## Particle Beam Diagnostics and Control Part 2: Transverse Phase Space

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- Introduction
- Emittance Diagnostics
- Profile Monitors


## Accelerator Key Parameters

- light source: spectral brilliance
> measure for phase space density of photon flux

$$
B=\frac{\text { Number of photons }}{[\sec ]\left[\mathrm{mm}^{2}\right]\left[\mathrm{mrad}^{2}\right][0.1 \% \text { bandwidth }]}
$$

> user requirement: high brightness
$\rightarrow$ lot of monochromatic photons on sample
> connection to machine parameters

$$
B \propto \frac{\mathrm{~N}_{\gamma}}{\sigma_{x} \sigma_{x^{\prime}} \sigma_{z} \sigma_{z^{\prime}}} \propto \frac{\mathrm{I}}{\varepsilon_{x} \varepsilon_{z}}
$$

- collider: luminosity L
> measure for the collider performance

$$
\dot{N}=\mathrm{L} \cdot \sigma
$$

relativistic invariant proportionality factor between cross section $\sigma$ (property of interaction) and number of interactions per second
> user requirement: high luminosity
$\rightarrow$ lot of interactions in reaction channel
> connection to machine parameters

$$
\left.L \propto \frac{I_{1} \cdot I_{2}}{\varepsilon}\right)
$$

for two identical beams with emittances $\varepsilon_{x}=\varepsilon_{z}=\varepsilon$
ii) small beam emittance
$>$ achieve small emittance (task of lattice designer)
$>$ preserve emittance (stability)
> measure small emittance

## Transverse Phase Space



- emittance
> 6-dim. phase space volume

$\rightarrow$ use areas of projection onto 3 orthogonal planes
> evolves out of Hamilton formalsim, hence based on canonical coordinates
$\rightarrow$ instead of phase space in ( $x, p_{x}$ ) use ( $x, x^{\prime}=p_{x} / p_{z}$ )
> constant of motion (Liouville's theorem)
$\rightarrow$ area preserved
$\rightarrow$ independent on location in accelerator


## - equation of motion

> Hill's type differential equation

$$
\begin{aligned}
& x^{\prime \prime}(s)+\left(\frac{1}{\rho^{2}(s)}-k(s)\right) \cdot x(s)=\frac{1}{\rho(s)} \Delta p / p \\
& z^{\prime \prime}(s)+k(s) \cdot z(s)=0 \quad k=\frac{e}{p} \frac{\partial B_{z}}{\partial x}
\end{aligned}
$$

- solution: quasi-periodic motion

$$
\begin{aligned}
x(s)= & \sqrt{\varepsilon \beta_{x}(s)} \cdot \cos (\Psi(s)+\Phi) \\
& \text { beam size }
\end{aligned}
$$



## Transverse Emittance

## - linear forces

> any particle moves on an ellipse in phase space ( $\mathrm{x}, \mathrm{x}$ ')
> ellipse rotates in magnets and shears between magnets
$\rightarrow$ but area is preserved: emittance
(acceleration: normalized emittance preserved)

phase space evolution

> general ellipse equation

$$
\varepsilon=\gamma \cdot x^{2}+2 \alpha \cdot x x^{\prime}+\beta \cdot x^{\prime 2}
$$

> $\alpha, \beta, \gamma, \varepsilon$ : Courant-Snyder or Twiss parameters
$\rightarrow \boldsymbol{\alpha}, \boldsymbol{\beta}, \gamma:$ functions of location s in accelerator
$\rightarrow \boldsymbol{\varepsilon}: \quad$ constant. Ellipse area $=\pi \varepsilon$
> 4 Twiss parameters, only 2 of them are independent:

$$
\gamma=\frac{1+\alpha^{2}}{\beta}
$$

## Emittance and Beam Matrix



- beam matrix

$$
\begin{gathered}
\sigma=\left(\begin{array}{ll}
\sigma_{11} & \sigma_{12} \\
\sigma_{21} & \sigma_{22}
\end{array}\right)=\left(\begin{array}{ll}
\left\langle x^{2}\right\rangle & \left\langle x x^{\prime}\right\rangle \\
\left\langle x x^{\prime}\right\rangle & \left\langle x^{\prime 2}\right\rangle
\end{array}\right)=\varepsilon\left(\begin{array}{cc}
\beta & -\alpha \\
-\alpha & \gamma
\end{array}\right) \\
\varepsilon=\sqrt{\operatorname{det} \sigma}=\sqrt{\sigma_{11} \cdot \sigma_{22}-\sigma_{12}^{2}}
\end{gathered}
$$

> transformation of beam matrix

$$
\sigma^{1}=\boldsymbol{R} \sigma^{0} \boldsymbol{R}^{T} \quad R=\left(\begin{array}{ll}
R_{11} & R_{12} \\
R_{21} & R_{22}
\end{array}\right)
$$

- via Twiss parameters

$$
\varepsilon=\gamma x^{2}+2 \alpha x x^{\prime}+\beta x^{\prime 2}
$$

- statistical definition
P.M. Lapostolle, IEEE Trans. Nucl. Sci. NS-18, No. 3 (1971) 1101
$\varepsilon_{r m s}=\sqrt{\left\langle x^{2}\right\rangle\left\langle x^{\prime 2}\right\rangle-\left\langle x x^{\prime}\right\rangle^{2}}$
$\underset{2^{2}}{2^{\text {nd }} \text { moment of beam }}$ distribution $\rho(\mathrm{x}) \quad\left\langle x^{2}\right\rangle=\frac{\int_{-\infty}^{\infty} \mathrm{d} x x^{2} \cdot \rho(x)}{\int_{-\infty}^{\infty} \mathrm{d} x \rho(x)}$
$>\varepsilon_{\mathrm{rms}}$ is measure of spread in phase space
> root-mean-square (rms) of distribution

$$
\sigma_{x}=\left\langle x^{2}\right\rangle^{1 / 2}
$$

$>\varepsilon_{\mathrm{rms}}$ useful definition for non-linear beams
$\rightarrow$ usually restriction to certain range
(c.f. $90 \%$ of particles instead of $[-\infty,+\infty]$ )

