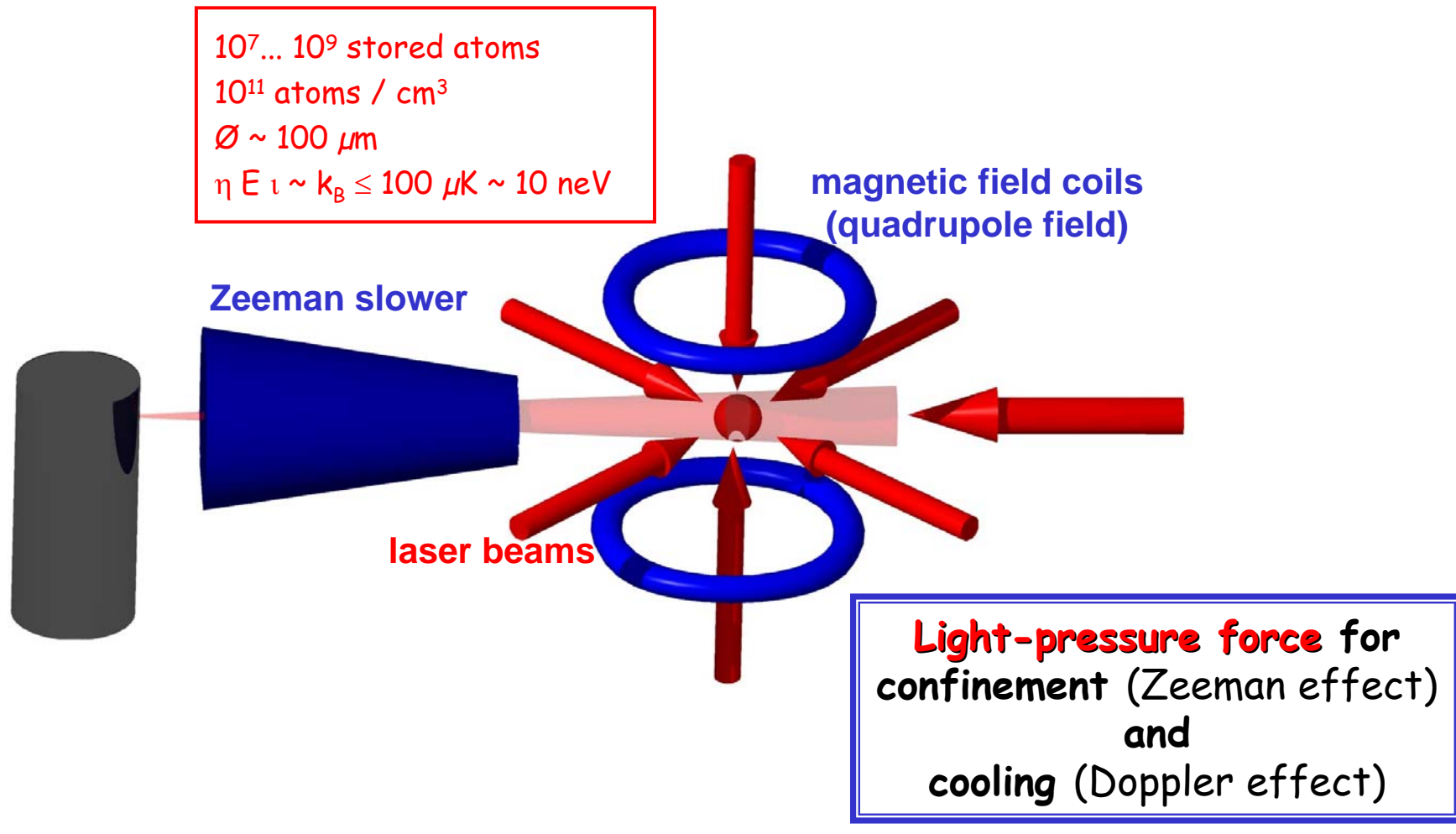
# The long, long road to the Ultracold



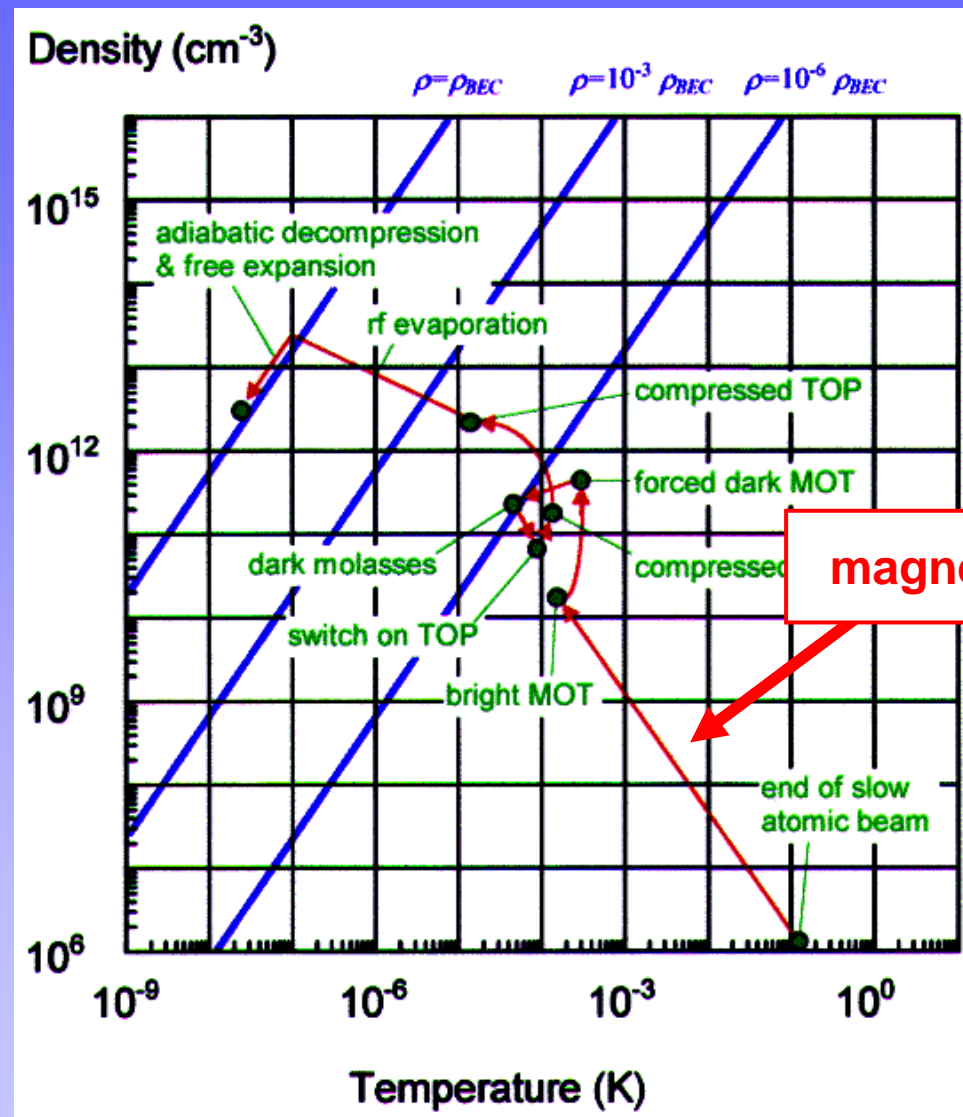
**laser cooling of  
an atomic beam**



