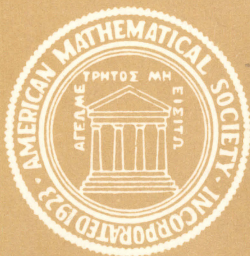


Notices

OF THE
AMERICAN
MATHEMATICAL
SOCIETY



VOLUME 14, NUMBER 5

ISSUE NO. 99

AUGUST, 1967

Notices

OF THE

AMERICAN MATHEMATICAL SOCIETY

Edited by Everett Pitcher and Gordon L. Walker

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MEETINGS

Calendar of Meetings

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

Meet- ing No.	Date	Place	Deadline for Abstracts*
649	October 28, 1967	Cambridge, Massachusetts	Sept. 14
650	November 17-18, 1967	Knoxville, Tennessee	Oct. 3
651	November 18, 1967	Albuquerque, New Mexico	Oct. 3
652	November 25, 1967	Urbana, Illinois	Oct. 3
653	January 23-27, 1968 (74th Annual Meeting)	San Francisco, California	Dec. 1
	August 26-30, 1968 (73rd Summer Meeting)	Madison, Wisconsin	
	January 23-27, 1969 (75th Annual Meeting)	New Orleans, Louisiana	
	August 25-29, 1969 (74th Summer Meeting)	Eugene, Oregon	
	January 22-26, 1970 (76th Annual Meeting)	Miami, Florida	

* The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadline dates for the by title abstracts are September 7, and September 26, 1967.



The *Notices* of the American Mathematical Society is published by the Society in January, February, April, June, August, October, November and December. Price per annual volume is \$12.00. Price per copy \$2.00. Special price for copies sold at registration desks of meetings of the Society, \$1.00 per copy. Subscriptions, orders for back numbers (back issues of the last two years only are available) and inquiries should be addressed to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904.

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Seventy-Second Summer Meeting
University of Toronto
Toronto, Ontario, Canada
August 29 - September 1, 1967

PROGRAM

The American Mathematical Society will hold its seventy-second summer meeting at the University of Toronto, Toronto, Canada, from Tuesday, August 29, through Friday, September 1, 1967. All sessions will be held in lecture rooms and classrooms of the university.

Professor Samuel Eilenberg of Columbia University will deliver the Colloquium Lectures on "Universal algebras and the theory of automata" at 2:00 p.m. on Tuesday and at 11:00 a.m. on Wednesday, Thursday, and Friday.

By invitation of the Committee to Select Hour Speakers for Summer and Annual Meetings, there will be two invited addresses. Professor David Gale of the University of California, Berkeley, will speak on "A mathematical theory of optimal economic development" at a joint session with the Econometric Society at 9:30 a.m. on Wednesday. Professor Michael Artin of the Massachusetts Institute of Technology will speak on "The topology of algebraic varieties" at 9:30 a.m. on Friday.

The Colloquium Lectures and invited addresses will be presented in Convocation Hall.

There will be sessions for contributed papers on Tuesday at 3:15 p.m., on Thursday at 9:00 a.m. and 2:00 p.m., and on Friday at 2:00 p.m. on the second floor of Sidney Smith Hall and on the first floor of the New Physics Building.

This meeting will be held in conjunction with meetings of several other organizations. The Mathematical Association of America and the Econometric Society will meet from Monday through Wednesday. The Association will sponsor the Hedrick Lectures by Professor Gian-

Carlo Rota of Rockefeller University on "Combinatorial analysis as a theory." At 8:00 p.m. on Wednesday the Society for Industrial and Applied Mathematics will present the von Neumann Lecture by Professor C. C. Lin of the Massachusetts Institute of Technology on "Some aspects of stellar dynamics and galactic structure." The Biennial Meeting of the Canadian Mathematical Congress will be held from Monday through Saturday. The Presidential Address to the Canadian Mathematical Congress will be delivered at 10:10 a.m. on Monday by Professor H. S. M. Coxeter on "The problem of Apollonius." In joint sessions of the Mathematical Association of America and the Canadian Mathematical Congress, Professor Joseph Lehner of the University of Maryland will speak on "The Picard theorems" at 11:10 a.m. on Monday, and Professor Pierre Samuel of the University of Paris will speak on "Unique factorization" at 11:00 a.m. on Tuesday. Pi Mu Epsilon and Mu Alpha Theta will meet concurrently with the Association.

COUNCIL AND BUSINESS MEETINGS

The Council of the Society will meet at 5:00 p.m. on Tuesday, August 29, in the University Room of the Park Plaza Hotel. The Business Meeting of the Society will be held on Thursday, August 31, at 10:15 a.m. in Convocation Hall. An amendment to the by-laws has been recommended by the Council and will be presented to the Society for action. The amendment accomplishes two things. First, it changes the terms of members of the Publications Committee of the Proceedings and the Transactions from three to four years.

Second, it describes the length of terms in a transitional period until the rotation with four-year terms is established.

REGISTRATION

The registration desk will be in the lobby of Sidney Smith Hall. It will be open on Sunday, August 27, from 2:00 p.m. to 8:00 p.m.; on Monday, August 28, from 8:00 a.m. to 5:00 p.m.; on Tuesday through Thursday, August 29 through August 31, from 9:00 a.m. to 5:00 p.m.; and on Friday, September 1, from 9:00 a.m. to 1:00 p.m.

The registration fees will be as follows:

Member	\$2.00
Member's family	0.50

for the first such registration and no charge for additional registrations.

Students	No charge
Others	\$5.00

EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register will be in Sidney Smith Hall on the first floor in Rooms 1070, 1072, 1074, 1084, 1086, and 1088. It will be open Tuesday through Thursday, August 29 through August 31, from 9:00 a.m. to 5:00 p.m. on each of the three days. Attention is invited to the announcement of the Employment Register on page 351 of the April issue of the Notices, in particular to the deadline dates for application and to the necessity for prompt registration at the Employment Register desk by both applicants and employers.

EXHIBITS

The exhibits will be displayed on the first floor of Sidney Smith Hall on Tuesday through Thursday, August 29-31, from 9:00 a.m. to 5:00 p.m. In addition to individual book and educational media, the following exhibits will also be displayed: free literature, joint book, joint educational media, joint journal, and poster.

BOOK SALE

Books published by the Society will be sold for cash prices somewhat below the usual prices when these books are sold by mail on invoice. In addition, there

will be a number of slightly damaged copies with bent bindings and other minor defects for sale at 25% of list price.

DORMITORY HOUSING

Dormitory rooms will be available. Married guests with or without children will be assigned to Victoria College and St. Michael's College. Others will be assigned to New College, University College, and Trinity College. Glendon Hall and the main York University campus will also be used if registration is high. Glendon Hall is approximately one-half hour drive from the University of Toronto, and the main York campus is approximately three-quarter hour drive away. A regular bus service will be provided free of charge.

A reservation form for dormitory and hotel accommodations was on page 570 of the June *Notices*. Those who have not yet made reservations are advised to do so immediately as space is limited.

Rooms will be available from 10:00 a.m. on Saturday, August 26, to 10:00 a.m. on Saturday, September 2. Bed linen, towels, and room cleaning service are provided. Soap is not provided. The rates are \$5.00 per day per person. There are a few rooms available in local Student Co-ops at \$12 per week per person. Cribs (available on special request only) are \$7.00 per week. Bedding for cribs will not be provided. High chairs will be available also for \$4.00 per week.

Upon arrival on campus all guests should check in at Sidney Smith Hall to register for dormitory rooms and receive housing information. Guests are urged to arrive on the campus during the normal registration hours. However, there will be members of the housing staff on duty 24 hours a day to issue dormitory keys to persons who have advance reservations. Payment for rooms should be made upon departure directly to the person in charge of each dormitory.

FOOD SERVICE

Meals will be served in the cafeterias maintained by some of the various dormitories. The service will be cafeteria style on a cash basis, and the hours for meals are as follows:

Breakfast 7:30 a.m. - 9:00 a.m.
Lunch 11:45 a.m. - 2:00 p.m.
Dinner 5:15 p.m. - 6:45 p.m.

The snack bar in New College will be open from 8:30 a.m. to 4:30 p.m., and the Refectory in University College will be open from 8:00 a.m. to 3:00 p.m.

A list of restaurants in Toronto will be available at the registration desk in Sidney Smith Hall.

HOTELS AND MOTELS

There are a number of hotels and motels in Toronto. Some of them are listed below with coded information which is explained at the end of the list. Participants are urged to make reservations early if they want off-campus housing as a large number of events are taking place in Toronto at the same time as our meeting and room accommodations will be limited. Rates quoted are minimum and subject to change. Extra person charge is for a roll-away cot. Age limit for children, under which there is no charge, is indicated in parentheses.

CANADIANA MOTOR HOTEL-2

Kennedy Road and Highway 401
Single \$10.00, Double \$14.00
Extra person (14) \$4.00
Code: RT, AC, CL, SP, FP, TV
Tel: 291-171

EXECUTIVE MOTOR HOTEL-3

621 King Street West (at Bathurst)
Single \$9.00, Double \$13.00
Extra persons (14) \$2.50
Code: RT, AC, CL, FP, TV
Tel: 362-7441

HOLIDAY INNS (East)-4. (West)-5

Highway 27 at Burnhamthorpe (West)
Highway 401 at Warden (East)
Single \$10.00, Double \$12.00, Twin \$15.00
Extra person (12) \$2.00
Code: RT, AC, CL, SP, FP, TV
Tels: 621-2121 (West) 293-8170 (East)

KING EDWARD SHERATON-6

37 King Street East
Single \$8.50, Double \$13.50, Twin \$13.50
Extra person, N. C.
Code: RT, E, AC, CL, FP, TV
Tel: 368-7474

ROYAL YORK HOTEL-7

Front and York streets (at Union Station)
Single \$9.50, Double \$13.50, Twin \$13.50
Extra person (14) \$4.00
Code: RT, AC, CL, TV, Parking - \$2.25
for 24 hours
Tel: 368-2511

SKYLINE HOTEL-8

655 Dixon Road (near International Airport)
Single \$10.50, Double \$13.50, Twin \$13.50
Extra person (12) \$4.00
Code: RT, AC, CL, SP, FP, TV
Tel: 244-1711

*WINDSOR ARMS HOTEL-9

22 St. Thomas (at Bay and Bloor)
Single \$7.50, Twin \$13.50
Extra person (12) \$3.50
Code: RT, AC, CL, FP, TV
Tel: 921-5141

YMCA (men only)-10

40 College Street
Single \$4.00, Double \$3.10 per day per
person

THE CONSTELLATION HOTEL-11

900 Dixon Road (at International Airport)
Single \$10.50, Double \$14.50, Twin \$15.00
Extra person (12) \$5.00
Code: RT, AC, CL, SP, FP, TV
Tel: 677-1500

*FOUR SEASONS MOTOR HOTEL-12

415 Jarvis (at Carlton)
Single \$9.50, Double \$13.50, Twin \$15.00
Extra person (12) \$5.00
Code: RT, AC, CL, FP, TV, SP
Tel: 924-6631

INN ON THE PARK-13

Leslie and Eglinton East
Single \$11.50, Twin \$16.50
Extra person (12) \$4.00
Code: RT, AC, CL, SP, FP, TV
Tel: 444-2561

LORD SIMCOE HOTEL-14

150 King Street West (at University Ave.)
Single \$6.50, Double \$10.50
Extra person (12) \$4.00
Code: RT, AC, CL, SP, FP, TV
Tel: 362-1848

SEAWAY TOWERS MOTOR HOTEL-15

2000 Lakeshore Boulevard, West
Single \$11.00, Twin \$15.50
Extra person (12) \$2.00
Code: RT, AC, CL, SP, FP, TV
Tel: 763-4521

WESTBURY HOTEL-16
475 Yonge Street
Single \$10.50, Double \$14.50, Twin \$14.50
Extra person (14) \$3.50
Code: RT, AC, CL, TV, Parking \$2.00 for
24 hours
Tel: 924-0611

*SUTTON PLACE HOTEL-17
Bay and Wellesley
Single \$13.50, Double \$18.50
Extra person (12) \$3.00
Code: RT, AC, SP, CL, FP, TV
Tel: 924-9221

*FORD HOTEL-18
595 Bay Street
Single \$4.50 (without bath) \$6.90 (with bath)
Twin \$9.50 to \$13.50
Code: RT, CL, TV in maximum rate rooms.
AC in \$13.50 rooms
Tel: 366-9911

*Indicates hotels in which blocks of rooms
have been reserved because of their prox-
imity to the university campus.

CODE

RT - Restaurant
E - European plan
AC - Air conditioned
CL - Cocktail lounge
SP - Swimming pool
FP - Free parking
TV - Television

A form for making hotel and motel
accommodations appeared on page 570 of
the June *Notices*.

CAMPING

There are no suitable camping sites
located near the University of Toronto
campus.

ENTERTAINMENT

The tentative plan calls for the fol-
lowing events:

Wednesday Evening. SIAM BEER PARTY
following the von Neumann Lecture.
The beer party will take place in
Hart House and should begin at ap-
proximately 9:30 p.m. Tickets will
be on sale in the registration area
for approximately \$2.00. While it is
requested that tickets be purchased
beforehand, the ticket sale will ex-
tend beyond the Wednesday closing
of registration.

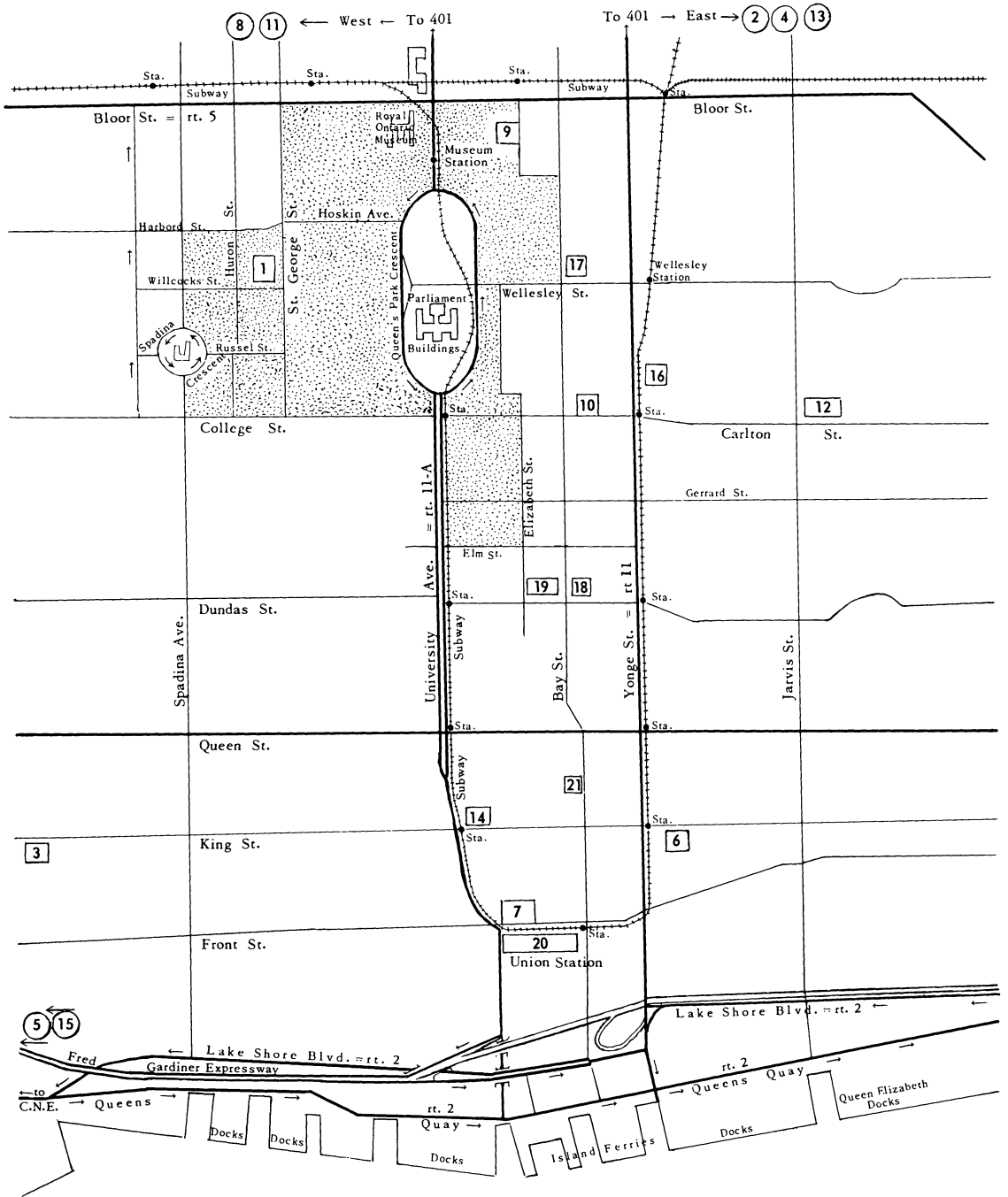
Thursday Evening. The University has
offered the participants a free con-
cert on Thursday evening in Concert
Hall. The artists will be Lorand
Fenyves, violinist, and Bela Siki,
pianist. The concert will begin at
8:30 p.m.

Wednesday and Thursday. All day bus
tours to Niagara. Busses will leave
Sidney Smith Hall at 9:40 a.m. on
each of the two days and return at
approximately 7:45 p.m. Tickets will
be on sale in the registration area,
and the charge will be: adults \$9.45;
children under 12 \$4.75.

In addition there will be a number of
local tours to points of interest in the
Toronto area. The Royal Ontario Museum,
attached to the University of Toronto, is one
of the largest museums in the Common-
wealth and is especially noted for its
Chinese collection. The Art Gallery of
Ontario is also close to the campus. During
the time of the meetings it will be showing
a special Centennial Exhibition of Canadian
art. The Toronto City Hall has attracted
considerable attention with its dramatic
design. Another architectural landmark is
Scarborough College of the University of
Toronto, which has been featured in *Time*
and other magazines. It is at a distance
of about 20 miles from the main campus.
Afternoon bus tours will be arranged.

There will be bus tours to several
other points of interest: Casa Loma,
Toronto's famous castle; David Dunlop Ob-
servatory, the research observatory of the
University; Yorkdale Shopping Centre, "the
largest enclosed shopping center in the
world"; Fort York, a monument of the War
of 1812 when Toronto was overrun by in-
vading Americans; Pioneer Village which
reconstructs the life of early settlers; the
Toronto Stock Exchange; Toronto Police
Headquarters; and local plants. None of
these tours require any advance notice;
participants can apply at the time of reg-
istration.

The Canadian National Exhibition
will be in progress during the week of the
meeting. No special excursions to the Exhi-
bition are planned, but it is easily acces-
sible by public transportation and will
surely interest some of the families of
participants.



- 1. Sidney Smith Hall
- 18. See numbers following the names of the hotels and motels on page 579 of these *Notices*
- 19. Bus terminal
- 20. Railroad station
- 21. Stock Exchange

STRATFORD AND MONTREAL

The rooms that the Committee on Local Arrangements reserved at Loyola College in Montreal have all been applied for, and no more rooms at the college are available.

A block of 80 rooms (50 with twin beds, 20 with double beds, and 10 singles) has been reserved in Stratford for those wishing to attend the Shakespearean Festival. These rooms have been reserved for Friday and Saturday, August 25 and 26, and cost at most \$8.00 per night. In addition, a block of 100 seats (at \$4.50 each) has been reserved for the performances on Friday and Saturday (matinee and evening). On Sunday, 50 seats have been reserved. When the initial letter has been received in Toronto requesting reservations, a brochure will be mailed back. This brochure will contain an accommodation order form and a ticket order form. The order form should be individually filled in and sent directly to Stratford with the payment for tickets and the first night's accommodation. Stratford will assign tickets and rooms from the reserved blocks and mail the tickets and room locations back to those applying. If accommodation on Sunday is required, this can be arranged for locally.

The performances are:

Friday, August 25, at 8:30 p.m.; Antony and Cleopatra
Saturday, August 26, at 2:00 p.m.; Richard III
Saturday, August 26, at 8:30 p.m.; Merry Wives of Windsor
Sunday, August 27, at 2:00 p.m.; B-Minor Mass (Bach)

TRAVEL

Toronto is served by Air Canada, American, and Mohawk Airlines, to Malton Airport. The airport bus connects to the Royal York Hotel which is across from the railroad station. Rail service to Toronto is by Canadian National and Canadian Pacific Railways with good connections from Detroit, Buffalo, and Montreal. It is a short subway ride from the railroad station to the campus. Limited access highways (the 401 and the Queen Elizabeth Way) connect

Toronto with Detroit, Buffalo, Kingston (The Thousand Islands), or Montreal.

PARKING

Parking throughout the campus is extremely limited as the campus was not designed for motor traffic. Parking stickers will be available at the registration desk. However, participants are urged to drive as little as possible between dormitories and the meeting area.

WEATHER

During the week of the meeting, Toronto usually enjoys fine, settled, and rather warm weather. Long term average temperatures are: Mean daily, 65° ; mean daily maximum 75° ; mean daily minimum, 56° . The probability of a rainfall-free day is 0.7 and of sunshine, 0.55. The relative humidity averages 83% at 0100 hours, 80% at 0700, 59% at 1300, and 69% at 1900. The pollen count is high and increasing at this time.

BOOKSTORE

The bookstore will be open from 8:45 a.m. to 4:30 p.m. from Monday through Friday.

LIBRARY

The university library will be open on Monday through Thursday from 8:30 a.m. to 8:00 p.m.; and on Friday from 8:30 a.m. to 5:00 p.m.

MEDICAL SERVICES

There will be no facilities for medical services on the campus, but a list of physicians and hospitals will be available at the registration desk.

ADDRESS FOR MAIL AND TELEGRAMS

Individuals may be addressed at: American Mathematical Society, Summer Meeting, Sidney Smith Hall, University of Toronto, Toronto 5, Canada.

CANADIAN MATHEMATICAL
CONGRESS

An announcement of the Congress
was contained on page 209 of the February

Notice). Those participants attending the
Seminar of the Congress and staying at
Glendon Hall on the York Campus can
retain their dormitory rooms for the week
of the mathematical meetings if they wish.

COMMITTEE

The Local Arrangements Commit-
tee for the Toronto Meeting is as follows:

H. L. Alder (ex officio)
D. V. Anderson
Bernhard Banaschewski
H. S. M. Coxeter
Chandler Davis (Chairman)
D. B. DeLury
G. F. D. Duff
Herbert Federer (ex officio)

T. E. Hull
Kenneth O. May
Mrs. Joan Robinson
R. A. Ross
D. C. Russell
F. A. Sherk
Gordon L. Walker (ex officio)

Herbert Federer
Associate Secretary

Providence, Rhode Island



TIME TABLE
(Eastern Daylight Time)

SUNDAY August 27	American Mathematical Society	Mathematical Association of America	Econometric Society
10:00 a. m. - 4:00 p. m.		Board of Governors New Physics Building 134	
2:00 p. m. - 8:00 p. m.		REGISTRATION - Sidney Smith Hall	
7:00 p. m. - 7:47 p. m.		<u>Concert Hall, Film Program</u>	
8:00 p. m. - 9:03 p. m.		MEASURES AND SET THEORY: A lecture by Stanislaw Ulam (A CEM Individual Lectures film in b & w)	
		NIM AND OTHER ORIENTED GRAPH GAMES: A lecture by Andrew M. Gleason (A CEM Individual Lectures film in b & w)	
MONDAY August 28	AMS	MAA	ES
8:00 a. m. - 5:00 p. m.		REGISTRATION - Sidney Smith Hall	
8:45 a. m. - 9:00 a. m.		<u>Convocation Hall</u>	
9:00 a. m. - 10:00 a. m.		Welcome on behalf of the University	
10:10 a. m. - 11:00 a. m.		The Earle Raymond Hedrick Lectures: Combinatorial Analysis as a Theory, Lecture I Gian-Carlo Rota	
11:10 a. m. - Noon		The Problem of Apollonius Presidential Address to the Canadian Mathematical Congress H. S. M. Coxeter	
1:20 p. m. - 3:45 p. m.		The Picard Theorems Joseph Lehner	Capital and the firm Victoria C. Lecture Hall
1:20 p. m. - 3:45 p. m.			Econometric studies Emanuel C. Lecture Hall
2:00 p. m. - 3:00 p. m.		Hedrick Lectures, Lecture II Gian-Carlo Rota	
3:10 p. m. - 4:30 p. m.		Panel Discussion on Geometry in the University Moderator: V. L. Klee	
4:00 p. m. - 5:45 p. m.		Presentation by Members of the Panel: Walter Prenowitz, Seymour Schuster, F. A. Sherk	Economic theory Victoria C. Lecture Hall
4:00 p. m. - 5:45 p. m.			Mathematical economics Emanuel C. Lecture Hall
4:30 p. m. - 5:00 p. m.		General Discussion by the Panel and the Audience	
7:00 p. m.		<u>Concert Hall, Film Program</u> Films of the College Geometry Project of the University of Minnesota (color)	

MONDAY August 28	American Mathematical Society	Mathematical Association of America	Econometric Society
7:00 p. m. - 7:12 p. m.		ORTHOGONAL PROJECTION by Daniel Pedoe	
7:13 p. m. - 7:23 p. m.		CENTRAL SIMILARITIES by Daniel Pedoe	
7:24 p. m. - 7:36 p. m.		DIHEDRAL KALEIDOSCOPIES by H. S. M. Coxeter	
7:37 p. m. - 7:54 p. m.		GEOMETRIC VECTORS by Wm. O. J. Moser and S. Schuster	
8:05 p. m. - 8:18 p. m.		INVERSION by Daniel Pedoe	
8:19 p. m. - 8:35 p. m.		CURVES OF CONSTANT WIDTH by J.D. E. Konhauser	
8:36 p. m. - 8:49 p. m.		CENTRAL PERSPECTIVITIES by S. Schuster	
9:00 p. m.		CEM Animated Calculus Films in Color	
9:00 p. m. - 9:15 p. m.		THE DEFINITE INTEGRAL by Charles E. Rickart	
9:16 p. m. - 9:24 p. m.		VOLUME BY SHELLS by George F. Leger	
9:25 p. m. - 9:39 p. m.		FUNDAMENTAL THEOREM OF THE CALCULUS by Morris Schreiber	
9:40 p. m. - 9:50 p. m.		THEOREM OF THE MEAN by Felix P. Welch	
TUESDAY August 29	AMS	MAA	ES
9:00 a. m. - 5:00 p. m.	REGISTRATION - Sidney Smith Hall		
9:00 a. m. - 5:00 p. m.	EMPLOYMENT REGISTER - Sidney Smith Hall		
9:00 a. m. - 5:00 p. m.	EXHIBITS - Sidney Smith Hall		
9:00 a. m. - 10:45 a. m.			Consumer theory Victoria C. Lecture Hall
9:00 a. m. - 10:45 a. m.			Monetary theory Emanuel C. Lecture Hall
		<u>Convocation Hall</u>	
9:00 a. m. - 10:00 a. m.		Hedrick Lectures, Lecture III Gian Carlo-Rota	
10:10 a. m. - 11:00 a. m.		Business Meeting; Pre- sentation of L. R. Ford Awards	
11:00 a. m. - 12:45 p. m.			Labor economics Victoria C. Lecture Hall
11:00 a. m. - 12:45 p. m.			Consumer behavior Emanuel C. Lecture Hall
11:10 a. m. - Noon		Unique Factorization Pierre Samuel	
12:15 p. m.	PI MU EPSILON Governing Council - Hart House		
2:00 p. m. - 3:00 p. m.	Colloquium Lectures: Universal algebras and the theory of automata Samuel Eilenberg Convocation Hall		

TUESDAY August 29	American Mathematical Society	Mathematical Association of America	Econometric Society
2:00 p. m. - 3:45 p. m.			Canadian tax reform: An econometric appraisal Victoria C. Lecture Hall
2:00 p. m. - 3:45 p. m.			Statistical theory Emanuel C. Lecture Hall
3:15 p. m. - 4:55 p. m.	Session on Topology I Sidney Smith Hall 2102		
3:15 p. m. - 4:55 p. m.	Session on Functional Analysis I Sidney Smith Hall 2117		
3:15 p. m. - 4:55 p. m.	Session on Matrix Theory Sidney Smith Hall 2118		
3:15 p. m. - 4:55 p. m.	Session on Group Theory Sidney Smith Hall 2135		
3:15 p. m. - 4:55 p. m.	Session on Numerical Analysis and Automata Sidney Smith Hall 2129		
3:15 p. m. - 4:40 p. m.	Session on Statistics and Probability Sidney Smith Hall 2127		
3:15 p. m.	PI MU EPSILON - Contributed papers - Lash Miller 162		
4:00 p. m. - 5:45 p. m.			Expectations and the stock market Victoria C. Lecture Hall
4:00 p. m. - 5:45 p. m.			Colloquium Lecture on Applied Mathematics and Economics Leonid Herwicz and Melvin Berger Emanuel C. Lecture Hall
5:00 p. m.	Council Meeting University Room Park Plaza Hotel		
6:00 p. m.	PI MU EPSILON Banquet - Hart House		
7:00 p. m.	SIAM Council Meeting - Sidney Smith Hall 2130		
		<u>Concert Hall, Film Program</u>	
7:00 p. m. - 8:03 p. m.		JOHN VON NEUMANN (A CEM Individual Lectures film in b & w)	
8:15 p. m.		CEM Animated Level I Films in Color	
8:15 p. m. - 8:29 p. m.		WHAT IS A SET?	
8:29 p. m. - 8:39 p. m.		ONE-TO-ONE CORRES- PONDENCE	
8:39 p. m. - 8:48 p. m.		COUNTING	
8:48 p. m. - 8:56 p. m.		UNION AND INTERSECTION	
8:56 p. m. - 9:04 p. m.		ADDITION AND SUBTRACTION	
9:04 p. m. - 9:12 p. m.		MULTIPLICATION AND DIVISION	
9:20 p. m.		CEM Individual Lectures films not previously shown	

WEDNESDAY August 30	American Mathematical Society	Mathematical Association of America	Econometric Society
8:00 a. m.	PI MU EPSILON Breakfast - Hart House		
9:00 a. m. - 5:00 p. m.	REGISTRATION - Sidney Smith Hall		
9:00 a. m. - 5:00 p. m.	EMPLOYMENT REGISTER - Sidney Smith Hall		
9:00 a. m. - 5:00 p. m.	EXHIBITS - Sidney Smith Hall		
9:00 a. m.	MU ALPHA THETA Governing Council - New Physics Building 118		
9:30 a. m. - 10:30 a. m.	Invited address: A mathematical theory of optimal economic develop- ment David Gale Convocation Hall Joint session with the Econometric Society		See AMS program Column 2
10:40 a. m.	PI MU EPSILON - Contributed papers - Lash Miller 162		
11:00 a. m. - Noon	Colloquium Lecture II Samuel Eilenberg Convocation Hall		
11:30 a. m. - 12:45 p. m.			Growth theory I Victoria C. Lecture Hall
11:30 a. m. - 12:45 p. m.			Econometric theory Emanuel C. Lecture Hall
2:00 p. m. - 3:45 p. m.			Econometric models Victoria C. Lecture Hall
2:00 p. m. - 3:45 p. m.			Growth theory II Emanuel C. Lecture Hall
2:00 p. m. - 3:30 p. m.		Convocation Hall	
2:00 p. m. - 3:30 p. m.		Panel Discussion on What to do till the Computer Scientist Comes Moderator: P.D. Lax	
3:30 p. m. - 4:00 p. m.		Presentation by Members of the Panel. G. E. Forsythe, T. E. Hull, Seymour Papert	
3:30 p. m. - 4:00 p. m.		General Discussion by the Panel and the Audience	
4:10 p. m. - 5:00 p. m.		Numerical Representation of Preference Orderings, with an Application to In- finite Consumption Sequences T. C. Koopmans Joint Session with Econom- etric Society	See MAA Program, Column 3
4:30 p. m.	SIAM Council Meeting - Sidney Smith Hall 2130		
8:00 p. m.	SIAM - VON NEUMANN LECTURE - Some aspects of stellar dynamics and galactic structure - C. C. Lin - Convocation Hall		
9:30 p. m.	SIAM BEER PARTY - Hart House		
THURSDAY August 31	AMS	MAA	ES
9:00 a. m. - 5:00 p. m.	REGISTRATION - Sidney Smith Hall		
9:00 a. m. - 5:00 p. m.	EMPLOYMENT REGISTER - Sidney Smith Hall		
9:00 a. m. - 5:00 p. m.	EXHIBITS - Sidney Smith Hall		
9:00 a. m. - 9:55 a. m.	Session on Topology II Sidney Smith Hall 2102		
9:00 a. m. - 9:55 a. m.	Session on Functional Analysis II Sidney Smith Hall 2117		

THURSDAY August 31	American Mathematical Society	Mathematical Association of America	Econometric Society
9:00 a. m. - 9:55 a. m.	Session on Complex Function Theory I Sidney Smith Hall 2118		
9:00 a. m. - 9:55 a. m.	Session on Algebra I Sidney Smith Hall 2135		
9:00 a. m. - 9:55 a. m.	Session on Semigroups Sidney Smith Hall 2129		
9:00 a. m. - 9:55 a. m.	Session on Graph Theory Sidney Smith Hall 2127		
9:00 a. m. - 9:55 a. m.	Session on Logic and Foundations New Physics Building 102		
10:15 a. m.	Business Meeting Convocation Hall		
11:00 a. m. - Noon	Colloquium Lecture III Samuel Eilenberg, Convocation Hall		
2:00 p. m. - 4:10 p. m.	Session on Topology III Sidney Smith Hall 2102		
2:00 p. m. - 4:40 p. m.	Session on Functional Analysis III Sidney Smith Hall 2117		
2:00 p. m. - 4:25 p. m.	Session on Geometry Sidney Smith Hall 2118		
2:00 p. m. - 4:25 p. m.	Session on Differential Equations Sidney Smith Hall 2135		
2:00 p. m. - 4:40 p. m.	Session on Analysis I Sidney Smith Hall 2129		
2:00 p. m. - 4:55 p. m.	Session on Number Theory and Algebra I Sidney Smith Hall 2127		
2:00 p. m. - 3:55 p. m.	Session on Applied Mathe- matics I New Physics Building 102		
8:30 p. m.	CHAMBER MUSIC CONCERT - Concert Hall		
FRIDAY September 1	AMS	MAA	ES
9:00 a. m. - 1:00 p. m.	REGISTRATION - Sidney Smith Hall		
9:30 a. m. - 10:30 a. m.	Invited address: The topology of algebraic varieties Michael Artin Convocation Hall		
11:00 p. m. - Noon	Colloquium Lecture IV Samuel Eilenberg Convocation Hall		
2:00 p. m. - 4:25 p. m.	Session on Topology IV Sidney Smith Hall 2102		
2:00 p. m. - 4:25 p. m.	Session on Functional Analysis IV Sidney Smith Hall 2117		
2:00 p. m. - 4:25 p. m.	Session on Complex Function Theory II Sidney Smith Hall 2118		
2:00 p. m. - 4:40 p. m.	Session on Algebra II Sidney Smith Hall 2135		
2:00 p. m. - 4:10 p. m.	Session on Analysis II Sidney Smith Hall 2129		
2:00 p. m. - 3:55 p. m.	Session on Applied Mathematics II Sidney Smith Hall 2127		

PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. To maintain the schedule as published, the time limit will be strictly enforced.

