PROBLEM SET 7 (DUE ON THURSDAY, MARCH 23)

(All Exercises are references to the December 31, 2022 version of *Foundations of Algebraic Geometry* by R. Vakil.)

If you like thinking about non-algebraically-closed fields, you might want to think about what modifications are needed to remove that assumption in problems 2-5.

Problem 1. Exercise 20.2.B (deleting a point makes a curve affine)

Problem 2. Suppose C is an integral regular projective curve of genus 1 over an algebraically closed field k. Let L be a line bundle of degree 4 on C. Show that L identifies C with the intersection of two quadric surfaces in $\mathbb{P}^3_k$.

Problem 3. Suppose C is an integral regular projective nonhyperelliptic curve over an algebraically closed field k. Let p_1, p_2, p_3 be distinct closed points in C. Show that p_1, p_2, p_3 are collinear in the canonical embedding of C if and only if there exists a degree 3 morphism $\pi : C \to \mathbb{P}^1_k$ with $\pi(p_1) = \pi(p_2) = \pi(p_3)$.

Problem 4. Suppose C is an integral regular projective curve of genus $g \geq 2$ over an algebraically closed field k. Prove that there exists a degree 1 line bundle L on C with $h^0(C, L) = 0$.

Problem 5. Suppose C is an integral regular projective curve of genus 2 over an algebraically closed field k. Prove that C is trigonal (i.e. there exists a degree 3 morphism $\pi : C \to \mathbb{P}^1_k$).