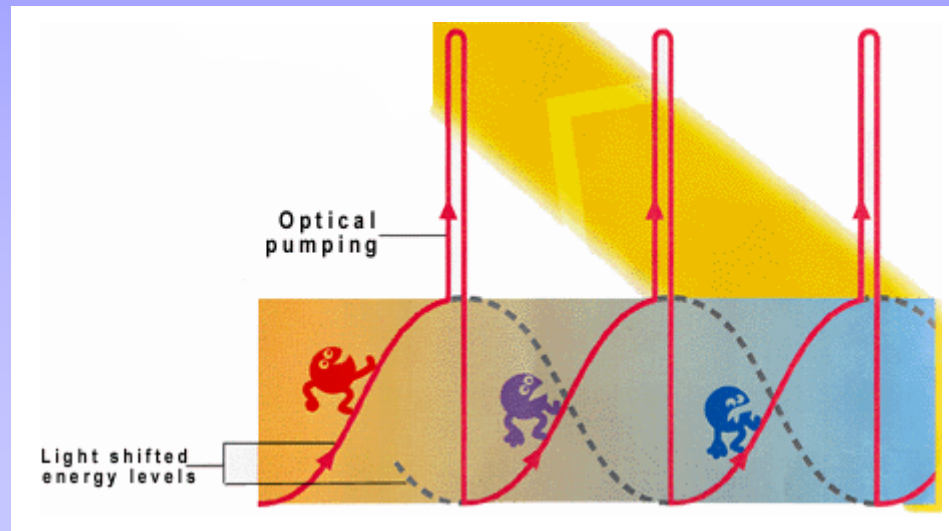
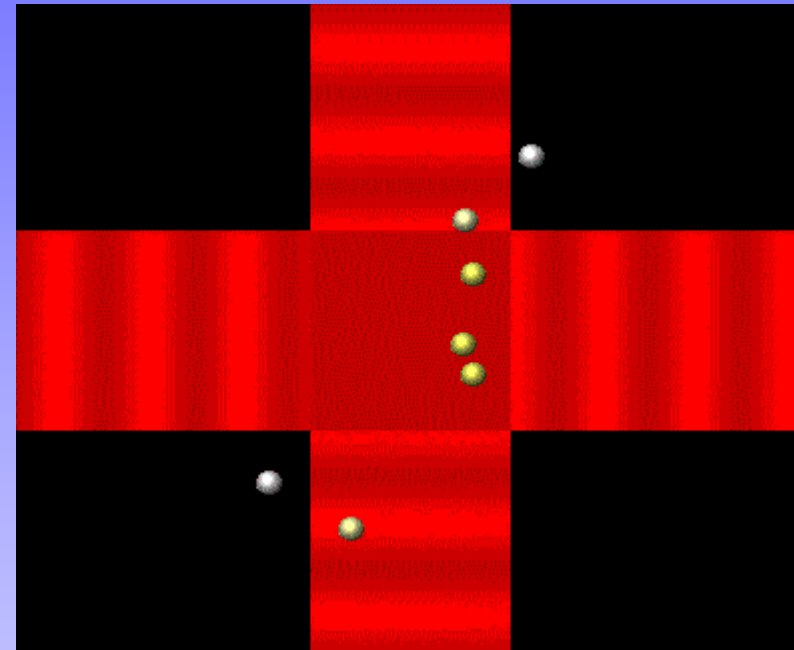
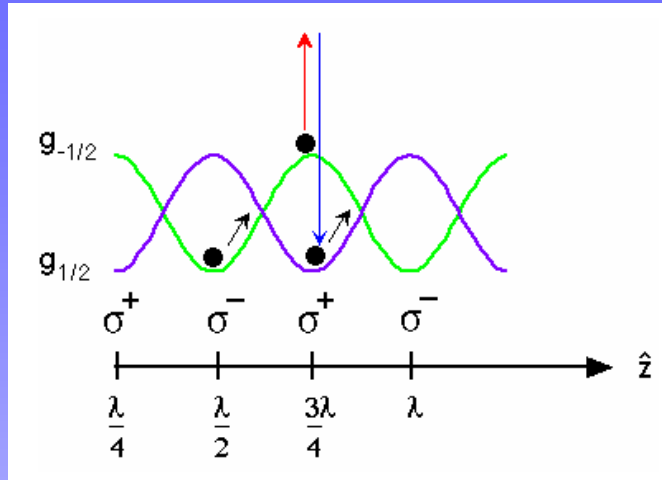
# Magneto-optical trap



