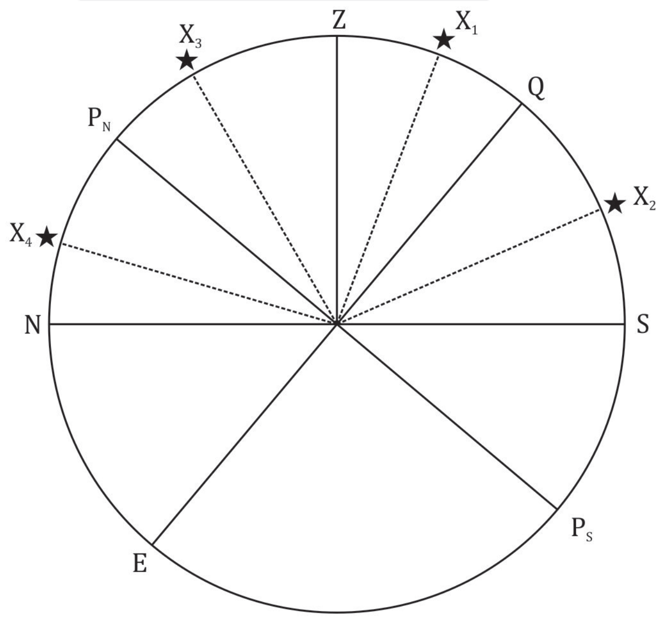
Latitude by Meridian Altitude

Consider an observer on a northern latitude observing heavenly bodies X_1, X_2, X_3 and X_4 which are in the same great circle of meridian as the observer, where X_1, X_2, X_3 are on the upper meridian, and X_4 is on the lower meridian.



Plane of the Observer's Meridian

Z	Zenith	EQ	Plane of the Equinoctial
QZ	Latitude of observer	NS	Plane of Rational Horizon
P_N	North poles	P_S	South pole

Let's name the altitude according to the bearing of the heavenly body from the observer, and name the zenith distance opposite from the altitude; e.g., if the body is on the South, the true altitude is South, and thus the zenith distance is named North.

Case 1 X_1 is on the upper meridian and on the south of the observer, so the zenith distance is named north and the body has north declination. In this case, the declination and zenith distance have same names.

$$ZQ = ZX_1 + X_1Q \quad \therefore \text{Latitude} = \text{Zenith Distance} + \text{Declination}$$

Case 2 X_2 is on the upper meridian and south of observer, so the zenith distance is north and the body is south of the equinoctial, so the declination is south. In this case, the zenith distance is greater than declination.

- Declination south, zenith distance north.

- Zenith distance is greater than declination.

$$ZQ = ZX_2 - X_2Q \quad \therefore \text{Latitude} = \text{Zenith Distance} - \text{Declination}$$

Case 3 X_3 is on the upper meridian and north of the observer, so it is named south and has north declination. But in this case, the declination is greater than the zenith distance.

- Declination north and zenith distance south.
- Declination is greater than zenith distance.

$$ZQ = X_3Q - ZX_3 \quad \therefore \text{Latitude} = \text{Declination} - \text{Zenith Distance}$$

Case 4 X_4 has north declination, but is on the lower meridian.

$$ZQ = 180° - \left(X_4Q' + ZX_4\right) = \left(90° - X_4Q'\right) + \left(90° - ZX_4\right)$$
$$\therefore \text{Latitude} = \text{Polar Distance} + \text{True Altitude}$$

Procedure to find the latitude by meridian passage

When the heavenly body is on the upper meridian

1. Find the declination of the body;
2. Find the true altitude of the body and name it North or South, according to the bearing from the observer;
3. Subtract true altitude from 90° to obtain the zenith distance, and name it opposite from the altitude;
4. Apply the declination to the zenith distance. If same names, then add; if opposite names, then subtract and name the latitude as the greater one.

When the heavenly body is on the lower meridian.

1. Find the declination;
2. Subtract declination from 90° to obtain Polar Distance;
3. Find the true altitude of the body;
4. Add the polar distance to the altitude to obtain the latitude, and always name the latitude the same as the declination.

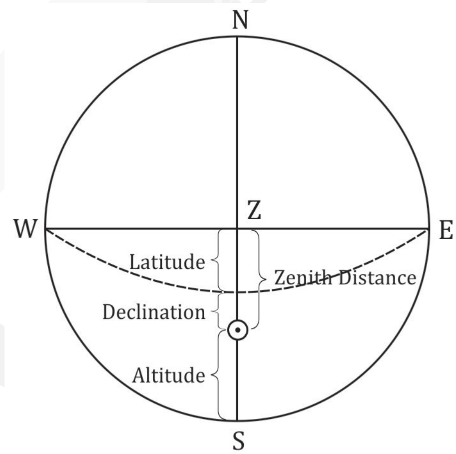
Procedure to obtain the latitude by meridian altitude of the sun

1. Find LMT and GMT of the meridian passage of the sun;
2. Use the GMT to extract the declination of the sun;
3. Take the altitude of the sun at LMT, make correction to obtain true altitude, and name it according to the bearing;
4. Calculate the zenith distance and name it opposite to the altitude;
5. Draw the sketch;
6. Apply the declination, zenith distance to obtain the latitude.

