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# Value at Risk (VaR) and its calculations: an overview.

## 1. Introduction

Modern financial markets adopt several major kinds of risk: credit risk, operational risk, liquidity risk and market risk. In recent years, researches and market practitioners have paid more attention to Value at Risk (VaR) in analyzing the market risk. According to the International Monetary Fund (IMF), the 2008 financial crisis has made an overall loss of $3.4 trillion among all major financial institutions over the world(Dattels,2009). It is an enormous economic decline since the 1930s. In the case of Black Monday, the world stock crashed in a very short time. The major quantitative measurement of market risk during such catastrophic events is the above-mentioned Value at Risk, or VaR in short. It attempts to measure the risk of unexpected changes in prices or log-return rate within a given period. It is a very simple and popular way of measuring market risk. Since it will provide an amount that summarizes the whole market risk faced by the company, VaR has become a necessary implementation in any professional corporate risk management. VaR is not only applicable in exploring the market risk but also in manage all other types of risk. This entire system is primarily designed for both risk management and regulatory purposes. It is broadly used by most financial institutions, commercial banks, and investment banks to estimate the potentially maximal loss of their portfolio during a given time period for a given market condition. On the other hand, from the viewpoint of regulatory committee, VaR can be defined as a set margin, which is the minimal loss under the confidence interval of a horizontal time.

By the basic definition of the VaR, it is the maximum expected potential loss on the portfolio over the given time horizon for a given confidence interval under normal market conditions(Jorion,2001). In other words, there are three key elements to describe the Value at Risk (VaR):

1. A time period.
2. The dollar amount of VaR(portfolio, assets ,etc. ).
3. A given normal market condition (or confidence interval).

For example: consider a \$ 100 million portfolio, suppose the confidence interval is 95% for a 1-month horizon. These are typical statements to calculate the VaR for a 1-month horizon (30 days). Overall, each of our models will be related to these three statements.

The objective of this paper is threefold. First of all, I briefly discuss the mathematical theory used to calculate VaR. Secondly; I intend to list the three different methodologies to estimate the risk. Those are Risk Metrics, time series to calculate VaR, and Extremely Value theory to measure it. I want to introduce how this concept can be used to count VaR The paper ends with a discussion of the strengths and weaknesses of calculation of the VaR in each of the approaches.

**2. 1 Mathematical definition of Value at Risk.**

Given a confidence level of $p \in (0,1)$, and assumed the time index of t and $t + \alpha$ , we want to find the change asset of the $\Delta V\ (\alpha)$ in the financial position over the time period $\alpha$. Let $F_\alpha(x)$ be the cumulative distribution function (CDF) of $\Delta V\ (\alpha)$. Since the financial position is

$\Delta V\ (\alpha) \leq 0$, then we can define the VaR of a long position over the time horizon $\alpha$ for a given $p$ as

$$p = \mathbb{P}[\ \Delta V\ (\alpha) \leq VaR] = F_\alpha(VaR)$$

Considering the holder of a short position, in a given time $\alpha$ with probability $p$, and the financial position $\Delta V\ (\alpha) \geq 0$, the VaR is define as

$$p = \mathbb{P}[\ \Delta V\ (\alpha) \geq VaR] = 1 - \mathbb{P}[\ \Delta V\ (\alpha) \leq VaR] = 1 - F_\alpha(VaR)$$

Next, we will define the $p$-quantile of $F_\alpha(x)$, that for any CDF of $F_\alpha(x)$ and the given confidence level of $p \in (0,1)$ is

$$VaR_p = x_p = \inf\{x|F_\alpha(x) \geq p\}$$

inf : The smallest real number

$x_p$: It also can write is $VaR_p$ if $F_\alpha(VaR)$ is known

Therefore, the tail behavior of the CDF of $F_\alpha(x)$ or its quantile is condition necessary for approaching VaR calculation.

In the application of VaR calculation, we have already listed three factors involve in the article.

1. A time period. Such as, the time horizon $\alpha$.
2. The dollar amount of VaR(portfolio, assets ,etc. ).
3. A given normal market condition (or confidence interval).Such as, a confidence level of $p \in (0,1)$

We could apply two more factors to estimate the VaR:

4. The frequency of the data.
5. The CDF of $F_\alpha(x)$ or its quantiles.

**2.2 An Easy Example for VaR Calculation.**

Since the definition of the log return $r_t$ is the effective daily returns with continuous compounding, we use $r_t$ to calculate the VaR. That is VaR= Value of amount financial position * VaR (of log return). We will solve the previous example we disuses at the beginning of the article.

