

# Mathematics Across the Iron Curtain

## A History of the Algebraic Theory of Semigroups

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Christopher Hollings

# **Mathematics Across the Iron Curtain**

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Theory of Semigroups**



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American Mathematical Society  
Providence, Rhode Island

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## Preface

A semigroup is a set that is closed under an associative binary operation. We might therefore regard a semigroup as being either a defective group (stripped of its identity and inverse elements) or a defective ring (missing an entire operation). Indeed, these are two of the original sources from which the study of semigroups sprang. However, to regard the modern theory of semigroups simply as the study of degenerate groups or rings would be to overlook the comprehensive and independent theory that has grown up around these objects over the years, a theory that is rather different in spirit from those of groups and rings. Perhaps most importantly, semigroup theory represents the abstract theory of transformations of a set: the collection of all not-necessarily-invertible self-mappings of a set forms a semigroup (indeed, a *monoid*: a semigroup with a multiplicative identity), but not, of course, a group. The development of the theory of semigroups from these various sources is the subject of this book. I chart the theory's growth from its earliest origins (in the 1920s) up to the foundation of the dedicated semigroup-oriented journal *Semigroup Forum* in 1970. Since the theory of semigroups developed largely after the Second World War, it might be termed 'Cold War mathematics'; a comparison of the mathematics of semigroup researchers in East and West, together with an investigation of the extent to which they were able to communicate with each other, is therefore one of the major themes of this book.

I believe that semigroup theory provides a particularly good illustration of these problems in East-West communication precisely because it is such a young theory. We are not dealing here with a well-established mathematical discipline, to whose traditions and methods mathematicians in East and West were already privy and had in common when the Iron Curtain descended. Instead, the foundations of many semigroup-theoretic topics were laid independently by Soviet and Western mathematicians who had no idea that they were working on the same problems. Thus, different traditions and priorities were established by the two sides from the earliest days of the theory.

The structure of the book is as follows. In Chapter 1, I set the scene by considering the status of algebra within mathematics at the beginning of the twentieth century. I discuss the coining of the term 'semigroup' in 1904 and give an overview of the broad strokes of the subsequent development of the theory of semigroups.

Chapter 2 is devoted to the major theme mentioned above: the East-West divide in mathematical research. I provide a general discussion of the extent to which scientists on opposite sides of the Iron Curtain were able to communicate with each other and the degree to which the publications of one side were accessible to the other.

The description of the development of semigroup theory begins in Chapter 3 with a survey of the work of the Russian pioneer A. K. Sushkevich. I investigate his



influences, the types of problems that he considered, and the legacy of his work, with an attempt to explain his lack of impact on the wider mathematical community.

Chapter 4 is the first of two chapters dealing with the semigroup-theoretic problems that arose in the 1930s by analogy with similar problems for rings. Thus, Chapter 4 concerns unique factorisation in semigroups, while Chapter 5 deals with the problem of embedding semigroups in groups. In these two chapters, we see how the investigation of certain semigroup-related questions began to emerge, although this was not yet part of a wider ‘semigroup theory’. The beginning of a true *theory* of semigroups is dealt with in Chapter 6, which concerns the celebrated Rees Theorem, together with a result proved by A. H. Clifford in 1941, which might be regarded as semigroup theory’s first ‘independent’ theorem: a result with no direct group or ring analogue.

Paul Dubreil and the origins of the French (or, more accurately, Francophone) school of ‘demi-groupes’ are the subject of Chapter 7, while Chapter 8 concerns the expansion of semigroup theory during the 1940s and 1950s, both in terms of the subjects studied and also through the internationalisation of the theory. I thus indicate the major semigroup-theoretic topics that emerged during this period and also give an account of the various national schools of semigroup theory that developed. Chapter 8 marks something of a watershed in this book: the material appearing before Chapter 8 represents the efforts of the early semigroup theorists to build up their discipline, while that coming after may be regarded as being part of a fully established theory of semigroups.

Chapter 9 concerns the development of the post-Sushkevich Soviet school of semigroup theory through the work of E. S. Lyapin and L. M. Gluskin. I pick up the discussion of Chapter 2 and try to give an indication of the extent to which Soviet semigroup theorists were aware of the work of their counterparts in other countries and of the level of knowledge of Soviet work outside the USSR.

In Chapters 10 and 11, I deal with two major aspects of semigroup theory that emerged in the 1950s: the theory of inverse semigroups and that of matrix representations of semigroups. Both of these remain prominent areas within the wider theory (though the latter was considerably slower in its development), and both furnish us with well-documented examples of the duplication of mathematical research across the Iron Curtain.

In the final chapter, I draw the book to a close by considering the first monographs on semigroups, the early seminars, and the first conferences.

The focus here is upon the history of the *algebraic* theory of semigroups, rather than that of the *topological* theory, which is dealt with elsewhere (see the references on page 10). I have, by necessity, been very selective in the material that I have included here, particularly in connection with the semigroup theory of the 1960s, which is simply too broad to cover in its entirety. A historical account that attempted to cover the whole of semigroup theory would be near-impossible to write and little easier to read. Nevertheless, the book is liberally sprinkled throughout with endnotes in which I give rough indications of other aspects of the theory that are not covered in the main text. One broad area that is perhaps somewhat conspicuous by its almost total absence is the theory of formal languages and automata, together with the Krohn–Rhodes theory of finite semigroups: when choosing which topics to include here, I decided that these theories were simply too large to do justice to within the confines of a book such as this.

I have tried to make this book accessible to as large an audience as possible. Thus, although I have supposed a general familiarity with abstract algebra on the part of the reader, I have not assumed any knowledge of the specifics of semigroup theory. Many elements of the relevant mathematics are introduced as we go along, but some of the more fundamental notions from semigroup theory are summarised in the appendix.

With regard to the notation used throughout the book, I have, as far as possible, retained the notation used by the original authors. Notable exceptions are those few cases where the original notation might prove to be confusing or ambiguous. Some of the authors considered here composed their functions from right to left, while others, following a convention often adopted in semigroup theory, composed from left to right (see the appendix). I have not imposed a uniform direction for the composition of functions but have again retained the conventions of the original authors. Nevertheless, I have endeavoured to make the particular direction clear in each case.

## Languages

The presentation of as full a picture as possible of the international development of semigroup theory necessarily involves the use of sources in many different languages. Wherever I have quoted from a foreign source, I have provided my own English translation (unless stated otherwise), together with the text of the original in an endnote. However, I have saved space in the bibliography by only giving the English translations of titles of foreign sources, except for those items in French or German (plus one or two in Spanish and Italian), for which I have only given the original: readers are, I estimate, likely to be able to translate these for themselves. With regard to non-English terminology, I have endeavoured to supply appropriate translations but in every case have provided the original term in parentheses at the point of the translation's first appearance in the text.

Regarding the Latin transliteration of Cyrillic characters, I have, for the most part, adopted the conventions of the journal *Historia Mathematica*. These conventions are summarised in Table 0.1. Notice in particular that the two silent letters, the hard (Ѣ) and soft (ѣ) signs, are omitted from transliterations altogether. Indeed, this latter point and the use of *i* instead of *ı* for *ı* are the only differences between the *Historia Mathematica* transliteration conventions and those employed in *Mathematical Reviews*. I have chosen the conventions in the table purely on the grounds of simplicity and aesthetics. I deviate from these conventions, however, in the cases of names that have commonly accepted Latin spellings. Thus, for example, 'Шмидт' is transliterated as 'Schmidt', rather than 'Shmidt', and 'Вагнер' as 'Wagner', rather than 'Vagner'.

Some of the Soviet authors whom I mention here have had their names transliterated in various ways, according to different conventions. Thus, for example, Сушкевич has appeared as 'Sushkevich', 'Suškevič', 'Suschkewitz', 'Suschkewitsch', and 'Suschkjewitsch', while Ляпин has been rendered as 'Lyapin', 'Ljapin', and 'Liapin', and Мальцев as 'Maltsev', 'Mal'tsev', 'Malcev', and 'Mal'cev'. In some cases, these authors published work under different transliterations of their names; these works have been listed in the bibliography according to the name under which they were published. Thus, for example, A. K. Sushkevich's work appears under both

TABLE 0.1. Conventions for transliteration from Cyrillic characters.

For Russian		
Aa = Aa	Кк = Kk	Xx = Kh kh
Бб = Bb	Лл = Ll	Цц = Ts ts
Вв = Vv	Мм = Mm	Чч = Ch ch
Гг = Gg	Нн = Nn	Шш = Sh sh
Дд = Dd	Оо = Oo	Щщ = Shch shch
Ее = Ee	Пп = Pp	ъ = -
Ёё = Ee	Рр = Rr	ы = y
Жж = Zh zh	Сс = Ss	ь = -
Зз = Zz	Тт = Tt	Ээ = Ee
Ии = Ii	Уу = Uu	Юю = Yu yu
Йй = Ii	Фф = Ff	Яя = Ya ya
Additional letters for Ukrainian		
Єє = Ee	Іі = Ii	İi = İi

‘Sushkevich’ and ‘Suschkewitsch’, while A. I. Maltsev’s is listed under ‘Maltsev’ and ‘Malcev’.

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# Notes

Unless stated otherwise, a cross-reference to a numbered note is to that numbered note in the same chapter.

## Chapter 1. Algebra at the Beginning of the Twentieth Century

### Section 1.1. A changing discipline

<sup>1</sup>The Oxford English Dictionary (accessed online, December 2012) lists the first appearance of ‘algebra’ in English as being in around 1400, when it was used to refer to the setting of fractured bones: the original sense of ‘al-jabr’/‘restoration’. The first use of ‘algebra’ in connection with the solution of equations is given as being in Robert Recorde’s *The pathway to knowledg* [sic] of 1551, while George Boole (in his *An introduction to the laws of thought* of 1854) is given the credit for first using the word in its ‘abstract algebra’ sense.

<sup>2</sup>Instead, for a general overview of their histories, see Katz (2009). On the development of group theory, see Wussing (1969). For Galois theory, see Neumann (2011) and the further references cited therein.

<sup>3</sup>In fact, a paper published by Galois in 1830 contains a treatment of what are essentially finite fields — see Neumann (2011, §II.4) and Stedall (2008, §13.2.1).

### Section 1.2. The term ‘semigroup’

<sup>4</sup>In line with the comments in the main text, the Oxford English Dictionary (accessed online, December 2012) cites Dickson (1904) as being the first appearance of ‘semigroup’ in English, although no comment is made on the different uses of the word, which is defined only in its modern sense.

<sup>5</sup>The Oxford English Dictionary (accessed online, December 2012) lists the first English use of ‘monoid’ in this sense as being in Claude Chevalley’s *Fundamental concepts of algebra* of 1956.

<sup>6</sup>See note 13 of Chapter 5.

### Section 1.3. An overview of the development of semigroup theory

<sup>7</sup>Knauer (1980) notes, for example, that systems of not-necessarily-invertible matrices were studied by Loewy (1903), Burnside (1905), and Frobenius and Schur (1906). To take just one of these papers, it seems that Frobenius and Schur dealt with semigroups simply because they found it unnecessary to postulate the existence of an identity and inverses (see the comments in Lawson 1992 and Petrich 1970). This paper is perhaps best classified as representation theory, or, along with the papers of Loewy and Burnside, as linear algebra.

<sup>8</sup>It was also around this time that semigroups received their first explicit mention in *Mathematical Reviews*; the following classifications were introduced in 1959:

- 06.70. ‘Ordered semigroups, other generalizations of groups’
- 20.90. ‘Semigroup algebras, representations, characters’
- 20.92. ‘Semigroups, general theory’
- 20.93. ‘Semigroups, structure, and classification’
- 22.05. ‘Topological semigroups and other generalizations of groups’
- 47.50. ‘Semigroups and groups of operators’
- 54.80. ‘Transformation groups and semigroups’

Further semigroup-related classifications were added in 1973. The bulk of research on algebraic semigroups is now classified under the heading 20M (‘Semigroups’), which is subdivided into a

range of specific topics, including, for example, ‘Inverse semigroups’ (20M18) and ‘Semigroups of transformations, etc.’ (20M20).

<sup>9</sup>It might also be said that this book focuses upon ‘classical’ semigroup theory, in the sense defined by Rhodes (1969a,b): broadly speaking, the early semigroup theory of Rees, Green, Clifford, etc., with strong ring theory connections and a focus upon the Rees matrix construction (see Chapter 6). ‘Classical’ semigroup theory stands in contrast with so-called ‘modern’ semigroup theory, which Rhodes deemed to have begun with the Krohn–Rhodes theory of finite semigroups and which has a much more group-theoretic flavour.

## Chapter 2. Communication between East and West

<sup>1</sup>In the interests of precision, we must remember that the ‘Iron Curtain’ did not exist before 1945. Since our story begins earlier than this, a discussion of ‘communication between East and West’ does not, strictly speaking, always equate with ‘communication across the Iron Curtain’. Nevertheless, ‘Iron Curtain’ will sometimes be used here as a convenient term for describing the divide between communist Central and Eastern Europe and the rest of the world, even before 1945.

<sup>2</sup>See, for example, Wolfe (2013) and also the articles in volume 101 (2010) of *Isis* and those in volume 31 (2001) of *Social Studies in Science*.

<sup>3</sup>Douglas Munn, private communication, 24th June 2008.

<sup>4</sup>For example, see Shabad (1986), Anon (1986, 1987), and Rich (1986) on (apparently spurious) claims that an American professor plagiarised a Russian electromagnetism textbook.

<sup>5</sup>“Когда я жил в СССР, то обижался, что часто западные математики не отражают приоритета советских учёных, не ссылаются на их работы. На Западе я увидел другую сторону медали. Подавляющее большинство западных математиков не ссылались на своих советских коллег только потому, что было почти невозможно что-то узнать об их результатах. Просьбы о присылке отписок, посланные в СССР, оставались без ответа. Посланные в СССР письма пропадали.” (Schein, 2008)

### Section 2.1. Communication down through the decades

<sup>6</sup>Just one Soviet delegate attended the 1920 congress in Strasbourg (Villat, 1921), probably because of the continuing civil war, while six appear to have attended the 1924 Toronto congress, though a further twelve were listed as ‘corresponding members’ (Fields, 1928). Distance and cost of travel probably account for the low Soviet turn-out at the 1924 congress. These figures should be contrasted with those from earlier ICMs. Twelve delegates at the 1897 Zürich congress are listed as having originated from ‘Rußland’ (Rudio, 1898), while 15 of those in Paris in 1900 came from ‘Russie’ (Duporcq, 1902). In both cases, the label ‘Rußland’/‘Russie’ was applied also to people from Ukraine and from other areas within the Russian sphere of influence (for example, Poland). It is a little more difficult to give exact numbers of ‘Russian Empire’ delegates at the 1904 (Heidelberg; see Krazer 1905), 1908 (Rome; see Castelnuovo 1909), and 1912 (Cambridge, UK; see Hobson and Love 1913) congresses since their proceedings give only cities of origin for the delegates, rather than countries; there appear to have been approximately 30, 19, and 30 delegates at the 1904, 1908, and 1912 ICMs, respectively, who originated from within the Russian Empire.

<sup>7</sup>For a short introduction to the Marxist philosophy of science, see Graham (1972, Chapter II) or Graham (1993, Chapter 5).

<sup>8</sup>On Soviet ideology of mathematics, see Vucinic (1999, 2000, 2002) or Graham and Kantor (2009); for a more compact and more elementary exposition, see Hollings (2013).

<sup>9</sup>See, for example, Demidov and Ford (1996), Demidov and Levshin (1999), Kutateladze (2007, 2013), Levin (1990), Lorentz (2002, §6), Shields (1987), and Yushkevich (1989).

<sup>10</sup>The word ‘prominent’ is used here, fairly arbitrarily, to mean a mathematician who features in the book Sinai (2003); ‘extensive’ indicates that their number of foreign publications was in double figures. All of the mathematicians in the table were publishing well before 1936, with Aleksandrov’s first listed publication dating from 1923, for example. The dates of the earliest listed publications of the other members of the table are as follows: Bernstein, 1917; Kantorovich, 1928; Khinchin, 1918; Lavrentev, 1924; Luzin, 1917; Menshov, 1922; Pontryagin, 1927; Smirnov, 1918; Tikhonov, 1925. In general, the figures in the table do not represent the total numbers of publications for these mathematicians, most of whom produced further works after the publication of Kurosh *et al.* (1959). Many of these later papers are listed in the follow-up bibliography Fomin and Shilov

(1969). The one exception is Luzin, who died in 1950 but whose publications continued to appear for a few years after this.

<sup>11</sup>For a general discussion of why Soviet authors stopped publishing abroad, including comments on the ‘Luzin affair’ and nationalistic considerations, see Aleksandrov (1996).

<sup>12</sup>See Graham (1993, p. 207). Gerovitch (2013), on the other hand, offers a rather more nuanced view of the success of Soviet mathematics, in which the ‘blackboard rule’ is just one factor.

<sup>13</sup>On the early development of Soviet mathematical publishing, see Bermant (1937).

<sup>14</sup>“Среди большинства советских математиков сохранилась традиция печатать свои лучшие работы в иностранных журналах. Больше того, существовала и пользовалась распространением точка зрения, усматривавшая в факте печатания большого количества наших работ за границей положительное явление . . . . Этот взгляд, конечно, неправилен: рассыпанная по журналам Германии, Франции, Италии, Америки, Польши и других буржуазных стран советская математика не выступает как таковая, не может показать собственного лица.

“Рост научных кадров внутри СССР . . . ставят перед нами задачу создания журнала отражающего эти сдвиги и организующего советскую математику в направлении активного участия в соцстроительстве.

“Группа московских математиков обратилась в редакцию с письмом, в котором принимает на себя обязательство печатать свои статьи, в первую очередь, в «Математическом сборнике» и призывает к этому других математиков Советского союза.” (Anon, 1931)

<sup>15</sup>On Dobzhanskii, see Ford (1977) or Ayala (1985); on Gamov, see Hufbauer (2009).

<sup>16</sup>This was by no means the only such statement of solidarity that was issued during the war — see note 36.

<sup>17</sup>Indeed, some went further and employed ideological language for their own ends: see Gerovitch (2002).

<sup>18</sup>The term ‘samizdat’ (‘самиздат’) is derived from the abbreviation of the Russian word ‘издательство’, meaning ‘publishing house’, together with the prefix ‘само-’ (‘self-’). It refers to manuscripts that were circulated privately within the Soviet Union, where the recipient would often retype a copy for him- or herself before passing on the original to another interested party. This was the only means of distributing materials (in particular, those dealing with politically sensitive subjects) that could not be published through official channels. Related to samizdat is ‘tamizdat’ (‘тамиздат’, from the Russian ‘там’, meaning ‘there’): the formal publication of samizdat texts outside the Soviet Union. An early usage of the term ‘tamizdat’ appears in Medvedev’s original essays. Indeed, their translator into English (Vera Rich) credited Medvedev with the coining of the word (Medvedev, 1971, p. 288). On samizdat, see Boiter (1972) and Johnston (1999).

<sup>19</sup>In a later book, Medvedev referred to ‘The Medvedev papers’ as a “rather trivial title invented by the publisher” but deemed the subtitle ‘The plight of Soviet science today’ to be “more relevant” (Medvedev, 1979, p. xi).

<sup>20</sup>Medvedev was also the author of a more general critique of Soviet science (Medvedev, 1979) and an account of Lysenkoism (Medvedev, 1969). The former contains further details of the communications difficulties of scientists across the Iron Curtain.

<sup>21</sup>Some similar problems with regard to mathematical conferences are mentioned briefly in Kline (1952, p. 84).

<sup>22</sup>A few years later, Ziman wrote ‘A second letter to an imaginary Soviet scientist’ (Ziman, 1973), in which he highlighted the plight of refusenik scientists in the USSR and wondered what action Western scientific organisations could take. Some suggestions were provided in a response by Medvedev (1973). The problems faced by dissident scientists in the USSR, and by refuseniks in particular, came to be discussed extensively in the pages of Western scientific publications during the 1970s and 1980s. See, for example, the series of articles in *Nature*: Rich (1976), Adelstein (1976), Meyers (1976), Levich (1976). Further references on the closely-related issue of anti-Semitism in Soviet academia may be found in note 51.

<sup>23</sup>More of Schein’s experiences with regard to international contacts may be found in Breen *et al.* (2011, pp. 7–8).

<sup>24</sup>Medvedev referred to ‘England’ but he almost certainly meant the UK as a whole since Scottish universities were mentioned elsewhere in his essay.

<sup>25</sup>See Petrovsky (1968), Lehto (1998, §8.2), or Curbera (2010). For a Western account of the congress, see Lorch (1967).