## Emittance Measurement: Principle

- emittance: projected area of transverse phase space volume
- not directly accessible for beam diagnostics

- measured quantity
> beam size

$$
\begin{aligned}
& \sqrt{\sigma_{11}}=\sqrt{\left\langle x^{2}\right\rangle}=\sqrt{\varepsilon \beta} \\
& \sqrt{\sigma_{22}}=\sqrt{\left\langle x^{\prime 2}\right\rangle}=\sqrt{\varepsilon \gamma}
\end{aligned}
$$

> beam divergence
> divergence measurements seldom in use
$\rightarrow$ restriction to profile measurements

- measurement schemes
> beam matrix based measurements
$\rightarrow$ determination of beam matrix elements: $\quad \varepsilon=\sqrt{\operatorname{det} \sigma}=\sqrt{\sigma_{11} \cdot \sigma_{22}-\sigma_{12}^{2}}$
> mapping of phase space
$\rightarrow$ restrict to (infenitesimal) element in space coordinate, convert angles $x^{‘}$ in position


## Beam Matrix based Measurements

- starting point: beam matrix

$$
\sigma=\left(\begin{array}{ll}
\sigma_{11} & \sigma_{12} \\
\sigma_{21} & \sigma_{22}
\end{array}\right)=\left(\begin{array}{ll}
\left\langle x^{2}\right\rangle & \left\langle x x^{\prime}\right\rangle \\
\left\langle x x^{\prime}\right\rangle & \left\langle x^{\prime 2}\right\rangle
\end{array}\right)=\varepsilon\left(\begin{array}{cc}
\beta & -\alpha \\
-\alpha & \gamma
\end{array}\right)
$$

- emittance determination
$>$ measurement of $\mathbf{3}$ matrix elements $\boldsymbol{\sigma}_{11}, \boldsymbol{\sigma}_{\mathbf{1 2}}, \boldsymbol{\sigma}_{\mathbf{2 2}}$
> remember: beam matrix $\sigma$ depends on location, i.e. $\sigma(\mathrm{s})$

$$
\varepsilon=\sqrt{\operatorname{det} \sigma}=\sqrt{\sigma_{11} \cdot \sigma_{22}-\sigma_{12}^{2}}
$$

$\rightarrow$ determination of matrix elements at same location required

## - access to matrix elements

$\Rightarrow$ profile monitor determines only $\sqrt{\sigma_{11}}$
${ }^{>}$other matrix elements can be inferred from beam profiles taken under various transport conditions
$\rightarrow$ knowledge of transport matrix R required $\quad \sigma^{b}=R \cdot \sigma^{a} \cdot R^{T} \quad R=\left(\begin{array}{ll}R_{11} & R_{12} \\ R_{21} & R_{22}\end{array}\right)$

- measurement of at least 3 profiles for 3 matrix elements

$$
\begin{array}{lll}
\sigma_{11}^{a} & & \\
\sigma_{11}^{b}=R_{11}^{2} \cdot \sigma_{11}^{a}+2 R_{11} R_{12} \cdot \sigma_{12}^{a}+R_{12}^{2} \cdot \sigma_{22}^{a} & >\text { measurement : profiles } & \sigma_{11}^{a}, \sigma_{11}^{b}, \sigma_{11}^{c} \\
\sigma_{11}^{c}=\bar{R}_{11}^{2} \cdot \sigma_{11}^{a}+2 \bar{R}_{11} \bar{R}_{12} \cdot \sigma_{12}^{a}+\bar{R}_{12}^{2} \cdot \sigma_{22}^{a} & >\text { deduced: matrix elements } & R, \bar{R} \\
\sigma_{11}^{a}, \sigma_{12}^{a}, \sigma_{22}^{a}
\end{array}
$$

$\rightarrow$ more than 3 profile measurements favourable, data subjected to least-square analysis

## Beam Matrix based Measurements

- "quadrupole scan" method
> use of variable quadrupole strengths
$\rightarrow$ change quadrupole settings and measure beam size in profile monitor located downstream

quadrupole transfer matrix

$$
Q=\left(\begin{array}{cc}
1 & 0 \\
\pm 1 / f & 1
\end{array}\right)
$$

drift space transfer matrix $\quad \rightarrow \quad \mathbf{R}=\mathbf{S Q}$

$$
S=\left(\begin{array}{ll}
1 & l \\
0 & 1
\end{array}\right)
$$

> $\sigma_{11}$ depends quadratically on quadrupole field strength


## Beam Matrix based Measurements

- "multi profile monitor" method
> fixed particle beam optics
$\rightarrow$ measure beam sizes using multiple profile monitors at different locations
> example:
emittance measurement setup at FLASH injector
courtesy: K. Honkavaara (DESY)



## Phase Space Mapping

- limitations for quadrupole scan / multi-screen method
> successive variations in parameter space required (perturbative profile measurements)
$\rightarrow$ no single-shot capability
> measuring principle relies on undisturbed phase space evolution (linear beam optics)
> low-energy beams: often space charge limited
$\rightarrow$ reduce space charge influence by cutting out beamlet
- "slit scanning" method
> slit generates vertical slice (beamlet) in transverse phase space
$\rightarrow$ reduced space charge influence
> convert angles into position through drift space L
$\rightarrow$ intensity at $u, x^{\prime}$ in $\left(x, x^{\prime}\right)$-space is mapped to position $u+\mathrm{Lx}^{\prime}$ defined by slit
> profile measurement to reconstruct angular distribution
$\rightarrow$ scan with intensity monitor
$\rightarrow$ spatial resolving intensity measurement
> successive scanning of slit to cover whole phase space

M.P.Stockli, Proc. BIW 2006, p. 25


## Phase Space Mapping


> moving through quadrupole
$\rightarrow$ no change in position
(vertical move in phase space)

