Szydagis

THE FORCES AMONGST *N* BODIES, PLUS CHAOS

1/ r^n forces, and their bound states, in classical mechanics. Plus some chaos theory.

But 1^{st}: Final Project Ideas

- Send me your plan
  - No later than Monday, March 25!
  - If you are in 577 (or, 477Y)
- Deadline i.e. due date
  - Monday May 6^{th} for 577: written (report and code). Oral on 4/30 optional

Suggested Final Projects (choose)

- Projectile motion with both wind + air resistance, and different distances from Earth, and on different planets, including escape velocity calculations and Corliolis force and any other relevant effects
- Orbital mechanics in 3 (or more! ☺) dimensions with 3 or more bodies
- Machine learning to solve a particular problem, such as recognizing words within a particular genre
- Sim of particle physics detector, weather/climate/atmosphere, reactor,…
- Looking for a signal buried in noise (acoustic, color in light, etc.)
- Image analysis: e.g., find the cancerous tumor
- Gravitational wave discovery from BH merger
- 2 birds with 1 stone: something in your own research
- Quantum entanglement calculator (q-computer coherence simulator)
- Geometric optimization (how many irregularly-shaped products in a box)
- Non-ideal gas in thermodynamics/stat mech
- Revisit universe simulator with even wilder/wider assumptions
- Meta-analysis of existing results in some field
- Contrasting REAL random numbers and better generators to the default
- Propagation of sound/light/heat in complicated media (inhomogeneous)
- Complicated, realistic, time-varying electric and magnetic fields
- ETC: Conjugate gradient topology, fractals, Brownian motion, fluids. May a thousand flowers bloom, and let your imagination run wild

Continued

These lists here are NOT exhaustive despite being very long. Particle physics: ATLAS/CMS, DUNE.

- Just like with revisiting universe: take any existing HW or example further
  - But would have to be MUCH further in order to qualify
- More projectile motion: look into oils/syrup/honey (viscous fluids), do initial velocity in all possible directions with additional acceleration, and an accelerating acceleration
- Nuclear physics: bombs and/or reactors and power plants. Fission, fusion
- Neural networks: Keras/TensorFlow, ROOT's TMultiLayerPerceptron
- Lattice QCD, QFT, QED. Ising model, solid state physics.
- More N-body sims, chaos theory (butterfly effect) e.g. double pendulum.
- Video game(s) (WITH PHYSICS). The Kessel Run?
- Neurology, neurophysiology, neuroscience, biophysics
- Quantum computing

Lastly: first come, first served! No dups. Need more ideas? Talk to me and/or Shane.

Intro to Verlet-Störmer Integration

AKA Leapfrog/Newton integration (called integration though DE solving; also called 'method').

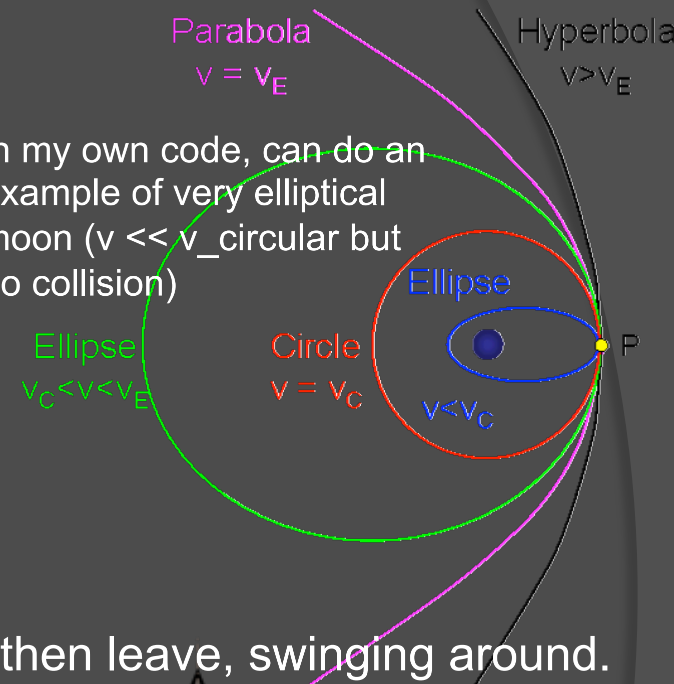
- Algorithm for solving the (coupled differential) equations of motion for N bodies, numerically
  - Basis for most if not all gravitational simulations in classical mechanics (orbital/celestial mechanics) Astrodynamics!
  - Coupled: no need to assume one mass is infinite and that then just one body is moving in a central potential
  - Basis for all video-game gravity-based physics too
- Very similar to semi-implicit (midpoint) Euler method, though generalizing to coupled, second-order differential equations. Discovered over & over
  - ~Conserves the total energy (kinetic+potential). stable
  - Similar accuracy to the great RK4 method but is faster.
  - Not limited to one force only; any kind / # should work
  - Easier than Hamiltonian or Lagrangian actually for many cases
- https://docs.microsoft.com/en-us/previous-versions/windows/internet-explorer/ie-developer/samples/dn528554(v=vs.85)?redirectedfrom=MSDN (review of the basic structure of the algorithm; read it then work for NASA :-)

