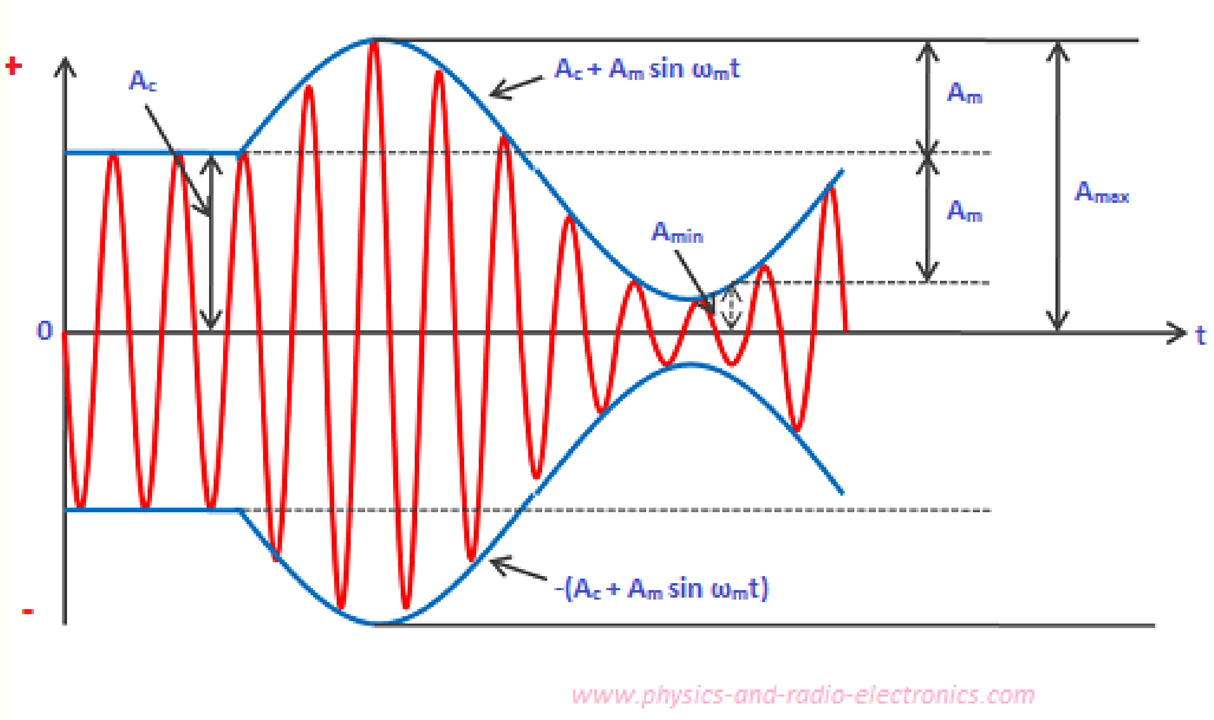
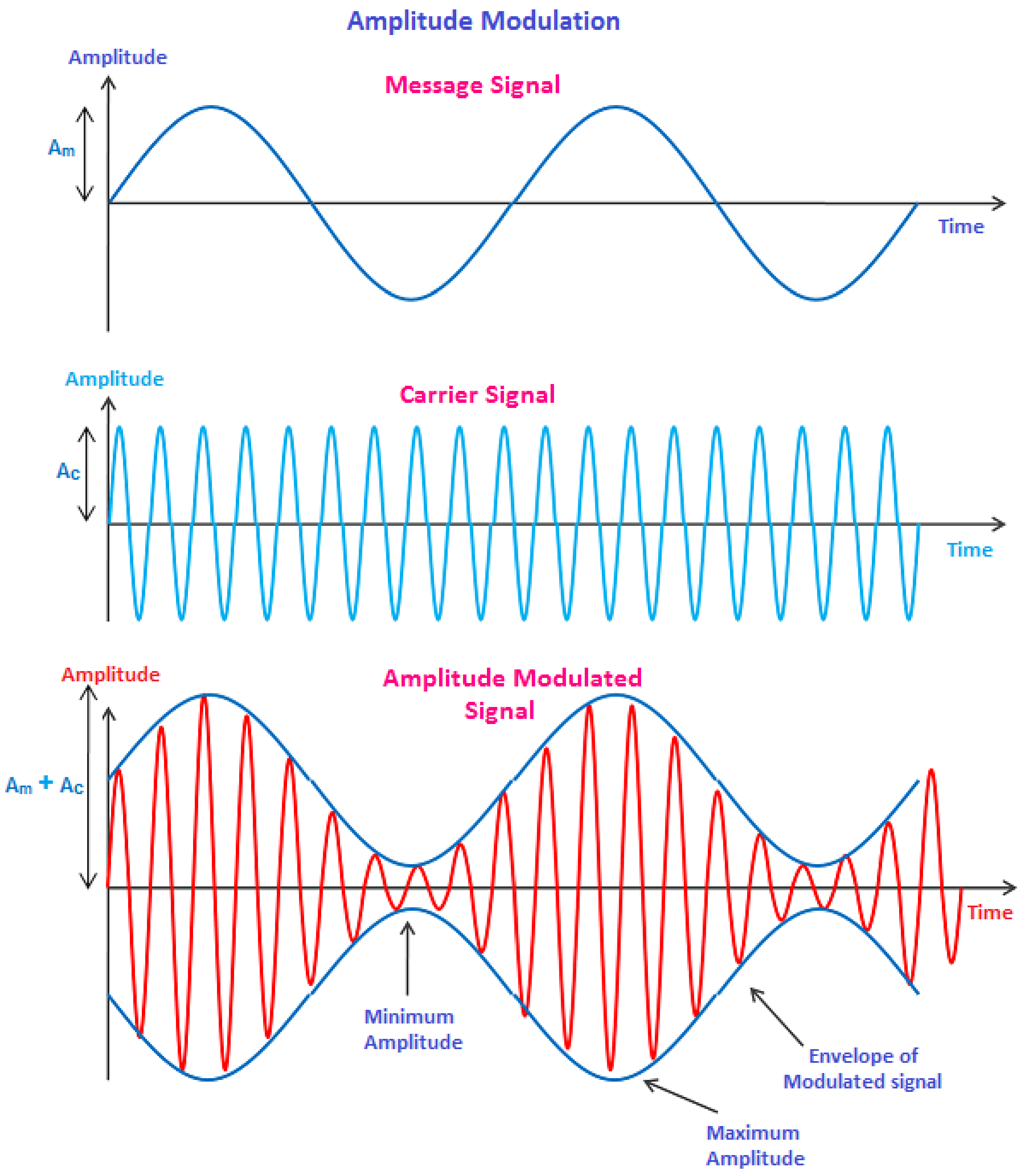
AMPLITUDE MODULATION



Amplitude Modulation

- Amplitude modulation is a type of modulation where the amplitude of the carrier signal is varied in accordance with the amplitude of the message signal keeping phase and frequency constant.
- Carrier signal contains no information.