<sup>26</sup>Demidov (2006, p. 796) goes so far as to assert that the 1966 Moscow ICM contributed to the growth of dissidence in the USSR:

An important event in the life of the Soviet mathematical community was the 1966 International Congress of Mathematicians in Moscow, which hosted a record number of participants (more than five and a half thousand!). At this congress our country declared itself one of the leading mathematical powers of the world, and, especially important, our mathematicians felt themselves to be competent and respected members of the world mathematical community. An awakened spirit of freedom found its expression in the growth of free thinking and even dissidence among Soviet mathematicians.

<sup>27</sup>A very nice example of an *ad hoc* exchange of mathematical news is provided by the experience of Peter M. Neumann (private communication, 30th April 2013). On his way to Canberra in the Summer of 1970, Neumann stopped over in Moscow, where he participated in a very well-attended seminar within the algebra section of the mathematics department of Moscow State University. During the seminar, which ran non-stop from the morning until around 4:30 pm, Neumann and his Russian counterparts exchanged news of what they, their students, and their colleagues were currently working on. Neumann relates that his notes from the seminar “created considerable excitement” when they were subsequently shared with colleagues in Canberra.

<sup>28</sup>Specific references are Aleksandrov and Kurosh (1959), Bari and Menshov (1959), and Shafarevich (1959). There were also reports in the same volume on particular areas of mathematics at the congress; see Kurosh (1959a), for example, for that on algebra.

<sup>29</sup>See instead the references in note 51.

## Section 2.2. Access to publications

### Section 2.2.1. Physical accessibility

<sup>30</sup>See Montagu *et al.* (1921), Schuster (1921), Anon (1921), and Gregory and Wright (1922).

<sup>31</sup>Indeed, Ziman’s second letter (see note 22) suffered a similar fate (Medvedev, 1973, p. 476).

<sup>32</sup>On which society, see Sintsov (1936), Akhiezer (1956), Marchevskii (1956b), and Ostrovskii (1999).

<sup>33</sup>The *Сообщения Харьковского математического общества* to which I am referring here is in fact the fourth series of the journal of that name. The first series consisted of 18 volumes (1879–1887), the second of 16 volumes (1887–1918), and the third of only 3 (1924–1926). The fourth series had its first volume in 1927, with volumes I and II being published under the name given at the beginning of this note. For volumes III–V, *и Украинского института математических наук* (and of the Ukrainian Institute of Mathematical Sciences) was added to the end of the title; this was replaced by *и Украинского научно-исследовательского института математики и механики* (and of the Ukrainian Scientific Research Institute of Mathematics and Mechanics) for volume VI. From volume VIII, *при Харьковском государственном университете* (for Kharkov State University) was also added to the title. The journal ceased publication in 1940 with volume XVIII; it resumed with volume XIX in 1948, now with the title *Записки научно-исследовательского института математики и механики и Харьковского математического общества* (Notes of the Scientific Research Institute of Mathematics and Mechanics and of Kharkov Mathematical Society). From volume XXII, ownership of the journal seems to have transferred from the Ukrainian Scientific Research Institute of Mathematics and Mechanics to the mathematics department of Kharkov State University, and this is reflected in yet another new title: *Записки математического отделения физико-математического факультета и Харьковского математического общества* (Notes of the Mathematics Department of the Physico-Mathematical Faculty and of Kharkov Mathematical Society). The final name change of which I am aware is that applied to volume XXIV in 1956, at which point it was evidently felt to be necessary to specify the full name of the university in the journal title: *Записки математического отделения физико-математического факультета Харьковского государственного университета им. А. М. Горького и Харьковского математического общества* (Notes of the Mathematics Department of the Physico-Mathematical Faculty of the A. M. Gorky Kharkov State University and of Kharkov Mathematical Society). To complicate matters further, the volumes of the journal from the 1930s are sometimes cited under a French title: *Communications de la Société mathématique de Kharkoff*.

<sup>34</sup>S. H. Gould was one of the translators into English of E. S. Lyapun's monograph *Semigroups* (see, in particular, Section 12.1.2).

<sup>35</sup>Such accounts, which in many cases were written merely as technical guides to Soviet scientific organisation, stand alongside works of a more 'academic' nature; we have, for example, Joravsky (1970), Graham (1972), Lewis (1972), Lubrano and Solomon (1980), and Berry (1988) from the Soviet era and many more from the last twenty years, including Birstein (2001), Gerovitch (2002), Graham (1993, 1998), Holloway (1994, 1999), Kojevnikov (2004), Kremontsov (1997), and Pollock (2006).

<sup>36</sup>An earlier symposium on Soviet science was that held at Marx House in London at Easter 1942, though this conference was rather more about propaganda than appraisal (Faculty of Science of Marx House, 1942). The conference proceedings cited here contain an appeal for solidarity between British and Soviet scientists that is reminiscent of the statement published in *Nature* in 1941 and reproduced here on page 19.

<sup>37</sup>The latter, in conjunction with questions of Soviet ideology, was also examined in a range of books and articles published in the 1940s, 1950s, and 1960s: see, for example, Bauer (1954), Feuer (1949), Joravsky (1961), London (1957), Muller (1954), Philipov (1954), Romanoff (1954), and Turkevich (1966).

<sup>38</sup>In its drive to document itself, the USSR also produced several surveys of the progress of Soviet mathematics, including, but not limited to, the books Aleksandrov *et al.* (1932), Kurosh *et al.* (1948), Kurosh *et al.* (1959), Shtokalo and Bogolyubov (1966), and Shtokalo *et al.* (1983). I use some of these surveys at the beginning of Chapter 9 to track the acceptance of semigroup theory into the Soviet mathematical canon.

<sup>39</sup>Other Western materials on Soviet education include Anisimov (1950), Bernstein (1948), Friese (1957), Joravsky (1983), Litchfield *et al.* (1958), Miller (1961), Sobolev (1973), and Vogeli (1965). We have also Gnedenko (1957), published in the West but written by a Soviet author. For Russian accounts of the development of Soviet mathematical education, see Lapko (1972) and Velmin *et al.* (1975); for latter-day academic research on this topic, see Karp (2006, 2012) and Karp and Vogeli (2010).

<sup>40</sup>Indeed, to these, we might add two further articles, this time written specifically for historians of Soviet science: Demidov (2007) and the 'Bibliographical essay' at the end of Graham (1993).

### Section 2.2.2. Linguistic accessibility

<sup>41</sup>"Советская математика может и должна иметь журнал международного значения. По-этому мы продолжаем обычай снабжать иностранными резюме статьи, написанные на русском языке, и печатаем статьи на иностранных языках. Опыт показал, что и математические статьи, написанные на русском языке, доходили до иностранного читателя." (Anon, 1931)

<sup>42</sup>'Foreign' in this context does not relate to an author's nationality (which I have not always been able to determine) but merely indicates that they gave an affiliation at a non-Soviet university.

<sup>43</sup>Medvedev (1979, p. 154) suggested that Western authors may also have been deterred from submitting papers to Soviet journals by the lengthy refereeing and publication process. Some of the delays that he records for Soviet biological journals in the mid-1970s (for example, one year from submission to publication) seem quite trivial when compared to the typical delays for modern mathematical journals.

<sup>44</sup>See the comments of O'Dette (1957) and also the conclusions of the report Litchfield *et al.* (1958, §11). Moreover, Soviet citizens applying to travel abroad were, by the 1970s, required to pass a foreign language exam (Medvedev, 1979, pp. 206–207).

<sup>45</sup>Medvedev (1971, p. 132) lamented the decline in the use of Russian at international (biological) congresses, citing the lack of Soviet delegates at such meetings during the middle decades of the twentieth century. His comments seem to imply that there was a tradition of using Russian as a working language at these conferences during the first half of the twentieth century. An examination of the proceedings of the International Congresses of Mathematicians for these decades, however, reveals that the same was not true in mathematics, although Russian was adopted as the third official language of the International Mathematical Union, alongside English and French, in 1958 (Lehto, 1998, p. 109, footnote 6). It should be noted that Soviet attendees of Western conferences did sometimes insist upon delivering their lectures in Russian, necessitating the use of an interpreter, even when they were fluent in a Western language (Kline, 1952, p. 83).

<sup>46</sup>More recent (post-Soviet) books include a new Russian-Ukrainian mathematical dictionary (Karachun *et al.*, 1995) and a glossary of Russian/Ukrainian/English mathematical phrases (Kotov *et al.*, 1992).

<sup>47</sup>The Amkniga Corporation (Furaev, 1974, English trans., p.67) had in fact been supplying American readers with Russian books in English translation since the early 1930s, but they appear to have dealt with literary works, rather than technical materials.

<sup>48</sup>On the Office of Naval Research's funding of such civilian projects and on the funding structures of post-war American science more generally, see Wolfe (2013, Chapter 2).

<sup>49</sup>Up until this point, the only translations of Russian scientific literature that had been available in the UK were those purchased by the UK government's Department of Scientific and Industrial Research from the USA (Anon, 1958b).

<sup>50</sup>A longer list of currently translated journals is available from the American Mathematical Society: <http://www.ams.org/msnhtml/trnjor.pdf> (last accessed 31 May 2013).

<sup>51</sup>Pontryagin accused Jacobson of having been part of a 'Zionist conspiracy' to take over the International Mathematical Union, which he (Pontryagin) claimed to have thwarted. The editor of *Russian Mathematical Surveys* gave Jacobson the opportunity to respond to this accusation and printed his reply at the end of the translation of Pontryagin's article. Further correspondence between Jacobson and the then-editor of *Uspekhi matematicheskikh nauk*, P. S. Aleksandrov, was subsequently printed in the June 1980 issue of *Notices of the American Mathematical Society*. Moreover, charges of anti-Semitism were levelled against Pontryagin within the pages of *Science* (see Kolata 1978 and Pontryagin 1979). For the political background to Pontryagin's accusations, see Lehto (1998, §10.1). Jacobson's correspondence in *Notices of the American Mathematical Society* formed a part of the discussion concerning anti-Semitism in Soviet academia (particularly Soviet mathematics) that had been going on in the letters to the editor since the publication, in the November 1978 issue, of an anonymous samizdat essay (see note 18) entitled *The situation in Soviet mathematics*, whose purpose was to draw attention to such discrimination. This discussion continued, on and off, for the next couple of years, though the focus shifted slightly from anti-Semitism in general to the plights of several individual refusenik mathematicians. On anti-Semitism in Soviet mathematics, see Freiman (1980), where the essay *The situation in Soviet mathematics* may also be found. See also note 35 of Chapter 10.

### Chapter 3. Anton Kazimirovich Sushkevich

<sup>1</sup>On the other hand, we note in passing that Sushkevich is *not* mentioned in one (non-Soviet) source where we might have expected to find him: in a list of group generalisations given by Wussing (1969, English trans., p. 292, note 239) in his book on the history of group theory.

#### Section 3.1. Biography

<sup>2</sup>There are several biographical articles on Sushkevich: Gluskin and Lyapin (1959), Anon (1962a), Gaiduk (1962), Gluskin and Schein (1972), Gluskin *et al.* (1972), Lyubich and Zhmud (1989), Zhmud and Dakhiya (1990), and Hollings (2009c). Parts of the final article in this list have been reused here. Note that the Soviet-era biographies can be limited in their usefulness owing to their often rather impersonal style, as discussed in note 5 of Chapter 9. Another useful resource on Sushkevich has been his Kharkov State University personnel file, which is held by the Ukrainian State Archives (Kharkov Region): Ф.Р-2782, ор. 20, спр. 572.

<sup>3</sup>I say 'tentative' because it is not absolutely certain that this evidence does indeed refer to Sushkevich's father. The evidence in question comes from the so-called *Memorial book of Voronezh Province (Памятная книжка Воронежской губернии)* for 1908: a directory which details the local governmental structure of the province, gives statistics on its population, and lists prominent citizens, etc. Within its pages, we find two passing references to a Kazimir Fomich Sushkevich (Казимиръ Фомичъ Сушкевичъ). I cannot say for certain that this was our Sushkevich's father, but his name is at least consistent with Sushkevich's patronymic 'Kazimirovich'. The different Cyrillic spelling of 'Sushkevich' that is found in the *Memorial book* is easily explained: it pre-dates the 1918 Russian spelling reforms, as a result of which the silent letter 'ъ', which had hitherto appeared at the ends of many Russian words, was largely dropped. The references to K. F. Sushkevich list him first as a member of the provincial committee of prison trustees (p. 6 of the *Memorial book*), and then as an assistant to I. I. Kharitonovich, a manager at the South-Eastern Railway Company (p. 41 of the *Memorial book*). This last reference is again consistent with other sources on Sushkevich: in his Kharkov State University personnel file, Sushkevich recorded that his father

had been an employee of South-Eastern Railways (Ф.Р-2782, оп. 20, спр. 572, арк. 1). The final point to address in arguing that this K. F. Sushkevich was indeed the father of our Sushkevich is the Voronezh connection: although Borisoglebsk now lies in Voronezh Province, it then lay in Tambov Province, so it is not clear why Sushkevich Senior should be involved in Voronezh provincial affairs. In fact, I have already noted that the Sushkevich family had some connections with Voronezh: A. K. Sushkevich was educated there, and the family appears, certainly by 1910, to have been living in the city: some lecture notes made by A. K. Sushkevich in that year are noted as having been written up in Voronezh (perhaps during a visit home from Berlin?).

<sup>4</sup>This claim is made by Pflugfelder (2000). Assuming that it is untrue, I offer the following suggestion as to how this notion may have arisen. In the Voronezh *Memorial book* (see note 3), K. F. Sushkevich, whom I argue is the father of A. K. Sushkevich, is listed with the title of Collegiate Secretary (коллежский секретарь, abbreviated as кскр). This is the 10th rank in Peter the Great's 'Table of Ranks': the scheme established by the tsar in 1722, whereby, in principal, anyone in his administration, no matter how low-born, could rise through the ranks on merit. Each rank came with a particular title and style of address: a Collegiate Secretary, for example, would be addressed as 'Ваше благородие' ('your nobleness'). Anyone achieving the 8th rank (later changed to the 5th) received a hereditary title, but those with lower ranks, such as K. F. Sushkevich, were awarded merely 'personal nobility' ('дворянство'). Thus, K. F. Sushkevich attained a certain degree of personal prestige, but this did not transfer in any formal way to his family.

<sup>5</sup>Many of Sushkevich's lecture notes survive in the mathematics library of Kharkov National University. The following list gives the courses represented by the lecture notes, in varying degrees of completion, arranged semester by semester:

- winter 1906–1907: differential calculus (H. A. Schwarz), analytic geometry (R. Lehmann-Filhés),
- summer 1907: integral calculus (H. A. Schwarz),
- winter 1907–1908: infinite series, products and continued fractions (G. Hettner), algebra (I. Schur), number theory (F. G. Frobenius), integral calculus (R. Lehmann-Filhés),
- summer 1908: theory of determinants (F. G. Frobenius),
- winter 1908–1909: algebra, part I (F. G. Frobenius), analytic geometry (F. G. Frobenius),
- summer 1909: algebra, part II (F. G. Frobenius), ordinary differential equations (I. Schur),
- winter 1909–1910: general mechanics (M. Planck),
- summer 1910: mechanics of deformable bodies (M. Planck),
- winter 1910–1911: theory of electricity and magnetism (M. Planck), integral equations (I. Schur),
- summer 1911: theory of optics (M. Planck).

Among Sushkevich's files may also be found some notes on lectures by Frobenius, which Sushkevich recorded as having been given in the mathematics seminar of Berlin University: Chebyshev's Theorem (November 1909), Bernoulli numbers (summer 1910), and the theory of matrices (winter 1910–1911, continuing in summer 1911). For further details on mathematics in Berlin in the early years of the twentieth century, see Rowe (1998) and Begehr (1998).

<sup>6</sup>Фонд 14, опись 3, дело 45212.

<sup>7</sup>“склав екстерном державний іспит” (Gaiduk, 1962, p. 4).

<sup>8</sup>A transcript of this certificate is held by the Ukrainian State Archives (Kharkov Region): Ф.Р-2782, оп. 20, спр. 572, арк. 4.

<sup>9</sup>“Исследования мои по этому предмету начались в 1918 году и, следовательно, проходили в весьма тяжелое время, часто прерываясь из-за посторонних причин на более или менее значительные промежутки времени.” (Sushkevich, 1922, p. 1).

<sup>10</sup>“была высоко оценена С. Н. Бернштейном и О. Ю. Шмидтом” (Zhud and Dakhiya, 1990, p. 23).

<sup>11</sup>“В 1926 г. я защищал докторскую диссертацию в Харькове (на Украине была тогда восстановлена ученая степень доктора) и получил степень доктора математики.” (Ukrainian State Archives, Kharkov Region: Ф.Р-2782, оп. 20, спр. 572, арк. 3).

<sup>12</sup>The 1927 lecture was ‘On non-uniquely invertible groups and their representation by generalised substitutions’ (‘Об однозначно необратимых группах и об их представлении посредством обобщенных подстановок’) (see Privalov 1927, p. 213), an account of the results of the paper Suschkewitsch (1926). The ICM talk was entitled ‘Untersuchungen über verallgemeinerte Substitutionen’; this appeared as a paper in the congress proceedings (Suschkewitsch, 1930). Both of these papers are studied in Section 3.3.1.

<sup>13</sup>I choose the term ‘cathedra’ to represent the Russian word ‘kafedra’ (‘кафедра’) both on aesthetic grounds and also to emphasise the word’s origins in the Latin ‘cathedra’ (derived in turn from the Greek ‘καθέδρα’), meaning ‘chair’ and now used in English to refer to a bishop’s throne (hence ‘cathedral’). Thus, the Russian ‘kafedra’ is somewhat akin to the English usage of ‘chair’ to mean a professorship. However, the Russian term tends to be used a little differently: to signify not the incumbent of the chair specifically, but the research group gathered around him/her. Thus ‘kafedra’/‘cathedra’ denotes a subdivision of an academic department or faculty.

<sup>14</sup>From the Ukrainian Голодомор, a reversal and contraction of ‘морити голодом’: literally ‘to kill by hunger’. For a succinct account of the Holodomor, see Snyder (2010, Chapter 1); for a more detailed treatment, see Conquest (1986).

<sup>15</sup>The specific references for these articles are Sushkevich (1934) and Sushkevich (1938b), respectively. Sushkevich’s interest in systems of numerals appears to pre-date the latter article by at least a decade. The mathematics library of Kharkov National University preserves a folder of notes made by Sushkevich on a range of books and papers. Most of these notes are meticulously dated, and the notes on Steinitz (1910), made in March 1927, are dated not only in modern Hindu-Arabic numerals, but also in two of the older forms of these numerals that feature in Sushkevich’s article. Another (broadly similar) version of Sushkevich’s numerals article was published a decade later (Sushkevich, 1948b).

<sup>16</sup>The term ‘steklograph edition’ (‘стеклографое издание’) is often found in Soviet publications lists of this period, in connection with informal (often handwritten) publications such as lecture notes; ‘steklograph’ appears to have been the Russian name for a photographic printing technique, possibly akin to what is known in the West as collotype.

<sup>17</sup>Other sources that comment on the new system are Medvedev (1971, p. 99), Medvedev (1979, p. 80), and Kojevnikov (2004, p. 95).

<sup>18</sup>Or, in Ukrainian: Вища атестаційна комісія.

<sup>19</sup>Ukrainian State Archives (Kharkov Region): Ф.Р-2782, оп. 20, спр. 572, арк. 5, 7.

<sup>20</sup>Ukrainian State Archives (Kharkov Region): Ф.Р-2782, оп. 20, спр. 572, арк. 6, 8.

<sup>21</sup>Ukrainian State Archives (Kharkov Region): Ф.Р-2782, оп. 20, спр. 572, арк. 3 зв.

<sup>22</sup>“Мені особисто запам’яталися дні кінця 1941 року, коли було зруйноване наше життя: замовкло радіо, погасла електрика, зникла вода — це означало, що прийшли німці. ... багато не пережило вже першу зиму 1941 року — померли, пригнічені голодом, хворобами і всілякими бідами ...” (Zaitsev and Migal, 2000)

<sup>23</sup>“Харьковские ученые благодарны ему и за спасение библиотеки института математики.” (Maznitsa, 1998).

<sup>24</sup>“Предание гласит, что в составе зондеркоманды, направленной в УФТИ, оказался майор, с которым Сушкевич учился в Германии. Они встретились. В память студенчества немец предложил Сушкевичу: «Сформулируй мне одну просьбу и я ее выполню, но только одну». Сушкевич, подумав, сказал: «Сохрани библиотеку». Библиотека работает и доныне.” (Maznitsa, 1998).

<sup>25</sup>“В настоящее время я работаю главным образом в области истории отечественного математики.” (Ukrainian State Archives, Kharkov Region: Ф.Р-2782, оп. 20, спр. 572, арк. 3 зв).

### Section 3.2. *The theory of operations as the general theory of groups*

<sup>26</sup>A short survey of Sushkevich’s dissertation may also be found in Gluskin and Schein (1972).

