CHAPTER Two

Hexatonic Cycles

Chapter 1 proposed that triads could be related by voice leading, independently of roots, diatonic collections, and other central premises of classical theory. This chapter pursues that proposal, considering two triads to be closely related if they share two common tones and their remaining tones are separated by semitone. Motion between them thus involves a single unit of work. Positioning each triad beside its closest relations produces a preliminary map of the triadic universe. The map serves some analytical purposes, which are explored in this chapter. Because it is not fully connected, it will be supplemented with other relations developed in chapters 4 and 5.

The simplicity of the model is a pedagogical advantage, as it presents a circumscribed environment in which to develop some central concepts, terms, and modes of representation that are used throughout the book. The model highlights the central role of what is traditionally called the chromatic major-third relation, although that relation is theorized here without reference to harmonic roots. It draws attention to the contrary-motion property that is inherent in and exclusive to triadic pairs in that relation. That property, I argue, underlies the association of chromatic major-third relations with supernatural phenomena and altered states of consciousness in the early nineteenth century. Finally, the model is sufficient to provide preliminary support for the central theoretical claim of this study: that the capacity for minimal voice leading between chords of a single type is a special property of consonant triads, resulting from their status as minimal perturbations of perfectly even augmented triads. The consequences of that claim are the focus of the final sections of this chapter.

A Minimal-Work Model of the Triadic Universe

We will say that two triads are in the *minimal-work relation* if motion between them involves the displacement of a single voice by semitone. According to this definition, each triad is in the minimal-work relation to two triads of the opposite mode. Each major triad is in the minimal-work relation with its parallel minor and with the minor triad whose root lies four semitones above it. For example, C major is in the specified relation with c minor and with e minor. Reciprocally, each minor triad is in the specified relation with its parallel major and with the major triad whose root lies four semitones below it. For example, c minor can reach both C major and A♭ major by a *single semitonal displacement*.

Each consonant triad is thus situated in a chain of alternating major and minor triads. C major is flanked by c minor and e minor, producing the three-element chain {c minor, C major, e minor}. That trio is nested within a five-element chain, {A♭ major, {c minor, C major, e minor}, E major}. That quintet is, in turn, nested within a seven-element chain, {a♭ minor, {A♭ major, {c minor, C major, e minor}, E major}, g♯ minor}. The external elements of that septet are identified under enharmonic equivalence, and so the outer elements are "glued together," converting the seven-element chain into a six-element cycle.

Figure 2.1 presents that cycle in its upper quadrant, along with three other cycles, each germinated in an analogous way from other triadic seeds. Together, the four cycles partition the twenty-four consonant triads.[1] Connections among the cycles will emerge as a central project of chapters 5 and 6. This chapter focuses on the internal workings of the individual cycles. Because they relate to each other by transposition, internal features proper to one are proper to all.

Two triads are situated in the same cycle if they have the same roots, or if their roots are four semitones apart. Classical theory presents various languages for identifying the collections of co-cyclic triadic roots. We might say that they form

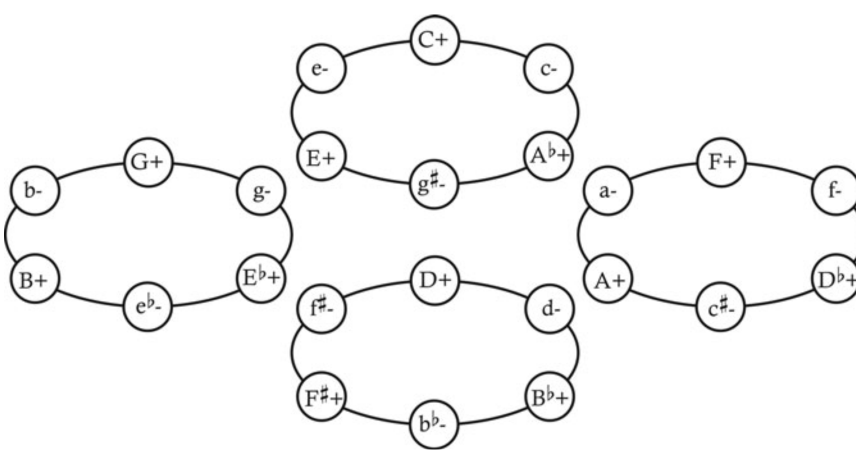


Figure 2.1. A graph of the twenty-four triads under single semitonal displacement, producing the four hexatonic cycles.

1. Derek Waller called attention to these cycles in a 1978 article in the *Mathematical Gazette*, which anticipated some aspects of my presentation in Cohn 1996.

an *augmented triad*. If we are concerned with projecting neutrality about enharmonic choices, we might say that the roots form a *symmetric division of the octave* into three equal parts. In circumnavigating the cycle in either direction, the progression consists of paired triads transposed by major third, or (again more neutrally) by an interval of four semitones. We might then be inclined to refer to it as a T_4 or T_8 cycle, where "T" refers to transposition and the subscripted number indicates the size of the transposition, measured in semitones. We might be inclined to view this progression in terms of a *chromatic sequence* whose *generating interval* is the major third or minor sixth, or whose generating transpositions are T_4 and T_8. All of these characterizations are more or less interchangeable, and I shall treat them as equivalent.

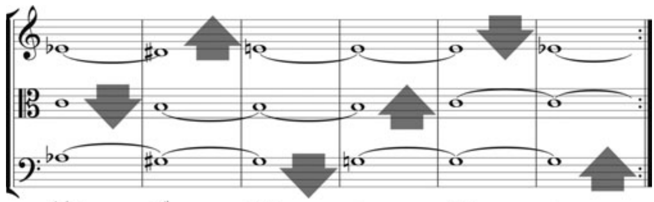
It has become standard to view the cyclic progressions portrayed in figure 2.1 in terms of "third relations" and thereby to throw them into the same pot with progressions that transpose by a series of minor thirds. This conflation, which is encouraged both by the labeling conventions for intervals and by the appearance of both types of cycles at roughly the same historical moment, obscures a fundamental distinction: cycles generated by major thirds exhibit *balanced voice leading*, alternating between up and down, whereas those generated by minor thirds lead their voices in a uniform direction. For example, in transposing by major third from C major to E major, under idealized voice leading one voice moves up (G → G♯) and one moves down (C → B), while the third voice, E, holds its place. In transposing by minor third from C major to E♭ major, again under idealized voice leading, both moving voices move down (C → B♭ and E → E♭) while G holds its place. The latter case is the norm. Under least-motion voice leading, recursive transposition by any interval other than major third generates uniformly directed voice leading. It is transposition by four or eight semitones that is special: these alone generate transposition cycles whose voice leading is balanced.[2]

A significant entailment of balanced voice leading is *contrary motion*: for every voice that rises, another falls by the same magnitude. Contrary motion among triads of the same mode thus inheres exclusively to triads that are transpositionally related by major third (Cohn 1996). This entailment, which is at the heart of the approach developed in this book, has gone unrecognized by the many scholars who have studied third relations during the last thirty years.[3] I will argue, in this chapter and beyond, that the contrary motion of major-third relations underlies both their central role in the syntax of pan-triadic progressions and their association with the semiotics of the supernatural. The reason that the major third has this special status is that *it divides the octave into as many equal parts as the triad has notes*. One job of this chapter is to show why.

2. Balanced voice leading corresponds to balls atop an arch. Some released balls will fall east, some west, and the behavior of one ball does not predict the behavior of its successor. Uniform voice leading corresponds to the situation of balls on a tilted plane. If one ball falls east, they all fall east; one can predict the behavior of all from the behavior of any one.

3. These include Proctor 1978; Krebs 1980; Taruskin 1985, 1996, 2005; Bailey 1985; Cinnamon 1986; Aldwell and Schachter 1989; Agawu 1989; Kraus 1990; Rosen 1995; Somer 1995; Kopp 2002; and Bribitzer-Stull 2006. Tymoczko 2011b is a recent and notable exception.