Mathematical Expression

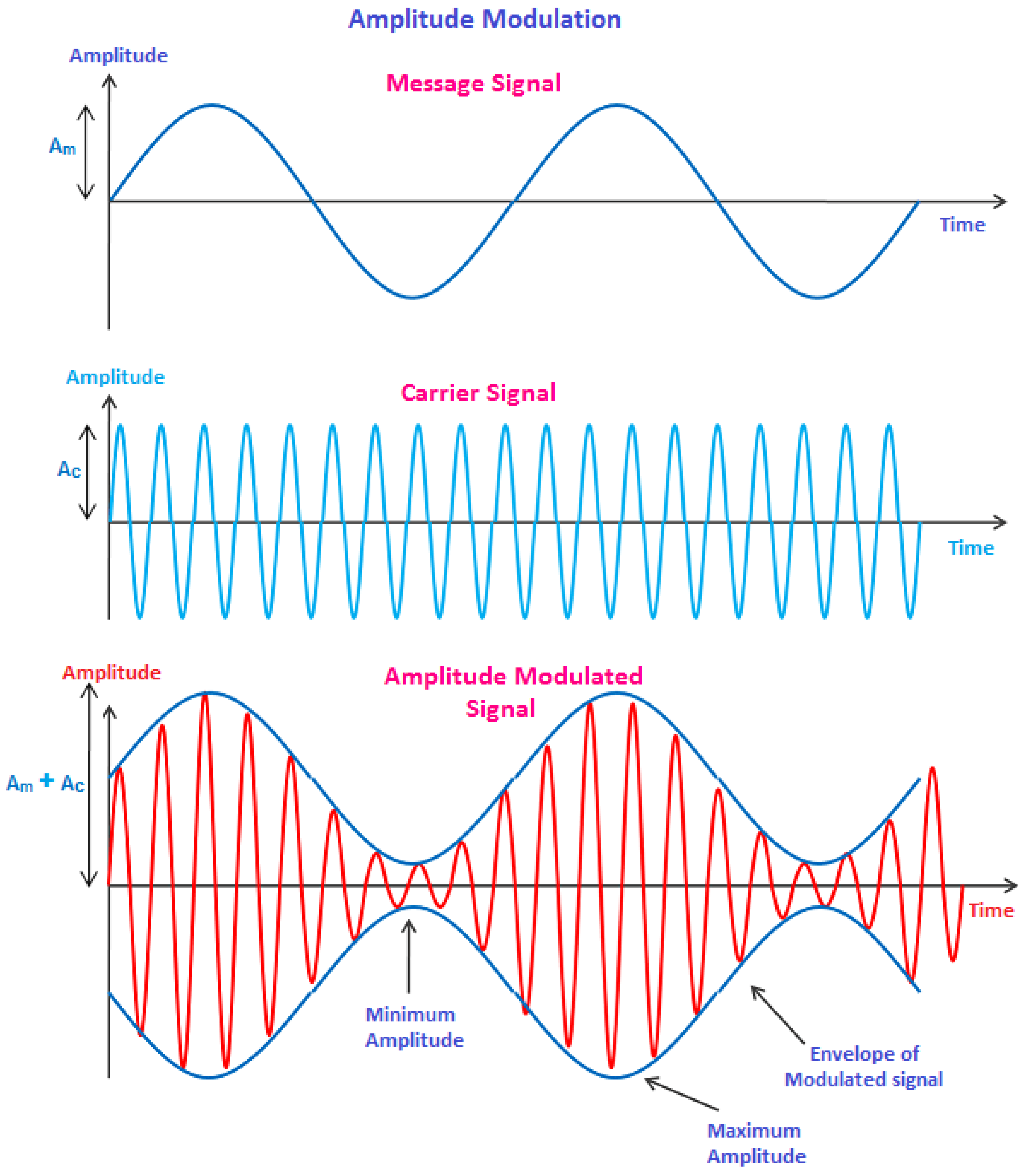
- sinusoidal modulating signal or message signal (am) of frequency (ωm) and amplitude (Am) given by:

$$am = Am \sin \omega mt \quad ...... (1)$$

Where, am is the modulating signal

Am = maximum amplitude of the message signal

ωm = frequency of the message signal



Mathematical Expression

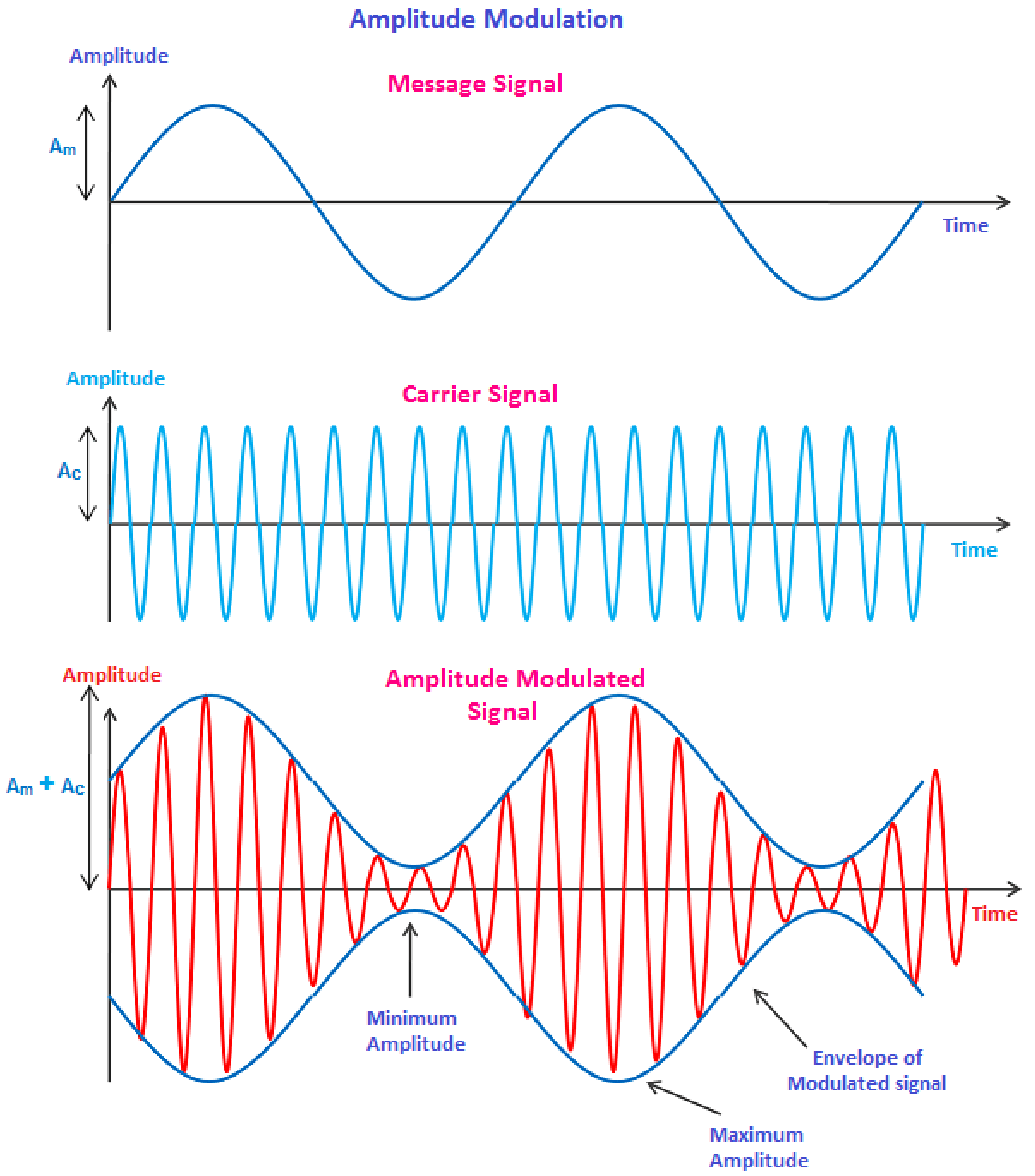
- carrier wave (ac) of frequency (ωc) and amplitude (Ac) given by:

$$ac = Ac \sin \omega ct \quad \ldots\ldots \quad (2)$$

Where, ac is the carrier signal

Ac = maximum amplitude of the carrier signal

ωc = frequency of the carrier signal

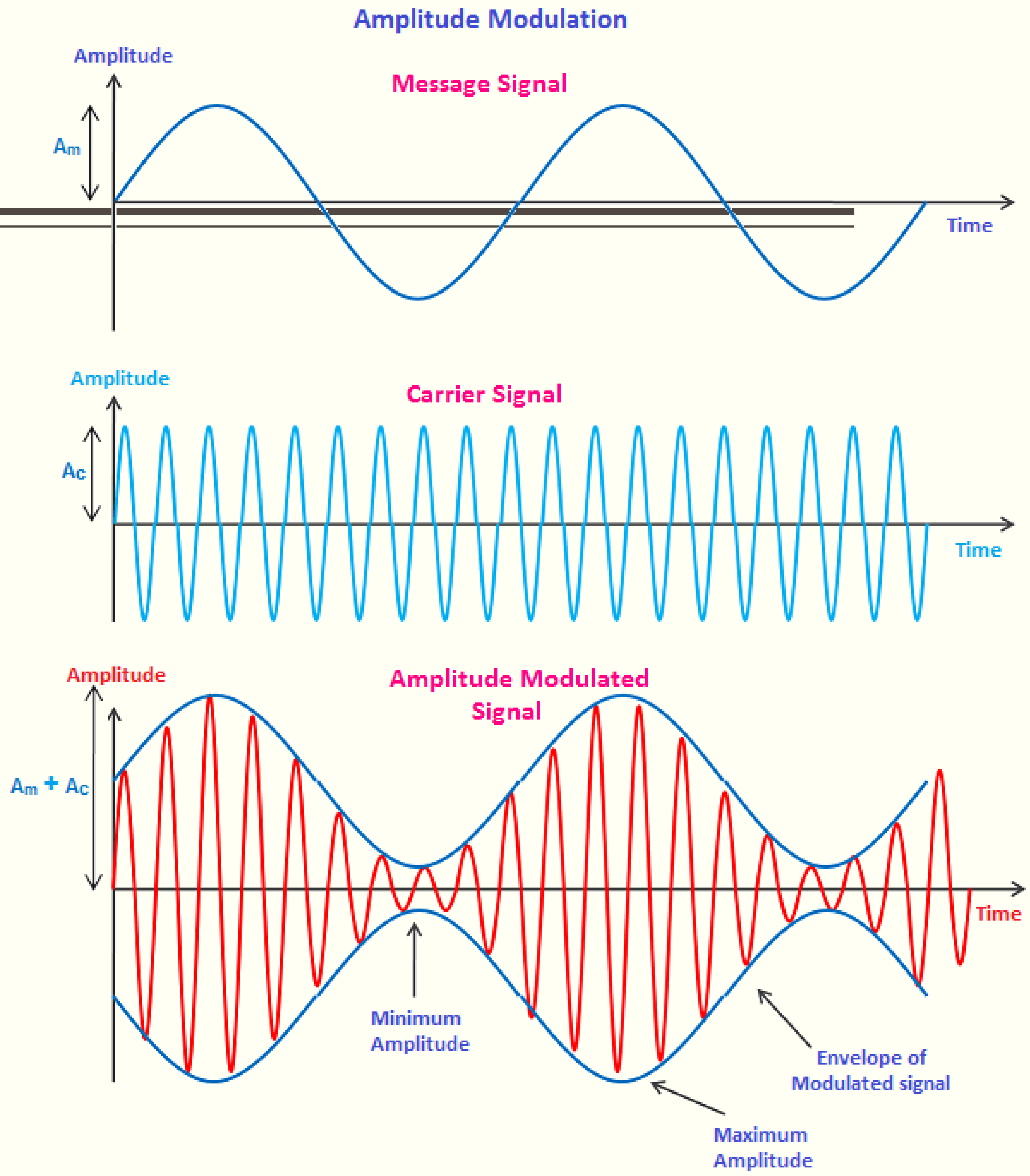


Modulation index of AM

- Modulation index or modulation depth describes how the amplitude, frequency or phase of the carrier signal and message signal affects the amplitude, frequency or phase of the modulated signal.
- Amplitude modulation index describes how the amplitude of the carrier signal and message signal affects the amplitude of the amplitude modulated (AM) signal.
- Amplitude modulation index is defined as the ratio of the maximum amplitude of message signal to the maximum amplitude of carrier signal.

$$\text{Modulation Index } (M_i) = \frac{A_m}{A_c}$$

Where,
Am is the maximum amplitude of the message signal
Ac is the maximum amplitude of the carrier signal



Mathematical Expression

$$am = Am \sin \omega mt \quad \ldots\ldots \quad (1)$$

$$ac = Ac \sin \omega ct \quad \ldots\ldots \quad (2)$$

Using the above mathematical expressions for message signal and the carrier signal, we can create a new mathematical expression for the complete modulated wave.

The amplitude modulated wave (A) is given as:

$$A = Ac + am.. \quad \ldots\ldots \quad (3)$$

Put am value from equation (1) into equation (3)

$$A = Ac + Am \sin \omega mt \ldots\ldots (4)$$

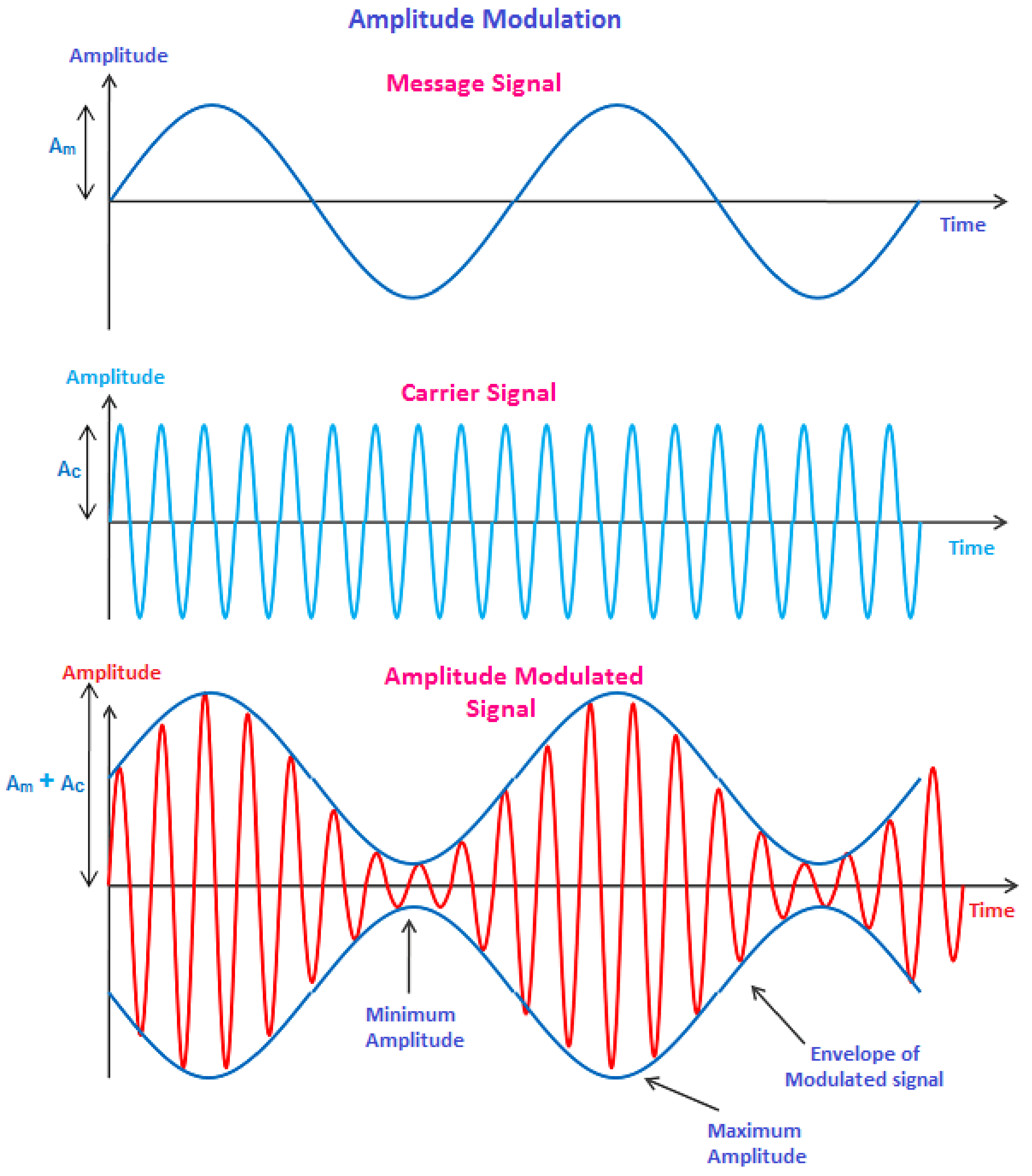
The instantaneous value of the amplitude modulated wave can be given as:

$$a = A \sin \theta$$

$$a = A \sin \omega ct \ldots\ldots (5)$$

Put A value from equation (4) into equation (5)