- multi-slit absorber

courtesy: U. Raich (CERN)
$>$ time consuming slit-scanning $\rightarrow$ multi-slit arrangement $\rightarrow$ 1-dim. single shot measurement
- Needs high resolution profile detector
- Must make sure that profiles dont overlap


$7 x 50 \mu \mathrm{~m}$
$500 \mu \mathrm{~m}$ spaced
courtesy: A. Cianchi (Tor Vergata, Roma \& INFN Frascati)


## Phase Space Mapping

- emittance analysis

> p: total number of slits
> $\mathrm{x}_{\mathrm{sj}}$ : $\mathrm{j}^{\text {th }}$ slit position
> $\mathrm{n}_{\mathrm{j}}$ : number of particles passing thgough $\mathrm{j}^{\text {th }}$ slit
$\rightarrow$ intensity weighting
, N : number of particles behind slits

$$
N=\sum_{j=1}^{p} n_{j}
$$

$\bar{x}=\frac{1}{N} \sum_{j=1}^{p} n_{j} x_{s j}$

$$
\begin{aligned}
\varepsilon_{r m s} & =\sqrt{\left\langle x^{2}\right\rangle\left\langle x^{\prime 2}\right\rangle-\left\langle x x^{\prime}\right\rangle^{2}} \\
\left\langle x^{2}\right\rangle & =\frac{1}{N} \sum_{j=1}^{p} n_{j}\left(x_{s j}-\bar{x}\right)^{2} \\
\left\langle x^{\prime 2}\right\rangle & =\frac{1}{N} \sum_{j=1}^{p} n_{j}\left[\sigma_{x^{\prime} j}^{2}+\left(\bar{x}^{\prime}{ }_{j}-\bar{x}^{\prime}\right)^{2}\right] \\
\left\langle x x^{\prime}\right\rangle & =\frac{1}{N}\left(\sum_{j=1}^{p} n_{j} x_{s j} \bar{x}^{\prime}{ }_{j}-N \bar{x} \bar{x}^{\prime}\right)
\end{aligned}
$$

, $\overline{\mathrm{x}}_{\mathrm{j}}$ : mean divergence of $\mathrm{j}^{\text {th }}$ beamlet

$$
\begin{aligned}
& \bar{x}_{j}^{\prime}=\frac{\bar{X}_{j}-x_{s j}}{L} \text { with } \bar{X}_{j}=\frac{1}{n_{j}} \sum_{i=1}^{n} X_{j i} \\
& \text { vergence of all beamlets }
\end{aligned}
$$

> $\overline{\mathrm{x}}$ : mean divergence of all beamlets

$$
\bar{x}^{\prime}=\frac{1}{N} \sum_{i=1}^{p} n_{j} \bar{x}_{j}
$$

> $\sigma_{x^{\prime} \mathrm{j}}: \quad$ rms divergence of $\mathrm{j}^{\text {th }}$ beamlet

$$
\sigma_{x^{\prime} j}=\frac{\sigma_{j}}{L} \quad \text { with } \sigma_{j}^{2}=\frac{1}{n_{j}} \sum_{i=1}^{n}\left(x_{j i}-\bar{X}_{j}\right)^{2}
$$

## Phase Space Mapping

## - multi-slit absorber: practical considerations

$>$ grid of dense material (tungsten, tantalum) inserted in beam path
$\rightarrow$ split beam in several beamlets
$\rightarrow$ beam has to be fully absorbed i.e. works reliable only for low energetic beams
> transverse position at which beamlets are created is known (position of grid)
$\rightarrow$ space coordinate
$>$ measurement of beamlet-size downstream (after drift space)
$\rightarrow$ access to beam divergence
b beam size and beam divergence
$\rightarrow$ combined to emittance
example for space-charged dominated beam:
low-energy ion beam at GSI @ Darmstadt (Germany) behind source
P. Forck, "Lecture Notes on Beam Instrumentation and Diagnostics", JUAS 2011


## Phase Space Mapping

- pepper-pot: 2-dim. extension
> matrix of holes in absorber plate
$\rightarrow$ single-shot capability for both planes
$\rightarrow$ investigation of shot-to-shot fluctuations
> destructive method:
tiny fraction of beam (ca. 1\%) passes holes
$\rightarrow$ heat load in absorber has to be considered
$\rightarrow$ requires very sensitive detection system
> also in use at Laser Plasma Accelerators
useful references

courtesy: P. Forck (GSI)
> M. Zhang, Emittance Formula for Slits and Pepper-pot Measurements, FERMILAB-TM-1988
>S. Jolly et al., Data Acquisition and Error Analysis for Pepperpot Emittance Measurements, Proc. DIPAC’09, Basel, WEOA03
- limitations of slit / pepper-pot systems
> best suited for low energy beams ( $\leq 100 \mathrm{MeV}$ )
high energetic electron beam requires very thick absorber material $\rightarrow$ diffraction inside slit/hole
> pepper-pot for 508 MeV electrons: N.Delerue et al., Proc. PAC‘09, Vancouver (Canada), 2009, p. 3597


## Emittance Diagnostics: Comments

- comparison of different methods
$\left.\begin{array}{|l|l|c|c|c|}\hline & \begin{array}{l}\text { high } \\ \text { energy }\end{array} & \begin{array}{l}\text { high space } \\ \text { charge forces }\end{array} & \begin{array}{l}\text { large energy } \\ \text { spread }\end{array} & \begin{array}{l}\text { single shot } \\ \text { capability }\end{array} \\ \hline \text { slit / pepper-pot } & - & + & + & + \\ \hline \text { quadrupole scan } & + & - & - \\ \text { (chromatic effects) }\end{array}\right]$
according to H. Braun, "Emittance Diagnostics", CAS on Beam Diagnostics, Dourdan 2008
> emittance diagnostics in Linacs: non-parasitic and destructive measurements
- emittance diagnostics in circular accelerators
> circular accelerator: periodic with circumference
$\rightarrow$ Twiss parameters $\alpha(\mathrm{s}), \beta(\mathrm{s}), \gamma(\mathrm{s})$ uniquely defined at each location in ring
> measurement at one location in ring sufficient to determine $\varepsilon$
$\rightarrow$ measured quantity: beam profile / angular distribution