Example 1

On 26th October 2008, in longitude 112°45´E, the sextant altitude of the sun's lower limb is 41°24.7´ on the meridian bearing south. Index error 1.4´ off the arc; height of eye 12 metres. Temperature 30°C, pressure 1000mb. Find the latitude and position line.

$$\begin{array}{rll} \text{LMT mer. pass.} & 26^{th} & 11^h 44^m \\ \text{Longitude in Time} & & -7^h 31^m \ (112°45'E) \\ \hline \text{UT Mer. Pass.} & 26^{th} & 04^h 13^m \quad \Rightarrow \quad \text{Declination} = 12°33.2'S \end{array}$$

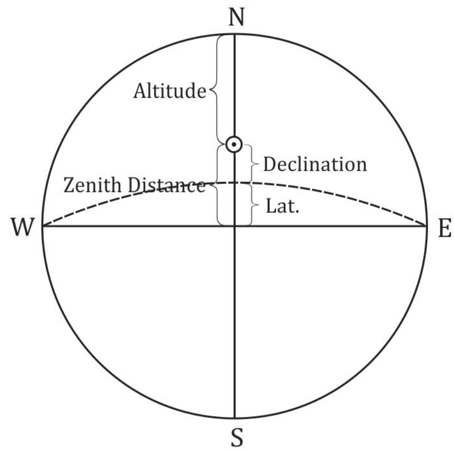


Sextant Altitude	41°24.7′	True Altitude	41°35.3′ S
Index Error	+1.4′		90°
Observed Altitude	41°26.1′	Zenith Distance	48°24.7′ N
Dip	−6.1′	Declination	12°32.2′ S
Apparent Altitude	41°20.0′	Latitude	35°52.5′ N
MainCorrection	+15.2′		
Additional Correction	+0.1′		
True Altitude	41°35.3′		

Position line runs 090° T / 270° T through position 35°52.5´N 112°45.0´E

Example 2 On 17th April 2008, in DR longitude 96°35´W, the sextant altitude of the sun's upper limb is 61˚25.0´ on the meridian bearing north; index error 1.4´ on the arc; height of eye 11.5 metres. Find the latitude and position line:

LMT mer. pass.	17^{th}	$11^h 59^m$
Longitude in Time		$+6^h 26^m$ (96°35′W)
UT Mer. Pass.	17^{th}	$18^h 25^m$ $\Rightarrow$ Declination = 10°48.2′N



Sextant Altitude	61°25.0′	True Altitude	61°01.0′ N
Index Error	−1.4′		90°
Observed Altitude	61°23.6′	Zenith Distance	28°59.0′ S
Dip	−6.0′	Declination	10°48.2′ N
Apparent Altitude	61°17.6′	Latitude	18°10.8′ S
MainCorrection	−16.6′		
Additional Correction	0.0′		
True Altitude	61°01.0′		

Position line runs 090° T / 270° T through position 18°10.8´ S 96°35.0´ W

Procedure to obtain the latitude by meridian altitude of the moon

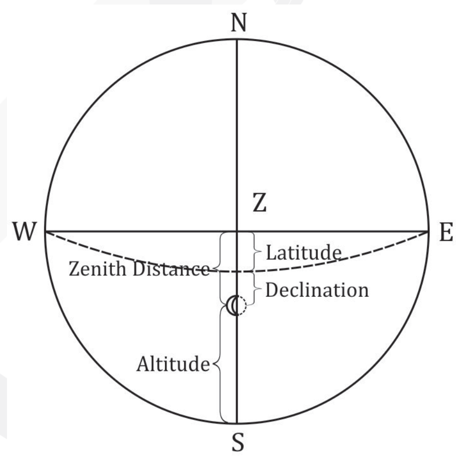
1. Find LMT and GMT of the meridian passage of the moon;
2. Use the GMT to extract the declination of the moon from Nautical Almanac;
3. Take the altitude of the moon at LMT, make correction to obtain true altitude and name it according to the bearing;
4. Calculate the zenith distance and name it opposite to the altitude;