TUESDAY, 2:00 P.M.

Colloquium Lecture I, Convocation Hall

Universal algebras and the theory of automata
Professor Samuel Eilenberg, Columbia University

TUESDAY 3:15 P.M.

Session on Topology I, Sidney Smith Hall 2102

3:15-3:25

- (1) A theorem concerning 3 dimensional manifolds
Mr. Andrew Connor, University of Wisconsin (648-31)

3:30-3:40

- (2) Mapping cubes with holes onto cubes with handles
Professor H. W. Lambert, University of Iowa (648-48)

3:45-3:55

- (3) Strong homotopy equivalence of 3-manifolds
Professor D. R. McMillan, Jr., University of Wisconsin (648-32)

4:00-4:10

- (4) Finding a boundary for a 3-manifold
Professor L. S. Husch, University of Georgia (648-107)

4:15-4:25

- (5) Uncountably many mildly wild non-Wilder arcs
Professor S. J. Lomonaco, Jr., Florida State University (648-40)

4:30-4:40

- (6) Open mappings on manifolds. Preliminary report
Mr. W. D. Nathan, Syracuse University (648-178)

4:45-4:55

- (7) Local triviality of Hurewicz fiber maps
Mr. Soon-kyu Kim, University of Illinois (648-151)

TUESDAY, 3:15 P.M.

Session on Functional Analysis I, Sidney Smith Hall 2117

3:15-3:25

- (8) Representations of commutative algebras with involution. Preliminary report
Professor Jesús Gil de Lamadrid, University of Minnesota (648-125)

3:30-3:40

- (9) Interpolation theorems for operators in function spaces
Professor G. G. Lorentz*, Syracuse University, and Professor T. Shimogaki,
Hokkaido University (648-143)

3:45-3:55

- (10) Splines, n -widths and optimal approximations
Professor Michael Golomb, Purdue University and Mathematics Research
Center (648-159)

4:00-4:10

- (11) On convolution and sequences of Banach spaces
Professor Jack Bryant, Texas A and M University (648-164)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

4:15-4:25

- (12) An interpolation method for positive operators
Professor A. M. Garsia, University of California, San Diego (648-189)

4:30-4:40

- (13) On a theorem of Mergelyan
Mr. John Garnett, Massachusetts Institute of Technology (648-90)

4:45-4:55

- (14) Lipschitz functions on abstract metric spaces
Mr. T. M. Jenkins, Oakland University (648-179)

TUESDAY, 3:15 P.M.

Session on Matrix Theory, Sidney Smith Hall 2118

3:15-3:25

- (15) An existence theorem for finite projective planes of even order
Professor K. A. Bush, Washington State University (648-6)

3:30-3:40

- (16) Construction and enumeration of symmetric involutory matrices over finite fields
Mr. J. D. Fulton, Oak Ridge National Laboratory, Tennessee (648-10)

3:45-3:55

- (17) Linear operators and their partition matrices
Mr. John E. de Pillis, University of California, Riverside (648-23)

4:00-4:10

- (18) On singular matrices
Professor J. B. Kim, West Virginia University, and Michigan State University (648-114)

4:15-4:25

- (19) Orthogonalization and the generalized inverse of a matrix
Professor W. E. Parr, University of Maryland (648-130)

4:30-4:40

- (20) Bounds for permanents of nonnegative matrices
Professor Henryk Minc, University of California, Santa Barbara (648-138)

4:45-4:55

- (21) A solution of Baxter's equation in the algebra of complex matrices
Professor N. A. Derzko, University of Toronto (648-183)

TUESDAY, 3:15 P.M.

Session on Group Theory, Sidney Smith Hall 2135

3:15-3:25

- (22) Finite groups of quaternions. Preliminary report
Professor H. S. M. Coxeter, University of Toronto (648-1)

3:30-3:40

- (23) Finite p -groups with k conjugate classes
Dr. John Poland, Carleton University (648-11)

3:45-3:55

- (24) On pseudo lattice ordered groups
Professor J. R. Teller, Georgetown University (648-33)

4:00-4:10

- (25) N -normalizers of finite solvable groups. Preliminary report
Dr. Avino'am Mann, University of Illinois (648-34)

4:15-4:25

- (26) Kostrikin's theorem on Engel conditions in groups of prime-power exponent
Professor Seymour Bachmuth* and Professor H. Y. Mochizuki, University of California, Santa Barbara (648-39)

4:30-4:40

- (27) Residually central elements in groups
Professor J. R. Durbin, University of Texas, Austin (648-58)

4:45-4:55

- (28) On the semiprimitivity of group rings
Mr. K.-T. Liu, State University of New York, Buffalo (648-170)

TUESDAY, 3:15 P.M.

Session on Numerical Analysis and Automata, Sidney Smith Hall 2129

3:15-3:25

- (29) Significance spaces and base conversion mappings
Mr. D. W. Matula, Washington University (648-50)

3:30-3:40

- (30) Evaluation of orthogonal polynomials and relationship to evaluating multiple integrals
Dr. P. M. Hirsch, International Business Machines, Houston, Texas (648-75)

3:45-3:55

- (31) Truncation error estimates for g -fractions
Mr. W. B. Gragg, Oak Ridge National Laboratory, Tennessee (648-139)

4:00-4:10

- (32) Secant methods for solving systems of nonlinear equations
Professor W. M. Kincaid, University of Michigan (648-152)

4:15-4:25

- (33) Generalized pair algebra with applications to nondeterministic automata
Professor R. T. Yeh, Pennsylvania State University (648-15)

4:30-4:40

- (34) Short wire theory. III: Categories of automata systems. Preliminary report
Mr. W. L. Hamilton, University of Utah (648-88)

4:45-4:55

- (35) Short wire theory. II
Professor J. H. Case, University of Utah (648-87)

TUESDAY, 3:15 P.M.

Session on Statistics and Probability, Sidney Smith Hall 2127

3:15-3:25

- (36) Epsilon entropy of the normal distribution
Mr. E. C. Posner*, Mr. E. R. Rodemich and Mr. Howard Rumsey, Jr.,
Jet Propulsion Laboratory, California Institute of Technology (648-198)

3:30-3:40

- (37) Some problems on measures of information and their characterization
Professor J. D. Aczel, University of Waterloo (648-21)

3:45-3:55

- (38) A characterization of pseudometrically generated probabilistic metric spaces
Professor Howard Sherwood, Illinois Institute of Technology (648-129)

4:00-4:10

- (39) On a random Uryson equation
Mr. Arunava Mukherjea, Eastern Michigan University (648-133)

4:15-4:25

- (40) Cantor-type interval dissections as random number tests
Mr. W. A. Beyer, Los Alamos Scientific Laboratory, New Mexico (648-185)

4:30-4:40

- (41) Diffuse semigroups (systems with a stochastic product)
Mr. W. W. Armstrong, Bell Telephone Laboratories, Inc., Holmdel, New Jersey (648-127)

WEDNESDAY, 9:30 A.M.

Invited Address, Convocation Hall

A mathematical theory of optimal economic development
Professor David Gale, University of California, Berkeley

WEDNESDAY, 11:00 A.M.

Colloquium Lecture II, Convocation Hall

Universal algebras and the theory of automata
Professor Samuel Eilenberg, Columbia University

THURSDAY, 9:00 A.M.

Session on Topology II, Sidney Smith Hall 2102

9:00-9:10

- (42) The Banach point structure of extensions of interior boundaries
Professor M. L. Marx, Vanderbilt University (648-60)

9:15-9:25

- (43) The Samelson product and the J-homomorphism
Professor P. S. Green and Professor Richard Holzsager*, University of Maryland (648-153)

9:30-9:40

- (44) Elementary ideals of linear graphs in a 3-sphere. Preliminary report
Professor S. Kinoshita, Florida State University (648-154)

9:45-9:55

- (45) Examples in the fixed point theory of finite polyhedra
Mr. William Lopez, University of Wisconsin (648-165)

THURSDAY, 9:00 A.M.

Session on Functional Analysis II, Sidney Smith Hall 2217

9:00-9:10

- (46) The support of ergodic measures
Mr. Ching Chou, University of Rochester (648-55)

9:15-9:25

- (47) Trace class and centralizers of an H^* -algebra. Preliminary report
Professor P. P. Saworotnow, Catholic University of America (648-113)

9:30-9:40

- (48) Generalizations and applications of a theorem of Mackey
Mr. V. S. Mandrekar* and Mr. M. G. Nadkarni, University of Minnesota
(648-147)

9:45-9:55

- (49) A characterization of $M(G)$
Mr. Roger Rigelhof, McMaster University (648-150)

THURSDAY, 9:00 A.M.

Session on Complex Function Theory I, Sidney Smith Hall 2118

9:00-9:10

- (50) On the derivative of an entire function
Professor Morris Marden, University of Wisconsin-Milwaukee (648-73)

9:15-9:25

- (51) Entire functions with no finite deficient value
Professor S. M. Shah, University of Kentucky (648-43)

9:30-9:40

- (52) On the removal of singularities of analytic sets
Mr. Bernard Shiffman, University of California, Berkeley (648-67)

9:45-9:55

- (53) A Lindelöf theorem and analytic continuation for functions of several variables, with an application to the Feynman integral
Professor R. H. Cameron* and Professor D. A. Storvick, University of Minnesota (648-119)

THURSDAY, 9:00 A.M.

Session on Algebra I, Sidney Smith Hall 2135

9:00-9:10

- (54) Products for the derived functors Cotor and Coext
Professor Frank Brenneman* and Professor Hiroshi Uehara, Oklahoma State University (648-94)

9:15-9:25

- (55) Triple cohomology in relative homological algebra
Professor Nobuo Shimada, Professor Hiroshi Uehara* and Professor Frank Brenneman, Oklahoma State University (648-100)

9:30-9:40

- (56) Reflectors as compositions of epi-reflectors
Dr. S. Baron, McGill University (648-155)

9:45-9:55

- (57) Pure projectivity
Mr. D. J. Fieldhouse, McGill University and Queen's University (648-161)

THURSDAY, 9:00 A.M.

Session on Semigroups, Sidney Smith Hall 2129

9:00-9:10

- (58) General products of a set and a semigroup. Preliminary report
Professor Takayuki Tamura, University of California, Davis (648-2)

9:15-9:25

- (59) An old familiar formula in a semigroup setting
Professor D. W. Hardy*, Colorado State University, and Professor R. J. Wisner, New Mexico State University (648-72)

9:30-9:40

- (60) The structure of medial semigroups
Mr. J. L. Chrislock, University of California, Santa Cruz (648-169)

9:45-9:55

- (61) Power semigroups
Professor Takayuki Tamura, University of California, Davis and Mr. John Shafer*, Amherst, Massachusetts (648-194)

THURSDAY, 9:00 A.M.

Session on Graph Theory, Sidney Smith Hall 2127

9:00-9:10

- (62) The group of an X-join of graphs
Professor R. L. Hemminger, Vanderbilt University (648-8)

9:15-9:25

- (63) Minimal singularities for graph realizations of incidence sequences
Professor A. B. Owens*, Naval Research Laboratory, Washington, D. C. and Dr. H. M. Trent, Deceased (648-9)

9:30-9:40

- (64) Point symmetric graphs with a prime number of points
Mr. James Turner, Stanford Research Institute, Menlo Park, California (648-63)

9:45-9:55

- (65) Convergence of sequences of graphs with variable support. Preliminary report
Professor B. J. Trawinski, University of Alabama (648-200)

THURSDAY, 9:00 A.M.

Session on Logic and Foundations, New Physics Building 102

9:00-9:10

(66) Restricted ω -rule for arithmetic

Professor C. F. Kent, Case-Western Reserve University (648-121)

9:15-9:25

(67) A language of order $1 \frac{1}{2}$

Professor H. B. Enderton, University of California, Berkeley (648-166)

9:30-9:40

(68) Initial segments of one-one degrees

Professor A. H. Lachlan, Simon Fraser University (648-157)

9:45-9:55

(69) Set-theories as algebras

Mr. Paul Fjelstad, University of Bern, Switzerland (648-204)

(Introduced by Professor Peter Wilker)

THURSDAY, 10:15 A.M.

Business Meeting, Convocation Hall

THURSDAY, 11:00 A.M.

Colloquium Lecture III, Convocation Hall

Universal algebras and the theory of automata

Professor Samuel Eilenberg, Columbia University

THURSDAY, 2:00 P.M.

Session on Topology III, Sidney Smith Hall 2102

2:00-2:10

(70) Shrinkability of certain decompositions of E^3 that yield E^3

Professor Steve Armentrout, University of Iowa (648-117)

2:15-2:25

(71) Extension of a result by J. H. Roberts

Dr. S. L. Jones, University of Wisconsin (648-101)

2:30-2:40

(72) Expansive homeomorphisms on homogeneous spaces

Professor Erik Hemmingsen*, Syracuse University, and Professor W. L. Reddy, State University of New York, Albany (648-123)

2:45-2:55

(73) Semigroup actions on specialized spaces. Preliminary report

Dr. David Stadlander, University of Florida (648-46)

3:00-3:10

(74) On compactifications and structure of topological groups. Preliminary report

Mr. R. T. Ramsay, North Carolina State University (648-49)

3:15-3:25

(75) Semigroups on coset spaces. Preliminary report

Mr. B. L. Madison, Louisiana State University, Baton Rouge (648-25)

3:30-3:40

(76) Homeomorphisms of the unit ball

Professor Jack Hachigian, State University of New York, Stony Brook (648-27)

3:45-3:55

(77) Decomposable circle-like continua

Professor W. T. Ingram, University of Houston (648-74)

4:00-4:10

(78) On the product of k -spaces

Professor E. A. Michael, University of Washington (648-191)

THURSDAY, 2:00 P.M.

Session on Functional Analysis III, Sidney Smith Hall 2117

2:00-2:10

- (79) Convex cones and pseudo-inverse in Hilbert space. Preliminary report
Professor M. A. H. Dempster, Balliol College, England (648-201)

2:15-2:25

- (80) Expansions for the inverse of a polynomial operator
Mr. M. V. Pattabhiraman* and Mr. Peter Lancaster, The University of
Calgary (648-47)

2:30-2:40

- (81) Fredholm theories in von Neumann algebras
Professor Manfred Breuer, University of Kansas (648-61)

2:45-2:55

- (82) Harmonic functions and their conjugates on Hilbert space
Dr. M. J. Fisher, Johns Hopkins University (648-77)

3:00-3:10

- (83) Decomposition of operator norms
Professor Chester Feldman, Kent State University (648-124)

3:15-3:25

- (84) Expansive automorphisms of Banach space. Preliminary report
Professor Murray Eisenberg, University of Massachusetts (648-126)

3:30-3:40

- (85) On the essential spectrum of multiplication operators, singular integral operators and symmetrizable operators
Professor J. I. Nieto, University of Maryland, College Park (648-140)

3:45-3:55

- (86) On power-bounded operators and operators satisfying a resolvent condition
Professor J. H. Miller, University of Massachusetts, Boston (648-158)

4:00-4:10

- (87) Inequalities involving the numerical radius of an operator
Professor J. A. R. Holbrook, University of California, San Diego (648-180)

4:15-4:25

- (88) On the solutions of a differential equation in a Banach space. Preliminary report
Mr. D. R. Colner* and Professor C. T. Taam, Georgetown University
(648-199)

4:30-4:40

- (89) Regular matrix summability and the Čech compactification of the rationals
Professor A. K. Snyder, Lehigh University (648-174)

THURSDAY, 2:00 P.M.

Session on Geometry, Sidney Smith Hall 2118

2:00-2:10

- (90) A cyclic involution of period nineteen contained on a rational surface in a space of twelve dimensions
Professor W. R. Hutcherson, University of Florida (648-18)

2:15-2:25

- (91) Representations of central convex bodies. Preliminary report
Professor N. F. Lindquist, Western Washington State College (648-54)

2:30-2:40

- (92) The intersection of maximal starlike sets
Mr. J. W. Kenelly and Mr. W. R. Hare, Jr.*, Clemson University (648-62)

2:45-2:55

- (93) The role of the first Bianchi identity in Riemannian geometry
Professor J. A. Thorpe, Haverford College (648-89)

3:00-3:10

(94) Transitivity of the automorphism groups of some geometric structures on manifolds

Professor W. M. Boothby, Washington University (648-95)

3:15-3:25

(95) Homogeneous nets and their fundamental regions

Mr. A. H. Schoen, Palos Verdes Peninsula, California (648-106)

(Introduced by Professor M. A. Perles)

3:30-3:40

(96) Order induced by a transformation group. Preliminary report

Mr. P. S. Marcus, Shimer College (648-141)

3:45-3:55

(97) A lattice characterization of convexity

Professor M. K. Bennett, University of Massachusetts, (648-182)

4:00-4:10

(98) Convexity and a certain property P_m

Professor David Kay*, University of Oklahoma, and Mr. Merle Guay,

University of New Hampshire (648-187)

4:15-4:25

(99) On order in geometry

Dr. R. G. Vinson, Huntingdon College (648-167)

THURSDAY, 2:00 P.M.

Session on Differential Equations, Sidney Smith Hall 2135

2:00-2:10

(100) A unique continuation theorem for mixed parabolic problems

Professor A. E. Hurd, University of California, Los Angeles (648-7)

2:15-2:25

(101) The coupled equation approach to the numerical solution of the biharmonic equation by finite differences

Professor Julius Smith, University of Tennessee (648-70)

2:30-2:40

(102) A new bifurcation theory for a class of nonlinear elliptic partial differential equations

Professor M. S. Berger, University of Minnesota and Courant Institute of Mathematical Sciences (648-132)

2:45-2:55

(103) Existence theorems for a nonlinear partial differential equation of viscous incompressible flow

Mr. A. R. Elcrat, General Electric Space Sciences Laboratory, King of Prussia, Pennsylvania, and Indiana University (648-173)

3:00-3:10

(104) Periodic solutions of parabolic partial differential equations

Dr. S. J. Farlow, Oregon State University (648-175)

(Introduced by Dr. J. M. Gonzalez-Fernandez)

3:15-3:25

(105) Convergent solutions of parabolic partial differential equations

Professor H. L. Turrittin, University of Minnesota (648-184)

3:30-3:40

(106) Resonance behavior of a perturbed system depending on a slow-time parameter

Dr. J. A. Morrison, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey (648-17)

3:45-3:55

(107) Bounds for solutions of nonlinear integro-differential equations

Dr. J. S. Muldowney*, University of Oklahoma, and Dr. J. S. W. Wong, University of Alberta (648-41)

4:00-4:10

- (108) On the uniqueness of solutions of linear differential equations
Dr. J. A. Cochran, Bell Telephone Laboratories, Inc., Whippany, New
Jersey (648-51)

4:15-4:25

- (109) A method of deriving differential equations of special functions
Professor A. M. Chak, West Virginia University (648-57)

THURSDAY, 2:00 P.M.

Session on Analysis I, Sidney Smith Hall 2129

2:00-2:10

- (110) On continuous singular functions
Professor K. M. Garg, University of Alberta (648-193)

2:15-2:25

- (111) Derivatives with respect to finite dimensional vector measures
Professor Milton Rosenberg, University of Kansas (648-19)

2:30-2:40

- (112) Coefficient properties of Fourier-Stieltjes series
Professor J. A. Siddiqi, University of Sherbrooke (648-104)

2:45-2:55

- (113) The Wiener-Dirichlet problem and the theorem of Evans
Professor M. G. Arsove, University of Washington (648-112)

3:00-3:10

- (114) Summability in topological groups. III. Preliminary report
Professor D. L. Prullage, University of Kentucky (648-118)

3:15-3:25

- (115) Bohr sets are finite unions of unrelated sets
Professor John Fournier, University of British Columbia (648-136)

3:30-3:40

- (116) A characterization of analyticity. III. Preliminary report
Professor K. O. Leland, Illinois Institute of Technology (648-144)

3:45-3:55

- (117) Variable parabolic convergence regions for continued fractions
Professor W. B. Jones* and Professor W. J. Thron, University of Colorado
(648-146)

4:00-4:10

- (118) A decomposition of flat functions
Professor R. D. Moyer, University of Kansas (648-148)

4:15-4:25

- (119) The existence of irregular Borel measures on dyadic spaces
Professor H. L. Peterson, University of Connecticut (648-156)

4:30-4:40

- (120) Singular integrals are Perron integrals of a certain type. Preliminary report
Professor W. F. Pfeffer, University of California, Davis (648-171)

THURSDAY, 2:00 P.M.

Session on Number Theory and Algebra I, Sidney Smith Hall 2127

2:00-2:10

- (121) On the Diophantine equations $n! = x^4 - y^4$
Dr. M. L. Faulkner, Western Washington State College (648-68)

2:15-2:25

- (122) On abelian difference sets having minus one as a multiplier
1/Lt. R. L. McFarland, Ohio State University and Department of Defense,
Maryland (648-102)

2:30-2:40

(123) A method in Diophantine approximation. II and III

Dr. C. F. Osgood, University of Illinois and National Bureau of Standards,
Washington, D. C. (648-120)

2:45-2:55

(124) On plane partitions, strictly monotonic on rows

Professor Basil Gordon, University of California, Los Angeles and Pro-
fessor Lorne Houten*, Washington State University (648-162)

3:00-3:10

(125) Coefficients of cyclotomic polynomials of index $3qr$

Sister Marion Beiter, Rosary Hill College (648-168)

3:15-3:25

(126) On the nonrecurrence of the logarithmic rank jump

Professor R. K. Wright, University of Vermont (648-176)

3:30-3:40

(127) Construction of Stone lattices. III

Professor C. C. Chen, Queen's University and Professor G. A. Grätzer*,
University of Manitoba (648-71)

3:45-3:55

(128) Subunitary transformations

Professor A. R. Amir-Moez, Texas Technological College (648-42)

4:00-4:10

(129) Incidence functions as generalized arithmetic functions. II

Professor D. A. Smith, Duke University (648-80)

4:15-4:25

(130) The ring of finite elements in a nonstandard model of the real numbers

Professor Harry Gonshor, Rutgers, The State University (648-91)

4:30-4:40

(131) On certain maps of modules which are direct sums of sub modules

Mr. R. A. Guy, University of Montreal and Queen's University (648-196)
(Introduced by Dr. S. H. Gould)

4:45-4:55

(132) Introduction to hyperboolean systems. II: Modules of N -tuples; hyperboolean
determinants and matrices

Professor John Hays, Inter American University of Puerto Rico (648-202)

THURSDAY, 2:00 P.M.

Session on Applied Mathematics I, New Physics Building 102

2:00-2:10

(133) Matching algorithms

Mr. D. R. Morrison, Sandia Corporation, Albuquerque, New Mexico (648-3)

2:15-2:25

(134) Particular proofs of ergodic problem in continuous media and reduction of
independent variables

Professor M. Z. v. Krzywoblocki, Michigan State University (648-4)

2:30-2:40

(135) An analysis of flexural vibrations for a circular ring with free edges

Professor J. S. Bakshi, State University College, Buffalo (648-5)

2:45-2:55

(136) Flexural vibrations of elliptical plates when transverse shear and rotary inertia
are considered

Professor W. R. Callahan, St. John's University (648-14)

3:00-3:10

(137) Inequalities related to Lidskiĭ's

Professor M. F. Smiley, State University of New York, Albany (648-59)

3:15-3:25

- (138) Determination of angle of attack from rotational body rates
Dr. Mark Lotkin, General Electric Company, Cherry Hill, New Jersey
(648-65)

3:30-3:40

- (139) Multi-order property of Runge-Kutta formulas and error estimation
Professor Diran Sarafyan, Louisiana State University, New Orleans (648-85)

3:45-3:55

- (140) A simple flexible rule for numerical integration
Dr. G. O. Peters* and Mr. C. E. Maley, General Electric Company,
Philadelphia, Pennsylvania (648-71)

FRIDAY, 9:30 A.M.

Invited Address, Convocation Hall

- The topology of algebraic varieties
Professor Michael Artin, Massachusetts Institute of Technology

FRIDAY, 11:00 A.M.

Colloquium Lecture IV, Convocation Hall

- Universal algebras and the theory of automata
Professor Samuel Eilenberg, Columbia University

FRIDAY, 2:00 P.M.

Session on Topology IV, Sidney Smith Hall 2102

2:00-2:10

- (141) (n,m) -arrangements. Preliminary report
Professor M. C. Gemignani, State University of New York, Buffalo (648-20)

2:15-2:25

- (142) The lattice of pretopologies on a set S
Mr. A. M. Carstens, Washington State University (648-92)

2:30-2:40

- (143) On countable paracompactness and normality
Mr. P. L. Zenor, University of Houston (648-99)

2:45-2:55

- (144) On fixed points of mappings contractive in a local sense
Professor Ludvik Janos, University of Florida (648-108)

3:00-3:10

- (145) Sequential space methods in general topological spaces
Professor P. R. Meyer, Hunter College of City University of New York
(648-116)

3:15-3:25

- (146) D -dimension. I. A new transfinite dimension
Professor D. W. Henderson, Cornell University (648-149)

3:30-3:40

- (147) On a class of spaces containing Arhangel'skiĭ's p -spaces
Dr. H. H. Wicke* and Dr. J. M. Worrell, Jr., Sandia Corporation, Albuquerque, New Mexico (648-188)

3:45-3:55

- (148) Filters in general
Professor P. C. Hammer, Pennsylvania State University (648-16)

4:00-4:10

- (149) Classification of Hausdorff spaces
Professor C. E. Aull, Virginia Polytechnic Institute (648-22)

4:15-4:25

(150) Introduction to general analysis

Professor R. G. Lintz, McMaster University (648-76)

FRIDAY, 2:00 P.M.

Session on Functional Analysis IV, Sidney Smith Hall 2117

2:00-2:10

(151) Setvalued additive functions. IV

Professor Dagmar Henney, George Washington University (648-13)

2:15-2:25

(152) Operators on the space of continuous functions

Dr. Jürgen Batt, Kent State University (648-35)

2:30-2:40

(153) A converse to Bishop's general Rudin-Carleson theorem. Preliminary report

Professor G. L. Seever, California Institute of Technology (648-79)

2:45-2:55

(154) Feller boundary induced by a transition operator

Mr. S. P. Lloyd, Bell Telephone Laboratories, Murray Hill, New Jersey
(648-109)

3:00-3:10

(155) Extreme points of some convex subsets of L^1

Professor J. V. Ryff, University of Washington (648-111)

3:15-3:25

(156) Decompositions of operator spaces

Professor William Ruckle, Lehigh University (648-134)

3:30-3:40

(157) An integral representation theorem

Professor Gregers Krabbe, Purdue University (648-135)

3:45-3:55

(158) On quadratic functional continuous along rays

Mr. J. A. Baker, University of Waterloo (648-205)
(Introduced by Professor J. Aczel)

4:00-4:10

(159) Full ideals of operators. Preliminary report

Mr. Peter Falley, The City University of New York (648-160)

4:15-4:25

(160) Regular functions on a Banach algebra associated with Laplace's equation

Professor H. H. Snyder, Southern Illinois University (648-195)

FRIDAY, 2:00 P.M.

Session on Complex Function Theory II, Sidney Smith Hall 2118

2:00-2:10

(161) Real singular points of Legendre series

Professor G. G. Walter, University of Wisconsin-Milwaukee (648-81)

2:15-2:25

(162) On the boundary of Teichmüller spaces

Dr. F. Gardiner, Harvard University, and Dr. Irwin Kra*, Massachusetts
Institute of Technology (648-84)

2:30-2:40

(163) A note on the perturbation of starlike functions

Mr. D. J. Wright, University of Kentucky (648-93)

2:45-2:55

(164) Large Cantor sets with very small subsets of relative harmonic measure one

Professor W. J. Schneider, Syracuse University (648-131)

3:00-3:10

- (165) Bounded analytic functions tending radially to zero
Professor M. Rosenfeld and Professor M. L. Weiss*, University of California, Santa Barbara (648-142)

3:15-3:25

- (166) Averages involving Fourier coefficients of nonanalytic automorphic forms.
Preliminary report
Professor V. V. Rao, University of Calgary (648-28)

3:30-3:40

- (167) On the convergence of Bernstein polynomials for some unbounded analytic functions. Preliminary report
Mr. P. C. Tonne, Emory University (648-52)

3:45-3:55

- (168) On commuting analytic functions with fixed points
Dr. S. P. Singh, University of Windsor (648-145)

FRIDAY, 2:00 P.M.

Session on Algebra II, Sidney Smith Hall 2135

2:00-2:10

- (169) Some open questions on minimal primes of a Krull domain
Mr. P. M. Eakin and Professor W. J. Heinzer*, Louisiana State University, Baton Rouge (648-24)

2:15-2:25

- (170) Rings with Noetherian spectrum
Professor Jack Ohm and Professor R. L. Pendleton*, Louisiana State University, Baton Rouge (648-26)

2:30-2:40

- (171) Inertial automorphisms of a class of wildly ramified V-rings
Professor Nickolas Heerema, Florida State University (648-36)

2:45-2:55

- (172) On the genera of irreducible lattices
Dr. K. W. Roggenkamp, University of Illinois (648-86)

3:00-3:10

- (173) A characterization of primes in simple rings over locally finite fields
Professor H. G. Rutherford, Montana State University (648-82)

3:15-3:25

- (174) Singular extensions and cohomology of Lie algebras
Dr. P. C. Morris, Oklahoma State University (648-177)

3:30-3:40

- (175) Solution of the "basis problem" for the finite-dimensional representations of a complex simple Lie algebra of type A
Professor D. -N. Verma, New Mexico State University (648-172)

3:45-3:55

- (176) Intrinsic characterization of polynomial transformations of vector spaces
Professor G. R. Blakley, University of Illinois and State University of New York, Buffalo (648-98)

4:00-4:10

- (177) Hereditary radicals and derivations of algebras. Preliminary report
Professor Tim Anderson, University of British Columbia (648-122)

4:15-4:25

- (178) Endomorphisms of universal algebras
Professor Maurice Chacron, University of Sherbrooke (648-53)
(Introduced by Dr. J. Siddige)

4:30-4:40

- (179) A counter-example in ring theory and homological algebra
Mr. A. V. Jategaonkar, University of Rochester (648-103)
(Introduced by Professor Newcomb Greenleaf)

FRIDAY, 2:00 P.M.

Session on Analysis II, Sidney Smith Hall 2129

2:00-2:10

- (180) Quasi-analytic collections containing Fourier series which are not infinitely differentiable
Professor J. W. Neuberger, Emory University (648-30)

2:15-2:25

- (181) Homomorphic summation methods
Professor G. U. Brauer, University of Minnesota (648-37)

2:30-2:40

- (182) On best approximations in several variables with linear superpositions
Professor D. A. Sprecher, University of California, Santa Barbara (648-38)

2:45-2:55

- (183) An inequality for integral norms
Professor H. W. McLaughlin* and Professor F. T. Metcalf, University of California, Riverside (648-56)

3:00-3:10

- (184) On the L^1 norm and the minimum value of a trigonometric polynomial
Professor L. C. Kurtz* and Professor S. M. Shah, University of Kentucky (648-66)

3:15-3:25

- (185) Matrix transformations which preserve regularity
Professor Gaston Smith, William Carey College (648-197)

3:30-3:40

- (186) Approximate solutions to nonlinear integral equations
Professor C. A. Bryan, University of Montana (648-78)

3:45-3:55

- (187) Discontinuous functions of bounded variation and change of variable in a Lebesgue integral
Mr. K. G. Johnson, East Carolina University, and Virginia Polytechnic Institute (648-83)

4:00-4:10

- (188) A note regarding Haar measure on the space of oriented lines
Professor R. C. Steinlage, University of Dayton (648-44)

FRIDAY, 2:00 P.M.