$$a = (Ac + Am \sin \omega mt) \sin \omega ct \ldots\ldots \quad (6)$$



Modulation index of AM

Now we have

$$a = (Ac + Am \sin \omega mt) \sin \omega ct \ldots . \quad (6)$$

We know that Mi = Am / Ac.
Hence we have Am = Mi Ac

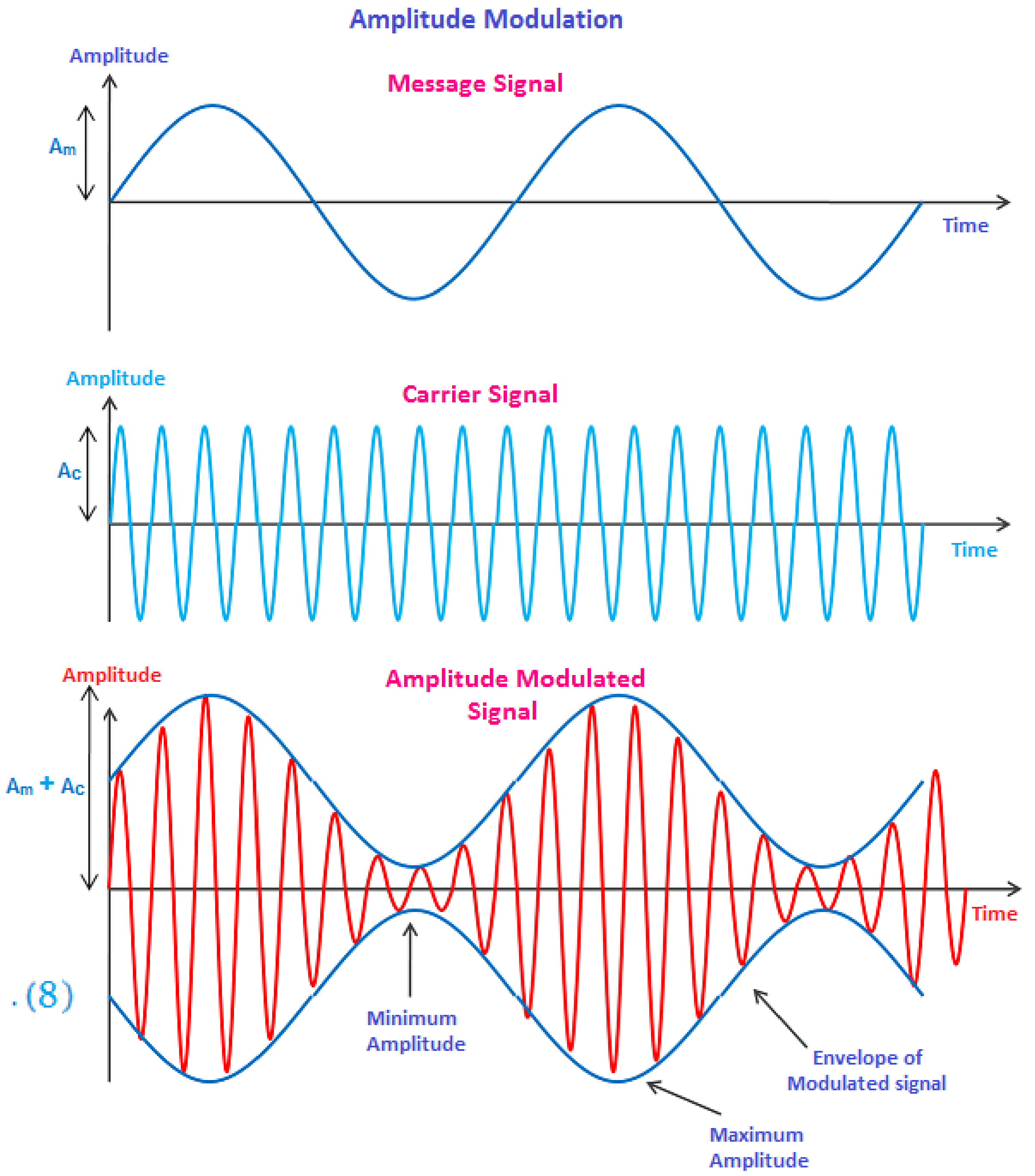
Putting this value of Am in above equation (6) we get,

$$\begin{aligned} a &= (Ac + Mi\, Ac \sin \omega mt) \sin \omega ct \\ &= Ac\,(1 + Mi \sin \omega mt) \sin \omega ct \\ &= Ac \sin \omega ct + Ac\, Mi \sin \omega mt \sin \omega ct \ldots \ldots . . (7) \end{aligned}$$

We know that $\sin(A) \sin(B) = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$

Applying this result to last term in above equation (7) we get,

$$a = A_c \sin \omega_c t + A_c M_i \frac{1}{2}\cos(\omega_c - \omega_m)t \; - A_c M_i \frac{1}{2}\cos(\omega_c + \omega_m)t \;..(8)$$



Equation of AM Wave

$$a = A_c \sin \omega_c t + A_c M_i \frac{1}{2} \cos (\omega_c - \omega_m) t - A_c M_i \frac{1}{2} \cos (\omega_c + \omega_m) t$$

In the above equation, the first term represents unmodulated carrier, the second term represents lower sideband and the last term represents upper sideband.

Note that ωc = 2πfc and ωm = 2πfm. Hence, the above equation can also be written as

$$a = A_c \sin 2\pi f_c t + A_c M_i \frac{1}{2} \cos 2\pi (f_c - f_m) t - A_c M_i \frac{1}{2} \cos 2\pi (f_c + f_m) t$$

$$a = A_c \sin 2\pi f_c t + A_c M_i \frac{1}{2} \cos 2\pi f_{LSB} t + A_c M_i \frac{1}{2} \cos 2\pi f_{USB} t$$

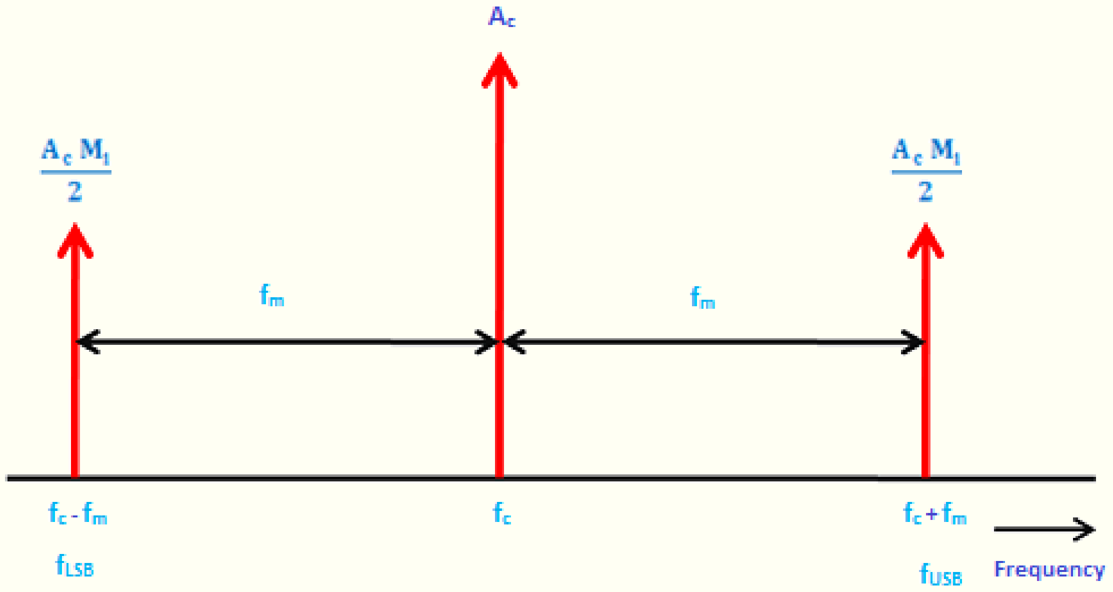
Equation of AM Wave

$$a = A_c \sin \omega_c t + A_c M_i \frac{1}{2} \cos (\omega_c - \omega_m) t - A_c M_i \frac{1}{2} \cos (\omega_c + \omega_m) t$$

$$a = A_c \sin 2\pi f_c t + A_c M_i \frac{1}{2} \cos 2\pi (f_c - f_m) t - A_c M_i \frac{1}{2} \cos 2\pi (f_c + f_m) t$$

$$a = A_c \sin 2\pi f_c t + A_c M_i \frac{1}{2} \cos 2\pi f_{LSB} t + A_c M_i \frac{1}{2} \cos 2\pi f_{USB} t$$

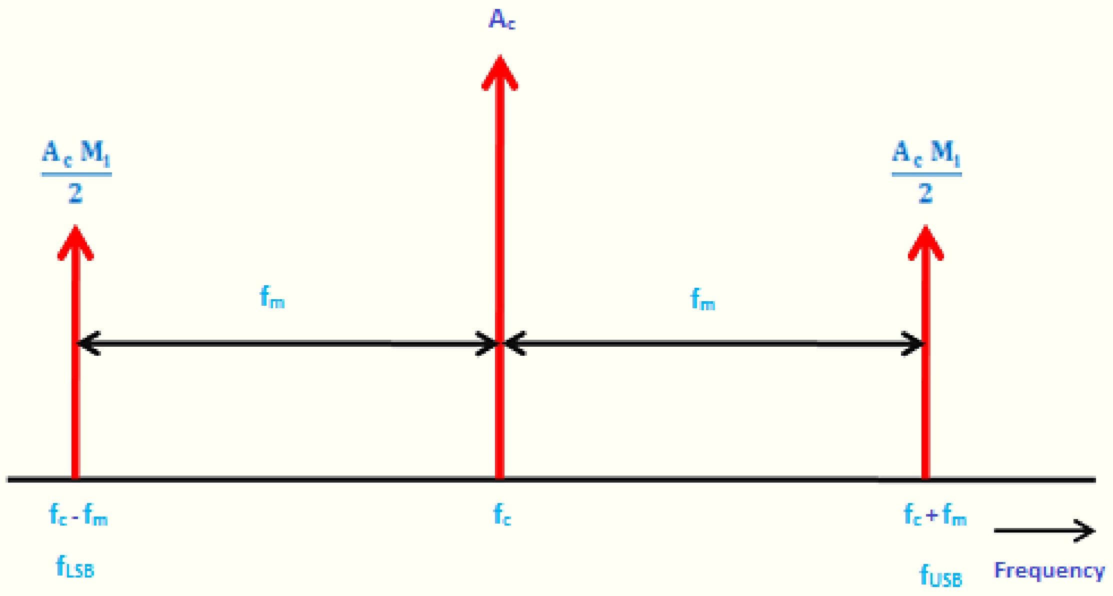
From these above equations we can prepare the frequency spectrum of AM wave as shown in the below figure.