Figure 2.2. Voice leading through a hexatonic cycle. Arrowheads indicate the direction of semitonal displacement between adjacent triads.



The Hexatonic Trance

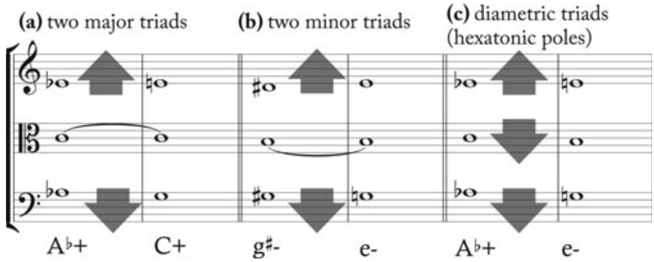
Figure 2.2 progresses clockwise about one of the cycles from figure 2.1, in a format that emphasizes the behavior of the individual voices. The repeat sign suggests a continuously cyclic process. (The features that we identify in this progression will also be present in its retrograde, as voice-leading features are independent of cyclic direction.) Arrowheads indicate the location and direction of semitonal displacements. Each individual voice holds its pitch for three "beats" and then displaces to a new pitch that is likewise sustained for three beats before returning to the original pitch. The entire cycle engages only six pitches, two for each of its three voices. Accordingly, the progression is referred to as a *hexatonic cycle*. Ordered linearly within an octave, the six tones form a *hexatonic scale*, alternating semitone and minor third.[4]

The triple periodicity within each individual voice interlocks with the duple periodicity resulting from their combination. This duple periodicity has both a directly perceptible aspect and a more abstract structural one. The direct aspect emerges in the periodic alternation of upward and downward motion, from one "beat" to the next. The more abstract aspect emerges from the relations between the three voices, each of which executes its displacements two beats later and four semitones lower than its predecessor (assuming octave equivalence). The voices thus combine to form a hocket canon, a structure familiar to music historians from the caccia of late-medieval polyphony (Bukofzer 1940). The interlocking of duple and triple periodicities, induced respectively within and between the individual voices, forms a 3:2 phasing (also referred to variously as polyrhythm, grouping dissonance, hemiola, and cross pattern). (Readers may find it useful at this point to kinetically engrave and aurally entrain these periodicities by playing through the repeating hexatonic cycle with one hand at a keyboard, or by arpeggiating the successive triads on a single-line instrument.)

Figure 2.3 juxtaposes nonadjacent triads of the same hexatonic cycle. Next-adjacent triads are of the same mode; either major (a) or minor (b).

4. Jazz players know the same pattern as the *augmented scale*, because the collection combines two adjacent augmented triads. It has also been referred to variously as the Ode to Napoleon collection, Miracle hexachord, Liszt model, source-set E, 1:3 collection, and set class 6–20.

Figure 2.3. Voice leading between nonadjacent triads within a hexatonic cycle.



Diametric triads, three positions apart, are of opposite mode (c). The number of semitonal displacements in each progression is equivalent to the cyclic distance of its constituents. In figure 2.3, (a) and (b) show the double displacements that occur in major-to-major and minor-to-minor juxtapositions, and (c) shows the triple displacement between nonadjacent triads of opposite mode. All three progressions involve balanced voice leading, in the sense that at least one voice moves in each direction. These progressions can be seen to compress the serial alternation of up and down in figure 2.2, transforming the directional alternation into a simultaneous contrary motion.

In music of the nineteenth century, and throughout the history of music for film, the progressions illustrated in figure 2.3 frequently depict sublime, supernatural, or exotic phenomena. In "Nacht und Traüme," Schubert slips directly from B major to G major to depict a nocturnal fixation on evanescent dreams (Schachter 1983b). In the *Ring*, Wagner uses the progression at figure 2.3(b) to portray the *Tarnhelm*, which makes its wearer invisible. Rimsky-Korsakov used the same progression, transposed through the minor triads of a hexatonic cycle, to depict the reclusive Antar adrift on the sands of the Sahara desert (see figure 3.6 in chapter 3). Wagner subsequently used the same cycle in *Parsifal* to depict the sorcerer Klingsor, another recluse in the Arabian wasteland. The diametric progression at figure 2.3(c) depicts a range of uncanny phenomena. The many examples cited in Cohn 2004 include Kundry's de-souling (*Parsifal*), the dead Siegfried shaking his fist (*Götterdämmerung*), Scarpia's murder (*Tosca*), Aase's arrival at St. Peter's Gates (*Peer Gynt*), Strauss's Salomé singing to the severed head of Jochanaan, and Schoenberg's self-portrait in death (String Trio, Op. 45).

In a chapter titled "Music and Trance," Richard Taruskin (2005) recognizes the semantic charge of chromatic progressions by major third but implies that their link to altered and uncanny states is conventional, relying on what Swiss linguist Ferdinand de Saussure referred to as the arbitrary bond between signifier and signified.[5] Although convention is certainly an element of this semiotic system, there is something else at work: the affective power of the progressions in figure 2.3 derives from a paradoxical characteristic that is inherent to them, when they are

5. Taruskin's association of chromatic progressions by major third with trance, the uncanny, etc., seems to be limited to the major mode, in which all of his examples occur. In minor, the submediant is "naturally" flat, and so ♭VI and "flat submediant" are not meaningful designations.

heard against the expectations of classical diatonic tonality. By default, the classically conditioned ear interprets a relation between two tones as diatonic rather than chromatic (Agmon 1986, 185; Temperley 2001, 128). The empty ear filling with music interprets a two-semitone interval as a major second rather than a diminished third, a three-semitone interval as a minor third rather than an augmented second, and so forth. (A canny composer cultivates various resources for reversing these defaults, just as an engineer can raise objects in physical space; but they nonetheless hold in an "everything else equal" context.) This same principle dictates that a single semitone be heard, again *ceteris paribus*, as a change of diatonic degree, rather than as a chromatic inflection of an invariant degree.

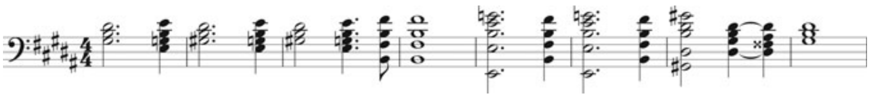
Applied to a perfect fifth whose voices move outward by semitone, as in the outer voices of the three progressions at figure 2.3, the default principle dictates that the interval between them increase by two diatonic degrees, producing a diminished seventh. Yet when heard in their own insular context, the default principle dictates that two tones separated by an interval of nine semitones express a major sixth.[6] The principle thus produces contradictory information: on the first application, the nine-semitone interval is dissonant; on the second, it is consonant. In the attempt to reconcile these interpretations, the ear is caught in a liminal space, where the binary distinction between consonance and dissonance is eroded. Such breakdowns in the division between otherwise securely demarcated categories, prototypically the boundary between reality and illusion, or life and death, are a mark of the psychological uncanny.[7] The capacity of chromatic root progressions by major third to signify altered and unstable mental states is thus based not on mere convention but on a homology between the signifying progression and the signified affect. In Peircean terms, the progressions in figure 2.3 are icons rather than symbols of altered or destabilized mental states.

Consider, for example, the *Tarnhelm* progression (figure 2.4), whose first two chords match those of figure 2.3(b). Warren Darcy (1993, 170) writes of the "aural sense of . . . eerie power" and of "radical disjunction. . . . the motif seems almost to have fallen in from another world." For Carolyn Abbate (2006, qtd. in Parly 2009, 166), that world is subterranean, "as if excavated from primeval time." Nila Parly (2009, 167) attributes this effect to the open B/F♯ fifth, an archaic symbol that evokes an "air of something 'uncanny,'" by virtue of incongruity, when embedded into Wagner's progressive harmonic language.