**Example 1:**

Consider a $1000 million portfolio of medium-term bonds. Suppose the confidence interval is 95%, what is the maximum monthly loss under normal markets over any month?

**Solution:**

We first look at the graph Fig. 1.1 and Fig. 1.2 we can find the 95% confidence interval, the lowest monthly log return$r_t = -1.7\%$. So the corresponding VaR for a monthly loss under the normal market over any month is,

$$VaR = \$1000 \text{ million} * 1.7\% = \$17 \text{ million}$$

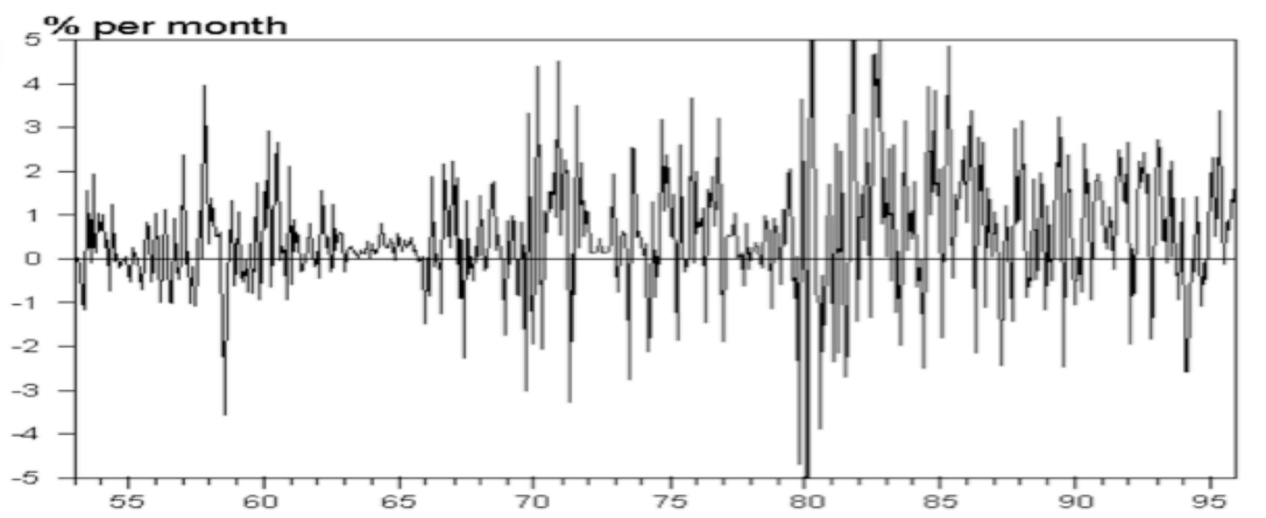


Figure 1.1 The Monthly Medium-term Bond Returns from 1953 to 1995

From the graph, we can get the lowest point is -6.5% and the highest point is 12%

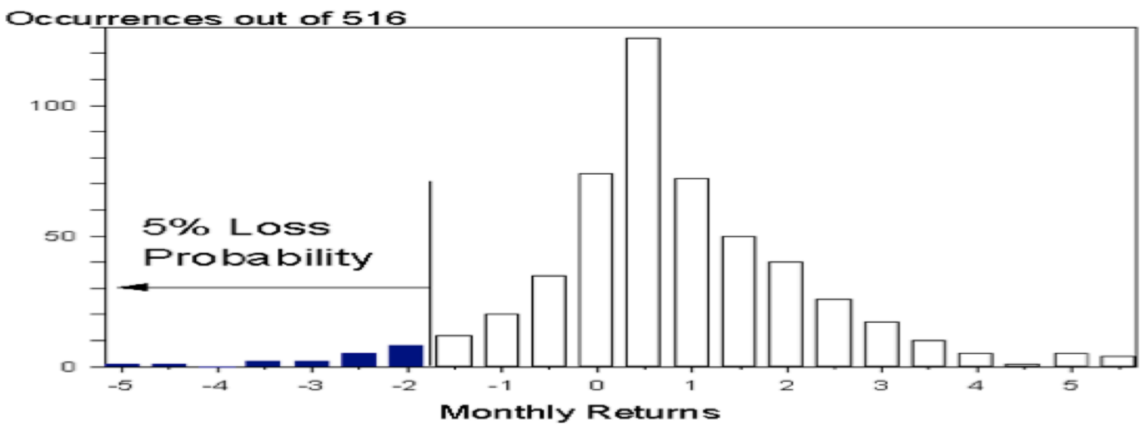


Figure 1.2 The Distribution of Medium Bond Returns.
From the picture, we get the lowest monthly is -1.7%.There are 26 months out of the 516 for which the monthly returns were lower than -1.7%. Therefore, log return is -1.7%

