T a g u n g s b e r i c h t 42/1997

"Einhüllende Algebren und Darstellungstheorie"

2. Nov. bis 8. Nov. 1997

Dies war die neunte Oberwolfacher Tagung über Einhüllende Algebren von Lie-Algebren nach den vorangegangenen in den Jahren 1973, 1975, 1978, 1982, 1985, 1987, 1990 und 1995.

Die Tagung fand unter der Leitung von W. Borho (Wuppertal), M. Duflo (ENS Paris), A. Joseph (Paris 6 und Rehovot) und R. Rentschler (Paris 6) statt.

Die Tagung "Einhüllende Algebren und Darstellungstheorie" war dazu bestimmt, die jüngsten Ergebnisse und laufende Entwicklungen aufzuzeigen. Die meisten Vorträge betrafen die halbeinfache Situation.
Die Tagung war geprägt durch neue Fragestellungen und Resultate, durch den Ausbau und die Verfeinerung bestehender Methoden, sowie durch das Auftreten neuer Methoden. Im Mittelpunkt des Interesses standen die folgenden Gebiete, die infolge ihrer Verbundenheit mit Untersuchungen Einhüllender Algebren und ihrer Darstellungen in enger Wechselwirkung miteinander stehen :

1. Differentialoperatoren auf Lie-Gruppen und auf Lie-Algebren
2. Neuere Entwicklungen in Bezug auf die Gelfand-Kirillov Vermutung (sowohl klassisch, wie auch im quantisierten Fall)
3. (Co-)adjungierte Orbiten und Verwendung ihrer symplektischen Struktur
4. Quantendeformationen Einhüllender Algebren und davon abgeleitete Algebren einschliesslich Quantenfunktionenalgebren
5. Spezielle Aspekte der Darstellungstheorie, insbesondere Kippmoduln (für Kac-Moody Algebren, für Quantengruppen mit Einheitswurzelparameter, sowie in Characteristik p) ; Struktur von $\wedge \mathfrak{g}$

6. Kanonische Basen

7. Fragen der Cohomologie

8. Multiplizitäten, affine Hecke Algebren, Kazhdan-Lusztig Polynome sowie Positivitätsfragen

9. Köcher und Hall-Algebren

Der Zusammenhang von Einhüllenden Algebren mit Differentialoperatoren ist nunmehr klassisch. Es wurden neue Resultate vorgestellt über gewisse invariante bezw. biinvariante Differentialoperatoren. Ausserdem wurden die neuesten Entwicklungen in Bezug auf die Gelfand-Kirillov Vermutung wiedergegeben (sowohl klassisch wie auch im quantisierten Fall).

In Bezug auf Orbiten wurde berichtet über die adjungierte Operation im unipotenten Radikal einer Parabolischen, sowie über Abelsche Systeme auf gewissen coadjungierten Orbiten (mit darstellungstheoretischen Konsequenzen und mit einer Beschreibung des ganzzahligen Momenten-Polytops).

Eine grosse Rolle spielen Quantendeformationen Einhüllender Algebren (Quantengruppen) sowie davon abgeleitete Algebren einschliesslich der sogenannten Quantenfunktionenalgebren (die sich aus dem Hopf-Dual ergeben). Einerseits stösst man im Vergleich zu den üblichen Einhüllenden Algebren auf neue oder verfeinerte Fragestellungen (R-Matrix, PI-Struktur bei Einheitswurzelparametern, Kippmoduln), zum anderen tauchen mit den Quantenfunktionenalgebren ganz neue Algebren auf , deren Spektrum sich oft in interessanter und natürlicher Weise stratifizieren lässt.

In Fragen der Darstellungstheorie wurden einerseits verschiedene Irreduzibilitätsresultate präsentiert; zum anderen Untersuchungen von Kipp-Moduln für Kac-Moody Algebren, für Quantengruppen (mit Einheitswurzelparameter) sowie in Characteristik p, einschliesslich von Zusammenhängen dieser Situationenen (insbesondere unter Verwendung von $\mathbb{Z}[v]_{(v,v-1)}$ als Grundring) . Präsentiert wurde ein detailliertes Resultat (Kostant) über die Struktur von $\wedge\mathfrak{g}$ mit Anwendung auf minimale K-Typen diskreter Serien.

Drei Vorträge befassten sich mit Fragen der Cohomologie : Ein neuer Zugang zur Ginzburg-Darstellung in der Cohomologie gewisser Springer-Fasern, ein Gegenbeispiel bezüglich Hanlon's Vermutung, sowie Fragen der endlichen Dimensionalität bezüglich des Cohomologierings von Lie-Superalgebren mit reduktivem geraden Anteil.

Relativ neu sind die sogenannten kanonischen (oder kristallinen) Basen (im nilpotent positiven Teil von Quantengruppen) . Sie stellen ein sehr wirksames Hilfsmittel dar, da sie insbesondere oftmals mit gewissen Untermodulstrukturen verträglich sind. Es wurden präsentiert Untersuchungen über das (eingeschränkt) multiplikative Verhalten von dual-kanonischen Basen, sowie die Konstruktion bemerkenswerter Basen für Darstellungen symmetrsierbarer Kac-Moody Algebren; eine Konstruktion, die etwa einen alternativen Beweis von Demazure's Charakterformel ermöglicht.