Gravity: A 1 / r^2 Force

- ~Unique: closed orbits. Few other power laws
  - 2nd power may be unique if counting back-reaction
- F = G m M / r ^2 in Newton, which can be generalized to constant / r ^ (d-1) where d is # of macroscopic spatial dimensions.
  - Careful tests of separation^-2 law indicate we live in 3 dimensions. Extra must be small scale
- Action at a distance of Newton vs. light speed limitation of information transfer of Einstein
  - Quantum Mechanics: it is a spin-2 boson, the graviton (undiscovered) which has zero mass
    - No mass: speed never different than c; gravitational force thus has an infinite reach. Would otherwise have extra “suppression term.”
  - General relativity: gravity is geometry of space-time

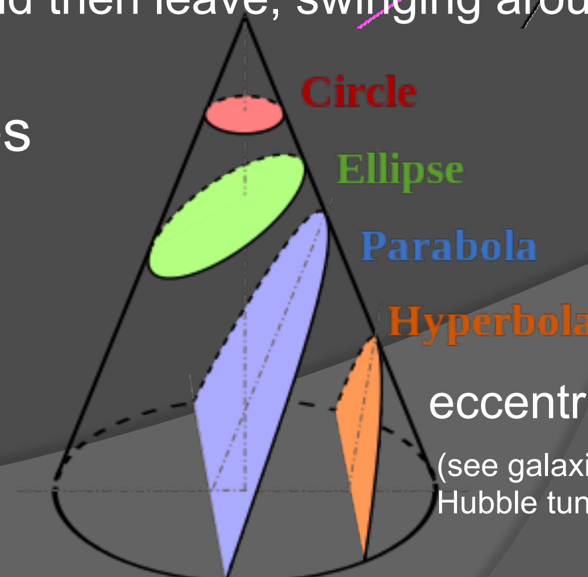
Possible Orbital Shapes

The conic sections (mostly)!



In my own code, can do an example of very elliptical moon (v << v_circular but no collision)

- Bound (collide / revolve around)
  - Straight line: boom (or away)
  - Circle (unstable! Slight perturbation creates an ellipse, such as disturbance of the bigger mass caused by back-reaction of gravity from smaller mass)
    - Transitional border for different foci
  - Ellipse: Not same as an oval!
- Unbound (escape) close-encounter and then leave, swinging around.
  - Parabola (turning point between ellipse and hyperbola; mass escapes to infinity with zero kinetic energy)
  - Hyperbola (@infinity with > 0 KE!)
- NEW with GR: non-Newtonian
  - Inward-going spiral
  - Counterintuitive that non-classical



eccentricity=?

(see galaxies -> Hubble tuning fork...)


20 Dec 2010
19 Mar 2011

An Aside

Story of elliptical dome in DC ("perfect" enough!)

- https://web.archive.org/web/20220605030226/http://mathematica.stackexchange.com/questions/63690/oval-or-bunimovich-stadium
- J. Kepler's three laws of planetary motion
  - Orbit of planet is ellipse with Sun at one focus
  - Equal areas over equal periods of time (this implies planets move faster when closer to Sun)
    - A changing acceleration: makes perfect sense since more force when nearer.
    - Useful for slingshot maneuver "boosts/kicks" like in "The Martian." Real method NASA uses in real life, such as for various interplanetary probes.
  - Square of orbital period proportional to cube of the semi-major axis of the ellipse that is the orbit
    - Also, F = ma but a = a of r, where a or little g=(-)G*M/r^2.
  - Empirical only! Not first principles until Newton
- Cassini used a moon of Jupiter as a clock to find distance to Mars, allowing all other values to follow

Lagrange Points L2, L5

- WMAP, Planck, (e)LISA
- space station colonies?
- Earth-Moon, Earth-Sun
- Trojan exoplanets exist. https://arxiv.org/abs/1511.01068

Addendum: Gravitational Waves!