6. Riemann (1890, 38) wrote that, in these cases, one "will more or less always feel the inclination . . . and indeed with good reason," to hear one of the triads as a dissonance, spelled as a consonance only for convenience. Kopp 1995, 141ff., contains a translation and exegesis. Prout (1903, 256) calls them "false triads." See also Louis and Thuille 1982 [1913], 409–10, and Hull 1915, 42. Louis and Thuille note that Liszt frequently spells mediant triads as dissonances. Lendvai (1988, I: 60) observes a specific instance in Verdi's *Otello*.

7. Although the analogy may initially seem far-fetched, it taps into a significant history of theorizing about consonance and dissonance. Heinrich Schenker considered consonances to uniquely possess life-generative capacities, in the form of the capacity for prolongation; in one passage, he refers to the tonic triad as the maternal womb. He also considered dissonant harmonies to be false and illusory. Thus, to confound consonance with dissonance was literally to confound musical reality with musical illusion. Similar passages can be found in writings of his contemporaries, Ernst Kurth and Alfred Lorenz. See Cohn 2004. I return to this theme in chapter 7, in connection with *Parsifal*.

Figure 2.4. The *Tarnhelm* progression from Wagner's Ring.



In his initial notation of the progression, Wagner struggled with enharmonic decisions (Darcy 1993, 168–69). He initially sketched the first chord as a♭ minor. He led cantus E♭ to F♭, as the diatonic logic suggests, and then retracted it in favor of E, as part of an e minor triad that supplies the G leading tone. The third presentation of that chord, though, is rewritten as an f♭ minor triad. The approach pursued here suggests that the indeterminate enharmonics and the uncanny and disjunctive semantics stem from the same source. The melodic E♭ ought to move diatonically to F♭; the bass A♭ ought to move diatonically to (tenor) G; and the interval between bass and melody ought to be a diatonic and consonant major sixth. One of these imperatives must be discarded so that the other two can survive.

But the effect is overdetermined. Hexatonic progressions are also able to depict the world of noumena by virtue of their tonal multistability. Each *Tarnhelm* triad contains the other's leading tone and hence signifies the other as tonic, like the Escher hands drawing each other's sleeves. A hexatonic cycle abjures the cadential resources of classical tonality, such as fifth-related roots, dissonances, and diatonic coordination. A composer can suggest one of its constituent triads as a tonic "factitiously by virtue of its recurrence" (Taruskin 1996, I: 259) but can secure it only by recruiting external syntactic routines. The six triads are equally likely recipients of rhetorical or cadential benefaction; the progression itself is neutral with respect to its potential tonics. In this way, hexatonic progressions resemble the so-called "standard" bell pattern (variously called *bembé* and *gagokoe*) of West Africa or the Caribbean, whose temporal "tonic" (i.e., downbeat) can occur at several points along the cycle. The cycle itself is neutral with respect to these bestowals and is in this sense multistable (Pressing 1983).

The analogy that I have just ventured takes us into a speculative terrain that merits light treading, because it is counterintuitive from a historical standpoint. Yet a small cluster of features line up suggestively in support of pursuing it at least around one or two bends in the road, before beating a hasty retreat back to the main path. We observed, in connection with figure 2.2, that a hexatonic cycle embeds a hocket canon that projects a 3:2 phasing. This particular combination of attributes is likewise identified with African musical traditions associated with spirit possession and trance. Rouget 1985 suggests that such concerns are not entirely remote from Europe. In the post-Enlightenment world, these concerns bore scientific trappings. Dr. Franz Mesmer's theories of animal magnetism and universal fluids, as well as his therapeutic practice of hypnosis, held considerable interest among artists and intellectuals in Biedermeier Vienna; Schubert's sustained encounter with them is documented in Feurzeig 1997. One can imagine, then, that a composer of the early nineteenth century might be interested in exploring ways to depict states or sensations associated with hypnosis, as an uncanny state liminally perched at the juncture between reality and illusion, or life and death, and might find in hexatonic progressions a number of homologous attributes.

Contrary Motion and Balance

How is it that contrary motion is a special case, when it is universally acknowledged as a prescriptive norm in voice-leading theory and pedagogy? The resolution to this peculiarity points to an overlooked circumstance that turns out to be fundamental to our understanding of pan-triadic harmony. When three-voice triads (i.e., lacking tone doublings) are connected by idealized voice leading, contrary motion is indeed a special case: it arises only between triads whose roots are related by major third and hence share membership in a hexatonic cycle. Any given triad (e.g., C major) can be juxtaposed with eighteen other triads that lie outside of its cycle. The progression to one of these triads (in the given case, A minor) involves motion in a single voice, to which the categories of paired motion do not apply. The progression to each of the remaining seventeen triads involves similar or parallel motion.[8]

The bidirectional voice leading in a hexatonic cycle, whether successive, as in the case of incremental motion through the cycle, or simultaneous, as in the case of motion between nonadjacent triads, invites an interpretation in terms of internal points of symmetry. In figure 2.2, each triad has a point of balance whose location is roughly constant throughout the progression, despite local fluctuations. This even balance results from the quasi-uniform size of the intervals of which the triad is composed, which fall within a narrow compass of from three to five semitones. The particular point of balance depends, of course, on the registral ordering of the voices, which is arbitrarily selected in figure 2.2. But the general feature of balance would remain under any other ordering of the pitches, provided that the ambitus remained within an octave. (Readers comfortable with representations of pitch-class space will recognize that the center of symmetry is more properly shown as a vector cutting through a cycle, but the basic point holds in pitch space as well.) As the triads move incrementally through the hexatonic cycle, the semitonal alternation of up and down causes the center of balance to toggle back and forth between two pitches separated by one-third of a semitone. Circumnavigation of a hexatonic cycle thus produces neither upward nor downward motion through the pitch spectrum. Like a walker or a waterfall, the incessant

8. This observation has a significant entailment for one aspect of the history of harmonic tonality. Late-medieval contrapuntal theory privileged small melodic intervals within voices and contrary motion between them. Under triadic tonality, these two principles come into conflict. Any three-voice progression between diatonic triads must violate one or the other of them, if it involves two moving voices. This conflict may have been a stimulant to collective creativity, a problem whose solution incidentally introduced a number of varietals that ultimately enriched the compositional soil of harmonic tonality: among them, the addition of a fourth voice that either doubled a triadic pitch class or introduced a dissonance that resolved in contrary motion to the other voices.

local fluctuations are underlain by a global stasis. This stasis, however, coalesces around the prolongation not of a tonic, in any standard construal of the term, but rather of a *zone of voice-leading space* (or *voice-leading zone*). What is meant by the italicized term is taken up in chapter 5, after several more elements that support it are put in place.

Hexatonic Progressions, *Tonnetz* Representations, and Triadic Transformations

Having observed hexatonic cycles in the laboratory, we are now in a position to study them in the field. Figures 2.5 through 2.8 present four of the earliest hexatonic cycles, from four decades of Viennese music. It is characteristic of these chronologically early examples that tonicizing dissonances buffer the stations of the cycle, which are presented at a thinly veiled layer of the middleground. Like the Schubert excerpt examined in chapter 1, each of these excerpts instantiates what Ramon Satyendra (1992) calls *layered tonality*: classically tonal at both the most global and the most local level but chromatic at the level of the local modulations (also see McCreless 1996, 102). The diatonic indeterminacy resides in the succession of tonics, as symptomized by the essential enharmonic transformation that a hexatonic cycle requires.