This contains the full carrier and both the sidebands. Hence, it is also called Double Sideband Full Carrier (DSBFC) system.

Bandwidth of Amplitude Modulation

The bandwidth of the signal can be obtained by taking the difference between the highest and lowest frequencies of the signal.



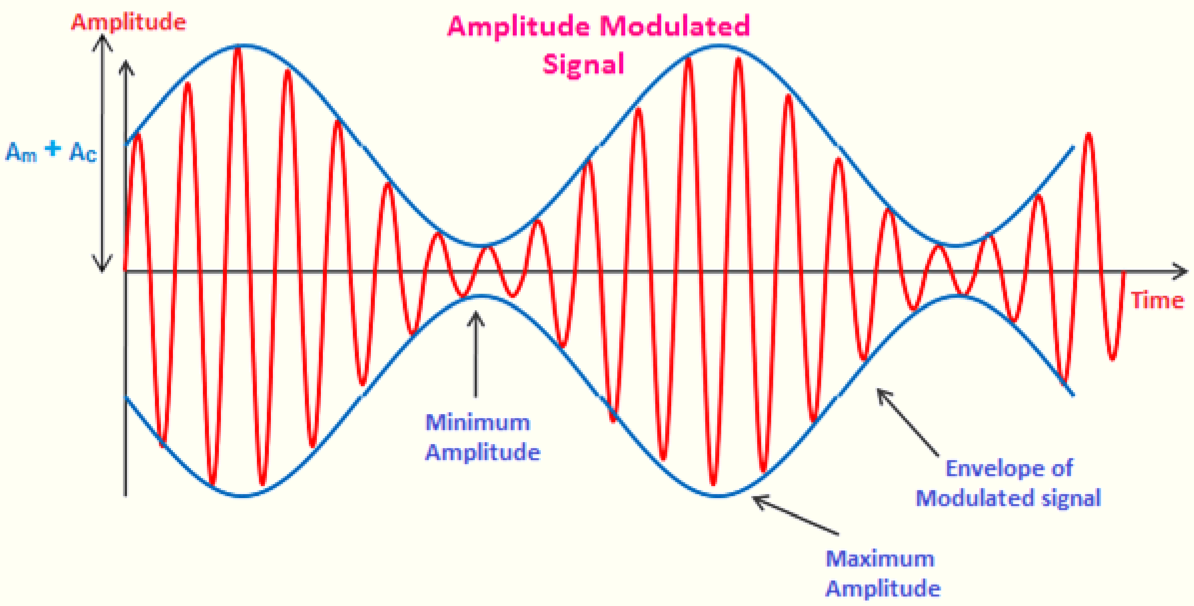
From the figure, we can obtain the bandwidth of AM wave as,