In this adapted question, the measurement slightly considers that the possible loss in value is under the "normal market risk". The maximum loss is 17 million. Overall, VaR could specifically calculate for an individual loss, a large investment project risk for a firm, and a portfolio of asset.

**3. Approaches to VaR Calculation**

I will discuss several methods of the VaR calculation are RiskMetrics, econometric modeling using volatility models, and extreme value theory to estimate VaR. Also, it relates some practical and simple question.

**3.1.1 RiskMetrics**

The Riskmetrics methodology is concerned with data sets and technique software used to calculate the VaR. This model was first established by JPMorgan, which was applied to expose the trading losses and explain the risks of their company in 1989(RiskMetrics). A few years later, J.P.Morgan launched this methodology and released the technical document freely available to all marketplaces. J.P Morgan develope the RiskMetrics method to VaR calculation under the normal distribution. We can denoted a daily report measure and explain the risk of the company.

A mathematical method to find the daily price in this model is: suppose the daily log return by $r_t$ and $F_{t-1}$ be the data at time $t-1$.The conditional normal distribution under the RiskMetrics is

$$r_t|F_{t-1} \sim N(\mu_t, \sigma_t^2),$$

where $\mu_t$ is the conditional mean and $\sigma_t^2$ is the conditional variance of $r_t$.This method is simple model that involve two quantities over time.

$$\mu_t = 0, \quad \sigma_t^2 = \alpha\sigma_{t-1}^2 + (1-\alpha)r_{t-1}^2, \text{ which } \alpha \in (0,1) \qquad (4.1.1)$$

This model satisfies the without drift IGARCH process with $p_t - p_{t-1} = a_t$, where $a_t = \delta_t \varepsilon_t$, and uses it to estimate the daily price $p_t$, which is $p_t = ln\ (P_t)$. Also, we can use the probability theory to find a multi-period log return to find the $k$-period horizon VaR of the portfolio under the IGARCH model. Assume $r_t[k]$ is a log return in a $k$-period horizon. We can write the equation as $r_t[k] = r_{t+1} + r_{t+2} + \cdots + r_{t+k}$. We want to find the conditional normal distribution of $r_t[k]|F_{t-1}$. Since the $r_{t+1}, r_{t+2}, \cdots, r_{t+k}$ are independent and identically distributed (i.i.d) random variable. Therefore, the mean of the conditional distribution of $r_t[k]|F_{t-1}$is $\mu_t[k] = 0$. To find the variance of this conditional distribution, we will use the forecasting mathematical method and combined equation of (4.1.1) to get the $\sigma_t^2[k]$=k$\sigma_{t+1}^2$. Hence, the conditional normal distribution is $r_t[k]|F_t \sim N\ (0, k\sigma_{t+1}^2)$.

**4.1.2 Example for estimate VaR**

Supposed in a long position, we will use RiskMetrics(RM) to estimate the risk of the portfolio under the conditional normal distribution. The 1-day horizon VaR of the portfolio is

VaR= Value of the financial position * (VaR (of log return)* $\sigma_{t+1}$),

and a $k$-period horizon VaR of the portfolio is

VaR(k)= Value of the financial position * (VaR (of log return)* $\sqrt{k}\sigma_{t+1}$), by combining these two equations, we can easily get VaR(k) using the RiskMetrics method

$$\text{VaR(k)}= \sqrt{k} * \text{VaR}.$$

**Example 2:**

Consider an investor had a $10 million portfolio of bonds in a long position Suppose the confidence interval is 95%. The actual daily standard deviation of the portfolio over one trading year is 3.67%, what is the daily VaR of this portfolio? What is the VaR for a 1-month horizon(30 days)?