Seit der Kazhdan-Lusztig-Vermutung (Ende der 70er Jahre) und ihres Beweises weiss man, dass die von Hecke-Algebren herrührenden Kazhdan-Lusztig Polynome ein aussagekräftiges Mittel zur Bestimmung von Multiplizitäten in verschiedenen Situationen darstellen. Sie eröffnen ein weites Feld neuer Fragestellungen ; beispielsweise bezüglich der Gestalt gewisser Verallgemeinerungen (Kostka Kompositions-Funktionen) im Rahmen sogenannter doppelter Hecke-Algebren : von besonderem Interesse sind (wie präsentiert) Resultate der Ganzzahligkeit und Fragen der Positivität der Koeffizienten.
Auch wenn Kazhdan-Lusztig-Polynome und ihre Koeffizienten erlauben, die Dimension gewisser Gewichtsräume zu bestimmen, so bleiben Fragen und Antworten (wie präsentiert) nach der Existenz kombinatorischer Formeln, d.h. von Formeln in denen die Terme offensichtlich ganzzahlig und nichtnegativ sind.

Ein neuer Zugang zu Quantendeformationen Einhüllender Algebren und zu den kanonischen Basen ist die Verwendung von Hall-Algebren im Zusammenhang mit Darstellungen gewisser Köcher, wobei in gewisser Weise die Anzahlen von Erweiterungen gezählt werden. Auf diese Weise ist beispielsweise ein neuer Zugang zu Lusztig's Automorphismen von Quantengruppen möglich.

Insbesondere die neuen Methoden der letzten Jahre : Quantengruppen, kanonische (kristalline) Basen, Kipp-Moduln, Hall-Algebren haben eine Reihe neuer und unerwarteter Impulse gegeben. Die Vorträge spiegelten die Reichhaltigkeit der Entwicklung, die Lebendigkeit des Themas, sowie die Zusammenhänge der verschiedenen Aspekte wider.

Vortragsauszüge

Jacques ALEV :

A Survey on the Gelfand Kirillov conjecture : classical and quantum

We present the Gelfand Kirillov conjecture and some recent developments about it : a class of counter examples (joint work with A. Ooms and M. Van den Bergh), the fact that it is true for algebraic Lie algebras of dimension ≤ 7 and also the fact that it is now proved for $U_q(\mathfrak{sl}(n))$ and $\mathbb{C}_q[G]$ by F. Millet and P. Caldero. We also present a non commutative analog classical Noether's problem in the spirit of the Weyl skew fields.

Henning Haahr ANDERSEN :

Tilting modules for algebraic groups and quantum groups

Let G be a reductive algebraic group over a field k of characteristic p. Fix a maximal torus T, set $X = X(T)$, the character group of T and choose a dominant chamber X^+.

For each dominant weight $\lambda \in X^+$ we have the Weyl module $\Delta(\lambda)$, the dual Weyl module $\nabla(\lambda)$ and the simple module $L(\lambda)$, all having highest weight λ. In addition there exists a unique indecomposable tilting module $T(\lambda)$ with highest weight λ. Here tilting is having both a Δ-filtration and a ∇-filtration. If $\{\lambda_1, \lambda_2, ..., \lambda_r\} = \{\nu | [T(\lambda) : \Delta(\nu)]\} \neq 0$ is ordered such that $\lambda_i > \lambda_j$ only if $i < j$ then $T(\lambda)$ has a unique Δ-filtration of the form

$$0 \subset \Delta(\lambda) = E_0 \subset E_1 \subset ... \subset E_r = T(\lambda)$$

with $E_i/E_{i-1} \simeq \Delta(\lambda_i)^{d_i}$. We recall the Ringel construction which gives $d_i = dim\, Ext^1_G(\Delta(\lambda_i), E_{i-1})$.

The very same procedure works for the quantum group U_q corresponding to G when $q \in \mathbb{C}$ is a p'th root of 1. Denoting the tilting modules T_q in this case we have :

Conjecture. Suppose $< \lambda + \rho, \check{\alpha} > < p^2$ for all roots α and $p > h$. Then $ch\, T(\lambda) = ch\, T_q(\lambda)$.

We show that this conjecture is equivalent to a statement about the $Ext^1_{U_A}(\Delta_q(\lambda_i), E_{i-1}), i = 1...r$ where $A = \mathbb{Z}[v]_{(v,v-1)}$.

Alexander BRAVERMAN :

On Ginsburg's Lagrangian construction of representations cf $GL(n)$ (joint work with D. Gaitsgory).

It was observed by V. Ginzburg that one could realize irreducible representations of the group $GL(n,\mathbb{C})$ in the cohomology of certain Springer's fibers for the group $GL(d)$ (for all $d \in \mathbb{N}$). However, Ginzburg's construction of the action of $GL(n,\mathbb{C})$ on the above cohomology was rather complicated (he constructed explicitly the action of the Chevalley generators of the Lie algebra $\mathfrak{gl}(n)$).

In this work we give a very simple construction of the action of $GL(n,\mathbb{C})$ on the above cohomology, which avoids checking the Serre relations for generators. As an application we reprove the theorem of Bressler, Finkelberg and Lunts about irreducibility of the characteristic cycle of intersection cohomology sheaves on Grassmannians.

Philippe CALDERO :

The Gelfand-Kirillov conjecture for quantum algebras

Let q be a complex not root of one and $\mathfrak{g}$ be a semi-simple Lie $\mathbb{C}$-algbra. Let $U_q(\mathfrak{g})$ be the quantified enveloping algebra of Drinfeld-Jimbo, $U_q(\mathfrak{n}^-) \otimes U^0 \otimes U_q(\mathfrak{n})$ be its triangular decomposition and $\mathbb{C}_q[G]$ the associated quantum group. We describe explicity Fract $U_q(\mathfrak{n})$ and Fract $\mathbb{C}_q[G]$ as a quantum Weyl field. A similar description is given for the quantified field of regular fonctions on a Schubert Variety. We use for this a quantum analogue of the Taylor Lemma. If q is a root of one, then we may extend these result for $U_q(\mathfrak{n})$. We obtain in this way the center $Z(U_q(\mathfrak{n}))$.

Jean-Yves CHARBONNEL :

On biinvariant differential operators on a Lie group.