Their discovery is not same as discovery of graviton (despite QM wave-particle duality)


The Washington Post

FRIDAY, FEBRUARY 12, 2016

Deadly prison riot in Mexico

Distraught relatives confront police at the Topo Chico prison in Monterrey, Mexico, where a clash between rival factions touched off a riot, leaving 49 inmates dead. It was one of the worst outbreaks of violence in a system plagued by crowding and corruption, and comes less than a week before Pope Francis is to visit a Mexican prison. Story, A15

U.S., Russia agree to a halt in Syrian war

'CESSATION' WILL LET IN HUMANITARIAN AID

Deal to take effect in days; Moscow to make airdrops

By Karen DeYoung

Ex-wife's tip led to gunfire at Md. Panera

One of two slain deputies 'never had a chance' to draw weapon, sheriff says

Fault line spotlighted in Wis. debate

By Dan Balz and Abby Phillip

Democratic contenders clash repeatedly but civilly

Oregon siege ends as four surrender

Standoff drew attention to growing movement of anti-government activists

Gravitational waves Einstein foresaw are detected

By Joel Achenbach and Rachel Feltman

California Institute of Technology professor Kip Thorne co-founded LIGO, which detected the waves.


The New York Times

Late Edition

NEW YORK, FRIDAY, FEBRUARY 12, 2016

$2.50

WITH FAINT CHIRP, SCIENTISTS PROVE EINSTEIN CORRECT

A RIPPLE IN SPACE-TIME

An Echo of Black Holes Colliding a Billion Light-Years Away

By DENNIS OVERBYE

A worker installed a baffle in 2010 to control light in the Laser Interferometer Gravitational-Wave Observatory in Hanford, Wash.


LA NACION

Viernes 12 de febrero de 2016 | lanacion.com

Los gremios llevaron al Gobierno sus planteos y salieron conformes

Griesa evalúa si facilita que el país salga del default

El dengue ya es una epidemia en Córdoba

CONFIRMAN LA ÚLTIMA PREDICCIÓN DE EINSTEIN

El Papa inicia su visita al México más crudo

Chicos cada vez más solos

Boca, en crisis, le dio un partido más al DT

Un hallazgo histórico abre una nueva era en la astronomía


Frankfurter Allgemeine

ZEITUNG FÜR DEUTSCHLAND

Ayrault wird Außenminister Frankreichs

Einstein hat wieder mal recht

Einsteins Beben

Zwischen den Fronten


theguardian

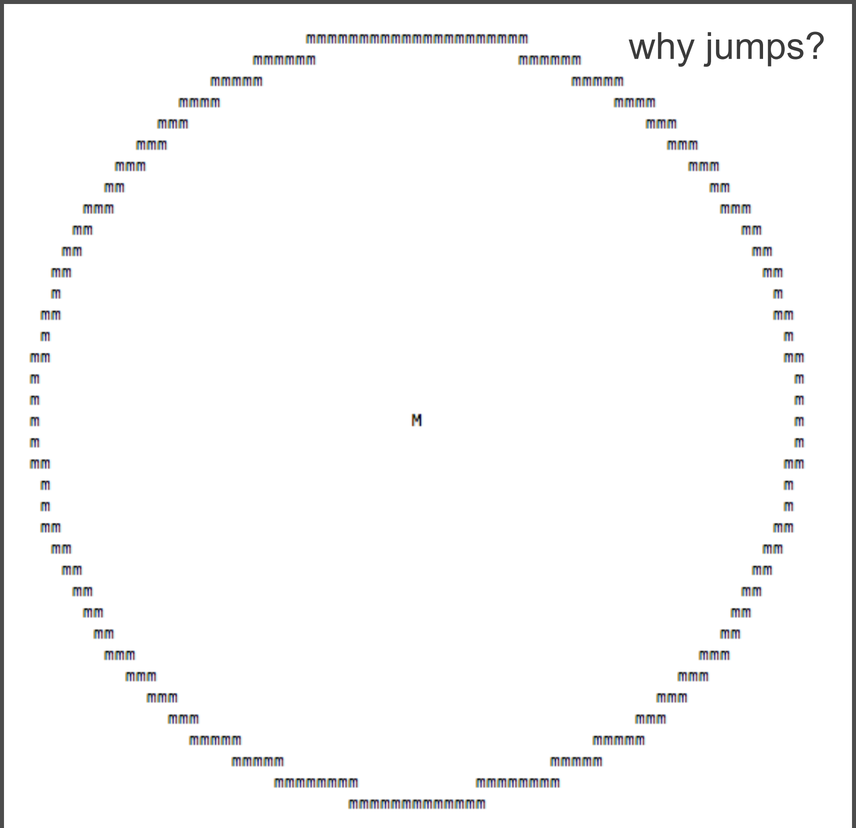
So it turns out Einstein was right all along ...