Session on Applied Mathematics II, Sidney Smith Hall 2127

2:00-2:10

- (189) Matrix scaling. Preliminary report
Dr. R. J. Arms, National Bureau of Standards, Gaithersburg, Maryland (648-96)

2:15-2:25

- (190) Toward a stochastic theory of spin. Preliminary report
Mr. T. G. Dankel, Jr., Princeton University (648-128)

2:30-2:40

- (191) Degeneracy in the discrete Tchebycheff problem
Mr. Murray Schechter, Lehigh University (648-137)

2:45-2:55

- (192) A method of fictitious "play" for linear economic exchange models
Professor Torsten Norvig, Wellesley College (648-163)

3:00-3:10

(193) On the computation of a schedule

Mrs. F. M. Carroll, Raytheon Company, Sudbury, Massachusetts (648-181)

3:15-3:25

(194) Investigations of a class of gravitational theories

Professor A. C. Sugar, Bradley University (648-186)

3:30-3:40

(195) On the Rayleigh assumption in scattering by a periodic surface. Preliminary report

Dr. R. F. Millar, National Research Council of Canada (648-190)

3:45-3:55

(196) Hybrid formulas with variable time step

Dr. C. R. Cassity, Environmental Research Corporation, Alexandria, Virginia (648-192)

Providence, Rhode Island

Herbert Federer
Associate Secretary



NEWS ITEMS AND ANNOUNCEMENTS

12TH INTERNATIONAL CONGRESS OF APPLIED MECHANICS

The 12th International Congress of Applied Mechanics will be held at Stanford University from August 26 through August 31, 1968. Like earlier Congresses, the present one will again encompass the entire field of the science of particle, solid and fluid mechanics, including applications but excluding computational methods as such. There will be five general lectures for which the speakers will be invited by the International Program Committee. In addition, not more than 300 contributed research papers will be presented.

Information on accommodations may be obtained from the 12th International Congress of Applied Mechanics, Post Office Box 5789, Stanford, California 94305. Limited funds will be available for those who cannot attend the Congress without financial aid, and applications should be sent to the Stanford address not later than February 2, 1968. There is also a possibility that there will be a charter flight from Europe to San Francisco; those interested in such transportation at reduced cost are requested to write to Professor W. Koiter, Secretary of the Congress Committee of IUTAM, Mekelweg 2, Delft, The Netherlands, before October 15, 1967.

Those who wish to present papers at the 12th Congress should send 500-word

summaries and 100- to 150-word abstracts, both in triplicate, to the Stanford address not later than February 2, 1968. A number of papers that appear particularly interesting because of their subject matter, method of treatment of the problem and results obtained will be designated as sectional lectures. Thirty minutes will be allotted to the sectional lecturers for presentation and ten minutes for discussion, while the remaining contributed papers on the program will be presented in fifteen minutes with five minutes of discussion.



Errata

In the June issue of these *Notices*, the news item, "Publication of Education in Applied Mathematics," was partly incorrect. The fourth paragraph should have stated, "Only a limited number of reprints are still available at the special price of \$2.40 each."

PRELIMINARY ANNOUNCEMENTS OF MEETINGS

Six Hundred Forty-Ninth Meeting Massachusetts Institute of Technology Cambridge, Massachusetts October 28, 1967

The six-hundred forty-ninth meeting of the American Mathematical Society will be held at the Massachusetts Institute of Technology on Saturday, October 28, 1967.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, Professor Edgar H. Brown, Jr. of Brandeis University will speak at 11:00 a.m. on "Cobordism groups," and Professor Arthur P. Mattuck of M. I. T. will speak at 2:00 p.m. on a topic in algebraic geometry. Both lectures will be presented in the Compton Auditorium, Room 26-100.

There will be sessions for contributed papers both morning and afternoon. Abstracts for contributed papers should be sent to the American Mathematical Society, Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of September 14, 1967.

Registration will be on the first floor of Building 2. It will open at 9:00 a.m.

Parking space will be available in the East Parking Garage on the Institute grounds for those travelling by automo-

bile. The entrance to this parking garage is at the corner of Main and Vassar Streets. If needed, additional parking space in the West Parking Garage (entrance on Vassar Street, west of Massachusetts Avenue) will also be available.

M.I.T. is a seven to ten minute walk from the Kendall Square station of the Cambridge-Dorchester subway. This subway may be boarded at various points, including South Station, Boston, and Harvard Square, Cambridge. The most convenient entrance for those coming by subway is at the northwest corner of the Hayden Memorial Library.

Those coming by taxicab or trackless trolley will find it convenient to use the main entrance, 77 Massachusetts Avenue. Most of the entrances except the two mentioned are closed on Saturday.

Lunch will be served in an M.I.T. cafeteria, and a list of nearby restaurants in Boston and Cambridge will be available.

Herbert Federer
Associate Secretary

Providence, Rhode Island



Six Hundred Fiftieth Meeting University of Tennessee Knoxville, Tennessee November 17-18, 1967

The six-hundred and fiftieth meeting of the American Mathematical Society will be held at the University of Tennessee, Knoxville, Tennessee, on Friday and Saturday, November 17-18, 1967.

Sessions for contributed papers will be held in the new Humanities and Social Science Tower.

By invitation of the Committee to Select Hour Speakers, Professor A. S.

Householder of Oak Ridge National Laboratory and the University of Tennessee, Professor Bjarni Jonsson, Vanderbilt University and Professor Pasquale Porcelli of Louisiana State University will address the Society. These addresses are scheduled to be held in the Music Auditorium.

Abstracts of contributed papers should be sent to the American Mathematical Society, Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of September 28, 1967.

The Registration Desk will be located in the Humanities and Social Science Tower. Registration hours will be 9:00 a.m.-5:00 p.m. Friday, November 17, and 9:00 a.m.-12:00 noon on Saturday, November 18.

No dormitory space is available.

Knoxville, Tennessee is served by United, American, Delta, Piedmont and Southern Airlines; by Southern and by Louisville and Nashville Railroads; and by Greyhound and Trailways buses. Interstate highways 40 and 75 provide major road access to Knoxville.

The following is a list of accommodations convenient to the University:

Admiral Benbow Inn, Pierce Parkway
Single \$ 9.00 to \$12.00
Double 16.00

Holiday Inn, 2000 Chapman Highway
Single \$9.00 to \$11.00
Double 13.00 (1 bed)
15.00 to 16.00 (2 beds)

Andrew Johnson Hotel, 918 S. Gay Street
Single \$ 8.00 to \$12.00
Double 12.50 to 17.50 (1 bed)
17.50 up (2 beds)
13.50 to 17.50 (twin beds)

Farragut Hotel, 530 S. Gay Street
Single \$7.50 to \$10.50
Double 8.50 to 11.50 (1 bed)
11.50 to 14.50 (twin beds)

University Inn, 17th and Clinch
Single \$10.50 to \$16.00
Double 14.00 to 18.00

O. G. Harrold, Jr.
Associate Secretary

Tallahassee, Florida



Six Hundred Fifty-Second Meeting

University of Illinois

Urbana, Illinois

November 25, 1967

The six hundred fifty-second meeting of the American Mathematical Society will be held on Saturday, November 25, 1967, at the University of Illinois, Urbana, Illinois.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be hour addresses by Professor Donald J. Lewis of the University of Michigan and by Professor Mark E. Mahowald of Northwestern University. By invitation of the same committee there will be a special session of twenty-minute papers on "Categorical algebra", arranged by Professor Saunders Mac Lane of the University of Chicago.

A further announcement containing information about accommodations and travel will appear in the October issue of these *Notices*.

On Friday, November 24, 1967, the University of Illinois will sponsor a symposium on the Theory of Finite Groups with the financial support of the G. A. Miller Endowment. The speakers will be Professors Reinhold Baer, Richard Brauer, Graham Higman, Robert Steinberg, John Thompson, and Helmut Wielandt.

Urbana, Illinois

Paul Bateman
Associate Secretary

Reorganization of Mathematical Reviewing*

F. K. Schmidt

Problems in the reviewing of science are approaching a crisis: the reviewing process is too slow, it requires increasingly large governmental support, and it is especially troublesome for the young mathematicians, who must now carry an almost unsupportable burden of writing reviews. Although the situation is similar in the other sciences, it is particularly alarming in mathematics, because of the difficulty of writing a mathematical review.

Consequently, many mathematicians are searching with considerable anxiety for new solutions. In particular, they hope that some sort of agreement among the three reviewing journals (German, American and Soviet) will lead to improvement. So a thorough examination should be made of the various possibilities for reorganization and for international cooperation. The following remarks are intended to examine this question from the point of view of Zentralblatt.

II. Summary of the possibilities.

Up to now essentially four types of reorganization have been considered for Zentralblatt.

(a) Preservation and improvement of the status quo, i.e. continuation of Zentralblatt in the present manner with the hope of overtaking and keeping up with Mathematical Reviews, whose reviews are similar in nature but are published more quickly.

(b) Independent continuation in the manner of the earlier "Fortschritte" with the purpose, in contradiction to Mathematical Reviews and the Soviet journal, of giving a critical survey of the development of the mathematical field in question, with consequent loss of promptness in reviewing.

(c) Independent continuation by means of author's summaries, with parti-

cular emphasis, in comparison with the other reviewing journals, on speed of publication.

(d) An international solution, i.e. continuation of the Zentralblatt with international cooperation.

Occasionally the first of these possibilities is combined with the variant discussed below; namely, continuation of Zentralblatt in its present form with greater restriction in the choice of articles in the borderline subjects.

III. Critical examination of these possibilities.

Any responsible attempts to plan for Zentralblatt necessarily involve an unprejudiced examination of all of these possibilities.

(a) Preservation and improvement of the status quo. Opinions vary on the prospect that Zentralblatt may overtake Mathematical Reviews within a few years.

The optimistic view is based on the hope that a new impetus will be given to Zentralblatt by the agreement for common publication through the Berlin and Heidelberg academies, by the formation and further development of the editorial offices in West Berlin, by the increasing role of machines, and by greater governmental support from both West and East Germany.

On the other side of the question there is the existing headstart of Mathematical Reviews, which is now of long standing and can easily be verified by mere counting; also there is the fact that although the existence during the last two years of the editorial offices in West Berlin has made it possible to remove some of the backlog, acceleration of current reviewing has been very slight, and it still remains true that thoroughgoing changes in the organization of Zentralblatt are faced with both practical and

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personal difficulties. Also we must reckon with an acceleration in Mathematical Reviews, which has just appointed a new executive editor, has just moved to a university whose interest in its problems is presumably greater, and has always been able, in view of its greater promptness in publishing reviews, to attract the best reviewers.

Again there is opposition to the idea that Zentralblatt should place still further restrictions on its coverage of borderline subjects, which are being given increasing attention in Mathematical Reviews. For example, theoretical physics is represented on the staff of Zentralblatt by its own scientific editor and its own group of reviewers. Complete removal of this field would not reduce the burden on the other fields, but would lead only to some reduction in overall administrative work. Again, the change would probably be regretted by mathematicians who attach particular importance to the completeness of a reviewing service or find particular stimulus in the borderline fields. In fact, there is a very definite desire in the opposite direction, toward further spreading out, in particular toward applications of statistical methods in biology, medicine, etc., which are handled in detail in Mathematical Reviews.

But even if we leave these differences of opinion to one side, there still remains the decisive question whether long-term competition between Mathematical Reviews and Zentralblatt is desirable, or even practicable, from the point of view of mathematical effort or governmental support. Against the argument that competition is the best protection against onesidedness we must point out the disproportionately large sacrifices entailed and the effectiveness of other possible steps. Many mathematicians consider it senseless to have two competing journals with the same purposes, in view of their unexpectedly large demands on intellectual and financial resources.

On the contrary, an attempt should be made to provide the Zentralblatt with such a special character of its own that its existence along with the other journals is thereby justified. Possibilities (b) and (c) lie in this direction.

(b) Independent continuation in the

manner of the "Fortschritte". At first glance there is something attractive in this plan; from the organizational point of view it is an obvious one, since the persons who now play a leading role in the publication of the Zentralblatt were formerly in charge of the "Fortschritte". Moreover, it would provide a much needed critical survey of present-day developments in mathematics. But the task of actually carrying it out involves almost insuperable difficulties.

The lack of mathematicians both able and willing to write a critical survey of the developments in a given mathematical field has become more pronounced since the discontinuance of the "Fortschritte". Today this lack forms a still greater obstacle to the realization of an ideal which even in former times was scarcely practicable. The truth of these remarks is shown by what has happened in the series "Ergebnisse der Mathematik". Originally its volumes were intended to take the place of the "Fortschritte" in providing a comprehensive survey of the development of mathematical fields of current interest; but today they have inevitably become a series of special monographs.

These difficulties represent a definite danger when one considers the vagueness necessarily attaching to resumption of the guiding principles of the "Fortschritte". In fact, the whole idea is sometimes reduced to a mere effort to arrange the reviews in better order. Since the end of the Second World War, this formulation of the idea has been put forward again and again, with the hope that Zentralblatt could be given a special character of its own, and there is no doubt that it has been of some service in this direction. But greater emphasis on such an idea involves the risk that Zentralblatt will drop still further behind with its reviews; and on the other hand, in view of the difficulty of writing critical reviews, the difference in character between Zentralblatt and Mathematical Reviews will probably remain about the same as at present. Since the plan does not envisage any decrease of governmental support, while the difficulty of writing the reviews will no doubt increase, it would seem that the final result will only be worse than the present situation.

(c) Independent continuation with authors' reviews. This plan is based on the same idea as the original founding of Zentralblatt, an idea which accounted for its success in comparison with the then still existing "Fortschritte"; i.e. rapid, reliable and comprehensive orientation in the mathematical literature with no emphasis on criticism. The plan corresponds to a realistic appraisal of what can be attempted by Zentralblatt and of what is in fact attained by the present system of reviewing by mathematicians other than the authors: a reviewer who does not happen to have a detailed acquaintance with the subject of the article under review requires a period of several weeks, which is very seldom available to him, if he wishes to immerse himself in the subject to such an extent that he can provide information beyond what is already given by the author. On the other hand, an author's review is much prompter, it removes the burden of reviewing from the shoulders of a third party, and in most cases it represents no loss of factual information. Moreover, in the long run the use of authors' reviews leads to a considerable decrease in the costs of publication. Consequently, in view of the great increase in present-day mathematical production, the method of authors' reviews must be given serious consideration.

In fact, in the various exact sciences, the system of authors' reviews works very well, and mathematicians can certainly make use of the experience of scientists in other fields. Most important for mathematics is the example of "Physics Abstracts", which appears to provide a satisfactory solution, at least

in the opinion of most theoretical physicists. The experience of "Physics Abstracts" will no doubt be of great value in many ways; for example, in the examination of an individual author's review to see whether it is in fact usable, in the treatment of authors' reviews written in some language other than German, English or French, and so forth.

Some fear has been expressed that the publishers of large scientific journals will feel that the almost simultaneous appearance of an author's review represents an undesirable competition with the original article; but here the prevailing opinion seems to be just the opposite. Many publishers welcome a prompt author's review as a stimulus to the public to examine the original article and therewith the journal in which it appears. In this respect, the interests of all concerned appear to coincide.

The changeover to authors' reviews opens up still another possibility. With the agreement of all concerned, the author's review could be published before the article itself. Since many mathematical journals now have a long waiting period, preliminary information of this sort will be highly desirable.

(d) International solution. Willingness on the part of Zentralblatt to make the change to authors' reviews can be very helpful towards an international agreement among the various reviewing journals; and conversely, such a changeover would itself be much facilitated by a preliminary international agreement.

In April 1965*, the Executive Committee of the International Mathematical

*Professor Otto Frostman, Secretary of IMU, has informed the editors of these *Notices* that the Committee was appointed in June 1967, to implement the following resolution of the Fifth General Assembly of IMU, Dubna, 1966: "The General Assembly is in favour of a project directed by the Executive Committee towards improving the existing system of reviewing mathematical papers along the following lines:

"The existing system of refereeing might be dovetailed with the system of reviewing. Referees of papers which are accepted by journals may be invited by the journals themselves to furnish a short review or preview on a voluntary basis. Such reviews may then be forwarded to any one or more of the reviewing journals for use at their discretion. In no case should the review of the paper be published before the definitive form of the paper is ready for printing; in cases where the author modifies his paper substantially after its acceptance, the editor of the journal should warn the referee who might wish to modify his review as a result."

The Committee consists of Professors Michael F. Atiyah, György Hajós and Edoardo Vesentini.

Union appointed an Advisory Committee to investigate the present situation in mathematical reviewing and to make suggestions for improvement. It is a natural idea that, while preserving their independence, the various reviewing journals should introduce greater differences into their respective procedures, and thereby increase their overall value. The changeover to authors' reviews on the part of Zentralblatt would be a step in this direction. In particular, it would bring mutual advantages to the Zentralblatt and Mathematical Reviews, and these advantages could be augmented by an agreement with the Soviet journal for increased exchange of reviews.

IV. Summary.

Among the various steps leading to

reorganization of Zentralblatt, the changeover to authors' reviews appears to be most worthy of serious consideration, particularly if such a change received international recognition and consequent financial support as a contribution to international cooperation. As a preliminary step it will be advantageous, on an experimental basis, to accept voluntary reviews from authors; in other words, authors publishing in a recognized mathematical journal should be given an opportunity to send to Zentralblatt an author's review, in German, English or French. The experience gained in this way will be useful in making subsequent decisions.

Heidelberg



NEWS ITEMS AND ANNOUNCEMENTS

SYMPOSIUM ON OPERATIONS ANALYSIS ON EDUCATION

Specialists in the fields of education and operations research will participate in a symposium on Operations Analysis of Education, sponsored by the U. S. Office of Education (OE), November 19-22, in Washington, D. C.

OE's National Center for Educational Statistics, under the direction of Alexander M. Mood, Associate Commissioner for Educational Statistics, will host the symposium through its Division of Operations Analysis. The symposium will include addresses by Charles J. Hitch, Vice President of the University of California, and former U. S. Commissioner of Education, Francis Keppel, currently Chairman of the Board, General Learning Corporation.

education, and specialized applications of computers.

The meetings will be held in the Washington-Hilton Hotel. Individuals interested in attending should request registration forms from the Symposium Committee, Division of Operations Analysis, U. S. Office of Education, Washington, D. C. 20202.

The impact of operations analysis and management sciences upon education and educational research will be examined in both general and specialized meetings. Special sessions will be devoted to such topics as multivariate analysis in educational research; models of educational processes; systems analysis in university planning; economics of education; cost-effectiveness and cost-benefit analysis of

LETTERS TO THE EDITOR

Editor, the *Notices*)

Before 1940 it was a tradition, if not a policy of the Society, that all non-research articles appearing in Society periodicals should be in English. Since the inauguration of the Mathematical Reviews this tradition has been largely abandoned. It is my opinion that the tradition is a sound one and deserves to be restored.

The non-research periodical publications of the Society are the Reviews, the *Notices*) and the departments of the Bulletin (including special issues) other than that of research announcements. To gain the full value of these publications the Society member should expect to be able to read them without language encumbrance, which means in his native or adopted tongue. In reading any single issue of the Reviews, the member will find himself faced with articles in three languages in addition to English.

I have no doubt that some will object to this view on the basis that the Reviews, in particular, has an international readership. To answer this objection it suffices to point out that foreign mathematicians would inevitably benefit by having the Reviews published in a single universally read language. At present there are two mathematical journals of review besides the Reviews: the *Zentralblatt*, which publishes in the same four languages sanctioned by the Reviews, and the *Referativnyi*, which publishes in Russian alone.

Recently, in a letter to the President of the Society I proposed that all non-research articles not in English suitable for publication in Society journals be translated into English at the Society's expense before being published. I urge all Society members who have an opinion to express on this proposal to communicate their views to the President.

In a recent issue of the *Notices*) there is a letter suggesting in effect that the Reviews be replaced by a journal of

abstracts. It is my opinion that such a change would be a mistake. We are living in a fascinating age of mathematics, one of the most exciting, in fact, since the great heyday of Euler. I am not just speaking of the feverish activity in contemporary mathematics; I am speaking rather of the quality of the best work being produced. The best mathematical work of the present age is equal to the best produced at any time during the past two centuries. I have only to mention the recent work of Feit and Thompson (*Pacific J. Math.*, vol. 13, 1963, pp. 775-1029), P. J. Cohen (*Proc. Nat. Acad. Sci.*, vol. 50, 1963, pp. 1143-1148; second part, vol. 51, 1964, pp. 105-110), and Ax and Kochen (*Amer. J. Math.*, vol. 87, 1965, pp. 605-630). What better justification for the Reviews is there than the masterly reviews of these three papers by Suzuki (*Math. Rev.*, vol. 29, 1965, no. 3538), Mostowski (*Math. Rev.*, vol. 32, 1966, no. 1118 and 2962), and Armitage (*Math. Rev.*, vol. 32, 1966, no. 2401)?

In a mathematical era such as the one in which we live it is urgent that our Society publish a periodical devoted to an analytical review of current mathematical research. The *Mathematical Reviews* serves this purpose and its continued publication deserves to be assured.

Eckford Cohen

Editor, the *Notices*)

The time and effort reports now being required result from Bureau of the Budget Circular A-21, as revised March 1965. They were agreed to after lengthy consultations between representatives of the Bureau of the Budget and Business Officers representing the universities. The Business Officers apparently did not realize the implication of these reports for academic personnel.

The Government's theory is that it

reimburses the universities for costs incurred for research, subject to criteria of allowability. The effort and time reports constitute such criteria. Not only professors but also deans and administrators grant that they should never have existed. However, if the professors do not sign these reports, the universities have been placed in such a position that they lose money already spent, and hence administrators have been pressuring professors into signing, even though "there isn't a single administrator in the country who does not realize that effort and time reports are double talk", as Dean Trottenberg of Harvard once said at an open meeting.

During this past year, I have written a number of letters (with cc list of more than 100 persons) listing in detail the practical and philosophical objections to the reports. Together with these letters, I have also distributed statements opposing the reports by chairmen of various math departments (Harvard, Brandeis, Yale, Cornell, Berkeley, etc.). I summarized these in my letter to Science Magazine, 17 February 1967.

The Association of Deans of Graduate Schools took a unanimous stand against the reports last fall, but instead of taking action, passed the buck to the University Presidents, who, as far as I am able to learn, have done nothing about the problem.

The threat to unfettered academic support does not only arise from certain pressures in the government, but from inadequate representation of academic interests by our own administrators. In the present instance, what seemed merely an unfortunate drift, and a mistake in good faith by the Business Officers, is threatening to become a clear betrayal of academic responsibilities by our top university administrators. Some sort of pressure at the professorial grass roots is therefore necessary.

Aside from writing my numerous letters, I have personally given up my contract last winter, even though at Columbia we have not yet been asked to sign the reports. Practically for me, this meant foregoing my 2/9th summer grant. I did this partly as a sympathetic gesture towards those colleagues in other universities who had been subjected to particularly obnoxious pressure by their administrators, and partly to strengthen my own position and influence, for whatever it is worth. Although so far I don't think any widespread cut back is necessary, I do believe that there should be a systematic refusal to sign the reports, for the reasons listed in my letter to Science, and to prevent the recognition of a *fait accompli*, from which no relief can be obtained.

Serge Lang



NEWS ITEMS AND ANNOUNCEMENTS

THE SECOND INTERNATIONAL SYMPOSIUM ON MULTIVARIATE ANALYSIS

The Aerospace Research Laboratories sponsored an International Symposium on Multivariate Analysis in June 1965. Several internationally known statisticians participated in that symposium.

The Aerospace Research Laboratories are now planning to sponsor the Second International Symposium on Multivariate Analysis at Dayton, Ohio during the period June 17-22, 1968. Only invited papers will be presented at the symposium. Several prominent statisticians working in different areas of multivariate analysis have either accepted or indi-

cated tentative acceptance of the invitations to present papers at the symposium. The papers will discuss both methodology and applications. It is expected that the Proceedings of the symposium will be edited and published as a bound volume subsequent to the symposium.

Attendance at the symposium is open to anyone interested. Further information may be obtained from Dr. P. R. Krishnaiah (Organizer), Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio 45433.

NEWS ITEMS AND ANNOUNCEMENTS

UNITED STATES--INDIA EXCHANGE OF SCIENTISTS AND ENGINEERS

The National Science Foundation has announced that the governments of India and the United States signed an agreement in February, 1967, for an exchange of scientists and engineers to promote increased communication between the scientific communities of the two countries. The national agencies which have been assigned responsibility for administering the program are the NSF and the Indian Council for Scientific and Industrial Research (CSIR).

Visits by scientists and engineers from one country to the other will be for periods of two weeks to several months, and the total man-days per year will be limited to 800 for each country. NSF will pay transportation costs to India for American participants and the CSIR will pay subsistence and travel costs within India. Similarly, NSF will host Indian visitors while in the United States.

Further information concerning the program may be obtained from the following address: United States--India Exchange, Office of International Science Activities, National Science Foundation, Washington, D. C. 20550.

DEPARTMENTAL SCIENCE DEVELOPMENT GRANTS

The National Science Foundation recently announced the award of the first four grants under the Departmental Science Development Program aimed at strengthening science and engineering programs at universities operating at the graduate level. Departmental Science Grants support a specific area of science or engineering in which strength already exists and is sufficient to serve as a base for significant further improvement to a higher level of capability.

One of the grants was awarded to the Department of Mathematics at the University of New Mexico. The project plan of the University of New Mexico is aimed at improving the Department of Mathematics through a program of expansion. The objective is to build a major center of research and instruction in applied mathematics, and the primary goal of the plan is to strengthen the mathematics faculty with the addition of nine permanent staff members. The program will also expand the mathematics library and the computer services for research.

NEW CHAIRMAN OF THE NAS-NRC DIVISION OF MATHEMATICAL SCIENCES

On July 1, R. H. Bing, Professor of Mathematics at the University of Wisconsin, succeeded Mark Kac as chairman of the Division of Mathematical Sciences, National Academy of Sciences--National Research Council.

Professor Bing, whose research interests have been chiefly in geometric topology, was elected to the National Academy of Sciences in 1965. He has been a member of the Institute for Advanced Study, vice president of the American Association for the Advancement of Science, chairman of the Conference Board of the Mathematical Sciences, and Council Member of the American Mathematical Society. At present he is vice president of the Society.

As a member of the Mathematical Association of America, Professor Bing served as president of the Association, chairman of the Wisconsin section, visiting lecturer and Hedrick lecturer, as well as representative of the Association as a member of the National Research Council. During the last decade he served on various committees of the National Research Council.

ANNUAL MEETING
OF THE DEUTSCHE
MATHEMATIKER VEREINIGUNG

The 1967 Annual Meeting of the Deutsche Mathematiker Vereinigung will be held in Karlsruhe, Germany, from September 11 to September 16, 1967. Further information on the meeting may be obtained from Professor W. L. Walter, Mathematisches Institut der Technischen Hochschule Karlsruhe, 75 Karlsruhe, Kaiserstr 12, Germany.

NRC-NBS POSTDOCTORAL ASSOCIATES

Fifteen outstanding young scientists have been selected for Postdoctoral Associateships in a program sponsored by the National Research Council of the National Academy of Sciences and the U. S. Department of Commerce's National Bureau of Standards. In the field of applied mathematics, George B. DeLancey of the University of Pittsburgh was selected.

The NRC-NBS program is designed to provide young scientists of exceptional ability with an opportunity for further study through advanced research under the guidance of senior NBS scientists. It is financially supported by NBS and administered by NRC.

SYMPOSIUM ON CODING THEORY

The Mathematics Research Center, U. S. Army, University of Wisconsin, will hold a symposium on coding theory on May 6-8, 1968, in Madison, Wisconsin. The symposium will emphasize recent advances in algebraic and combinatorial problems of coding theory. The chairman of the program committee is Professor H. B. Mann.

FIFTY-SIX SENIOR FOREIGN
SCIENTISTS RECEIVE FELLOWSHIPS
FOR U.S. WORK

The National Science Foundation recently announced that 56 senior foreign scientists will be teaching and conducting

research in universities throughout the United States through NSF fellowship awards. These scientists come to the U.S. from 17 countries in 5 continents. The purpose of bringing these scholars to the United States is to enable them to lend their talents to the improvement of scientific research and science education in the U. S. Fellowship holders selected under this program have distinguished records in formal education, research, and teaching.

Fellowship periods run from 5 to 12 months. Stipends are proportional to salaries of U. S. faculty members of similar status at the host institutions.

Three fellowship recipients will teach and participate in research in mathematics. Pierre E. Dolbeault, Professor of Mathematics, University of Poitiers, Biard, France, will conduct a seminar and do research on complex analytic manifolds at Northeastern University for 6 months starting September 1, 1967. Also starting September 1, 1967, Heinz Dombrowski, Professor of Mathematics, University of Göttingen, Germany, will visit Carnegie University (previously Carnegie Institute of Technology) for 12 months. He will conduct a seminar, deliver special lectures, and work with faculty members on the applications of modern mathematical methods to basic problems in physics. Morio Obata, Professor of Mathematics, University of Manitoba, Canada, will visit Lehigh University for 12 months beginning September 1968. He will teach graduate courses and supervise graduate students, conduct a weekly seminar, and participate in research in differential geometry.

NEW DIVISION CREATED IN
UNIVERSITY OF COLORADO
SCHOOL OF MEDICINE

The Board of Regents of the University of Colorado recently created a Division of Biostatistics and Epidemiology in the CU School of Medicine and named Dr. Strother H. Walker as its head. The new division will create an administrative structure for work already under way in these fields. It will be involved in educational and research efforts in the fields of biostatistics and epidemiology and in related disciplines such as biomathematics,

statistical methods and other biomedical studies. The division will be a part of the Department of Preventive Medicine and Comprehensive Health Care.

Organizing Committee, The George Washington University, Washington, D. C. 20006.

BERNARD FRIEDMAN MEMORIAL PRIZE

LECTURE SERIES IN MODERN ANALYSIS AND APPLICATIONS

The Consortium of Universities (American, Catholic, Georgetown, George Washington and Howard) in Washington, D. C., and the University of Maryland, in conjunction with the Air Force Office of Scientific Research, shall sponsor a lecture series in modern analysis and applications. It will consist of eight sessions of three lectures each, given on the first Saturdays of October, December, March, and May in the academic years 1967-1968 and 1968-1969. Each session will be devoted to an important active area of contemporary analysis which is important in application or shows potential application. Each lecture will present a survey and a critical review of certain aspects of that area, with emphasis on new results, open problems and applications. The following subject areas have been proposed:

- Modern methods and new results in complex analysis
- Banach algebras (topological algebras) and applications
- Topological linear spaces, distributions and their applications with particular emphasis on partial differential equations
- Analytical theory of semi-groups and applications
- Integration in function spaces and applications
- Potential theory and related areas (integral representations in convex sets and applications)
- Modern harmonic analysis and applications
- Singular perturbation theory

Preprints will be available shortly after each session for distribution to participants, and the entire series will be published by a commercial publisher. Additional information may be obtained from Professor C. T. Taam, Chairman,

The first Bernard Friedman Memorial Prize has been awarded to Ralph Ta-Shun Cheng, who has just received the Ph.D. degree from the Department of Mechanical Engineering at the University of California, Berkeley. Dr. Cheng was cited for the mathematical quality of his dissertation, "An investigation of the laminar flow around the trailing edge of a flat plate."

The prize consists of a cash award bestowed annually and is supported by a fund donated by friends of Professor Friedman. Additional contributions are welcome; they should be made out to The Bernard Friedman Memorial Fund and addressed to the Chairman, Department of Mathematics, University of California, Berkeley.

NEW MEXICO STATE UNIVERSITY GRADUATE PROGRAM IN COMPUTER SCIENCE

A computer science program leading to an M. S. degree has been initiated at New Mexico State University beginning with the 1967 fall semester. Both the program and the staff have an interdisciplinary nature with specialization areas in theory of algorithms, automata theory, logical design, threshold logic, numerical analysis, programming language and systems, and information systems.

The program will be given in conjunction with the University Computer Center, through which students will have access to CDC 3300 and IBM 1130 computing systems. Financial assistance is available both through the Computer Center and through teaching assistantships.

Further information about the program may be obtained from Dr. J. Mack Adams, Director, University Computer Center, New Mexico State University, University Park, New Mexico.

PERSONAL ITEMS

Professor SHREERAM ABHYANKAR of Purdue University has been appointed the Marshall Professor of Mathematics.

Professor E. S. ALLEN of Iowa State University has been appointed a visiting professorship at Wartburg College for the academic year 1967-1968.

Dr. F. L. ALT of the National Bureau of Standards, Washington, D. C. has accepted a position as Director of the Computer Applications Division of the American Institute of Physics, New York, New York.