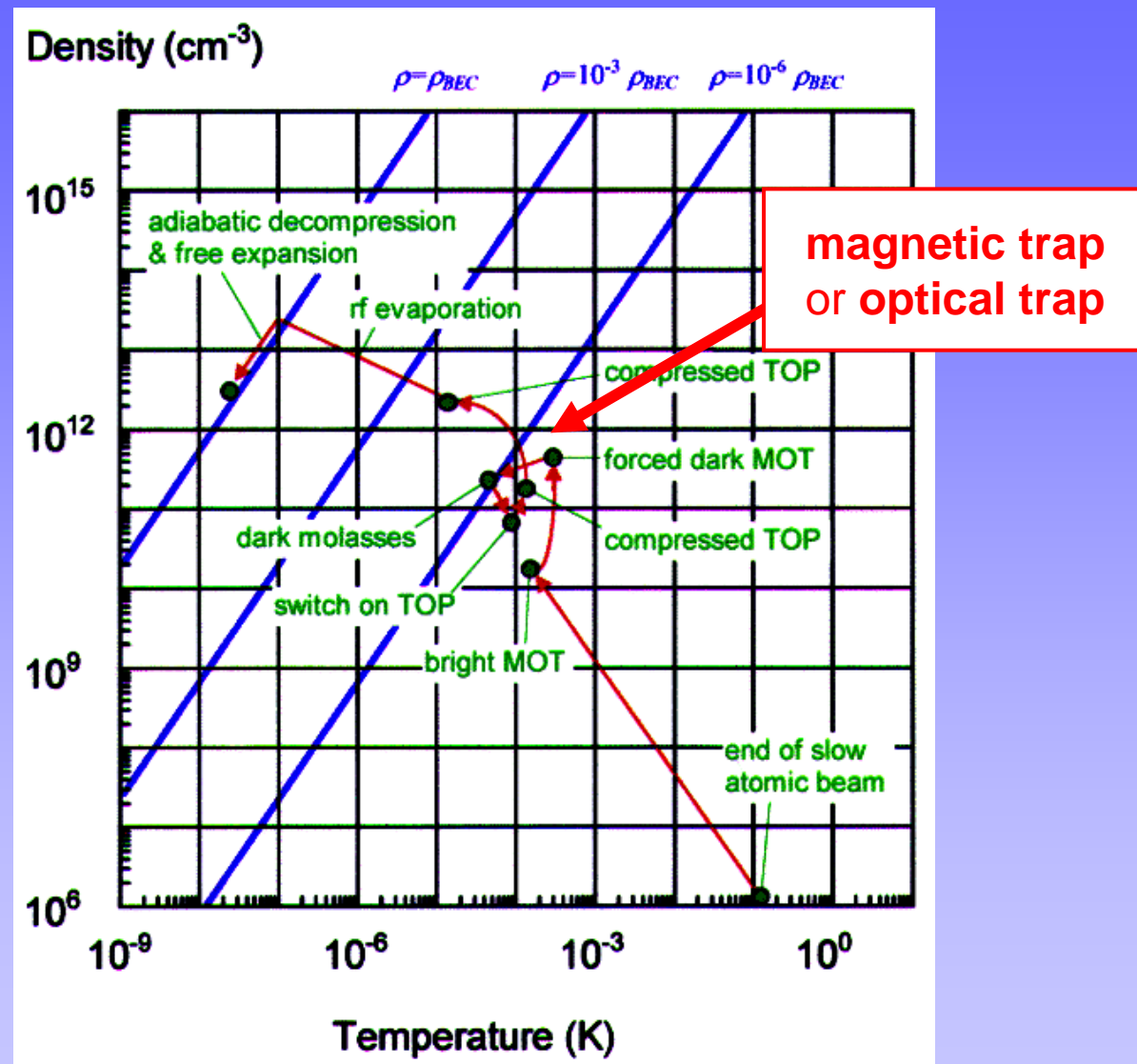
# The long, long road to the Ultracold



# Optical molasses

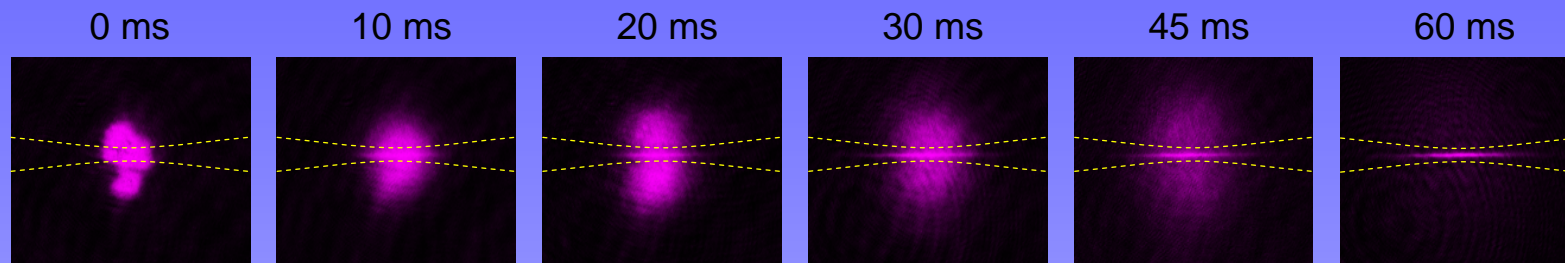


# The long, long road to the Ultracold



# Transfer into optical dipole trap

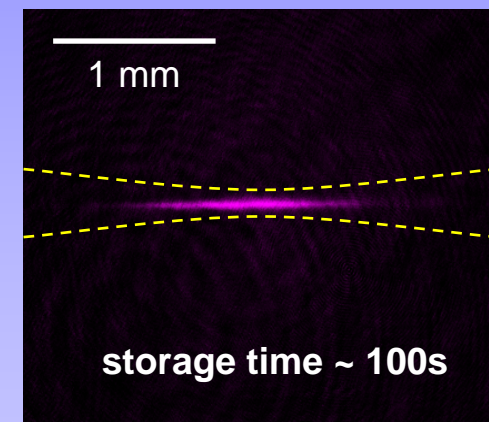
## Transfer into the optical dipole trap (absorption images)



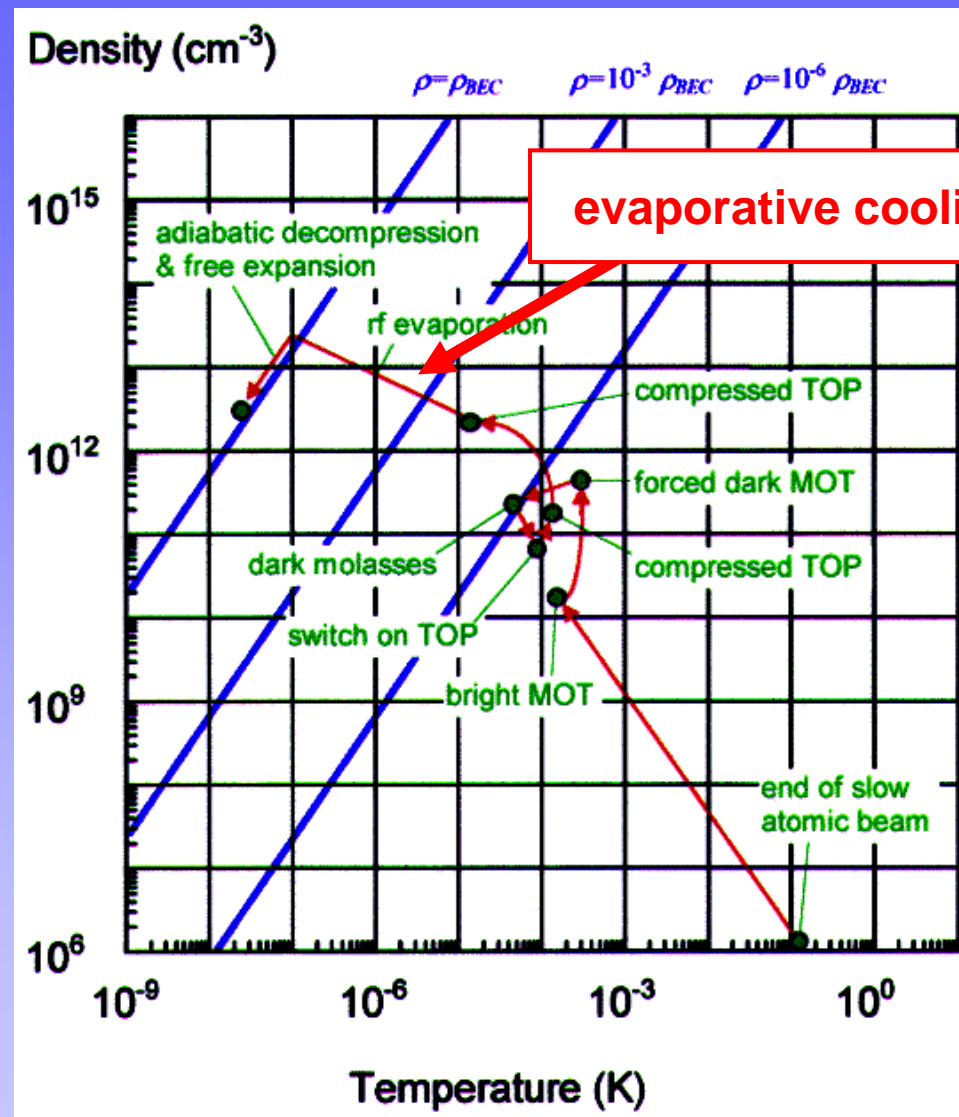
### Trap parameters

	Cs	Li
trap depth	1000 $\mu\text{K}$	400 $\mu\text{K}$
eff. temperature	30 $\mu\text{K}$	$\sim 100 \mu\text{K}$
# of stored atoms	$\sim 10^6$	$\sim 10^5$
transfer efficiency	7%	0.03%
peak density	$\sim 10^{12} \text{ cm}^{-3}$	$\sim 10^{10} \text{ cm}^{-3}$