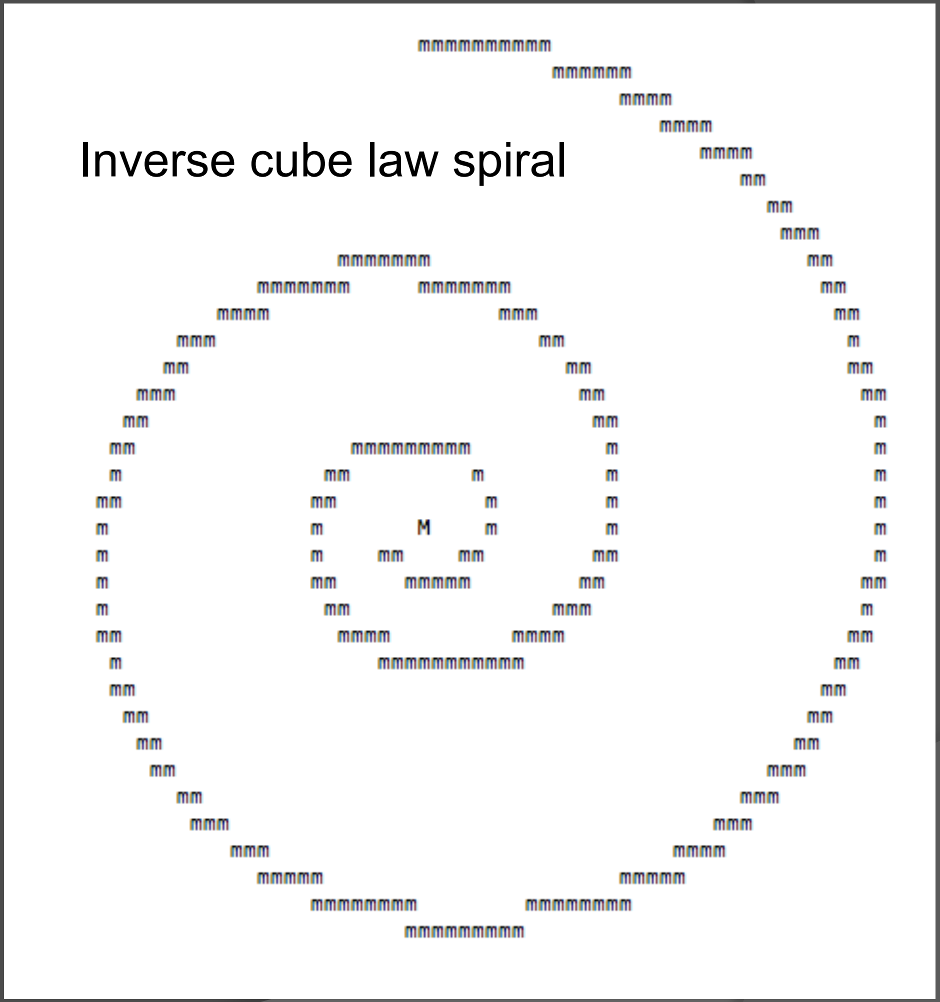
BUT, limit was set on graviton mass (maybe it's like neutrino? Small but non-zero 'm')

Pause for Code Running

- Plus, an in-class code checkup: are you able to (always) run my examples, close to real time?

More Examples from My Code





https://en.wikipedia.org/wiki/Newton%27s_theorem_of_revolving_orbits

Importance of Extra Dimensions

In Physics

$$\Delta\Phi = 4\pi G\rho$$

$$\Delta^{(D-1)}\Phi^{(D)} = 4\pi G^{(D)}\rho^{(D)} \qquad \Delta^{(D-1)} = \partial_1^2 + \cdots + \partial_{D-1}^2$$

$$[G^{(4)}\rho^{(4)}] = [G^{(D)}\rho^{(D)}]$$

$$[G^{(D)}] = [G^{(4)}]\frac{[\rho^{(4)}]}{[\rho^{(D)}]} = [G^{(4)}]\frac{ML^{D-1}}{ML^3} = [G^{(4)}]L^{D-4}.$$