Professor T. W. ANDERSON, JR. of Columbia University has been appointed to a Professor of Statistics and Economics at Stanford University. He will be on leave at the Imperial College of Science and Technology in London during the academic year 1967-1968.

Dr. D. C. BOSSARD of Dartmouth College has accepted a position as an Associate with David H. Wagner Associates, Paoli, Pennsylvania.

Mrs. F. M. CLARKE CARROLL of the Mitre Corporation, Bedford, Massachusetts has accepted a position as Senior Engineer in Operations Research at the Raytheon Company, Sudbury, Massachusetts.

Professor W. W. COMFORT of the University of Massachusetts has been appointed to a professorship at Wesleyan University.

Professor J. L. B. COOPER of the University of Toronto has been appointed to the Chair of Mathematics in the University of London tenable at Chelsea College of Science and Technology.

Dr. N. H. EGGERT of Colorado State College has been appointed to an assistant professorship at Montana State University.

Professor JOHN ERNEST of Tulane University has been appointed to an associate professorship at the University of California, Santa Barbara.

Professor W. T. FORD of the University of Houston has been appointed to an associate professorship at Texas Tech-

nological College.

Dr. J. L. GAMMEL of Austin, Texas has joined the staff of the Los Alamos Scientific Laboratory, Los Alamos, New Mexico to work in the Theoretical Physics Division.

Dr. R. T. HANSEN of Washington State University has been appointed to an assistant professorship at Montana State University.

Professor T. L. HICKS of Illinois State University has been appointed to an assistant professorship at the University of Missouri at Rolla.

Dr. J. C. HOLLADAY of the Institute of Defense Analyses, Arlington, Virginia, has been appointed to a professorship at the University of California, Irvine.

Professor R. A. HUNT of the University of Chicago has been appointed to an assistant professorship at Princeton University.

Dr. G. R. INGRAM of Montana State University has returned after a two year leave of absence and will be Director of the Computing Center.

Dr. KENKICHI IWASAWA of the Massachusetts Institute of Technology has been appointed to a professorship at Princeton University.

Dr. TATSUJI KAMBAYASHI of the University of Pisa, Italy has been appointed to a professorship at Northern Illinois University.

Dr. B. L. McALLISTER of the South Dakota School of Mines and Technology has been appointed to an associate professorship at Montana State University.

Professor J. S. MACNERNEY of the University of North Carolina at Chapel Hill has been appointed to a professorship at the University of Houston.

Professor J. K. MOSER of New York University has been appointed Director of the Courant Institute of Mathematical Sciences.

Dr. R. E. PEINADO of the University of Iowa has been appointed to an associate professorship at the University of Puer-

Rico.

Dr. L. V. QUINTAS of The City University of New York has been appointed to a professorship at Pace College.

Professor DURGA RAY of McGill University has been appointed to an assistant professorship at the University of Illinois.

Dr. B. H. RHODES of Lehigh University has accepted a position as an Associate with David H. Wagner Associates, Paoli, Pennsylvania.

Dr. G. H. RYDER of Clarkson College of Technology has been appointed to an assistant professorship at Montana State University.

Professor B. T. SIMS of San Jose State College has been appointed to an associate professorship at Eastern Washington State College.

Professor T. I. SEIDMAN of Wayne State University will be on leave for the academic year 1967-1968 as a Visiting Associate Professor at Carnegie University.

Dr. L. D. STONE of Purdue University has accepted a position as an Associate with David H. Wagner Associates, Paoli, Pennsylvania.

Dr. K. H. TIAHRT of Oklahoma State University has been appointed to an assistant professorship at Montana State University.

Professor D. M. TOPPING of the University of Washington has been appointed to an associate professorship at Indiana University.

Professor K. W. WEGNER of Carleton College will be on leave from September to mid-February at Virginia Union University in a special Inter-College Exchange Program.

Professor R. W. WEST of the University of California, Los Angeles, has been

appointed to an assistant professorship at the University of California, Irvine.

Professor R. J. WHITLEY of the University of Maryland has been appointed to an associate professorship at the University of California, Irvine.

PROMOTIONS

To Professor. University of Hawaii: E. H. MOOKINI.

To Associate Professor. University of Hawaii: W. J. LEAHEY, Z. Z. YEH; Lafayette College: A. E. LIVINGSTON, W. R. JONES; Lehigh University: S. L. GULDEN.

To Assistant Professor. Wisconsin State University—Whitewater: T. L. MC-FARLAND.

To Instructor. University of Chicago: TSIT-YUEN LAM; Fairmont State College: D. A. MUEHLBAUER; University of Wisconsin-Waukesha: G. S. GLAZER.

DEATHS

Professor Emeritus C. M. HEBBERT of Brooklyn Polytechnic Institute died on April 3, 1967 at the age of 76. He was a member of the Society for fifty two years.

Professor J. P. RUSSELL of Brooklyn Polytechnic Institute died on April 15, 1967 at the age of 45. He was a member of the Society for eighteen years.

Professor C. C. TORRANCE of the U. S. Naval Post-graduate School, died on May 2, 1967 at the age of 65. He was a member of the Society for forty years.

Mr. G. C. WOLPIN of the University of California, Berkeley, died on February 13, 1967 at the age of 22.

NEWS ITEMS AND ANNOUNCEMENTS

HELP WANTED

The Society wishes to announce the availability of a position as Associate Editor of Mathematical Reviews, in Ann Arbor. The responsibilities of an Associate Editor include the mathematical classification of journal articles and the assign-

ing of them to appropriate reviewers. Fluency in reading Russian is an essential requirement for the position. Persons interested in obtaining further details should contact a member of the Mathematical Reviews Editorial Board or the Executive Editor.

MEMORANDA TO MEMBERS

Backlog of Mathematical Research Journals

Information on this important matter is being published twice a year, in the February and August issues of the *Notices*, with the kind co-operation of the respective editorial boards.

It is important that the reader should interpret the data with full allowance for the wide and sometimes meaningless fluctuations which are characteristic of them. Waiting times in particular are affected by many transient effects, which arise in part from the refereeing system. Extreme waiting times as observed from the published dates of receipt of manuscripts may be very misleading, and for that reason, no data on extremes are presented in the table at the bottom of this page.

Some of the columns in the table are not quite self-explanatory, and here are some further details on how the figures were computed.

Column 2. These numbers are rounded off to the nearest 50.

Column 3. For each journal, this is the estimate as of the indicated dates, of the total number of printed pages which will have been accepted by the next time that manuscripts are to be sent to the printer, but which nevertheless will not be sent to the printer at that time. (Pages

received but not yet accepted are being ignored.)

Column 4. Estimated by the editors (or the Editorial Department of the American Mathematical Society in the case of the Society's journals) and based on these factors; manuscripts accepted, manuscripts received and under consideration, manuscripts in galley, and rate of publication. There is no fixed formula.

Column 5. The first quartile (Q_1) and the third quartile (Q_3) are presented to give a measure of the dispersion which will not be too much distorted by meaningless extreme values. The median (Med.) is used as the measure of location. The observations were made from the latest issue received in the Headquarters Offices before the deadline date for this issue of the *Notices*. The waiting times were measured by counting the months from receipt of manuscript in final revised form, to month in which the issue was received at the Headquarters Offices. It should be noted that when a paper is revised, the waiting time between receipt by editors of the final revision and its publication may be much shorter than is the case for a paper which is not revised, so these figures are to that extent distorted on the low side.

JOURNAL	1 No. issues per year	2 Approx. no. pages per year	3 BACKLOG		4 Est. time for paper submitted currently to be published (in months)	5 Observed waiting time in latest published issue (in months)		
			6/31/67	12/30/66		Q_1	Med.	Q_3
			year	year				
American J. Math.	4	1200	923	810	15	14	16	20
Annals of Math.	6	1200	0	200	10	8	9	12
Annals of Math. Stat.	6	2000	0	0	5-9	6	7	10
Arch. Hist. Exact Sciences	not fixed	400	0	0	5	***	12	***
Arch. Rational Mech. Anal.	not fixed	1300	0	0	4	6	7	9
Canad. J. Math.	6	1300	1000	500	18	14	16	17
Duke Math. J.	4	700-800	478	304	12-15	12	13	13
Illinois J. Math.	4	725	800	510	22	14	15	17
J. Math. Analysis and Appl.	12	2310	1300	1800	7	**	**	**
J. Math. and Mech.	12	1200	200	970	6	8	8	9
J. Math. and Physics	4	400	350	500	8-12	13	13	18
J. Mathematical Physics	12	NR*	NR*	NR*	NR*	10	12	13
J. Symbolic Logic	4	650	0	300	12	10	13	22
Math. of Comp.	4	750	0	0	8	7	8	9
Michigan Math. J.	4	500	250	NR*	15	10	12	15
Pacific J. Math.	12	2250	NR*	NR*	12-15	15	16	22
Proceedings of AMS	6	1152	770	340	16	12	13	14
Quarterly of Appl. Math.	4	400	200	NR*	12	11	12	14
SIAM J. on Appl. Math.	6	1550	375	360	8-10	8	10	11
SIAM J. on Control	4	700	0	0	6	6	7	7
SIAM J. on Numer. Anal.	4	725	125	130	5-8	8	8	12
SIAM Review	4	725	25	50	4-6	9	10	11
Transactions of the AMS	12	2750	1000	400	14	13	16	20

* NR means that no response was received to a request for information

** Dates of receipt of manuscripts not indicated in this journal

*** The most recent issue of this journal consisted of one long article

Visiting Foreign Mathematicians

The following list contains the names and addresses of foreign mathematicians who will be visiting in the United States in 1967-1968. It has been compiled from the response received to the request for names which appeared in these *Notices* for June, 1967. These names will also be included in the more complete annual list to be published in these *Notices* for November, 1967.

NAME AND HOME COUNTRY	HOST INSTITUTION	PERIOD OF VISIT
Aczel, Peter (England)	University of Wisconsin, Madison	Sept. 1967-June 1968
Adjan, S. I. (Russia)	University of Illinois	
Aubin, Jean-Pierre (France)	Purdue University	Sept. 1967-June 1968
Bender, Helmut (Germany)	University of Illinois	Sept. 1967-June 1968
Bhargava, Mira (India)	University of Florida	Sept. 1967-June 1968
Brelot, Marcel E. (France)	Rutgers, The State University	Sept. 1967-Jan. 1968
Britton, John L. (England)	University of Illinois	Feb. 1968-April 1968
Brown, Gavin (England)	University of Illinois	Sept. 1967-June 1968
Camina, Alan R. (England)	University of Illinois	Sept. 1967-June 1968
Charatonik, J. J. (Poland)	University of Kentucky	Aug. 1967-May 1968
Dinghas, Alexander (Germany)	Wayne State University	Sept. 1967-March 1968
Dowson, Henry R. (England)	University of Illinois	Sept. 1967-June 1968
Edmunds, Gwilym (England)	Purdue University	Sept. 1967-June 1968
Essen, Matts (Sweden)	University of Illinois	Sept. 1967-June 1968
Farahat, H. K. (England)	University of Illinois	Sept. 1967-June 1968
Fullerton, Gordon H. (England)	University of Illinois	Sept. 1967-June 1968
Gagen, Terence M. (Australia)	University of Illinois	Sept. 1967-June 1968
Garbe, Dietmar (Germany)	University of Notre Dame	Sept. 1967-June 1968
Groh, Hansjoachim (Germany)	University of Florida	Sept. 1967-June 1968
de Groot, J. (Netherlands)	University of Florida	Nov. 1967-April 1968
Hanna, Azmi I. (Lebanon)	University of Florida	Sept. 1967-June 1968
Helling, Heinz (Germany)	University of Notre Dame	Sept. 1967-June 1968
Igari, Satoru (Japan)	University of Wisconsin, Madison	Sept. 1967-June 1968
Knebusch, Manfred (Germany)	University of Notre Dame	Sept. 1967-June 1968
Koszul, Jean-Louis (France)	University of Notre Dame	Sept. 1967-Nov. 1967
Kunz, Ernest (Germany)	Purdue University	Sept. 1967-June 1968
Lamont, Patrick (England)	University of Notre Dame	Sept. 1967-June 1968
Laudal, Olav A. (Norway)	University of Illinois	Feb. 1968-June 1968
Leviatan, Dany (Israel)	University of Illinois	Sept. 1967-June 1968
Lickorish, W. B. R. (England)	University of Wisconsin, Madison	Sept. 1967-June 1968
Londen, Stig-Olaf (Sweden)	University of Wisconsin, Madison	Sept. 1967-June 1968
Mahler, K. (Australia)	University of Arizona	Sept. 1967-Jan. 1968
Mann, Avino'am (Israel)	University of Illinois	Sept. 1967-June 1968
Maric, Vojislav (Yugoslavia)	University of Kentucky	Aug. 1967-Dec. 1967
Meier, Kurt (Switzerland)	Wayne State University	Sept. 1967-March 1968
Mennicke, Jens (Germany)	University of Notre Dame	Sept. 1967-June 1968
Meyer-Konig, Werner M. (Germany)	University of Wisconsin, Milwaukee	Sept. 1967-June 1968
Murase, Ichiro (Japan)	University of Florida	Sept. 1967-June 1968
Novikov, P. S. (Russia)	University of Illinois	
Oberschelp, Walter (Germany)	University of Illinois	Sept. 1967-June 1968
Petersson, Hans (Germany)	University of Notre Dame	Sept. 1967-June 1968
Pittnauer, Franz (Germany)	University of Wisconsin, Madison	Sept. 1967-June 1968
Popp, Herbert (Germany)	Purdue University	Sept. 1967-June 1968
Pretzel, Oliver (England/Germany)	University of Illinois	Sept. 1967-June 1968
van der Put, Marius (Netherlands)	Purdue University	Feb. 1968-June 1968

NAME AND HOME COUNTRY	HOST INSTITUTION	PERIOD OF VISIT
Richberg, Rolf (Germany)	University of Notre Dame	Sept. 1967- June 1968
Rieger, Georg J. (Germany)	State University of New York, Buffalo	June 1967- June 1968
Schmidt, Roland (Germany)	University of Illinois	Sept. 1967- June 1968
Stark, Richard (Austria)	University of Florida	Sept. 1967- June 1968
Stetter, Franz (Germany)	University of Chicago, Argonne National Laboratory	May 1967-May 1968
Tobiasson, Leif (Sweden)	East Carolina College	Sept. 1967-May 1968
Varadarajan, K. (India)	University of Illinois	Sept. 1967- June 1968
Varma, A. K. (Canada)	University of Florida	Sept. 1967- June 1968
Verdenius, Wibbe (Netherlands)	University of Florida	Sept. 1967- June 1968
Yamazaki, K. (Japan)	University of Florida	Sept. 1967- June 1968

DOCTORATES CONFERRED IN 1966

(Supplementary List)

UNIVERSITY OF CALIFORNIA, LOS ANGELES (1)

Kugler, Lawrence D.
Non-standard analysis of almost periodic functions

UNIVERSITY OF COLORADO (12)

Callas, Nicholas Phillip
Minor: Physics
Singular points of certain functions represented by C-fractions

Davis, Ronald Webb
Minor: Physics
On the Matrix Congruence $X^e = (\text{mod } pk)$

Dooher, Terrence Enroth
Proximity relations on an abstract lattice

Franzen, Norman Ray
Some contributions to the theory of Padé tables

La Grange, Robert Hamor, Jr.
Minor: Philosophy
Some problems concerning Boolean algebras

McKenzie, Ralph Nelson Whitfield
The representation of relation algebras

Reed, Ellen Elizabeth
Minor: Physics
Uniform structures

Stevenson, Frederick William
Uniform spaces with linearly ordered bases

Department of Applied Mathematics

Elliott, Donald Dale
On a class of singular integral operators

Gibson, Archie Gail
Triples of operator-valued functions related to the unit circle

Hagin, Frank Gordon
Invariant imbedding and the asymptotic behavior of solutions to initial value problems

Modeer, James Richard
Lattice and ring characterizations of Nevanlinna space

THE JOHNS HOPKINS UNIVERSITY (2)

Department of Biostatistics

Francis, Mildred E.
A stochastic model for the estimation of platelet survival in vivo

Sykes, Zenas M., Jr.
The variance of population projections

UNIVERSITY OF MICHIGAN (7)

Department of Communication Sciences

Allen, George Douglas
Two behavioral experiments on the location

of syllable beat in conversational American English

Birdsall, Theodore Gerald
The theory of signal detectability: ROC curves and their character

Boyd, John Paul
The algebra of kinship

Give'on, Jehoshafat
On some categorical algebra aspects of automata theory: The categorical properties of transition systems

Hedetniemi, Stephen
Homomorphism of graphs and automata

Rothenberg, Martin
The breath-stream dynamics of simple-released-plosive production

Stanat, Donald Ford
Nonsupervised pattern recognition through decomposition of probability functions

UNIVERSITY OF PENNSYLVANIA (6)

Department of Electrical Engineering, and Computer and Information Sciences

Amoroso, Serafino
A theory of a parallel acting automaton

Beizer, Boris

A new theory for analysis, synthesis, cutting and splicing of sequential switching networks

Booth, Theodore
Hazards in sequential relay circuits

Castle, James
Automatic generation of command and control programs

Korsh, James
Compound decision theory and discrete time Markov processes

Van Dam, Andries
A study of digital processing of pictorial data

UNIVERSITY OF TEXAS (30)

Boullion, Thomas Loris
Contributions to the theory of pseudo inverses

Bowman, Robert H.

Minor: Physics
Contribution to the theory of extensive differentiation and related topics

Carlisle, Charles H.
Minor: Physics
A lattice of partitions on a group

Cude, Joe E.
Minor: Chemistry
Compact integral domains with finite characteristics

Davis, Roy Dale
Minor: Physics

- Concerning the sides from which sequences of arcs converge to a compact irreducible continuum
- Davis, Tommy F.
Minor: Education
 ϕ -derivations on a ring
- Falconer, David Ross
Minor: Physics
Contributions to the theory of numerical simulation
- Gray, Henry L.
Minor: Education
The Holmgren-Riesz transform and its applications
- Hadlock, Frank Owen
Minor: Engineering
Realization of sequential machines with threshold elements
- Harrell, Carl E.
Minor: Business Administration
Concerning sequence to sequence transformation
- Kowalik, Virgil C.
Minor: Physics
Concerning the nonlinear differential equation $H'(x) = \phi(x;H(x))$, with elements in E^2
- Lewis, Truman O.
Application of the theory of generalized matrix inversion to statistics
- Mayes, Vivienne, L. M.
Minor: Chemistry
A structure problem in asymptotic analysis
- McEwen, Henry N.
Minor: Biology
Automorphisms of a class of finite groups
- Meicler, Marcel
Weighted generalized inverses with minimal p and q norms
- Miller, Max Karlson
Minor: Physics
Numerical inversion of the Laplace transform by the use of Jacobi polynomials
- Morris, Peter Don
Spaces of continuous functions on scattered spaces
- Peek, Darwin E.
Minor: Physics
Pointwise limits of sequences of continuous functions
- Reed, Coke Stevenson
Minor: Education
Concerning pointwise limits of sequences functions
- Roach, Francis A.
Minor: Business Administration
Concerning the value regions associated with a certain type of continued fraction
- Rogers, Jack W.
Minor: Business Administration
A space whose regions are the simple domains of another space
- Scheiblich, H. Edward
Minor: Actuarial Science
Congruences on an inverse semi-group
- Secker, Martin D.
Minor: History
Reversibly continuous bisensed transformations of an annulus into itself
- Sheffield, Miller
Minor: Petroleum Engineering
Block partitioned matrix theory and solution of systems of certain partial differential equations.
- Sister Mary Molloy of the Assumption
Minor: Physics
Substitution theorem for a Stieltjes integral
- Steib, Michael Lee
Minor: Business Administration
Linear transformations in a function space
- Tatikonda, Lakshmi
Interval estimation of a stimulus levels of order in sensitivity testing
- Tucker, Charles Thomas, II
Minor: English
Pointwise limits of quasi-continuous functions
- Williams, Bennie B.
Minor: Education
An integral equation and the related differential equation boundary problem
- Wulbert, Daniel E.
Continuity of metric projections-approximation theory in a normed linear lattice

YALE UNIVERSITY (1)

Department of Statistics

Olshen, Richard A.

Asymptotic properties of the periodogram of a discrete stationary process

Errata

The following is a correction of the information given under Georgetown University in the June issue of these *Notices*.

Welch, John N.

Compact solutions of nonlinear differential equations in Banach spaces

NEW AMS PUBLICATIONS

MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY

Number 72

THE METASTABLE HOMOTOPY OF S^n
By Mark Mahowald

84 pages; List Price \$1.60; Member Price \$1.20.

The stable homotopy of S^n refers to the collection of homotopy groups $\pi_{j+2n}(S^n)$, $-1 \leq j \leq n-3$. In this range the homotopy groups depend on n but do exhibit some regular behavior. The purpose of this Memoir is to study this behavior. Some general results are obtained and, in addition, a lot of calculations are made. The particular $[i_n, \theta]$ is determined for n large and $|n-2^i| > 10$ for all i and $\theta \in \pi_j^S$, $j < 20$. A few ambiguities remain if n is near a power of 2. In addition the stable groups $\pi_{j+n}^S(\mathbb{R}P/\mathbb{R}P^{n-1})$ are computed for $j \leq 29$. These results give the first 29 unstable groups of $SO(n)$, n large. Unstable groups for $n \leq 40$ of S^n are given which extend the calculations of Toda and others.

Number 73

TWO PAPERS ON SIMILARITY OF CERTAIN VOLTERRA INTEGRAL OPERATORS

By Stanley J. Osher

72 pages; List Price \$1.60; Member Price \$1.20.

Volterra integral operators play a role in the general theory of non-selfadjoint operators which is analogous to that of nilpotent matrices in the theory of

finite dimensional non-selfadjoint operators. In these papers the similarity properties of a certain class of these operators are examined. Necessary and sufficient conditions for two such operators to be similar are obtained, and consequently some results on invariant subspaces are inferred.

Number 74

THE MOD 2 COHOMOLOGY STRUCTURE OF CERTAIN FIBRE SPACES

By W. S. Massey and F. P. Peterson

100 pages; List Price \$1.70; Member Price \$1.28.

This Memoir is devoted to the problem of determining the mod 2 cohomology of certain fibre spaces in terms of the cohomology of the base space, the fibre, and certain invariants of the fibre space. Special attention is given to the study of the operation of the Steenrod algebra on the mod 2 cohomology algebras of the various spaces involved. In Part I, the authors complete in an essential respect the main theorem of their previous joint paper on this topic (Topology, Vol. 4 (1965), pp. 47-65). In Part II these results are used to discuss the cohomology structure of 2-stage Postnikov systems. In Part III the results are applied to the construction of an unstable version of the Adams spectral sequence.

Volume 87

UNSTEADY MOTIONS OF COMPRES-
SIBLE MEDIA WITH BLAST WAVES

By L. I. Sedov et al.

124 pages; List Price \$5.80; Member
Price \$4.35.

The authors of the four articles in
this book develop effective methods for the

solution of nonlinear problems in the
motion of gases and liquids for cases where
thermodynamical properties play an im-
portant role. Numerical tables and graphs
are given for many of the functions neces-
sary in practical applications. The methods
and results are in continuation of Sedov's
monograph on similarity and dimensional-
ity in mechanics.

Recent Reprints

MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY

Number 8

THE REPRESENTATION PROBLEM FOR
FRECHÉT SURFACES

By J. W. T. Youngs

143 pages; List Price \$3.20; Member
Price \$2.40.

The subject matter of this Memoir
deals with the representation problem for
a collection of Frechét varieties known
as Frechét surfaces; however, an active
effort is made to offer more than a solu-
tion to the representation problem. In the
first chapter a measure of attention is
directed towards an indication of how the
pattern of research on this problem de-
veloped during the decades following the
initial major assault on the problem by
Kerékjártó. In addition, an attempt is made
to indicate those difficulties which initiated
the introduction of the topological tools
here employed.

Active use is made of both analytic
and algebraic topology; specifically, of the
cyclic element theory and the Čech and
locally compact cohomology theories.

Number 12

GEOMETRY OF MOMENT SPACES

By S. Karlin and L. S. Shapley

93 pages; List Price \$2.30; Member
Price \$1.73.

The purpose of this Memoir is to
present an approach through the geometry

of convex sets to the theory of distribution
functions and orthogonal polynomials.
The method provides an interesting con-
trast to the more usual techniques involv-
ing continued fractions and complex vari-
ables. The work is concerned primarily
with distribution functions on a finite
interval.

Number 18

THE LATTICE THEORETIC BACK-
GROUND OF THE DIMENSION THEORY

By L. H. Loomis

36 pages; List Price \$1.60; Member
Price \$1.20.

It is the purpose of this Memoir
to give the von Neumann-Murray dimen-
sion theory an abstract setting which is
as simple and natural as possible. A com-
plete orthocomplemented lattice is studied
which bears a completely additive con-
gruence relation (\sim) satisfying certain
natural axioms suggested by the operator
algebra prototype. The theory includes
most of the standard theorems of the type
I, II, III, classification theory. However,
it fails to reduce to continuous geometry
in the finite cases, and for this reason it
includes also the corresponding part of the
work of Maharam on the classification of
Boolean algebras under an equivalence
relation.

Supplementary Program-Number 47

During the interval from April 28 to June 30, 1967, the papers listed below were accepted by the American Mathematical Society for presentation by title. After each title on this program there is an identifying number. The abstracts of the papers will be found following the same number in the section on Abstracts of Contributed Papers in this issue of these *Notices*.

One abstract presented by title may be accepted per person per issue of these *Notices*. Joint authors are treated as a separate category; thus in addition to abstracts from two authors individually one joint abstract by them may be accepted for a particular issue.

- (1) On the cofinality of ordinal numbers
Professor Alexander Abian, Iowa State University (67T-538)
Dr. M. Bhaskaran, University of Illinois (67T-536)
(Introduced by Professor R. R. Rao)
- (2) On the problem of characterizing hyperelliptic Riemann surfaces in terms of vanishing theta-nulls. Preliminary report
Professor R. D. M. Accola, Brown University (67T-480)
- (3) Concerning a class of linear transformations
Professor W. D. L. Appling, North Texas State University (67T-501)
- (4) Sigma-finite invariant measures
Dr. L. K. Arnold, Daniel H. Wagner, Associates, Paoli, Pennsylvania (67T-488)
- (5) A Tauberian theorem for a class of entire functions. Preliminary report
Mr. Albert Baernstein, II, Wisconsin State University - Whitewater (67T-498)
- (6) Products of positive definite matrices. III
Professor C. S. Ballantine, Oregon State University (67T-544)
- (7) Powers of nonnegative matrices with special symmetries
Professor Sami Beraha, Ball State University (67T-472)
- (8) On adjoints of functors between functor categories
Mr. Pierre Berthiaume, University of Montreal (67T-505)
- (9) Continuity properties of operator spectra. Preliminary report
Dr. N. J. Bezak and Professor M. Eisen, Gulf Research and Development Company, Pittsburgh, Pennsylvania (67T-543)
- (10) Diagonal forms over P-adic rings. Preliminary report
Professor J. R. Blum, University of New Mexico, and Dr. J. A. Schatz, Sandia Corporation, Albuquerque, New Mexico (67T-557)
- (11) On orthogonal arrays of odd index
Professor C. J. R. Borges, University of California, Davis (67T-500)
- (12) On continuously semimetrizable spaces and stratifiable spaces
Professor R. C. Bose, University of North Carolina (67T-496)
- (13) Descendents of strongly regular graphs
Génération des matroides binaires et de leurs orthogonaux
Mr. C. P. Bruter, University of Waterloo (67T-459)
(Introduced by Professor T. T. Tutte)
- (14) On the lattice of partitions of finite degree
Professor Maurice Chacron, University of Sherbrooke (67T-513)
- (15) Fourier-Bessel series of generalized functions
Professor C. M. Chambers and Mr. G. L. Finch, University of Alabama (67T-508)
- (16) Almost uniquely complemented lattices
Mr. C. C. Chen, Queen's University (67T-458)
- (17) Construction of Stone lattices. II
Mr. C. C. Chen, Queen's University, and Professor G. A. Grätzer, University of Manitoba (67T-525)
- (18) The cardinality of the set of left invariant means on a semigroup
Mr. Ching Chou, University of Rochester (67T-467)

- (20) Characteristic manifolds in non-equilibrium hydrodynamics
Professor Nathaniel Coburn, University of Michigan (67T-463)
- (21) On injective envelopes of Banach spaces
Professor H. B. Cohen, University of Pittsburgh and Mr. H. E. Lacey, University of Texas (67T-494)
- (22) Norm preserving extensions of linear operators on Hilbert spaces. Preliminary report
Professor M. G. Crandall, Stanford University (67T-554)
- (23) Universes of Ehresmann-Dedecker
Professor N. C. A. da Costa, University of Paraná, Brazil (67T-515)
- (24) On Jaśkowski's discursive propositional calculus
Professor N. C. A. da Costa, University of Paraná, Brazil and Mr. L. Dubikajtis, University N. Copernicus, Toruń, Poland (67T-516)
- (25) The Dubois-Reymond and Euler-Lagrange equations for higher order derivatives
Professor John DeCicco, Illinois Institute of Technology, and Mr. Arun Walvekar, Bombay, India (67T-521)
- (26) Some results on central and parallel fields of force in a Euclidean space of several dimensions
Professor John DeCicco, Illinois Institute of Technology, and Professor G. E. Culbertson, U. S. Naval Academy, Annapolis, Maryland (67T-455)
- (27) Complex analysis methods applied to the study of branching processes
Dr. Giacomo Della Riccia, Indiana University (67T-549)
- (28) Some extremal properties not possessed by regular polygons
Mr. C. F. Dunkl, Princeton University, and Mr. J. F. Fournier, University of Wisconsin (67T-528)
- (29) Continuity of polynomial operators in Banach spaces. Preliminary report
Professor E. J. Eckert, California State College, Los Angeles (67T-546)
- (30) A Weierstrass-Stone theorem for compact convex sets
Dr. D. A. Edwards, University of Oxford, England, and Mr. G. Vincent-Smith, University of Copenhagen, Denmark (67T-493)
- (31) Multiplications on additive groups
Professor Steven Feigelstock, Polytechnic Institute of Brooklyn (67T-490)
- (32) Envelopes of holomorphy on Stein spaces
Mr. E. R. Fernholz, Columbia University (67T-520)
- (33) Intuitionistic models and independence results in set theory
Mr. M. C. Fitting, Yeshiva University (67T-497)
- (34) Generalized compactification for weak topologies
Dr. Isidore Fleischer, Fleischer Foundation, Brooklyn, New York (67T-461)
- (Introduced by Professor R. H. McDowell)
- (35) Measure and integration in non-archimedean Banach spaces
Mr. Martin Fried and Professor George Bachman, Polytechnic Institute of Brooklyn (67T-511)
- (36) Some properties of the concept of class
Dr. Baruch Gershuni, Amsterdam, Netherlands (67T-486)
- (37) Irredundant intersections of valuation rings
Professor R. W. Gilmer, Florida State University, and Professor William Heinzer, Louisiana State University, Baton Rouge (67T-470)
- (38) Equational spectra and reduction of identities
Professor G. A. Grätzer, The University of Manitoba, and Dr. R. N. McKenzie, University of California, Berkeley (67T-469)
- (39) Hill equations with coexisting periodic solutions
Professor H. W. Guggenheimer, University of Minnesota (67T-478)
- (40) A faithful matrix representation for certain soluble groups. Preliminary report
Mrs. C. K. Gupta, Australian National University and University of Manitoba (67T-474)
- (Introduced by Professor B. H. Neumann)
- (41) Periodicity of the commutator subgroup of a certain group
Dr. N. D. Gupta, Australian Na-