Let G be a connected Lie group whose Lie algebras is $\mathfrak{g}$. For x in $\mathfrak{g}$, the inverse image by the exponential map of the left invariant vector field generated by x, is denoted by $L(x)$. The map L is a Lie algebra homomorphism from $\mathfrak{g}$ to the underlying Lie algebra of the algebra of differential operators on $\mathfrak{g}$ with formal power series coefficients. This map has a natural extension to the enveloping algebra of $\mathfrak{g}$. It will be denoted by the same symbol L. For x in $\mathfrak{g}$, the vector field on $\mathfrak{g}$ defined by the adjoint action of x is denoted by $W(x)$. The map W is a Lie algebra homomorphism from $\mathfrak{g}$ to the underlying Lie algebra of the algebra of differential operators on $\mathfrak{g}$ with polynomial coefficients. We are interested in the following problem:

Let z be a point in the center of the enveloping Lie algebra and p_z its image by the Duflo's isomorphism. Let j be the usual analytic $\mathfrak{g}$-invariant function at the neighbourhood of 0 on $\mathfrak{g}$. Does the element $jL(z)j^{-1} - p_z$ belong to the left ideal of the algebra of differential operators on $\mathfrak{g}$ with formal power series coefficients, generated by the elements $W(x) + tr(ad(x))$, where x is in $\mathfrak{g}$.

In the solvable case it is a consequence of a Kashiwara-Vergne result about Campbell-Hausdorff formula. In the semi-simple case, it is a corollary of a Levasseur-Stafford result. In this lecture, we prove that it is true in the case of an algebraic Lie algebra. As in the Levasseur-Stafford work, the Harish-Chandra result about invariant eigen-distributions plays an important role. The second step of the proof is to prove that a differential operator on $\mathfrak{g}$, with polynomial coefficients, belongs to the left ideal generated by the elements $W(x) + tr(ad(x))$, if and only its Fourier transform annihilates the distributions defined by the invariant measures on all coadjoint orbits.

Vyjayanthi CHARI :

Kostant-Lusztig forms for quantum affine algebras

Let $U_q(\hat{\mathfrak{g}})$ be the quantized universal enveloping algebra of the affine algebra $\hat{\mathfrak{g}}$ and let $U_A(\hat{\mathfrak{g}})$ be the $A = \mathbb{C}[q, q^{-1}]$-subalgebra of $U_q(\hat{\mathfrak{g}})$ generated by the q-divided powers of the Chevalley generators. We prove a triangular decomposition for $U_A(\hat{\mathfrak{g}})$ and use this to classity irreducible finite-dimensional representations of quantum affine algebras at a root of unity.

Maria GORELIK :

The prime spectrum of a quantum Bruhat cell translate.

We study the prime and primitive spectra of a class of non-commutative rings which are parametrized by a split semisimple Lie algebra and an element w of the corresponding Weyl group W.

Such an algebra is a quantum analogue of the ring of regular functions $\mathbb{C}[wB^-B/B]$ on the open Bruhat cell translate.

The prime spectrum is presented as a disjoint union of strata $X(y,z)_w$ with $y, z \in W$ satisfying $y \leq w \leq z$. The stratum $X(y,z)_w$ turn out to be essentially independent of w.

These results generalize the results of A. Joseph in the case $w = e$.

Caroline GRUSON :

Cohomology of simple Lie superalgebras with reductive even part

The aim of this talk is to describe the cohomology ring of simple Lie subalgebras with reductive even part with trivial coefficients.

For basic classical cases, $\mathfrak{sl}(m,n)$ $m \neq n$, $\mathfrak{osp}(m,2n)$, $G(3)$, $F(4)$ and $D(2,1,\alpha)$, and for $\mathfrak{q}(2n)$, we have a theorem stating that the homology into (some) finite dimensional modules is a finite dimensional vector space. It is possible to compute the cohomology with trivial coefficients. It has already been computed by Fuks for the cases $\mathfrak{sl}(m,n)$, $m \neq n$ and $\mathfrak{osp}(m,2n)$.

For the remaining cases, $\mathfrak{q}(2n+1)$, $\mathfrak{p}(n)$ and $\mathfrak{psl}(n,n)$, the cohomology with trivial coefficients is no longer a finite dimensional vector space but a ring with Krull dimension 1.

Malek KEBE :

Sur la classification des $\mathcal{O}$-algebre quantiques.

Nous considérons ici une généralisation des algèbres enveloppantes quantiques de Drinfeld-Jimbo. Il est important de noter qu'une algèbre de ce type, notée $\check{U}_q$, ne correspond pas toujours à une quantification de l'algèbre enveloppante d'une algèbre de Lie de Kac-Moody. On se propose alors de donner, dans une certaine mesure, une classification des analogues quantiques des $\mathcal{O}$-algèbres semipremières introduites par Joseph. Soient $\check{T}$ l'ensemble des éléments de type groupe de $\check{U}_q^0$ et $\check{U}_q(\mathfrak{p}_{\pi'}^-)$ la sous-algebre de Hopf de $\check{U}_q$ similaire a l'algèbre enveloppante d'une sous-algèbre de Lie parabolique standard associée a un sous-ensemble $\pi' \subset \pi$ de racines simples. Le résultat principal est alors le suivant :

Théorème : 1) Toute $\mathcal{O}$-algèbre admet un quotient maximal avec la propriété d'être une $\mathcal{O}$-algebre semipremière primitivement finie.

2) Toute $\mathcal{O}$-algèbre semipremière primitivement finie est une somme directe finie de $\mathcal{O}$-algèbres $\check{T}$-premières primitivement finies.