## Slice Emittance

- commment: emittance at FELs
$>$ electrons slip back in phase with respect to photons by $\mathrm{I}_{\mathrm{r}}$ each undulator period
$>$ FEL integrates over slippage length $\rightarrow$ slice emittance of importance projected emittance

- measurement with transverse deflecting cavity

$\rightarrow \quad$ discussion in $3^{\text {rd }}$ lecture together with bunch length diagnostics


## Profile Measurements

- emittance diagnostics
> underlying principle is profile measurement
> profile contains information about angular/spatial distribution
- profile monitor: principles
> consider beam with (for simplification 1-dim.) spatial distribution $\rho(\mathrm{x})$
> generate secondary signal $\Sigma(\mathrm{x})$ with intensity proportional to $\rho(\mathrm{x})$
> $\Sigma(\mathrm{x})$ is generated via
$\rightarrow$ interaction of beam with matter
$\rightarrow$ interaction of beam with photons
$\rightarrow$ separating particle beam electromegnetic field
$>\Sigma$ can be flux of
$\rightarrow$ charged particles
$\rightarrow$ electromagnetic radiation
$>$ spatial resolution for detection of $\Sigma(\mathrm{x})$
$\rightarrow$ scanning of conversion target or/and detector
$\rightarrow$ spatial resolving detector
wire scanner, scintillation screen, residual gas monitor, secondary electron emission (SEM)... laser wire scanner, ... synchrotron radiation, OTR, ODR, ...
secondary electrons, ...
visible light, $\gamma$-rays ...
wire, ...
optical detectors (CCD or CMOS sensors) , ...


## Wire Scanner

## - advantages

> direct and reliable measurement
> resolution down to $1 \mu \mathrm{~m}$ achieveable
> minimum inivasive method

## - operation principle

> scanning of thin wire across the beam
> detection of beam-wire interaction as function of wire position


- wire and mechanics
> material: $\mathrm{C}, \mathrm{W}, \mathrm{Be}, \ldots$
> wire size: down to few $\mu \mathrm{m} \rightarrow$ limits resolution
> fast wire movement ( $5-20 \mathrm{~m} / \mathrm{sec}$ ) for intense \& high brilliant beams
$\rightarrow$ minimize emittance blow-up (circular hadron accelerators)
$\rightarrow$ reduce heat-load on wire: high melting temperature \& low Z material
> high speed movement
$\rightarrow$ rotationary wire movement: limited resolution down to $10-100 \mu \mathrm{~m}$


PIIURE 3. Failed $15 \mu \mathrm{~m}$ diameter tungsten wire showing the rough surface
resulting from many discharges.
$\rightarrow$ linear movement: speed limited by vacuum bellow stress properties
> high precision encoder for position readout

## Wire Scanner

## - signal detection

> measure scattered beam particles and/or bremsstrahlung outside vacuum chamber
$\rightarrow$ fast scintillation counter (able to resolve single bunches)
$\rightarrow$ signal can depend on wire scanner location: Monte Carlo studies recommended
> secondary electron emission in wire
$\rightarrow$ often used at low energy beams:

- scattered particles cannot penetrate vacuum chamber
- intensity of bremsstrahlung to low

$\rightarrow$ if wire temperature exceeds thermionic threshold: thermal electrons superimpose SEM signal


## - comments

, wire scanner measures projected beam profiles
$\rightarrow$ typically one scanner for each transverse plane required
> option: wire scanner for both projections
$\rightarrow$ scanning under $45^{\circ}$ geometry
> information of $x-y$ coupling
$\rightarrow$ installation of $3^{\text {rd }}$ wire

(bi-Gaussian with tilt: $\sigma_{x}, \sigma_{y}, \theta \rightarrow$ requires 3 d.o.f. )

## Wire Scanner for XFEL



》 wire scanners in combination with screen monitors
> scan speed $1 \mathrm{~m} / \mathrm{sec}$

- design based on commercial linear motor
- drive and unit can be separated
$\rightarrow$ ease of maintenance
$\rightarrow$ flexibility of wire scanner distribution (COTR)

wires and fork
$\rightarrow 3$ wires for fast scan ( $\mathrm{W}, 10-30 \mu \mathrm{~m}$ )
$\rightarrow 2$ diagonal wires for
optional coupling studies



## Harps

## profile diagnostics for hadron linacs

) beam profiles in the order $\geq \mathrm{mm}$
> instead of moving wire
$\rightarrow$ grid of wires: harp
$\rightarrow$ wire spacing down to a few hundreds of $\mu \mathrm{m}$
> measurement of SEM current

courtesy: U. Raich (CERN)

J. Douglas et. al, LANSCE Harp Upgrade, Proc. BIW2010, p. 132

## Laser Wire Scanner

- advantages
$>$ in principle resolution down to $1 \mu \mathrm{~m}$ (and better?) achievable
> non-invasive profile measurement
- principle
> photons from Q-switched high power Nd:YAG laser collide with beam
$\rightarrow$ generation of forward Compton-scattered $\gamma^{\prime}$ s
> transverse scanning of laser beam across electron beam
$\rightarrow \gamma$ intensity as function of laser poition: beam profile

- device for reliable daily operation (?)
> two test set-ups
$\rightarrow$ PETRA III@ DESY
$\rightarrow$ ATF2@ KEK
> PETRA experience
$\rightarrow$ so far expert system

T. Aumeyr et. al, Proc. IPAC'10, Kyoto (Japan), 2010 p. 1137


Figure 7: Horizontal scan ( $\sigma_{x}=293.6 \pm 18.1 \mu \mathrm{~m}$ )