$$BW = fUSB - fLSB$$

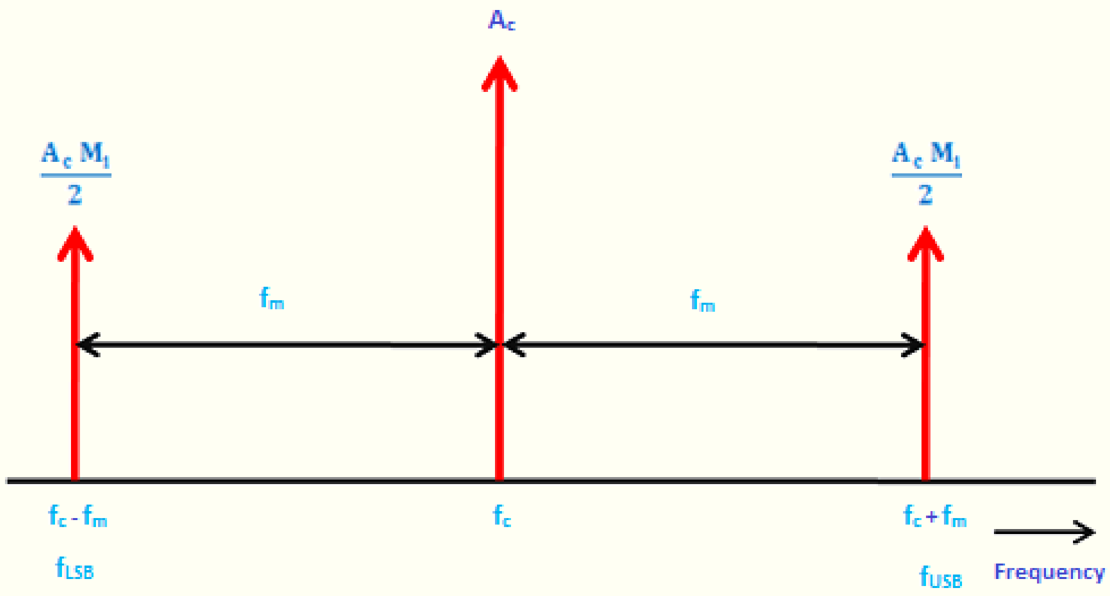
$$= (fc + fm) - (fc - fm)$$

$$BW = 2\ fm$$

Time domain & frequency domain representation of AM wave

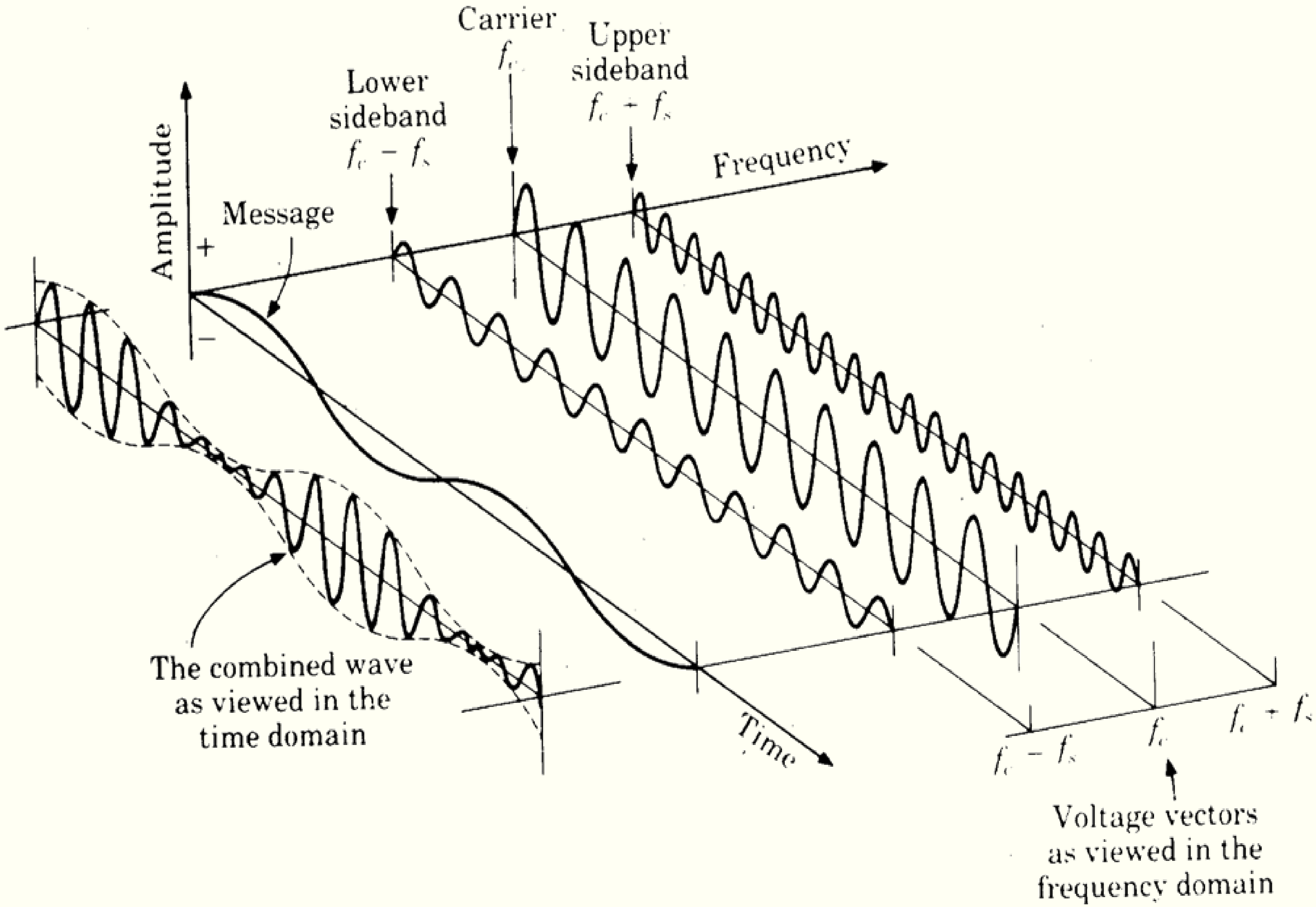


AM wave in time domain



AM wave in frequency domain

Combined Time Domain & Frequency Domain View



Calculation of Modulation Index from Amplitude Modulated (AM) waveform

$$A_{max} = 2A_m + A_{min}$$

$$A_m = \frac{A_{max} - A_{min}}{2} \ldots\ldots\ldots\ldots(1)$$

$$A_c = A_{max} - A_m \ldots\ldots\ldots\ldots(2)$$

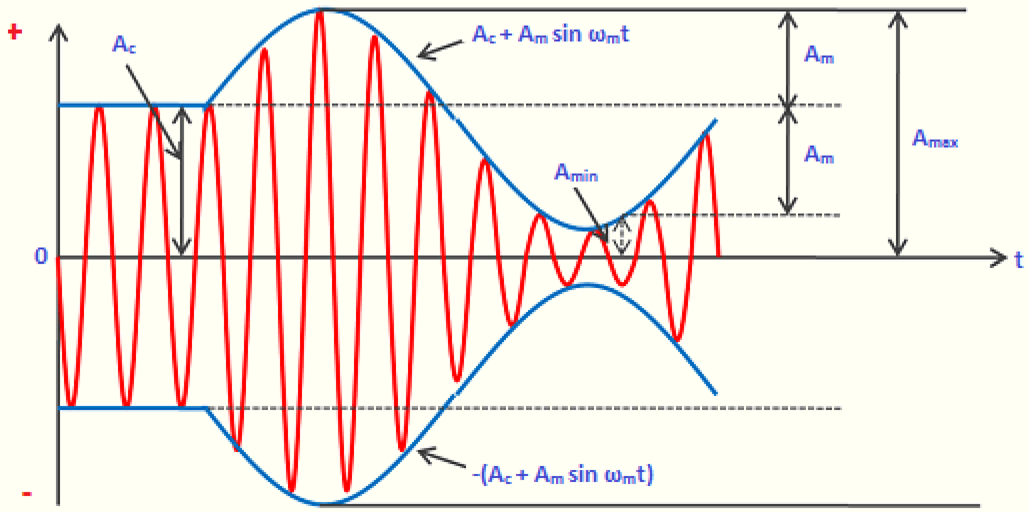
Put A_m value from eq (1) into eq (2), then we get

$$A_c = A_{max} - \frac{A_{max} - A_{min}}{2} \ldots\ldots\ldots\ldots(3)$$

$$A_c = \frac{A_{max} + A_{min}}{2} \ldots\ldots\ldots\ldots(4)$$

Taking the ratio of equation (1)and (4),

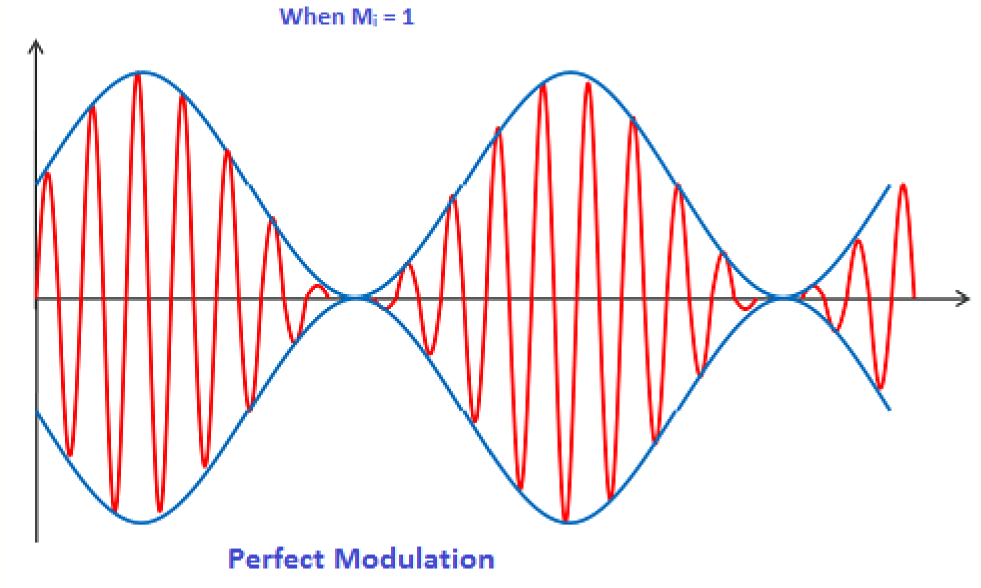
$$M_i = \frac{A_m}{A_c}$$



$$M_i = \frac{\frac{A_{max} - A_{min}}{2}}{\frac{A_{max} + A_{min}}{2}}$$

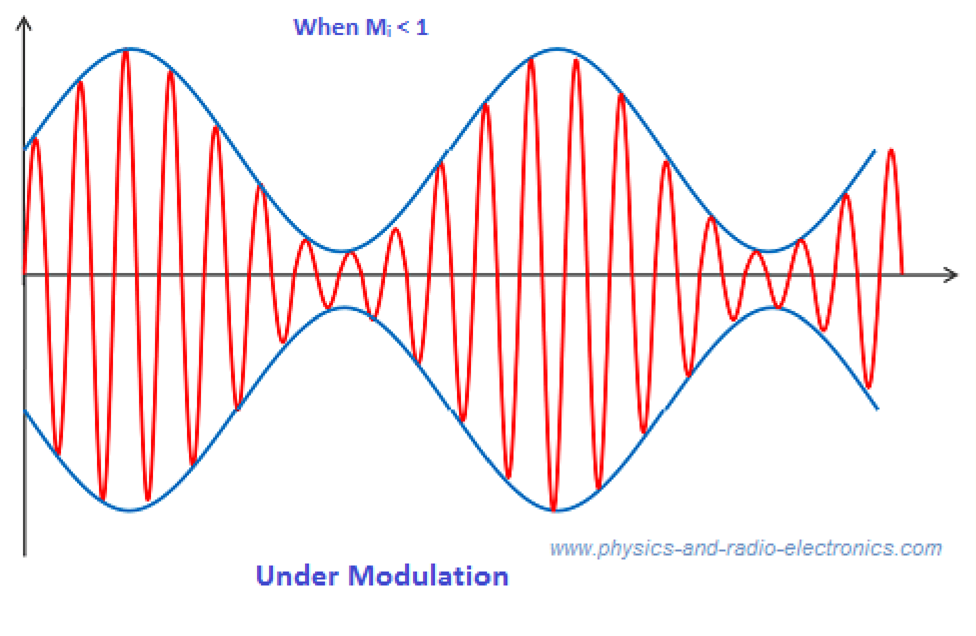
$$M_i = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} \ldots\ldots\ldots\ldots(5)$$