3) Pour toute $\mathcal{O}$-algèbre $\mathcal{A}$, $\check{T}$-première primitivement finie, il existe un sous-ensemble π' de π, un quotient fini Γ de $\check{T}$ compatible avec π' et un $\check{U}_q(\mathfrak{p}_{\pi'}^-)$-module V de dimension finie tels que $\mathcal{A}$ soit isomorphe à l'algèbre $(\delta M_{\pi'}(0) \rtimes K\Gamma^\wedge) \otimes End(V)$

D'autre part les méthodes utilisées permettent également de généraliser le résultat classique de Joseph.

Alexander KIRILLOV, Jr. :

Kazhdan-Lusztig polynomials and canonical basis for quantum $\mathfrak{sl}(n)$

Let V be the fundamental representation of $U_q(\mathfrak{sl}(k))$. Then $V^{\otimes n}$ is naturally a module over the Hecke algebra $\mathcal{H}_n$. We show that Lusztig's canonical basis in $V^{\otimes n}$ coïncides with the Kazhdan-Lusztig basis for parabolic representations of the Hecke algebra. We also use this together with known results about canonical basis for $U_q(\mathfrak{sl}(2))$ to get explicit formulas for KL polynomials for Grassmanians. The formulas we get coïncide with those of Lascoux-Schützenberger and Zelevinsky.

Friedrich KNOP :

Composition Kostka Functions

For any finite root system, Cherednik constructed a double Hecke algebra $\mathcal{DH} = \mathbb{Q}_{t,q}[P] \otimes \mathcal{H}_f \otimes \mathbb{Q}_{t,q}[Q^\vee]$. Here $\mathcal{H}^\vee = \mathbb{Q}_{t,q} \otimes \mathcal{H}_f$ and $\mathcal{H} = \mathcal{H}_f \otimes \mathbb{Q}_{t,q}[Q^\vee]$ are both affine Hecke algebras. Let V be the induced module $\mathcal{DH} \otimes_{\mathcal{H}} \mathbb{Q}$. Then V can be identified with the Laurent polynomial ring $\mathbb{Q}[P]$. The action of $\mathbb{Q}[Q^\vee]$ on V can be diagonalized. The eigenvectors are the non-symmetric Macdonald polynomials E_λ. As an $\mathcal{H}$-module, V is induced from the parabolic subalgebra $\mathcal{H}_f$. This equips it with a standard basis M_λ, dual standard basis $\tilde{M}_\lambda$, and Kazhdan-Lusztig basis C_λ. Define a (non-symmetric) bilinear form $<,>$ on V by $< M_\lambda, \tilde{M}_\mu >= \delta_{\lambda,\mu}$. The composition Kostka functions are $K_{\lambda,\mu}(q,t) = < C_\lambda, E_\mu >$ (where E_μ has to be suitably normalized). In the A_n-case, we restrict to $\lambda, \mu \in \mathbb{N}^n \subset \mathbb{N}^{n+1} \subset \dots$

Theorem : $K_{\lambda,\mu} = lim_{n\to\infty} K^{(A_n)}_{\lambda,\mu}$ exists in $\mathbb{Q}((q,t))$ and is in fact in $\mathbb{Z}[q,t]$.

Conjecture : $K_{\lambda,\mu} \in \mathbb{N}[q,t]$.

Evidence : 1) If λ, μ are partitions then $K_{\lambda,\mu}$ is just Macdonald's (partition) Kostka function. This follows from the fact that C_λ is just a Schur polynomial. Thus our conjecture generalizes Macdonald's conjecture.

2) $K_{\lambda,\mu}|_{q=0}$ are (ordinary) Kazhdan-Lusztig polynomials, hence positive.

3) Thousands of examples computed.

Steffen KÖNIG :

Schur algebras and blocks of cyclic defect

In 92, C. C. Xi classified blocks of Schuralgebras $S_p(p,p)$ by quiver and relations. A more general result can be obtained as a consequence of the classification of blocks of cyclic defect (of finite groups) in the following way : Using the double centralizer property, one first has to look at blocks of $R\Sigma_n$ (R a discrete valuation ring, Σ_n the symmetric group). In case of cyclic defect, such a block is a Green order (which generalizes Brauer tree algebras). Since $R\Sigma_n$ is cellular, the associated tree is a line with no exceptionel vertex.

Since $S_R(n,r)$ is quasi-hereditary, its structure (in case of cyclic defect) can be read off from the of $R\Sigma_n$. A block looks as follows :

(R is a discrete valuation ring with (π) = rad R ; the ties are representing congruences mod π).

Bertram KOSTANT :

The structure of $\wedge\mathfrak{g}$ and discrete series

Let $\mathfrak{g}$ be a complex simple Lie algebra. Let l = rank($\mathfrak{g}$) and let $\mathfrak{b}$ be a Borel subalgebra. If $\mathfrak{a} \subset \mathfrak{g}$ is a k-dimensional subspace let $[\mathfrak{a}] \subset \wedge^k\mathfrak{g}$ be the 1-dimensional subspace spanned by $x_1 \wedge ... \wedge x_k$ where $\{x_i\}$ is a basis of $\mathfrak{a}$. Let $M \subset \wedge\mathfrak{g}$ be the span of all $[\mathfrak{a}]$ where $\mathfrak{a} \subset \mathfrak{g}$ is an abelian Lie subalgebra. Let Ξ be an index set for the set of all abelian ideals in $\mathfrak{b}$ and for $\xi \in \Xi$ let $\mathfrak{a}_\xi$ be the corresponding abelian ideal. Let $M_\xi \subset M$ be the irreducible $\mathfrak{g}$ -module with highest weight space $[\mathfrak{a}_\xi]$. Then $M = \sum_{\xi\in\Xi} M_\xi$ is the unique complete reduction of M. The eigenvalue of the Casimir operator in M_ξ equals the integer dim $\mathfrak{a}_\xi$. The representations M_ξ generalize a family of representations introduced by Deligne. A Theorem of Dale Peterson asserts that card $\Xi = 2^l$. Using his results we relate Ξ to the elements of order 2 of a maximal torus. One may then construct all $\mathfrak{a}_\xi$ from the θ stable Borel subalgebras using all equal rank real forms of $\mathfrak{g}$. Applications can be given to the minimal K-types for all the discrete series.