Figure 2.5 presents a passage from the final movement of Mozart's Symphony in E♭ major, K. 543 (1788). The excerpt initiates the developmental core of a monothematic sonata-form movement. Beginning in A♭ major with a partial quotation of the principal theme, the music proceeds, via sostenuto wind chords, through a g♯ minor triad to E major at m. 115. A series of violin/cello stretti, based on the

Figure 2.5. Mozart, Symphony in E♭ major, K. 543, finale, mm. 109–26.

Figure 2.6. Haydn, Symphony no. 98, finale, mm. 148–98.

one-measure head, unfolds in the subsequent measures, connecting through e minor and C major to c minor at m. 123, and then intensifies through a diatonic sequence that ultimately prolongs c minor.

Figure 2.6 presents a passage from a Haydn symphony composed four years later; the passage likewise constitutes the developmental core of a final movement. The development begins with a series of four-measure thematic segments that tonicize A♭ major, its submediant f minor, and its subdominant D♭ major, the latter tonicization precipitating a sudden caesura. For four measures, the first violin paws at the rising-third incipit of the principal theme, finally grabbing hold of a *Sturm und Drang* arpeggiation of C♯ minor, the enharmonic parallel of the immediately previous tonic. G♯ → A leads to a thematic statement in A major, from which vantage point we can reinterpret f minor → D♭ major, initially heard in A♭ major diatonic space, as the first step in a journey through a hexatonic cycle that is completed by an a minor arpeggiation (m. 178) that ultimately leads to F major at m. 190.

The Mozart and Haydn passages are harmonically open and texturally diverse, as befits a developmental core. In the following two passages, similar progressions are harmonically closed, unfold at a leisurely pace, and involve block transpositions of lyrical thematic material. Figure 2.7 presents the sixteen-measure period in the center of the ternary-form Adagio of Beethoven's "Spring" Sonata, Op. 24 (1802).[9] After a cadence in B♭ major, the antecedent phrase arises from a modal mutation to b♭ minor and leads after eight measures to a cadence in its submediant, G♭ major. The consequent phrase likewise begins with a modal mutation to

9. Schenker models this passage in *Free Composition*, figure 100.6(b), commenting only that "here we have a descending register transfer by means of three major thirds" (1979 [1935], 82). Proctor 1978, 175–76, provides an astute commentary.

Figure 2.7. Beethoven, Sonata for Violin and Piano, Op. 24, 2nd mvt., mm. 38–54.

f♯ minor and leads to a tonicization of its submediant D major, but this tonal motion is transacted in half the time. This acceleration leaves a balance of four measures, which are filled by yet a third transposition of the same progression, leading from d minor to B♭ major for the start of the final section of the movement.

Figure 2.8 presents the first of three phrases from the conclusion of the immense initial movement of Schubert's Piano Trio in E♭ major (1828). The unmelodied accompaniment (or "vamp") mutates a major chord to its parallel minor, leading to an eight-measure period that modulates down a major third to the latter's submediant. The phrase given in figure 2.8 is followed twice by its exact transposition, beginning first in B major and then in G major, the latter modulating back to the closing E♭ major tonic.

Figure 2.9 models these four hexatonic passages in a graphic format that brings out some features obscured by the cyclic graphs of figure 2.1. Points represent individual tones, rather than the triads formed by their combination, and edges connect tones that form consonant intervals. This graphic format is a fragment of the *Tonnetz* ("tonal network" in German), a planar figure that coordinates axes representing the consonant interval classes. In the version that will be used

Figure 2.8. Schubert, Piano Trio in E♭ major, Op. 100, 1st mvt., mm. 584–595.

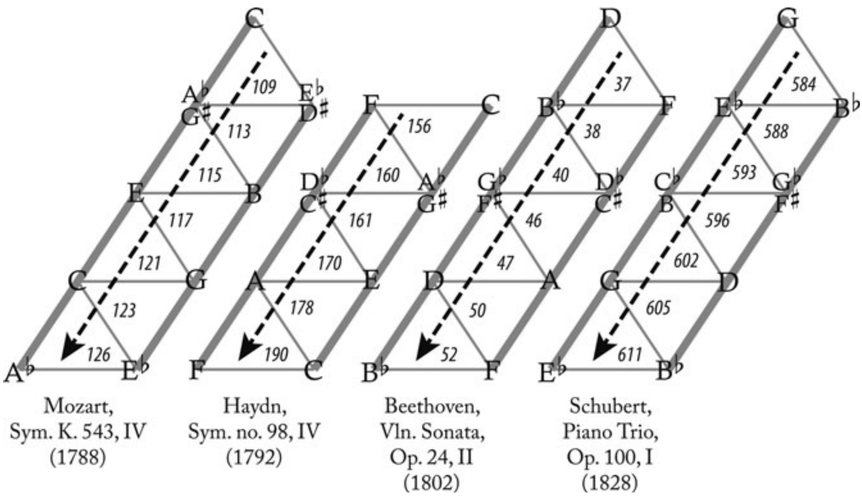


Figure 2.9. Tonnetz models of figures 2.5–2.8.

throughout this book, perfect fifths rise from left to right along the horizontal axis, minor thirds rise from northwest to southeast, and major thirds from southwest to northeast.[10] What appears as clockwise motion in figure 2.1 is converted in figure 2.9 to downward motion through a strip whose external boundaries form augmented triads. Each strip's interior is tiled into triangles, representing consonant triads. Major triads extend upward, and minor triads subtend downward, from their shared perfect-fifth edge. Internal edges represent shared dyads. The motion of each passage through the strip is modeled by an arrow. Numbers inside the triangles reference bar numbers of the score.

These planar graphs suffer from a Bering Strait flaw. Each tone and edge at the top of a strip reappears at its bottom, masking identities and distorting distances. Were these identities honored by "gluing together," the strip would convert to a cylinder. Our failure to so honor them is a concession to the dimensional limits of the printed page. These limitations are worth tolerating because of the many advantages that the triangularly tiled planar representations afford. A few of those advantages will become apparent in this chapter; many more will accumulate as we penetrate more deeply into the heart of the model.

10. The *Tonnetz* was first presented by Leonhard Euler in 1739. It was revived by German harmonic theorists in the second half of the nineteenth century and was independently reinvented numerous times, and for numerous reasons, by late-twentieth-century music theorists, psychologists, and dilettantes interloping from other academic fields. The angled format was introduced by Ottakar Hostinský in 1879 and adopted by Hugo Riemann in his later publications. For more on the history of the *Tonnetz*, see Vogel 1993 [1975], Mooney 1996, Gollin 2006, and Cohn 2011. One innovation adopted here, following an idea of Daniel Harrison (2002b), is the double labeling of nodes that correspond to enharmonic exchanges in the score being modeled (e.g., the A♭/G♯ along the left border of the Mozart *Tonnetz* in figure 2.9). This makes it easier to identify triads and track their progressions.

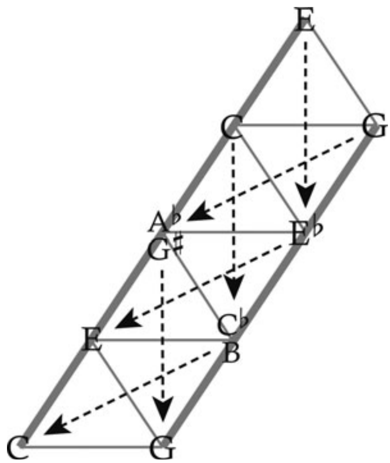


Figure 2.10. A progression on a *Tonnetz* strip, as incremental moves through a hexatonic cycle.

