Lecture Outline

Strengthening Induction Hypothesis.

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Strengthening Induction Hypothesis.

Strong Induction

Lecture Outline

Strengthening Induction Hypothesis.

Strong Induction

Well ordered principle.

Tutoring Option.

How does tutoring work?

1. (Ideally) You work on homework and solve (most of) it.
2. You *do not* need to write-up or turn in.
3. You read and understand homework solutions.
4. You see a tutor, who gives you a short oral quiz.
    4.1 If you do well.

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Questions?

Strenthening Induction Hypothesis.

Theorem: The sum of the first n odd numbers is a perfect square.
Theorem: The sum of the first n odd numbers is k^2.

kth odd number is $2(k-1)+1$.

Base Case: 1 (1th odd number) is 1^2.

Induction Hypothesis: Sum of first k odds is perfect square $a^2 = k^2$.

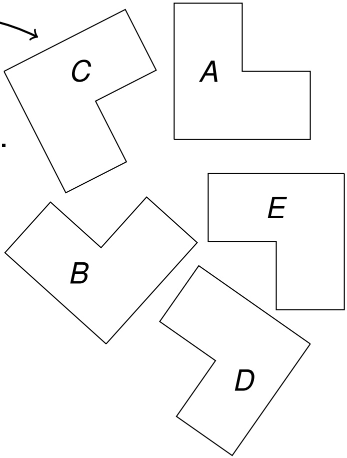
Induction Step:

1. The $(k+1)$st odd number is $2k+1$.
2. Sum of the first $k+1$ odds is
   $a^2+2k+1 = k^2+2k+1$
3. $k^2+2k+1 = (k+1)^2$
   ... P(k+1)!

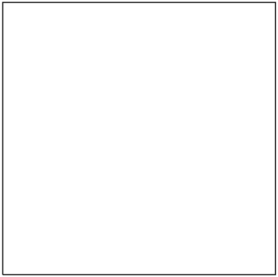
□

Tiling Cory Hall Courtyard.

Use these L-tiles.



To Tile this 4×4 courtyard.



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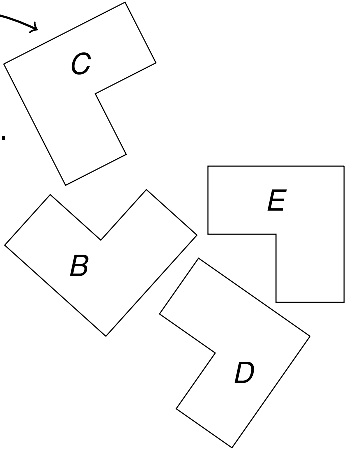

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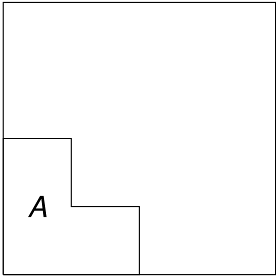

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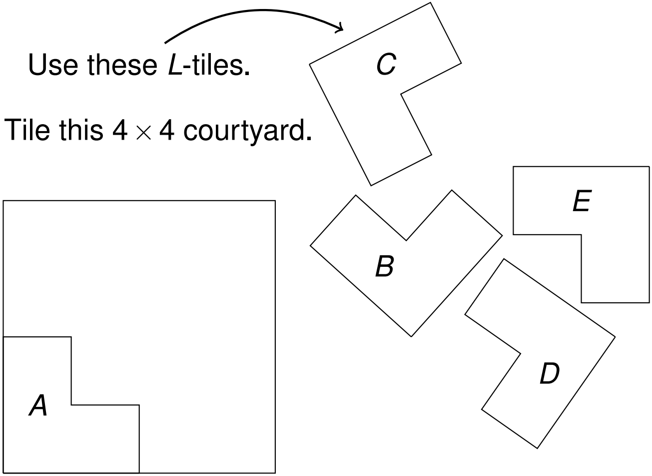

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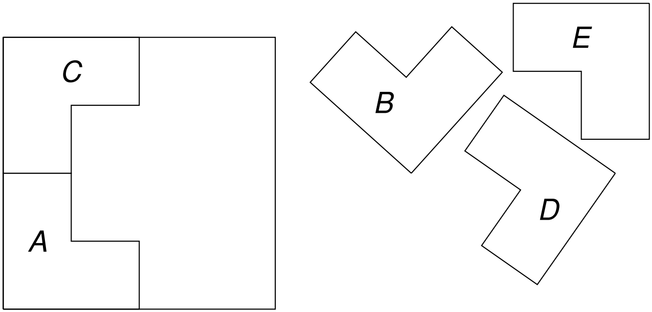