<sup>27</sup>“В довольно богатой литературе, которая была мне доступна, я не нашел и следа тех обобщений понятия о группе, о которых я говорю. Я пытаюсь заполнить этот пробел и дать примеры групп относительно действий, существенно отличных от обычного действия классических групп.” (Sushkevich, 1922, p. 1).

<sup>28</sup>“В заключенные считаю своим долгом выразить искреннюю благодарность бывшему профессору Харьковского Университета А. П. Пшеборскому за интерес, который он проявил к

моим исследованиям в мою бытность в Харькове, и который побуждал меня к дальнейшей работе” (Sushkevich, 1922, p. 1).

<sup>29</sup>The relevant references are Weber (1882, 1893), Frobenius (1895), Huntington (1901a,b, 1903, 1905), Moore (1902, 1905), Pierpont (1900), Burnside (1911), and Dickson (1905a,b).

<sup>30</sup>Frobenius (1895, p. 81): “In der Theorie der endlichen Gruppen betrachtet man ein System von Elementen, von denen je zwei,  $A$  und  $B$ , ein drittes  $AB$  erzeugen. Über die Operation, durch welche  $AB$  aus  $A$  und  $B$  hervorgeht, wird nur vorausgesetzt, dass sie folgenden Bedingungen genügt ... Sie soll sein

1. eindeutig. Ist  $A = A'$  und  $B = B'$ , so ist  $AB = A'B'$ .
2. eindeutig umkehrbar. Ist  $AB = A'B'$ , so ist jede der beiden Gleichungen  $A = A'$ ,  $B = B'$  eine Folge der anderen.
3. associativ, aber nicht notwendig [*sic*] commutativ. Es ist also  $(AB)C = A(BC)$ , aber im Allgemeinen nicht  $AB = BA$ .
4. begrenzt in ihrer Wirkung, so dass aus einer endlichen Anzahl der gegebenen Elemente durch beliebig oft wiederholte Anwendung der Operation nur eine endliche Anzahl von Elementen erzeugt wird.”

<sup>31</sup>“отвлеченная теория действия занимается изучением групповых свойств действия вообще и различных частных случаев действий.” (Sushkevich, 1922, p. 21).

### Section 3.3. Generalised groups

#### Section 3.3.1. The 1920s

<sup>32</sup>See note 12.

<sup>33</sup>“Erstens, führt er das Studium unserer abstrakten Gruppen zum Studium des konkreten Falles der verallgemeinerten Substitutionsgruppen zurück, was in mancher Hinsicht leichter ist.” (Suschkewitsch, 1926, p. 372).

<sup>34</sup>“Zweitens, zeigt er einen inneren Zusammenhang zwischen dem associativen Gesetz und den Substitutionen: bei der Komposition der Substitutionen gilt bekanntlich das associative Gesetz; wir können aber jetzt auch umgekehrt sagen: in allen Fällen, wo das associative Gesetz gilt, hat man mit der Komposition der Substitutionen zu tun.” (Suschkewitsch, 1926, p. 372).

<sup>35</sup>Note that here and elsewhere (most notably, Section 6.3), I have replaced Sushkevich’s ‘+’ for union by the modern ‘ $\cup$ ’, in the interest of clarity.

<sup>36</sup>In the early days of semigroup theory, similar studies of the powers of an element in a semigroup were carried out independently by a number of authors: not just Sushkevich, but also Poole (1937), Rees (1940), Schwarz (1943), and Climescu (1946), for example. Indeed, a very brief such study had been carried out by Frobenius (1895, pp. 633–634) even earlier, though he did not consider the elements of a group or semigroup, but the complexes (subsets) of a finite group, multiplied in the usual way (that is,  $A^2 = \{ab : a, b \in A\}$ , etc.). Nowadays, such material appears somewhere in the early pages of any semigroup textbook. Howie (1995b, §1.2), for example, gives the following treatment: consider an element  $a$  of a semigroup  $S$ . We have the collection of all powers of  $a$ :  $\langle a \rangle = \{a, a^2, a^3, \dots\}$ . This is clearly a subsemigroup of  $S$ , which we call the *monogenic subsemigroup* of  $S$  (or *cyclic subsemigroup*, in the terminology of Clifford and Preston 1961, §1.6); we may of course also speak of *monogenic semigroups* (*cyclic semigroups*)  $\langle a \rangle$  independently, that is, without regarding them as subsemigroups of some other semigroup. If  $S$  is infinite, then  $\langle a \rangle$  may also be infinite, that is, it contains no repetitions. If this is the case, then  $\langle a \rangle$  is evidently isomorphic to  $(\mathbb{N}, +)$ . On the other hand, suppose that  $\langle a \rangle$  does contain repetitions, and suppose also that  $m$  is the smallest repeated power of  $a$ , that is to say,  $m$  is the least element of the set

$$\{x \in \mathbb{N} : a^x = a^y, \exists y \in \mathbb{N} \setminus \{x\}\}.$$

Following Clifford and Preston (1961, §1.6), Howie terms this  $m$  the *index* of  $a$ . It follows immediately that the set

$$\{x \in \mathbb{N} : a^{m+x} = a^m\}$$

is non-empty and therefore also has a least element, which Howie denotes by  $r$  and terms the *period* of  $a$ . The minimality of  $m$  and  $r$  imply that the powers

$$a, a^2, a^3, \dots, a^m, a^{m+1}, \dots, a^{m+r-1}$$

are distinct, and so  $a$  is said to have *order*  $m+r-1$ . We note in particular that the subset

$$(*) \quad \{a^m, \dots, a^{m+r-1}\}$$

is a cyclic subgroup of  $\langle a \rangle$ . The elements of a semigroup may then be classified in terms of their indices and periods; in particular, a monogenic semigroup is completely determined by its index and period. For further comments on the study of powers of elements in semigroups, see note 22 of Chapter 6.

### Section 3.3.2. The 1930s

<sup>37</sup>The definition of a ‘distributive group’ has some features in common with that of a *rack*, as used in knot theory (see Fenn and Rourke 1992): a set  $R$  together with a binary operation for which the equation  $ax = b$  is uniquely soluble for any  $a, b \in R$  and for which  $a(bc) = (ab)(ac)$ , for any elements  $a, b, c$ . Thus, a rack is in some sense a ‘one-sided’ version of a ‘distributive group’.

<sup>38</sup>“Im folgenden betrachte ich Matrizen, deren Rang kleiner als Ordnung ist; für die Komposition (Multiplikation) solcher Matrizen gilt bekanntlich das assoziative Gesetz, im Allgemeinen aber nicht das Gesetz der eindeutigen Umkehrbarkeit. Diese Matrizen eignen sich also zur Darstellung der Gruppen ohne das Gesetz der eindeutigen Umkehrbarkeit.” (Suschkewitsch, 1933, p. 27).

<sup>39</sup>Similar terminology was used by Kurt Hensel in his 1913 book *Zahlentheorie*, where an early notion of ring was defined to be an object that satisfied all the field axioms except ‘the axiom of unrestricted and uniquely determined division’ (‘das Gesetz der unbeschränkten und eindeutigen Division’), a condition that postulates the existence of a multiplicative identity, multiplicative inverses, and the lack of zero divisors (Corry, 1996, 2nd ed., pp. 207–208). However, I have no evidence that Sushkevich had seen Hensel’s book.

<sup>40</sup>In particular, Baer and Levi drew upon the axioms given by Hasse (1926), Loewy (1910, 1915), and Weber (1882).

<sup>41</sup>Such substitutions were also studied by an Italian mathematician, Giulio Andreoli (1915), although Sushkevich gave no indication of being aware of this. Andreoli later followed this up with some work on other types of transformations: in a paper of 1940, he considered ‘generalised substitutions’ (‘sostituzioni generalizzate’), though he used the term a little differently from Sushkevich. Rather than being well-defined transformations, Andreoli’s ‘generalised substitutions’ were multi-valued functions on a set. Nevertheless, he considered collections of these that are closed under an appropriate composition, terming such collections ‘generalised groups’ (‘gruppi generalizzate’). However, Andreoli did not subsequently undertake a systematic study of such ‘generalised groups’: his 1940 paper appears to have been his only published contribution to this area.

<sup>42</sup>“die ... Elementen sind miteinander ... nach einigen nicht sehr einfachen Regeln verknüpfbar” (Zbl 0013.05503).

<sup>43</sup>“ $\mathfrak{R}$  ist ... eine weitere Verallgemeinerung von Semigruppen, die bei den Verallgemeinerungen endlicher Gruppen der sogen. „Kerngruppe“ entspricht.” (Suschkewitsch, 1935, pp. 94–95).

<sup>44</sup>“Obwohl wir im Folgenden mit den Matrizen operieren werden, wollen wir doch jetzt, um die Sache möglichst allgemein anzufassen, axiomatisch verfahren ...” (Suschkewitsch, 1935, p. 89).

<sup>45</sup>“яка була відома ще на початку XX сторіччя” (Sushkevich, 1936, p. 49).

<sup>46</sup>“Крім того я ще дослідив один цікавий тип узагальнених безконечних груп, який не має аналогії в теорії скінчених узагальнених груп ...” (Sushkevich, 1936, p. 49).

<sup>47</sup>“де всі вищезазначені дослідження будуть докладно викладені” (Sushkevich, 1936, p. 50).

<sup>48</sup>“виявити характерні особливості того типу узагальнених груп, який може бути представлений через матриці скінченного порядку” (Sushkevich, 1936, p. 50).

### Section 3.3.3. The 1940s

<sup>49</sup>This is not an entirely arbitrary choice: the Grave memorial volume is freely available online and so should be accessible to the reader; as observed in Section 2.2.1, however, the journal of the Kharkov Mathematical Society is not particularly easy to get hold of outside Ukraine.

<sup>50</sup>Around this time, the Dnepropetrovsk Mathematical Society provided funds to bring guest lecturers to the city; Sushkevich was one of several lecturers who came — see Nikolskii (1983). Note that ‘Dnepropetrovsk’ (‘Днепропетровск’) is the Russian name for the city, which appears on the cover of Sushkevich’s lecture notes (which are in Russian). The city is now more commonly known by the Ukrainian version of its name: Dnipropetrovsk (Дніпропетровськ). On the term ‘steklograph’, see note 16.

### Section 3.4. Sushkevich's impact

<sup>51</sup>“Авторы обязаны Б. М. Шайну за указание на пионерские работы А. К. Сушкевича.” (Clifford and Preston, 1967, Russian trans., vol. 2, p. 304). This line does not appear in the original English edition.

<sup>52</sup>Fedoseev's paper contains as an example the system of real numbers together with the operations of addition and that of taking the maximum of two numbers, subject to certain conditions. This is a very early appearance of what is now termed the *tropical semiring*, an object of widespread and popular study in modern mathematics — see, for example, Speyer and Sturmfels (2009).

<sup>53</sup>Ukrainian State Archives (Kharkov Region): Ф.Р-2782, оп. 20, спр. 572, арк. 13.

<sup>54</sup>It has been asserted (in Pflugfelder 2000, for example) that the main reason for Sushkevich's work having passed into obscurity was the fact he was barred from supervising students as a result of the authorities' suspicion of him (he had studied abroad, and then lived under — hence 'collaborated with' — Nazi rule in Kharkov, etc.). Indeed, I have repeated these assertions myself (Hollings, 2009c). However, in the years since writing this article on Sushkevich, I have found no particular evidence for this 'suspicion'; the Soviet authorities were probably no more or less suspicious of Sushkevich than they were of any other citizen. In particular, the list of students cited on page 74 (details in note 53 above) effectively disproves the claim that Sushkevich was not permitted to supervise dissertations.

## Chapter 4. Unique Factorisation in Semigroups

<sup>1</sup>I use the term 'multiplicative system' rather loosely, to mean a set with a binary operation ('multiplication') defined upon it. The vague terms 'multiplicative system' and 'domain' are used more or less interchangeably.

<sup>2</sup>Brief accounts of Noether's wider contributions to commutative ring theory can be found in Gilmer (1981) and Kaplansky (1973). Another account, this time within the context of the development of modern algebra, may be found in Corry (1996, 2nd ed., Chapter 5).

<sup>3</sup>In the early decades of the twentieth century, a parallel development of arithmetical theories for hypercomplex numbers and more general algebras was also taking place, though I do not attempt to describe this here; I instead refer the reader to Fenster (1998) for a comprehensive account.

<sup>4</sup>The main perpetrator was E. T. Bell (see Section 4.2). He considered the positive rationals ( $\mathbb{Q}^+$ ,  $\times$ ) to have unique decomposition since he was only interested in the factorisation properties of  $\mathbb{Z}^+$  within  $\mathbb{Q}^+$ . Intuitively, the positive integers should be the 'integral' elements of the positive rationals, but notice that they are not in fact 'integral' in the sense of being non-units since every element of  $\mathbb{Q}^+$  is a unit.

### Section 4.1. Postulational analysis

<sup>5</sup>The work of C. S. Peirce may also have had some influence: see Ewald (1996, vol. 1, Chapter 15).

<sup>6</sup>See MacDuffee (1936), Hildebrandt (1940), and Siegmund-Schultze (1998) for further details on Moore and Barnard (1935).

### Section 4.2. E. T. Bell and the arithmetisation of algebra

<sup>7</sup>See note 4.

### Section 4.3. Morgan Ward and the foundations of general arithmetic

<sup>8</sup>For biographical details on Ward, see Bohnenblust *et al.* (1963); for a discussion of his work, see Lehmer (1993).

<sup>9</sup>This condition is given as: “[t]here exists an element  $i$  of  $\Sigma$  such that  $i \circ i = i$ ”. However, Ward proved almost immediately that such an  $i$  is in fact an identity.

### Section 4.4. Alfred H. Clifford

<sup>10</sup>This section has been compiled with the particular help of three biographical articles on Clifford: Miller (1974, 1996) and Rhodes (1996). A list of Clifford's publications may be found in Anon (1996a); see Preston (1974) for a survey of Clifford's work up to 1974, and see Preston (1996) for a discussion of his work on semigroups which are unions of groups (to be dealt with



in Section 6.6). An obituary of Clifford may be found in Anon (1993a). Note that ‘Hoblitzelle’ was Clifford’s mother’s maiden name and is of Swiss German origin. It should be pronounced to rhyme with ‘gazelle’.

<sup>11</sup>On the Institute for Advanced Study around this time, see Aspray (1989).

#### Section 4.5. Arithmetic of ova

<sup>12</sup>The theorem first appeared in Noether (1927), but the formulation given here is a blend of that of Ore (1933b, p. 741), and Clifford (1938, p. 595).

<sup>13</sup>For biographical details on König, see *J. C. Poggendorffs biographisch-literarisches Handwörterbuch zur Geschichte der exacten Wissenschaften*, volume IV (1885–1900), p. 777, and volume V (1904–1922), p. 651.

<sup>14</sup>“der Geist der Kroneckerschen Methoden” (König, 1903, p. IV).

<sup>15</sup>“eine systematische Darstellung der Theorie — oder genauer ausgedrückt ihrer Fundamentalsätze” (König, 1903, p. III).

<sup>16</sup>This is not to be confused with the way in which this notation is sometimes used in number theory, where, for  $p$  a prime,  $p^h | a$  means that  $p^h | a$  but  $p^{h+1} \nmid a$ .

<sup>17</sup>Klein-Barmen was born in Barmen, North Rhine-Westphalia, Germany, and studied in Marburg, Munich, and Kiel, before obtaining his PhD (*Über die Anzahl der Lösungen von gewissen Kongruenzsystemen*) from the Friedrich-Schiller-Universität Jena in 1925. He then moved to Wuppertal (close to his native Barmen, which was in fact incorporated into Wuppertal around 1930), where he became a high school teacher but continued to conduct research. Much of his work was in the early theory of lattices, with an emphasis on axiomatics (Schlimm, 2011). Up to 1933, his name and affiliation appeared on his papers as ‘Fritz Klein in Wuppertal-Barmen’ (see Klein 1932, for instance) but then changed to ‘Fritz Klein-Barmen in Wuppertal’ in Klein-Barmen (1933), which is how it appeared thereafter, apart from one reversion to ‘Fritz Klein in Wuppertal’ in Klein (1935). Klein-Barmen’s entry in volume VI (p. 1330) of *J. C. Poggendorffs biographisch-literarisches Handwörterbuch zur Geschichte der exacten Wissenschaften* lists him simply as ‘Klein, Friedrich (Fritz) Wilhelm’; the entry in volume VIIa (p. 769) lists him as ‘Klein, Friedrich Wilhelm’ but notes ‘Klein-Barmen, Fritz’ as a *nom de plume*. Klein’s reason for adding ‘Barmen’ to his name is not clear, though it may have been in memory of his hometown, which, by 1933, no longer existed as an independent entity. In the bibliography, I have listed Klein-Barmen under the specific name used in each of his papers, but in the text, I refer to him consistently as ‘Klein-Barmen’.

<sup>18</sup>“Der Begriff der *Verknüpfung* ist von grundlegender Bedeutung für den Aufbau der gesamten Mathematik und Logik. ... Unter einer *abstrakten* Verknüpfung insbesondere verstehe ich eine Verknüpfung, bei der von der Eigenart der verknüpften Elemente abgesehen wird.” (Klein, 1931, p. 308).

<sup>19</sup>For an account of the connections between the work of Klein-Barmen and Clifford, see Hintzen (1957).

<sup>20</sup>Both in Clifford’s thesis and in his 1934 summary thereof, a typo appears in the ‘completely prime’ condition: it is stated that an element  $p$  is completely prime if, for any  $n \in \mathbb{N}$ ,  $p^n | ab$  implies that either  $p^n | a$  or  $p^n | b$ . However, if we examine the proofs in the thesis, we see that the correct version of the condition is used throughout. The fact that Clifford (1934) contained statements of results but no proofs led the paper’s *Jahrbuch über die Fortschritte der Mathematik* reviewer, B. H. Neumann, to observe that Clifford must have made a mistake somewhere since  $(\mathbb{N}, \times)$  does not satisfy condition (III). The typo does not appear in Clifford (1938).

<sup>21</sup>This paper seems to be (part of) a thesis of the same title, defended at Moscow State University in 1929 (Andronov, 1967). What type of thesis it was is not clear: as we saw in Section 3.1, it would not have been possible for Arnold to have gained a formal degree at this time. Note that the paper is one of those appearing within the figures in Table 2.3 on page 35.

<sup>22</sup>“Die Zerlegungssätze der Idealtheorie in Ringbereichen beziehen sich auf die multiplikative Struktur der Elemente. Es liegt deshalb nahe, von der Addition gänzlich abzusehen und den Sachverhalt in rein multiplikativen Bereichen zu untersuchen.” (Arnold, 1929, p. 401).

<sup>23</sup>“ihr liebenswürdiges Entgegenkommen” (Arnold, 1929, p. 401).

<sup>24</sup>Incidentally, the first publications of Preston (1953, 1954a,b) concerned the notion of ideals for universal algebras and featured a universal algebra version of one of Noether’s decomposition theorems.

### Section 4.6. Subsequent developments

<sup>25</sup>See Bogart (1995) and Bogart *et al.* (1990) for biographical details on Dilworth.

<sup>26</sup>See Fenstad (1996), Ljunggren (1963), and Nagell (1963) for biographies of Skolem.

<sup>27</sup>See Lehmer (1974) and Greenwood *et al.* (1974) for biographies of Vandiver.

<sup>28</sup>See Weaver (1956b) and Morgan (2008) for biographies of Weaver.

<sup>29</sup>See Schein (1992) for a comparison of various notions of coset for semigroups, including that of Weaver.

<sup>30</sup>In a joint paper, Vandiver and Weaver (1956, p. 136) made the comment: “we begin a detailed examination of the structure of [correspondences], we think, for the first time”. They cited a paper by Suschkewitsch (1928) as being the first appearance of correspondences in the literature but noted that “he did not treat them with much detail”.

<sup>31</sup>Besides those items already cited, some further references for Vandiver’s research programme are Vandiver (1934a,b, 1940a,b).

### Chapter 5. Embedding Semigroups in Groups

<sup>1</sup>Among the authors to be considered here, there was no great consistency in the use of the term for a ‘non-commutative field’: some authors used ‘skew field’, while others used ‘division ring’ (which is also used as a catch-all term to cover both fields and skew fields: see, for example, Gouvêa 2012, Definition 5.1.3); yet others opted simply for ‘non-commutative field’. I use the term ‘skew field’ since it appears in results that are adapted from theorems concerning fields.

<sup>2</sup>For some comments on this paper and its place within Ore’s wider work, see Corry (1996, 2nd ed., p. 264).

<sup>3</sup>In fact, as Clifford and Preston (1961, §1.10) noted, this result may be stated in a slightly stronger manner: a commutative semigroup can be embedded in a group if and only if it is cancellative.