5. Apply the declination, zenith distance to obtain the latitude.

Example 3 On 20th July, 2008, in longitude 062˚45´E, the sextant altitude of the moon's lower limb was 61˚35.9´ on the south of observer; index error 0.7´ off the arc; height of eye 12 metres; temperature 35˚C, pressure 1010mb. Find the latitude of the observer:

LMT mer. pass. Long. 0°	20th	01^h27^m
LMT mer. pass. Long. 0°	19th	00^h40^m
different		47^m

LMT mer. pass. Long. 0°	20th 01^h27^m		
Longitude correction	$+8^m$		
LMT mer. pass. (62°45′E)	20th 01^h35^m	Declination	16°34.8′ S
Longitude in time	4^h11^m	d = 12.0′	–4.9′
UT	19th 21^h24^m		16°29.9′ S



Sextant Altitude	61°35.9′	True Altitude	62°11.7′ S
Index Error	+0.7′		90°
Observed Altitude	61°36.6′	Zenith Distance	27°48.3′ N
Dip	–6.1′	Declination	16°29.9′ S
Apparent Altitude	61°30.5′	Latitude	11°18.4′ N
Main Correction (Part 1)	+37.7′		
(Part 2)	+3.5′		
Add. Corr. for Refraction	0.0′		
True Altitude	62°11.7′		

Procedure to obtain the latitude by meridian altitude of the star

Due to the declination of the star being constant over a long period of time, the meridian passage and GMT are not necessary for the calculation to find the declination. Practically, however, the LMT of the meridian passage should be calculated in advance so the observer knows exactly the time when the star is over the meridian, the approximate altitude can be computed, the sextant can be set ready so the altitude can be taken quickly and accurately, and also the observer knows in advance the best time for the observation. The declination of the star can be directly extracted from the daily page of the Nautical Almanac at the appropriate date.

1. Find the LMT of meridian passage of the star;

2. Extract the declination from Nautical Almanac at the appropriate date;

3. Take the altitude of the star at LMT, make a correction to obtain true altitude, and name the altitude according to the bearing;

4. Calculate the zenith distance and name it opposite to the altitude;

5. Apply the declination, zenith distance according to the sketch to calculate the latitude.

Example 4

On 19th July, 2008 at DR position 44°N 138°E, Star Arcturus was observed at altitude 65°07.4´ on the same meridian as the observer, bearing south; index error 1.1´ on the arc, eye height 10.0 metres. Find the latitude of the observer:

When star Arcturus is on same meridian as observer, but on easterly longitude, then:

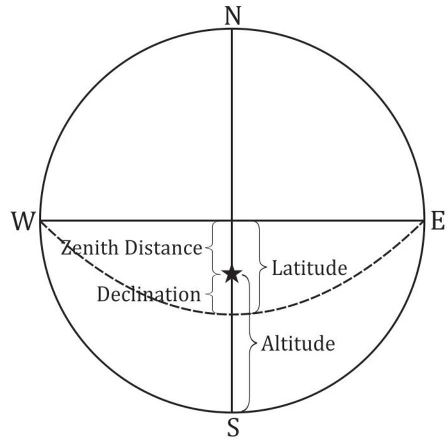
$$\text{GHA}^* = 360° - \text{Longitude} = 360° - 138° = 222°$$

$$\begin{array}{rr} \text{GHA}^* & 222°00.0' \\ \text{SHA}^* & \underline{145°59.0'} \\ \text{GHA}^{\Upsilon} & 76°01.0' \\ \text{GHA}^{\Upsilon} \text{ on } 19^{\text{th}} \text{ at } 9^{\text{h}}00^{\text{m}} & \underline{72°31.9'} \\ & 3°29.1' \quad = 13^{\text{m}}54^{\text{s}} \end{array}$$

$$\therefore \text{UT mer. pass. on } 19^{\text{th}} \text{ is } 09^{\text{h}}13^{\text{m}}54^{\text{s}}$$

$$\begin{array}{rrr} \text{UT mer. pass.} & 19^{\text{th}} & 09^{\text{h}}13^{\text{m}}54^{\text{s}} \\ \text{Longitude in time (138°E)} & & \underline{9^{\text{h}}12^{\text{m}}00^{\text{s}}} \\ \text{LMT mer. pass.} & 19^{\text{th}} & 18^{\text{h}}25^{\text{m}}54^{\text{s}} \end{array}$$

Depending on the longitude in time, choose the date to extract the GMT for the GHA of Aries so it will give the required local date.