Figure 2.10 extracts one of these graphs for closer study. It breaks the single arrow into a series of local ones that indicate individual pitch-class displacements. Downward arrows, indicating chromatic-semitone descents, alternate with diagonal ones, indicating diatonic-semitone ascents. To distinguish these two species of local progression, we draw on a tradition initiated by Arthur von Oettingen (1866), who identified several exchange operations (*Wechsel*) that connect opposite-mode triads. These include a leading-tone exchange (*Leittonwechsel*) that connects opposite-mode triads that share a minor-third dyad, as exemplified by the diagonal arrows in figure 2.10. Oettingen's exchange operations were developed by Riemann (1880), and some of them were revived and formalized a century later by David Lewin (1982, 1987).[11] Following Brian Hyer's 1989 adaptation from Lewin, I will use the letter **P** (parallel major/minor) to indicate the motion between triads that share two common tones and a common root, and **L** (*Leittonwechsel*) to indicate triads that share two common tones and whose roots are a major third apart. Both operations are involutions, which is to say that they "undo themselves": two consecutive applications produce an identity. Table 2.1 summarizes the information about these two transformations.

Figure 2.11 presents *Tonnetz* models of four additional passages, which traverse hexatonic cycles in a manner more characteristic of the nineteenth century. Figure 2.11(a) models the passage presented at figure 2.12, from the first movement of Brahms's Concerto for Violin and Cello, Op. 102 (1887). The passage resembles those studied above in connection with Figures 2.5 through 2.9, but the diatonic buffers have been removed. Each station along the path is approached

11. The history of transformations, on their own and as they relate to the *Tonnetz*, is documented in Klumpenhouwer 1994, Mooney 1996, Gollin 2000, Kopp 2002, and Engebretsen 2002. The exchange operations were theorized as contextual inversions in Clough 1998.

Table 2.1. Incremental hexatonic transformations

Name	Symbol	Root motion	Common dyad	Semitonal species	Planar angle on *Tonnetz*
Parallel	**P**	No change	Perfect fifth	Chromatic	0°
Leittonwechsel	**L**	Major third	Minor third	Diatonic	120°

directly from its predecessor, and as a result the passage sounds only the six tones of the cycle's associated hexatonic scale.

Figure 2.11(b) represents the second sentence of Liszt's Consolation no. 3 (1840s), for which a score is available at Web score 8.3. The passage presents upward motion through the strip, equivalent to counterclockwise cyclic motion, for the first time.[12] A second new feature is the omission of one member of the cycle, c♯ minor, resulting in the direct juxtaposition of two triads of the same mode (see figure 2.3(a)). Although the name for this transformation, **LP**, suggests a compound transformation, with intermediate c♯ minor deleted, I prefer to think of the transformation (and its inverse, **PL**) as a unitary *Gestalt* whose name happens to have two syllables.[13]

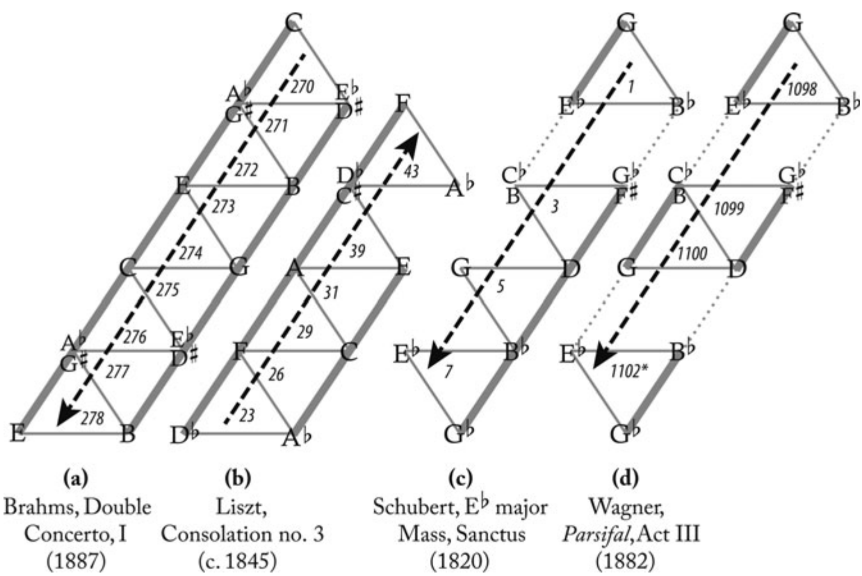


Figure 2.11. Four hexatonic passages from the nineteenth century.

12. For analyses, see Lewin 1967, Gollin 2000, Kopp 2002, Santa 2003, and chapter 8 of this book.
13. Kopp (2002, 159) suggests that **LP** is a compound operation by virtue of its compound name. Gollin 2000 (6–12) argues that this need not be so; any compound transformation, such as "retrograde inversion," can be furnished with a unary name (he suggests "George"). Although there is heuristic value in the compound name, there is no necessary significance to it. The same is true in natural language, where words like *breakfast* and *handicap* autonomously accrue and shed meanings apart from their compound origins.

Figure 2.12. Brahms, Concerto for Violin and Cello, Op. 102, 1st mvt., mm. 270–79.

The final two passages modeled in figure 2.11 are presented at figure 2.13: the opening of the Sanctus movement from Schubert's E♭ major Mass (1828) and a chromatic Grail distortion from Wagner's *Parsifal* (1882).[14] Both passages begin with E♭ major → b minor, diametrically across a hexatonic cycle. Two intermediate triads on the hexatonic cycle are omitted, and all three voices move simultaneously by semitone, as in figure 2.3(c). I will describe this relation as a *hexatonic pole*, and the corresponding transformation with the label **H** (after Cook 1994). Like all mode-switching operations, **H** is an involution that "undoes itself" when performed twice consecutively.

Both passages also continue by indirectly connecting b minor → e♭ minor, but they do so in different ways. Wagner interpolates G major, which shares two

Figure 2.13. Two similar hexatonic progressions.

(a) Schubert, Mass in E♭ major, Sanctus, mm. 1-7

(b) Wagner, *Parsifal*, act 3, mm. 1098-1102

14. For a consideration of the Schubert passage, see Salzer and Schachter 1969 (215–18). Concerning the Grail distortion, see Lewin 1984 and 1992, Clampitt 1998, Lerdahl 2001, Cohn 2006, Lerdahl and Krumhansl 2007, Brower 2008, and Rings 2011.

common tones with its predecessor and none with its successor. As a result, the closing G major → e♭ minor transposes the opening **H** progression, E♭ major → b minor. Schubert interpolates g minor, which shares one tone with both of its neighbors, dividing the four units of voice-leading work evenly between the two progressions. The progression into and out of g minor transposes a minor triad *downward* by major third, meriting the label **LP**.

That label is familiar from the Liszt Consolation (figure 2.11(b)), where it identified the transposition of a major triad *upward* by a major third. Here we arrive at a circumstance that has come to be viewed by some as the Achilles heel of triadic transformational theory. In furnishing g minor → e♭ minor, which terminates the Schubert progression, with the same label as A major → D♭ major, which terminates the Liszt one, we are implicitly claiming that the two progressions are in some sense equivalent. Three sorts of objections have been raised against this claim. First, the claim is seen to violate the intuition that progressions, in order to be considered equivalent, ought to move roots not only by the same magnitude but also in the same direction (Kopp 2002; Tymoczko 2009b). Second, the claim is seen to result solely from the way that the transformational logic plays out. The **LP** operations, and their **PL** inverses, become the boorish in-laws that need to be tolerated if we want to marry into the otherwise attractive and well-behaved kinship system. Third, the claim seems to entail a commitment to the harmonic dualism under whose banner it first arose in the writing of Oettingen and Riemann: the notion that major and minor triads are generated by equal but opposite metaphysical or physical forces. Such a commitment would be embarrassing on a priori grounds, since the metaphysics is obsolete and the physics apocryphal (Harrison 1994; Rehding 2003).