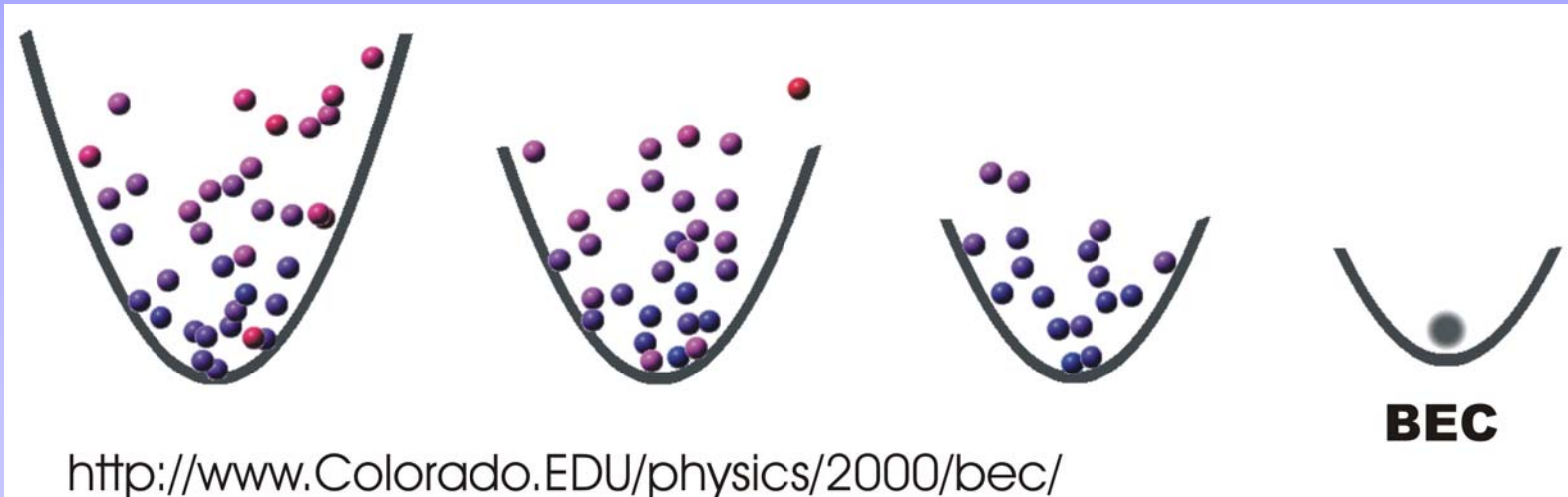
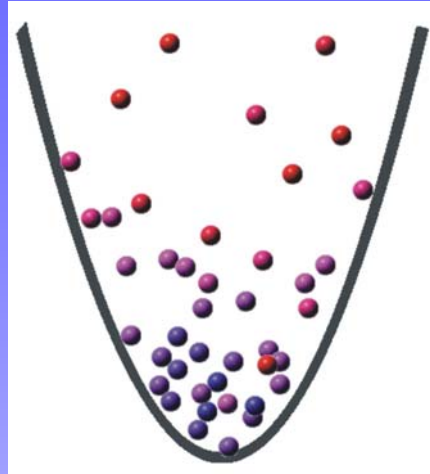
### density distribution



# The long, long road to the Ultracold



# Evaporative cooling



# Contents of the lectures

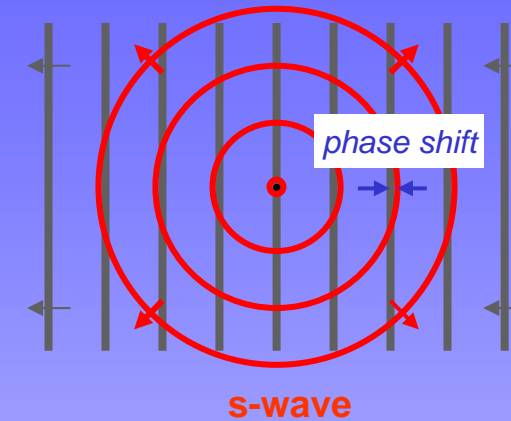
0. Primer on light-matter interactions
1. The way to absolute zero – cooling and trapping methods for atoms **Lecture 1**
- 2. Cold collisions**
3. Bose-Einstein condensation **Lecture 2**
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# Short reminder on binary collisions

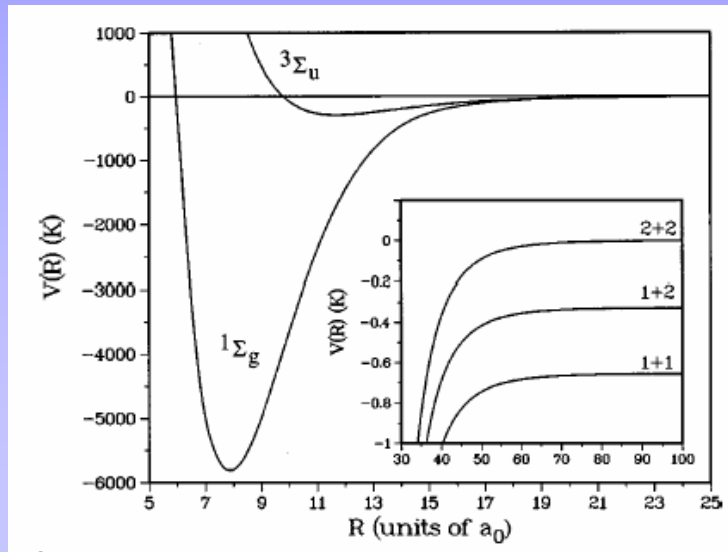
## Cross section

$$\begin{aligned}\sigma(E) &= \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1) |T_{\ell}(E)|^2 \\ &= \frac{4\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1) \sin^2 \eta_{\ell}\end{aligned}$$



phase shift induced by interaction potential

**Rb<sub>2</sub>**



**Effective potential**  
(incl. centrifugal energy):

$$V_{\text{eff}}(R) = V_g(R) + \frac{\hbar^2 \ell(\ell + 1)}{2\mu R^2}$$

centrifugal barrier

# Cold collisions

## Scaling laws

For small energies, all partial waves (except  $\ell=0$ ) freeze out due to centrifugal barrier