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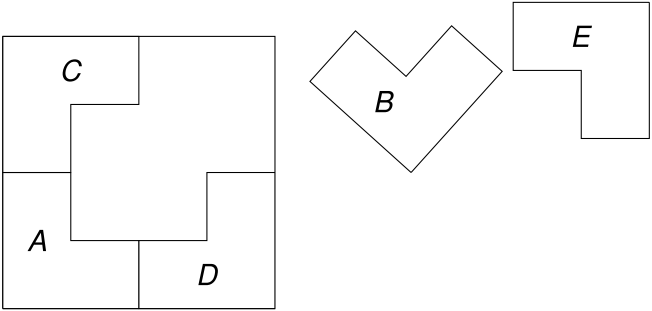

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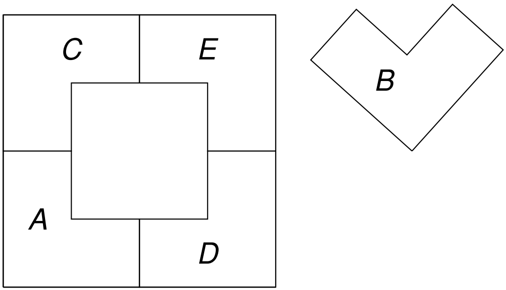

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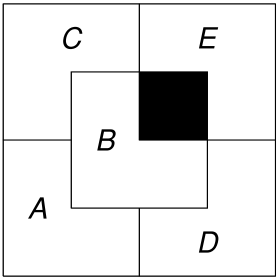

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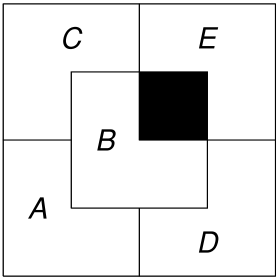

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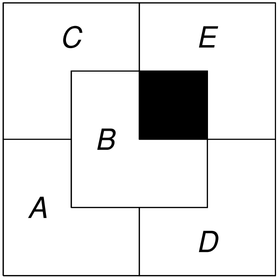


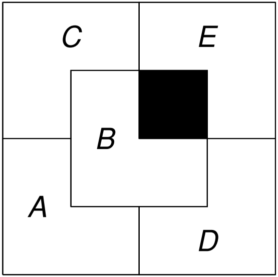
Alright!

Tiled 4×4 square with 2×2 L-tiles.

Tiling Cory Hall Courtyard.

Use these L-tiles.

To Tile this 4×4 courtyard.



Alright!

Tiled 4×4 square with 2×2 L-tiles.

with a center hole.

Tiling Cory Hall Courtyard.

Use these L-tiles.

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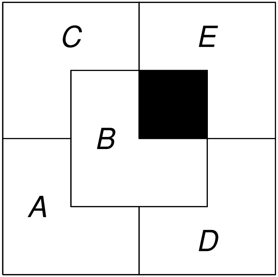

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Can we tile any $2^n \times 2^n$ with L-tiles (with a hole)

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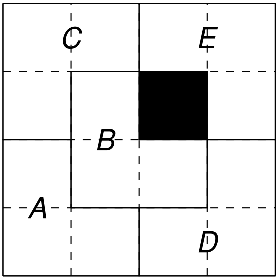

Alright!

Tiled 4×4 square with 2×2 L-tiles.

with a center hole.

Can we tile any $2^n \times 2^n$ with L-tiles (with a hole) for every n!

Hole have to be there? Maybe just one?

Theorem: Any tiling of $2^n \times 2^n$ square has to have one hole.

Hole have to be there? Maybe just one?

Theorem: Any tiling of $2^n \times 2^n$ square has to have one hole.

Proof: The remainder of 2^{2n} divided by 3 is 1.

Hole have to be there? Maybe just one?

Theorem: Any tiling of $2^n \times 2^n$ square has to have one hole.

Proof: The remainder of 2^{2n} divided by 3 is 1.

Base case: true for $k = 0$.

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Ind Hyp: $2^{2k} = 3a + 1$ for integer a.

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$$2^{2(k+1)}$$

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$$2^{2(k+1)} = 2^{2k} * 2^2$$

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\begin{aligned}
2^{2(k+1)} &= 2^{2k} * 2^2 \\
&= 4 * 2^{2k}
\end{aligned}
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a integer $\implies$ $(4a + 1)$ is an integer.

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a integer $\Longrightarrow$ $(4a + 1)$ is an integer. □

Hole in center?

Theorem: Can tile the $2^n \times 2^n$ square to leave a hole adjacent to the center.

Proof:

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The hole is adjacent to the center of the 2×2 square.

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Induction Hypothesis:
Any $2^n \times 2^n$ square can be tiled with a hole at the center.