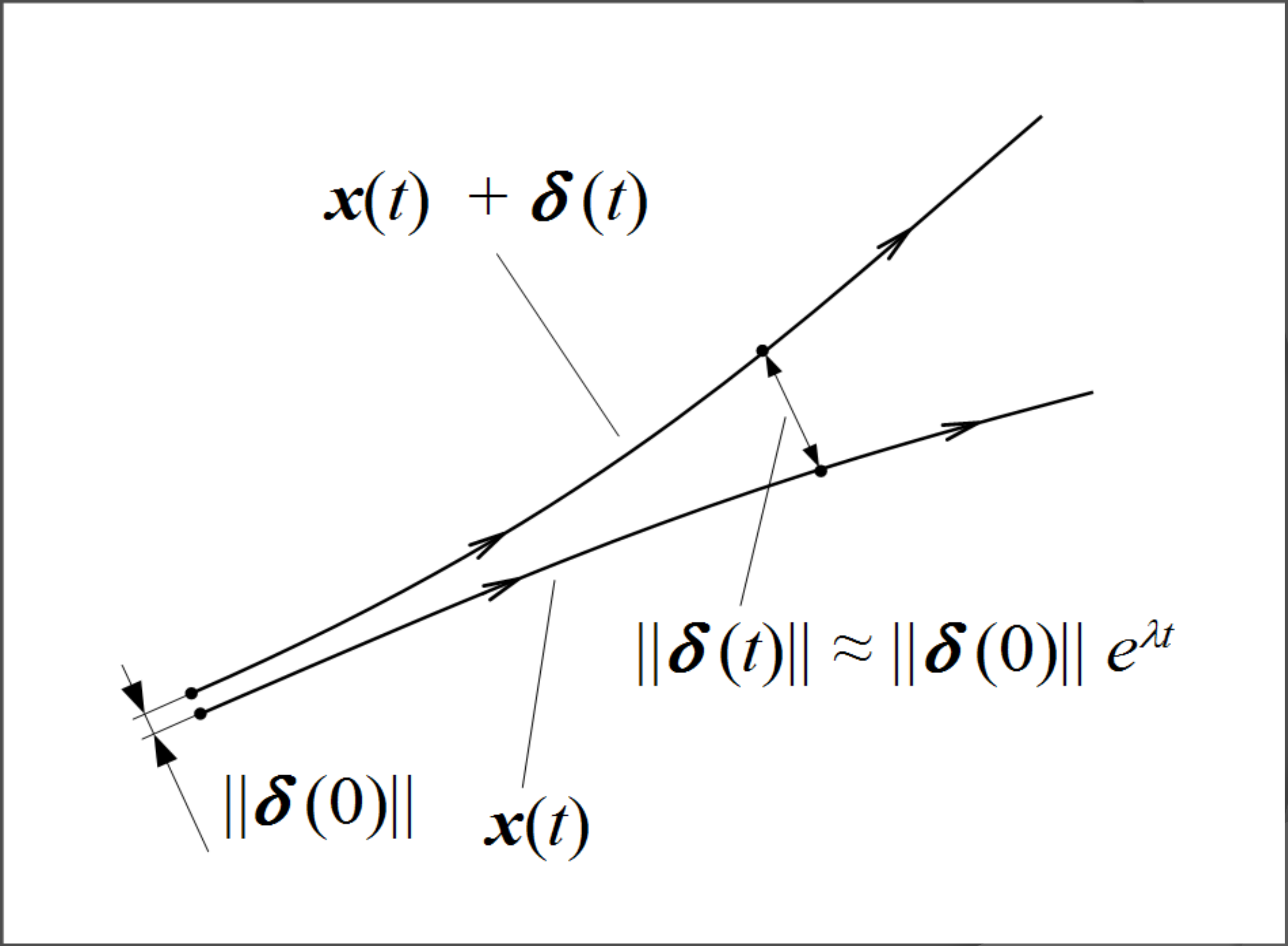
$$G^{(5)} = \ell_C G^{(4)}$$

Time permitting: Do 1/r^(D-2) on board.