→ pure s-wave scattering

### Inelastic scattering:

(bad collisions)

$$\sigma_{\text{inel}}(E) \propto 1/k$$

(Wigner  $1/v$ -law)

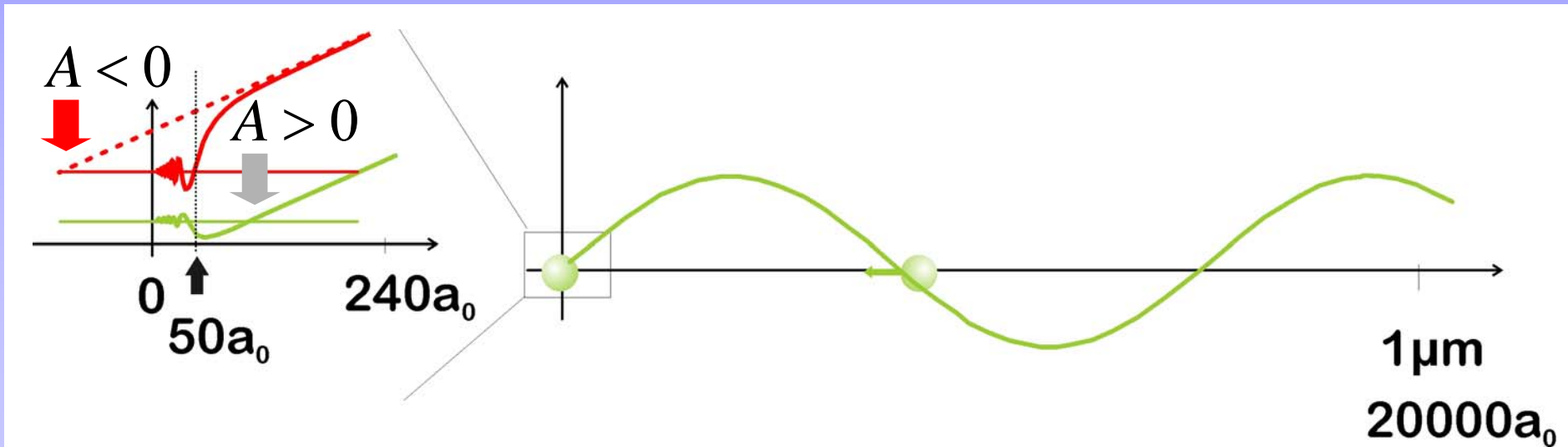
### Elastic scattering:

(good collisions)

$$\sigma_{\text{el}}(E) \simeq 4\pi A^2$$

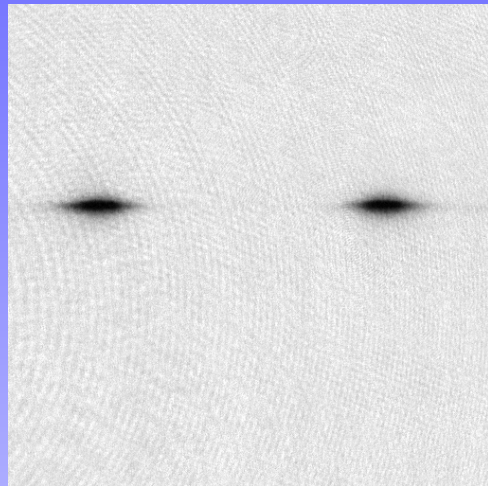
scattering length

(identical particles:  $\sigma_{\text{el}}(E) \simeq 8\pi A^2$ )

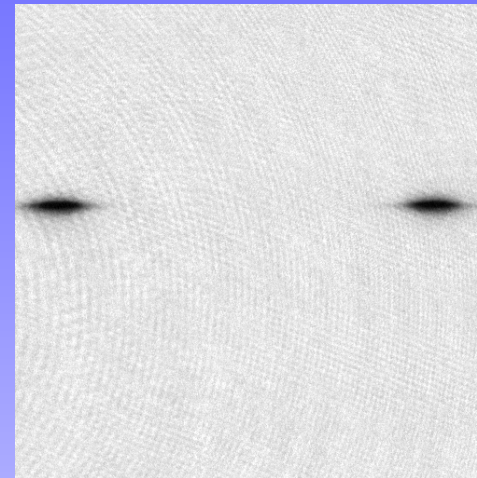


# Direct imaging of elastic scattering

## Collision of two ultracold clouds



$$E_c/k_B = 138 \mu\text{K}$$



$$E_c/k_B = 1230 \mu\text{K}$$

courtesy Jook Walraven (University of Amsterdam)

Ch. Buggle, *et al.*, Phys. Rev. Lett. **93**, 173202 (2004)

N. R. Thomas, *et al.*, Phys. Rev. Lett. **93**, 173201 (2004)

# Sympathetic cooling

Mean energy transfer per collision:

$$\Delta E = k_B \Delta T \frac{4 m_{\text{Li}} m_{\text{Cs}}}{(m_{\text{Li}} + m_{\text{Cs}})^2}$$