## Optical Profile Diagnostics

- taking advantage of the huge market for commercial available sensors
> nowadays: high quality of optical light sensors for reasonable price
$\rightarrow$ small pixel size: high spatial resolution
$\rightarrow$ good linearity
$\rightarrow$ high dynamic range
> optical beam diagnostics
$\rightarrow$ change from pure „visualization" to high resolution measurements
- available light sensors
> 1-D sensors
$\rightarrow$ photo diode array, line scan camera, segmented photomultiplier, ...
$\rightarrow$ can be fast, up to hundreds of MHz
> 2-D sensors
$\rightarrow$ area scan CCD \& CMOS sensors, segmented photomultiplier, ...
$\rightarrow$ usually slow ( $\approx 50 \mathrm{~Hz}$ ), possible up to 100 kHz

2-D sensors preferable because 2-D image contains whole spatial information

## Profile Monitor Considerations

- point of interest: monitor resolution
> consider single charged particle
> calculate „point spread function"
(PSF) in image plane
> convolute beam profile with PSF
- tasks
> radiation generation
$\Longrightarrow$ electromagnetic field distribution of radiation field
> radiation propagation
$\Longrightarrow$ aperture limitations
> focusing optics (lens)

$\Longrightarrow$ thin lens approximation
focusing lens
(focal length f)


## Optical Propagation

- ...in frame of scalar diffraction theory
> source plane to lens entrance

$$
E_{x_{t}, y_{l}}^{l_{\text {lor }}}\left(\vec{r}_{l}, \omega\right)=-i \frac{e^{i k a}}{\lambda a} \cdot e^{i \frac{k}{2 a}\left(x_{t}^{2}+y_{t}^{2}\right)} \iint_{\text {source }} d x_{S} d y_{S} E_{x_{s}, y_{s}}^{S}\left(\vec{r}_{S}, \omega\right) \cdot e^{i \frac{k}{2 a}\left(x_{s}^{2}+y_{s}^{2}\right)} \cdot e^{-i k \frac{x_{s} x_{t}+y_{t} y_{l}}{a}}
$$

> lens input to lens output (thin lens approximation)

$$
E_{x_{i}, y_{l}}^{l_{a}}\left(\vec{r}_{l}, \omega\right)=E_{x_{i}, y_{l}}^{l_{\text {air }}}\left(\vec{r}_{l}, \omega\right) \cdot e^{-i \frac{k}{2 f}\left(x_{i}^{2}+y_{i}^{2}\right)} \quad \quad \text { with } \quad \frac{1}{f}=\frac{1}{a}+\frac{1}{b}
$$

> lens output to image plane (CCD detector)

$$
E_{x_{i}, y_{i}}^{i}\left(\vec{r}_{i}, \omega\right)=-i \frac{e^{i k b}}{\lambda b} \cdot e^{i \frac{k}{2 b}\left(x_{i}^{2}+y_{i}^{2}\right)} \iint_{l e n s} d x_{l} d y_{l} E_{x_{i}, y_{l}}^{l_{\text {ol }}}\left(\vec{r}_{l}, \omega\right) \cdot e^{i \frac{k}{2 b}\left(x_{i}^{2}+y_{l}^{2}\right)} \cdot e^{-i k \frac{x_{l} x_{i}+y_{i} y_{l}}{b}}
$$

- measured quantity
> spatial intensity distribution

$$
\left(\frac{\mathrm{d}^{2} W}{\mathrm{~d} \omega \mathrm{~d} \Omega}=\frac{\mathrm{c}}{4 \pi^{2}}\left(\left|\vec{E}_{x_{i}}^{i}\left(\vec{r}_{i}, \omega\right)\right|^{2}+\left|\vec{E}_{y_{i}}^{i}\left(\vec{r}_{i}, \omega\right)\right|^{2}\right)\right.
$$

## Radiation Generation: Considerations

- radiation generation via particle interaction with matter
> luminescent screen monitors
- radiation generation via particle electromagnetic field

> relativistic contraction characterized by Lorentz factor

$$
\gamma=E / m_{0} c^{2} \quad \begin{aligned}
& E: \text { total energy } \\
& m_{0} c^{2}: \text { rest mass energy }
\end{aligned}
$$

$$
\begin{array}{ll}
\text { proton: } & m_{p} c^{2}=938.272 \mathrm{MeV} \\
\text { electron: } & m_{e} c^{2}=0.511 \mathrm{MeV}
\end{array}
$$

$\gamma \rightarrow \infty$ : plane wave
> $\mathrm{mc}^{2}=0 \mathrm{MeV}: \quad$ light $\rightarrow$,,real photon"
> ultra relativistic energies: idealization $\rightarrow$,virtual photon"

## Separation of Particle Field

- electromagnetic field bound to particle observation in far field (large distances)
separate field from particle
- separation mechanisms
> bending of particle via magnetic field synchrotron radiation

$$
\Longrightarrow \quad \text { circular accelerators }
$$

linear accelerators: no particle bending ???

> diffraction of particle electromagnetic field via material structures exploit analogy between real/virtual photons:

- light reflection/refraction at surface $\leftrightarrow$ backward/forward transition radiation (TR)
- light diffraction at edges $\quad \leftrightarrow \quad$ diffraction radiation (DR)
- light diffraction at grating $\leftrightarrow \quad$ Smith-Purcell radiation
- light (X-ray) diffraction in crystal $\leftrightarrow$ parametric X-ray radiation (PXR) $\ldots$


## Optical Transition Radiation

- transition radiation: electromagnetic radiation emitted when a charged particle crosses boundary between two media with different optical properties
- visible part: Optical Transition Radiation (OTR)
- beam diagnostics: backward OTR (reflection of virtual photons) typical setup: image beam profile with optical system
$\rightarrow$ beam image and measurements of beam shape and size
- advantage: fast single shot measurement, linear response (neglect coherence !)
- disadvantage: high charge densities may destroy radiator, limitation on bunch number angular distribution