**Solution:**

Since the confidence level is 95%, RiskMetrics uses 1.645, as the z-score for 95%.And the standard deviation is 3.67%. According to the method discussed above, we can easily get the 5% VaR of a 1-day horizon is

VaR=$10 million * 1.645 * 3.67% = $603,715

The VaR of a 1-month horizon(30 days) for the investor is

VaR=$10 million * 1.645* $\sqrt{30}$ *3.67%=$3,306,683

**4.2 Econometric Model to Calculate VaR**

Using a time series economic model to measure a company's risk is an econometric approach to VaR calculation. We will use the GARCH model to discuss this approach. This methodology mainly uses statistical techniques and economic concepts. $r_t$ is a log return of an asset. According to Tsay(2002) can write the following equations to represent $r_t$with time series model:

$$r_t = \varphi_0 + \sum_{i=1}^{p} \varphi_i r_{t-i} + a_t - \sum_{j=1}^{q} \theta_j \, a_{t-j}, \qquad (4.2.1)$$

$$a_t = \delta_t \varepsilon_t, \sigma_t^2 = \alpha_0 + \sum_{i=1}^{u} \alpha_i \, a_{t-i}^2 + \sum_{j=1}^{v} \beta_j \, \sigma_{t-j}^2, \qquad (4.2.2)$$

where the equation of $r_t$ is the mean and the equation of $\sigma_t^2$is the volatility of $r_t$.

Assume that $\varepsilon_t$is Gaussian, and all parameters are known. Then using the 1-step ahead forecast method to find the condition mean, $\widehat{r_t}(t)$ and variance of $r_t$, $\widehat{\sigma_t^2}(t)$. We have

$$\widehat{r_t}(t) = \varphi_0 + \sum_{i=1}^{p} \varphi_i r_{t+1-i} + a_t - \sum_{j=1}^{q} \theta_j \, a_{t+1-j},$$

$$\widehat{\sigma_t^2}(t) = \alpha_0 + \sum_{i=1}^{u} \alpha_i \, a_{t-i}^2 + \sum_{j=1}^{v} \beta_j \, \sigma_{t+1-j}^2,$$

we also assume that $\varepsilon_t$is Gaussian, we get the conditional distribution of $r_{t+1}$ given the data at time t is $r_{t+1}|F_t \sim N\left[\widehat{r_t}(1), \widehat{\sigma_t^2}(1)\right]$, where the mean $\widehat{r_t}(1) \neq 0$. Hence, the calculation of value at risk(VaR) is concerned both with the mean and standard deviation. So, the daily VaR of the asset using this method at time t is VaR=Amount of position * (mean –VaR(log return) * standard deviation) which writing in mathematical notation becomes:

$$VaR = Amount\ of\ assest * \left[\left(\widehat{r_t}(1)\right) - \text{VaR(log return)} * \widehat{\sigma_t^2}(1)\right]$$

Also, we need suppose that $\varepsilon_t$is Gaussian, and all parameter are known. We are interested in finding the log return of $r_t$ of a k-horizon VaR of the portfolio at time z using this approach. Thus, the k-period horizon log return is $r_z[k] = r_{z+1} + r_{z+2} + \cdots + r_{z+k}$.

If the individual log return $r_t$ follows the time series model discussed above, then the conditional distribution of $r_z[k]|F_z$ can still use the forecasting method to find it. After the calculation of this model, we get the normal distribution of $r_z[k]$ given $F_h$ with a mean of $k\mu$ and a variance of $Var\ (e_z[k]|F_z\ )$, which we write following

$$Var(e_z[k]|F_z) = \sum_{i=1}^{k} \sigma_z^2\ (i)\ ,$$

where $i$ is the $i$-step ahead, and $e_z[k]$ is the sum of 1-step to k-step forecast errors of $r_t$ at the forecast origin $z$. Hence, this results in $r_z[k]|F_z \sim N[\ k\mu\ , \sum_{i=1}^{k} \sigma_z^2\ (i)\ ]$. Now, we get the equation to estimate the VaR of a k-period horizon starting at the forecast origin z is VaR=Amount of position * (mean –VaR(log return) * standard deviation), which we can use notation to represent as

$$r_z[k] = k\mu, \quad Var(e_z[k]|F_z) = \sum_{i=1}^{k} \sigma_z^2\,(i),$$

$$VaR = Amount\ of\ assest * \left[(r_z[k]) - \text{VaR(log return)} * \sum_{i=1}^{k} \sigma_z^2\,(i),\right],$$

However, the mean and standard deviation are totally different from the 1-day conditional normal distribution. Therefore, we need to find the two variables first based on our model and compute the VaR of the portfolio..