Modulation Depth Examples



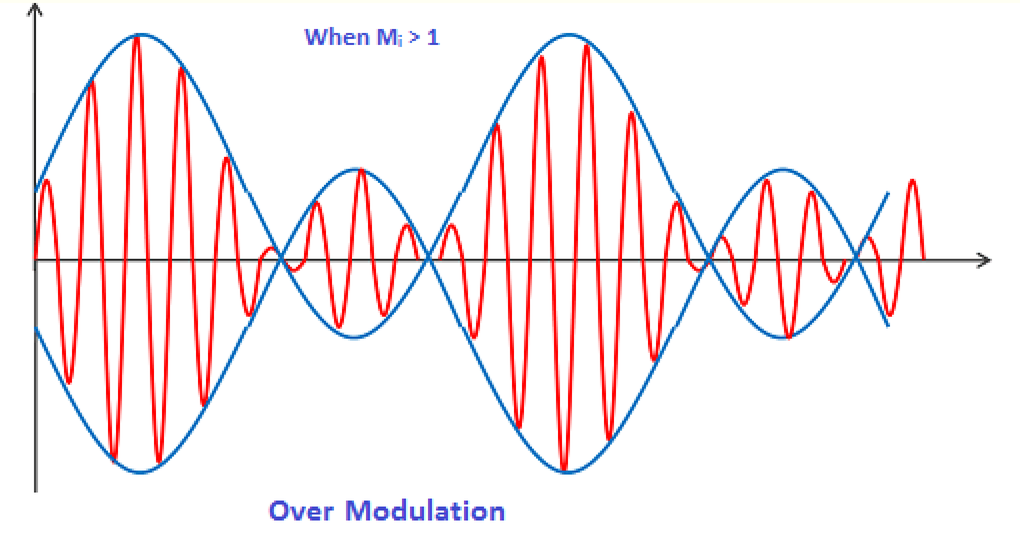
Em = 2V,   Ec = 2V

Modulation Depth Examples



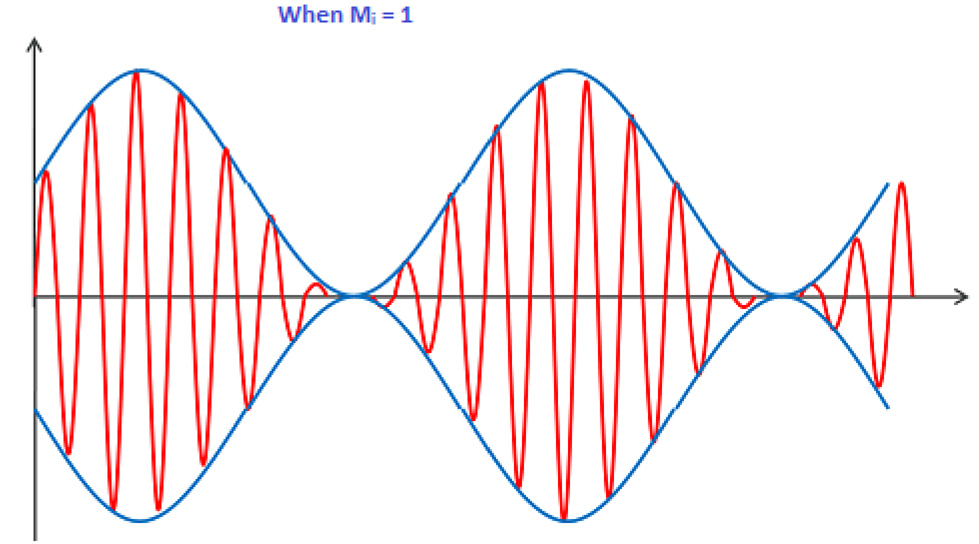
Em = 2V,   Ec = 3V

Modulation Depth Examples

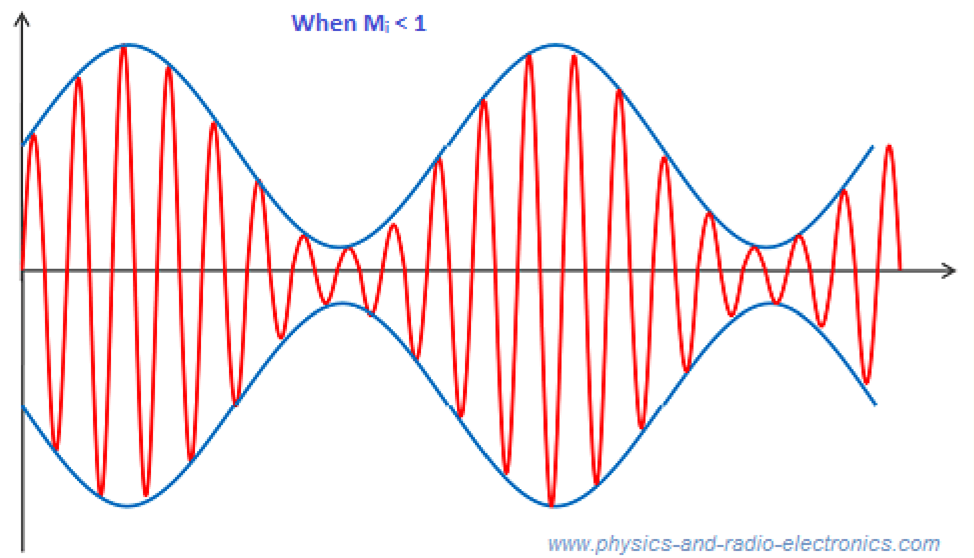


Em = 3V,   Ec = 2V

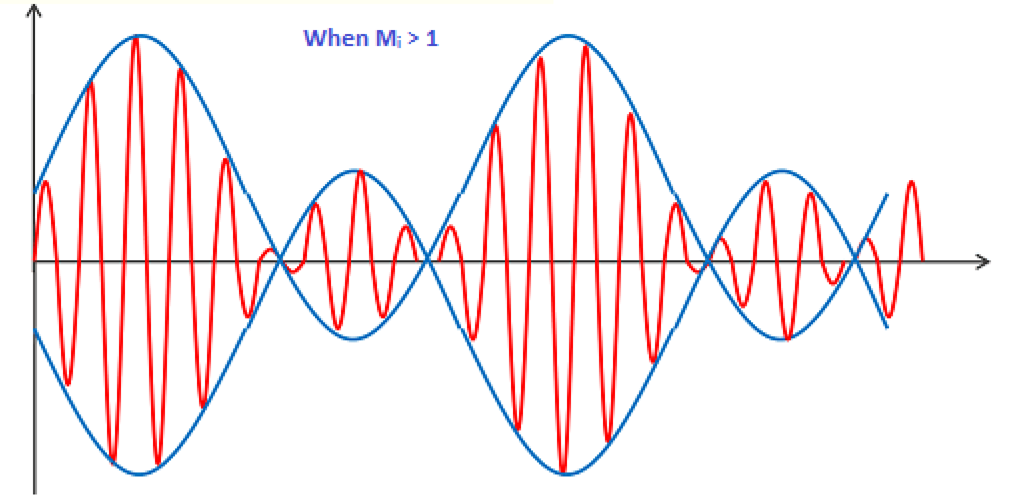
Modulation Depth Examples



Perfect Modulation



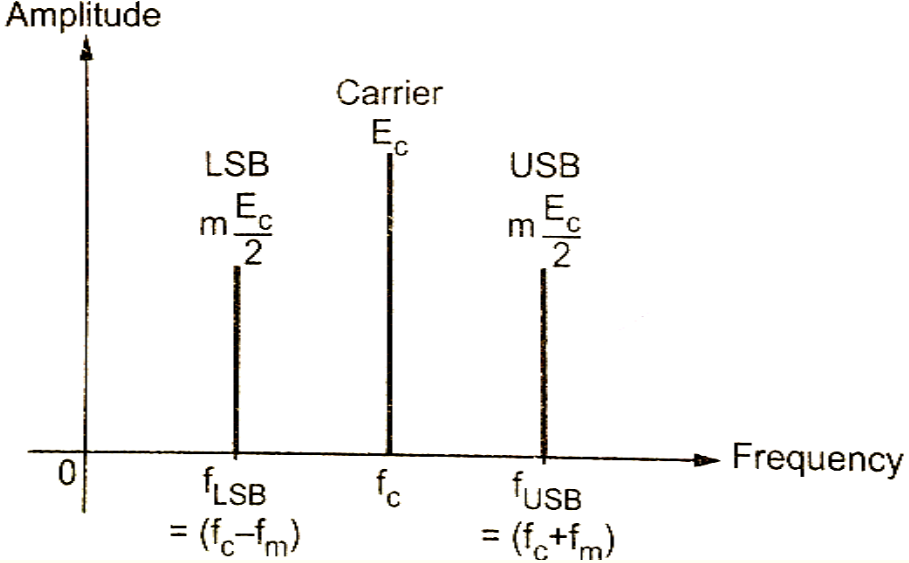
Under Modulation



Over Modulation

Average Power in AM Wave

$$e_{AM} = \underbrace{E_c \sin\omega_c t}_{\text{carrier}} + \underbrace{\frac{mE_c}{2}\cos(\omega_c - \omega_m)t}_{\text{Lower side band}} - \underbrace{\frac{mE_c}{2}\cos(\omega_c + \omega_m)t}_{\text{Upper side band}}$$



Power of AM wave is equal to the sum of powers of carrier, upper sideband, and lower sideband.

$$P_{Total} = P_c + P_{USB} + P_{LSB}$$

$$= \frac{E_{carr}^{\ 2}}{R} + \frac{E_{USB}^{\ 2}}{R} + \frac{E_{LSB}^{\ 2}}{R}$$

Where all three voltages represent r.m.s. values & resistance R is a characteristic impendence antenna