Shrawan KUMAR :

Hanlon's conjecture and Lie algebra homology

Let $\mathfrak{g}$ be either a semisimple Lie algebras or the nil-radical of a parabolic subalgebra of $\mathfrak{g}$. Then Hanlon conjectured that for any positive integer k,

$$H_*(\mathfrak{g} \otimes \mathbb{C}[t]/(t^k)) \approx H_*(\mathfrak{g})^{\otimes k}$$

I show that this conjecture is false in general for the case when $\mathfrak{g}$ is the nil-radical of the Borel subalgebra of a semisimple algebra. The counter example exists e.g. in the case of $\mathfrak{g}$ being the 4x4 upper triangular matrices.

Peter LITTELMANN :

Bases for representations of symmetrizable Kac-Moody algebras

For a complex irreducible representation V_λ of heighest weight, λ one has a "canonical" embedding $V_{l\lambda} \hookrightarrow V_\lambda^{\otimes l}$.

Let now $U_\zeta(\mathfrak{g}^t)$ be the quantum group of the dual Kac-Moody algebra at a $2l$-th root of unity. Then one has an embedding of representations $V_\lambda \hookrightarrow \hat{V}_\lambda^{\otimes l}$, where $\hat{V}_\lambda$ is the $U_\zeta(\mathfrak{g}^t)$- representation. Going to the dual representation, one has a restriction map

$$\hat{V}_\lambda^{*\otimes l} \rightarrow V_\lambda^*$$

One the left side one has a certain set of "canonical" vectors : the monomials (or tensors) of extremal weight vectors in V_λ^*.

We show that one can choose an l such that to any L-S path π of shape λ one can associate in a natural way such a monomial in $\hat{V}_\lambda^{*\otimes l}$. Denote the restriction to V_λ^* by p_π. The set of vectors p_π form a basis of V_λ^*.

This basis has a lot of nice geometrical properties, for example it is compatible with the filtration by kernels of the form $V_\lambda^* = H^0(G/B, \mathcal{L}_\lambda) \rightarrow H^0(X(\tau), \mathcal{L}_\lambda)$, one can use it to give an alternative proof of the Damazure character formula, (the construction holds in any characteristic), etc .

Leonid MAKAR-LIMANOV :

On representations of Lie algebras by locally nilpotent derivations

Let R be a finitely generated algebra over the field $\mathbb{C}$ of complex numbers. A set d_1, d_2,..., d_n of locally nilpotent derivations of R generates a finite dimensional Lie subalgebra of the Lie algebra of derivations of R if and only if the orbit of any $r \in R$ under the action of these derivations belongs to a finite dimensional space.

It is sufficient to check this condition on a set of generators of R.

Olivier MATHIEU :

Application of Verlindes formula

Let $\mathfrak{g}$ be a simple Lie algebra/$\mathbb{C}$, let $L(\lambda)$ be the simple $\mathfrak{g}$-module with highest weight λ. By Kazhdan-Lusztig conjecture, there is a formula

$$\dim(L(\lambda))_\mu = \sum_{w \in W} c_w K(w(\lambda+\rho) - (\mu+\rho))$$

where K( ) is Kostant partition function and c_w are some integers closely connected with KL polynomials. Unfortunately the numbers c_w can be < 0, thus this formula is not "combinatorial" (it means that it not clear why the RS is ≥ 0). For $L(\lambda)$ of finite dimension, there are also combinatorial formulas : for $\mathfrak{g} = \mathfrak{gl}(n)$, there is a combinatorial formula based on semi-standard tableaux (work of Littelwood and Richardson).
In a common work with Papapadopoulo, we extend the combinatorics of semi-standard tableaux for certain infinite dimensional $L(\lambda)$. Our approach is based on a formula for certain tensor product multiplicities for modular groups (Georgiev and M. 1992), called Verlinde's formula.

Florence MILLET-FAUQUANT :

Quantization of the Dixmier localization of $U(\mathfrak{sl}(n))$

Consider a complex simple finite dimensional Lie algebra $\mathfrak{g}$ and a parabolic subalgebra $\mathfrak{p}$. Let denote π (resp π') the set of simple roots associated to $\mathfrak{g}$ (resp.to $\mathfrak{p}$). To the "simply-connected" Drinfeld-Jimbo quantized enveloping algebra $U_q(\mathfrak{g}) = U$ one may associate a Hopf subalgebra $P = U_q(\mathfrak{p})$ corresponding to $\mathfrak{p}$. Let $Y(P)$ be the space generated by the semiinvariants of P under the adjoint action, $W(\pi)$ (resp. $W(\pi')$) the Weyl group associated to π (resp. to π') and w_0 (resp. w_0') the longest element of $W(\pi)$ (resp. $W(\pi')$). Let ϖ_i be the fundamental weights associated to the simple roots $\alpha_i \in \pi$ and λ be a dominant weight. We show that :

$$dim((adP)\tau(-4\lambda) \cap Y(P)) \leq 1$$

and

$$dim((adP)\tau(-4\lambda) \cap Y(P)) = 1 \Longleftrightarrow w_0'\lambda - w_0\lambda \in \sum_{\alpha_i \in \pi - \pi'} \mathbb{Z}\varpi_i$$

Consider the case of $\mathfrak{g} = \mathfrak{sl}_{n+1}$ and $\mathfrak{p}$ the maximal parabolic subalgebra obtained by suppressing essentially the last row. Then the above equation implies that $Y(P)$ is a polynomial ring of one variable d. Extending a classical result of Dixmier, we show that $U_d \cong P_d \otimes Z(U)$ where U_d (resp. P_d) is the localized of U (resp. P) by the Ore subset $\{d^n\}_{n \in \mathbb{N}}$ and $Z(U)$ is the centre of U. This is a much finer result than the quantum analogue of the Gelfand-Kirillov conjecture for $U_q(\mathfrak{sl}(n))$.