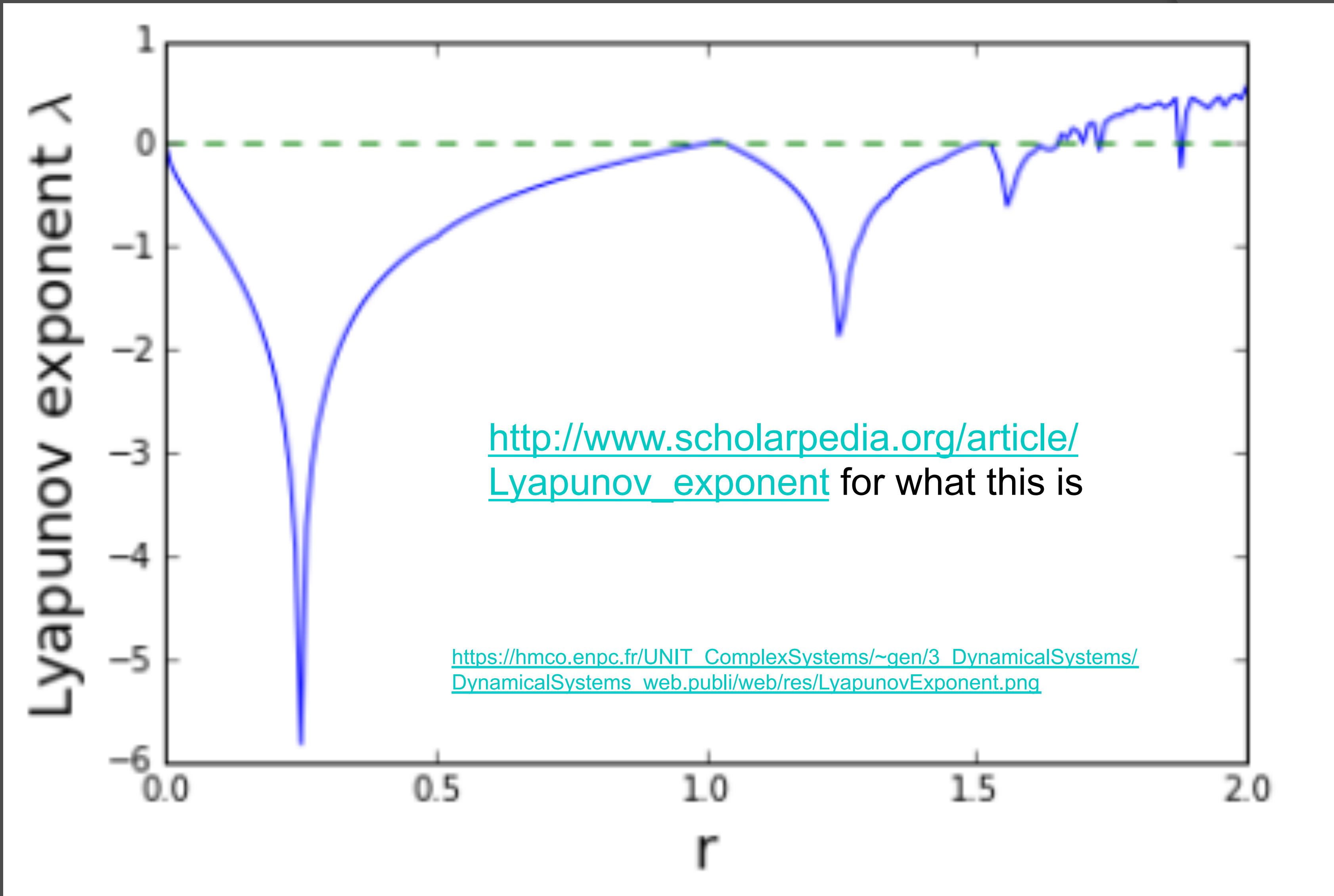
Source: https://physics.stackexchange.com/questions/86508/gravitational-constant-in-higher-dimensions (answer by user “joshphysics”)

Chaos Theory  Rounding Effect

- No computer can store an infinite number of significant figures of course
  - However, chaos theory goes back to 17th century even, way before computers! Classical effect
- In certain systems, the final results are so badly dependent on the initial conditions that the smallest amount of rounding can lead to enormous errors
  - In so-called “chaotic” systems, the error grows exponentially with time quantified by max Lyapunov exponent (error is uncertainty: compared to reality)
- A **classic** example: >= **3** ~equal-mass objects



On board -- write down exponential. Comparing to reality: there is NO right answer, neither analytic nor even numerical in some cases. Must use nature's truth instead



http://www.scholarpedia.org/article/Lyapunov_exponent for what this is

https://hmco.enpc.fr/UNIT_ComplexSystems/~gen/3_DynamicalSystems/DynamicalSystems_web.publi/web/res/LyapunovExponent.png

What is ‘r’? Let’s pretend it is (+)step size (But this is a generic example!) Positive lambda is chaos (if always + even if step -> 0), negative is stable, and zero is border