Collision rate:

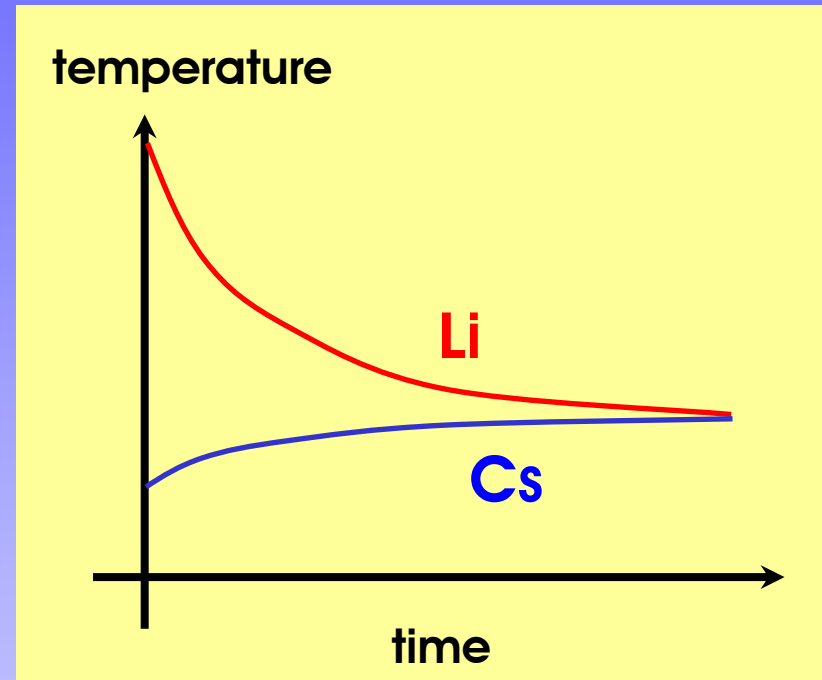
$$\Gamma_{\text{coll}} = \frac{N_{\text{Li}} N_{\text{Cs}}}{V_{\text{eff}}} \sigma_{\text{LiCs}} \bar{v}$$

Thermalization rate:

$$\Gamma_{\text{therm}} \simeq \Gamma_{\text{coll}} \frac{\Delta E}{\Delta T} / C_V$$

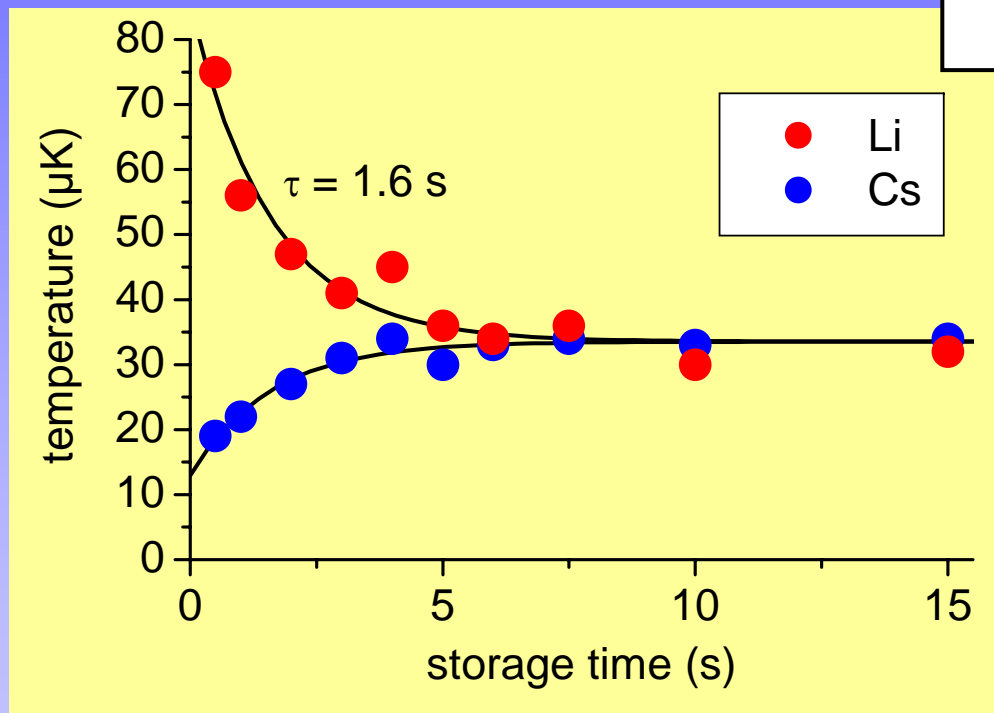
Heat capacity in harmonic trap:

$$C_V = 3 N k_B$$



# Sympathetic cooling

$$\frac{d(T_{\text{Li}} - T_{\text{Cs}})}{dt} = -\Gamma_{\text{therm}}(T_{\text{Li}} - T_{\text{Cs}})$$
$$[\Gamma_{\text{therm}} \propto \sigma_{\text{LiCs}}]$$