- tional University and University of Manitoba (67T-487)
- (Introduced by Professor B.H. Neumann)
- (42) On f -injective modules and semihereditary rings
Mr. R. N. Gupta, Ramjas College, Bombay, India (67T-552)
- (Introduced by Professor P. C. Jain)
- (43) Positive operator products
Professor K. E. Gustafson, Battelle Institute, Geneva, Switzerland (67T-531)
- (44) On Lie algebras of derivations. Preliminary report.
Mr. Charles Hallahan, Rutgers, The State University (67T-540)
- (45) Topologies for certain integer-valued measures
Dr. T. E. Harris, University of Southern California, Los Angeles (67T-485)
- (46) Generalizations of weakly compact operators
Dr. R. H. Herman, University of Rochester (67T-553)
- (47) Quotient uniformities. II
Professor Charles Himmelberg, University of Kansas (67T-558)
- (48) Projectivities of free products
Mr. C. S. Holmes, Miami University (67T-471)
- (49) Ordinary representations of algebraic linear groups
Professor J. E. Humphreys, University of Oregon (67T-507)
- (50) Backward continuous dependence for mixed parabolic problems
Professor A. E. Hurd, University of California, Los Angeles (67T-466)
- (51) Continuity of derivations and a problem of Kaplansky
Dr. B. E. Johnson and Mr. A. M. Sinclair, The University, Newcastle upon Tyne, England (67T-479)
- (52) Degrees of undecidability of number theories. Preliminary report
Mr. J. P. Jones, University of Washington (67T-532)
- (Introduced by Professor R. W. Ritchie)
- (53) Equational classes of lattices
Professor Bjarni Jonsson, Vanderbilt University (67T-491)
- (54) Semiderivation is a representable functor. Preliminary report
Dr. Tatsuji Kambayashi, University of Pisa (67T-504)
- (55) On the global existence of solutions and Liapunov functions
Professor J. Kato, Tohoku University, Japan and Professor Aaron Strauss, University of Maryland (67T-556)
- (56) On mappings with diminishing orbital diameters
Professor W. A. Kirk, University of California, Riverside (67T-527)
- (57) A complete R_Δ -harmonically immersed surface in E^3 on which $H \neq 0$
Professor Tilla Klotz, University of California, Los Angeles (67T-519)
- (58) The density of meteorite impacts on the moon
Professor A. M. Krall, Pennsylvania State University (67T-465)
- (59) A generalization of Beth's theorem on definability
Mr. D. W. Kueker, University of California, Los Angeles (67T-535)
- (60) On completion of measure spaces
Professor Y.-L. Lee, Kansas State University (67T-503)
- (61) Generalized quantifiers and weak direct products
Mr. L. D. Lipner, University of California, Berkeley (67T-537)
- (62) On Poincaré duality for generalized homology theories. Preliminary report
Professor M. C. McCord, University of Georgia (67T-547)
- (63) Lower bound for constrained minimization problems
Dr. O. L. Mangasarian, Shell Development Company, Emeryville, California (67T-477)
- (64) On the derivative of canonical products
Professor Morris Marden, University of Wisconsin-Milwaukee (67Y-514)
- (65) An application of the Bergman-Whittaker operator
Professor V. S. Maric, University of Kentucky (67T-534)
- (66) WITHDRAWN.
- (67) A necessary condition for a Banach space to be projective
Professor Nick Metas, Queens

- College (67T-551)
- (68) A countable locally connected quasi-metric space
Mr. Gary Miller, University of Missouri at Kansas City (67T-541)
(Introduced by Professor B. J. Pearson)
- (69) A proximate fixed-point theorem for multifunctions
Mr. T. B. Muenzenberger, University of Florida (67T-530)
(Introduced by Dr. R. E. Smithson)
- (70) Fixed points and proximate fixed points
Mr. T. B. Muenzenberger and Dr. R. E. Smithson, University of Florida (67T-529)
- (71) A necessary and sufficient condition for a family of sample operators on a separable Banach space to be a random operator
Mr. Arunava Mukherjea, Eastern Michigan University (67T-464)
- (72) Applications of number theory to aesthetics. II
Captain A. A. Mullin, U. S. Department of Defense (67T-545)
- (73) On the lattice $D(X)$
Professor Philip Nanzetta, Case-Western Reserve University (67T-502)
- (74) A new representation of Hurwitz's determinants in the expansion of certain continued fractions
Professor Israel Navot, Israel Institute of Technology, Haifa (67T-523)
- (75) Minimal elements in the many-one degrees of the predicates $H_a(x)$. Preliminary report
Mr. G. G. Nelson, Case-Western Reserve University (67T-483)
- (76) Surfaces with local mapping cylinder neighborhoods are tame
Professor V. A. Nicholson, University of Iowa (67T-475)
(Introduced by Professor T. M. Price)
- (77) A Phragmén-Lindelöf theorem for a plane half strip
Mr. J. K. Oddson, University of Maryland (67T-462)
- (78) Two identities for lattices
Mr. R. Padmanabhan, Madurai University, India (67T-468)
(Introduced by Professor G. A. Grätzer)
- (79) Hypersimplicity as a necessary and sufficient condition for nonindependent axiomatization. I
Professor M. B. Pour-El, University of Minnesota (67T-524)
- (80) Weakly almost periodic functions on discrete groups
Mr. D. E. Ramirez, University of Washington (67T-481)
- (81) A class of spaces determined by sequences with their cluster points. Preliminary report
Mr. T. W. Richel, University of Pittsburgh (67T-473)
(Introduced by Professor J.-I. Nagata)
- (82) Isomorphisms of measure algebras
Mr. R. P. Rigelhof, McMaster University (67T-489)
- (83) Quasi-subordinate functions. Preliminary report
Professor M. S. Robertson, University of Delaware (67T-539)
- (84) Geometric conditions for algebraic varieties
Professor Walter Rudin, University of Wisconsin (67T-542)
- (85) Nörlund summability of derived Fourier series
Professor B. N. Sahney, University of Calgary and Queen's University (67T-550)
- (86) On intermediate extensions. III
Professor Martin Schechter, Yeshiva University (67T-518)
- (87) Compact and weakly compact operators on $C(S)\beta$
Mr. F. D. Sentilles, Louisiana State University, Baton Rouge, (67T-509)
- (88) The Dirichlet problem for nonuniformly elliptic partial differential equations
Professor James Serrin, University of Minnesota (67T-517)
- (89) On the existence of subsonic flows
Professor Max Shiffman, Stanford University (67T-548)
- (90) Embedding of association schemes
Professor S. S. Shrikhande and Bhagwandas, University of Bombay and University of Wisconsin (67T-512)
- (91) Recursion theory and Dedekind cuts
Mr. R. I. Soare, Cornell University (67T-526)
- (92) An improvement of Aronszajn's inequality
Mr. William Stenger, University of Maryland (67T-482)

- (93) Inequalities expressing degree of convergence of rational functions
Professor J. L. Walsh, University of Maryland (67T-555)
- (94) On some sign reverse rule property of first passage probabilities
Professor P. C. C. Wang, Michigan State University (67T-456)
- (95) A generalization of the regular representation of finite Abelian groups. II
Professor Charles Wells, Case-Western Reserve University (67T-510)
- (96) Infinite algebras satisfying the descending chain condition. Preliminary report
Mr. T. P. Whaley, Vanderbilt University (67T-522)
- (97) Isotopisms of loops of prime order. Preliminary report
Professor R. L. Wilson, Jr., University of Wisconsin (67T-484)
- (98) Generalized Bloch theorem
Professor H. -H. Wu, University of California, Berkeley (67T-476)
- (99) A weaker separation axiom than T_1 implies a normal space is T_4
Professor Y. C. Wu and Professor S. M. Robinson, Union College (67T-457)
- (100) Generalized free-products of Boolean algebras
Professor F. M. Yaqub, American University of Beirut, Lebanon (67T-492)
- (101) The area and variation of linearly continuous functions on E^n
Professor W. P. Ziemer, Indiana University (67T-495)
- (102) Sufficiency conditions for implication among the axioms of choice for finite sets. Preliminary report
Mr. Martin Zuckerman, New York University and Yeshiva University (67T-460)



NEWS ITEMS AND ANNOUNCEMENTS

NATO SENIOR FELLOWSHIPS IN SCIENCE

The National Science Foundation and the Department of State recently announced that twenty American Scientists have been awarded the first North Atlantic Treaty Organization Foreign Fellowships in Science. The scientists will study new scientific developments in research and educational institutions in other NATO nations and in countries cooperating with NATO. Fellowships normally carry short-term tenures from one to three months, and Fellows receive a subsistence allowance and a travel allowance.

Two members of the AMS were awarded Fellowships in mathematics. Professor Daniel Drucker, Brown University, received a Fellowship in Engineering Mechanics. He will visit the Technical University, Istanbul, Turkey; the National Laboratory for Civil Engineering, Lisbon, Portugal; and the Technical University, Athens, Greece. Professor James Jans, University of Washington, will visit the University of Munich, Ger-

many. His field is Mathematical Algebra.

NEW NSF MATHEMATICAL SCIENCES SECTION HEAD

Dr. William H. Pell has been appointed Head of the Mathematical Sciences Section, Division of Mathematical and Physical Sciences, at the National Science Foundation. He succeeds Dr. Milton Rose, who was recently appointed to head the new Office of Computer Activities at the Foundation.

Dr. Pell has served as Program Director for the Foundation's Applied Mathematics and Statistics Program since August, 1965. Before coming to the Foundation, he was Chief of the Mathematical Physics Section at the National Bureau of Standards from 1960 to 1965. Other positions held include Associate Professor of Applied Mathematics, Brown University; Mathematician, National Bureau of Standards; and Chairman, Department of Mathematics and Astronomy, University of Kentucky.

ABSTRACTS OF CONTRIBUTED PAPERS

The June Meeting in Missoula, Montana

June 17, 1967

647-22. E. J. COCKAYNE, University of Victoria, Victoria, British Columbia. Steiner problem in Minkowski space.

The Steiner problem in E^2 may be stated: Given distinct points a_1, \dots, a_n ($n \geq 3$) in the plane, to construct the shortest tree(s) whose vertices include a_1, \dots, a_n and any set of k additional points s_1, \dots, s_k ($k \geq 0$). Let $|x|$ be the set of all vertices sending branches to the vertex x and $w(x)$ be their number. A minimising tree of the Steiner problem has the properties: (1) It is non self-intersecting. (2) $w(s_i) = 3$, $i = 1, \dots, k$. (3) $w(a_j) \leq 3$, $j = 1, \dots, n$. (4) $0 \leq k \leq n - 2$. (5) Each s_i is the Steiner point of the triangle formed by $\{s_i\}$. It is shown that similar properties are enjoyed by minimising trees of the Steiner problem in plane Minkowski metric spaces. (Received May 8, 1967.)

The August Meeting in Toronto, Ontario, Canada

August 29 - September 1, 1967

648-1. H. S. M. COXETER, University of Toronto, Toronto 5, Ontario, Canada. Finite groups of quaternions. Preliminary report.

Every finite group of (real) quaternions is either cyclic or binary polyhedral [see S. A. Amitsur, Trans. Amer. Math. Soc. 80 (1955), 385]. The binary polyhedral groups, of orders $4p$, 24 , 48 , 120 , have the presentation $S^p = T^q = (ST)^2$, where $(p, q) = (p, 2)$ or $(3, 3)$ or $(4, 3)$ or $(5, 3)$. For instance, the classical quaternion group of order 8 has the presentation $i^2 = j^2 = (ij)^2$ [see Coxeter and Moser, Generators and relations for discrete groups, Springer, Berlin, 1963, pp. 8, 69]. It has been found possible to express the generators in a form that covers all four cases at once, in terms of the integer h defined by $\cos^2(\pi/h) = \cos^2(\pi/p) + \cos^2(\pi/q)$ [see Coxeter, Regular polytopes, Macmillan, New York, 1963, p. 19]. The expressions are: $S = \cos(\pi/p) + i \sin(\pi/h) + k \cos(\pi/q)$, $T = \cos(\pi/q) + j \sin(\pi/h) + k \cos(\pi/p)$. (Received November 14, 1966.)

648-2. TAKAYUKI TAMURA, University of California, Davis, California. General products of a set and a semigroup. Preliminary report.

Let $\Theta(S)$ be the set of all groupoid operations defined on a set S . For each element a of S we define two binary operations a^* and $*a$ as follows: For $\theta, \eta \in \Theta(S)$, $x(\theta a^* \eta)y = (x\theta a)\eta y$ and $x(\theta^* a \eta)y = x\theta(a\eta y)$, $x, y \in S$. Then $\Theta(S)$ is a semigroup with respect to a^* and $*a$ for all a of S . Let T be a semigroup. Suppose that a mapping Φ of $T \times T$ into $\Theta(S)$, $(\alpha, \beta)\Phi = \theta_{\alpha, \beta}$, satisfies $\theta_{\alpha, \beta} a^* \theta_{\alpha, \beta, \gamma} = \theta_{\alpha, \beta \gamma}^* a \theta_{\beta, \gamma}$ for all $\alpha, \beta, \gamma \in T$ and for all $a \in S$. Let $U = \{(x, a); x \in S, a \in T\}$. A binary operation on U is defined as follows: $(x, a)(y, \beta) = (x \theta_{\alpha, \beta} y, a\beta)$. U is called a general product of a set S and a semigroup T . A subsemigroup V of U is called a sub-general product of S to T if V is a subsemigroup of U and the projection of V into T is equal to T . Clearly every sub-general product of S to T is homomorphic onto T .

Every semigroup which is homomorphic onto T is obtained as a sub-general product of S to T for some set S. As a special case the concept of semigeneral product of S and T $((x, \alpha)(y, \beta) = (x\theta_{\alpha}y, \alpha\beta))$ is discussed. Related to this problem the author studies the structure of $\Theta(S)$. (Received December 2, 1966.)

648-3. D. R. MORRISON, Sandia Corporation, Sandia Base, Albuquerque, New Mexico.

Matching algorithms.

A graph M is a matching if every point is incident on exactly one line, every line on exactly two points. The following problems arise in scheduling interviews and other kinds of matchings. Given a bipartite graph B, (1) find a matching M in B, with a maximum number of lines; (2) find a matching M in B, which includes all the points in B whose number of incident lines is maximal, and a minimal number of other points; (3) find a sequence M_1, M_2, \dots, M_R of matchings in B, whose union is B and whose number is minimal. Algorithms are described to solve these and other problems efficiently. The key tools are (a) a class of graphs called ionizations, more general than matchings and more special than bipartite graphs, and (b) a class of transformations called improvements of the ionizations in a bipartite graph B. The matchings in an ionization and the improvements applicable to it are particularly easy to describe and to catalog. The algorithms consist of applying improvements to an ionization until no more are applicable and then extracting from the ionization the required matching. (Received November 21, 1966.)

648-4. M. Z. v. KRZYWOBLOCKI, Michigan State University, East Lansing, Michigan 48823.

Particular proofs of ergodic problems in continuous media and reduction of independent variables.

The author presents proofs of ergodic theorem in a few particular problems in the mechanics of continuous media. The technique in question works well in some one-dimensional examples. It was successful in one case of a two-dimensional problem, but it seems to fail in two- and more-dimensions. To overcome this difficulty, the author proposes a combination of the ergodic-proof-technique with the reduction of independent variables. The latter is also at the beginning of its development. A discussion of the new problems connected with the combination of the two techniques closes the work. (Received November 16, 1966.)

648-5. J. S. BAKSHI, State University College, Buffalo, New York. An analysis of flexural

vibrations for a circular ring with free edges.

The partial differential equations of isotropic and elastic plates, which include the coupling between flexural and shear motions, are analysed for the case of circular ring with free edges. The solution of the partial differential equation is represented by an infinite series of product solutions in terms of Bessel functions. A comparison between the Lagrange theory and the theory which includes rotatory inertia and shear deformation correction is made. The resulting frequency spectrum for a limiting case is compared with the corresponding case given by the Lagrange theory. (Received December 1, 1966.)

648-6. K. A. BUSH, Washington State University, Pullman, Washington 99163. An existence theorem for finite projective planes of even order.

We show that the existence of a finite projective plane of order $2t$ implies the existence of an orthogonal matrix of order $4t^2$ with these properties: (i) symmetry; (ii) the matrix admits a partitioning into submatrices of order $2t$ such that every "off-diagonal" submatrix contains t entries of $+1$ and t entries of -1 in each of its rows and columns; (iii) in the partitioned form each "diagonal" submatrix consists solely of positive ones. (Received December 1, 1966.)

648-7. A. E. HURD, University of California, Los Angeles, California 90024. A unique continuation theorem for mixed parabolic problems.

It is shown that solutions $u(x,t)$ of a parabolic partial differential equation $\partial u/\partial t + A(x,t)u = 0$ ($A(x,t)$ being an elliptic operator), which are subject to homogeneous boundary conditions on the lateral boundary $\partial\Omega \times (a,b)$ of a cylindrical region $\Omega \times (a,b)$ ($\Omega \subset E^n$), satisfy a forward unique continuation property in the following sense: if $u(x,c)$, $a < c < b$, vanishes for x in an open set in Ω , then $u(x,t)$ vanishes identically in Ω for $c < t < b$. The result is established under weak differentiability assumptions on the coefficients of $A(x,t)$ as functions of t . The backward uniqueness result of Lions and Malgrange (Math. Scand. 8 (1960), 277-286) can then be used to obtain a forward and backward unique continuation theorem, thus answering a question posed in their paper. (Received April 27, 1967.)

648-8. R. L. HEMMINGER, Vanderbilt University, Nashville, Tennessee. The group of an X-join of graphs.

Let X and Y_x , $x \in X$, be graphs (undirected and without loops or multiple edges). The X-join of $\{Y_x\}_{x \in X}$ is the graph Z obtained from X by replacing the vertex x by the graph Y_x for each $x \in X$ and then either inserting all or none of the possible edges between vertices of Y_a and Y_b depending on the existence or nonexistence of an edge between a and b in X . There are some "natural automorphisms" of Z ; namely, those obtained by permuting the Y_x 's according to a permutation of the subscripts determined by an automorphism of X , followed by an arbitrary automorphism of each Y_x . In this paper we give necessary and sufficient conditions for the set of all automorphisms of Z to be precisely those "natural" ones. If $Y_x = Y$ for all $x \in X$ then $Z = X \circ Y$ is the lexicographic product of X and Y and the theorem gives necessary and sufficient conditions for $G(X \circ Y)$ to be the wreath product of $G(X)$ and $G(Y)$ where $G(X)$ is the group of automorphisms of X . Therefore this paper is an extension of prior results of the author [The lexicographic product of graphs, Duke Math. J. 33 (1966), 499-502]. (Received November 28, 1966.)

648-9. A. B. OWENS and H. M. TRENT, Solid State Division, Naval Research Laboratory, Washington, D. C. 20390. Minimal singularities for graph realizations of incidence sequences.

To each sequence $\pi = (a_1, \dots, a_p)$ of positive integers with an even sum corresponds some linear graph with p vertices and local degrees a_1, \dots, a_p . Such a linear graph is said to be a realization of π . The collection of all realizations is denoted $\mathcal{G}(\pi)$. A given linear graph $G(\pi)$ in $\mathcal{G}(\pi)$ may or may not possess multiple edges or loops. Algorithms have been developed which will determine, (a) the mini-

imum number of multiple edges needed to realize π as a linear graph (without loops), (b) the minimum number of loops needed to realize π as a linear graph (without multiple edges), and (c) the minimum number of loops and/or multiple edges needed to realize π as a linear graph. (Received November 25, 1966.)

648-10. J. D. FULTON, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830.

Construction and enumeration of symmetric involutory matrices over finite fields.

For terminology see Levine and Nahikian, Amer. Math. Monthly 69 (1962), 267-272. Symmetric, n th order involutory matrices of signature s , $s = 0, 1, 2, \dots, n - 1$, over $GF(q)$, q odd, are characterized by the following: Theorem. The $n \times n$ involutory matrix A of signature s over $GF(q)$, q odd, is symmetric if and only if A can be decomposed as $A = I_n - 2P^T D_\mu P$, where P is $s \times n$ of rank s , where D_μ is the $s \times s$ diagonal matrix $D(I_{s-1}, \mu)$ with $\mu = 1$ or μ a fixed but unspecified nonsquare of $GF(q)$, and where $PP^T = D_\mu^{-1}$. These n th order symmetric involutory matrices are enumerated by signature s as $\sum_\mu N_n^s(q; P, D_\mu^{-1}) / N_s^s(q; D_\mu^{-1}, D_\mu^{-1})$, where $\mu = 1$ or else μ is a fixed but unspecified nonsquare of $GF(q)$, where $N_n^s(q; P, D_\mu^{-1})$ is the number of $s \times n$ matrices P such that $PP^T = D_\mu^{-1}$, and where $N_s^s(q; D_\mu^{-1}, D_\mu^{-1})$ is the number of automorphs of D_μ^{-1} . (Received November 29, 1966.)

648-11. JOHN POLAND, Carleton University, Ottawa, Canada. Finite p -groups with k conjugate classes.

Let G be a group of order p^m ($m = 2n + e$, p prime, m and n nonnegative integers, $e = 0$ or 1) having $k(G) = k$ conjugate classes, and let $f_r(p^m)$ denote the expression $(n + r(p - 1))(p^2 - 1) + p^e$. A simple proof is given that there exists some nonnegative integer r such that $k = f_r(p^m)$. If $k = f_0(p^m)$ it may be shown that G has maximum nilpotent class and the p th power of any element not in the derived group is in the second center. Consequently if $m \geq p + 3$ then $k \geq f_1(p^m)$. In general if G has nilpotent class c and $k(G) = f_r(p^m)$, $m > 3$, then $c + r \geq m/2 + 1$. The proofs depend mainly upon the simple lemma that if N is a normal subgroup of order p of G and $k(G) = f_r(p^m)$ then $k(G/N) \leq f_r(p^{m-1})$ with equality implying a number of conditions--in particular, either the center of G has order p , or m is even and the center has order p^2 . (Received March 7, 1967.)

648-12. WITHDRAWN.

648-13. DAGMAR HENNEY, 6912 Prince Georges Avenue, Takoma Park, Maryland. Set-valued additive functions. IV.

Let $C(I, R)$ denote the class of all realvalued continuous bounded functions f defined on a topological space I and let D be the set of all $f \in C(I, R)$ for which $f(t) \geq 0$ for $t \in I$. Define the norm of an element in $C(I, R)$ by $\|f\| = \sup_{t \in I} |f(t)|$. Let Y be a linear topological space and C the family of all nonempty convex sets of the space Y . The family C forms a semilinear space under the operation of algebraic addition of sets and multiplication of a set by a scalar. One introduces a uniform topology on C by means of the uniformity $\{(B_1, B_2) \in C \times C : B_1 \subset B_2 + V \text{ and } B_2 \subset B_1 + V\}$ where V denotes any symmetric neighborhood of zero in the space Y . Let A be an additive function from D into the

space C. One then proves the following Theorem. If A is continuous at $f = 0$ then the set-valued function A is uniformly continuous on D. (Received March 8, 1967.)

648-14. W. R. CALLAHAN, St. John's University, Jamaica, New York. Flexural vibrations of elliptical plates when transverse shear and rotary inertia are considered.

In this paper, the author solves the partial-differential equations governing the motion of plates, which were derived by Mindlin, Reissner, and Uflyand, by using product solutions and finds, by taking an infinite series of product solutions, that the boundary conditions are satisfied. The elimination of the arbitrary constants occurring in each boundary problem leads to the frequency equation for the normal modes of vibration. The frequency equation for each problem is an infinite determinant and each element in it is an infinite series of Mathieu functions containing as unknown the frequency. A method is described permitting one to find the roots of the infinite determinant. These roots represent the normal modes of vibration for the elliptical plate. Since the classical (Lagrange) theory of plate is good only for plate when the wave length is large in comparison with the thickness of the plate, it is restricted to low-frequency vibrations. The present theory gives good results for high-frequency vibrations, essentially because it includes coupling between flexural and shear motions. (Received November 17, 1966.)

648-15. R. T. YEH, The Pennsylvania State University, 650 East Prospect Avenue, State College, Pennsylvania 16801. Generalized pair algebra with applications to nondeterministic automata.

(For terminologies see Universal algebra by P. M. Cohn, and Algebraic structure theory of sequential machines by J. Hartmanis and R. E. Stearns.) Let R and S be relations of finite ranks; define $\bar{R} = \{(x_1, \dots, x_k) | (x_1, \dots, x_k) \in R\}$, $R^{-n} = \{(y_1, \dots, y_m, x_1, \dots, x_n) | (x_1, \dots, x_n, y_1, \dots, y_m) \in R\}$, and $R \circ_n S = \{(x_1, \dots, x_l, z_1, \dots, z_m) | (\exists y_1, \dots, y_n) [(x_1, \dots, x_l, y_1, \dots, y_n) \in R \wedge (y_1, \dots, y_n, z_1, \dots, z_m) \in S]\}$. If M and M' are relational structures over Ω with carriers X and X' respectively, then $H \subseteq X \times X'$ is a generalized pair algebra (GPA) on $M \times M'$ iff $(\forall a \in \Omega(n)) [\omega(a)^{-n} \circ_n H \circ_n \omega(a)' \subseteq H]$. Theorem 1. H is a GPA on $M \times M'$ iff $(\forall a \in \Omega(n)) [\omega(a)' \subseteq H^{(2n)-n} \circ_n \omega(a) \circ_1 \bar{H}]$, where $H^{(2n)} = \{(x_1, \dots, x_n, x'_1, \dots, x'_n) | (\forall 1 \leq i \leq n) [(x_i, x'_i) \in H]\}$. Theorem 2. Let L_1 and L_2 be finite lattices. Then H is a pair algebra on $L_1 \times L_2$ iff H is a GPA on $L_1 \times L_2$ such that $\Delta_{L_1} \supseteq H \circ H^{-1}$, $\Delta_{L_2} \subseteq H^{-1} \circ H$, and $(O_1, I_2) \in H$. Theorem 3. If M is a relational structure over Ω with carrier X, X' is an arbitrary set, and $H \subseteq X \times X'$. Then there is a relational structure M' over Ω with carrier X' such that H is a GPA on $M \times M'$. Theorem 4. A mapping $h: S \rightarrow S'$ is a homomorphism from a state machine $M = \langle S, I, \delta \rangle$ to a state machine $M' = \langle S', I, \delta' \rangle$ iff h is a GPA on $M \times M'$. (Received March 30, 1967.)

648-16. P. C. HAMMER, The Pennsylvania State University, University Park, Pennsylvania 16802. Filters in general.

Filters were introduced into topology by H. Cartan in 1937. Essentially these serve to separate the power set of a space into two classes. The particular properties of topological filters restrict their applicability unduly. Two kinds of generalization are considered. The first kind results from a function mapping a set into $\{0,1\}$ which produces an equivalence relation in the set which is either the maximum or a dichotomy of the set. The second kind results from a function mapping a set into

another set. This kind can yield any equivalence relation. The second kind of filter can be interpreted as an intersection of a collection of filters of the first kind. A theory of isotonic filter spaces in which there is no requirement that points be associated with the filter sets is developed. Applications are given. (Received April 3, 1967.)

648-17. J. A. MORRISON, Bell Telephone Laboratories, Murray Hill, New Jersey 07971.
Resonance behaviour of a perturbed system depending on a slow-time parameter.

A perturbed second order system of differential equations, depending on a slow-time parameter, and containing a rapidly rotating phase, is considered. The zero order rate of change of one variable vanishes when the second variable, which is slowly changing, assumes a particular value, which is a function of the slow-time. Such a system arises in the consideration of resonance phenomena in the essentially nonlinear forced vibrations of a system which depends on slow-time. Under suitable conditions, the behavior of the system, in a certain small neighborhood of resonance, is analyzed for a finite interval of slow-time, by means of a double averaging procedure. The formal results are justified by establishing precise bounds on the solution, for sufficiently small values of the perturbation parameter. In some instances when the averaged motion is damped, and in particular when the slow-time parameter does not occur explicitly in the original system, the results may be extended to the infinite time interval. The relation of the results obtained in this paper to those obtained by others is discussed. (Received April 10, 1967.)

648-18. W. R. HUTCHERSON, University of Florida, Gainesville, Florida 32601. A cyclic involution of period nineteen contained on a rational surface in a space of twelve dimensions.

A rational surface [W. R. Hutcherson and J. W. Kenelly, A rational surface generated by an involution of period seventeen, Revista Matematica Fisica Teorica (Tucuman) 13 (1960), 30-35] containing an involution of period nineteen is found. The ten equations defining the irrational surface, to which the involution belongs, were simple compared to the eleven equations needed to show the desired surface was rational. The last such equation involved coefficients of the thirtieth degree. The cases for prime periods 3, 5, 7, 11, 13, 17 have been found within the last fifty years. (Received April 24, 1967.)

648-19. MILTON ROSENBERG, The University of Kansas, Lawrence, Kansas 66044.
Derivatives with respect to finite dimensional vector measures.

Let $M = [M_i]_{i=1}^m$ and $N = [N_j]_{j=1}^n$ be column-vectors of complex measures, M_i, N_j , on the same measurable space (Ω, \mathcal{O}) . Let Φ and Ψ_k denote \mathcal{O} -measurable $m \times n$ matrix-valued functions on Ω , i.e., $\Phi = [\phi_{ij}]$, $\Psi_k = [\psi_{ij}^{(k)}]$, where the ϕ 's and ψ 's are \mathcal{O} -measurable functions. Let each M_i, N_j be $\ll \llcorner$ a σ -finite nonnegative measure μ , and let $M'_\mu = [dM_i/d\mu]$, $N'_\mu = [dN_j/d\mu]$. We define (1) $\int_B \Phi d\mu = [\int_B \phi_{ij} d\mu]$ if each R.H.S. entry integral exists, (2) $\int_B \Phi \cdot dN = \int_B (\Phi \cdot N'_\mu) d\mu$ if the R.H.S. integral exists. We say that $M \llcorner \llcorner N$ modulo $(\Psi_k)_{k=1}^p$ iff there exists a Φ so that (1) $\Phi(\omega) \in \langle \Psi_k(\omega) \rangle_{k=1}^p$ (the linear space spanned by the $\Psi_k(\omega)$'s) a.e. (μ) , and (2) $M(B) = \int_B \Phi \cdot dN$ each B in \mathcal{O} . Let $(\xi_l(\omega))_{l=1}^q$ denote a family of m -tuple column-vector-valued functions so that $\langle \xi_l \rangle_1^q \cap \langle \Psi_k \cdot N'_\mu \rangle_1^p = \{0\}$ and $\langle \xi_l \rangle_1^q + \langle \Psi_k \cdot N'_\mu \rangle_1^p = \mathcal{L}^m$ a.e. (μ) . We say that M is singular to N modulo $(\Psi_k)_{k=1}^p$ along

$(\xi_l)_1^q$ iff $M'_\mu(\omega) \in (\xi_l(\omega))_1^q$ a.e. (μ) . A unique Lebesgue decomposition of M exists relative to $\{N, (\Psi_k)_1^p, (\xi_l)_1^q\}$. Our main tool is the generalized inverse of a matrix, cf. Penrose, Proc. Cambridge Philos. Soc. 51 (1955). These results extend immediately to arbitrary measures valued in finite-dimensional Banach spaces. (Received April 10, 1967.)

648-20. M. C. GEMIGNANI, State University of New York, Buffalo, New York 14214.

(n,m)-arrangements. Preliminary report.

In Topological geometries and a new characterization of R^m , Notre Dame J. Formal Logic, January (1966), and in A characterization of the m-sphere by means of topological geometries, to appear in the same journal, the notions of an m-arrangement and a spherical m-arrangement are defined. Both these concepts are special cases of a more general concept, an (n,m)-arrangement. Specifically, a space X with geometry G of length $m - 1$ is said to be an (n,m)-arrangement if (i) Each 0-flat consists of exactly n points. (ii) If f is a $k-1$ -flat contained in a k -flat g , then f disconnects g into two convex components which are open in g , $1 \leq k \leq m$. (iii) Each 1-flat is connected. (iv) If f is an $m-1$ -flat, then we call the components of $X-f$ half-spaces of X . The collection of half-spaces of X forms a sub-basis for the topology of X . An m-arrangement is a (1,m)-arrangement and a spherical m-arrangement is a (2,m)-arrangement. The m-sphere and euclidean m-space can be characterized as special cases of an (n,m)-arrangement. (Received April 6, 1967.)

648-21. J. D. ACZEL, University of Waterloo, Waterloo, Ontario, Canada. Some problems

on measures of information and their characterization.

(I) Let f be differentiable in $(0,1)$; (1) $\sum p_j f(p_j) = \min(\sum p_j f(q_j) : q_j > 0, \sum q_j = 1)$ for all complete distributions of positive probabilities iff up to an additive and a positive multiplicative constant the left hand side of (1) is Shannon's entropy (J. Aczel-J. Pfanzagl). Is this also true if f is only supposed to be continuous? (II) If (2) $I(p_j) = g^{-1}(\sum f(p_j)g(-\log_2 p_j) / \sum f(p_j))$, f, g continuous, f nonnegative, g strictly monotonic, and (3) $I(p_j q_k) = I(p_j) + I(q_k)$, p_j, q_k ($j = 1, 2, \dots, m; k = 1, 2, \dots, n$) being arbitrary in an interval, m, n arbitrary positive integers, then and only then $f(x) = kF(x)/(cG(x) + d)$, $g(x) = (aG(x) + b)/(cG(x) + d)$, where a, b, c, d, k are constants with $k(c^2 + d^2)$ ($ad - bc$) $\neq 0$ and either $F(x) = x^B$ with $G(x) = x$ or ($B \neq A, B \neq 0$) $G(x) = 2^{(B-A)x}$, or again $F(x) = x^B \cos(C \log x)$ with $G(x) = \tan Cx$, ($C \neq 0$), i.e. (2) is a generalization of Shannon's entropy or of the entropy of order A/B (replacing in both cases p by $p^{A/B}$) or a new kind of entropy (defined only for $p \in (2^{-\pi/(2C)}, 2^{\pi/(2C)})$) respectively (J. Aczel-Z. Daroczy) Is this also true if (3) is supposed only for $p_j > 0, q_k > 0, \sum p_j \leq 1, \sum q_k \leq 1$ (not necessarily complete distributions of positive probabilities)? (III) Further problems. (Received April 7, 1967.)

648-22. C. E. AULL, Virginia Polytechnic Institute, Blacksburg, Virginia 24061. Classification

of Hausdorff spaces.