Maxim NAZAROV :

Irreducibility of tensor products of Yangian modules

There are several series of representations with similar conditions of irreducibility : the spherical principal series representations of a real semi-simple Lie group G (Kostant'69), the eigenspace representations of a non-compact symmetric space G/K (Helgason'76), the principal series representations of an affine Hecke algebra (Kato'83). For each of these series the irreducibility condition has the form $e(\lambda)e(-\lambda) \neq 0$, where $e(\lambda)$ is the numerator of the Harish-Chandra C-funtion . The proof for each series are however different.

We give another approach to the same phenomenon, based on the representation theory of Yangians. For the Yangian of the Lie algebra $\mathfrak{gl}(N)$ the above series correspond to a tensor product of the simplest representations of the form $\mathbb{C}^N(h)$ with $h \in \mathbb{C}$. We generalize this to the representations of the form $V(h)$ where $V = S^k(\Lambda^l(\mathbb{C}^N))$. The case k=1 corresponds to the Bernstein-Zelevinsky criterion of irreducibility of parabolically induced representations of the p-adic $GL(N)$, and to the recent results of Akasaka-Kashiwara on $U_q(A_N^{(1)})$.

Vladimir POPOV :

Orbits of parabolic subgroups acting on its unipotent radicals

Let P be a parabolic subgroup of a complex semi simple algebraic group. It is well known that there is a dense open orbit in the unipotent radical P_u of P with respect to the adjoint action of P. This makes the problem of describing the orbital decomposition of this action rather delicate.

We address this problem from the point of view of modality of action. This leads to the monotonicity results and the explicit classifications of parabolics of small modality (on particular, modality 0, i.e., with only finitely many P-orbits in P_u).

Markus REINEKE :

Multiplicative properties of dual canonical bases

Let U^+ be the positive part of the quantized enveloping algebra U over $\mathbb{Q}(v)$ of type A_n. Let B be Lusztig's canonical basis in U^+ and let B^* be the dual of B with respect to the standard inner product of U^+.

Berenstein and Zelevinsky conjecture that quasicommuting elements of B^* (i.e. elements commuting up to some power of v) are mutiplicative (i.e. their product belongs to B^* up to some v^D). They proved this for type A_2, A_3; in these cases all elements of B^* are products of quasicommuting quantum minors.

The aim of the talk was to prove parts of this conjecture for general A_n :

1. two quasicommuting elements of B^* are multiplicative if one of them is a product of "small" quantum minors,

2. one "chamber" of B^* (i.e. given by linear inequalities of B^* is parametrized appropiately) always consists of products of quasicommuting quantum minors.

Our main working tool is Ringel's Hall algebra approach parametrizing B^* by representations of quivers. In particular, we use the connection between B and degenerations of representations of quivers and a description of the crystal graph structure of B in terms of representations of quivers.

Wulf ROSSMANN :

Abelian systems and weight multiplicities

Il define an Abelian system of rank r on a complex (affine algebraic) Poisson variety Z to be an r-dimensional linear system of functions on Z,

$$\mathcal{A} : J = \theta_1 J_1 + \ldots + \theta_r J_r \ \ (\theta_1, \ldots, \theta_r \in \mathbb{C})$$

with the following three properties.

(a) commutative : $\{J_i, J_j\} = 0$,

(b) periodic : the Hamiltonian flow of J_i is periodic of period one,

(c) algebraic : J_i is a (multiple-valued) algebraic function on Z.

An Abelian system $\mathcal{A}$ has a moment map $\Phi : Z \to \mathcal{A}^* \approx \mathbb{C}^r$ and a weight lattice $\mathcal{A}^*(\mathbb{Z}) \approx \mathbb{Z}^r$. I assume given a Lie subalgebra $\mathfrak{h}$ of (Z) whose Hamiltonian vector fields generate a compact group H. Let $\mathcal{A}$ be an Abelian system on Z, Poisson-commuting with $\mathfrak{h}$ in $\mathcal{O}(Z)$, and maximal with this property. H-invariant functions on $\mathfrak{h}^*$ give H-invariant functions on Z which belong to the associative algebra generated by $\mathcal{A}$, resulting in a map

$$p : \mathcal{A}^* \to H \backslash \mathfrak{h}^*$$

dual to $\mathcal{O}(H\backslash\mathfrak{h}^*) \to \mathcal{O}(A^*)$. The integral moment polytope is
$\Pi := \Phi(\Gamma) \cap \mathcal{A}^*(\mathbb{Z})$ (Γ is a real form of Z).

Theorem. Let G:=compact classical, H:=maximal torus, $\lambda \in \mathfrak{h}^*(\mathbb{Z})$, $Z := Ad^*(G_{\mathbb{C}}).\lambda$, $\Gamma := Ad^*(G).\lambda$, $\Pi := \Phi(\Gamma) \cap \mathcal{A}^*(\mathbb{Z}), \chi_\lambda$:=the irreducible character of G with highest weight λ. There is a maximal Abelian system $\mathcal{A}$ on Z commuting with $\mathfrak{h}$ so that

$$\chi(expY) = \sum_{\lambda \in \Lambda} m(\Theta, \lambda) e^{\lambda(Y)}$$

where

$$m(\Theta, \lambda) := \#\{\Theta \in \Pi | \ \Theta|\mathfrak{h} = \lambda\}$$

and II consists of the Gelfand-Tsetlin tables for χ_λ.