<sup>4</sup>See Clifford and Preston (1961, Theorem 1.23). Clifford and Preston phrased their statement of the result in terms of the property of ‘right reversibility’, which is equivalent to Ore’s common right multiples condition and is due to Paul Dubreil (see Section 5.3 and also Section 7.3). Ore’s proof of this theorem (that is, by means of ordered pairs) carries over easily to the semigroup case, but Clifford and Preston presented a later proof due to Rees (1947), which employs partial bijections — see Section 10.6.

<sup>5</sup>Indeed, Thoralf Skolem (1951b) made much the same mistake some years later; he subsequently realised his mistake and produced (apparently independently of the other authors mentioned in this chapter) the semigroup version of Ore’s Theorem (Skolem, 1952).

### Section 5.1. The theorems of Steinitz and Ore

<sup>6</sup>“Welche kommutativen Ringe besitzen einen Quotientenkörper? Oder, was auf dasselbe hinauskommt, welche lassen sich überhaupt in einen Körper einbetten?” (van der Waerden, 1930, p. 47).

<sup>7</sup>“Die Möglichkeit der Einbettung nichtkommutativer Ringe ohne Nullteiler in einen sie umfassenden Körper bildet ein ungelöstes Problem, außer in ganz speziellen Fällen.” (van der Waerden, 1930, p. 49). By ‘full’ (‘umfassend’ = ‘comprehensive’, ‘extensive’), van der Waerden presumably meant a ring with multiplicative inverses: as noted on page 108, a non-commutative ring must necessarily be embedded in a non-commutative object.

<sup>8</sup>“distingue par sa simplicité et son élégance” (Dubreil, 1946, 3rd ed., p. 267).

<sup>9</sup>Biographical references for Ore are Anon (1970a) and Aubert (1970).

<sup>10</sup>The condition is labelled  $M_V$  because it is the fifth condition in Ore’s list to relate to multiplication.

<sup>11</sup>As with Clifford’s ‘regular ova’ in Section 4.5, this use of ‘regular’ should not be confused with its modern usage (on which, see Section 8.6). What Ore called a regular ring (with identity) is now termed a *right Ore domain* (Coutinho, 2004, p. 258). In a subsequent paper, Ore gave examples of such regular rings in terms of non-commutative polynomials (Ore, 1933a).

<sup>12</sup>Coutinho (2004, §2) observes that Ore was not the only person to arrive at the notion of a skew field of quotients around this time: similar ideas appeared in the work of D. E. Littlewood and J. H. M. Wedderburn. Inspired by considerations from quantum mechanics, Littlewood (1933) obtained a (non-commutative) ring of quotients (the ‘algebra of rational expressions’) for the Weyl algebra, using a version of the condition  $M_V$  (Coutinho, 2004, §2.2.3). Wedderburn (1932)

obtained a ring of quotients for a (non-commutative) Euclidean domain, again using a version of condition  $M_V$  (Coutinho, 2004, §2.3.1). I have focused upon Ore’s version of these results since this seems to have been the best known (at least among the authors we are considering) and therefore had the greatest influence.

### Section 5.2. Embedding according to Sushkevich

<sup>13</sup>As noted in Section 1.2, the modern German term for ‘semigroup’ is ‘Halbgruppe’; besides Sushkevich, the only author I can find who used the term ‘Semigruppe’, at least in German, is Fritz Klein-Barmen (1943) (see Section 8.1). We note however that ‘semigruppe’ is used in both Norwegian and Danish to correspond to the modern English sense of ‘semigroup’, while Swedish uses ‘semigrupp’, with ‘halvgrupp’ as an alternative. While the paper Suschkewitsch (1934b) is in German, Sushkevich (1935a) is in Ukrainian, although it has a German summary appended. Sushkevich continued to use the term ‘Semigruppe’ in this summary, which carries a German version of the paper’s title, ‘Über die Erweiterung der Semigruppe bis zur ganzen Gruppe’, by which the paper is sometimes cited. In the Ukrainian text, on the other hand, Sushkevich used the term ‘півгрупа’, in contrast to the modern Ukrainian ‘напівгрупа’ (‘пів-’ and ‘напів-’ both being Ukrainian prefixes denoting ‘half-’ or ‘semi-’). Incidentally, a later (Czech) author on the embedding problem used similarly unusual terminology: in the Russian version of his work (Pták, 1952), Vlastimil Pták employed the term ‘семигрупа’ (‘semigruppa’) for semigroup, rather than the usual Russian term ‘полугруппа’ (‘polugruppa’). For a few brief comments on Pták’s work, see Section 5.5.

<sup>14</sup>Sushkevich presented Steinitz’s proof himself in his short Ukrainian book *Elements of new algebra* (*Елементи нової алгебри*) (Sushkevich, 1937a, §14), as well as in the third Russian edition of his *Foundations of higher algebra* (*Основы высшей алгебры*) (Sushkevich, 1931a, 3rd ed., §236). No mention was made in either instance of the semigroup case.

<sup>15</sup>“gewiß nicht trivial”.

<sup>16</sup>“Verf. gibt zwei vermeintliche Beweise für die Behauptung, dass jede Semigruppe sich in eine Gruppe einbetten lässt. Doch ist diese Behauptung inzwischen von Malcev ... durch ein Gegenbeispiel widerlegt worden.” (JFM 61.1014.02).

<sup>17</sup>Ф.Р-2782, оп. 20, спр. 572, арк. 10–12. In fact, this file contains two lists of Sushkevich’s publications, neither of which features Sushkevich (1935a). Both lists are handwritten, in what appears to be Sushkevich’s own writing (by comparison with other documents), and each was certainly signed and dated by him: the first is dated 1 June 1946, while the second is an updated version of 20 November 1952. The only other true omission is a one-paragraph abstract (which were customarily included in Soviet publications lists) of his talk at the 1927 All-Russian Congress of Mathematicians (see note 12 of Chapter 3). Strictly speaking, there is also one further omission, but this is the journal version of the paper ‘Investigations on infinite substitutions’ (see page 71), so this was probably a deliberate omission on Sushkevich’s part.

<sup>18</sup>Holvoet (1959) later provided a counterexample to demonstrate that Condition Z is not sufficient in general.

### Section 5.3. Further sufficient conditions

<sup>19</sup>“A cause de la guerre, je ne les connus que plus tard.” (Dubreil, 1981, p. 61).

<sup>20</sup>“Ce Mémoire et sa traduction m’ont été aimablement communiqués par MM. B. L. van der Waerden et H. Richter, auxquels j’exprime mes sincères remerciements.” (Dubreil, 1943, p. 626, footnote 2).

<sup>21</sup>“Mais un autre résultat, d’un degré de généralité intermédiaire, et particulièrement intéressant par sa maniabilité et ses possibilités d’application, a été donné dès 1931 par O. Ore ...” (Dubreil, 1943, p. 626).

<sup>22</sup>Dubreil (1946, p. 137, footnote 1) also acknowledged the work of Wedderburn mentioned in note 12.

### Section 5.4. Maltsev’s immersibility conditions

<sup>23</sup>A non-exhaustive list of biographical articles on Maltsev is Aleksandrov *et al.* (1968), Anon (1989), Bokut (1989, 2003), Cheremisin (1984), Dimitrić (1992), Gainov *et al.* (1989), Glushkov (1964), Goryaeva (1986), Khalezov (1984), Kolmogorov (1972), Kurosh (1959b), Lavrov (2009), Malcev (2010), Marchuk *et al.* (1973), Nikolskii (1972, 2005), Pontryagin (1946), Rosenfeld (1974,

2007). Maltsev also features in Sinai (2003, pp. 559–560). Regarding the transliteration of Maltsev’s name, see the comments on page ix.

<sup>24</sup>I choose to translate ‘включение’ as ‘immersion’ here, rather than ‘inclusion’ or ‘insertion’, in deference to Maltsev’s own English terminology in Malcev (1937). Note that another often-used Russian word for ‘embedding’/‘imbedding’ is ‘вмещение’ (‘containment’) but that this word only seems to have come into use in this context with later papers (see Schein 1961, for example). Other terms are ‘погружение’ (‘immersion’), as used in Lyapin (1960a), and ‘вложение’ (‘enclosure’), which is used in the Russian translation of Clifford and Preston (1961, 1967).

<sup>25</sup>“Множество элементов алгебраического кольца относительно умножения образует ассоциативную систему. Этим объясняется значение теории ассоциативных систем для изучения алгебраических колец.” (Maltsev, 1939, p. 331).

<sup>26</sup>“Некоторые проблемы теории групп также связаны со свойствами ассоциативных систем. Однако, для решения этих проблем необходимо более тщательное изучение условий, при которых данная ассоциативная система может быть рассматриваема как часть некоторой группы. В настоящей заметке указываются необходимые и достаточные условия для возможности включения ассоциативных систем в группы.” (Maltsev, 1939, p. 331).

<sup>27</sup>Maltsev (1939, p. 335); see also Bush (1963, Theorem 2) and Clifford and Preston (1967, Theorem 12.17).

<sup>28</sup>“... мы видим, что для возможности включения [ассоциативной] системы в группу должно выполняться бесконечное множество условий.” (Maltsev, 1939, p. 336).

<sup>29</sup>“Если потребовать, чтобы выполнялась только часть этих условий, то получится ассоциативная система, более или менее приближающаяся к группе.” (Maltsev, 1939, p. 336).

<sup>30</sup>“ассоциативная система, более близкая к группе, но все еще не включаемая в нее” (Maltsev, 1939, p. 336).

<sup>31</sup>“Для строгого проведения намеченной здесь классификации необходимо исследовать независимость указанных условий. Такая независимость легко изучается для простейших цепочек, например, содержащих только один идеальный элемент 1-го рода. Однако, в общем виде вопрос остается открытым.” (Maltsev, 1939, p. 336).

<sup>32</sup>“Для удобства ссылок” (Maltsev, 1939, II, p. 251).

<sup>33</sup>Maltsev (1939, II, Theorem 4); see also Bush (1961, Theorem 6.2) and Clifford and Preston (1967, §12.8).

### Section 5.5. Other embedding problems

<sup>34</sup>“в определенном родстве с задачей о погружении полугруппы в группу” (Lyapin, 1956, p. 373).

<sup>35</sup>See Schein (1982) for biographical details. Shutov’s main publications on potential properties comprise Shutov (1963a, 1964, 1965, 1966, 1968, 1980, 1981). Note that Shutov’s name is transliterated as ‘Šutov’ in some of the English translations of his papers.

<sup>36</sup>As stated, for example, in Bokut (1969b, English trans., p. 706). Bokut noted that the problem was also formulated in Cohn (1965).

## Chapter 6. The Rees Theorem

<sup>1</sup>A comparison of various generalisations of this theorem, including that of Rees, may be found in Steinfeld and Wiegandt (1967).

<sup>2</sup>“Dieser Satz von REES ist ein klassisches Ergebnis der Halbgruppentheorie.” (Steinfeld and Wiegandt, 1967, p. 153).

<sup>3</sup>Much of the material of this chapter is drawn from Hollings (2009b).

### Section 6.1. Completely (0-)simple semigroups

<sup>4</sup>See Gouvêa (2012, pp. 127–128, 198, 206) for comments on different uses of the word ‘simple’ in connection with rings.

<sup>5</sup>Private communication, 3 July 2008.

### Section 6.2. Brandt groupoids

<sup>6</sup>An overview of Brandt’s work in general and of groupoids in particular may be found in Fritzsche and Hoehnke (1986). See also Anon (1955).

<sup>7</sup>“Der Verf. hält für derartige Systeme einen besonderen Terminus für notwendig, verfällt dabei aber unglücklicherweise auf den vom Ref. in ganz anderem Sinne eingeführten und so in Gebrauch

gekommenen Ausdruck Gruppoid (groupoid). Er vermehrt dadurch die schon bestehende Verwirrung im Gebrauch dieser Bezeichnung.” (Zbl 0025.24501).

<sup>8</sup>“Verf. benützt für die multiplikativen Systeme nach dem Vorbild von B. A. Hausmann und Oystein Ore den Ausdruck Gruppoid, der vor 15 Jahre von dem Ref. für einen ganz andern Begriff eingeführt worden ist. Da dieser Begriff für die Zahlentheorie der hyperkomplexen Systeme unentbehrlich und auch sonst nützlich ist, hat sich die vorgeschlagene Bezeichnung im In- und Ausland eingebürgert und unter anderm auch Eingang in die Neuausgabe des ersten Teils der mathematischen Enzyklopädie gefunden. Falls der Verf. für die multiplikativen Systeme einen besondere Ausdruck für notwendig hält, muß daher zur Vermeidung von Begriffsverwirrungen von ihm erwartet werden, dass er die Benennung ändert, unabhängig davon, wie sich der ohnehin in manchen Punkten abweichende anglikanische Sprachgebrauch entwickelt.” (Zbl 0024.29901).

<sup>9</sup>“Hier wäre die Einführung eines neuen Elementes Null als Symbol für bisher nicht existierende Produkte möglich, aber im allgemeinen doch von geringem Vorteil, weshalb wir davon absehen.” (Brandt, 1926b, p. 360).

<sup>10</sup>“eine naturgemäße und sogar notwendige Ergänzung zur gewöhnlichen Gruppentheorie” (Brandt, 1926b, p. 360).

### Section 6.3. Sushkevich’s ‘Kerngruppen’

<sup>11</sup>“In der vorliegenden Abhandlung habe ich den Versuch gemacht eine abstrakte Theorie der endlichen Gruppen, deren Operation nicht eindeutig umkehrbar ist, zu konstruieren. Freilich sind in der mathematischen Literatur solche Gruppen in konkreter Form schon betrachtet worden. Als Beispiel solcher konkreten Gruppen kann man die Theorie der nicht-kommutativen Ringe, speziell auch die Theorie der hyperkomplexen Zahlen anführen . . . Dabei werden aber zugleich zwei Operationen betrachtet: die „Addition“ und die „Multiplikation“. Es entsteht nun die Frage nach der Verallgemeinerung, die man erhält, wenn man die eine Operation — nämlich die Addition — wegläßt und bloß die andere — die Multiplikation — beibehält, die als eindeutig, assoziativ, aber nicht eindeutig umkehrbar vorausgesetzt wird.” (Suschkewitsch, 1928, p. 30).

<sup>12</sup>“Ich bin auf diese Arbeit erst nach Fertigstellung der meinigen durch einen freundlichen Hinweis von Fr. E. Noether aufmerksam geworden.” (Suschkewitsch, 1928, p. 30, footnote 1).

<sup>13</sup>In the Russian of Sushkevich (1937b), these were referred to in the modern way, as *идемпотентные элементы* (*idempotent elements*).

<sup>14</sup>“... zu der dieses Element „gehört“ . . .” (Suschkewitsch, 1928, p. 34).

<sup>15</sup>Namely,  $(*)$  in note 36 of Chapter 3, with  $k = 1$ .

<sup>16</sup>Besides Sushkevich and Clifford (Section 6.4), several other authors have obtained the ‘direct product’ characterisation of left/right groups independently; these include Schwarz (1943), Mann (1944), Ballieu (1950), Skolem (1951b), and Thierrin (1954a).

### Section 6.4. Clifford’s ‘multiple groups’

<sup>17</sup>In the original German (van der Waerden, 1930, p. 15):

- (3) Es existiert (mindestens) ein (linksseitiges) *Einselement*  $e$  in  $\mathfrak{G}$  mit der Eigenschaft:  $ea = a$  für alle  $a$  von  $\mathfrak{G}$ .
- (4) Zu jedem  $a$  von  $\mathfrak{G}$  existiert (mindestens) ein (linksseitiges) *Inverses*  $a^{-1}$  in  $\mathfrak{G}$ , mit der Eigenschaft:  $a^{-1}a = e$ .

### Section 6.5. The Rees Theorem

<sup>18</sup>For obituaries of Rees, see Sharp (2013a,b,c). For a longer biography, see Lawson *et al.* (to appear).

<sup>19</sup>With regard to Preston’s comment that Hall’s lectures did not find their way into print, we note that at least some of Hall’s material eventually appeared in Cohn’s *Universal algebra* of 1965, in the preface of which we find:

As with other fields, there is now a large and still growing annual output of papers on universal algebra, but a curiously large portion of the subject is still only passed on by oral tradition. The author was fortunate to make

acquaintance with this tradition in a series of most lucid and stimulating lectures by Professor Philip Hall in Cambridge 1947–1951, which have exercised a much greater influence on this book than the occasional reference may suggest. (Cohn, 1965, p. xv)

<sup>20</sup>See note 36 of Chapter 3 and also note 22 below.

### Section 6.6. Unions of groups and semigroups

<sup>21</sup>For surveys of this topic, see Clifford (1972) and Preston (1996).

<sup>22</sup>Poole (1937, Theorem 14). Recall from Section 4.2 that Poole was another student of Bell. Poole's PhD thesis (Poole, 1935) and a subsequent paper based thereupon (Poole, 1937) concerned the detailed study and classification of elements of a finite commutative semigroup (or finite ovum, as Poole termed it) according to the properties of their powers; this is very much akin to the theory sketched in note 36 of Chapter 3. Using (and, indeed, probably coining) the terminology of that note, Poole identified four possibilities for the index and period of an element of a finite ovum: (1) index = period = 1; (2) index > 1, period = 1; (3) index = 1, period > 1; (4) index > 1, period > 1. The elements of case (1) are evidently idempotents; in cases (2), (3), and (4), Poole referred to *elements of types A, B, and C*, respectively. He then studied various semigroups containing different combinations of elements of types A, B, and C. Among other things, Poole noted the now well-known fact that any element of a finite semigroup has an idempotent power; in particular, he proved that there is precisely one idempotent in the list of powers of an element of type B — he termed this its *period element*. Near the end of the paper, Poole defined an *ovum of type 2* to be finite ovum with at least one element of type B, but no elements of types A or C. The theorem generalised by Clifford in his 1941 paper is the following:

*THEOREM. Every ovum of type 2 is either a group or consists of sub-ova which have no element in common and each of which is a group. Each of these groups consists of an idempotent element and all the type B elements which have this idempotent element for period element.*

## Chapter 7. The French School of ‘Demi-groupes’

<sup>1</sup>Some comments on the figures in the table are in order. As noted in the caption, the figures given are those recorded on ‘MathSciNet’ (<http://www.ams.org/mathscinet/index.html>) as of January 2013. The citations tool, however, only seems to count citations from the early 1950s onwards. Thus, for example, Clifford (1941) does not appear on the list of papers that cite Rees (1940), although it does do so. The citation figures are therefore skewed very slightly towards citations that appeared in the longer term; if, say, Dubreil's paper provoked a flurry of interest in the few years after its publication, then the consequent citations are not recorded here. Nevertheless, I think that the figures give an indication of the relative impacts of these three papers. The numbers in the second part of the table are much more dramatic. In each instance, I searched for the exact strings given, rather than the individual words. In the case of Rees (1940), I felt that the term ‘completely simple semigroup’ was sufficiently representative of the content of the paper that I did not also need to search for ‘completely 0-simple semigroup’. In connection with Clifford (1941), I searched for the modern term ‘completely regular semigroup’ (p. 156); Clifford's term ‘semigroup admitting relative inverses’ returned just three results.

### Section 7.1. Paul Dubreil and Marie-Louise Dubreil-Jacotin

<sup>2</sup>For biographies of Dubreil, see Lesieur (1994) and Lallement (1995). See also Dubreil's own article on the early development of semigroup theory in France (Dubreil, 1981), in which he described his entry into the theory.

<sup>3</sup>For details of the French education system, see Lewis (1985).

<sup>4</sup>It seems to have been very common at this time for young French mathematicians to travel widely in Europe — see Mashaal (2002, English trans., p. 46). Dubreil later wrote about his Rockefeller travels in Dubreil (1983).

<sup>5</sup>From the conclusion to the article: “Ayant apprécié, tout au long de ma vie scientifique (y compris en Théorie des Demi-groupes ... qui n'existait pas à l'époque!), l'énorme bénéfice que j'ai retiré de mes contacts de jeunesse avec les algébristes allemands, surtout avec le trio Emmy Noether, Artin, Krull, je suis très reconnaissant à P. Dugac de m'avoir proposé de rendre aujourd'hui cet hommage à la mémoire d'Emmy Noether et j'adresse mes plus chaleureux remerciements à tous ceux qui sont venus s'y associer.” (Dubreil, 1986, p. 27).

<sup>6</sup>This seminar bore Dubreil's name from 1945 to 1979. The proceedings were published under the title *Séminaire Dubreil: Algèbre et théorie des nombres* until 1971, at which time it became simply *Séminaire Dubreil: Algèbre*. Details of the seminar, under its various different names, may be found in Anderson (1989, pp. 34, 44–47).

<sup>7</sup>For biographies of Dubreil-Jacotin, see Leray (1974) and Lesieur (1973).

<sup>8</sup>Translation of Leray (1974) by Jean O'Connor.