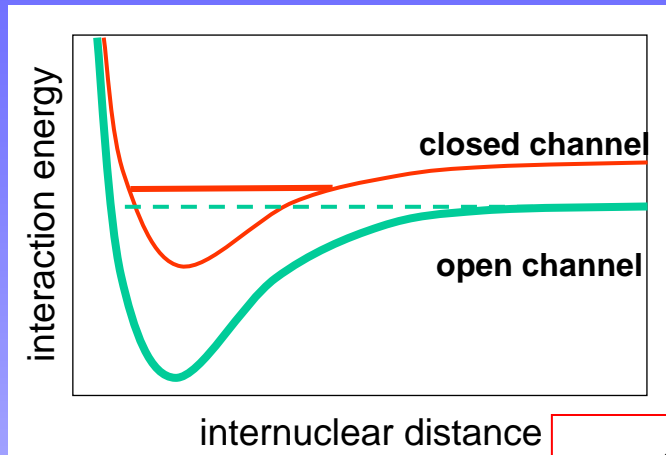
Sympathetic cooling takes place on a timescale of  $\sim 1$  s

**large cross section  $\sigma_{\text{LiCs}} \sim (300 \text{ \AA})^2$**

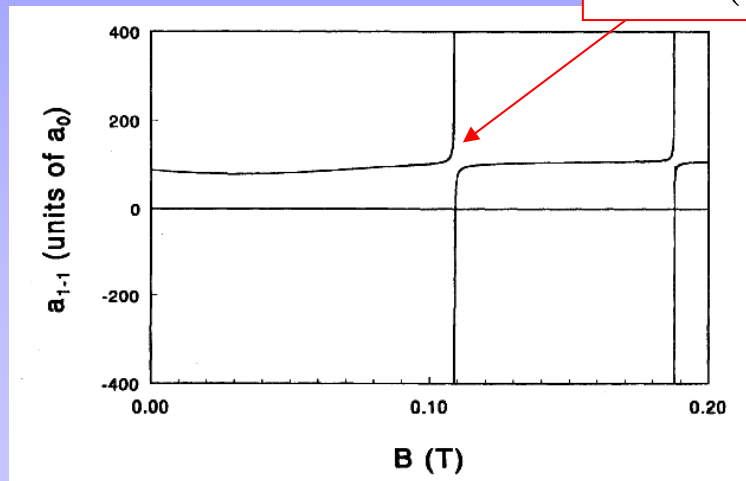
M. Mudrich *et al.*, Phys. Rev. Lett. **88**, 253001 (2002)

# Controlling the scattering length

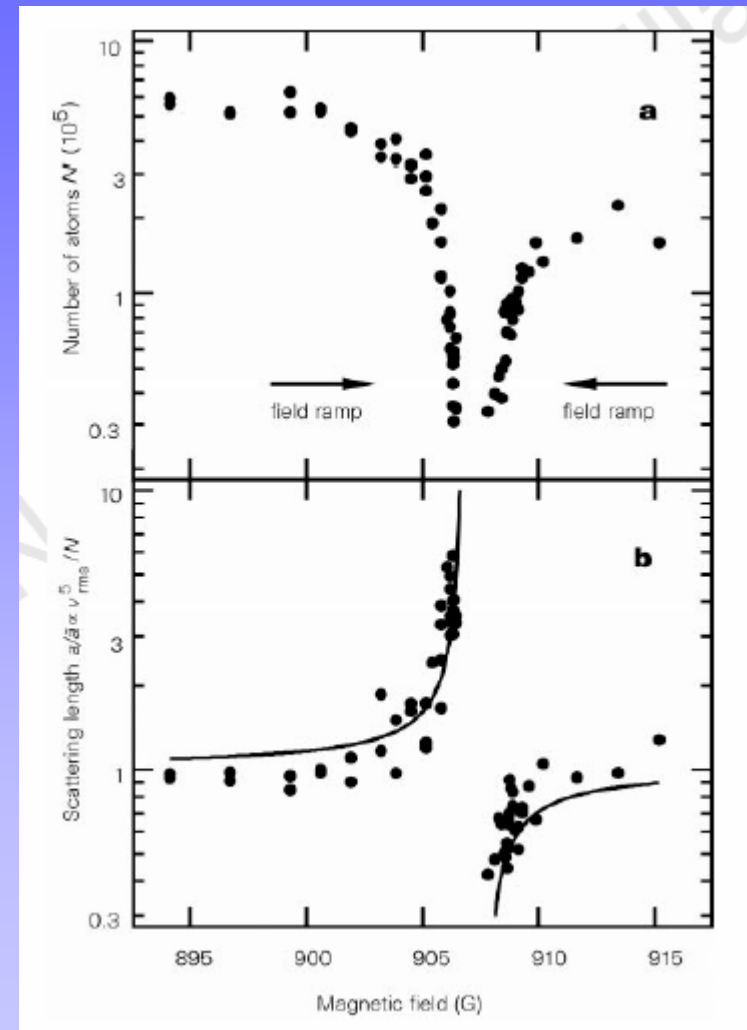
## Feshbach resonance



$$A = \hat{A} \left( 1 - \frac{\Delta}{B - B_0} \right)$$



A.J. Moerdijk *et al.*, Phys. Rev. A **51**, 4852 (1995)



S. Inouye *et al.*, Nature **392**, 151 (1998)

# Summary of Lecture 1

## ➤ Lorentz atom and forces on atoms

- force in a magnetic field
- force in an oscillating electric field (optical dipole force)

## ➤ Magnetic traps and optical dipole traps

- quadrupole, Joffe-Pritchard and chip traps
- FORT and QUEST, blue- and red-detuned traps

## ➤ Cooling techniques and increase of phase-space density

- laser cooling and optical molasses
- evaporative cooling
- sympathetic cooling

# Summary of Lecture 1 (cont'd)

## ➤ **Cold collisions**

- pure s-wave collisions at ultralow temperatures
- inelastic collisions → trap loss and heating
- elastic collisions → thermalization and momentum exchange
- control of the elastic scattering cross section (Feshbach resonance)



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