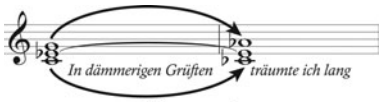
All three objections are neutralized by an appeal to voice leading, a dimension of experience that is, in principle, independent of root motion. What C major → E major shares with c minor → a♭ minor is the behavior of each individual voice: the G voice moves up by semitone, the C voice moves down by semitone, and the remaining voice holds constant. More generally, any **LP** operation sends its perfect-fifth dyad to a major sixth (or perfect fourth to minor third), and any **PL** operation does the reverse.

Consider figure 2.14, which juxtaposes two passages from Richard Strauss's "Frühling" of 1949. A score is available at Web score 4.19. After an orchestral alternation between c minor and a♭ minor, the singer enters on the wave of that same progression at m. 5. The second stanza opens on a C major triad that moves directly to E major in $^6_4$ position. The two passages feature major-third transpositions but in opposite directions. What they share is their voice leading: both progressions lead C down by semitone and G up by semitone while keeping their third voice invariant. It is this voice-leading equivalence that the label **LP** captures.[15]

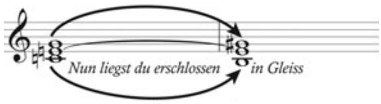
Although there is no appeal to harmonic dualism here, there is, nonetheless, a more benign *melodic dualism* lurking about in the wings, whose implications are treated below.

15. See also Lewin 1992, figure 3; for a similar case involving **PL**, see Cohn 1999.

Figure 2.14. Two **LP** transformations in Strauss's "Frühling" (*Four Last Songs*).



(a) stanza 1



(b) stanza 2

Near Evenness, Minimal Voice Leading, and the Central Role of Augmented Triads

Our work with the *Tonnetz* strips (figures 2.9 and 2.11) has focused on the relations between the triangles at their interior. In a move that will have significant repercussions for the remainder of this book, we now attend to the augmented triads that bound the strips. The role that they play, in the hexatonic passages that those strips represent, is not immediately apparent. In none of the passages does an augmented triad appear as a surface harmony. One of the augmented triads, on the left boundary of the strip, does have a certain salience in the bass register, where its tones slowly unfold one by one. We might be inclined, by habit, to say that this augmented triad is *arpeggiated*, or even *prolonged*. But we may not be prepared to take on board some implications of such a claim. Are the tones of the augmented triad fused into a *corps sonore*, which generates the passage by distributing its components across time and sprouting triads from each one? The invocation of *arpeggiation* and *prolongation* in this context has a metaphorical component whose heuristic value has been a site of heated controversy among Schenkerian theorists since the 1950s.[16]

The notion that augmented triads are "prolonged" is particularly problematic in the current context, since it suggests that the smooth voice leading of these passages is a by-product. If we consider semitonal voice leading as primary, then such a conception places the tail where the head should be. It is the semitonal voice leading that stands at the core. The disjunct bass is merely running about town making calls where its services are needed (see Schachter 1983b, 75). Moreover, even if we are comfortable with assigning fundamental status to the augmented triad at the left boundary of each strip, what is the role of its partner at its right boundary?

In proposing an equal role for the two augmented triads that bound a *Tonnetz* strip, we arrive at a major theoretical claim of this book: when triadic progressions are pursuing the logic of smooth voice leading rather than that of acoustic

16. For a nuanced discussion of this issue, see Proctor 1978, 157ff.

consonance, augmented triads play a central role in their syntax, even when occluded from the music's surface and hence not directly accessible to perception. By virtue of their status as *perfectly even* trisections of the octave, augmented triads are the invisible axes about which pan-triadic progressions spin. Consonant triads acquire their distinctive voice-leading features in chromatic space by virtue of their status as *minimal perturbations* of the perfectly even augmented triads. The crucial property that consonant triads bear is one that Dmitri Tymoczko has named *near evenness*.[17] Major triads are nearly even because they can be formed from an augmented triad by a single semitonal displacement downward; minor triads, conversely, are formed by a semitonal displacement upward. A consonant triad is thus like a wheel that is dented just enough to affect a wobble but not so much that it is knocked off its rotation. It is this property that makes possible the minimal voice leading that hexatonic progressions feature.

Why does a nearly even chord bear the capacity to connect to an equivalently structured chord by minimal voice leading? Why does it uniquely bear that capacity? More generally, what is the connection between degree of evenness (proper to a chord's internal structure) and voice-leading magnitude (proper to its external relations)? We can begin to explore these questions by inspecting figure 2.15(a), a circle that intersects the vertices of an equilateral triangle. The vertices are labeled with the tones of an augmented triad. Each vertex is flanked by two black circles, labeled in two ways: with the tone that semitonally displaces a member of the augmented triad, and with the major or minor triad that results when that tone is combined with those of the remaining fixed vertices.[18]

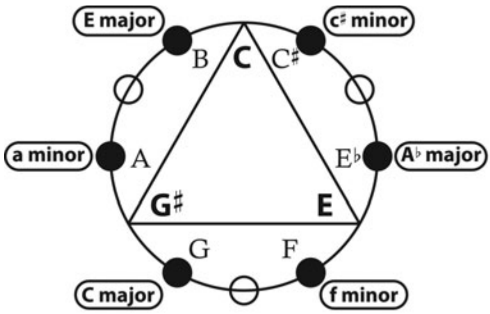
Figure 2.15(b) shows how a sample triad, C major, is generated from the augmented triad by the displacement G♯ → G. Our interest is in exploring the behavior of this triad as it is subjected to those shape-preserving transformations that involve minimal change. When the triangle is transformed in such a way that two vertices are preserved, how far is the third vertex displaced?

The shape-preserving transformations are of two types: rotation, equivalent to pitch-class transposition; and reflection, equivalent to pitch-class inversion. As the triangle in figure 2.15(b) is scalene, it cannot be rotated in such a way that two of its vertices are invariant. (This corresponds to the impossibility of transposing a consonant triad such that two tones are preserved.) Reflection, though, is another matter: to preserve two vertices, exchange their position by reflecting them about an axis halfway between them.

As there are three pairs of vertices, there are three axes around which such a reflection is possible. They are positioned as broken lines in the three components of figure 2.16. In each component, the C major triangle is presented in half-tones, and its inversion about the axis is presented in full tones. A double-headed arrow indicates the exchange of the two common tones. A single-headed arrow indicates

17. The connection between smooth voice leading and near evenness was initially suggested to me by John Clough in 1993, mentioned in Cohn 1996, 39n40, and elaborated in Cohn 1997, although using different terms. Tymoczko 2011b (14, 61, 85–93) positions the connection between near evenness and voice leading within a very broad framework with many concrete applications.

18. Figure 2.15(a) resembles some graphs that appear in the teaching materials of jazz guitarist Pat Martino (Capuzzo 2006). See also Siciliano 2005a.



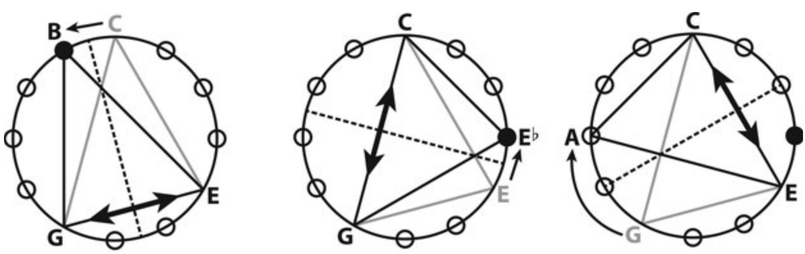
(a)
six triads derivable from CEG♯

(b)
derivation of C major from CEG♯

Figure 2.15. Derivation of consonant from augmented triads via single semitonal displacement.

the path from the remaining tone of the C major triad to the tone that replaces it as a result of inversion about that axis. That path's magnitude is double the distance that separates the tone from the axis. This is, of course, how inversion/reflection works: the farther the distance of some object from the axis, the farther that object is projected by inversion about it.