<sup>9</sup>*Ibid.*

### Section 7.2. Equivalence relations

<sup>10</sup>“...  $\bar{E}$  est homomorphe à  $E$  ...” (Dubreil and Dubreil-Jacotin, 1937a, p. 705).

<sup>11</sup>“Ces propositions peuvent être regardées comme des généralisations du théorème du homomorphie et du premier théorème d'isomorphie.” (Dubreil and Dubreil-Jacotin, 1937a, p. 706).

<sup>12</sup>“une théorie systématique des relations d'équivalence” (Dubreil and Dubreil-Jacotin, 1939, p. 63).

<sup>13</sup>See, for example, Dubreil (1950a,b), Dubreil-Jacotin (1950a,b), Dubreil-Jacotin and Croisot (1952).

### Section 7.3. Principal equivalences and related concepts

<sup>14</sup>“... la double empreinte laissée dans mon esprit par les leçons d'ARTIN (théorie d'ARTIN-PRÜFER) et par celles d'Emmy NOETHER (utilisation systématique des homomorphismes) m'a suggéré qu'un autre procédé pour obtenir un groupe à partir d'un deami-groupe [*sic*] (quelconque cette fois) était la recherche de ce groupe comme image homomorphe.” (Dubreil, 1981, p. 61).

<sup>15</sup>“Le problème est facile quand  $D$  est abélien, classique et élémentaire quand  $D$  est lui-même un groupe, et, dans tous les cas, il admet la solution triviale dans laquelle l'image est d'ordre 1: la question paraissait abordable.” (Dubreil, 1981, p. 62).

<sup>16</sup>“Mes réflexions en étaient là le premier septembre 1939 ...” (Dubreil, 1981, p. 62).

<sup>17</sup>An account of much of the material of this paper may be found in Clifford and Preston (1967, §§10.2–10.3).

<sup>18</sup>“L'objet du présent travail est de montrer que certaines propriétés fondamentales des groupes s'étendent, avec les modifications convenables, aux demi-groupes ou à certaines catégories de demi-groupes. Ce sont essentiellement les propriétés qui concernent les sous-groupes et les sous-groupes invariants, et surtout les décompositions en classes, ainsi que les équivalences correspondantes.” (Dubreil, 1941, p. 1).

<sup>19</sup>“... fournit une propriété caractéristique des sous-groupes invariants qui est susceptible de généralisation.” (Dubreil, 1941, pp. 2–3).

<sup>20</sup>“... cette notion de quotients ne différant pas, au fond, de celle des quotients d'idéaux.” (Dubreil, 1941, pp. 3–4).

<sup>21</sup>“Avec le théorème [7.8], le théorème précédent caractérise complètement les équivalences régulières à droite et simplifiable à droite dans un demi-groupe strict à droite: ce sont les équivalences principales à droite définies par des complexes forts. En outre, d'après le théorème [7.9], toutes les classes définies par une telle équivalence jouent des rôles symétriques. Comme nous allons le voir, ces propriétés ne diffèrent pas essentiellement de celles qui ont lieu dans les groupes.” (Dubreil, 1941, p. 20).

<sup>22</sup>“On voit que la théorie des équivalences principales contient les théorèmes fondamentaux de la Théorie des Groupes ...” (Dubreil, 1941, p. 22).

<sup>23</sup>In the statement of this result in Dubreil's paper,  $F$  is assumed merely to be an arbitrary semigroup, but, as Clifford and Miller (1948, p. 123) pointed out, the surrounding considerations indicate that  $F$  should in fact be cancellative.

### Section 7.4. Subsequent work

<sup>24</sup>See the references in note 13.

<sup>25</sup>The 1966 volume (fasc. 1) of *Annales scientifiques de l'Université de Besançon (3<sup>e</sup> Série — Mathématiques)* consists entirely of a tribute to Croisot in two parts: the first features transcripts of speeches delivered (by Dubreil, for example) at Croisot's funeral and at the inauguration of the 'Robert Croisot amphitheatre' in Besançon, while the second part is a survey of Croisot's mathematical work, authored by Lesieur.

<sup>26</sup>“Les équivalences principales de P. Dubreil (à droite ou à gauche) sont parfaitement adaptées à l'étude des équivalences régulières à droite ou à gauche; elles le sont un peu moins à celle

des équivalences régulières des deux côtés; les équivalences principales bilatères sont exactement adaptées à cette étude.” (Croisot, 1957, p. 374).

<sup>27</sup>“le maniement des équivalences principales bilatères est plus compliqué que celui des équivalences principales” (Croisot, 1957, p. 375).

<sup>28</sup>“... alors qu’il est très facile de voir sur la table d’opération d’un demi-groupe  $D$  si un complexe de  $D$  est fort ou non, il est beaucoup plus malaisé de déterminer s’il est bilatèrement fort ou non.” (Croisot, 1957, p. 375).

<sup>29</sup>“Le problème de la recherche des *groupes homomorphes* à un demi-groupe a été complètement résolu en utilisant les équivalences principales. Les équivalences principales bilatères n’apportent donc rien de plus que les équivalences principales sur ce problème; elles permettent simplement d’en donner une solution un peu différente ...” (Croisot, 1957, p. 375).

<sup>30</sup>Note that Croisot’s ‘bilaterally neat’ is *not* the same as Dubreil’s neat (that is, both left and right neat): the latter is defined in terms of  $\mathcal{R}_H$  and  ${}_H\mathcal{R}$ , while the former relates to  $\mathcal{R}'_H$ .

<sup>31</sup>For an obituary of Lallement, see Almeida and Perrin (2009).

<sup>32</sup>For biographies of Schützenberger, see Lallement and Perrin (1997) and Wilf (1996). For a list of his publications, see Lallement and Simon (1998). For an interview with him concerning his controversial views on Darwinism, see Anon (1996b). Finally, for a short account of his semigroup-theoretic work, see Pin (1999).

<sup>33</sup>For further details on the theories of formal languages and automata, see Howie (1991).

## Chapter 8. The Expansion of the Theory in the 1940s and 1950s

### Section 8.1. The growth of national schools

<sup>1</sup>For biographies of Hoehnke, see Márki *et al.* (1996) and Denecke (2008a,b).

<sup>2</sup>In connection with radicals in semigroups, see also the comment on Munn’s work on page 292 and the references in note 10 of Chapter 11.

<sup>3</sup>See, for example, Čupona (1958). It is difficult to see where Čupona’s ‘semigroup influence’ came from, as the papers by him that I have seen contain no references.

### Section 8.2. The Slovak school

<sup>4</sup>There is a very large number of biographies of Schwarz, among them Kolibiar (1964), Jakubík and Kolibiar (1974, 1984, 1994), ZnáM and Katriňák (1979), Mišík (1981), Jakubík *et al.* (1984), Grošek *et al.* (1994), Dvurečenskij (1996), and Riečan (1997).

<sup>5</sup>A short introduction to the activities of the Society for the Protection of Science and Learning can be found in the Bodleian Library’s introductory booklet on the society’s archive (which is housed in Oxford): Baldwin (1988).

<sup>6</sup>On which, see Anon (1953b). The same issue of the journal also features an article by Schwarz on the need for a separate Slovak Academy of Sciences and its importance for the teaching of mathematics (Schwarz, 1953c).

<sup>7</sup>See Gerretsen and de Groot (1957, vol. 1, p. 82), James (1975, vol. 1, p. xliii), and Lehto (1980, vol. 1, p. 38), respectively.

<sup>8</sup>For an abstract of the Amsterdam talk, ‘Characters of commutative semi-groups’, see Schwarz (1957); in the case of the Vancouver talk, we have only a title: ‘Ideal structure of  $C$ -semigroups’ (James, 1975, vol. 2, p. 596).

<sup>9</sup>The *Czechoslovak Mathematical Journal* that was launched in 1951 was the mathematical continuation of the mathematics and physics journal *Časopis pro pěstování matematiky a fyziky* (*Journal for the Cultivation of Mathematics and Physics*), which had been published in Prague by the Union of Czech Mathematicians and Physicists (on which organisation, see Bečvářová 2013) since 1872 but whose “publication was interrupted for several years owing to the criminal interference of Hitler’s fascists” (Anon, 1951, p. 1). Following five post-war volumes (1946–1950), the journal was taken over by the Czechoslovak Academy of Sciences, which split it into two separate journals: one for mathematics (the *Czechoslovak Mathematical Journal*, though it was also published in two other versions, each under a different name — see note 10) and one for physics (*Československý časopis pro fyziku*). Publication of the mathematical journal was taken over by Springer in 1997. The fact that the *Czechoslovak Mathematical Journal* is the continuation of an older journal means that references to papers in the journal sometimes feature two volume



numbers: one for the refounded journal, the other for the original. Thus, for example, the volume number for Jakubík and Kolibiar (1984) is given as **34(109)**, where, strictly speaking, **34** is the volume number for the *Czechoslovak Mathematical Journal* and **109** is that for *Časopis pro pěstování matematiky a fyziky*. The reason for giving both numbers is presumably to lend an extra respectability to the journal by reminding readers that it is older than it would at first appear to be. See also notes 10 and 15.

<sup>10</sup>Following on from the comments in note 9, we observe that the English/French/German version of the journal was published under the name *Czechoslovak Mathematical Journal*, while the Russian version was published under the direct Russian translation *Чехословацкий математический журнал*. The papers from this journal that are cited here are in fact a mixture of the Russian and non-Russian versions since my source for the full text of the journal has been the Czech Digital Mathematics Library (<http://dml.cz/>), which seems to consist of an amalgam of the Russian and non-Russian versions, though all presented under the English name. In addition, a Czech and Slovak version of the journal was also published, under a modified version of the original title: *Časopis pro pěstování matematiky*. The aim of this further edition was “to improve the professional and ideological knowledge of those interested in mathematics at home and to serve the propagation of mathematics in Czechoslovakia” (Anon, 1951, p. 2). However, it is not clear in what sense this latter journal is an ‘edition’ of the *Czechoslovak Mathematical Journal* (as is claimed in Anon 1951) since its contents appear to be different. *Časopis pro pěstování matematiky* changed its name to *Mathematica Bohemica* in 1991. These comments on the tangle of Czechoslovak mathematical journals should go some way towards explaining why many of the articles cited in note 4 have more than one version listed in their entry in the bibliography. Indeed, some of the articles cited in note 4 were also reprinted in the entirely separate journal *Mathematica Slovaca*. See also note 15.

<sup>11</sup>“Potreby algebry, číselnej teórie a topologie si vynútily v posledných rokoch nutnosť štúdia systémov všeobecnejších ako sú grupy. ... Štúdium takýchto systémov je v poslednom čase predmetom viacerých prác. V predloženej práci som si vzal za úlohu odvodiť vlastnosti a vyšetrit’ štruktúru takzvaných pologrúp.” (Schwarz, 1943, p. 3).

<sup>12</sup>“Platí táto základná veta ...” (Schwarz, 1943, p. 8).

<sup>13</sup>In the terminology of note 36 of Chapter 3, an element has preperiod if its index is strictly greater than 1.

<sup>14</sup>See instead the articles cited in note 4.

<sup>15</sup>Following on from notes 9 and 10, on the subject of Czechoslovak mathematical journals and their myriad incarnations, I mention that the journal *Matematicko-fyzikálny časopis* (or, more fully, *Matematicko-fyzikálny časopis, Slovenská akadémia vied = Mathematico-Physical Journal, Slovak Academy of Sciences*), in which the Slovak version of these papers appeared, was founded by Schwarz in 1951 as *Matematicko-fyzikálny zborník*. The name *Matematicko-fyzikálny časopis* was used from 1953 until 1966, at which point, as with *Časopis pro pěstování matematiky a fyziky* before it, the mathematics and physics strands of the journal were split into two separate publications: *Matematický časopis* and *Fyzikálny časopis*, respectively. Each of these eventually underwent a further name change, the mathematics journal becoming *Mathematica Slovaca* in 1976, and the physics journal *Acta Physica Slovaca* in 1974. The volume numbering remained constant throughout these name changes, so that, for example, *Matematicko-fyzikálny časopis*, volume 16, was followed by *Matematický časopis* and *Fyzikálny časopis*, volumes 17. The full text of the pre-split journal, together with that of the subsequent physics strand, may be found on the current *Acta Physica Slovaca* website (<http://www.physics.sk/aps/>), where, rather perversely, the latest name change has been applied retrospectively. Thus, for example, although the bibliographical information printed on the Slovak version of the paper cited here identifies it as having been published in volume 3 of *Matematicko-fyzikálny časopis*, it is filed on the website under volume 3 of *Acta Physica Slovaca*, even though, strictly speaking, the latter journal never had such a volume. Similarly, the Czech Digital Mathematics Library (see note 10) features the full text of the pre-split journal, together with the subsequent mathematical strand, all listed under the name *Mathematica Slovaca*.

<sup>16</sup>For biographies, see Grošek and Satko (1998) and Jakubík and Šmarda (1992), respectively.

<sup>17</sup>For a biography, see Horák (1985).

<sup>18</sup>For biographies, see Černák (2003) and Katriňák (1996), respectively.

### Section 8.3. The American school

<sup>19</sup>Although such a categorisation is by no means impossible: see Preston (1974).

<sup>20</sup>Clifford and Miller used the term *universally minimal* to denote a right ideal that is contained in every other right ideal, and *locally minimal* for a right ideal that contains no proper right ideals. That the two notions are distinct was demonstrated by an example of a semigroup with two disjoint locally minimal right ideals (Clifford and Miller, 1948, p. 118). The result quoted here concerns universally minimal ideals.

<sup>21</sup>Rich completed a PhD dissertation, entitled *Factorization of partially ordered groups*, at Johns Hopkins University in 1950. The 1949 paper cited here appears to be his only mathematical publication. Rich's completion of a thesis on partially ordered groups was probably connected with Clifford's interest in this topic: see, for example, Clifford (1952b). Clifford's interests extended also to linearly ordered groups (Clifford, 1952a).

<sup>22</sup>See instead the comment on Munn's work on page 292, and the references in note 10 of Chapter 11.

<sup>23</sup>This was in fact a specialisation of a result of David McLean (1954). Inspired by Clifford's 1941 paper, McLean showed (his Theorem 1) that any band is a semilattice of anticommutative bands, where an *anticommutative band* is a band in which  $ab = ba$  implies that  $a = b$ . McLean used this to prove that any finitely generated free idempotent semigroup has finite order (his Theorem 2). This is something that also follows from the results of one of Rees's few forays into semigroups. In a paper co-authored with Green (Green and Rees, 1952),  $S_{nr}$  was defined to be the semigroup generated by  $n$  elements, subject to the single relation  $x^r = x$ , for every element  $x$ . The semigroup  $S_{nr}$  was defined by analogy with the group  $B_{n,r-1}$ , generated by  $n$  elements and subject to the relation  $x^{r-1} = 1$ , for every element  $x$ . Green and Rees showed that the statement ' $S_{nr}$  is finite for all  $n$ ' is equivalent to an appropriate version of the Burnside conjecture (see Green and Rees 1952, p. 35). The connection with McLean's work arises from the fact that Green and Rees found a formula for the (finite) order of  $S_{nr}$  for the case  $r = 2$ .

<sup>24</sup>See, for example, Fountain (1977), where certain semigroups (those alluded to in note 40 of Chapter 10) are characterised as semilattices of left cancellative monoids.

<sup>25</sup>For comprehensive surveys of the study of semigroups of transformations, see Sullivan (1978, 2000). For specific details on the representation of semigroups by transformations, see Clifford and Preston (1967, Chapter 11). For a survey article with a much broader viewpoint (namely semigroups of binary relations), see Schein (1969). See also the books Lipscomb (1996) and Ganyushkin and Mazorchuk (2009).

<sup>26</sup>For a brief obituary of Stoll, see Anon (1991).

<sup>27</sup>See Hollcroft (1944, p. 21); an abstract may be found in Anon (1943, p. 850).

<sup>28</sup>Sometimes called an *S-act*, *S-system*, *S-operand* (Howie, 1995b, §8.1), or *S-polygon*, the latter term being found more often in the work of Eastern European authors. On the general theory of *S*-sets, see Kilp *et al.* (2000).

<sup>29</sup>See the references in note 25.

<sup>30</sup>In connection with Wallace and Tulane, the exclusion of topological semigroups from the present book may in fact be to its detriment. To quote Karl H. Hofmann (private communication, 18 January 2013):

In one sense your rigorous exclusion of, say topological semigroups, understandable as a strict discipline of drawing boundaries, is regrettable as in this way your book will never capture the vibrant and vital semigroup life at Tulane which really flourished through the interaction of the Wallace people and the Clifford people.

<sup>31</sup>See Clifford and Preston (1961, §1.3, Ex. 7a and §1.8, Ex. 4). The former result cited by Clifford and Preston was also proved by Takayuki Tamura (1955a).

<sup>32</sup>See Clifford and Preston (1961, §1.11, Exx. 7–9 and §2.5, Ex. 9).

<sup>33</sup>See Clifford and Preston (1961, §1.9, Ex. 1 and §2.2 — Theorem 2.9 in particular).

### Section 8.4. The Japanese school

<sup>34</sup>For a biography of Tamura, see Hamilton and Nordahl (2009); for an obituary, see Anon (2009).

<sup>35</sup>See the comments in note 41.

<sup>36</sup>See Stedall (2008, §13.1.4). Cayley determined all groups of orders 4 and 6, presenting the former in what we now term ‘Cayley tables’.

<sup>37</sup>Note that although the number of distinct (up to isomorphism and anti-isomorphism) semigroups of order 10 is unknown, it is known that there are 52,991,253,973,742 *monoids* of order 10 (Distler and Kelsey, 2009).

<sup>38</sup>In addition to those sources cited so far, I give a (non-exhaustive) list of further references concerning the enumeration of finite semigroups:

- Plemmons (1970) (general comments on algorithms for the computation of finite semigroups);
- Kleitman *et al.* (1976) (estimates for the number of distinct semigroups of order  $n$ );
- Jürgensen (1977) (survey of computer applications in the study of finite semigroups);
- Jürgensen (1989) (annotated tables of semigroups of orders 2–7);
- Grillet (1995b) (upper bound for the number of commutative semigroups of order  $n$ );
- Grillet (1996) (on improvements to existing algorithms);
- Distler and Kelsey (2009) (calculation of monoids of orders 8, 9, and 10);
- Distler (2010) (among other things, new results on semigroups of order 9 and monoids of order 10);
- Distler and Kelsey (2014) (semigroups of order 9 and their automorphism groups).

<sup>39</sup>An example of another topic treated by Tamura but not dealt with here is his study of finite semigroups in which the order of every subsemigroup divides the order of the semigroup (Tamura and Sasaki, 1959). These were later dubbed *Lagrange semigroups* by Mertes (1966) and were the subject of the paper by the Chinese authors that was mentioned at the end of Section 8.1.

<sup>40</sup>Kimura’s time at Tulane coincided with Preston’s visit there (see Section 12.1.3). Indeed, Preston served as chair of examiners for Kimura’s viva; the other examiners were P. F. Conrad and P. S. Mostert.

<sup>41</sup>Prior to this, particularly, in the 1930s, Japanese mathematicians (and scientists more generally) appear to have been heavily influenced by their German counterparts, even going so far as to publish a great deal of work in German. On the ‘Prussianisation’ of Japanese science, see Parshall (2009, p. 98).

<sup>42</sup>There is some overlap between these papers and some work by both Cohn (1956a, 1958) and Gluskin (1955a) (for Gluskin’s work, see Section 9.4).

### Section 8.5. The Hungarian school

<sup>43</sup>For English biographies of Rédei, see Márki (1985) and Anon (1981); an English survey of his mathematical work can be found in Márki *et al.* (1981). For Hungarian biographies, see Anon (1972a), Steinfeld and Szép (1970), and Wiegandt (1998); Pálffy and Szép (1982) gives a survey of his group-theoretic work.

<sup>44</sup>Such semigroups are now the objects of study of an active Spanish group of researchers on numerical semigroups.

<sup>45</sup>For a biography, see Márki (1991).

<sup>46</sup>“Es ist eine wichtige Frage, wie weit  $R$  durch die eine der Strukturen  $R^+$ ,  $R^\times$  bestimmt ist.” (Rédei and Steinfeld, 1952, p. 146).

<sup>47</sup>For biographies, see Csákány *et al.* (2002) and Fried (2004).

<sup>48</sup>See also the comments in note 13 of Chapter 12.

<sup>49</sup>See note 33 of Chapter 9.

<sup>50</sup>On rings, see Pollák (1961); on semigroups, see Megyesi and Pollák (1968).

<sup>51</sup>For a biography, see Forgó (2005).

### Section 8.6. British authors

<sup>52</sup>See note 23.

<sup>53</sup>See also note 33 of Chapter 9 on some other types of ‘normal subsemigroup’.

<sup>54</sup>For greater detail, see Clifford and Preston (1961, §2.1) and Howie (1995b, §2.1).