## OTR Monitor Resolution

- calculation of point spread function in image plane

- OTR resolution
> resolution definition according to classical optics:
$\Longrightarrow$ first minimum of PSF $(\rightarrow$ diameter of Airy disk $)$
G. Kube, TESLA-FEL Report 2008-01

> parameters of calculation

$$
\begin{aligned}
& \mathrm{E}=\mathbf{1} \mathrm{GeV} \\
& \lambda=\mathbf{5 0 0} \mathrm{nm} \\
& \mathrm{f}=\mathbf{2 5 0} \mathbf{~ m m} \\
& \mathbf{a}=\mathrm{b}=\mathbf{5 0 0} \mathbf{~ m m ~ ( 1 : 1 ~ i m a g i n g ) ~} \\
& \text { lens-Ø }=\mathbf{5 0 . 8} \mathbf{~ m m}
\end{aligned}
$$

$$
R_{i 0} \approx 1.12 \frac{M \lambda}{\theta_{m}} \left\lvert\, \begin{aligned}
& \text { M: magnification } \\
& \theta_{\mathrm{m}}: \text { lens acceptance angle }
\end{aligned}\right.
$$

## OTR Monitors at FLASH


K. Honkavaara et al.,

Proc. PAC 2003, p. 2476


## Example of Beam Images (matched)




## XFEL OTR Monitors

- monitor setup

courtesy: Ch. Wiebers, D. Nölle (DESY)
- optics: Scheimpflug principle
from large format photography

sharp image of an entire plane, if
- lens
- image
- object plane cut in a single line
picture by Linhof



## Optical Diffraction Radiation (ODR)

- problem OTR: screen degradation/damage
$\rightarrow$ limited to only few bunch operation, no permanent observation
- ODR: excellent candidate to measure beam parameters parasitically
> DR generation via interaction between the EM fields of the moving charge and the conducting screen
$\rightarrow$ diffraction of "virtual photons"
> extension of EM field of a relativistic particle is flat circle
$\rightarrow$ radius $\lambda \beta \gamma / 2 \pi$
> radiation intensity scales proportional to $|\mathrm{E}|^{2}$ :

$$
I \propto e^{-a / h_{\mathrm{inf}}} \quad \text { with } h_{\mathrm{int}}=\frac{\lambda \beta \gamma}{4 \pi}
$$


$\Rightarrow$ dependency on impact parameter $\mathrm{h}_{\text {int: }}$

$$
\begin{aligned}
& a \gg h_{\text {int }}: \text { no radiation } \\
& a \cong h_{\text {int }}: \mathrm{DR} \\
& a \ll h_{\mathrm{int}}: \mathrm{TR}
\end{aligned}
$$

## Principle of ODR Diagnostics

- imaging with ODR: no beam image, illuminated slit
$\rightarrow$ seems not suitable for beam diagnostics nevertheless, attempts to use ODR imaging: A. Lumpkin et al., Proc. BIW 2008, TUPTPF061
- exploit ODR angular distribution: visibility of interference fringes can be used to determine transverse size of a bunch of electrons crossing the slit
$\rightarrow$ increasing $\sigma_{y}$ both the peak intensity and the central minimum increase
- research project @ FLASH



## ODR Evidence

- CCD image


Beam transport optimization
$>0.7 \mathrm{nC}$
> 25 bunches
$>2 \mathrm{~s}$ exposure time
$>\mathrm{E}_{\text {beam }}($ nominal $)=680 \mathrm{MeV}$
$>800 \mathrm{~nm}$ filter and polarizer in

- comparison

Simulation parameters:
$>\mathrm{a}=0.5 \mathrm{~mm}$
$>$ Gaussian distributed beam
$>\sigma_{\mathrm{y}}=80 \mu \mathrm{~m}$
$>\sigma_{y}^{\prime}=125 \mu \mathrm{rad}$
$>\mathrm{E}_{\text {beam }}=610 \mathrm{MeV}$

E. Chiadroni, M. Castellano, A. Cianchi, K. Honkavaara, G. Kube, V. Merlo, F. Stella,

Non-intercepting electron beam transverse diagnostics with optical diffraction radiation at the DESY FLASH facility NIM B 266 (2008) 3789-3796 and Proc. of PAC 2007, p. 3982

## ODR Interferometry (ODRI)

- reduction of synchrotron radiation background
$\rightarrow$ stainless steel shield in front of ODR screen with larger cut


In the case of a wavelength of 800 nm and 1 GeV beam energy the 1 mm cut is not large enough to prevent the production of ODR in the forward direction, reflected by the screen and interfering with the backward ODR produced by the screen itself.

An ODR analogous of the Wartski interferometer used for OTR, with the difference that in this case the two interfering amplitudes are different in intensity and angular distribution


Single particle going through the center of both slits with 900 MeV energy and 800 nm wavelength

## ODRI Measurements

 courtesy: E. Chiadroni (INFN)| +175 um |
| :--- |
| +125 um |
| +100 um |
| +75 um |
| +50 um |
| +25 um |
| center |
| -25 um |
| -50 um |
| -75 um |
| -100 um |
| -125 um |
| -150 um |

- transverse scan within the slit
- fit of ODRI angular distribution

E. Chiadroni, M. Castellano, A. Cianchi, K. Honkavaara, G. Kube, Optical diffraction radiation interferometry as electron transverse diagnostics Proc. DIPAC 2009, p. 151
$\Longrightarrow$ ongoing experiment


## OTR/ODR Diagnostics: Pitfalls

- Linac Coherent Light Source (LCLS) @ SLAC

- OTR monitor observation with $B C 1, B C 2$ switched on
- OTR 12

> OTR 22

$\Longrightarrow$ measured spot is no image of beam


## Coherent OTR (COTR)

- interpretation: coherent OTR emission
$\rightarrow$ strong compression in bunch compressors
- simulation
H. Loos et al., Proc. FEL 2008, p. 485




(8)nkin
long bunch $\left(\lambda<\sigma_{z}\right)$
short bunch $\left(\lambda>\sigma_{z}\right)$