**4.3.1 Extreme Value Theory**

The mathematical theorem of Extreme Value Theory (EVT) was first explored by Fisher Tippett(1928). His works shows that this concept leads to find the limiting distribution of the normalized maximum. Firstly, we assume that $\{r_1, r_2, \cdots r_n\}$ is a sequence of independent and identically distributed $(i.i.d)$ random variables with a common distribution function $F(x)$, satisfying $r_{(1)} \le r_{(2)} \le \cdots \le r_{(n)}$. Also, we know

$$VaR_p = \inf\{x: F(x) \ge 1 - p\}, \text{ where } 1 - p = \mathbb{P}(r_t \le\ VaR_p)$$

And the range of the log return $r_t$ is $[-\infty, +\infty]$. If we get the distribution of $r_{(n)}$, then we can write that

$$1 - p^* = \mathbb{P}\left[\, r_{n,i} \le r^*_{(n)}\right] = \left[\mathbb{P}(r_t \le\ r^*_{(n)})\right]^n,$$

where $r^*_{(n)}$ is the $1 - p^*$ quintile of $r_{(n)}$. Denoted the CDF of $r_{(1)}$ is $F_{n,1}(x)$, we get the equation

$$\begin{aligned} F_{n,1}(x) &= \mathbb{P}\ (r_t \le\ \text{x}) \\ &= \mathbb{P}\ (r_1 \le \text{x}, r_2 \le \text{x}, \cdots, r_t \le \text{x}) \\ &= \prod_{i=1}^{n} \mathbb{P}\ (r_i \le\ \text{x}), \text{ since } r_t \text{ is } i.i.d \\ &= \prod_{i=1}^{n} F(x) \\ &= [F\ (x)]^n, \end{aligned}$$

Thence, we need to have two appropriate normalization sequences a position sequence of $\{\beta_n\}$ and a scale sequence $\{\alpha_n\} > 0$. Also, to consider the limiting distribution of the $F_{n,1}(x)$, this distribution holds:

$$r_{(n*)} = \frac{r_{(n)} - \beta_n}{\alpha_n} \overset{n\to\infty}{\Longrightarrow} F_*(x),$$

where the limit of a probability for $n \to \infty$ denoted by $F_*(x)$ is the *generalized extreme value* (GEV). According to the Jenkinson (1955) and von Mises(1954 ), it defines as

$$\text{If } \varepsilon \neq 0, \text{then } F_*(x) = e^{-(1+\varepsilon x)^{1/\varepsilon}},$$

$$\text{If } \varepsilon = 0, \text{then } F_*(x) = e^{-e^{-x}},$$

where the symbol * is the maximum, and $\varepsilon$ is the shape parameter which can determine the tail index x .Also, the probability density function(pdf) of the generalized limiting distribution becomes

$$\text{If } \varepsilon \neq 0, then\ f_*(x) = (1+\varepsilon x)^{-\frac{1}{\varepsilon-1}} e^{[-(1+\varepsilon x)^{-\frac{1}{\varepsilon}}]}$$
$$\text{If } \varepsilon = 0, \text{then } f_*(x) = e^{-x-e^{-x}}$$

There are three specific types of the limiting distribution of Gnedenoko(1943) based on the extreme value distribution, which are generalized as the *generalized extreme value* (GEV), is

Case I: $\varepsilon = 0.$ $It\ is\ the\ Gumbel$ family, $with$

$$F_*(x) = e^{-e^{-x}}, \quad x \in \Re$$

Case II: $\varepsilon > 0$.It is F$rechet$ family, with

$$F_*(x) = e^{-(1+\varepsilon x)^{-1/\varepsilon}}, \ x > -\frac{1}{\varepsilon},$$

$$F_*(x) = 0, \quad x \leq -\frac{1}{\varepsilon}$$

Case II: $\varepsilon < 0$.It is $Weibull$ family, with

$$F_*(x) = e^{-(1+\varepsilon x)^{-1/\varepsilon}}, \ x > -\frac{1}{\varepsilon},$$

$$F_*(x) = 1, \qquad x \leq -\frac{1}{\varepsilon}$$

Gnedenko(1943) states that the tail of $F(x)$ decays that could determines the limiting distribution $F_*(x)$. The exponentially decaying tails of the $Gumbel$ family consists of the thin-distribution the example for example normal and lognormal distribution. In the F$rechet$ family, the distribution has a polynomials decaying tail that declines like a power function such as the Pareto distribution, Student's t-test and mixture distribution. Finally, the tail of the distribution declines is finite endpoint distribution for the $Weibull$ family.