<sup>55</sup>For very brief biographies of Green, see Anon (1984, 1998, 2002). For an obituary, see Erdmann (2014).

<sup>56</sup>Generalisations of Green’s relations may be found in Wallace (1963), Anscombe (1973), Márki and Steinfeld (1974), Pastijn (1975), McAlister (1976), Carruth and Clark (1980), Cripps (1982), Lawson (1991), Yang and Barker (1992), Shum *et al.* (2002), and Guo *et al.* (2011), some

of which are surveyed in Hollings (2009a). Generalised Green’s relations are a major tool of the active Chinese school of semigroup theory; for surveys of Chinese work in this direction, see Guo *et al.* (2010) and Shum *et al.* (2010). In contrast to early semigroup theory, where, as we have seen, ideas from rings were applied to semigroups, Green’s relations have also been applied to rings (Petro, 2002).

<sup>57</sup>For biographies of Howie, see Munn (2006), Robertson (2012), and Shaw (2012).

### Chapter 9. The Post-Sushkevich Soviet School

<sup>1</sup>A comment on the use of the label ‘Soviet’ in such phrases as ‘Soviet semigroup theory’, ‘Soviet mathematics’, etc.: it is used in this chapter, as throughout the rest of this book, merely as a convenient single term by which we may refer to the work of (mainly) Russian and Ukrainian authors; it should not be taken to have any political connotations. Moreover, the term ‘post-Sushkevich’ is used simply in a chronological sense and does not imply any kind of continuity.

<sup>2</sup>For example, the article Gluskin (1968) in volume 3 of Shtokalo and Bogolyubov (1966) and the article Slipenko (1983) in the volume Shtokalo *et al.* (1983).

<sup>3</sup>“В 1984 г. исполняется 70 лет выдающемуся советскому алгебраисту профессору Ленинградского государственного педагогического института Евгению Сергеевичу Ляпину. Е. С. Ляпин является одним из создателей важного направления общей алгебры — теории полугрупп. Его первая работа, посвящённая этой теории, вышла в 1947 году. К этому времени понятие полугруппы уже сформировалось в математике, однако оно рассматривалось просто как один из возможных вариантов обобщения понятия группы и самостоятельного значения не имело. Главным образом благодаря трудам Е. С. Ляпина из разрозненных работ, посвящённых полугруппам, выросло новое направление в общей алгебре — теория полугрупп. Появление в 1960 г. первой в мировой литературе монографии Е. С. Ляпина по теории полугрупп оказало решающее влияние на формирование этой теории и выдвинуло советскую полугрупповую школу на передовые позиции.” (Gluskin *et al.*, 1984).

#### Section 9.1. Evgenii Sergeevich Lyapin

<sup>4</sup>Soviet-era biographies are Budyko *et al.* (1975), Gluskin *et al.* (1985), and Wagner *et al.* (1965), written for Lyapin’s 50th, 60th, and 70th birthdays, respectively. A slightly more candid (post-Soviet) biography was written for Lyapin’s 80th birthday by his student J. S. Ponizovskii (1994). Since Lyapin’s death, several further articles have appeared: a brief ‘official’ Russian obituary (Gordeev *et al.*, 2005), a memorial article by his student A. Ya. Aizenshtat and his sometime-collaborator B. M. Schein (2007), and an article by former students V. A. Makaridina and E. M. Mogilyanskaya (2008). Another source that has proved particularly useful in the compilation of this section has been Khait (2005), which was apparently written with the input of Lyapin’s family; it discusses his family background and has a great deal to say about his life in the 1940s; it is the only one of the cited biographies to deal with Lyapin’s wartime activities and ideological persecution in any detail, though it is also the only Russian article cited in this note that is not available in English translation. However, I drew heavily upon Khait while writing my own article on Lyapin (Hollings, 2012), parts of which have been reused here.

<sup>5</sup>These Soviet-era biographies are very impersonal affairs, which could almost have been written in the form of bullet points. Although I have yet to conduct a full survey of Soviet mathematical biographies, it is my impression that they (particularly those published in *Uspekhi matematicheskikh nauk*) usually have the following structure:

- An introductory paragraph consisting of a single sentence, giving the name and status of the person about whom the article has been written; we are also told the reason for the article (e.g., death or significant birthday, for which a date is given). Thus, for example, the English translation of Wagner *et al.* (1965) begins: “The eminent Soviet algebraist and Professor at the Leningrad Pedagogical Institute E.S. Lyapin had his 50th birthday on September 19th, 1964.”
- A list of facts about the subject’s life: place of birth, social background, university education, dissertations submitted, institutions at which the subject has worked. If the subject is of an appropriate age, we might also find one or two sentences here about their activities during the ‘Great Patriotic War’ (i.e., the Second World War).
- A sketch of the subject’s mathematical work. Depending on the length of the article, this could be anything from a couple of lines (often the case for obituaries) to several pages.

- The ‘usefulness to the state’ paragraph. Typical statements might be that the subject has authored  $m$  textbooks and that they have supervised  $n$  research students. We might also be told that the subject has been instrumental in the training of generations of mathematics teachers and that their students teach at schools throughout the USSR. The committees upon which the subject has sat will be listed here.
- Awards and honours (not always present). The Order of Lenin and the title of ‘Honoured Scientist of the RSFSR (Russian Soviet Federative Socialist Republic)’ are the honours most often found here.

<sup>6</sup>The names Saint Petersburg (1703–1914 and 1991–), Petrograd (1914–1924), and Leningrad (1924–1991) are used interchangeably and without further comment.

<sup>7</sup>I choose to transliterate ‘Герцен’ as ‘Herzen’ since this appears to be the generally accepted Latin spelling.

<sup>8</sup>At that time, the full name of this institution was Ленинградский государственный педагогический институт имени А. И. Герцена (‘Leningrad State Pedagogical Institute, named for A. I. Herzen’). It is now the ‘Russian State Pedagogical University, named for A. I. Herzen’ (Российский государственный педагогический университет имени А. И. Герцена).

<sup>9</sup>Also known as the Leningrad Blockade, following the Russian: блокада Ленинграда.

<sup>10</sup>In Russian, we have, for example, Karasev (1959) and Sirota (1960), while some books in English are Goure (1962), Jones (2008), Pavlov (1965), and Salisbury (2000). Nikitin (2002) is a book of photographs from the siege which is not for the squeamish.

<sup>11</sup>Adamovich and Granin (1982, English trans., p. 60). Further references to Lyapun may be found on pages 43, 167, and 373 of that book. For an account of the siege from another mathematician, see Lorentz (2002, §7).

<sup>12</sup>“расчет прочности ледовой трассы по Ледожскому озеру” (Khait, 2005, p. 15). Perhaps Lyapun had a hand in the compilation of Table 23 on p. 136 of Pavlov (1965), which expresses the expected rate of the thickening of the ice in different temperatures?

<sup>13</sup>Not all publications lists for Lyapun include the meteorological papers, but they may be found, for example, in that given by Budyko *et al.* (1975).

<sup>14</sup>For some background to these objections, see Gerovitch (2002, pp. 34–35). See also the comments of Lorentz (2002, p. 218).

<sup>15</sup>For biographies of Shanin, see Artemov *et al.* (2010), Maslov *et al.* (1980), Matiyasevich *et al.* (1990) and Vsemirnov *et al.* (2001). None of these articles, however, mention the ideological attack.

<sup>16</sup>See Veksler *et al.* (1979) for a biography of Vulikh.

<sup>17</sup>See Borovkov *et al.* (1969) for a biography of Sanov. Sanov’s claim to mathematical fame was his solution, at the age of 21, of Burnside’s problem for exponent 4 (Sanov, 1940).

<sup>18</sup>“для разоблачения идеологически чуждых и неверных явлений” (Khait, 2005, p. 16).

<sup>19</sup>“оторвавшихся от жизни и не приносящих никакой пользы социалистическому обществу” (Khait, 2005, p. 16).

<sup>20</sup>“... очень эмоционально возражал тем, кто мешал науке двигаться вперед путем выдвижения новых идей и направлений” (Khait, 2005, p. 16).

<sup>21</sup>“... успех науки требует выдвижения новых идей ...” (Khait, 2005, p. 16).

<sup>22</sup>Soloveichik had in fact worked there since 1933; his research interests seem to have been in fluid mechanics and attendant areas of mathematics: see, for example, Kurosh *et al.* (1959, vol. 2, p. 655). For a biography of Soloveichik, see Prudinskii (2011).

<sup>23</sup>“... далеких от нужд народного хозяйства” (Khait, 2005, p. 16).

<sup>24</sup>“математика служит производственным целям!” (Khait, 2005, p. 16).

<sup>25</sup>“и прочие «измы»” (Khait, 2005, p. 16).

<sup>26</sup>“... сделало вид, что не знает о произошедшем в Университете” (Khait, 2005, p. 17).

## Section 9.2. Lyapun’s mathematical work

### Section 9.2.1. Normal subsystems and related concepts

<sup>27</sup>“Основами современной теории групп бесспорно являются теория гомоморфизмов (включающая теорию нормальных делителей) ...” (Lyapun, 1945, p. 3).

<sup>28</sup>“За последние годы в математической литературе не раз делались попытки обобщить современную теорию групп, перенести те или иные групповые результаты на различные виды «полугрупп», т.е. на системы с одним действием, более общие, чем группы. Были получены

также некоторые результаты, специфические для «обобщенных групп», не имеющие аналогии или тривиальные для обычных групп.» (Lyapin, 1947, p. 497).

<sup>29</sup>“... теория групп есть не что иное, как абстрактное учение об обратимых преобразованиях...” (Lyapin, 1947, p. 497).

<sup>30</sup>“[л]юбая физическая теория, любая отрасль математики дают бесчисленные примеры чрезвычайно важных необратимых преобразований.” (Lyapin, 1947, p. 498).

<sup>31</sup>“Изучение преобразований необратимых требует теории более широкой, нежели теория групп.” (Lyapin, 1947, p. 498).

<sup>32</sup>“... [в] настоящее время общая теория ассоциативных систем только начинает развиваться и еще находится в зачаточном состоянии. ... Поэтому естественно начинать построение общей теории ассоциативных систем с разбора, тех вопросов, решение которых послужило основой успешного развития теории групп.” (Lyapin, 1947, p. 498).

<sup>33</sup>Besides Lyapin's, and that of Rees that we saw in Section 8.6, another notion of 'normal subsemigroup' had in fact already been introduced by F. W. Levi in the paper in which he obtained a characterisation of the free semigroup (Levi 1944: see Section 5.3). Given an arbitrary semigroup  $S$ , Levi termed a subsemigroup  $N$  of  $S$  *normal* if it satisfies 'condition N': for  $\alpha, \beta, \gamma \in S$ , if any two of the three elements  $\alpha\beta\gamma$ ,  $\alpha\gamma$ ,  $\beta$  belong to  $N$ , then all three belong to  $N$ . Levi focused his attention on so-called *R-semigroups*; these are semigroups that satisfy what he called 'condition R', or the 'condition of refinement': if  $a, c$  and  $a', c'$  are any two distinct pairs of elements of  $S$  such that  $ac = a'c'$ , then there exists  $b \in S$  for which at least one of the following two sets of conditions holds:

$$\{a' = ab, c = bc'\}, \quad \{a = a'b, c' = bc\}.$$

In an **R**-semigroup  $R$  with normal subsemigroup  $N$ , Levi defined two elements  $a, b$  to be *equivalent* if there exists an element

$$(\dagger) \quad \omega = \alpha_0 a_1 \alpha_1 \cdots a_n \alpha_n = \beta_0 b_1 \beta_1 \cdots b_m \beta_m$$

in  $R$ , such that  $a_1 \cdots a_n = a$  and  $b_1 \cdots b_m = b$ , where the  $\alpha_i$  and  $\beta_j$  are either elements of  $N$  or else 'empty symbols' which may be omitted from the factorisation in  $(\dagger)$ . This equivalence is in fact a congruence, so we may factor by it to obtain a new semigroup, which Levi denoted by  $R/N$ ; he termed the elements of  $R/N$  *cosets of  $N$  in  $R$* , the 'coset' of  $a \in R$  being denoted by  $(a)$ . With this set-up, Levi was able to prove the following theorem (Levi, 1944, I, Theorems 1 and 2):

**THEOREM.** *The semigroup  $R/N$  is an **R**-semigroup with  $N$  as its identity element, and the mapping  $a \mapsto (a)$  is a homomorphism.*

Yet another type of 'normal subsemigroup' was studied by István Peák (1960). In this instance, a subsemigroup  $N$  of a semigroup  $H$  is *left normal* if  $H$  may be written in the form  $H = N \cup \alpha N \cup \beta N \cup \cdots$ , for  $\alpha, \beta, \dots \in H$ , and  $(\alpha H)(\beta H) = \gamma H$ , for some  $\gamma \in H$ . Peák compared his notion of normality with that of Lyapin.

<sup>34</sup>A better literal translation would perhaps be 'moving' or 'shifting' (from the verb 'передвигать' = to move/shift), but the term 'removing' is used in the English summary that appears at the end of Lyapin's paper.

<sup>35</sup>“[с]овокупность неособенных квадратных матриц  $n$ -я порядка над произвольным полем образует группу относительно умножения. Эта группа подвергалась многочисленным исследованиям и в настоящее время хорошо изучена. Естественно, возникает вопрос об исследовании совокупности всех (особенных и неособенных) квадратных матриц  $n$ -я порядка. Относительная трудность такого исследования объясняется тем, что эта совокупность уже, очевидно, не образует группы относительно умножения; она является лишь ассоциативной системой. Между тем, теория ассоциативных систем развита еще очень мало.” (Sivertseva, 1949, p. 101).

<sup>36</sup>In connection with the notation used here, it is interesting to observe that Lyapin made use of some very limited logical symbolism in all three of his 1950 papers (' $\rightarrow$ ' for implication and ' $\leftrightarrow$ ' for equivalence), yet, as far as can be ascertained, he did not have the same difficulties as those experienced by Wagner when trying to use logical notation in the same journal a couple of years later: see page 262 and also note 28 of Chapter 10.

<sup>37</sup>“Хорошо известна большая роль простых групп и значение вопроса о простоте в теории групп. Поэтому естественно поставить аналогические вопросы и в теории ассоциативных систем...” (Lyapin, 1950b, p. 275).

<sup>38</sup>The term that Lyapun used for an element with a power equal to zero was ‘нульстепенный’ (Lyapun, 1950b, p. 276), which we might translate literally as ‘null-powered’. There is a strong temptation to translate Lyapun’s ‘нульстепенный’ as ‘nilpotent’, particularly in light of the fact that this is precisely how the term ‘nilpotent’ is used in modern semigroup theory (Howie, 1995b, p. 70). However, this would not give an accurate rendering of Lyapun’s terminology. The Russian for ‘nilpotent’ is ‘нильпотентный’, and this term was already in use at the time that Lyapun was writing: it was a term that had only recently been used in connection with nilpotent groups (Kurosh and Chernikov, 1947) — Lyapun may thus have been avoiding this term in his own work since he would have been using ‘нильпотентный’ in a different sense.

<sup>39</sup>The restriction to surjective homomorphisms does not appear to be stated explicitly but is necessary: an associative system  $S$  has infinitely many non-surjective homomorphisms, namely, to pick some silly examples, the injections into  $S^1$ ,  $(S^1)^0$ ,  $((S^1)^0)^1$ ,  $\dots$ , where  ${}^1$  and  ${}^0$  denote the adjunction of an identity and a zero, respectively (see the appendix).

<sup>40</sup>Contrast the fact that a simple commutative ring is necessarily a field.

<sup>41</sup>“[о]казалось, что за исключением нескольких особо просто устроенных систем, все системы являются непростыми . . .” (Lyapun, 1950c, p. 367).

### Section 9.2.2. Semigroups of transformations

<sup>42</sup>Indeed, Medvedev (1971, p. 127) noted that there were restrictions on the size of each issue of every Soviet scientific journal, although he did not state the reasons for this. It may have been connected with what appears to have been a chronic shortage of paper in the USSR.

<sup>43</sup>“объясняет важность изучения полугруппы  $S_\Omega$ ” (Lyapun, 1955, p. 8).

<sup>44</sup>In contrast to the situation discussed in note 38, Lyapun was by this stage employing the term ‘nilpotent’ (‘нильпотентный’) in its modern semigroup-theoretic sense.

<sup>45</sup>Lyapun went on to study arbitrary partial transformations in a later paper (Lyapun, 1960b), where he determined the general form of a homomorphic representation of a semigroup by means of partial transformations. An isomorphic representation followed in Lyapun (1961), but it is rather more involved than the other similar characterisations given in this section, so I do not reproduce it here.

### Section 9.3. Lazar Matveevich Gluskin

<sup>46</sup>There are rather fewer biographical sources for Gluskin than for Lyapun: one ‘official’ obituary (Belousov *et al.*, 1987), published in the USSR and written in the impersonal Soviet style (see note 5), another Soviet-era biography (Lyapun *et al.*, 1983), and two further articles by Gluskin’s long-term friend and colleague, Boris M. Schein (1985, 1986a).

<sup>47</sup>On anti-Semitism in Soviet academia, see note 51 of Chapter 2 and also note 35 of Chapter 10.

<sup>48</sup>Recall from note 54 of Chapter 3 that a wider claim that Sushkevich was not officially permitted to supervise *any* students does not hold water.

<sup>49</sup>See also note 35 of Chapter 10.

<sup>50</sup>Six name changes later, this institution is now Kharkiv National University of Radioelectronics.

### Section 9.4. Gluskin’s mathematical work

<sup>51</sup>For an indication of Gluskin’s wider semigroup-theoretic work, the reader is directed to the biographical articles cited in note 46 and also to Gluskin’s own survey articles, cited in the introduction to this chapter.

<sup>52</sup>‘Avtoreferaty’ (‘авторефераты’) are formal documents that must be submitted in advance of candidate and doctor of science dissertations. The avtoreferat of Gluskin’s candidate dissertation is undated but must have been submitted prior to the eventual approval of the dissertation in 1952; the doctoral avtoreferat, on the other hand, is dated 1960 — the corresponding dissertation was defended on 28 April 1961. Parts of the second avtoreferat were published as Gluskin (1962).

#### Section 9.4.1. Homomorphisms

<sup>53</sup>“[н]ачало общей теории ассоциативных систем было положено работами А. К. Сушкевича . . . Изучению гомоморфизмов ассоциативных систем посвящен ряд работ Е. С. Ляпина, который ввел понятия нормального комплекса . . . и нормальной подсистемы . . . ассоциативной системы. Настоящая работа является развитием некоторых исследований А. К. Сушкевича и Е. С. Ляпина.” (Gluskin, 1952, avtoreferat, p. 3).

### Section 9.4.2. Semigroups of matrices

<sup>54</sup>“внутренняя характеристика” (Gluskin, 1954, p. 17).

<sup>55</sup>“более естественное” (Gluskin, 1958, p. 441).

<sup>56</sup>In fact, such conditions had already been obtained by Khalezov (1954a,b), but Gluskin provided a new proof, using the theory of completely simple semigroups.

### Section 9.4.3. Semigroups of transformations

<sup>57</sup>On Bourbaki and his ‘structures’, see Corry (1992, 2001) and also Corry (1996, Chapter 7).

<sup>58</sup>Early instances of Gluskin’s study of transformations of sets with extra structure may be found in Gluskin (1959d, 1961b), where he gave, for instance, an abstract characterisation of the semigroup of isotone transformations of a partially ordered set. A detailed account of Gluskin’s work on semigroups of topological transformations may be found in Chapter III of his doctoral dissertation (Gluskin, 1961c).

### Section 9.5. Other authors

<sup>59</sup>In Cyrillic: Л. Рыбаков; this is transliterated as ‘Rybakoff’ in the French summary at the end of the paper.

<sup>60</sup>For a biography of Lesokhin, see Kublanovsky (1999). For further comments on characters for semigroups, see page 297 and also note 12 of Chapter 11.

<sup>61</sup>For a biography of Vorobev, see Korbut and Yanovskaya (1996).

<sup>62</sup>Other early Soviet papers on the word problem in semigroups are those of Adyan (1960) (see Lallement 1988). Furthermore, I take this opportunity to note V. M. Glushkov’s work on automata; see, for example, the survey Glushkov (1961). Unlike cybernetics, the study of formal languages and automata does not seem to have flourished in the USSR to the same extent that it did in the West; this may have been for ideological reasons — see Gerovitch (2001, 2002). Nevertheless, Glushkov’s work may have had an international influence, given that the survey cited above was not only translated into English, but also into German and Hungarian.

<sup>63</sup>For a biography of Liber, see Ermakov *et al.* (1985).

<sup>64</sup>For a biography of Shevrin, see Volkov (2008).