Definition. M is an n -subset of a topological space if by means of interior and closure operators one can obtain n distinct subsets (M is always included among the n -distinct subsets). A topological space is an n -space if it contains an n -subset but not an $(n + 1)$ -subset. Among Hausdorff spaces there are no 3 or 6-spaces. Theorem. For Hausdorff spaces without isolated points, a 4-space is both extremally disconnected and 0I. A 5-space satisfies exactly one of these properties and 7-space has

neither of these properties. A topological space without isolated points is OI if every open subset is not resolvable in its relative topology or equivalently the interior of a dense subset is dense. See Hewitt, A problem in set theoretic topology, Duke Math. J. 10 (1943), 309-333. (Received April 10, 1967.)

648-23. J. E. de PILLIS, University of California, Riverside, California. Linear operators and their partition matrices.

Let p be an $m \times m$ matrix whose ij th entry is the $n \times n$ matrix A_{ij} , and set $p_{(q)}$ equal to the $m \times m$ matrix whose ij th entry is $E_q(A_{ij})$, where E_q is the q th elementary symmetric function of A_{ij} , $1 \leq q \leq n$. Suppose P is that linear operator on mn -dimensional unitary space \mathcal{A}_{mn} whose matrix (relative to a certain o.n. basis of \mathcal{A}_{mn}) is p . We characterize the linear operator whose matrix is $p_{(q)}$ -- specifically, we have the Theorem. Let $\Lambda^q \mathcal{A}_{mn}$ be the subspace spanned by vectors of the form $x_1 \wedge x_2 \wedge \dots \wedge x_q$, $x_i \in \mathcal{A}_{mn}$, and let $\mathcal{B}(\Lambda^q \mathcal{A}_{mn})$ be the algebra of linear operators on $\Lambda^q \mathcal{A}_{mn}$. Set \mathcal{M} equal to the (orthogonal projection onto the) subspace of $\mathcal{B}(\Lambda^q \mathcal{A}_{mn})$ spanned by the set $\{C_q(\mathcal{N}^1), C_q(\mathcal{N}^2), \dots, C_q(\mathcal{N}^m)\}$, where \mathcal{N}^i is a certain partial isometry on $\Lambda^q \mathcal{A}_{mn}$, and $C_q(\mathcal{N}^i)$ is the q th compound of \mathcal{N}^i . If $P \in \mathcal{B}(\mathcal{A}_{mn})$ is the linear operator whose matrix is $p = (A_{ij})$, then $\mathcal{M} \mathcal{L} C_q(P) \mathcal{M}$, restricted to \mathcal{M} , is the linear operator (in $\mathcal{B}(\mathcal{B}(\Lambda^q \mathcal{A}_{mn}))$) whose matrix is $p_{(q)} = (E_q(A_{ij}))$, where $\mathcal{L} C_q(P)$ is the left-hand multiplication operator defined by $\mathcal{L} C_q(P)(X) = C_q(P) \cdot X$ for all $X \in \mathcal{B}(\Lambda^q \mathcal{A}_{mn})$. Corollary. If p is positive semidefinite, then $p_{(q)}$ is positive semidefinite. (Received April 13, 1967.)

648-24. P. M. EAKIN and W. J. HEINZER, Louisiana State University at Baton Rouge, Baton Rouge, Louisiana 70803. Some open questions on minimal primes of a Krull domain.

In Algebra commutative, Chapter 7, p. 83, Bourbaki indicates a method for constructing a two dimensional Krull domain with a nonfinitely generated minimal prime. The following result, however, shows that a Krull domain obtained in the manner suggested by Bourbaki must be noetherian. Proposition. Let A_0 be a Dedekind domain with quotient field K_0 and let $J_0 = A_0[[X^2]]$. Suppose that $K = K_0(a_1, a_2, \dots)$ is a separable algebraic extension of K_0 , that A is the integral closure of A_0 in K , and that L_0 is the quotient field of J_0 . Then if J is the integral closure of J_0 in $L_0(a_1X, a_2X, \dots) = L$, the following are equivalent: (1) J is a Krull domain, (2) J is noetherian, (3) A is a Dedekind domain. If k is a field and $\{X_i\}_{i=1}^{\infty}$ is a set of indeterminates over k , then $k[[X_i X_j]]$ is a Krull domain which has minimal primes which are not finitely generated. The following question appears to be open. If D is a finite dimensional Krull domain is each minimal prime of D finitely generated? (Received April 13, 1967.)

648-25. B. L. MADISON, Louisiana State University, Baton Rouge, Louisiana 70803. Semigroups on coset spaces. Preliminary report.

A space X is reducible if the identity map on X is homotopic to a map from X to a proper subset of itself. A limit manifold (K. H. Hofmann and P. S. Mostert, Elements of compact semigroups, Charles E. Merrill Books, Columbus, Ohio) is not reducible. Let (S, \cdot) be a compact connected arc-

wise connected semigroup with identity. Since a compact connected group space is a limit manifold, it follows that (S, \cdot) is a group if and only if S is not reducible. (A more general definition of reducible allows one to remove the arcwise connected restriction on S .) Using the above idea it is seen that if S is the base space of certain types of fiber spaces $\{G, p, S\}$ where G is a compact connected group, then (S, \cdot) is a group. In particular, if S is homeomorphic to certain types of coset spaces of compact groups, then (S, \cdot) is a group. (Received April 14, 1967.)

648-26. JACK OHM and R. L. PENDLETON, Louisiana State University, Baton Rouge, Louisiana 70803. Rings with Noetherian spectrum.

Let R and R' be commutative rings with identity. An intersection of maximal ideals is called a J -radical ideal. Theorem. If R' is a finite R -algebra and R has the ascending chain condition (a.c.c.) on J -radical ideals, then R' has the a.c.c. on J -radical ideals. Example. If $R' = R[X]$, then R may have the a.c.c. on J -radical ideals while R' does not. (Received April 17, 1967.)

648-27. JACK HACHIGIAN, State University of New York, Stony Brook, New York. Homeomorphisms of the unit ball.

Let B^n be the unit ball in R^n , Euclidean n -space, i.e. $B^n = \{x : x \in R^n, d(x, 0) \leq 1\}$. If f and g are any two functions of B^n to itself define $\|f - g\| = \sup\{d[f(x), g(x)] : x \in B^n\}$. We prove the following: Theorem. Let $f : B^2 \rightarrow B^2$, onto and continuous such that f fixes three points on ∂B^2 and such that $f \neq I$ then $\inf_k \|f^k - I\| > 0$. Here f^k is the k th iterate of f e.g. $f \circ f = f^2$. The theorem is a conjecture of J. Ax. (Received April 4, 1967.)

648-28. V. V. RAO, The University of Calgary, Calgary, Alberta, Canada. Averages involving Fourier coefficients on nonanalytic automorphic forms. Preliminary report.

Let $f(\tau)$ be a complex valued function analytic in the upper half plane $\text{Im}(\tau) > 0$ and $f(\tau)$ a function with signature $(\lambda, \kappa, \gamma)$ in the sense of E. Hecke (Math. Ann. 112 (1936), 664-699). It then follows that $f(\tau) = a_0 + \sum_{n=1}^{\infty} a_n \exp(2\pi i n \tau / \lambda)$, with the infinite series converging absolutely in the upper half plane $\text{Im}(\tau) > 0$. It is well known that $\sum_{0 < n \leq x} a_n (x - n)^\mu$, x being a real number, can be expressed as an absolutely convergent series of Bessel functions of the first kind, for $\mu > \mu_0$, where μ_0 is a number depending on $f(\tau)$. The work of C. L. Siegel on indefinite quadratic forms has led to the study of nonanalytic automorphic forms and these functions have been studied by H. Maass and others. Let $f(z, w)$ be a nonanalytic automorphic form of the type $\{G; \alpha, \beta, \nu\}$ in the sense of H. Maass (Math. Ann. 125 (1937), 237), where G is the group generated by the linear transformations $\tau \rightarrow \tau + \lambda$ and $\tau \rightarrow -1/\tau$. We then obtain an absolutely convergent series expansion for $\sum_{0 < t \leq x} a_t (x - t)^\mu$, where a_t (t not necessarily integral) denotes the t th Fourier coefficient of the nonanalytic automorphic form $f(z, w)$, for $\mu > \mu_0$, μ_0 being a number depending on $f(z, w)$. (Received April 20, 1967.)

648-30. J. W. NEUBERGER, Emory University, Atlanta, Georgia 30322. Quasi-analytic collections containing Fourier series which are not infinitely differentiable.

If g is a bounded real function whose domain includes $[u,v]$, then $\limsup_{n \rightarrow \infty} \left[\sum_{t=0}^n \binom{n}{t} | \sum_{s=0}^t \binom{t}{s} (-1)^{t-s} f(u + s(v-u)/n) | \right]^{1/n} \leq 3$. Suppose that I is a connected nondegenerate number set, $\Delta = \{\delta_q\}_{q=1}^{\infty}$ is a positive number sequence which has a subsequence converging to 0, $K = \{N(q)\}_{q=1}^{\infty}$ is a sequence of increasing positive integer sequences ($N(q) = \{n_{q,j}\}_{j=1}^{\infty}$, $q = 1, 2, \dots$) and $G(\Delta, K, I)$ is the set of all continuous real valued functions on I such that if x is in I there is an open interval S containing x such that if u, v are in both I and S and $|u - v| = \delta_q$ for some positive integer q , then $\limsup_{n \rightarrow \infty (n \in N(q))} \left[\sum_{t=0}^n \binom{n}{t} | \sum_{s=0}^t \binom{t}{s} (-1)^{t-s} f(u + s(v-u)/n) | \right]^{1/n} < 3$. Theorem 1. No two members of $G(\Delta, K, I)$ agree on any sub-interval of I . Theorem 2. $G(\Delta, K, I)$ contains all functions on I which are real-analytic. Theorem 3. If there is a positive number L and a positive number sequence $\{\epsilon_q\}_{q=1}^{\infty}$ such that for infinitely many positive integers p , $|(pL\delta_q/n_{q,j}) - (2t-1)| \geq \epsilon_q$ for all positive integer triples q, j, t , then $G(\Delta, K, I)$ contains Fourier series which are not infinitely differentiable. (Received February 23, 1967.)

648-31. ANDREW CONNOR, University of Wisconsin, Madison, Wisconsin. A theorem concerning 3 dimensional manifolds.

Definition. Suppose that g is a simple closed curve in the interior of the handlebody M with n handles and H_1, \dots, H_n are handles of M such that $g \cap \bigcup_{i=1}^n H_i = g \cap H_i =$ a point p , and g pierces H_i , then g is a generator of M . Theorem. Suppose M is a 3 dimensional manifold such that $M = T_1 \cup T_2$ where T_1 and T_2 are handlebodies, $T_1 \cap T_2 = \text{Bd}T_1 = \text{Bd}T_2$ and there exist a generator g in the interior of T_1 which bounds a disk in M then g bounds a disk D in M such that $D \cap T_1$ is an annulus. This theorem combined with results of Haken has significant implication on the Poincaré conjectures. (Received December 1, 1966.)

648-32. D. R. McMILLAN, JR., University of Wisconsin, Madison, Wisconsin 53706. Strong homotopy equivalence of 3-manifolds.

In what follows, let M and N denote closed 3-manifolds, and let f be a continuous mapping of M onto N such that $f^{-1}(y)$ is a compact absolute retract, for each $y \in N$ (only a weaker property of $f^{-1}(y)$ is required: for each open set $U \subset M$ such that $f^{-1}(y) \subset U$, there is an open set V such that $f^{-1}(y) \subset V$

$\subset U$ and V is contractible to a point in U). Such a mapping is known to be a homotopy equivalence. Theorem 1. For all but a finite number of $y \in N$, $f^{-1}(y)$ is cellular in M . Theorem 2. M minus the interiors of a finite disjoint collection of polyhedral homotopy 3-cells is homeomorphic to N minus the interiors of a finite disjoint collection of polyhedral 3-cells. Theorem 3. If there is also a mapping of N onto M of the above type, then M and N are homeomorphic. Theorem 4. If M and N are homeomorphic then f is a cellular map, that is, $f^{-1}(y)$ is cellular for each $y \in N$. The proofs use a recent theorem of Armentrout (the converse of Theorem 4). (Received April 24, 1967.)

648-33. J. R. TELLER, Georgetown University, Washington, D. C. 20007. On pseudo lattice ordered groups.

Let G be an abelian partially ordered group. A subgroup M of G is a value of $0 \neq g \in G$ if M is an o -ideal of G that is maximal without g . Let $\bigcap \mathcal{M}(g)$ denote the intersection of all values of g . Two positive elements $a, b \in G$ are pseudo disjoint if $a \in \bigcap \mathcal{M}(b)$ and $b \in \bigcap \mathcal{M}(a)$, and G is called pseudo lattice ordered if each $g \in G$ can be written $g = a - b$ where a and b are pseudo disjoint. Theorem. An abelian partially ordered group G is pseudo lattice ordered if and only if (i) G is a Riesz group (Abstract 619-35, these *Notices*) 12 (1965), 65) and (ii) for each $g \in G$ there is $0 \leq a \in G$ such that $a \geq g$ and $a - x \in \bigcap \mathcal{M}(g)$ whenever $a \geq x \geq c$, $x \geq 0$. A Riesz group is an antilattice if $\{g \in G | g > 0\}$ is lower directed. We also give necessary and sufficient conditions that a pseudo lattice ordered group G be a small lexico-sum of antilattices. (Received April 24, 1967.)

648-34. AVINO'AM MANN, University of Illinois, Urbana, Illinois 61801. N -normalizers of finite solvable groups. Preliminary report.

Let G be a finite solvable group. For each prime p dividing $|G|$, choose a normal subgroup N_p of G , and let T_p be a p -complement of N_p . Then $\bigcap N(T_p)$ is termed an N -normalizer of G . This generalizes both the concepts of a relative system normalizer and an F -normalizer (Carter and Hawkes, *J. of Algebra* 5 (1967), 175-202). If a system of normal subgroups N_p as above is chosen in each subgroup of G , and certain natural conditions are met, then one can also define (and show the existence of) N -covering subgroups of G . Most of the results from the theory of F -normalizers and F -covering subgroups can be carried over to the present situation. (Received April 25, 1967.)

648-35. JÜRGEN BATT, Kent State University, Kent, Ohio 44240. Operations on the space of continuous functions.

Since F. Riesz discovered his famous representation theorem for the linear bounded functionals on the space $C(I)$ of continuous functions on a compact interval I of the reals in 1909, representation theorems in similar situations have always been of high interest. Using a representation theorem for the linear bounded transformations on the space $C(I, E)$ of the continuous functions on I with values in a Banach space E in some other Banach space F , it is proven that every such linear bounded transformation is weakly compact if E is reflexive and F weakly complete. This is a partial improvement of a result of Bartle, Dunford and Schwartz (See *Canad. J. Math.* 7 (1955), 289-305). Furthermore, a representation theory for a class of nonlinear transformations is developed. (Received April 27, 1967.)

648-36. NICKOLAS HEEREMA, Florida State University, Tallahassee, Florida. Inertial automorphisms of a class of wildly ramified V-rings.

Let $R_p = R[\pi]$ be an Eisenstein extension of degree $p (\neq 2)$ of an unramified V ring R having residue field k . Let $G_1 \supseteq H_1 \supseteq G_2 \supseteq \dots$ be the ramification series of R_p and let G_D be the group of automorphisms $\alpha_D = \sum_{i=0}^{\infty} D_i$ where $\{D_i\}$ is a convergent higher derivation on R_p . Thus $\pi^p + p u = 0$ where $\bar{u} \neq 0$ (\bar{u} being the residue of u). If $\bar{u} \in k^p$, π can be chosen so that $\pi^p + p = 0$ or $\pi^p + p(1 + \pi^t v) = 0$, $t > 0$, $\bar{v} \neq 0$. Using convergent higher derivations to construct automorphisms the following theorems are proved where $G(R_p : R)$ is the group of automorphisms of R_p over R . Theorem 1. $G_1 = G_D$ unless $\bar{u} \in k^p$ and $t = p - 1$ in which case the following are equivalent (a) \bar{v} is a $(p - 1)$ th root in k , (b) R_p is Galois over R , (c) $G_2 \neq H_2$, (d) G_2/H_2 is the group of order p . If R_p is Galois over R then $G(R_p : R) \subset G_D$ if and only if $\bar{v} \notin k^p$. In any case $G_1 = G_D \cdot G(R_p : R)$. Theorem 2. If $\bar{u} \notin k^p$ and $i \geq 1$, H_i/G_{i+1} is isomorphic to the additive group of those derivations on k which left to R_p . Also G_i/H_i is isomorphic to the additive group of k . If $\bar{u} \in k^p$, H_i/G_{i+1} is isomorphic to the group of derivations on k unless $t = p$ and $i = 1$. If $t = p$, H_1/G_2 is isomorphic to the subgroup of those derivations δ on $k \ni \delta(\bar{v}) = 0$. Also, $G_i = H_i$, $i \geq 1$, unless $t = p - 1$, $i = 2$ and one of the four equivalent conditions of Theorem 1 holds. (Received November 30, 1966.)

648-37. G. U. BRAUER, University of Minnesota, Minneapolis, Minnesota 55455.

Homomorphic summation methods.

Let s and t be two bounded sequences and let $B = (b_{nk})$ be a positive regular triangular summation matrix such that $b_{n,n-r}$ tends to zero as n tends to infinity for each r . We define the product $s*t$ as the sequence $\{\sum_{k=0}^n b_{nk} s_k t_{n-k}\}$. If a regular summation ϕ has the property that $\phi(s*t) = \phi(s)\phi(t)$ for all sequences s and t which are evaluated by ϕ to $\phi(s)$ and $\phi(t)$ respectively, ϕ is called a regular B homomorphism. If the matrix $A = (a_{nk})$ is a regular B homomorphism which evaluates all periodic sequences, then $\max |a_{nk}|$ tends to zero as n tends to infinity. If $b_{n,k} = 0$ when k lies outside the interval $[n/2 - k_n, n/2 + k_n]$ and the numbers b_{nk} are essentially equal when k lies inside this range (where k_n is a sequence of numbers tending to infinity slower than n), then for each sequence s the values of $\phi(s)$ range over a continuum as ϕ ranges over the regular B homomorphisms. If C_0 is the space of bounded sequences whose differences tend to zero, the product of each two bounded sequences tends to zero provided $\lim \sum_{k=0}^n |b_{n,k} - b_{n,k+1}| = 0$. (Received May 1, 1967.)

648-38. D. A. SPRECHER, University of California, Santa Barbara, California 93106. On best approximations in several variables with linear superpositions.

Consider the linear space C_m of real valued continuous functions defined on the unit cube in m -dimensional Euclidean space. Our main result is this: If the vector valued function $\psi = (\psi_1, \dots, \psi_n)$, $n \geq m$, with fixed members $\psi_p \in C_m$, imbeds the unit cube E_m homeomorphically in E_n , then each function $f \in C_m$ has a best approximation of the form $g = \sum g_p \circ \psi_p$. In the proof we extend to the situation at hand the leveling process of Diliberto and Straus which they applied to the superpositions $\sum g_p(x_p)$, in which the variables are separated. (Received May 4, 1967.)

648-39. SEYMOUR BACHMUTH and H. Y. MOCHIZUKI, University of California, Santa Barbara, California 93106. Kostrikin's theorem on Engel conditions in groups of prime-power exponent.

Theorem. Let $p \geq 3$ be a prime and $e \geq 1$ any integer. Then there exists a group \mathcal{G} which has exponent p^e and Engel length $e(p^e - p^{e-1}) + (p - 3)/2$. If $e = 1$, this reduces to a Theorem of Kostrikin [On Engel properties of groups with the identical relation $x^{p^a} = 1$, Dokl. Akad. Nauk SSSR 135 (1960), 520-523; = Soviet Math. Dokl. 1 (1961), 1278]. Our method, different from that of Kostrikin, yields the additional information that \mathcal{G} is a solvable group of class at most $k + 1$, where k is the least integer such that $2^{k-1} \geq p - 2$. (Received May 8, 1967.)

648-40. S. J. LOMONACO, JR., Florida State University, Tallahassee, Florida 32306. Uncountably many mildly wild non-Wilder arcs.

R. H. Fox and O. G. Harrold in The Wilder arcs, Proc. Top. Inst. (1962), 184-187, have defined a Wilder arc as a mildly wild L. P. U. (locally peripherally unknotted) arc and have completely classified such arcs. In this paper, to show that the L. P. U. condition is essential, uncountably many mutually nonequivalent mildly wild non-L. P. U. arcs are constructed. (This paper corrects Abstract 636-57, these *Notices* 13 (1966), 592.) (Received May 22, 1967.)

648-41. J. S. MULDOWNEY, University of Oklahoma, Norman, Oklahoma 73069, and J. S. W. WONG, University of Alberta, Edmonton, Alberta, Canada. Bounds for solutions of nonlinear integro-differential equations.

Bounds and asymptotic bounds are given for solutions of the nonlinear integro-differential equations: (*) $\dot{x}(t) - \dot{x}(0) + \int_0^t a(s)g(x(s))ds = h(t)$, $t \in [0, \infty)$ where "dot" denotes differentiation with respect to t , by methods of integral inequalities and construction of Lyapunov-like functions. The following theorem is typical of results presented here: Theorem. Assume that: (i) $a(t)$ is positive and absolutely continuous on bounded intervals, (ii) there exists an odd nondecreasing continuous function f on $(-\infty, \infty)$ such that $0 < \mu xf(x) \leq xg(x) \leq xf(x)$ if $x \neq 0$, where μ is a constant ≤ 1 . Then any solution $x(t)$ of (*) may be continued throughout $[0, \infty)$ and there exists a positive constant γ such that $|x(t)| \leq 2 \int_0^t |h| + (\gamma + \int_0^t |h|)\mu^{-1} \exp(\int_0^t a - /a)$ and $|\dot{x}(t)| \leq |h(t)| + (2a(t)F(\gamma + \int_0^t |h|))^{1/2} \cdot \exp(1/2 \int_0^t a - /a)$, $F(x) = \int_0^x |f|$. This result extends recent work of Hastings, Proc. Amer. Math. Soc. 17 (1966), 40-47. (Received May 3, 1967.)

648-42. A. R. AMIR-MOEZ, Texas Technological College, Lubbock, Texas 79409. Subunitary transformations.

A linear transformation A on a unitary space E is called subunitary if $A^* = A^-$, the generalized inverse of A . We show that a nonzero proper value m of A satisfies $0 < |m| \leq 1$; to each subunitary transformation A corresponds a subunitary transformation B such that $A + B$ is unitary. A few other properties of subunitary transformations are given and results are applied to subpolar decompositions of a linear transformation. (Received May 4, 1967.)

648-43. S. M. SHAH, University of Kentucky, Lexington, Kentucky 40506. Entire functions with no finite deficient value.

In this paper a theorem giving a class of entire functions having no finite Valiron deficient value is proved. Theorem (i) Let $\Psi(r)$ and $\theta(r)$ be two functions tending to ∞ with r . Suppose that for large r , $\Psi(r) \uparrow$, $\theta(r) \uparrow$, $\Psi(r) \geq \log r$, $\Psi(r\theta(r)) \leq c\Psi(r)$ where $c (> 1)$ is a positive constant. Assume also that $\lim_{r \rightarrow \infty} \Psi(r)/\log \theta(r) = \infty$. If $\phi(r)$ is any function tending to ∞ , however slowly with r , and if ultimately $(\Psi(r)\phi(r))/\log \theta(r) \leq \log M(r, f) \leq \Psi(r)$, then we have for every complex number a (x) $\lim_{r \rightarrow \infty} \log M(r, f)/N(r, a) = 1$ and (xx) $\lim_{r \rightarrow \infty} \log M(r, f)/n(r, a) = \infty$. (ii) Let $\phi(r)$ be any function tending to ∞ with r and such that $\phi(r)/r \downarrow 0$. Let $L(r)$ be a slowly oscillating function, and define $L_1(r) = \int_1^r \{L(t)/t\} dt$, $\xi(r) = L_1(r)/L(r) \{\phi(L_1(r)/L(r))\}$. Suppose that $\lim_{r \rightarrow \infty} L(r) = \infty$. If ultimately $L_1(r)/L(r)$ is nondecreasing and $0 < L(r)/\xi(r) \leq n(r, 0) \leq L(r)$ then (x) and (xx) hold for every a . (iii) If $\lim_{r \rightarrow \infty} n(r, 0) > 0$ and $n(r, 0) = o(N(r, 0))$ then (x) and (xx) hold for every a . (Received May 5, 1967.)

648-44. R. C. STEINLAGE, University of Dayton, Dayton, Ohio 45409. A note regarding Haar measure on the space of oriented lines.

The existence and uniqueness of Haar measure on the space of oriented lines in E_3 (Mickle and Rado, Duke Math. J. 15 (1948), 169-180 and Trans. Amer. Math. Soc. 93 (1959), 492-508) is shown to be a consequence of the fundamental theorem of Andre Weil (Integration dans les groupes topologiques et ses applications, p. 45). The proof does not utilize the so called "5r condition" nor does it depend on a relaxation of the concept of invariant distance (see the second reference above). Theorem. Let G be a group, each element of which defines a function from a group H into H . Assume that G and H are T_2 topological groups such that G is compact and H is locally compact. If $G \times H$ is a topological group under the operation $(g_1, h_1)(g_2, h_2) = (g_1 g_2, h_1 g_1(h_2))$, then both right and left Haar measures on H are also invariant under the action of G . Furthermore, if H is unimodular, then $G \times H$ is likewise unimodular. This theorem is used twice in order to show that a certain coset space of the group of rigid motions has a unique Haar measure. Then the space of oriented lines with the Mickle-Rado topology is shown to be homeomorphic to this coset space in such a way that the Haar measure can be carried over. (Received May 12, 1967.)

648-45. WITHDRAWN.

648-46. DAVID STADTLANDER, University of Florida, Gainesville, Florida 32601. Semigroup actions on specialized spaces. Preliminary report.

Let X be a Hausdorff space and T a topological semigroup. An act is a continuous function $\mu: T \times X \rightarrow X$ which is onto and satisfies $\mu(st, x) = \mu[s, \mu(t, x)]$. A clan is a topological semigroup with identity which is compact and connected. The author investigates clan actions on specialized continua. Typical of the theorems obtained are the following. Theorem 1. Let the clan T act on the space X with some element acting as a constant mapping. If X is a compact continuum irreducible between two points, then X is an arc. Theorem 2. Let the clan T , with minimal ideal K , act on the compact continuum X irreducible from KX a point b not in KX . Then X/KX is an arc and KX is the underlying space of an abelian topological group. Theorem 3. Let the clan T act on the compact n -indecomposable continuum X (so that $Tx = X$ for some $x \in X$). Then for some idempotent e in the minimal ideal of T , $(H(e), X)$ is a topological transformation group (which is transitive on X). (Moreover, $n = 1$.) (Received May 15, 1967.)

648-47. M. V. PATTABHIRAMAN and PETER LANCASTER, The University of Calgary, Calgary, Alberta, Canada. Expansions for the inverse of a polynomial operator.

Let $D(\lambda) = A_0 \lambda^l + A_1 \lambda^{l-1} + \dots + A_l$ be a polynomial operator where the A_i^s are bounded linear operators from a Banach space B into itself, A_0 has a bounded inverse and λ is a complex parameter. We say that λ_j is a characteristic value of $D(\lambda)$ if there exists a nontrivial $x \in B$ for which $D(\lambda_j)x = 0$. In the finite dimensional case it is known that there exists an expansion of $D^{-1}(\lambda)$ in terms of the matrices of right and left eigen vectors and the diagonal matrix of eigen values. We extend the results to the infinite dimensional case under the following assumptions: (a) $D(\lambda)$ has a pure point spectrum with the possible exception of zero (b) the characteristic values λ_n are isolated, denumerable in number, tend to zero and have index one (c) $\{(|\lambda_n| + |\lambda_{n+1}|)^r / (|\lambda_n| - |\lambda_{n+1}|)\}$ tends to zero for $r = 2$ or 3 . Typical of these expansions is $\lambda^l D^{-1}(\lambda) = \sum_{j=1}^{\infty} (\lambda_j^l / (\lambda - \lambda_j)) P_j + A_0^{-1}$ where $P_j = (1/(2Di)) \int_{P_j} D^{-1}(\lambda) d\lambda$. (Received May 15, 1967.)

648-48. H. W. LAMBERT, University of Iowa, Iowa City, Iowa 52240. Mapping cubes with holes onto cubes with handles.

In connection with some recent work by W. Haken on the Poincaré conjecture in dimension 3, R. H. Bing raised the following question. If T is any cube with 2 holes (n holes), does there exist a continuous map f of T onto a cube with 2 handles (n handles) C such that $f|BdT$ is a homeomorphism onto BdC ? For the case $n = 1$, J. Hemple answered the question in the affirmative. We show that the question has a negative answer for $n = 2$ (and hence for each $n > 2$). (Received May 15, 1967.)

648-49. R. T. RAMSAY, North Carolina State University, Raleigh, North Carolina. On compactifications and structure of topological groups. Preliminary report.

If G is a locally compact group with center Z such that G/Z is compact then $G = V \times H$ where V is a vector group and H has an open compact normal subgroup, G is maximally almost periodic, and the right and left uniformities of G are equal. These results were stated by S. Grosser and M. Moskowitz [Bull. Amer. Math. Soc. 72 (1966), 826-830]. More generally, if G is any topological

group such that G/Z is compact and Z is maximally almost periodic, then G is maximally almost periodic. (It is well known that a group G is maximally almost periodic if and only if there is a continuous isomorphism of G into a compact group.) It is known that if G is a locally compact connected group, then the following are equivalent; (1) G/Z is compact, (2) G is maximally almost periodic, and (3) G has equal uniformities. A similar result can be stated for the compactly generated groups for which the closure of the commutator subgroup is compact. Various other results relate the maximally almost periodic groups to the groups with equal uniformities. (Received May 18, 1967.)

648-50. D. W. MATULA, Washington University, St. Louis, Missouri 63130. Significance spaces and base conversion mappings.

The popular concept of significant-digit or floating-point numbers involves those numbers representable by a truncated radix representation and an arbitrary integral power of the radix or base. We characterize the set of real numbers representable with n significant-digits to the base β as follows: $S_{\beta}^n = \{\pm (\alpha_1 \beta^{j-1} + \alpha_2 \beta^{j-2} + \dots + \alpha_n \beta^{j-n}) | \alpha_i, j \text{ integers, } 0 \leq \alpha_i \leq \beta - 1\}$ is a significance space with base β and significance n . Truncation conversion of a nonnegative real number x into a member of S_{β}^n is defined by the following mapping: $T_{\beta}^n(x) = \max\{y | y \leq x, y \in S_{\beta}^n\}$, and for negative x , $T_{\beta}^n(x) = -T_{\beta}^n(-x)$. We show that if $\beta^i \neq \nu^j$ for any integers $i, j > 0$, then $T_{\nu}^m : S_{\beta}^n \rightarrow S_{\nu}^m$ is one-to-one iff $\nu^{m-1} \geq \beta^n - 1$ and is onto iff $\beta^{n-1} \geq \nu^m - 1$. Under the same conditions on β and ν it is also shown that $S_{\beta}^n \cap S_{\nu}^m$ has only a finite number of elements, thus $\dots T_{\beta}^n T_{\nu}^m \dots T_{\beta}^n T_{\nu}^m(x)$ need not converge in a finite number of steps. Finally, we show that although $S_{\beta}^n = S_{\nu}^m$ iff $\beta = \nu$ and $n = m$, the base β rationals, $B_{\beta} = \bigcup_{n=1}^{\infty} S_{\beta}^n$, possess the following relationship: $B_{\beta} = B_{\nu}$ iff $p|\beta \Leftrightarrow p|\nu$ for all primes p , (e.g., $B_{12} = B_{18}$). (Received May 18, 1967.)

648-51. J. A. COCHRAN, Bell Telephone Laboratories, Inc., Whippany, New Jersey 07981. On the uniqueness of solutions of linear differential equations.

The possibility of representing the two-point boundary-value problem as a pseudo initial-value problem, and hence as an equivalent pseudo Volterra integral equation, is exploited in this paper. This permits the derivation of necessary and sufficient conditions (NASC) for uniqueness of solutions of the given problem in terms of the resolvent kernel associated with this integral equation. For example, the Dirichlet problem associated with the equation $[p(x)u'(x)]' + q(x)u(x) = r(x)$ on $0 \leq x \leq 1$ has a unique solution if and only if $R(1,0)/q(0) \neq 0$, where $R(x,x')$ is the above-mentioned resolvent kernel. A fundamental application of these results is made to singular perturbation theory. (Received May 18, 1967.)

648-52. P. C. TONNE, Emory University, Atlanta, Georgia 30322. On the convergence of Bernstein polynomials for some unbounded analytic functions. Preliminary report.

If f is a function from a set containing $[0, 1]$ to the complex numbers and n is a positive integer and z is a complex number, then $B_n^f(z) = \sum_{p=0}^n f(p/n) \binom{n}{p} z^p (1-z)^{n-p}$. B_n^f is the n th Bernstein polynomial for f . Theorem. If A is a bounded complex number sequence and f is a function such that, for each complex number z with modulus less than 1, $f(z) = \sum_{p=0}^{\infty} A_p z^p$ and $f(1) = 0$, then the Bernstein

polynomial sequence B^f converges uniformly to f on each closed subset of the unit disc. (Received May 18, 1967.)