OTR22, $250 \mathrm{pC}, 4.3 \mathrm{GeV}$
$10 \mu \mathrm{~m}$ bunch length
$100 \mu \mathrm{~m}$ rms beam size Super-Gaussian


- summary of present knowledge
S. Wesch and B. Schmitt, Summary of COTR Effects, Proc. DIPAC‘11 Hamburg (Germany), 2011, WEOA01


## Consequences

- LCLS: coherent emission compromise use of OTR as reliable beam diagnostics
> wire scanner for transverse beam diagnostics instead of OTR monitors
- FLASH: COTR observed after modifications to linearize longitudinal phase space




SMATCH screen before FLASH undulator
> COTR also expected for E-XFEL

- alternative schemes for transverse profile diagnostics
$>\mathrm{TR}$ at smaller wavelengths (EUV-TR): L.G. Sukhikh, G. Kube, Y. Popov et al., Proc. DIPAC‘11 Hamburg, WEOA02
$>$ screen monitors:
widely used at hadron accelerators, nearly no information available for high energy electron machines
B.Walasek-Höhne and G.Kube, Scintillation Screen Applications in Beam Diagnostics, Proc. DIPAC‘11 Hamburg, WEOB01


## $\Longrightarrow$ ongoing R\&D project at DESY

## Inorganic Scintillators

## - properties

> radiation resistant $\rightarrow$ widely used in high energy physics, astrophysics, dosimetry,...
$>$ high stopping power $\rightarrow$ high light yield
$>$ short decay time $\rightarrow$ reduced saturation

- generation of scintillation light
$\rightarrow$ energy conversion (characterstic time $10^{-18}-10^{-9} \mathrm{sec}$ )
Formation of el. magn. shower. Below threshold of $\mathrm{e}^{+} \mathrm{e}^{-}$pair creation relaxation of primary electrons/holes by generation of secondary ones, phonons, plasmons, and other electronic excitations.
$\Rightarrow$ thermalization of seconray electrons/holes $\quad\left(10^{-16}-10^{-12} \mathrm{sec}\right)$
Inelastic processes: cooling down the energy by coupling to the lattice vibration modes until they reach top of valence resp. bottom of conduction band.
> transfer to luminescent center

$$
\left(10^{-12}-10^{-8} \mathrm{sec}\right)
$$


$>$ photon emission ( $\left.>10^{-10} \mathrm{sec}\right)$
radiative relaxation of excited luminescence centers

```
A - Doped ion, or intrinsic defect B - self- trapped exciton
C - crossluminescence
```


## Implication on Transverse Resolution

## Which effects may affect transverse resolution?

> light generation: energy conversion $\rightarrow$ transverse range of ionization
$>$ light propagation $\rightarrow$ total reflection at scintillator surface

- energy conversion

》 „thick target" : formation of electromagnetic shower
(thickness in the order of radiation length $\mathrm{X}_{0}$ )
> transverse shower dimension: Molière radius as scaling variable
$\rightarrow$ containing $90 \%$ of shower energy
F. Schmidt, "CORSIKA Shower Images", http:// www.ast.leeds.ac.uk/~fs/showerimages.html

$$
R_{M} \approx 0.0265 X_{0}(Z+1.2)
$$

$X_{0}$ : radiationlength, $Z$ : atomic number

## Implication on Transverse Resolution

- energy loss
> Bethe-Bloch (collision)
> Bremsstrahlung (radiative)
- energy deposition in "thin target"
> ignore radiative contribution
- thickness $/ \mathrm{X}_{0} \approx 10^{-2}$
- small amount of re-absorption in material

- ionization: interaction of particle em. field with lattice
, particle field

$$
\left\langle\frac{\mathrm{d} E}{\mathrm{~d} x}\right\rangle_{\text {„relativistic rise" }} \propto \ln \beta^{2} \gamma^{2}
$$

virtual photons, in classical picture transverse evanescent waves
> relativistic rise
incease of transverse field extension
> Fermi plateau
cancellation of incoming particle field by induced polarization field of electrons in medium

$\rightarrow$ saturation range as scaling variable $R_{\delta}$

## Implication on Transverse Resolution

- extension radius
> limiting value:

$$
R_{\delta}=\frac{c}{\omega} \sqrt{1-\varepsilon(\omega)}
$$

> approximation as free electron gas (Drude model)

$$
R_{\delta}=\frac{\hbar c}{\hbar \omega_{p}}
$$

$\varepsilon(\omega)$ : complex dielectric function

$$
\begin{aligned}
& \omega_{\mathrm{p}}: \text { plasma frequency } \\
& \quad \hbar \omega_{p}=28.816 \sqrt{\rho\langle Z / A\rangle} \mathrm{eV}
\end{aligned}
$$

- light propagation
> light generated inside scintillator has to cross surface


## refractiveindex

inorganic scintillators
$\rightarrow$ high n, i.e. large contribution of total reflection


## Scintillator Material Properties



- scintillators under investigation
> BGO: 0.5 mm
> PWO: 0.3 mm
> LYSO: $0.8 \mathrm{~mm}, 0.5 \mathrm{~mm}$
(Prelude 420)
> YAG: $1.0 \mathrm{~mm}, 0.2 \mathrm{~mm}$, phosphor

|  | $\boldsymbol{\rho}[\mathbf{g} /$ <br> $\left.\mathbf{c m}^{3}\right]$ | $\hbar \omega_{\mathrm{p}}$ <br> $[\mathrm{eV}]$ | $\mathbf{R}_{\mathrm{M}}$ <br> $[\mathbf{c m}]$ | $\lambda_{\max }$ <br> $[\mathbf{n m}]$ | yield $[\mathbf{1 / /}$ <br> $\mathbf{k e V}]$ | $\mathbf{n} @ \lambda_{\max }$ | $\mathbf{R}_{\mathbf{\delta}}$ <br> $[\mathbf{n m}]$ |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| BGO | 7.13 | 49.9 | 2.23 | 480 | 8 | 2.15 | 3.95 |
| PWO | 8.28 | 53.3 | 2.00 | 420 | 0.1 | 2.16 | 3.70 |
| LSO:Ce | 7.1 | 51.3 | 2.08 | 420 | 32 | 1.82 | 3.85 |
| YAG:Ce | 4.55 | 45.5 | 2.77 | 550 | 11 | 1.95 | 4.34 |