HW7: DUE Th Mar. 28 at 11:59PM

- Explore a 3rd body: throw in an in-between mass
  - Use my Earth-Moon sample as your “launching point”!
- Animation optional (it was only for fun anyway)
  - If you do animation, take screen caps or upload saved video (in case code doesn’t run on Shane’s machine)
- You can make simplifying assumptions to help
- The bottom line: Demonstrate chaos, *qualitatively*
  - Run the sim for a fixed amount of time *t*, and show that the final conditions (positions and/or speeds and/or accelerations) vary wildly depending on the initial conditions: 3D position & speed of new body

More Detail code + report

- Correctness: Does your system obey the Virial Theorem? Why or why not? Prove it or show it
- Derive *U* from *F* as you learned in Physics 101
  - Plot the KE and PE or U (kinetic and potential energies) of the system with time for a sample case and verify that the Virial Theorem is being followed
  - Don't make 100 plots: combine (colors, shapes, ...)
- EXTRA CREDIT: *Quantify* the deviation of your trajectories vs. the # of decimal places in the initial conditions (xyz location, *v*, total *E*, pick)
  - do this multiple times (for 1 of those quantities)
  - derive some Lyapunov coefficients for the system and judge how much chaos you have got within it

Virial's Theorem

- <KE> = (n/2) * <PE>. True not just for grav (n = -1)
  - Not re-deriving here from classical mechanics

-2 for force means -1 for V or U

$$P.E.(system) \simeq -\frac{1}{2} G \frac{N^2 m^2}{R_{tot}} = -\frac{1}{2} G \frac{M_{tot}^2}{R_{tot}}$$

$$\frac{1}{2} M_{tot} v^2 = +\frac{1}{4} G \frac{M_{tot}^2}{R_{tot}}$$

$$M_{tot} \simeq 2 \frac{R_{tot} v^2}{G}$$

https://web.archive.org/web/20231111083754/http://hosting.astro.cornell.edu/academics/courses/astro201/vt.htm

R_tot is the size of the "cloud" of "stuff." You can do this more precisely for only 2-3 bodies.

- If a system "virializes," that means that it started out with the eq. being false, and then this becomes ~true (if the large system is spherically symmetric)

Steps Zero and One

Start with gravity.cpp (works?)

- Come up with an initial position: experiment!
  - Not so close your three bodies collide in seconds
  - Not so far away that unbound (i.e. open orbits)
- Initial velocity will also require experimentation
  - Not so slow that, again, collisions occur too soon
  - Not so fast that the bodies blast apart (escape ‘v’)
- Orbits have to be *bound* even if not *closed*, and stay on screen. Again, start with just TWO
  - Use paper, and 2-body mechanics and central potentials, to estimate ords of mag (Fermi problem)
- Email me and/or come to office hours (mine, TA’s) if you are having difficulties, as always!

Further Hints, Tips, Tricks

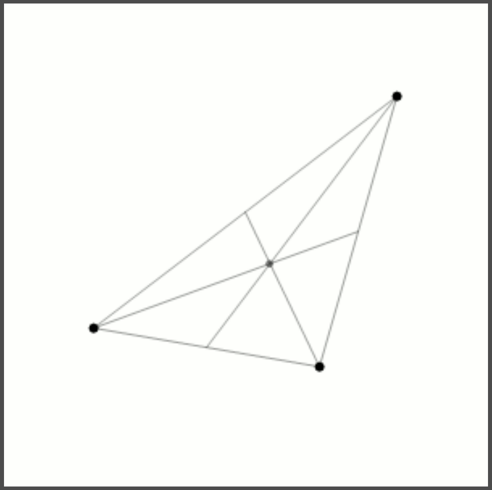
- Initial velocities cannot exceed escape of course! Would lead to unbounded (open) system
  - But, order of magnitude of v_circular = sqrt[G*M/R]

  Make initial net momentum p equal to 0! M1*v1x + M2*v2x + M3*v3x = 0 and similarly for y axis!! This will mean that the center of mass is *fixed*

  gravity
- Re-calculate scaling factors from original Earth-Moon example, to keep everything on the screen and start with 1.9, 2.1: small changes to square
- Use the extra day to NOT be up until 3am again