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Theorem: Can tile the $2^n \times 2^n$ square to leave a hole adjacent to the center.
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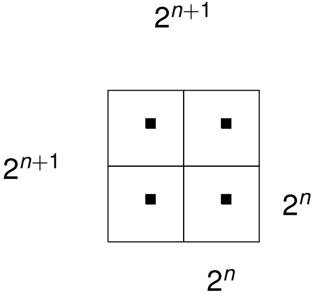

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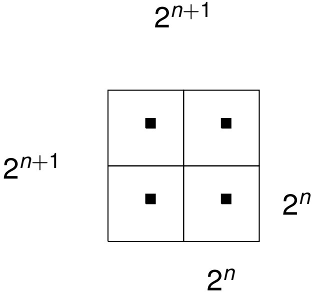

What to do now???

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Theorem: Can tile the $2^n \times 2^n$ to leave a hole adjacent *anywhere.*

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Base case: Sure. A tile is fine.
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Induction Hypothesis:
"Any $2^n \times 2^n$ square can be tiled with a hole **anywhere.**"
Consider $2^{n+1} \times 2^{n+1}$ square.

Hole can be anywhere!

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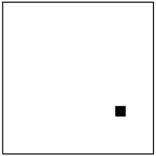

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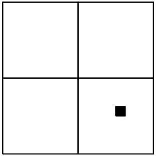

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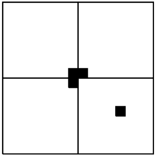

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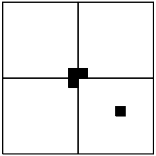

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Use L-tile and ...

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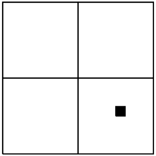

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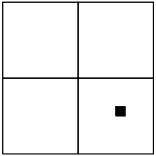

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Strong Induction.

Theorem: Every natural number $n > 1$ can be written as a product of primes.

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$P(n) =$ "n can be written as a product of primes. "

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E.g. Reduced form is "smallest" representation of the representations a/b that represent a single quotient.

Tournaments have short cycles

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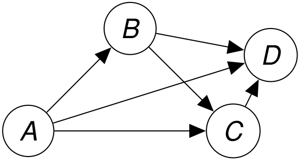

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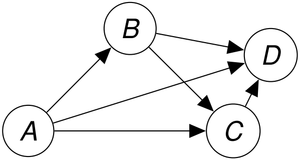

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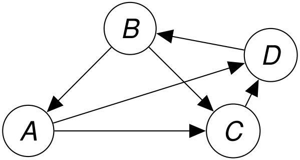

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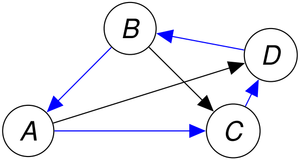

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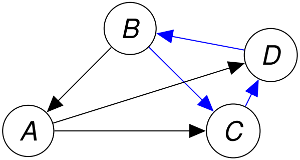

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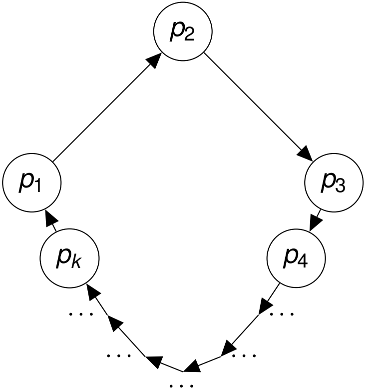

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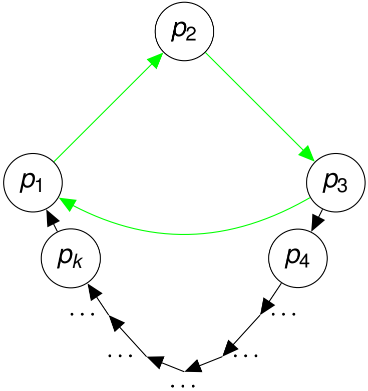

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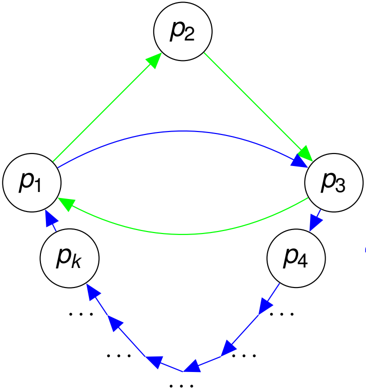

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"$p_1 \to p_3$" $\Longrightarrow$ $k - 1$ length cycle!

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Induction step $P(k+1)$?

First k have same color by $P(k)$. $1, 2$

Second k have same color by $P(k)$. $1, 2$

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No horse in common!

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As we will see, it is more subtle to catch errors in proofs of correct theorems!!

Strong Induction and Recursion.

Thm: For every natural number $n \geq 12$, $n = 4x + 5y$.

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Slight differences: showed for all $n \geq 16$ that $\wedge_{i=4}^{n-1} P(i) \implies P(n)$.

Summary: principle of induction.

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Strengthen theorem statement.
Sum of first n odds is n^2.
Hole anywhere.
Not same as strong induction.

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Induction $\equiv$ Recursion.