## Beam Images

## - measurement and analysis: $\mathrm{I}=46 \mathrm{pA} \quad 5$ signal and 1 background frame



## Results

- vertical beam size

- horizontal beam size



## Top-view



## favorites: $\quad \mathrm{BGO}$ and LYSO

$\Longrightarrow$ dependency on observation geometry
G. Kube et al., Proc. IPAC’ 10 , Kyoto (Japan), 2010, p. 906

## Results

- results confirmed in additional experiment
> screen thickness: 0.3 mm
resolution of different materials


Top-view


## Observation Geometry

- beam diagnostics
$\rightarrow$ popular OTR-like observation geometry:
- scintillator tilt versus beam axis

$45^{\circ}$ tilt of screen
observation under $90^{\circ}$
$\rightarrow$ turns out to be bad!

BGO crystal
micro-focused beam
$\mathrm{I}=3.8 \mathrm{nA}$

- measured beam spots

> experimental results confirmed by simulation

G. Kube et al., Proc. IPAC'10, Kyoto (Japan), 2010, p. 906


## Observation Geometry Influence



## R\&D Program on Screens @ DESY

- OTR generation at scintillation screen
> boundary between scintillation screen and vacuum
$\rightarrow$ (C)OTR generation
$\rightarrow$ may be reflected to detector
- investigation of temporal suppression of COTR on screen surface
> basic idea: OTR emission is instantaneous process, scintillation light emitted with delay

Top-view


FLASH(13SMATCH section)
LuAG screen:
COTR \& scintillation light

LuAG screen
+100 ns delay
only scintillation light

- spatial suppression under investiagtion


## Storage Rings (Electrons)

- transverse profile / emittance
> imaging with synchrotron radiation (SR)
$\rightarrow$ non-destructive profile diagnostics
) HERA e beam size: $\mathrm{s}_{\mathrm{hor}}=1200 \mathrm{~mm}, \mathrm{~s}_{\text {vert }}=250 \mathrm{~mm}$
$\rightarrow$ resolution with optical SR sufficient
> problem: heat load on extraction mirror (X-ray part of SR)
$\rightarrow$ material with low absorption coefficient (Be)
$\rightarrow$ cooling of extraction mirror
$\rightarrow$ not sufficient to prevent image distortion...

Photon Factory, LEP: adaptive optics
> solutions:

HERA e:
observation out
of orbit plane

courtesy: T.Mitsuhashi (KEK)

PEP II: slotted mirror

A.S.Fisher et al., Proc. EPAC 1996 , TUP098L

## Light Sources: Emittance Diagnostics

- emittance
typical value
$e_{x}=1 \mathrm{p} \mathrm{nm} \mathrm{rad}$ and $1 \%$ emittance coupling
p principle: synchrotron radiation based diagnostics
> example: $\mathrm{s}_{\mathrm{hor}}=40 \mathrm{~mm}, \mathrm{~s}_{\text {vert }}=20 \mathrm{~mm}$ (PETRA III @ DESY)
SR based imaging: resolution limit (uncertainty principle)

optical imaging: $I=500 \mathrm{~nm}$ and $D \Psi \approx 1.7 \mathrm{mrad}$.. $\quad D \mathrm{~s}_{\text {vert }}=150 \mathrm{~mm}$
n resolution fully limited by uncertainty principle (diffraction limited)
> widely used schemes for emittance diagnostics

.. dedicated diagnostics beamline


## Proton Synchrotron Radiation

- SR generated in dipole fringe field
setup:

screen shot:

dynamics study:
moving collimators towards the beam

G. Kube et al., Proc. of BIW06 (2006), Batavia, Illinois, p. 374


## Simulation of Light Propagation



## Analysis:

- ZEMAX calculation of 2-dim PSF
- calculation of 2-dim beam profile
- convolution of PSF and beam profile
- horizontal / vertical projection of resulting distribution
- determinatiuon of $2^{\text {nd }}$ moment (standard deviation)
$>+22.5^{\circ}$


Signal in ROI


$\Rightarrow 0^{\circ}$


$\sigma_{x}=48.95 \mathrm{mu}$


## Comparison



- satisfactory agreement between simulation and measurement
$\rightarrow \quad$ simulation reproduces observed trend in beam size
- measured beam size systematically larger than simulated one
$\rightarrow \quad$ effect of extension radius not included in calculation $\rightarrow$ increase in PSF
> results summarized in IPAC' 10 proceedings:
G. Kube, C. Behrens, W. Lauth, MOPD088


## SR Single Particle Time Structure

- geometrical interpretation


$$
\Delta \mathrm{t}=\frac{4 \rho}{3 \gamma^{3} c}
$$

$\Delta t$ : distance in travel time between photon and particle

- radiation field in time domain

consequence:
time interval from maximum to zero crossing defines spectrum $\left(w_{c}\right)$


## SR Single Particle Time Structure

- comparison



## SR Single Particle Time Structure

- characteristic time: $\Delta t_{c}=\frac{L}{c} \cdot(1-\beta) \approx \frac{L}{2 \gamma^{2} c}$
with L: "formation length"


$$
\frac{\mathrm{d}^{2} N}{\mathrm{~d} \Omega \mathrm{~d} \omega / \omega} \propto\left|\vec{E}_{\omega}\right|^{2}
$$

with



$\vec{E}_{\omega}=\frac{1}{\sqrt{2 \pi}} \int_{-\infty}^{+\infty} \mathrm{d} t \vec{E}(t) e^{i \omega t}$