I believe that this kind of reduction rule is valid in much greater generality.

Marc ROSSO :

Lyndon bases and the product formula for universal R-matrices

We give an elementary, combinatorial derivation of the formula describing the universal R-matrix of a quantized enveloping algebra as an ordered product of q-exponentials.

We use the quantum shuffle approach to quantum groups and the formalism of Lyndon bases (adapted to the braided framework). This allows for a natural definition of a root for a quantized enveloping algebra (without using analogy with the classical limit), and does not use Lusztig's braid automorphisms.

Wolfgang SOERGEL :

Characters formulas for tilting modules on Kac-Moody-Algebras

Let $\mathfrak{g} = \oplus \mathfrak{g}_i$ a $\mathbb{Z}$-graded Lie algebra over k with finite dimensional homogeneous pieces. Suppose $\mathfrak{g}$ is generated by $\mathfrak{g}_{-1} \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_1$ and there is a character $\gamma : \mathfrak{g}_0 \to k$ such that $\gamma([X,Y]) = tr(adXadY : \mathfrak{g}_0 \to \mathfrak{g}_0)\ \forall\ X \in \mathfrak{g}_1, Y \in \mathfrak{g}_{-1}$.

Theorem : The category of all $\mathbb{Z}$-graded $\mathfrak{g}$-modules which are graded free of finite rank over $U(\mathfrak{g}_{<0})$ is equivalent to its own opposed category.

For an irreducible $\mathfrak{g}_0$-module E of finite dimension our equivalence transforms the Verma module
$U(g) \otimes_{U(\mathfrak{g}_{\geq 0})} E$ to $U(\mathfrak{g}) \otimes_{U(\mathfrak{g}_{\geq 0})} (E^* \otimes k_{-\gamma})$.

Or is not hard to see that such an equivalence has to exchange tilting modules and projectives. On this way it leads to character formulas for tilting modules.

J. Toby STAFFORD :

Invariant differential operators on the tangent space of symmetric spaces (joint with T. Levasseur)

Let $\mathfrak{g}$ be a complex, semisimple Lie algebra, with an involutive automorphism θ and set $\mathfrak{k} = Ker(\theta - I)$, $\mathfrak{p} = Ker(\theta + I)$. We consider the differential operators, $\mathcal{D}(\mathfrak{p})^K$, on $\mathfrak{p}$ that are invariant under the action of the adjoint group K of $\mathfrak{k}$. Write $\tau : \mathfrak{k} \to Der(\mathfrak{p})$ for the differential of this action. Then we prove, for the class of symmetric pairs $(\mathfrak{g}, \mathfrak{k})$ considered by Sekiguchi that $\{d \in \mathcal{D}(\mathfrak{p}) : d((\mathfrak{p})^K) = 0\} = \mathcal{D}(\mathfrak{p})\tau(k)$.

One significance of this result is that is easily implies the following result of Sekiguchi : Let $(\mathfrak{g}_0, \mathfrak{k}_0)$ be a real form of one of these symmetric pairs $(\mathfrak{g}, \mathfrak{k})$, and suppose that T is a K_0-invariant eigendistribution on $\mathfrak{p}_0$ that is supported on the singular set. Then, $T = 0$. In the diagonal case $(\mathfrak{g}, \mathfrak{k}) = (\mathfrak{g}' \oplus \mathfrak{g}', \mathfrak{g}')$ this is a well-known result due to Harish-Chandra.

Michel VAN DEN BERGH :

On the double of a Hall algebra

If Q is a quiver then the Hall algebra $H(Q)$ is an algebra with basis the isoclasses of Q-representations. The multiplication defined by counting extensions is an appropriate way. By using the Fourier transform one shows that $H(Q)$ is independent of the orientation of Q. A natural question is to define reflection homomorphisms on Hall algebras, however it is easy to see that this is not possible. One has to apply a "double construction" which yields an algebra $U(Q)$. It turns out that one can correctly define reflections on $U(Q)$. Combining these with the Fourier transform yields a new construction of Lusztig automorphisms on $U_q(\mathfrak{g})$.

Berichterstatter : R. Rentschler

Tagungsteilnehmer

Prof.Dr. Jacques Alev
Dept. de Mathematiques
Universite de Reims
B.P. 347

F-51 062 Reims Cedex

Prof.Dr. Philippe Caldero
Departement de Mathematiques
Universite Claude Bernard de Lyon I
LAN Bat. 101
43, Bd. du 11 Novembre 1918

F-69622 Villeurbanne Cedex

Prof.Dr. Henning Haahr Andersen
Matematisk Institut
Aarhus Universitet
Ny Munkegade
Universitetsparken

DK-8000 Aarhus C

Prof.Dr. Jean-Yves Charbonnel
U. F. R. de Mathematiques
Case 7012
Universite de Paris VII
2, Place Jussieu

F-75251 Paris Cedex 05

Prof.Dr. Michel Van den Bergh
Department of Mathematics
Limburgs Universitair Centrum
Universitaire Campus

B-3590 Diepenbeek

Prof.Dr. Vyjayanthi Chari
Dept. of Mathematics
University of California

Riverside , CA 92521-0135
USA

Prof.Dr. Walter Borho
Fachbereich 7: Mathematik
U-GHS Wuppertal

42097 Wuppertal

Dr. Sophie Chemla
Institut de Mathematiques Pures
UMR-9994
Universite Pierre et Marie Curie
4, place Jussieu

F-75252 Paris Cedex 05

Prof.Dr. Alexander Braverman
Department of Mathematics
School of Mathematical Sciences
Tel Aviv University
Ramat Aviv, P.O. Box 39040