Carrier Power

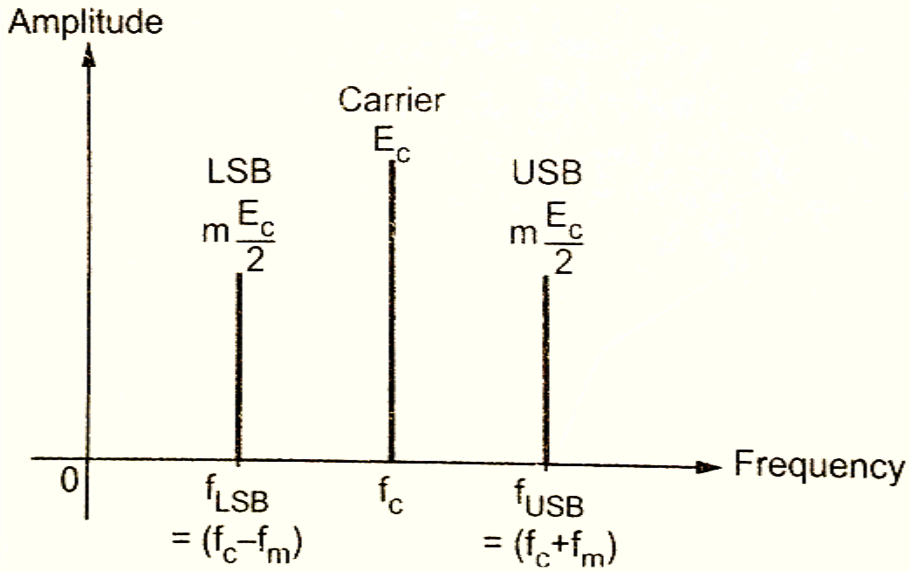
The Carrier Power is Given as:

$$P_c = \frac{E_{carr}^{\ 2}}{R}$$

As the voltage represent r.m.s. value

$$\therefore \quad \text{The average carrier power} = \frac{\left(E_c / \sqrt{2}\right)^2}{R}$$

$$P_c = \frac{E_c^{\ 2}}{2R}$$



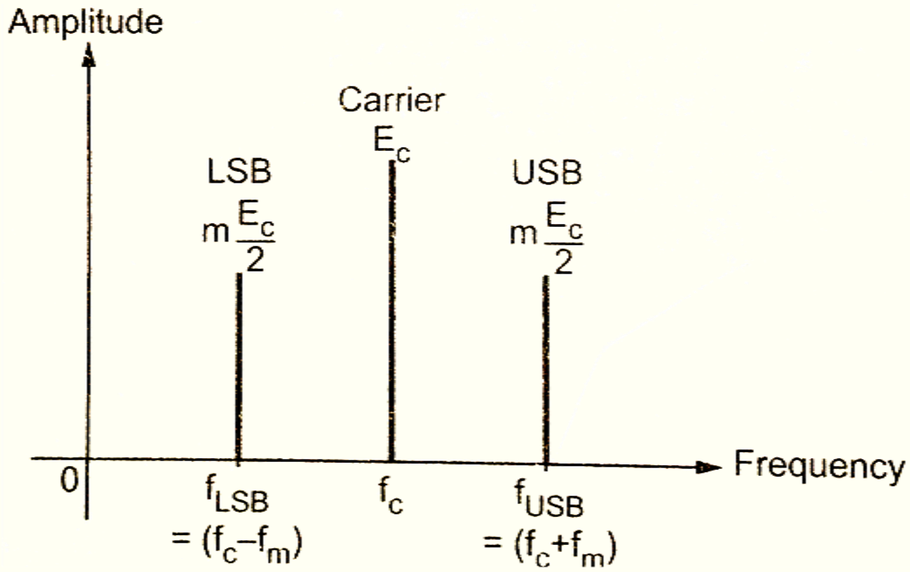
Power in Sidebands

average power for two sidebands can be given as

$$P_{LSB} = P_{USB} = \frac{E_{SB}^{2}}{R}$$

$$= \left(\frac{\frac{mE_c}{2}}{\sqrt{2}} \right)^2 \times \frac{1}{R} \qquad \because E_{SB} = \frac{mEc}{2}$$

$$P_{LSB} = P_{USB} = \frac{m^2 E_c^2}{8R}$$



Average Total Power in AM Wave

$$P_{Total} = P_c + P_{USB} + P_{LSB}$$

$$P_{Total} = \frac{E_c^2}{2R} + \frac{m^2 E_c^2}{8R} + \frac{m^2 E_c^2}{8R} = \frac{E_c^2}{2R}\left(1 + \frac{m^2}{4} + \frac{m^2}{4}\right)$$

$$= \frac{E_c^2}{2R}\left(1 + \frac{m^2}{2}\right)$$

$$P_{Total} = P_c\left(1 + \frac{m^2}{2}\right)$$

The maximum possible value of modulation index m is 1. if we put m= 1 in the power equation, we get the equation of power as:

$$Ptotal = \frac{3}{2}Pc = 1.5Pc$$

Transmission Efficiency

Transmission efficiency of an AM wave is the ratio of the transmitted power which contains the information (i.e. the total sideband power) to the total transmitted power.

$$\eta = \frac{P_{LSB} + P_{USB}}{P_t} = \frac{[\frac{m^2}{4}P_c + \frac{m^2}{4}P_c]}{[1+\frac{m^2}{2}]P_c}$$

$$\eta = \frac{\frac{m^2}{2}}{1+\frac{m^2}{2}} = \frac{m^2}{2+m^2}$$

The percent transmission efficiency is given by ,

$$\eta = \frac{m^2}{2+m^2} \text{ x } 100\%$$

A 400 watt carrier signal is modulated to the depth of 80%. Calculate the total power of the modulated wave.

$$P_{Total} = P_c\left(1+\frac{m^2}{2}\right) = 400\left(1+\frac{(0.8)^2}{2}\right)$$

$$= 528 \text{ watts}$$

A broadcast transmitter radiate 20kwatt when modulation % is 75. how much of this is carrier power? Also calculate power in each side band.

$$P_{Total} = P_c\left(1+\frac{m^2}{2}\right)$$

$$P_c = \frac{P_{Total}}{\left(1+\frac{m^2}{2}\right)} = \frac{20}{\frac{1+(0.75)^2}{2}} = \frac{20}{1.28} = 15.6\ \text{kW}$$

$$P_{SB} = P_c\left(\frac{m^2}{4}\right) = 15.6\left(\frac{(0.75)^2}{4}\right) = 2.2\ \text{kW}$$

$$P_{USB} = P_{LSB} = 2.2\ \text{kW}$$

An audio frequency signal $10 \sin 2\Pi \times 500t$ is used to amplitude modulate a carrier of $50 \sin 2\Pi \times 105\, t$ calculate

I. Modulation index
II. Sideband frequencies
III. Amplitude of each sideband frequencies
IV. Bandwidth required
V. Total power delivered to the load of 600 Ω
VI. Transmission efficiency.

An audio frequency signal 10 sin 2Π × 500t is used to amplitude modulate a carrier of 50 sin 2Π × 105 t

Sol. : i) **Modulation Index :**

Given

$$e_m = 10 \sin 2\pi \times 500\, t$$

$$e_c = 50 \sin 2\pi \times 10^5\, t$$

$$E_m = 10 \text{ and } E_c = 50$$

$$\text{Modulation Index } m = \frac{E_m}{E_c} = \frac{10}{50}$$

$$= 0.2$$

$$\text{percentage modulation} = 20\%$$

An audio frequency signal 10 sin 2Π × 500t is used to amplitude modulate a carrier of 50 sin 2Π × 105 t

ii) Sideband Frequencies:

$$\omega_m = 2\pi \times 500 \quad f_m = 500\ \text{Hz}$$

$$\omega_c = 2\pi \times 10^5 \quad f_c = 100\ \text{kHz}$$

$$f_{USB} = f_c + f_m = 100\ \text{kHz} + 500$$
$$= 100500\ \text{Hz} = 100.5\ \text{kHz}$$

$$f_{LSB} = f_c - f_m = 100\ \text{kHz} - 500$$
$$= 99500\ \text{Hz} = 99.5\ \text{kHz}$$

An audio frequency signal 10 sin 2Π × 500t is used to amplitude modulate a carrier of 50 sin 2Π × 105 t

iii) Amplitude of each sideband frequencies:

= mEc/2

= 0.2*50/2 = 5V

iv) Bandwidth Required

BW = Fusb – Flsb

= 1000Hz

An audio frequency signal 10 sin 2Π × 500t is used to amplitude modulate a carrier of 50 sin 2Π × 105 t

v) Total Power delivered to the load of 600ohm

$$P_{Total} = \frac{E_c^2}{2R}\left(1+\frac{m^2}{2}\right)$$

$$= \frac{(50)^2}{2\times 600}\left(1+\frac{(0.2)^2}{2}\right)$$

$$= 2.125 \text{ watts}$$

vi) Transmission Efficiency:

$$\eta = \frac{m^2}{2+m^2} \qquad = 1.96\%$$

Reference:

https://www.physics-and-radio-electronics.com/blog/amplitude-modulation/
Electronic communication system : Kennedy , Davis
Communication Engineering : Bakshi, Godse