### 4.3.2 Statistical Methodology

There are three parameters in the extreme value distribution. They could use to be either parametric or nonparametric methods. $-\varepsilon$ is refereed to as the shape, $\beta_n$ is related to as the location, and $\alpha_n$ is represented to as the scale parameter. Tsay(2002) assumed that there are $M$ returns $\{r_j\}_{j=1}^{M}$. And each of them is divided as

$$\{r_1, r_2, \cdots r_n | r_{n+1}, \cdots, r_{2n} | r_{2n+1,} \cdots, r_{3n} | \cdots | r_{(g-1)n+1}, \cdots, r_{ng}\}$$

and let $r_{in+j}$ is the observed return where $1 \le j \le n$ and $\forall i = 1,2,\cdots,g$.Then there are $n$ sizes of the subsample and supposed that the largest return of the $ith$ subsample is $r_{n,i}$.Let n go to infinite, the extreme value distribution apply$\frac{r_{n,i}-\beta_n}{\alpha_n} = x_{n,i}$, the maximization of the collection of the subsample define as

$$r_{n,i} = \max_{1 \le j \le i}\{r_{(i-1)n+j}\}, \qquad \forall i = 1,\cdots,g$$

we used the extreme value theory implement for estimating the unknown variable in the extreme value distribution when n is sufficiently large. The choice of the subsample size n could determine the estimators. We can use this assumption into either parametric or nonparametric methods to count VaR.

There are two parametric approaches maximum likelihood and regression methods. The regression method supposes that $\{r_{n,i}\}_{i=1}^{g}$ is a $i.i.d$ random variable sequence from the common cumulative distribution function $F(x)$ under the Gumbel distribution properties. We can denotes that

$$r_{n(1)} \leq r_{n(2)} \leq \cdots \leq r_{n(g)}$$

is the collection of subperiod maxima. Under the properties of the Gumbel distribution, we have the following

$$E\left[F_*\left(r_{n(i)}\right)\right] = \frac{i}{g+i}, \forall i = 1, \cdots, g$$

If $\varepsilon_n \neq 0$, it follows the generalized extreme value theory. We have

$$F_*\left(r_{n(i)}\right) = e^{-(1+\frac{\varepsilon_n\left(r_{n,i}-\beta_n\right)}{\alpha_n})^{-\frac{1}{\varepsilon_n}}}$$

we can apply both equation to get

$$\frac{i}{g+i} = e^{-(1+\frac{\varepsilon_n\left(r_{n,i}-\beta_n\right)}{\alpha_n})^{-\frac{1}{\varepsilon_n}}}, \text{for } \forall i = 1, \cdots, g$$

taking the logarithm twice in this equation, we have

$$\ln\left[-\ln\left(\frac{i}{g+i}\right)\right] = -\frac{1}{\varepsilon_n} ln(1 + \frac{\varepsilon_n\left(r_{n,i}-\beta_n\right)}{\alpha_n}), \ \ \forall i = 1, \cdots, g.$$

After we calculate previous equation, we will have error between these two quantities, denoted by $e_i$. Then we have the regression equation to get the $\varepsilon_n, \alpha_n\ and\ \beta_n$ that is

$$\ln\left[-\ln\left(\frac{i}{g+i}\right)\right] = -\frac{1}{\varepsilon_n} ln\left(1 + \frac{\varepsilon_n\left(r_{n,i}-\beta_n\right)}{\alpha_n}\right) + e_i, \ \ \forall i = 1, \cdots, g$$

If $\varepsilon_n = 0$, we can write the equation is

$$\ln\left[-\ln\left(\frac{i}{g+i}\right)\right] = \frac{1}{\alpha_n} r_{n(i)} + \frac{\beta_n}{\alpha_n} + e_i, for\ \forall i = 1, \cdots . g$$