Sextant Altitude	65°07.4′	True Altitude	65°00.2′ S
Index Error	−1.1′		90°
Observed Altitude	65°06.3′	Zenith Distance	24°59.8′ N
Dip	–5.6′	Declination	19°08.3′ N
Apparent Altitude	65°00.7′	Latitude	44°08.1′ N
Correction	–0.5′		
True Altitude	65°00.2′		

Procedure to obtain the latitude by lower meridian altitude

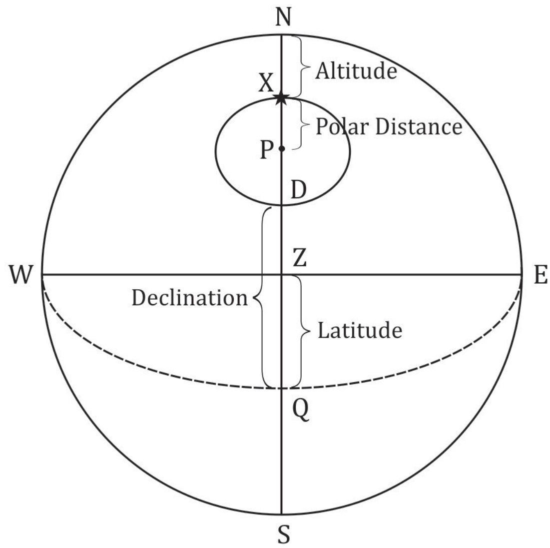
As all heavenly bodies circle around the pole, they will cross the observer's meridian and also the observer's anti-meridian. Bodies that remain visible to the observer the whole time are called **circumpolar** bodies. Such a body is visible to the observer when it crosses the anti-meridian, or lower meridian passage. In this case, the declination of the body is always greater than the latitude, or the latitude is always greater than the **polar distance** of the body.

1. Find the LMT of the anti-meridian passage of the body;
2. Extract the declination from the Nautical Almanac;
3. Subtract declination from 90° to obtain the polar distance;
4. Take the altitude of the body at LMT, make correction to obtain true altitude;
5. Add the altitude to the polar distance to obtain the latitude. Name the latitude the same as the declination.

$$\left.\begin{array}{l} QZ + PZ = 90^\circ \\ PN + PZ = 90^\circ \end{array}\right\} \quad \therefore QZ = PN \quad \because PN = PX + NX \quad \therefore QZ = PX + NX$$

Latitude = Polar Distance + Altitude

$PX = PD = 90° - QD \qquad \therefore \text{Polar Distance} = 90° - \text{Declination}$

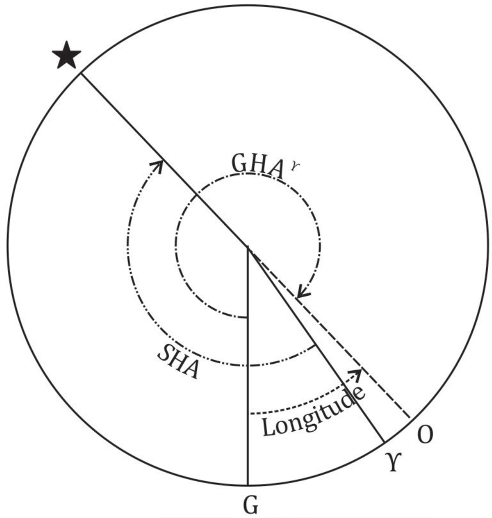


Example 5 On 22nd July, 2008, in DR position 49°40.0´S 50°00.0´E, Star Hadar was observed on a meridian below the pole with altitude 20°18.4´; index error 0.9´ on the arc; height of eye 10.5 metres. Find the latitude:

When the observer is on an easterly longitude, and the celestial body is on the same upper meridian as the observer, the GHA of the celestial body will be 360°–Longitude. But if the celestial body is on a lower meridian, which is 180° away from the observer; the GHA of the celestial body in the lower meridian is 180° different from the GHA of the celestial body when on an upper meridian.

	360°00.0′
Longitude	50°00.0′
GHA*	310°00.0′ (upper meridian)
	180°00.0′
GHA*	130°00.0′ (lower meridian)
SHA*	148°53.4′
GHA♈	341°06.6′
GHA♈ 22nd 02h	330°12.1′
	10°54.4′ = 43m31s

∴ UT mer. pass. on 22nd is 02h 43m31s



UT mer. pass.	22nd	$2^h 43^m 31^s$
Longitude in time (50°E)		$3^h 20^m 00^s$
LMT mer. pass.	22nd	$6^h 03^m 31^s$

Sextant Altitude	20°18.4′		
Index Error	–0.9′		
Observed Altitude	20°17.5′	Declination	60°25.2′ S
Dip	–5.7′		90°
Apparent Altitude	20°11.8′	Polar Distance	29°34.8′
Correction	–2.6′	True Altitude	20°09.2′
True Altitude	20°09.2′	Latitude	49°44.0′ S