648-53. MAURICE CHACRON, University of Sherbrooke, Quebec, Canada. Endomorphisms of universal algebras.

In an earlier work, we determined classes of semimodules not necessarily finite such that the semigroup of the endomorphisms consists only of singular endomorphisms or automorphisms. In this paper, we extend our results to universal algebras. We find that the efficiency of the finitary conditions which we considered earlier remain unchanged. From the descending chain condition on the lattice of the subuniversal algebra, we prove that the surjectivity of an endomorphism is equivalent to the bijectivity and also the dual result. Furthermore, we show that if the semigroup of endomorphisms is extended by an almost periodic semigroup (i.e. semigroup with involution such that every self-conjugate element generates a finite subsemigroup (periodic element) then every epimorphism or monomorphism permutable with the conjugate must be a unity of the semigroup which is the same as an automorphism of the universal algebra. (Received May 18, 1967.)

648-54. N. F. LINDQUIST, Western Washington State College, Bellingham, Washington 98225. Representations of central convex bodies. Preliminary report.

In Euclidean n -space $E(n)$ let $E(2)$ denote any 2-dimensional subspace and denote by $\Omega(j)$ the unit spherical surface in $E(j)$, $j = 2$ or n . For $\bar{v} \in \Omega(2)$ define $L(\bar{v}) = \{v \in \Omega(n) | (v, \bar{v}) > 0 \text{ and } (v - \bar{v}, u) = 0 \text{ for all } u \in \Omega(2)\}$. The surface element at v on $\Omega(n)$ is $d\omega(v)$ and the surface element at v on $L(\bar{v})$ is $d\sigma(v)$. Theorem. In order that $H(u) = \int_{\Omega(n)} |(u, v)| h(v) d\omega(v)$ be a representation of a support function of a central convex body, it is necessary and sufficient that h satisfy $\int_{L(\bar{v})} (v, \bar{v})^2 h(v) d\sigma(v) \geq 0$ for all $\bar{v} \in \Omega(2)$ and for all subspaces $E(2)$ of $E(n)$. (Received May 15, 1967.)

648-55. CHING CHOU, University of Rochester, Rochester, New York 14627. The support of ergodic measures.

Let N be the set of all positive integers and $\hat{N} = \beta N \setminus N$. Let τ be the homeomorphism of \hat{N} onto \hat{N} induced by the mapping $n \rightarrow n + 1$ ($n \in N$). Denote by M the set of τ -invariant probability measures on \hat{N} and by $\text{ex } M$ the set of extreme points of M . It is known that M is weak* compact and convex in $C(\hat{N})^*$ and $\phi \in \text{ex } M$ if and only if ϕ is ergodic. Theorem 1. $M \neq \emptyset \{ \sum_{i=1}^{\infty} c_i \phi_i : \sum_{i=1}^{\infty} c_i = 1, c_i \geq 0, \phi_i \in \text{weak}^* \text{ cl } (\text{ex } M) \}$. A subset K of \hat{N} is called minimal if it is closed, nonempty and τ -invariant (i.e., $\tau K \subseteq K$) and is minimal with respect to these three properties. It is known that each minimal set supports ergodic measures. But one has the following Theorem 2. There exists ergodic measure ϕ such that the support set of ϕ is not minimal. These theorems answer questions asked by R. G. Douglas and R. A. Raimi, respectively. (Received May 10, 1967.)

648-56. H. W. MCLAUGHLIN and F. T. METCALF, University of California, Riverside, California. An inequality for integral norms.

Let (i) m be a positive integer; (ii) $t = (t_1, \dots, t_m)$ and $a = (a_1, \dots, a_m)$ m -tuples of positive real

numbers, with $\sum_{i=1}^m \alpha_i = 1$; (iii) $H(t; \alpha) = (\sum_{i=1}^m \alpha_i t_i^{-1})^{-1}$, the harmonic mean of the numbers t_1, \dots, t_m , with weights $\alpha_1, \dots, \alpha_m$; (iv) (X, A, μ) a measure space, with $\mu(X) > 0$. For $f \in L_\tau(X, A, \mu)$, $\tau > 0$, define $\|f\|_\tau = [\int_X |f|^\tau d\mu]^{1/\tau}$. Theorem. Let r be a positive real number. Then a necessary and sufficient condition that $\|f\|_r \leq \prod_{i=1}^m \|f\|_{t_i}^{\alpha_i}$, independently of the choice of (X, A, μ) and of $f \in L_{t_i}(X, A, \mu)$ ($i = 1, \dots, m$), is that $r = H(t; \alpha)$. Furthermore, let $g(y_1, \dots, y_m)$ be a real-valued function, defined for $y_i \geq 0$ ($i = 1, \dots, m$), such that $g(y_1, \dots, y_m) \leq \prod_{i=1}^m y_i^{\alpha_i}$ whenever y_1, \dots, y_m are nonnegative. Then a necessary and sufficient condition that $\|f\|_{H(t; \alpha)} \leq g(\|f\|_{t_1}, \dots, \|f\|_{t_m})$, independently of the choice of (X, A, μ) , $f \in L_{t_i}(X, A, \mu)$ ($i = 1, \dots, m$), and t , is that $g(y_1, \dots, y_m) = \prod_{i=1}^m y_i^{\alpha_i}$ whenever y_1, \dots, y_m are nonnegative. The inequality contained in this theorem, in the special case of finite sums, is of particular interest in light of the inequalities appearing in Beckenbach and Bellman [Inequalities, Springer, Berlin, 1965, p. 19]. (Received May 19, 1967.)

648-57. A. M. CHAK, West Virginia University, Morgantown, West Virginia 26505. A method of deriving differential equations of special functions.

The need to find a general method of deriving differential equations of the so-called special functions of analysis from their generating functions arose because of the fact that the differential equation which the generalized Humbert polynomials $P_n \equiv P_n(m, x, y, p, C)$ satisfy is conspicuous by its absence in the paper by H. W. Gould [Duke Math. J. 32 (1965), 697-712] in which he studies them in great detail. He defines them by $(C - mx + yt^m)^p = \sum_{n=0}^{\infty} t^n P_n(m, x, y, p, C)$ where $m \geq 1$ is an integer and the other parameters are in general unrestricted. By an iterative use of the linear differential operator $axD + b$ and a simple lemma it is shown that the missing differential equation is $\{(m-1)xD + n - mp + m(m-2)\} \{(m-1)xD + n - mp + m(m-3)\} \dots \{(m-1)xD + n - mp\} \{xD - n\} P_n = yC^{m-1} P_n^{(m)}$. It has also been shown in this paper that the method used to derive this differential equation of the m th order from just two recurrence relations is so general that it can with profit be used to get the 2nd order and higher order differential equations of all the well-known classical special functions. (Received May 19, 1967.)

648-58. J. R. DURBIN, The University of Texas, Austin, Texas 78712. Residually central elements in groups.

An element x in a group G is said to be residually central if either $x = e$ or there exists a homomorphism ϕ of G onto a group H such that $x\phi$ is a nontrivial element of the center of H . Certainly every element of a Z -group is residually central. Theorem 1. A finite group is nilpotent if and only if each of its elements is residually central. Theorem 2. If G is a finite group and the set of all residually central elements of G is a subgroup $R \neq G$, then $G' = G$ and R is the center of G . The situation with respect to weaker finiteness conditions is also discussed. (Received May 22, 1967.)

648-59. M. F. SMILEY, State University of New York at Albany, Albany, New York 12203. Inequalities related to Lidskiĭ's.

Simple proofs are presented for certain linear inequalities involving the eigenvalues of the sum of two hermitian operators in a finite dimensional unitary space. The inequality of V. Lidskiĭ [Dokl. Akad. Nauk SSSR 75 (1950), 769-772; see also H. Wielandt, Proc. Amer. Math. Soc. 6 (1955),

106-110] is proved first and then used to derive the inequalities of Amir-Moëz [Duke Math. J. 23 (1956), 463-476] and those of J. Hersch and B. P. Zwahlen [C. R. Acad. Sci. Paris 252 (1962), 254-257, see also B. P. Zwahlen, Comm. Math. Helv. 40 (1966), 81-116]. Our method of proof is closely related to that of A. Horn [Pacific J. Math. 12 (1962), 225-242, Theorem 5] and uses only the classical Poincaré-Fischer-Weyl-Courant maximum-minimum principle. The relation of Lidskiĭ's inequality (L) to a recent result of F. John [Proc. Amer. Math. Soc. 17 (1966), 1140-1145] is discussed. It is shown that F. John's result is a direct consequence of (L) and that a slightly stronger statement is (trivially) equivalent to (L). (Received May 22, 1967.)

648-60. M. L. MARX, Vanderbilt University, Nashville, Tennessee 37203. The branch point structure of extensions of interior boundaries.

Let $D \subset E^2$ be an open set such that \bar{D} is compact and boundary D is an oriented Jordan curve. A mapping $f: \bar{D} \rightarrow E^2$ is said to be properly interior if f is light, open, and sense-preserving on D , and a local homeomorphism (relative to \bar{D}) at each point of boundary D . A theorem of Stoilow states that if f is properly interior, then at each point p of D there exists a closed two-cell neighborhood N of p on which f is topologically equivalent to $w = z^n$ on $|z| \leq 1$ for some positive integer n . The integer $n - 1$ is called the multiplicity of f at p . A branch point is a point at which the multiplicity is strictly larger than zero. Suppose $\delta: \text{Boundary } D \rightarrow E^2$ is continuous and there exists a properly interior $f: \bar{D} \rightarrow E^2$ such that $f|_{\text{Boundary } D} = \delta$; then f has only finitely many branch points, say z_1, \dots, z_r with multiplicities $\mu(1), \dots, \mu(r)$. It is well known that $1 + \mu(1) + \dots + \mu(r)$ is always equal to the tangential winding number of δ . What has not been considered is what minimal value r can take for a given δ and what multiplicities can occur for the minimal r . An algorithm is given that answers this question in case δ is normal (see Abstract 619-153, these *Notices* 12 (1965), 102). (Received May 29, 1967.)

648-61. MANFRED BREUER, University of Kansas, Lawrence, Kansas 66044. Fredholm theories in von Neumann algebras.

Let A be a von Neumann algebra of operators of a Hilbert space H . Let A' be its commutant. To each projection E of A one can canonically associate a representation \hat{E} of A' with representation space $E(H)$. The equivalence classes of representations of A' arising from finite projections of A generate a semiring (w.r. to $\oplus, \otimes, <$). The corresponding universal ring $I(A)$ is the index ring of A . Call $T \in A$ finite if its support is finite; compact if it is a limit in the norm of finite elements; Fredholm if (i) the null projections of T, T^* are finite and (ii) the range projection of T is the supremum of all projections $E \in A$ s.t. $E(H) \subseteq A(H)$ and $1 - E$ finite. The index of a Fredholm element T of A can be defined as an element $\nu(T) \in I(A)$. Much of the classical Fredholm theory can be generalized. E.g., one of the most elementary and basic results is that $1 - T$ is Fredholm of index zero if T is compact. Also what is called the Riesz theory of compact operators (see Dieudonné, Foundations of modern analysis) can be generalized to a certain extent to compact elements of von Neumann algebras. (Received May 29, 1967.)

648-62. J. W. KENELLY and W. R. HARE, JR., Clemson University, Clemson, South Carolina 29631. The intersection of maximal starlike sets.

The intersection of the maximal convex subsets of a compact set S is the convex kernel, provided the intersection is nonempty. Since compact sets have maximal starlike sets, the analogous question of the structure of their intersection is raised. A partial answer is given by the following: Theorem. Let S be compact simply-connected in the plane. Then the intersection of all the maximal starlike subsets of S is 2-polygonally connected. (Received May 31, 1967.)

648-63. JAMES TURNER, Stanford Research Institute, Menlo Park, California 94025. Point symmetric graphs with a prime number of points.

A graph is said to be point-symmetric if its automorphism group acts transitively on its vertex set. Point-symmetric graphs having a prime number of vertices are considered. A starred polygon is a regular polygon with inscribed chords such that the chords from each vertex span the same number of vertices. Theorem 1. Every connected point-symmetric graph having a prime number of vertices can be drawn as a starred polygon. Theorem 2. Two graphs satisfying the hypotheses of Theorem 1 are isomorphic if and only if adjacency matrices which represent these graphs have the same eigenvalues. Using the above theorems together with the Pólya enumeration theorem, point-symmetric graphs having a prime number of points are enumerated. Finally, point-symmetric and line-symmetric graphs having a prime number of lines are characterized. (Received May 31, 1967.)

648-64. WITHDRAWN.

648-65. MARK LOTKIN, 115 Hedgerow Drive, Cherry Hill, New Jersey. Determination of angle of attack from rotational body rates.

There are presented in this paper new formulations for the ordinary differential equations of the total angle of attack and the relative roll angle which contain as inputs the angular rates of the flying object. These formulations are inherently more accurate and economical than other sets of differential equations that have been employed widely in the past. The new formulations have been

found to be capable of producing the attitude angles of interest even in cases of utilization of unsmoothed angular rates to accuracies of better than five percent. (Received June 1, 1967.)

648-66. L. C. KURTZ and S. M. SHAH, University of Kentucky, Lexington, Kentucky 40506.

On the L^1 norm and the minimum value of a trigonometric polynomial.

A strictly increasing sequence $\{m_k\}$ of positive integers is said to be admissible if $m_k - m_j + m_l - m_p \neq 0$ if $k \neq j$, $k \neq p$ and $j \neq l$. Let $e(x) = e^{2\pi i x}$, $S_n(x) = \sum_{k=1}^n c_k e(m_k x)$, $R_n = \sum_{k=1}^n |c_k|^2$ and $T_n = \sum_{k=1}^n |c_k|^4$. **Theorem 1.** If $\{m_k\}$ is an admissible sequence, then $\int_0^1 |S_n(x)| dx \geq (R_n/2)^{1/2}$. **Theorem 2.** Let $T_n(x) = \sum_{k=1}^n a_k \cos 2\pi(m_k x + \alpha_k)$, $a_k \geq 0$, α_k real. If $\{m_k\}$ is admissible then (i) $\int_0^1 |T_n(x)| dx \geq (\sum_{k=1}^n a_k^2)^{1/2}/4$, (ii) $\min_{0 \leq x \leq 1} T_n(x) \leq -(1/8) (\sum_{k=1}^n a_k^2)^{1/2}$. **Corollary.** If $\{m_k\}$ is admissible, then $\min_{0 \leq x \leq 1} \sum_{k=1}^n \cos 2\pi m_k x \leq -(1/8) n^{1/2}$. Examples of admissible sequences are given. (Received May 25, 1967.)

648-67. BERNARD SHIFFMAN, 1709 Berkeley Way, Apartment A, Berkeley, California 94703.

On the removal of singularities of analytic sets.

A subset A of a domain U in C^n is said to be analytic in U if A is a closed complex analytic subvariety of U . The following is a generalization of a result of Remmert and Stein [Über die wesentlichen Singularitäten analytischer Mengen, Math. Ann. 126 (1953), 263-306]: **Theorem.** Let U be open in C^n , and let E be closed in U . Let A be analytic in $U - E$ of pure dimension k , and let A' be the closure of A in U . If E has Hausdorff $(2k - 1)$ -measure zero, then A' is analytic in U of pure dimension k . The proof makes use of elementary integral geometry, some methods of Bishop [Conditions for the analyticity of certain sets, Mich. Math. J. 11 (1964), 289-304], and a measure-theoretic condition on the removability of singularities of bounded holomorphic functions of several complex variables. (Received May 26, 1967.)

648-68. M. L. FAULKNER, Western Washington State College, Bellingham, Washington 98225.

On the diophantine equation $n! = x^4 - y^4$.

P. Erdős and R. Obláth (Acta Sci. Math. 8 (1937), 241-255) settled the question of the solvability of the diophantine equations $n! = x^m - y^m$ for $(x,y) = 1$, and $m > 2$ with $m \neq 4$. For $m = 4$, the equation was shown to be unsolvable for n sufficiently large. In the present paper, a proof is given of the inequality $\sum (\log p/p^r) < (1/2) \log n - .59 + 1/\log n$ for $n \geq 961$, where the sum is taken over primes $\equiv 1 \pmod{4}$, $p^r \leq n$ and $r \geq 1$. With the aid of this result and a modification of the argument of Erdős and Obláth, the equation $n! = x^4 - y^4$ is shown to have no solutions in positive integers n , x , and y with $(x,y) = 1$. (Received May 26, 1967.)

648-69. WITHDRAWN.

648-70. JULIUS SMITH, The University of Tennessee, Knoxville, Tennessee 37916. The coupled equation approach to the numerical solution of the biharmonic equation by finite differences.

The boundary problem $\Delta\Delta u = f$ in a rectangle R where u and $\partial u/\partial n$ are known on the boundary of R may be reduced to the study of the system $c\Delta u = v$ $\Delta v = cf$ with the same boundary conditions. A reduction to a system of difference equations by the usual techniques leads to two discrete Poisson equations in which, however, v is not given a priori on the boundary. This difficulty has been overcome by several workers with the aid of a scheme involving an "inner" and "outer" iteration. In this paper the rate of convergence of such a scheme is studied, and the eigenvalues of the iteration matrix are shown to be related to the minimization of $\iint(\Delta u)^2 dx dy + \int(\partial u/\partial n)^2 ds$. Asymptotic estimates are obtained for the eigenvalues as the mesh-size, h , approaches zero. (Received May 22, 1967.)

648-71. C. C. CHEN, Queen's University, Canada, and G. A. GRÄTZER, University of Manitoba, Winnipeg, Manitoba. Construction of Stone lattices. III.

For terminologies used, see Abstract 67T-364, these *Notices* 4 (1967), 527. Let L, M be Stone algebras. Then: (7) L is a relative Stone lattice iff D_L is so; (8) $\theta: P \rightarrow [P]$ is a one: one mapping from $S(B_L) \cup S(D_L)$ onto $S(L)$, where for a distributive lattice N , $S(N)$ denotes the Stone space of N . The subspace $S(B_L)\theta$ is homeomorphic to $S(B_L)$, and is an open (resp. closed) subset of $S(L)$ iff D_L has a least element (resp. $B_L = L$); (9) $S(D_L)$ is homeomorphic to the subspace $S(D_L)\theta$ of $S(L)$ under θ iff for each $x \neq D_L$ in L , $\{d \in D_L \mid d \geq x\}$ has a least element; (10) Let N be a pseudo-complemented distributive lattice and D the dual ideal of N consisting of all dense elements of N . Then N is a Stone lattice iff the join of any two distinct prime ideals of N disjoint from D is N ; (11) L is complete iff B_L is complete, D_L is conditionally complete and for each subset E of D_L , $\{b \in B_L \mid \bigwedge_{d \in E} d \rho_b \text{ exists}\}$ always has a greatest element; (12) Let B be a complete Boolean algebra. Then, there exists a homomorphism $\phi: D \rightarrow I(D)$ for every conditionally complete distributive lattice D with 1 such that $L \langle B, D, \phi \rangle$ is complete iff B contains an atom. (Received April 28, 1967.)

648-72. D. W. HARDY, Colorado State University, Fort Collins, Colorado, and R. J. WISNER, New Mexico State University, Box 1993, University Park Br., Las Cruces, New Mexico 88001. An old familiar formula in a semigroup setting.

Let (S, \cong) be an abelian, cancellative, well ordered semigroup with identity. Write $a|b$ if there exists $c \in S$ such that $ac = b$. Define $(a,b) = \sup\{x \in S: x|a \text{ and } x|b\}$ to be the greatest common divisor, and $[a,b] = \inf\{x \in S: a|x \text{ and } b|x\}$ to be the least common multiple of a and b . Theorem 1. $ab \cong (a,b)[a,b]$ for all $a,b \in S$. Theorem 2. $ab = (a,b)[a,b]$ for all $a, b \in S$ if and only if S is a free abelian semigroup. (Received June 2, 1967.)

648-73. MORRIS MARDEN, University of Wisconsin, Milwaukee, Wisconsin. On the derivative of an entire function.

The following new representation is found for the derivative f' of an entire function f of finite order ρ having the zeros $b_1, b_2, \dots, b_m, a_1, a_2, a_3, \dots$, where $0 < |a_1| \leq |a_2| < \dots$. If $\zeta_1, \zeta_2, \dots, \zeta_n$ are

any $n \geq m + [\rho]$ zeros of f' : (*) $\phi(z)f'(z) = f(z)\psi(z)\sum_{j=1}^{\infty} \phi(a_j)[\psi(a_j)(z - a_j)]^{-1}$ for all $z \neq a_j$, with $\phi(z) = \prod_{k=1}^m (z - b_k)$, $\psi(z) = \prod_{k=1}^n (z - \zeta_k)$. Let $S(T, \nu)$ denote the set of points from which set T subtends an angle of at least ν . From (*) follows that, if $a_j \in T$ all j and if $b_k \notin S(T, \beta < \pi/m)$ for $k = 1, 2, \dots, m$, then all but at most $m + [\rho]$ zeros of f' lie in $S(T, \nu)$ where $\nu = (\pi - m\beta)/(n + 1)$. From (*) also follows a simple, direct proof of Laguerre-Borel's Theorem on the zeros of the derivative of a real entire function with only m nonreal zeros. (Received June 2, 1967.)

648-74. T. INGRAM, University of Houston, Houston, Texas 77004. Decomposable circle-like continua.

Theorem. Suppose A and B are chainable continua which intersect. (1) $A + B$ is a chainable continuum if and only if $A + B$ is atriodic and $A \cdot B$ is connected. (2) $A + B$ is a nonchainable circle-like continuum if and only if $A + B$ is atriodic and $A \cdot B$ is not connected. Note that (1) strengthens a theorem of J. B. Fugate [Decomposable chainable continua, Trans. Amer. Math. Soc. 123 (1966), 460-468]. (Received May 26, 1967.)

648-75. P. M. HIRSCH, International Business Machines, 6900 Fannin Street, Houston, Texas 77025. Evaluation of orthogonal polynomials and relationship to evaluating multiple integrals.

Three methods are shown to calculate orthogonal polynomials in several variables. It is also proved that if an integration formula of precision m has less than $C_{m+n, n}$ points in E^n then there exists an orthogonal polynomial of degree K , $[m/2] < K \leq m$ with the evaluating points at zeros. (Received June 2, 1967.)

648-76. R. G. LINTZ, McMaster University, Hamilton, Ontario, Canada. Introduction to general analysis.

This paper is part of a general program in nondeterminist mathematics and deals with the extension of the concept of derivative to general spaces. For that a new kind of structure is introduced in a topological space X by means of what is called a standard family of coverings \mathcal{F} . The pair (X, \mathcal{F}) is called a Gauss space and contains what is needed to develop a theory of derivatives, in the same way that a topological structure contains what is needed to speak about continuity, in general spaces. A paracompact space is a Gauss space. Several properties of generalized derivatives are proved, including a "chain-rule" and the paper finishes with some applications to mechanics by introducing the idea of movement, speed, etc. in Gauss spaces. Among several theorems proved in the paper the following one shows the position of the new concept relative to the usual concept of derivative. Theorem. Let $\phi: R \rightarrow R$ be a real function of one real variable defined and differentiable for all real numbers. Then there exists a continuous generalized function f generating ϕ such that its generalized derivative Df generates a function $\psi: R \rightarrow R$, with the property: $\forall x \in R \Rightarrow \psi(x) = |\phi'(x)|$. (Received June 2, 1967.)

648-77. M. J. FISHER, Johns Hopkins University, Baltimore, Maryland 21218. Harmonic functions and their conjugates on Hilbert space.

Let H be a real separable Hilbert space and $y \rightarrow T_y$ be the regular representation of the additive group of H acting on $L^p(H, \text{normal distribution})$, $1 < p < \infty$. Let B be a one-one Hilbert-Schmidt operator on H and $\{e_j\}$ an orthonormal basis for H . For $f \in L^p(H)$ let $P_z(f) = \int_0^\infty H_t(f) N_t(z) dt/t$ where $H_t(f) = \int_H T_y f dn_t \circ B^{-1}(y)$, n_t is the normal distribution with variance parameter $t/2$, and $N_t(z) = z/(\pi t)^{1/2} \exp[-z^2/t]$. P_z is the Poisson integral of f . Let $q_{jt}(f) = \int_{H_B} T_{B_y} f(e_j, y) dn_t(y)$ where H_B is the completion of H in the norm $\|Bx\|$. The $Q_{jz}(f) = \int_0^\infty q_{jt}(f) N_t(z) dt/t$ are the conjugate Poisson integrals of f . $Q_{jz}(f)$ is a bounded operator on $L^p(H)$ and there is a singular integral operator R_j on $L^p(H)$ such that $Q_{jz} = R_j \circ P_z$. Set $p = 2$. Let A_j be the infinitesimal generator of the semigroup $T_{tB e_j}$, $t \geq 0$. A function $U: (0, \infty) \rightarrow L^2(H)$ is said to be harmonic if $D_z^2 U(z) + \sum_{j=1}^\infty A_j^2 U(z) = 0$ for all $z > 0$. If $f \in L^2(H)$, $P_z(f)$ is harmonic. If $U(z)$ is harmonic and if $\|U(z)\|_2 \leq M < \infty$ for $z > 0$, there is $f \in L^2(H)$ such that $U(z) = P_z(f)$. For f in $L^2(H)$, set $U(z) = P_z(f)$, $v_j(z) = Q_{jz}(f)$, $V(z) = (v_1(z), v_2(z), \dots)$, and $F(z) = (U(z), V(z))$. Then $|F(z)| \in L^2(H)$ for $z > 0$ and $|F(z)|$ is subharmonic if $|F(z)| > 0$ almost everywhere for each $z > 0$. (Received June 2, 1967.)

648-78. C. A. BRYAN, University of Montana, Missoula, Montana. Approximate solutions to nonlinear integral equations.

Given a nonlinear integral equation of the form $x(s) - \int_0^1 K(s, t, x(t)) dt = h(s)$ or equivalently $x - Kx = h$, consider the quadrature analog $x(s) - \sum_{j=1}^n \omega_j K(s, t_j, x(t_j)) = h(s)$ or $x - \bar{K}x = h$. Then if $\Gamma_0 = (I - \bar{K}^{-1}(x_0))^{-1}$ exists and is bounded, $A = \|\bar{K}'(x_0)K'(x_0) - (K'(x_0))^2\| < 1/\|\Gamma_0\|$, $\|x_0 - Kx_0 - h\| \leq \eta$, $\|K''(x)\| \leq M$ on $S(x_0, r)$, $h = B^2 M \eta \leq 1/2$ where $B = (1 + \|\Gamma_0\| \|K'(x_0)\|)/(1 - \|\Gamma_0\| A)$, and $r_0 = (1 - (1 - 2h)^{1/2})B\eta/h < r$; there exists x^* in $S(x_0, r)$ such that $x^* - Kx^* = h$ and x^* is the unique solution in $S(x_0, r) \cap S(x_0, r_1)$ where $r_1 = (1 + (1 - 2h)^{1/2})B\eta/h$. (Received June 5, 1967.)

648-79. G. L. SEEVER, California Institute of Technology, Pasadena, California. A converse to Bishop's general Rudin-Carleson Theorem. Preliminary report.

Theorem. Let X be a compact Hausdorff space, A a closed subalgebra of $C(X)$, and let Σ be a σ -ideal in the σ -ring of Borel subsets of X . Then the following conditions are equivalent.
 (a) $E \in \Sigma$, $\mu \in A^\perp \Rightarrow \mu(E) = 0$. (b) $S \in \Sigma$, S compact, $p \in C(X)$, $p(x) > 0$ for all $x \in X$, $f \in C(S)$, $|f(x)| < p(x)$ for all $x \in S \Rightarrow \exists F \in A \ni F|_S = f$, $|F(x)| < p(x)$ for all $x \in X$. (c) $\exists M > 0$, $r \in (0; 1/2) \ni S, T \subset X$, S, T compact, $S \cap T = \emptyset$, $S \in \Sigma \Rightarrow \exists f \in A \ni \|f\| \leq M, \|1 - f\|_\infty \leq r, \|f\|_T \leq r$. (Received June 5, 1967.)

648-80. D. A. SMITH, Duke University, Durham, North Carolina 27706. Incidence functions as generalized arithmetic functions. II.

This paper is a sequel to the one announced in Abstract 642-74, these *Notices* 14 (1967), 83, referred to below as (I). The results of (I) concerning factorability and additivity in incidence algebras on lattices are extended to a larger class of p.o. sets. Imbeddings of incidence algebras and of their groups of units induced by imbeddings of the underlying p.o. sets are studied. These are used to identify large classes of incidence algebras with subalgebras of various well-known algebras. A general construction of a class of important subalgebras is given, generalizing two different constructions in (I). Sufficient conditions are given for such a subalgebra to be commutative, lack zero-divisors, etc. For commutative subalgebras of this sort, we introduce a logarithmic operator, which leads to a general theory of exponentiation of incidence functions, and also some isomorphisms among additive and multiplicative groups in incidence algebras. These results generalize recent results of D. Rearick for algebras of arithmetic functions. (Received June 5, 1967.)

648-81. G. G. WALTER, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53201. Real singular points of Legendre series.

Let $f(z)$ be an analytic function given for $|z| < r$ by the power series $\sum a_n z^n$. Nehari has shown that a singular point z_0 of f is related to a singular point t_0 of the associated Legendre series $\sum a_n P_n(t)$ by the formula $t_0 = (1/2)(z_0 + 1/z_0)$. Gilbert and Howard have proved a similar result for Sturm-Liouville expansions. Both results hold only when $r > 1$. It is possible to use a modification of their methods to extend the first result to the case $r = 1$ by introducing an analytic representation \hat{P}_n of the P_n and thus get the Theorem. The series $\sum a_n P_n(t)$ converges in $(-1, 1)$ to a distribution g ; the series $\sum a_n \hat{P}_n(t)$ converges for $\text{Im } t \neq 0$ to an analytic representation \hat{g} of g ; the function \hat{g} has a singular point at $t_0 = (1/2)(z_0 + 1/z_0)$ in $(-1, 1)$ if and only if f has a singular point at z_0 and z_0 is not $+1$. A similar result holds for certain Sturm-Liouville series. (Received June 5, 1967.)

648.82. H. G. RUTHERFORD, Montana State University, Bozeman, Montana. A characterization of primes in simple rings over locally finite fields.

Let R be a ring with unit 1. A preprime of R is a nonempty subset T of R which is closed under addition, closed under multiplication, and does not contain -1 . A prime of R is a maximal preprime of R . This concept was defined by Harrison and for a number field K (i.e., a finite field extension of the rational number) he showed that the primes of K are exactly the useful prime divisors of algebraic number theory. Let k be a locally finite field (i.e., one in which every element is contained in some finite subfield), V a finite dimensional vector space over k and $G = \text{Hom}_k(V, V)$ the full ring of linear transformations of V over k . For subspaces W and L of V with $W \subset L$, we let $T(L, W) = \{\alpha \in G: \alpha(L) \subset W\}$. When $W \subsetneq L$, $T(L, W)$ is a preprime; when the codimension of W in L is 1, $T(L, W)$ is a prime of G . The main result here is that every prime of G has the form $T(L, W)$, $W \subset L$ subspaces of V , codimension of W in L being 1. (Received June 5, 1967.)

648-83. K. G. JOHNSON, East Carolina University, Greenville, North Carolina. Discontinuous functions of bounded variation and change of variable in a Lebesgue integral.

An affirmative answer to each of two questions of Varberg in Amer. Math. Monthly 72 (1965), 841, is contained in the following result. Theorem 1. Let ϕ be real and of bounded variation on a closed interval $[t_1, t_2]$ and let M be a measurable subset for which $m(\phi(M)) = 0$ where m is Lebesgue measure. Then $\phi'(t) = 0$ almost everywhere on M . An application is in proving the following general change of variable formula, other versions of which require ϕ either to be absolutely continuous or monotone continuous. Theorem 2. Let f be extended real valued and summable over $[a, b]$. Let $F(x) = \int_a^x f(u) du$. Let ϕ be of bounded variation on a closed interval $[t_1, t_2]$ such that $\phi([t_1, t_2]) \subseteq [a, b]$, and such that $F(\phi)$ is absolutely continuous on $[t_1, t_2]$. Let ϕ' be a function equal almost everywhere to the derivative of ϕ . Then $f(\phi)\phi'$ is summable over $[t_1, t_2]$ and $\int_{\phi(t_1)}^{\phi(t_2)} f(u) du = \int_{t_1}^{t_2} f(\phi(t))\phi'(t) dt = F(\phi(t_2)) - F(\phi(t_1))$. (Received June 6, 1967.)

648-84. F. GARDINER, Harvard University, Cambridge, Massachusetts 02139 and IRWIN KRA, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. On the boundary of Teichmüller spaces.