Extended Resources

Three-Body Chaos

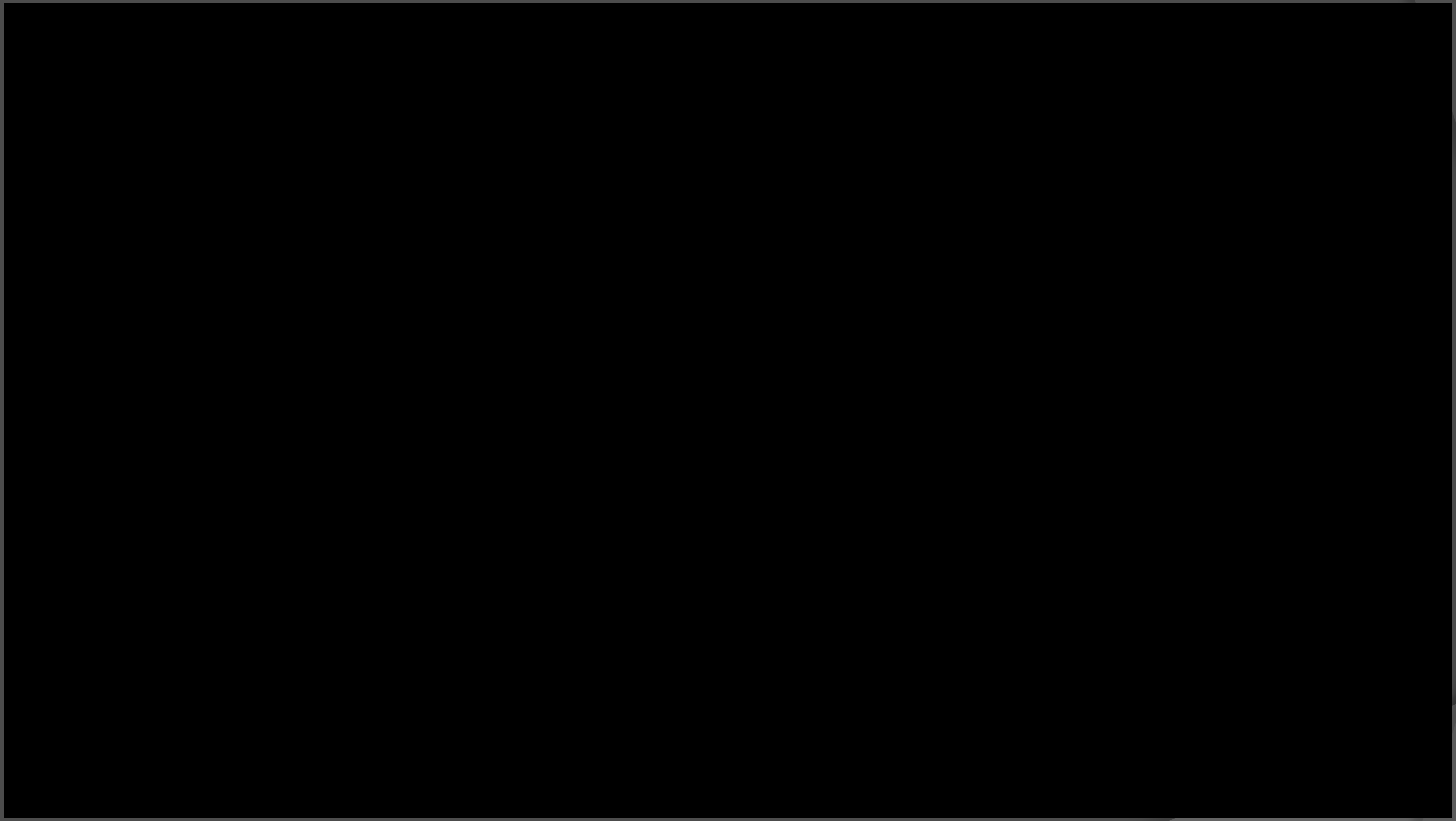


In general, for three bodies: escape (or collision) may be inevitable if initial conditions not so pretty (like all v_is = 0 in a triangular setup)

- https://en.wikipedia.org/wiki/Three-body_problem
- https://www.wolframscience.com/nks/notes-7-4--three-body-problem/
- https://medium.com/@mikeharrisNY/misconceptions-about-the-three-body-problem-and-its-relation-to-forecasting-c0c0a2bf44cc
- Google
- YouTube: https://www.youtube.com/watch?v=et7XvBenEo8

Note that this chaos is still deterministic -- NOT stochastic (random). This is *NOT* QM! Aperiodicity isn't randomness

Ex: A Very Old 577 Final Project



Credit: former UAlbany graduate student Bertrand Carado (advisor: Prof. Kevin Knuth)