## Chapter 10. The Development of Inverse Semigroups

### Section 10.1. A little theory

<sup>1</sup>For a discussion of various notions of generalised inverses, see Ben-Israel and Greville (2003). These authors deal, in particular, with so-called *Moore–Penrose (pseudo-)inverses* for matrices. These are somewhat akin to the generalised inverses used here in the inverse semigroup context: the Moore–Penrose inverse of a (possibly rectangular) complex matrix  $A$  is the unique solution  $A^\dagger$  of the equations  $AA^\dagger A = A$ ,  $A^\dagger AA^\dagger = A^\dagger$ ,  $(AA^\dagger)^* = AA^\dagger$ ,  $(A^\dagger A)^* = A^\dagger A$ , where  $*$  denotes conjugate transpose. This notion was explored by Roger Penrose in a paper of 1955, where it was used, for example, to provide a necessary and sufficient condition for the solubility of the matrix equation  $AXB = C$ . Initially unknown to Penrose, such generalised inverses had earlier been introduced by E. H. Moore (1920), though with a rather differently phrased definition: see Ben-Israel (2002). On the subject of generalised inverses, see also Rao (2002).

<sup>2</sup>Besides its appearance in the work of Wagner (Section 10.4) and Preston (Section 10.6), this notion of generalised invertibility also appeared briefly in a paper by Thierrin (1952) (see Section 7.4), who referred to inverse elements as *reciprocals (réciproques)*.

### Section 10.2. Pseudogroups and conceptual difficulties

<sup>3</sup>See Klein (1893), or Haskell (1892) for an English translation. For further comments on the place of the Erlanger Programm within mathematics and on its influence, see Birkhoff and Bennett (1988), Hawkins (1984), and Rowe (1983). See also Wussing (1969, §III.2).

<sup>4</sup>Veblen and Whitehead (1932, p. 38). Veblen and Whitehead hyphenated ‘pseudo-group’ but I omit the hyphen in deference to later usage. Lawson (1998, p. 7) notes that there was subsequently “little consensus in the literature” as to the definition of a pseudogroup. He comments that the notion given in Definition 10.2 is “about the most generous”.

<sup>5</sup>On Schouten, see Gołąb (1972), Nijenhuis (1972), and Struik (1989). On Haantjes, see Schouten (1956).



<sup>6</sup>At the time of writing this paper, Gołąb was an associate professor at the Kraków Mining Academy. By the time that it had appeared in print, however, Kraków had been occupied by German troops and Gołąb had been arrested, along with several other Kraków professors. He was imprisoned in Breslau (now Wrocław), before being moved first to the concentration camp at Dachau and then to that at Sachsenhausen. He was released in December 1940 and spent the rest of the war working as a bookkeeper in the forestry administration (Kucharzewski, 1982, p. 3).

<sup>7</sup>“nicht befriedigen . . . vom theoretischen Standpunkte” (Gołąb, 1939, p. 768).

<sup>8</sup>“Das Ziel dieser Untersuchung ist es nun, in axiomatischer Form eine Präzisierung des Begriffes der Pseudogruppe von Transformationen zu geben.” (Gołąb, 1939, p. 768).

<sup>9</sup>“Pseudogruppen im weiteren Sinne” (Gołąb, 1939, p. 773).

<sup>10</sup>“[f]ür die Zwecke der Theorie der geometrischen Objekte” (Gołąb, 1939, p. 768).

<sup>11</sup>“Pseudogruppen im engeren Sinne” (Gołąb, 1939, p. 775).

<sup>12</sup>These axioms may be found on pp. 774–775 of Gołąb (1939), but it takes some effort to ‘unpack’ the conditions in Gołąb’s highly formalised presentation. A much more transparent presentation can be found in Haantjes’s *Zentralblatt* review of Gołąb’s paper (Zbl 0021.04903). Since Gołąb’s symbolism differs considerably from familiar notation, I have translated it into something a little more modern.

### Section 10.3. Viktor Vladimirovich Wagner

<sup>13</sup>As noted on page ix, I choose to transliterate ‘Вaгнер’ as ‘Wagner’, not least because this seems to have been his own preference (Schein, 2002, p. 152).

<sup>14</sup>‘Bullet-point biographies’ (see note 5 of Chapter 9) were published to commemorate Wagner’s 50th birthday (Liber *et al.*, 1958) and his 70th (Efimov *et al.*, 1979). There is also a Soviet-published obituary (Vasilev *et al.*, 1982) and an article of memories of Wagner (Ermakov *et al.*, 1981). Some very brief reminiscences concerning Wagner may be found in Rosenfeld (2007, pp. 88–89). Schein (1981) is a rather more candid Western-published obituary of Wagner; although ostensibly a review of a book on inverse semigroups, Schein (2002) features some biographical anecdotes on Wagner. A much more recent publication about Wagner is the booklet Losik and Rozen (2008) published by Saratov State University in connection with a conference held to commemorate the 100th anniversary of his birth (Rozen, 2009); this booklet contains a short biography of Wagner, as well as reminiscences by several of his students and a comprehensive publications list.

<sup>15</sup>On the Soviet passport system, see Medvedev (1971, pp. 183–194).

<sup>16</sup>For details on the general algebraic work carried out in Saratov, see Gluskin (1970). A brief description of the wider mathematical work can be found in Liber and Chudakov (1963).

<sup>17</sup>For a biography of Schein, see Breen *et al.* (2011). See also note 35.

### Section 10.4. Wagner and generalised groups

<sup>18</sup>The reason for the translation is probably the same as that for the translation around the same time of Hilbert’s *The foundations of geometry*, Hilbert and Ackermann’s *The foundations of theoretical logic*, and Tarski’s *Introduction to logic*: a concerted effort was being made to promote the study of the foundations of mathematics in the USSR — see Vucinich (2000, p. 71).

<sup>19</sup>“известному советскому геометру” (Veblen and Whitehead, 1932, Russian trans., p. 6).

<sup>20</sup>“Проявлением порочных философских и методологических установок авторов является и их мнение о невозможности научного, объективного обсуждения самого вопроса о предмете и задачах геометрии.” (Veblen and Whitehead, 1932, Russian trans., pp. 6–7).

<sup>21</sup>“идеалистической точки зрения” (Veblen and Whitehead, 1932, Russian trans., p. 7).

<sup>22</sup>“Вопрос заключается, конечно, в том, как понимать объективное определение геометрии (вообще, всякой математической науки). Если, следуя авторам, рассматривать геометрию только как сложившуюся формально-логическую систему — оторваино и от ее исторической реальной базы и от ее современных реальных конкретизации, — мы, действительно, не в состоянии будем такого определения дать.” (Veblen and Whitehead, 1932, Russian trans., p. 31).

<sup>23</sup>“они являютс*я* [sic] лишь продуктом длинной цепи абстракций, восходящей к эвклидовой геометрии и дальше” (Veblen and Whitehead, 1932, Russian trans., pp. 31–32).

<sup>24</sup>A passing gibe about Veblen and Whitehead’s “false, metaphysical, idealistic conception” (“ложное, метафизическое, идеалистическое представление”) of geometry is also made in Aleksandrov *et al.* (1956, vol. 1, p. 69). All such attacks on Western mathematicians were removed in the English translation (see Gerovitch 2001, p. 280). By contrast, Soviet ideologues

seem to have been rather keen on the Erlanger Programm. Ernst Colman [Kolman] (p. 16) certainly regarded it as a success; he described geometry as “a science that is more material than mathematics”, which had therefore “detached itself less from reality than the latter”. He went on to make the colourful and rather moralistic comment that “[g]eometrical methods and problems have had a wholesome effect upon mathematics by drawing it back to “sinful mother earth” ...” (Colman, 1931, p. 12).

<sup>25</sup>“Поэтому мы сочли целесообразным дополнить книгу Веблена и Уайтхеда систематическим изложением общей теории объектов, в частности геометрических объектов” (Veblen and Whitehead, 1932, Russian trans., p. 135).

<sup>26</sup>“Il n’est pas de chapitre de mathématiques où la notion de relation d’équivalence ne joue un rôle.” (Riguet, 1948, p. 114).

<sup>27</sup>“Важность этой теоремы состоит в том, что из нее следует, что абстрактная теория симметричных полугрупп взаимно-однозначных частичных преобразований, рассматриваемых как множества, в которых кроме алгебраической операции заданы отношение порядка и симметричное преобразование, сводится к изучению некоторого специального класса абстрактных полугрупп.” (Wagner, 1952a, p. 654).

<sup>28</sup>Wagner’s notation was in fact rather harmless and consisted of familiar symbols like  $p'$  for the negation of a statement  $p$ ,  $p \wedge q$  for the conjunction of statements, and  $p \vee q$  for the disjunction. Implication and equivalence were denoted by arrows  $\rightarrow$  and  $\leftrightarrow$ , respectively. Wagner (1953, p. 549) indicated that his notation was “nonessentially distinct” (“несущественно отличаются”) from that of Lorenzen (1951) and was chosen for its symmetry ( $\wedge$  vs.  $\vee$ , etc.) and the fact that it accorded with the corresponding notation in set theory. In contrast, Lyapun does not appear to have had any trouble with the use of logical symbolism in his 1950 papers for *Izvestiya Akademii nauk SSSR* (note 36 of Chapter 9), although his logical notation was rather more limited. For further comments on the use of logical symbolism by Soviet mathematicians, see Vucinic (1999).

<sup>29</sup>“В последнее время все большее значение начинает приобретать изучение алгебраическими методами формальных свойств определяемых в теории ... операций над множествами и бинарными отношениями между элементами множеств. При этом применение алгебраических методов при изучении теоретико-множественных операций естественным образом приводит к построению соответствующих абстрактных алгебраических теорий. Получаемые таким образом абстрактные алгебраические теории, очевидно, имеют более важное значение, чем те, которые получаются в результате чисто формальных обобщений уже существующих абстрактных алгебраических теорий путем соответствующих изменений положенных в их основу систем аксиом. Действительно, для абстрактной алгебраической теории, в которой изучаются алгебраические операции, допускающие представление при помощи теоретико-множественных операций, очевидна возможность ее приложений в теории множеств, а следовательно, и в других областях математики.” (Wagner, 1953, p. 545).

<sup>30</sup>“Как известно, весьма существенное значение имеет теоретико-множественная операция умножения бинарных отношений между элементами двух различных или совпадающих множеств. Отсюда вытекает важность тех абстрактных алгебраических теорий, которые возникают в связи с изучением формальных свойств этой операции.” (Wagner, 1953, p. 545).

<sup>31</sup>Wagner adopted the notation  $\bar{\rho}$  for the inverse binary relation in order to draw a distinction with  $\rho^{-1}$ , which he used later to denote the inverse of  $\rho$  in the case where  $\rho$  is a partial bijection.

<sup>32</sup>The term *heap* has a somewhat tortuous etymology in this context. The study of ternary operations of this form seems to have originated with Prüfer (1924) and been continued by Baer (1929), who defined the ternary operation  $[x y z] = xy^{-1}z$  in a group, effectively giving an ‘affine’ concept of group, in which the role of the identity is diminished (Bertram and Kinyon, 2010). Baer’s name for a system with a ternary operation satisfying (10.8) and (10.9) was *Schar* (German: band, company, crowd, flock; the term ‘Schar’ had earlier been used by Sophus Lie to mean simply a set/class/collection of elements — see Wussing 1969, pp. 218, 220). When the study of such objects was taken up by Sushkevich (1937b), he elected to translate this into Russian as *gruda* (*груда*), meaning ‘heap’ or ‘pile’. Sushkevich was evidently exploiting the phonetic similarity between ‘gruda’ and the Russian word for group, ‘gruppa’ (‘группа’). Wagner adopted Sushkevich’s terminology and expanded it by inventing the terms *polugruda* (*полугруда*) and *obobshchennaya gruda* (*обобщенная груда*), which have been translated accordingly as *semiheap* and *generalised heap*, respectively. Schein (1979), however, coined a new English term, *groud*, and therefore referred also to *semigroups* and *generalised groups*. He commented:

The advantages of the new term are that it is not overloaded semantically, it is phonetically similar to “group”, and it is more euphonious than “heap”. (Schein, 1979, pp. 101–102)

Nevertheless, he did acknowledge that ‘groud’ is “not as appealing to the mathematical imagination” as ‘heap’ (Schein, 1992, p. 207). He noted further that, unlike ‘heap’, ‘groud’ “has no connotations” (Schein, 1981, pp. 194–195), these connotations presumably being the implication of lack of structure. However, apart from the possible ‘political’ issue of not wanting to put people off one’s research through use of unattractive terminology, I do not feel that these connotations are of any significance: in mathematics, a word is simply taken to mean what we define it to mean. Why should the word ‘group’ imply any kind of structure? Why should it imply any more structure than the word ‘set’? It only does so because we define it so. For this reason, I choose to retain the term ‘heap’ for Sushkevich’s ‘gruda’. Moreover, I eschew the word ‘groud’ on aesthetic grounds. Despite its intended phonetic similarity to ‘group’, whenever I see ‘groud’, I want to pronounce it ‘growd’ (graʊd). In French, Behanzin (1958) used the terms *amas*, *demi-amas* and *amas généralisé* (‘amas’ = heap, pile). Bruck (1958, p. 40) noted that other English names that have been used for a heap are ‘flock’, ‘imperfect brigade’ (see page 307), and ‘abstract coset’. This last name is explained by the observation that if we take a group  $G$  and define in it the ternary operation  $[x y z] = xy^{-1}z$  in order to obtain a heap, then  $H \subseteq G$  forms a subheap of  $G$  if and only if  $H$  is a coset of some subgroup of  $G$ . Finally, as if the above plethora of names were not enough, Bertram and Kinyon (2010) also record the terms ‘torsor’, ‘herd’, ‘principal homogeneous space’, and ‘pregroup’. Brzeziński and Vercruyse (2009) favour ‘herd’, which gives them an excuse to introduce the terms ‘shepherd’ and ‘pen’ for related concepts.

<sup>33</sup>“Задачей настоящей работы является построение абстрактной теории обобщенных гroud и обобщенных групп в их взаимной связи.” (Wagner, 1953, p. 549).

<sup>34</sup>“... понятие обобщенной груды и обобщенной группы, тесно связанные между собой, возникают не в результате чисто формальных обобщений каких-либо известных алгебраических теорий, а в результате применения алгебраических методов к изучению важных теоретико-множественных операций, связанных с рассмотрением ... частичных преобразований множеств. При этом построение абстрактной теории обобщенных гroud и обобщенных групп так же целесообразно, как и построение абстрактной теории групп, которая возникла аналогичным образом из теории групп преобразований.” (Wagner, 1953, p. 549).

<sup>35</sup>Schein’s candidate dissertation was *Абстрактная теория полугрупп взаимно однозначных преобразований* (*Abstract theory of semigroups of one-one transformations*) (Schein, 1962a). He also submitted a doctoral dissertation, *Relation algebras* (*Алгебры отношений*), some years later. However, the degree was never awarded, although the results eventually found their way into print in other ways. Being from a Jewish background, Schein suffered from Soviet institutional anti-Semitism (see the references in note 51 of Chapter 2 and also the comments on page 239): his doctoral dissertation was unfairly rejected as containing “a large number of uninteresting theorems and extremely cumbersome formulations” (Freiman, 1980, p. 76). This was in direct contradiction to the glowing appraisals provided by Wagner, Gluskin, and Kurosh. Indeed, Wagner described the dissertation in the following terms:

The dissertation is a fundamental piece of research, of an unusual richness of content, which contains a large quantity of major results. This important scientific work is a valuable contribution to modern algebra and establishes its author as a talented scientist. (Freiman, 1980, p. 75)

For more information on anti-Semitism in Soviet mathematics, see Freiman (1980). Like the work of Zhores A. Medvedev that was used in Section 2.1, the Freiman book cited here is an example of ‘tamizdat’ — see note 18 of Chapter 2.

### Section 10.5. Gordon B. Preston

<sup>36</sup>For biographies of Preston, see Howie (1995a) and Hall (1991); see also the autobiographical article Preston (1991).

### Section 10.6. Preston and inverse semigroups

<sup>37</sup>OP-20-G representatives at GC&CS’, The US National Archives and Records Administration (NARA), College Park, RG 457, Historic Cryptographic Collection, Box 808, NR 2336, CBLL51 entitled: “BRITISH COMMUNICATIONS INTELLIGENCE”.

<sup>38</sup>In an interview about his time at Bletchley Park, another American mathematician and cryptologist, Howard Campaigne, commented: “. . . we had a liaison officer who was Al Clifford at the time, but I was over there, not as liaison but as a working member . . .” (Farley, 1983, p. 17). The contrast that is being drawn here suggests that Clifford was not active as a cryptologist while at Bletchley and therefore may not have come into contact with the people mentioned above, such as Preston. However, Clifford is mentioned in passing in the reminiscences of Peter Hilton, who recalled the “happy and relaxed cooperation” that the British code-breakers enjoyed with “several American cryptanalysts” (Hilton, 1988, p. 298). Furthermore, Jack Good (1993, p. 160) recalled receiving a book as a gift while at Bletchley Park and that this book was signed by a number of people, including Clifford, which would seem to suggest that Clifford was part of the social life at Bletchley and, moreover, that he came into contact with people (namely Good) from the Newmanry. I end this note with an intriguing comment from Shaun Wylie (2011, p. 603):

Several analysts were seconded to us from the US Army and one from the US Navy; we also had highly professional advice at our tea parties [discussion sessions] from a US liaison officer.

However, Wylie did not name the officer.

<sup>39</sup>For other notions of ‘normal subsemigroup’, see Sections 8.6 and 9.2, as well as note 33 of Chapter 9.

<sup>40</sup>The fact that (M1) and (M2) are independent has given rise to two different approaches to the generalisation of inverse semigroups. The first, in which (M1) is retained but (M2) is dropped, is the study of regular semigroups, on which, see Section 8.6. The second approach deals with semigroups in which idempotents commute; for a discussion of some of the classes of semigroups which come under this study, see Hollings (2009a). See also Ren and Shum (2012).

<sup>41</sup>Representations of this type went on to be studied by Munn: see Howie (1995b, §5.4) and Fountain (2010, §2).

## Chapter 11. Matrix Representations of Semigroups

<sup>1</sup>A forthcoming article by Stanislav Kublanovskii and Eugenia Mogilyanskaya will provide more details on Ponizovskii’s life.

### Section 11.1. Sushkevich on matrix semigroups

<sup>2</sup>See note 5 of Chapter 3.

<sup>3</sup>The main Ukrainian text reads (Sushkevich, 1937d, p. 83): “As our *elements* we take pairs of vectors  $a$  and  $a'$  together with a scalar factor  $\alpha$  from  $P$ .” (“За наші *елементи* ми вважаємо пари векторів  $a$  і  $a'$  разом із скалярним множителем  $\alpha$  із  $P$ .”) The French summary at the end of the paper, however, is a little more specific about  $\alpha$ : “. . . ‘the scalar factors’  $\alpha$  are elements of  $P$  or square roots of its elements.” (“. . . ‘les facteurs scalaires’  $\alpha$  sont des éléments de  $P$  ou des racines carrées de ces éléments.”)

### Section 11.2. Clifford on matrix semigroups

<sup>4</sup>“Verf. betrachtet die Darstellung der Gruppen ohne Gesetz der eindeutigen Umkehrbarkeit, insbesondere der Kerngruppen, die Verf. schon in einer früheren Note [reference] behandelt hat. Zur Darstellung kommen Matrizen in Frage, deren Rng [*sic*] kleiner ist als Ihre Ordnung.” (JFM 59.0145.03).

<sup>5</sup>Evidence for this may be found in surviving letters from Munn to Preston of the mid-1950s (as cited in Sections 1.2, 11.3, and 12.1.3), in which correspondence with Clifford is mentioned.

### Section 11.3. W. Douglas Munn

<sup>6</sup>The details of this section are drawn from Howie (1999) and Reilly (2009), as well as from private correspondence with Professor Munn. For several short obituaries, see Churchhouse (2009), Duncan (2008a,b), Hickey (2008, 2009), and Howie (2008).

<sup>7</sup>Private communication, 17 June 2008.

<sup>8</sup>Private communication, 2 July 2008.

<sup>9</sup>*Ibid.*

### Section 11.4. The work of W. D. Munn

<sup>10</sup>Concerning radicals in semigroups, see also the comments on pages 186, 192, 196, and 211 in connection with the work of Hoehnke, Šulka, Schwarz, and Green, respectively. For a survey on radicals in semigroups, see Clifford (1970) or Roĭz and Schein (1978); see also Clifford and Preston (1967, §11.6).