- In figure 2.16(a), the axis (represented as a broken line intersecting the center of the circle) is positioned halfway between E and G, so that those two tones reflect into each other (as indicated by the double-headed arrow). Under reflection about that same axis, the remaining tone, C, is replaced by B (as indicated by the single-headed arrow). Interpreted in pitch-class space, this reflection models the application of the **L** transformation on C major to produce e minor.



(a)
C+ ⟹ e-
axis splits E, G
C ⟶ B

(b)
C+ ⟹ c-
axis splits C, G
E ⟶ E♭

(c)
C+ ⟹ a-
axis splits C, E
G ⟶ A

Figure 2.16. Double common tone transformation of C major, depicted as inversion about an axis that holds two of its tones invariant.

- In figure 2.16(b), the axis mediates C and G, which invert into each other. The remaining tone, E, reflects into E♭. This reflection models **P** as it transforms C major to c minor.
- In figure 2.16(c), the axis mediates C and E, which invert into each other. The remaining tone, G, reflects into A. As this transformation involves two units of work—small, but not minimal—it has not arisen in this chapter; we shall study it in chapter 4.

For comparison, figure 2.17 shows the behavior under reflection of two other chord types, which are, respectively, more and less even than the C major triad of figure 2.16. Both chords share the CE dyad with C major; what defines their degree of evenness is the position of the third voice. In figure 2.17(a) that third voice is subject to a "null" perturbation, remaining on G♯ and creating a perfectly even augmented triad. At (b), the third voice is perturbed by three units, moving to B and creating a relatively uneven [015]-type trichord. In the perfectly even case, the axis that reflects C and E into each other also reflects G♯ into itself. Consequently, the perfectly even augmented triad cannot be distinguished from its reflection, and the transformation is a phantom. In the uneven case, reflection about the same axis slings F a tritone away, to B. This exercise demonstrates that, in order to create a small but recognizable displacement of a single voice under inversion, the trichord must be as even as possible, but not perfectly even.

What is true of nearly even trichords in a chromatic space of twelve tones is equally true of nearly even chords of any cardinality in a chromatic space of any size.[19] I mention here briefly several other cases of near evenness that are familiar to music theorists, or that we will have occasion to refer to in chapter 7.

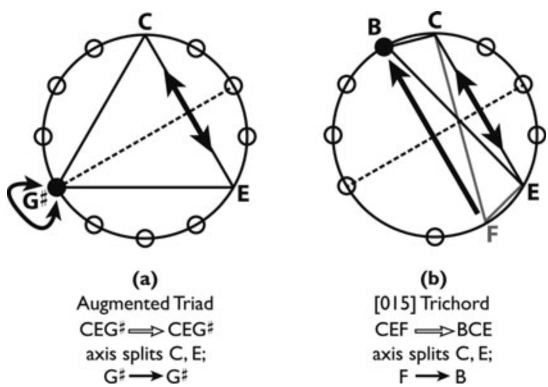


Figure 2.17. Double common tone transformations of two dissonant chords.

19. The general principles are established, in different ways, in Douthett 2008 and in Tymoczko 2011b.

- Dominant and half-diminished seventh chords, as minimal displacements of perfectly symmetric fully diminished seventh chords, voice lead to each other more smoothly than any other tetrachord type.
- Mystic chords, as minimal displacements of whole tone collections, have a similar capacity among hexachords.
- Diatonic collections, which are maximally but not perfectly even (Clough and Douthett 1991), communicate with each other via minimal displacement, a circumstance that underlies the system of key signatures at the heart of musical notation and theory of modulation. The same is true of their pentatonic complements (Kopp 1997).
- Diatonic triads, which are maximally but not perfectly even *in seven-tone diatonic space*, communicate with each other by single stepwise motion, a circumstance that underlies the system of diatonic third-relations central to some theories of harmonic function (e.g., Agmon 1991, 1995).

For the more circumscribed case that concerns us in this chapter, the central point is this: Via single semitonal displacement, each major triad communicates with two minor triads, and each minor triad with two major triads, precisely because major and minor triads are nearly even. One can thus draw a direct connection between near evenness and the unique ability of triads to participate in hexatonic chains and cycles. The role of near evenness, with respect to the participation of triads in hexatonic progressions, is thus analogous to that of consonance, with respect to their participation in diatonic ones. This suggests that the role of the augmented triad in the former case is analogous to that of the harmonic series in the latter: it is the concealed noumenon that gives rise to the revealed phenomenon. We might then find value in the claim that augmented triads *generate* pan-triadic syntax by way of nearly even trichords, just as many theorists have found value in the claim that the harmonic series generates diatonic syntax by way of consonant triads. This conjecture is given close scrutiny in chapter 3, which focuses more closely on the relationship between consonant and augmented triads in the music and the music theory of the nineteenth century.

Remarks on Dualism

The above discussion places us in a position to refine our understanding of the way that the system of triadic transformations rest on a dualistic basis. David Kopp (2002, 155) frames an extensive discussion of my early work on this topic under the subtitle "A Dualist Transformation System" and suggests without comment that the dualism is a limitation. Dmitri Tymoczko (2009a) makes a more sweeping claim: that the system of triadic transformations is "fundamentally dualist" and hence ripe for wholesale rejection. It is hardly necessary to explain why attributions of dualism are a priori problematic, since the term is identified with the discredited harmonic dualism of Oettingen and Riemann under which the triadic transformations were initially conceived.

There are two points to make in response. The first is that concepts may be detached from the framework in which they were initially conceived, in principle. This point will be familiar to music theorists, who identify Schenkerian prolongations and progressions without fear that they will be suspected of subscribing to the ideas of German racial superiority in support of which those ideas were evidently conceived. The second point is that relations that may appear to be fundamentally dualist may arise as epiphenomena of other relations. This is the central argument of Tymoczko 2011a: "Nineteenth-century composers were not explicitly concerned with inversional relationships as such; instead, these relationships appear as necessary by-products of a deeper and more fundamental concern with efficient voice leading (253)." The same argument applies to twenty-first-century theorists. Major and minor triads constitute a fundamental and coherent class of objects not because they are related to each other by inversion about an axis. They are related because they share the property of near evenness, and degree of evenness is invariant under inversion. Their inversional relation is a consequence of the capacity of the tones of an augmented triad to be semitonally perturbed in two directions, up and down.

What is true of the objects is true of their transformations: any hexatonic transformation introduced in this chapter can be defined with respect to its structure as a minimally displaced augmented triad. For example, **L** can be defined as the transformation that acts upon the tone that is further from the displaced tone, moving the former a semitone closer to the latter; and **P** as the transformation that acts upon the tone that is nearer to the displaced tone, moving the former a semitone further from the latter. Similar formulations can be used to characterize any of the remaining transformations introduced in this chapter and, indeed, any of the transformations and transformation classes introduced in chapters 4, 5, and 7.[20]

In adopting this position, we are not washing our hands of harmonic dualism, which Henry Klumpenhouwer characterizes as "music-theoretical work that accept[s] the absolute structural equality of major and minor triads as objects derived from a single, unitary process that structurally contains the potential for twofold, or binary, articulation" (2002, 459). Instead, we are viewing harmonic dualism as the product of a more fundamental melodic dualism, which posits that melodic motion proceeds in two opposite directions, which we figure in our culture as "up" and "down,"[21] and that there exists a sense in which it is productive to grant equivalent status to directed motions of equivalent magnitude but not equivalent direction. This dualism is so familiar as to be transparent; we invoke

20. To be sure, these formulations are cumbersome, so I often use inversion with reference to the triadic transformations. But, as I stressed in note 13 above, heuristics should not be confused with ontology. One can refer to inversional relations without believing that they are essential, just as one can refer to a sunset while still believing that the sun's position is fixed and the earth's is variable.