These estimates are consistent but less efficient than another parametric approach, which is maximum likelihood method. Assumed that $\{r_{n,i}\}$ is the subsample maximum. The pdf of $\frac{r_{n,i}-\beta_n}{\alpha_n} = x$ follows the generalized extreme value distribution, which we write as

$$\text{If } \varepsilon \neq 0, then\ f(r_{n,i}) = \frac{1}{\alpha_n}(1+\frac{\varepsilon_n(r_{n,i}-\beta_n)}{\alpha_n})^{-\frac{1+\varepsilon_n}{\varepsilon_n}} * e^{-(1+\frac{\varepsilon_n(r_{n,i}-\beta_n)}{\alpha_n})^{-\frac{1}{\varepsilon_n}}}$$

$$\text{If } \varepsilon = 0,\ \ then\ f(r_{n,i}) = \frac{1}{\alpha_n} e^{[-\frac{r_{n,i}-\beta_n}{\alpha_n} - e^{\left(-\frac{r_{n,i}-\beta_n}{\alpha_n}\right)}]}.$$

The maximum likelihood could estimate the $\varepsilon_n, \alpha_n\ and\ \beta_n$. These estimator are unbiased, more efficient and having the minimum variance. We will use the method to calculate the VaR. Let $p^*$ be a small upper tail probability and $r_n^*$ be the quantile of the subperoid maximum with $(1-p^*)$th. Also, we plug $\frac{r_n^*-\beta_n}{\alpha_n} \leq x$ into the CDF of the maximum likelihood methods function under the limiting generalized extreme value distribution. Then we have $1-p^*$ equal to

$$\text{If } \varepsilon \neq 0,\ then\ \ 1-p^* = e^{[-(1+\frac{\varepsilon_n(r_n^*-\beta_n)}{\alpha_n})^{-\frac{1}{\varepsilon_n}}]}$$

$$\text{If } \varepsilon = 0,\ \text{then } 1-p^* = e^{-e^{\frac{r_n^*-\beta_n}{\alpha_n}}}$$

After taking natural logarithm and transformation this equation, we have the quantile as

$$\text{If } \varepsilon \neq 0,\ then\ \ r_n^* = \beta_n - \frac{\alpha_n}{\varepsilon_n}[1-[-\ln(1-p^*)]^{-1/\varepsilon_n}$$

$$\text{If } \varepsilon = 0,\ then\ \ r_n^* = \beta_n - \alpha_n \ln[-\ln(1-p^*)].$$

Next we will show the relationship between the observed return $r_t$ series and the subperiod maximum for a given upper tail probability $p^*$ and the quantile $r_n^*$. We obtain

$$1-p^* = \mathbb{P}(r_{n,i} \leq r_n^*) = [\mathbb{P}(r_t \leq r_n^*)]^n.$$

Given the specified small upper probability $p$, and $(1-p)th$ quantile of $r_t$ is $r_n^*$.Thus we have the equation of the VaR portfolio is

$$\text{If } \varepsilon \neq 0,\ then\ \ VaR = \beta_n - \frac{\alpha_n}{\varepsilon_n}[1-[-n\ln(1-p)]^{-\varepsilon_n}$$

$$\text{If } \varepsilon = 0,\ then\ \ VaR = \beta_n - \alpha_n \ln[-n\ln(1-p)],$$

where n is the length of subperiod.

## 6. Conclusion

In this paper we have three different models to calculate value at risk and estimate the potential value of a portfolio during a given time period for a given market condition. In the first, we introduce the method of the RiskMetric. The advantage of this method is straightforward. It is very easy for people to understand and bring into the financial market to measure the risk of a company. Also, it makes the risk much easier to clarify. However, when the conditional mean is not equal to zero, we can not use the RiskMetric approach to estimate VaR. We need to consider using the econometric method to calculate the VaR. We briefly review the statistical techniques and economic concepts applied in this method. In the third approach, we discuss using the extreme value theory to compute the VaR of a portfolio. The extreme value distribution obtains three different parameters and select the length of the subperoid. Also, we need to check the adequacy of the fitted extreme value model. Since the statistical testing could fail to apply the daily log returns under the independent assumption, the extreme value theory may not return a relatively accurate VaR. All three Value at Risk approaches measure the value of the portfolio for analysis and suggestions for the company. Finally, we understand the mathematical and statistical concepts of calculating Value at Risk.

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