<sup>11</sup>Munn also reported in this letter that he had, in addition, received a preprint of the paper Thrall (1955), which concerns a particular class of matrix algebras — this is presumably the paper that Munn was referring to in the introduction to his thesis when he stated, in connection with certain matters arising from the study of semigroup algebras, that

[t]he author has learned from a private communication that a similar situation, arising in a different context, has recently been investigated by Professor R. M. Thrall; this work is to be published shortly. (Munn, 1955a, p. ii)

Thrall's work was rooted in that of W. P. Brown (1955) on matrix algebras which arise in connection with orthogonal groups; a short account of these may be found in the proceedings of the 1954 Amsterdam ICM (Brown, 1957). Although the algebra studied by Brown was, in Munn's words, "pretty nearly a semigroup algebra" (letter to Preston, 30 January 1955), the work of Brown and Thrall has been omitted from the account of semigroup algebras given here since, for the most part, both authors studied their algebras *qua* algebras, without indicating whether, or how, they might be obtained as the algebras of semigroups.

<sup>12</sup>Recall from Section 9.5 that Lyapin's student Lesokhin (1958) also developed some character theory for semigroups, as did Schwarz (1954a,b,c), who spoke about this work at the 1954 Amsterdam ICM (Schwarz, 1957), at which congress both Hewitt and Munn were present.

<sup>13</sup>This is stated explicitly in a letter from Munn to Preston, dated 12 March 1955.

### Section 11.5. The work of J. S. Ponizovskii

<sup>14</sup>I have deviated here from the transliteration conventions set down in the preface by writing 'Йосиф' as 'Josif' — this is to reflect the fact that in his papers in English, Ponizovskii usually, though by no means always, styled himself 'J. S. Ponizovskii'.

<sup>15</sup>"После того как рукопись настоящей статьи была отослана в редакцию «Математического сборника», автору стала известна статья Мунна, опубликованная в Proc. of Cambr. Phil. Soc. . . . Статья Мунна содержит наиболее существенные предложения . . . а также некоторые другие утверждения . . . нашей статьи. Следует отметить, однако, что все результаты нашей статьи . . . получены автором в 1952 г. и в июне 1953 г., за полтора года до опубликования статьи Мунна; по этим результатам автором была защищена кандидатская диссертация . . ." (Ponizovskii, 1956, p. 241).

<sup>16</sup>"Автор выражает глубокую благодарность Е. С. Ляпину за предложенную задачу и ценные советы в процессе ее решения." (Ponizovskii, 1956, p. 241).

<sup>17</sup>For an account of Ponizovskii's work that has been rephrased in terms of semisimplicity of semigroup algebras, see Hewitt's summary for *Mathematical Reviews* (MR0081292).

## Chapter 12. Books, Seminars, Conferences, and Journals

### Section 12.1. Monographs

#### Section 12.1.1. Sushkevich's *Theory of generalised groups* (1937)

<sup>1</sup>"Настоящая монография представляет собой, быть может, первое по времени, связанное изложение теории всех типов обобщенных групп. Сюда вошли как мои собственные исследования . . . так и исследования других математиков, посвященные обобщенным группам." (Sushkevich, 1937b, p. 3).

<sup>2</sup>It has been stated (see, for example, Schein 1994, p. 397) that very few copies of this book remain, the majority having been destroyed in the fighting over Kharkov during the Second World War. Indeed, I have played a role in perpetuating this idea (see Hollings 2009c). However, in the years since writing the article cited here, I have become much less convinced that this is the case: the libraries that I used on a visit to Kharkov appeared to retain many books from the relevant

period. Moreover, there are many copies of Sushkevich's book circulating within the thriving online trade in Russian second-hand books. At any rate, the tools of our electronic age obviate the problems of accessibility: a scanned version of the book is freely available online.

<sup>3</sup>“Это объясняется тем, что группы без закона однозначной и неограниченной обратимости разработаны в математической литературе подробнее всех других типов обобщенных групп, и теория их приведена в некоторых своих частях к известной законченности.” (Sushkevich, 1937b, p. 3).

<sup>4</sup>“кроме общей математической культуры” (Sushkevich, 1937b, p. 3).

<sup>5</sup>“знакомство с классической теорией обычных групп” (Sushkevich, 1937b, p. 3).

<sup>6</sup>“Настоящий труд предназначен для всех любителей групп, начиная от студентов старших курсов физматов и кончая квалифицированными математиками.” (Sushkevich, 1937b, p. 3).

<sup>7</sup>“анализ общих законов действия” (Sushkevich, 1937b, p. 39).

<sup>8</sup>“главные законы действия и их обобщения” (Sushkevich, 1937b, p. 42).

<sup>9</sup>“К обобщенным гурдам Бера можно причислить и „бригады“, рассматриваемые Sorgal'em, „бригадою“ Sorgal называет совокупность подстановок  $n$  символов, имеющую то свойство, что, если  $A, B, C$  три любые (не непременно различные) подстановки этой совокупности, то к ней же принадлежит и подстановка  $ABC$  (в случае так называемой „совершенной“ бригады) или подстановка  $AB^{-1}C$  (в случае „несовершенной“ бригады).” (Sushkevich, 1937b, p. 171).

<sup>10</sup>If such was indeed the case: see note 2.

### Section 12.1.2. Lyapun's *Semigroups* (1960)

<sup>11</sup>“В конце 50-х годов стало очевидным, что теория полугрупп выросла в самостоятельную область общей алгебры, имеющую свои задачи и методы. Появились монографик, посвященные этой теории. Среди них первая в мировой литературе книга Е. С. Ляпина «Полугруппы», сыгравшая значительную роль в развитии теории полугрупп. В опубликованных к этому времени работах о полугруппах был заметный разнобой в подходах к изучению, методах, системах построений и терминологии. В книге Ляпина впервые в связной форме излагались основные направления алгебраической теории полугрупп, выдвигались общие точки зрения и намечались некоторые перспективы развития.” (Gluskin, 1968, p. 324).

<sup>12</sup>“важные результаты” (Lyapun, 1953b, p. 302).

<sup>13</sup>Varieties of algebras were introduced by Garrett Birkhoff (1935), while the study of group varieties was initiated by V. H. Neumann in his 1935 Cambridge PhD thesis *Identical relations in groups* (published as Neumann 1937). On varieties of groups, see the article Neumann (1967a) or the monograph Neumann (1967b). In connection with semigroup varieties, see also the work of Pollák, cited on page 208.

<sup>14</sup>This may have been connected with the paper shortages mentioned in note 42 of Chapter 9.

<sup>15</sup>“Die Behandlungsweise ist bis zum Ende ganz einfach, leicht lesbar, manchmal vielleicht ein wenig ausführlich.” (Zbl 0100.02301).

<sup>16</sup>“Schade, daß Verf. die mit den behandelten Stoffteilen zusammenhängenden wichtigsten, noch offenen Probleme nicht darlegt, die zugleich die Richtung für die weiteren Forschungen zeigen könnten.” (Zbl 0100.02301).

<sup>17</sup>“тогда только зарождавшейся” (Gluskin, 1961a, p. 248).

<sup>18</sup>“не охватывающие даже содержания упомянутой выше книги А. К. Сушкевича” (Gluskin, 1961a, p. 248).

<sup>19</sup>“где впервые в мировой литературе дана систематизация большого материала по теории полугрупп” (Gluskin, 1961a, p. 248).

<sup>20</sup>“назрела необходимость” (Gluskin, 1961a, p. 248).

<sup>21</sup>“Автор монографии — один из ведущих математиков в области теории полугрупп, и, понятно, серьезное место в книге занимает изложение его собственных результатов и результатов его учеников.” (Gluskin, 1961a, p. 249).

<sup>22</sup>“книга богато иллюстрирована примерами полугрупп преобразований” (Gluskin, 1961a, p. 249).

<sup>23</sup>“В целом книга окажется весьма полезной как для алгебраистов, так и для других математиков, которым в их исследованиях приходится встречаться с полугруппами.” (Gluskin, 1961a, p. 250).

### Section 12.1.3. Clifford and Preston's *The algebraic theory of semigroups* (1961, 1967)

<sup>24</sup>Indeed, the first paragraph of Clifford and Preston's volume 1 might be regarded as the origin of the present book: the references there to de Séguier, Dickson, and Sushkevich were the starting point for my investigation of the development of semigroup theory.

<sup>25</sup>“Авторы монографии — известные специалисты по теории полугрупп, обогатившие ее рядом первоклассных достижений. Профессор А. Клиффорд — американский алгебраист, являющийся одним из пионеров теории полугрупп . . . Представитель более молодого поколения английский алгебраист профессор Г. Престон, ныне живущий в Австралии, известен своими важными работами по инверсным полугруппам.” (Clifford and Preston, 1961, Russian trans., p. 5).

<sup>26</sup>“В развитие теории полугрупп немалый вклад внесли советские алгебраисты. Некоторые результаты советских математиков вошли в книгу . . . Но в общем знакомство авторов с советскими работами по теории полугрупп было недостаточным и исследования, проведенные в СССР, отражены в книге Клиффорда и Престона непропорционально мало.” (Clifford and Preston, 1961, Russian trans., p. 6).

<sup>27</sup>“Мы надеемся, что этот перевод будет так же хорошо встречен советскими математиками, как английский перевод книги Е. С. Ляпина «Полугруппы» — в западных странах. Эти две работы скорее дополняют, нежели дублируют одна другую; книга профессора Е. С. Ляпина охватывает более широкий материал, в нашей книге более детально изложены некоторые темы.” (Clifford and Preston, 1961, Russian trans., p. 9).

### Section 12.1.4. Other books

<sup>28</sup>The list of semigroup-related textbooks and monographs given in this subsection is, of course, far from exhaustive. Aside from those already mentioned (both here and in earlier chapters), we also have Petrich (1974, 1977), Lallement (1979), Higgins (1992), Grillet (1995a), and Rhodes and Steinberg (2009). Further references may be found in the preface to Howie (1995b).

<sup>29</sup>In their preface, Petrich and Reilly (1999, p. ix) describe their text as “one of the proud grandchildren of Sushkevich's book”.

### Section 12.2. Seminars on semigroups

<sup>30</sup>Karl H. Hofmann, private communication, 24th November 2012. Indeed, some NSF funding also seems to have been available in the late 1950s, for this funded (at least part of) Preston's visit to Tulane in 1956–1958 (Section 12.1.3) — see, for example, the acknowledgement in Preston (1959).

### Section 12.3. Czechoslovakia, 1968, and *Semigroup Forum*

<sup>31</sup>The following is a decidedly non-exhaustive list of conferences on (algebraic) semigroups and related topics, up to 2013. More general algebraic conferences are omitted, as are general conferences with semigroup splinter sessions, such as the British Mathematical Colloquium, or the International Congress of Mathematicians (thus, for instance, the 1970 Nice semigroup conference mentioned in Section 12.2 is omitted from this list); a loose criterion for inclusion in the list is that the conference contain the word ‘semigroup’ in its title. However, some conferences of a more computational or automata- and formal language-related nature have been omitted. Details of each conference are given in as abbreviated a form as possible: location, month, year, reference (‘reference’ is an appropriate published source on the conference: announcement, report, or proceedings). The title of a conference and fuller details are given only in those cases where no reference is available.

- (1) Smolenice, Czechoslovakia, June 1968 (Bosák 1968; Lyapin and Shevrin 1969; Hofmann 1995).
- (2) Detroit, Michigan, USA, June 1968 (Folley, 1969).
- (3) Sverdlovsk, USSR, February 1969 (Shevrin, 1969b).
- (4) Szeged, Hungary, August/September 1972 (Anon, 1972b).
- (5) DeKalb, Illinois, USA, February 1973 (Anon, 1973).
- (6) Szeged, Hungary, August 1976 (Pollák, 1979).
- (7) Sverdlovsk, USSR, June 1978 (Shevrin, 1979).
- (8) New Orleans, Louisiana, USA, September 1978 (Nico, 1979).

- (9) Oberwolfach, FRG, December 1978 (Jürgensen *et al.*, 1981).
- (10) Melbourne, Australia, October 1979 (Hall *et al.*, 1980).
- (11) Oberwolfach, FRG, May 1981 (Hofmann *et al.*, 1983).
- (12) Szeged, Hungary, August 1981 (Anon, 1982; Pollák *et al.*, 1985).
- (13) Siena, Italy, October 1982 (Migliorini, 1983).
- (14) Milwaukee, Wisconsin, USA, September 1984 (Byleen *et al.*, 1985).
- (15) Greifswald, GDR, November 1984 (Hoehnke, 1985).
- (16) Oberwolfach, FRG, February/March 1986 (Jürgensen *et al.*, 1988).
- (17) Baton Rouge, Louisiana, USA, March 1986 (Koch and Hildebrandt, 1986).
- (18) Chico, California, USA, April 1986 (Goberstein and Higgins, 1987).
- (19) Kariavattom, Trivandrum, India, July 1986 (Nambooripad *et al.*, 1985).
- (20) Lisbon, Portugal, June 1988 (Almeida *et al.*, 1990).
- (21) Berkeley, California, USA, July/August 1989 (Rhodes, 1991).
- (22) Melbourne, Australia, July 1990 (Hall *et al.*, 1991).
- (23) Oberwolfach, FRG, July 1991 (Howie, 1992; Howie *et al.*, 1992).
- (24) Luino, Italy, June 1992 (Lallement 1993; Bonzini *et al.* 1993).
- (25) Qingdao, China, May 1993 (Shum and Zhou, 1992).
- (26) York, UK, August 1993 (Fountain, 1995).
- (27) Colchester, UK, August 1993 (Higgins, 1993).
- (28) Hobart, Australia, January 1994 (Trotter, 1993).
- (29) New Orleans, Louisiana, USA, March 1994 (Hofmann and Mislove, 1996).
- (30) Kočovce, Slovakia, May 1994 (Grošek and Satko, 1995).
- (31) Amarante, Portugal, June 1994 (Almeida, 1994).
- (32) Saint Petersburg, Russia, June 1995 (Ponizovskii, 1994a).
- (33) Kunming, China, August 1995 (Shum *et al.*, 1998).
- (34) Prague, Czech Republic, July 1996 (Demlova, 1995).
- (35) Tartu, Estonia, August 1996 (Kilp, 1996).
- (36) St Andrews, UK, July 1997 (Ruškuc and Howie, 1996).
- (37) Lincoln, Nebraska, USA, May 1998 (Birget *et al.*, 1998).
- (38) Braga, Portugal, June 1999 (Smith *et al.*, 2000).
- (39) Lisbon, Portugal, November 2002 (Araújo *et al.*, 2004).
- (40) Lisbon, Portugal, July 2005 (André *et al.*, 2007).
- (41) Fountainfest: “Semigroups, categories and automata” — A conference celebrating John Fountain’s 65th birthday and his mathematical achievements, University of York, UK, 12–14 October 2006.
- (42) Tartu, Estonia, June 2007 (Laan *et al.*, 2008).
- (43) Workshop on Groups, Semigroups and Applications (A day to remember W. Douglas Munn on the occasion of his 80th birthday), Centro de Álgebra da Universidade de Lisboa, Portugal, 24 April 2009.
- (44) International Conference on Geometrical and Combinatorial Methods in Group Theory and Semigroup Theory, Department of Mathematics, University of Nebraska, Lincoln, USA, 17–21 May 2009.
- (45) Porto, Portugal, July 2009 (Costa *et al.*, 2011).
- (46) Groups and Semigroups: Interactions and Computations, Faculdade de Ciências, Universidade de Lisboa, Portugal, 25–29 July 2011.
- (47) Workshop on Semigroups (To remember John M. Howie on the occasion of his 76th birthday), Centro de Álgebra da Universidade de Lisboa, Portugal, 23–25 May 2012.
- (48) Semigroups and Applications, Department of Mathematics, Uppsala University, 30 August – 1 September 2012.
- (49) Workshop on Semigroup Representations, International Centre for Mathematical Sciences, Edinburgh, UK, 10–12 April 2013.
- (50) The 4th Novi Sad Algebraic Conference in conjunction with a Workshop on Semigroups and Applications, Novi Sad, Serbia, 5–9 June 2013.
- (51) International Conference on Geometric, Combinatorial and Dynamics aspects of Semigroup and Group Theory on the occasion of the 60th birthday of Stuart Margolis, Bar Ilan, Israel, 11–14 June 2013.



### Section 12.3.1. The first international conference

<sup>32</sup>See items (9), (11), (16), and (23) in the list in note 31.

<sup>33</sup>The expository lectures from the conference were published in the book Arbib (1968); many of the remaining lectures were published in other places — see the appendix of Arbib (1968) for details.

<sup>34</sup>Private communication, 15 September 2010.

<sup>35</sup>The brief account of the conference that is given here is drawn principally from the report published by Lyapin and Shevrin (1969) in *Uspekhi matematicheskikh nauk* the following year, together with some details kindly supplied by Paul Mostert. See Bosák (1968) for another report of the conference. It should be noted, however, that the reports Lyapin and Shevrin (1969) and Bosák (1968) differ on some details. For a definitive account of the conference, we must await that being prepared by Mostert to mark its 45th anniversary.

<sup>36</sup>Paul S. Mostert, private communication, 16 January 2013.

<sup>37</sup>Lyapin and Shevrin (1969) indicated that the lengths of lectures were 60 and 30 minutes, although the conference programme gives the timings as 45 and 30 minutes. Paul S. Mostert recalls that the “talks nearly always went over the allotted time” (Paul S. Mostert, private communication, 16 January 2013).

<sup>38</sup>Paul S. Mostert, private communication, 17 January 2013.

### Section 12.3.2. A dedicated journal

<sup>39</sup>Paul S. Mostert, private communication, 15 September 2010.

<sup>40</sup>*Ibid.*

# Bibliography

In order to save space, the titles of certain frequently cited journals are given in a highly abbreviated form: a key to these abbreviations appears below.

An (R) in a bibliographic reference indicates that a source is in Russian.

## List of abbreviations of journal titles

AHES	<i>Archive for History of Exact Sciences</i>
AJM	<i>American Journal of Mathematics</i>
AM	<i>Annals of Mathematics</i>
AMM	<i>The American Mathematical Monthly</i>
AMST	<i>American Mathematical Society Translations</i>
ASM	<i>Acta Scientiarum Mathematicarum (Szeged)</i>
BAMS	<i>Bulletin of the American Mathematical Society</i>
BLMS	<i>Bulletin of the London Mathematical Society</i>
BSMF	<i>Bulletin de la Société mathématique de France</i>
CJM	<i>Canadian Journal of Mathematics</i>
CMJ	<i>Czechoslovak Mathematical Journal</i>
CMZ	<i>Chekhoslovatskii matematicheskii zhurnal</i>
CR	<i>Comptes rendus hebdomadaires des séances de l'Académie des sciences de Paris</i>
DAN	<i>Doklady Akademii nauk SSSR</i>
HM	<i>Historia Mathematica</i>
IAN	<i>Izvestiya Akademii nauk SSSR. Seriya matematicheskaya</i>
IVUZM	<i>Izvestiya vysshikh uchebnykh zavedenii. Matematika</i>
JA	<i>Journal of Algebra</i>
JGTU	<i>Journal of Sciences of the Gakugei Faculty, Tokushima University</i>
JLMS	<i>Journal of the London Mathematical Society</i>
JRAM	<i>Journal für die reine und angewandte Mathematik</i>
KMSR	<i>Kōdai Mathematical Seminar Reports</i>
MA	<i>Mathematische Annalen</i>
MFC	<i>Matematicko-fyzikálny časopis. Slovenská akadémia vied</i>
MM	<i>Mathematics Magazine</i>
MN	<i>Mathematische Nachrichten</i>
MS	<i>Matematicheskii sbornik</i>
MSl	<i>Mathematica Slovaca</i>
MZ	<i>Mathematische Zeitschrift</i>
OMJ	<i>Osaka Mathematical Journal</i>
PAMS	<i>Proceedings of the American Mathematical Society</i>
PCPS	<i>Proceedings of the Cambridge Philosophical Society</i>
PJA	<i>Proceedings of the Japan Academy</i>
PLMS	<i>Proceedings of the London Mathematical Society</i>
PNAS	<i>Proceedings of the National Academy of Sciences of the USA</i>
QJM	<i>Quarterly Journal of Mathematics, Oxford</i>
RMS	<i>Russian Mathematical Surveys</i>
SCD	<i>Séminaire Châtelet–Dubreil; partie complémentaire: demi-groupes</i>
SF	<i>Semigroup Forum</i>
SD	<i>Séminaire Dubreil. Algèbre et théorie des nombres</i>
SMD	<i>Soviet Mathematics: Doklady</i>
SMJ	<i>Siberian Mathematical Journal</i>
SMZ	<i>Sibirskii matematicheskii zhurnal</i>
SKMO	<i>Soobshcheniya Kharkovskogo matematicheskogo obshchestva</i>
TAMS	<i>Transactions of the American Mathematical Society</i>
UMN	<i>Uspekhi matematicheskikh nauk</i>
UZLGPI	<i>Uchenye zapiski Leningradskogo gosudarstvennogo pedagogicheskogo instituta</i>
ZKMO	<i>Zapiski Kharkovskogo matematicheskogo obshchestva</i>

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