21. In other cultures it is figured as old/young, sharp/flat, skinny/plump, etc. See Zbikowski 2002, 67–68.

it implicitly whenever we refer to arpeggiation, passing or neighboring motion, or refer to an interval without specifying its direction. On this position, the unitary process to which Klumpenhouwer refers is not a harmonic one, as it was for nineteenth-century German theorists, but rather a melodic one: the single semitonal displacement of the augmented triad. The binary articulation involves neither overtones/undertones nor having/being, but rather an acknowledgment that such displacements may proceed either up or down.

Triadic Structure Generates Pan-Triadic Syntax

> Material may suggest what process it should be run through (content suggests form), and processes may suggest what sort of material should be run through them (form suggests content).
>
> —Steve Reich, *Writings about Music*

One of the desirable qualities of a theory is the ability to demonstrate a relationship between the internal properties of an object and its function within a system. A successful model of triadic music ought to give a coherent account not only of triadic behavior but also of why composers have selected triads to do the behaving. Accordingly, one of the most enduring features of classical tonal theory is its capacity to generate syntax from the phonological properties of its constituent objects. Remarkably, the conviction that phonological consonance generates syntactic proximity is held by consensus across the many denominations of classical theory, which conceive and represent their subject in distinct and often competing ways. Riemann's functions, Piston's Roman numerals, Schoenberg's structural functions, Schenker's *Ursätze*, and Lerdahl's pitch-space grids say different things about tonal syntax, but the acoustic properties of major and minor triads are foundational to each.

If triads are nothing but quintessentially consonant objects, why should they be asked to generate a syntax that is not predicated on their consonance? The good composer listens to the musical object, identifies its properties and tendencies, and recognizes the transformations that will extract the dynamic life from the object's interior, just as the good sculptor recognizes and extracts the form latent in the stone or the fallen log. It is a poor composer who runs any old object through any old machine and calls the result "art." A passage from Daniel Harrison's "Three Essays on Neo-Riemannian Theory," written in 2001 but only recently published (2011), gives some sense of what is at stake:

> Transformational theory in general requires a separation of object and activity, of what something is and what is done with it. . . . Objects are inert and without tendency, and all activity and meaning are supplied by transformations applied to them. From this far vantage point, transformational theory appears to model the metaphor of musical motion by constructing a ventriloquist's dummy; it only appears to be alive, but is in fact a construction of lifeless parts that are made to move by some external force. (552)

Although transformational theory, in its broadest outlines, may suffer from Harrison's gruesome vision, that branch of transformational theory that takes consonant triads for its objects, and subjects those objects to transformations that minimize voice leading, is immune from it, by virtue of the work we have done in this chapter. Establishing near evenness as a unique feature of consonant triads places us in a position to see that the structure of triads, as objects, is intimately related to their function, as participants in hexatonic (and, more broadly, pan-triadic) syntax.

Triads Are Homophonous Diamorphs

Identifying the triad as an optimal voice-leading structure by virtue of its near evenness does not detract from its status as an optimal acoustic structure by virtue of its consonance. What it suggests is that the triad is a homophonous diamorph: one sound, two forms.[22] There are two distinct, independent reasons for selecting major and minor triads as primary structures on which to build a musical syntax.[23] Even in some alternative universe where major and minor triads were acoustically dissonant, there would still be a musical motivation for inventing them and basing a musical system upon their properties. Dmitri Tymoczko (2011b, 64) makes a similar point with a Deist parable:

> Suppose God asked you, at the dawn of time, to choose the chords that humanity would use in its music. There are two different choices you might make. You might say, ". . . I'd like some nearly even chords that allow us to combine efficient voice leading and harmonic consistency. . . ." And God would hand you a suitcase containing nearly even chords, including the perfect fifth, the major triad, and dominant-seventh chord. On the other hand, you might say ". . . I'd really like to hear chords that *sound good*—chords whose intrinsic consonance will put a smile on my face." In this case, God would hand you a suitcase containing . . . the perfect fifth,

22. The term is appropriated from linguistic theories of code switching (Muysken 2000, 123), whose connections with music are explored in chapter 9.
23. The assertion that near evenness is independent of consonance is complicated by the strong correlation between the two properties. Tymoczko points out that nearly even chords in the twelve-note universe are among the most consonant of their cardinality (2011b, 14), and that "for small chords, maximal consonance implies near evenness" (61). The implication goes only one way—it is not the case that near evenness implies maximal consonance. This is true whether one assesses consonance by an overtone method or by an interval vector method. With an overtone method, the maximally consonant chord of cardinality *n* is the one that most closely approximates the first *n* odd partials of some generating fundamental. On this standard, the [02469] major ninth chord is the maximally consonant pentachord; but it is less even than the [02479] "usual" pentatonic. With an interval vector method, the maximally consonant chord of a given cardinality is the one whose interval vector entries for classes 3, 4, and 5 sum to the highest value. On this standard, the [0347] split third is the maximally consonant tetrachord, but it is less even than the nearly even [0258] dominant/half-diminished chord. The divergence between the two properties becomes more acute in larger "microtonal" universes, where the nearly even dyad moves away from the perfect fourth and closer to the tritone.

the major triad, the dominant-seventh chord. In other words, he would hand you *the very same chords*, no matter which choice you made.

The general phenomenon described here turns up frequently in the natural and social worlds, where it is referred to as *overdetermination*, *robustness*, or *Babylonianism*. These terms characterize “the use of multiple means of determination to ‘triangulate’ on the existence and character of a common phenomenon, object, or result” (Wimsatt 1981, 125).[24] These might include “using different assumptions, models, or axiomatizations to derive the same result or theorem” (127).

Human physiology presents many easily accessible examples of overdetermination. Mouths are for eating, talking, and breathing; ears serve auditory and vestibular functions; male urethrae channel both excretory and reproductive fluids. Organs transform seamlessly between the different functions that they fulfill. In humans, these organs are housed within a body that also includes an organ responsible for achieving and articulating awareness of the world within, without, and beyond. And yet these transformations evade recognition by that organ, under ordinary circumstances. Organs fulfill their overdetermined potentials well beneath the threshold of awareness.

But music is different, as its active production and passive experience necessarily involve, in some measure, the participation of a conscious, aware mental faculty. How, then, does triadic music execute the transformation between the multiple potentials of its constituent objects? This question will emerge as explicitly central in the closing chapters of this book. In the meantime, our concern will be directed toward refining the model of triadic syntax introduced in this chapter, which is predicated exclusively on fulfilling the triad’s syntactic potential as a nearly even object.

We will nonetheless be unable to forget that the triad is also something else that we have long known it to be. We will preserve that memory in our terms of reference: I will continue to refer to them collectively as consonant triads, even though we are more interested in their extensionally identical but intensionally distinct status as nearly even trichords. I will continue to name them individually by their roots, even though roots have no theoretical status in the theory of pan-triadic syntax. (Heuristics ≠ ontology; see notes 13 and 20.) And, in discussions of particular passages, I shall continue to casually invoke all manner of overlearned ascriptions and categories, in ways that contain unspoken implications about how composers move between or overlay diatonic and pan-triadic syntax, the nearly even trichord with the consonant triad. What I defer, until chapters 8 and 9, is the explicit theorization of this process. Our docket is otherwise full, as we seek to refine the notion of how the nearly even trichord generates pan-triadic syntax on its own terms, independently of diatonic tonality.

24. Wimsatt 1981 traces the origin of the tradition to Aristotle, “who valued having multiple explanations of a phenomenon” (125). Freud’s *Interpretation of Dreams* (1900) is the moment when overdetermination emerges to prominence in modern thought. Feynman 1965, 46, referred to this approach to knowledge as “Babylonian.”