Tel Aviv 69978
ISRAEL

Prof.Dr. Michel Duflo
Departement de Mathematiques et
d'Informatique
Ecole Normale Superieure
45, rue d'Ulm

F-75005 Paris Cedex

Prof.Dr. Maria Gorelik
Dept. of Mathematics
The Weizmann Institute of Science
P. O. Box 26

Rehovot 76 100
ISRAEL

Prof.Dr. Malek Stefan Kebe
Laboratoire de Mathematiques
Fondamentales
Univ. Pierre et Marie Curie,UFR 921
4, place Jussieu, case no 191

F-75252 Paris Cedex 05

Dr. Caroline Gruson
UFR de Mathematiques, URA 748
Universite Denis Diderot - Paris 7
Case 7012
2, place Jussieu

F-75251 Paris Cedex 05

Prof.Dr. Aleksander Al. Kirillov
Dept. of Mathematics
Room 2 - 179
MIT

Cambridge , MA 02139
USA

Karen Günzl
Mathematisches Institut
Universität Freiburg
Eckerstr. 1

79104 Freiburg

Prof.Dr. Friedrich Knop
Dept. of Mathematics
Rutgers University
Busch Campus, Hill Center

New Brunswick , NJ 08903
USA

Prof.Dr. Jens Carsten Jantzen
Matematisk Institut
Aarhus Universitet
Ny Munkegade
Universitetsparken

DK-8000 Aarhus C

Dr. Steffen König
Fakultät für Mathematik
Universität Bielefeld
Postfach 100131

33501 Bielefeld

Prof.Dr. Anthony Joseph
Laboratoire de Mathematiques
Fondamentales
Univ. Pierre et Marie Curie,UFR 921
4, place Jussieu, case no 191

F-75252 Paris Cedex 05

Prof.Dr. Bertram Kostant
Department of Mathematics
Massachusetts Institute of
Technology

Cambridge , MA 02139-4307
USA

Prof.Dr. Shrawan Kumar
Dept. of Mathematics
University of North Carolina
Phillips Hall CB 3250

Chapel Hill , NC 27599-3250
USA

Prof.Dr. Leonid Makar-Limanov
Department of Mathematics
Wayne State University

Detroit , MI 48202
USA

Dr. Bernard Leclerc
Dept. de Mathematiques
Universite de Caen

F-14032 Caen Cedex

Prof.Dr. Olivier Mathieu
Institut de Mathematiques
Universite Louis Pasteur
7, rue Rene Descartes

F-67084 Strasbourg Cedex

Dr. Gail Letzter
Dept. of Mathematics
Virginia Polytechnic Institute and
State University
460 McBryde Hall

Blacksburg , VA 24061-0123
USA

Prof.Dr. Florence Millet
La Rochere

F-69700 Loire-sur Rhone

Prof.Dr. Thierry Levasseur
Mathematiques
Universite de Poitiers
40, Avenue du Recteur Pineau

F-86022 Poitiers Cedex

Dr. Maxim L. Nazarov
Dept. of Mathematics
University of York

GB-York YO1 5DD

Prof.Dr. Peter Littelmann
Institut de Mathematiques
Universite Louis Pasteur
7, rue Rene Descartes

F-67084 Strasbourg Cedex

Prof.Dr. Freddy van Oystaeyen
Dept. of Mathematics
Universitaire Instelling Antwerpen
Universiteitsplein 1

B-2610 Wilrijk

Dr. Gadi Perets
Departement de Mathematiques
Universite Claude Bernard de Lyon I
LAN Bat. 101
43, Bd. du 11 Novembre 1918

F-69622 Villeurbanne Cedex

Prof.Dr. Vladimir L. Popov
Musy Dzalilya 17-2-246

Moscow 115580
RUSSIA

Prof.Dr. Andrew Nicholas Pressley
Department of Mathematics
King's College London
University of London
Strand

GB-London WC2R 2LS

Markus Reineke
Fachbereich 7: Mathematik
U-GHS Wuppertal

42097 Wuppertal

Prof.Dr. Rudolf Rentschler
16, rue du Champ de Mars

F-75007 Paris

Prof.Dr. Alvany Rocha
Mathematics
Graduate School and Univ. Center
City University of New York
33 West 42nd Street

New York , NY 10036
USA

Prof.Dr. Wulf T. Rossmann
Department of Mathematics
University of Ottawa
585 King Edward

Ottawa , Ont. K1N 6N5
CANADA

Prof.Dr. Marc Rosso
Institut de Mathematiques
Universite Louis Pasteur
7, rue Rene Descartes

F-67084 Strasbourg Cedex

Dr. Steen Ryom-Hansen
Mathematisches Institut
Universität Freiburg
Eckerstr. 1

79104 Freiburg

Prof.Dr. Wolfgang Soergel
Mathematisches Institut
Universität Freiburg
Eckerstr. 1

79104 Freiburg

Prof.Dr. Tonny A. Springer
Mathematisch Instituut
Rijksuniversiteit te Utrecht
P. O. Box 80.010

NL-3508 TA Utrecht

Prof.Dr. J. Toby Stafford
Department of Mathematics
The University of Michigan
3220 Angell Hall

Ann Arbor , MI 48109-1003
USA

Prof.Dr. Michela Varagnolo
19, rue Ledru Rollin

F-94200 Ivry sur Seine

Prof.Dr. Eric Vasserot
Departement de Mathematiques
Universite de Cergy-Pontoise
Batiment Saint-Martin
2, avenue Adolphe Chauvin

F-95302 Cergy-Pontoise Cedex

Shmuel Zelikson
Laboratoire de Mathematiques
Fondamentales, U. E. R. 48
Universite Paris VI
4, Place Jussieu

F-75252 Paris Cedex 05