Let T_4 be the set of Schwarzian derivatives of univalent functions on the lower half-plane L , and T_1 the Schwarzians of those univalent functions that have quasiconformal extensions to the complex sphere. It is known that T_1 (Universal Teichmüller space) is an open set [L. Bers, Acta. Math. 116 (1966), 113-134] and T_4 is a closed set in the Banach space, $B_2(L)$, of bounded holomorphic forms of weight (-4) on L . Bers has conjectured that the closure of T_1 is T_4 . In this paper we analyze this and several other natural conjectures about the boundary of Teichmüller space. The space $B_2(L)$ is the dual of another space. We show that T_4 is the weak $*$ closure of T_1 . An example shows that the technique used to obtain the theorem on weak $*$ closure will not work in the norm topology. Examples of univalent functions on L whose Schwarzians are in the boundary (with respect to the norm topology) of T_1 are constructed. As a by-product of this investigation the following strange fact is established. There exists a one-to-one holomorphic mapping Γ of T_1 into the boundary of T_4 . Furthermore if α is a linear functional on $B_2(L)$ of norm 1 such that there is a $\Psi_0 \in \text{range } \Gamma$ for which $|\alpha(\Psi_0)| = \|\Psi_0\|$, then $\alpha(\Psi) = \alpha(\Psi_0)$ for all $\Psi \in \text{range } \Gamma$. (Received June 6, 1967.)

648-85. DIRAN SARAFYAN, Louisiana State University, New Orleans, Louisiana. Multi-order property of Runge-Kutta formulas and error estimation.

It will be shown that Runge-Kutta formulas are multi-order or multi-value. This property will be used for estimation of errors and determination of appropriate step-sizes. This error estimation method is competitive with the well known and widely used Richardson's extrapolation method. However, the latter requires three applications of a Runge-Kutta formula while the former requires only one. In other words, no additional substitutions or functional evaluations are necessary with this new method. Detailed literature will be made available to participants. (Received June 7, 1967.)

648-86. K. W. ROGGENKAMP, University of Illinois, Urbana, Illinois 61801. On the genera of irreducible lattices.

Let R be a Dedekind domain with quotient field K and $A = (D)_n$ the full matrix-ring of dimension n^2 over D , a skewfield of finite dimension over K . Assume that G is an R -order in A such that $C = D \cap G$ is the unique maximal R -order in D (this is especially the case if K splits A). Two G -lattices M, N are said to lie in the same genus if $R_P \otimes_R M \cong R_P \otimes_R N$ for every prime ideal P in R , R_P is the localization at P . Theorem. (i) There are as many different genera of irreducible G -lattices as there are different maximal R -orders in A containing G . (ii) Each genus of irreducible G -lattices contains exactly h isomorphism classes of irreducible G -lattices, where h is the number of classes of left C -ideals in D . (iii) The irreducible G -lattices can be given explicitly in terms of lattices of maximal orders over G in A . Remark. With an appropriate change on the condition for the order this theorem can be generalized to orders in separable finite dimensional K -algebras. (Received June 12, 1967.)

648-87. J. H. CASE, University of Utah, Salt Lake City, Utah 84112. Short wire theory. II.

Short Wire Theory. I (SWT. I) by Case and Stewart (ACM Repository January 3, 1966) (also submitted as an ACM Monograph) contains a theory of information processing systems which takes account of time of functioning of individual components, complexity of components, size of components, and the speed of propagation of signals between components. In SWT. I a new universality theorem was proven in a straightforward but lengthy manner (a 141 page proof). The paper reported here contains a redefinition of all of the principal systems of the theory in terms of input-output conditions, a category theoretic formulation of the theory, and proofs of generalized versions of the first theorems of SWT. I in this context. In a later paper results of SWT. II together with category theoretic results of Hamilton in SWT. III will be used to give a sequence of relatively easy theorems leading to a slightly stronger universality theorem than that given in SWT. I. Although, this new proof of the universality theorem is far shorter than that in SWT. I, it omits the intuitive universal parallel programming concepts found there. (Received June 9, 1967.)

648-88. W. L. HAMILTON, University of Utah, Salt Lake City, Utah 84112. Short Wire Theory. III: Categories of automata systems. Preliminary report.

Categories of the systems and the systems mappings defined by Case in Short Wire Theory. II are developed, and a special category equivalence between categories of systems that preserve all of the properties of systems and system mappings concerning input-output conditions is defined. The category of all systems is shown to be equivalent in this sense to the category of binary systems, to the category of systems of machines with spaces consisting of points with nonnegative integral coordinates, and to several categories of systems that are generalizations of the tessellation spaces of Von Neumann and of Moore. (Received June 8, 1967.)

648-89. J. A. THORPE, Haverford College, Haverford, Pennsylvania 19041. The role of the first Bianchi identity in Riemannian geometry.

A curvature tensor on a real inner product space V is an alternating bilinear map $R: V \times V \rightarrow \text{Hom}(V, V)$ such that, for $u, v \in V$, $R(u, v) \in \text{Hom}(V, V)$ is a skew-symmetric linear transformation and such that, for $u_i \in V$, $\langle R(u_1, u_2)u_3, u_4 \rangle = \langle R(u_3, u_4)u_1, u_2 \rangle$. The set \mathcal{R} of curvature tensors on V forms a vector space, with a natural inner product, on which the orthogonal group $O(V)$ of V acts as a group of isometries. Those curvature tensors which satisfy the Bianchi identity

$\sum_{\sigma} R(u_{\sigma(1)}, u_{\sigma(2)})u_{\sigma(3)} = 0$, where the sum ranges over the cyclic permutations of $(1, 2, 3)$, form an $O(V)$ -invariant subspace \mathcal{B} of \mathcal{R} . The orthogonal complement \mathcal{B}^{\perp} of \mathcal{B} in \mathcal{R} is also $O(V)$ -invariant. Theorem. $R \in \mathcal{B}^{\perp}$ if and only if the sectional curvature function determined by R is identically zero. It follows that the Bianchi identity is precisely the algebraic condition necessary to ensure the validity of the classical result that the sectional curvature function determines the curvature tensor uniquely. (Received June 9, 1967.)

648-90. JOHN GARNETT, Massachusetts Institute of Technology, Cambridge, Massachusetts 03129. On a theorem of Mergelyan.

When X is a compact plane set denote by $A(X)$ the algebra of functions continuous on X and analytic on the interior and let $R(X)$ be the subalgebra of $A(X)$ consisting of uniform limits of rational functions. It is a theorem of Mergelyan that $R(X) = A(X)$ if X has finitely many complementary components. Glicksberg and Wermer have given a functional analytic proof of this theorem in the case of one complementary component. Here the general theorem is reduced to that case using the fact that the approximation problem is a local one. This yields a short measure theoretic proof of the Mergelyan theorem. (Received June 9, 1967.)

648-91. HARRY GONSHOR, Rutgers, The State University, 100 Memorial Parkway, New Brunswick, New Jersey. The ring of finite elements in a nonstandard model of the real numbers.

We initiate a study of the ideal theory in the ring of finite elements in a nonstandard model of the real numbers. Sample theorems. For every ideal I there is a least primary ideal \bar{I} containing I and a greatest primary ideal \tilde{I} included in I . If I is principal, \tilde{I} is countably generated. On the other hand, if the nonstandard model is given by an ultraproduct on a countable set, then if I is principal, \bar{I} is not countably generated. There is a natural one-one order preserving correspondence between ideals and prime ideals. (Received June 13, 1967.)

648-92. A. M. CARSTENS, Washington State University, Pullman, Washington 99163. The lattice of pretopologies on a set S .

Let $(\mathcal{F}(x), \leq)$ be the lattice of filters on S satisfying $\mathcal{F} \leq \hat{x}$ ordered by set inclusion, where \hat{x} is the principle filter generated by $\{x\}$. Let $\underline{\mathcal{F}} = \prod_{x \in S} \mathcal{F}(x)$, ordered by $(\mathcal{F}_x)_{x \in S} \leq (\mathcal{G}_x)_{x \in S}$ iff $\mathcal{F}_x \leq \mathcal{G}_x$ for all $x \in S$. Theorem. $(\underline{\mathcal{F}}, \leq)$ is order isomorphic to the lattice of pretopologies on S . Corollary. The lattice of pretopologies on S is complete, modular, distributive, atomic, coatomic, and compactly generated. It is cocompactly generated and complemented iff S is finite. (Received June 12, 1967.)

In this paper the sum of two functions, one of which is starlike in the unit disk, is considered. Theorem 1. Let $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$ be starlike in $|z| < 1$, and let $g(z) = \sum_{k=n+1}^{\infty} b_k z^k$ ($n \geq 1$) be regular in $|z| < 1$. If $|g(z)| \leq |f(z)|$ for $|z| < 1$, then $h(z) = f(z) + g(z)$ is starlike in $|z| < r_n$ where r_n is the smallest positive root of $p_n(x) = 1 - x - nx^n - nx^{n+1} - (n+1)x^{2n} - (n-1)x^{2n+1}$. Furthermore, $r_n \rightarrow 1$ as $n \rightarrow \infty$. The result is sharp for $n = 1, 2$. Theorem 2. Let $f(z)$ be starlike of order at least $1/2$, and let $g(z) = \sum_{k=n+1}^{\infty} b_k z^k$ ($n \geq 1$) be regular in $|z| < 1$. If $|g(z)| \leq |f(z)|$ for $|z| < 1$, then $h(z) = f(z) + g(z)$ is starlike in $|z| < r_n$ where r_n is the smallest positive root of $p_n(x) = 1 - nx^n - nx^{n+1} - (n+1)x^{2n} - nx^{2n+1}$. Furthermore, $r_n \rightarrow 1$ as $n \rightarrow \infty$. The result is sharp for $n = 1, 2$. (Received June 12, 1967.)

Let Λ be a R -coalgebra (all coalgebras, modules and comodules are graded) where R is a commutative ring with unity and let \mathcal{E}^0 be the injective class of all coexact sequences in ${}^{\Lambda}\mathfrak{M}$ which are split exact in \mathfrak{M} . Theorem. If $N \in \mathfrak{M}^{\Lambda}$ ($M \in {}^{\Lambda}\mathfrak{M}$) then $N \square _ : {}^{\Lambda}\mathfrak{M} \rightarrow \mathfrak{M}(\text{Hom}_{\Lambda}(M, _)) : {}^{\Lambda}\mathfrak{M} \rightarrow \mathfrak{M}$ is an \mathcal{E}^0 -left exact functor. Therefore, the derived functors $\text{Cotor}_{\Lambda}^*(N, _)$ (Moore, Séminaire H. Cartan, 12e année) and $\text{Coext}_{\Lambda}^*(M, _)$ are axiomatized as cohomology theories relative to \mathcal{E}^0 over $N \square _$ and $\text{Hom}_{\Lambda}(M, _)$, respectively. Theorem. If Λ and Λ' are augmented R -coalgebras, then there exists a bigraded R -homomorphism $\phi : \text{Cotor}_{\Lambda}^*(M, N) \otimes \text{Cotor}_{\Lambda'}^*(M', N') \rightarrow \text{Cotor}_{\Lambda \otimes \Lambda'}^*(M \otimes M', N \otimes N')$ for $M \in \mathfrak{M}^{\Lambda}$, $N \in {}^{\Lambda}\mathfrak{M}$, $M' \in \mathfrak{M}^{\Lambda'}$ and $N' \in {}^{\Lambda'}\mathfrak{M}$. Theorem. If Λ is an augmented R -coalgebra, then there exists a bigraded R -homomorphism $\psi : \text{Coext}_{\Lambda}^*(M, N) \otimes \text{Coext}_{\Lambda}^*(N, L) \rightarrow \text{Coext}_{\Lambda}^*(M, L)$ for $M, N, L \in {}^{\Lambda}\mathfrak{M}$. (Received June 19, 1967.)

Let Γ be an infinite Lie pseudogroup acting on open subsets of R^m . A manifold M of dimension m is called a Γ -manifold if it has a differentiable structure in which the change of coordinates are given, as mappings of open subsets of R^m , by elements of Γ . In some important cases the group Γ acts transitively as a local transformation group; the purpose of this paper is to study the corresponding global group of automorphisms of a Γ -manifold M for a few of the locally transitive Γ . The group of automorphisms is not always transitive on M . It is shown that it is transitive on M in at least the following three cases: (i) $m = 2n$ and Γ is the group leaving invariant the quadratic differential form $\sum_1^n dx_j \wedge dx_{j+n}$; (ii) $m = 2n + 1$ and Γ is the group leaving the pffafian form $dx_0 + \sum_1^n x_j dx_{j+n}$ invariant to within a scalar multiple; and (iii) m arbitrary and Γ the group leaving invariant the Euclidean volume element $dx_1 \wedge \dots \wedge dx_m$. (Received June 14, 1967.)

648-96. R. J. ARMS, National Bureau of Standards, Computer Services Division 453.20 Gaithersburg, Maryland. Matrix scaling. Preliminary report.

Let A and B be square matrices, let $X = \text{diag}(x_i)$, $Y = \text{diag}(y_i)$, $x_i > 0$, $y_i > 0$. Put $\phi(X, Y, A, B) = \|X^{-1}AY\| \|Y^{-1}BX\|$ where $\|A\|$ is the maximum absolute row sum. Result 1. $\inf_{X, Y} \phi(X, Y, A, B) = \lambda_{\max}(A^+B^+)$ where $A^+ = (|a_{ij}|)$. Result 2. If A^+B^+ and B^+A^+ are nondecomposable, the X and Y are uniquely determined up to a multiple as maximal eigenvectors of A^+B^+ and B^+A^+ . With B taken as A^{-1} the expression ϕ is a condition number of A connected with error in digital inversion. In the above given sense of matrix norm, X and Y become a pair of "best scalars" for preconditioning A for inversion. (Received June 15, 1967.)

648-97. WITHDRAWN.

648-98. G. R. BLAKLEY, State University of New York at Buffalo, Buffalo, New York 14214. Intrinsic characterization of polynomial transformations of vector spaces.

V and W are vector spaces over a field K of characteristic zero. A. M. Gleason's results (Amer. Math. Monthly 73 (1966), 1049) generalize to polynomial transformations $T: V \rightarrow W$. For each integer degree $p \geq 2$ the author has proved a Theorem H_p characterizing homogeneous p th degree polynomial (diagonal restrictions of p linear) transformations and a Theorem I_p characterizing polynomial transformations of degree at most p . These identities, reminiscent of the law of inclusion and exclusion, characterize polynomial transformations in the way the identity $T(ra + sb) = rTa + sTb$ characterizes linear (homogeneous first degree polynomial) transformations. Gleason's paper contains H_2 and I_2 when $W = K$. For lack of space we state only Theorems H_3 , I_3 and only in their finite dimensional cases FH_3 , FI_3 . Theorem FH_3 . Each component of Tx in W is a homogeneous polynomial of degree three in the components of x in V if and only if T is Euler homogeneous of degree three and for each a, b, c in V , each scalar s it is true that $T(a + b + sc) - T(-a + b + sc) - T(a - b + sc) - T(a + b - sc) = s [T(a + b + c) - T(-a + b + c) - T(a - b + c) - T(a + b - c)]$. Theorem FI_3 . Each component of Tx in W is a polynomial of degree at most three in the components of x in V if and only if for each a, b, c in V , each scalar s it is true that $T(a + b + sc) - T(a - b + sc)$

- $T(a + b - sc) + T(a - b - sc) = s[T(a + b + c) - T(a - b + c) - T(a + b - c) + T(a - b - c)]$. (Received June 15, 1967.)

648-99. P. L. ZENOR, University of Houston, Houston, Texas 77004. On countable paracompactness and normality.

Let \mathcal{A} denote the collection of topological spaces to which S belongs if and only if S is a T_2 space and every closed subset of S is a G_δ set. Theorem. In \mathcal{A} the following conditions are equivalent: (i) Every countably paracompact space is normal. (ii) If $\{X_i, \pi_i^1\}$ is an inverse system with bonding maps onto and for each i , X_i is countably paracompact, then $\text{inv lim } \{X_i, \pi_i^1\}$ is hereditarily countably paracompact. (iii) Every countably paracompact space is hereditarily countably paracompact. (Received June 15, 1967.)

648-100. NOBUO SHIMADA, HIROSHI UEHARA, and FRANK BRENNEMAN, Oklahoma State University, Stillwater, Oklahoma 74074. Triple cohomology in relative homological algebra.

Let \mathcal{A} be an arbitrary category and \mathcal{B} be an abelian category. Let $(\tilde{G}, \tilde{\beta}, \tilde{\Delta})$ be a cotriple on $\mathcal{B}^{\mathcal{A}}$, where the functor $\tilde{G}: \mathcal{B}^{\mathcal{A}} \rightarrow \mathcal{B}^{\mathcal{A}}$ is assumed to be kernel-preserving. Let \mathcal{C}_0 be the class of all split exact sequences in $\mathcal{B}^{\mathcal{A}}$ and let $\tilde{\mathcal{C}} = \tilde{G}^{-1}(\mathcal{C}_0)$. Theorem. (a) The class $\tilde{\mathcal{C}}$ is a projective class in the sense of Eilenberg and Moore. (b) For any object $J \in \mathcal{B}^{\mathcal{A}}$, the standard complex $\tilde{\mathcal{C}}(J)$ of the cotriple $(\tilde{G}, \tilde{\beta}, \tilde{\Delta})$ is an $\tilde{\mathcal{C}}$ -projective resolution of J (S. Mac Lane, 645-26, these *Notices* 14 (1967), 388). (c) For any functor $T: \mathcal{B}^{\mathcal{A}} \rightarrow \text{Ab}$, the triple homology (cohomology) $H(T\tilde{\mathcal{C}})$ is the left (right) derived functor of T with respect to $\tilde{\mathcal{C}}$. Corollary. Given a cotriple (G, β, Δ) on \mathcal{A} (resp. \mathcal{B} with G kernel-preserving). Define a cotriple $(\tilde{G}, \tilde{\beta}, \tilde{\Delta})$ on $\mathcal{B}^{\mathcal{A}}$ by setting $\tilde{G}(J) = JG$, $\tilde{\beta}(J) = J\beta$, and $\tilde{\Delta}(J) = J\Delta$ (resp. $\tilde{G}(J) = GJ$ etc.) for $J \in \mathcal{B}^{\mathcal{A}}$. If a contravariant functor $\Phi: \mathcal{B} \rightarrow \text{Ab}$ is given, then the triple cohomology $H(\Phi JG(A))$ (resp. $H(\Phi GJ(A))$) is regarded as the derived functor $H(\Phi E_A \tilde{\mathcal{C}}(J))$, where $E_A: \mathcal{B}^{\mathcal{A}} \rightarrow \mathcal{B}$ is the evaluation functor. For example it is expected that known cohomology theories for Lie algebras \mathfrak{J} can be understood in the unified form $H(\Phi E_{\mathfrak{J}} \tilde{\mathcal{C}}(J))$. (Received June 15, 1967.)

648-101. S. L. JONES, University of Wisconsin, Madison, Wisconsin 53706. Extension of a result by J. H. Roberts.

In 1936 J. H. Roberts [Duke Math. J., vol. 2] showed that there exists no upper semicontinuous (usc) collection of arcs filling the plane. The Vietoris mapping theorem gives the following Lemma. If X is a compact subset of E^n , G is a usc decomposition of X into arcs and points, and Y is a closed subset of X such that Y intersects each element of G in a nonempty continuum, then X separates E^n if and only if Y separates E^n . Using this lemma and related ideas, one obtains the following Theorem. There exists no usc collection of arcs filling E^n . (Received June 15, 1967.)

648-102. R. L. McFARLAND, 13906 Briarwood Drive, Apt. 2321, Laurel, Maryland 20810.
On abelian difference sets having minus one as a multiplier.

It is proved that an abelian difference set with the Hadamard parameters $(*) (v, k, \lambda, n) = (4m^2, 2m^2 - m, m^2 - m, m^2)$ cannot have minus one as a multiplier if m is exactly divisible by an odd power of a prime $p \neq 2, 3$. On the other hand, it is observed that a class of difference sets constructed by P. K. Menon (On difference sets whose parameters satisfy a certain relation, Proc. Amer. Math. Soc. 13 (1962), 739-745) with the parameters $(*)$ for all m of the form $m = 2^r 3^s$, $r \geq s - 1$, $r \geq 0$, $s \geq 0$ have minus one as a multiplier. Noncyclic abelian difference sets with parameters $v = q^3(q^2 + q + 2)$, $k = q^2(q^2 + q + 1)$, $\lambda = q^2(q + 1)$, $n = q^4$ are constructed for all prime powers q . However, only for $q = 2, 5$ does this construction yield difference sets having minus one as a multiplier. Excepting the Hadamard parameters $(*)$ and the parameters corresponding to $q = 5$ above, the nonexistence theorems of E. C. Johnsen (The inverse multiplier for abelian group difference sets, Canad. J. Math. 16 (1964), 787-796) and the author imply the nonexistence of abelian difference sets having minus one as a multiplier for all but six of the (v, k, λ, n) parameter values with $n \leq 10^8$. (Received June 16, 1967.)

648-103. A. V. JATEGAONKAR, University of Rochester, Rochester, New York 14627.
A counter-example in ring theory and homological algebra.

Let ρ be a mono-endomorphism of a domain D . $D[X, \rho]$ denotes the domain of 'left' polynomials in X over D ; the multiplication is defined by the rule $Xd = \rho(d)X$ for every $d \in D$. A domain R is a generalized left twisted extension of a domain D if there exists an ordinal $\alpha > 0$ and a set of subdomains $\{R_\beta : \beta < \alpha\}$ of R such that (i) $D = R_0$; (ii) $R = \bigcup_{\beta < \alpha} R_\beta$; (iii) $R_\beta = (\bigcup_{\gamma < \beta} R_\gamma) [X_\beta, \rho_\beta]$ for $0 < \beta < \alpha$. Localizing at a suitable monoid in a suitable generalized left twisted extension of a s field, an example is constructed to prove the following Theorem. For an arbitrary ordinal τ , there exists a local left principal ideal domain D with Jacobson radical J such that (1) $J^\tau \neq 0$; (2) $r \cdot gl \cdot \dim D = \infty$; (3) D is not UFD in the sense of P. M. Cohn, [Non-commutative unique factorization domains, Trans. Amer. Math. Soc. 109 (1963), 313-331]. The proof uses a theorem of B. L. Osofsky, [Global dimension of valuation rings, Trans. Amer. Math. Soc. 127 (1967), 136-149]. (Received June 18, 1967.)

648-104. J. A. SIDDIQI, University of Sherbrooke, Sherbrooke, Province of Quebec, Canada.
Coefficient properties of Fourier-Stieltjes series.

It is shown that if f is an almost periodic function in the sense of H. Bohr and $\Lambda = (\lambda_{n,k})$ an infinite matrix satisfying certain regularity conditions, then $\lim_{n \rightarrow \infty} \sum_{k=0}^{\infty} \lambda_{n,k} f(x + kq)$ exists uniformly in x and is a q -periodic function. Applying this, the following theorem is proved: Let $\sum \lambda_{n,k} e^{ikt}$ and $\sum |\lambda_{n,k}| e^{ikt}$ tend to zero as $n \rightarrow \infty$ for all $t \not\equiv 0 \pmod{2\pi}$ and let $dF \sim \sum C_n e^{inx}$ where C_n denote the Fourier coefficients of dF . Then for F to be continuous it is necessary that $\lim_{n \rightarrow \infty} \sum_{k=0}^{\infty} |\lambda_{n,k}| |C_{k+p}|^2 = \lim_{n \rightarrow \infty} \sum_{k=0}^{\infty} |\lambda_{n,k}| |C_{k+p}| = 0$ for $p = 0, 1, 2, \dots$, uniformly in p and sufficient that the sequence $\{|C_n|^2\}$ or $\{|C_n|\}$ be summable Λ to zero. This theorem contains as particular cases a classical theorem of Wiener asserting that "F is continuous if and only if $\{|C_n|\}$ or $\{|C_n|^2\}$ is summable (C,1) to zero" and also various generalizations thereof. (Received June 19, 1967.)

648-106. A. H. SCHOEN, 5126 Elkmont Drive, Palos Verdes Peninsula, California 90274.
Homogeneous nets and their fundamental regions.

For the 5 2-dim and 12 3-dim homogeneous nets (connected infinite periodic arrays of nodes, with edges joining each node to Z of the Z' nearest nodes [$3 \cong Z \cong Z'$], with all nodes and edges symmetrically equivalent), a fundamental region may be chosen which encloses one node, has Z congruent (or mirror-symmetric) faces of zero mean curvature, assumes k orientations (k = no. of nodes per unit cell), has no edges not symmetry axes of space-filling assembly of such regions (hence symmetric space-linkages, or flexible closed chains of such regions hinged at edges, can be constructed), and has the point symmetry of the net at the enclosed node. This "symmetry domain" is the cell of a homogeneous honeycomb and is constructed as follows: span all edge-circuit polygons of the net, in order of no. of edges, by minimal surfaces, filling the space with closed cells (interstitial domains), without reducing volume of already closed cells; join centroids of adjoining cells by line segments; span smallest polygons of new (reciprocal) net by minimal surfaces, generating congruent cells, or symmetry domains. In 7 nets, interstitial and symmetry domains are saddle polyhedra (cf. P. Pearce, Syneustructics, Graham Foundation, 1966); in 5 3-dim nets, the symmetry domain is the Dirichlet region. (Received June 28, 1967.)

648-107. L. S. HUSCH, University of Georgia, Athens, Georgia 30601. Finding a boundary for a 3-manifold.

The following theorem is proved. Let M be a connected, open, orientable 3-manifold with one end. The interior of M is homeomorphic to the interior of a compact 3-manifold if and only if there exists a positive integer N such that every compact subset of M is contained in the interior of a compact 3-manifold M' with connected boundary such that (1) $\pi_1(M - M')$ is finitely generated, (2) genus (bdry M') $\leq N$, (3) Every contractible 2-sphere in $M - M'$ bounds a 3-cell. (Received June 19, 1967.)

648-108. LUDVIK JANOS, University of Florida, Gainesville, Florida. On fixed points of mappings contractive in a local sense.

Let X be a metrizable topological space, \mathfrak{M} the set of all metrics on X inducing the given topology and $\phi: X \rightarrow X$ a continuous mapping. If $a \in X$ is a fixed point of ϕ we will say a is of contractive character, if for some $\rho \in (0,1)$ there exists a neighborhood $N(a)$ of a invariant under ϕ and a metric $\rho \in \mathfrak{M}$ such that $\rho(\phi(x), \phi(y)) \leq \alpha\rho(x,y)$ for all $x, y \in N(a)$. Theorem. Let X be compact and connected and $\phi: X \rightarrow X$ such that for some $\rho \in \mathfrak{M}$ and some $\epsilon > 0$ the following condition holds: $0 < \rho(x,y) < \epsilon \Rightarrow \rho(\phi(x), \phi(y)) < \rho(x,y)$. Then ϕ has a unique fixed point a , a is of contractive character and $\phi^n(x) \rightarrow a$ for all $x \in X$. (Received June 19, 1967.)

648-109. S. P. LLOYD, Bell Telephone Laboratories, Murray Hill, New Jersey 07971. Feller boundary induced by a transition operator.

Let T be an operator on AL space L such that $T \geq 0$, $\|T\mu\| = \|\mu\|$ if $\mu \geq 0$. We show that the space of invariant vectors of T^* is isomorphic to $C(B)$, with B hyperstonian compact Hausdorff. We call B the Feller boundary induced by T , since it is the Feller boundary in the case of a Markov chain. Several Markov processes are exhibited such that B is a subset of the state space. When L is separable there is a closed subset B_0 of the Cantor set and a measure π_0 with B_0 as closed support such that $C(B)$ is isomorphic to $L_\infty(B_0, \pi_0)$. If a Markov process with state space (X, \mathcal{F}) has \mathcal{F} countably generated then we are able to attach such a boundary B_0 to X so that convergence with probability 1 to a point of the boundary holds. Another model has convergence in probability instead, and does not necessarily collapse null recurrent classes. (Received June 19, 1967.)

648-110. WITHDRAWN.

648-111. J. V. RYFF, University of Washington, Seattle, Washington 98105. Extreme points of some convex subsets of L^1 .

A partial order $<$ between elements of $L^1 (= L^1(0,1))$ is defined by $g < f$ whenever $\int_0^s g^* \leq \int_0^s f^*$ with equality holding when $s = 1$. Here, g^* and f^* represent the decreasing rearrangements of g and f . The orbit of f is defined to be the set of all g such that $g < f$. The orbit is convex and weakly compact. In (Trans. Amer. Math. Soc. 117 (1965), 92-100) it was shown that any g equimeasurable with f is an extreme point of the orbit of f . The converse was conjectured and is now shown to be correct. Each extreme point is equimeasurable with f . Those familiar with doubly stochastic matrices will note the analogy here. To say that g is equimeasurable with f means that the Lebesgue measure of the sets $\{g > y\}$ and $\{f > y\}$ is the same for each real number y . Some other observations concerning exposed and support points of the orbits can also be given. (Received June 19, 1967.)

648-112. M. G. ARSOVE, University of Washington, Seattle, Washington 98105. The Wiener-Dirichlet problem and the theorem of Evans.

New and simplified derivations are given for the following theorems of classical potential theory: (1) Kellogg-Evans theorem (the irregular boundary points form a Borel F_σ set of capacity zero); (2) Phragmén-Lindelöf maximum principle (a subharmonic function bounded above and having upper limit ≤ 0 on the boundary, except perhaps for a set of inner harmonic measure zero, must be ≤ 0); (3) Evans theorem (every compact set K of capacity zero supports a positive mass distribution with potential $+\infty$ on K); (4) equivalence of harmonic null sets and sets of harmonic measure zero (a boundary set E has harmonic measure zero if and only if there exists a positive harmonic function on the region tending to $+\infty$ on E). Proofs are based on the Wiener approach to the generalized Dirichlet problem, without appeal to the Perron method or to subharmonic function theory. (Received June 22, 1967.)

648-113. P. P. SAWOROTNOW, The Catholic University of America, Washington, D. C. 20017. Trace class and centralizers of an H^* -algebra. Preliminary report.

Let A be a proper H^* -algebra and let $R(A)$ be the set of all right centralizers on A (a right centralizer is a bounded operator S on A such that $S(ab) = (Sa)b$ for all $a, b \in A$); let $\tau(A) = \{ab \mid a, b \in A\}$ be the trace class of A (see Abstract 643-17, these *Notices* 14 (1967), 246) and let $C(A)$ be the closed linear subspace of $R(A)$ generated by the operators of the form $La : x \rightarrow ax, a \in A$. It is shown that $\tau(A) = C(A)^*$ and $R(A) = \tau(A)^*$, i.e. $\tau(A)$ can be identified with the set of all bounded linear functionals on $C(A)$ and $R(A)$ is isomorphic to the conjugate space of $\tau(A)$. This gives an alternative proof of completeness of $\tau(A)$. (Received June 22, 1967.)

648-114. J. B. KIM, West Virginia University, Morgantown, West Virginia. On singular matrices.

Theorem. $M_n(F)$ denotes the set of all n by n matrices over a field F . Any singular matrix in $M_n(F)$ is a finite product of idempotent matrices in $M_n(F)$. (Received June 22, 1967.)

648-115. WITHDRAWN.

648-116. P. R. MEYER, 790 Madison Avenue, New York, New York 10021. Sequential space methods in general topological spaces.

The theory of sequential spaces and Frechet spaces (e.g., see Franklin, Fund. Math. 57 (1965), 107-115) is extended so that it applies to any topological space. Let X be a set and \mathcal{L} be a class of net pairs on X (i.e., pairs of the form (Q, x) where Q is a net in X and $x \in X$). If $(\{x_\nu, \nu \in D\}, x) \in \mathcal{L}$ and E is a cofinal subset of D , assume that $(\{x_\nu, \nu \in E\}, x) \in \mathcal{L}$. There is a largest topology (smallest convergence class) on X in which nets in \mathcal{L} are convergent; \mathcal{L} is called a "convergence subbasis" for this topology. If, for any subset Y of X , every limit point of Y can be reached by a \mathcal{L} -net in Y , then \mathcal{L} is called a "convergence basis". A space which has a convergence subbasis (resp., convergence basis) consisting of m -nets is called an m -sequential (m -Frechet) space. (An m -net is a net whose directed set has cardinality $\leq m$.) These generalize the usual definitions. A new characterization of the notion of quotient topology is given, but the primary application is to function spaces in which pointwise convergent m -nets are of interest (e.g., see Poppe, Math. Nachr. 29 (1965), 247-253). (Received June 2, 1967.)

648-117. STEVE ARMENTROUT, University of Amsterdam, Amsterdam, Holland. Shrinkability of certain decompositions of E^3 that yield E^3 .

If G is an upper semicontinuous decomposition of E^3 , then E^3/G denotes the associated decomposition space, P denotes the projection map, and H_G denotes the union of all the nondegenerate elements of G . Suppose that G is an upper semicontinuous decomposition of E^3 such that $P[H_G]$ is 0-dimensional. G is shrinkable if and only if for each open set U containing H_G and each positive number ϵ , there is a homeomorphism h from E^3 onto E^3 such that (1) if $x \in E^3 - U$, $h(x) = x$ and (2) if $g \in G$, $(\text{diam } h[g]) < \epsilon$. A well-known theorem of Bing's (Ann. of Math. 65 (1957), 363-374) states that if G is a monotone decomposition of E^3 such that $P[H_G]$ is 0-dimensional and G is shrinkable, then E^3/G is homeomorphic to E^3 . The main result of this paper is the following theorem which provides a converse, in the case of pointlike decompositions of E^3 , to the theorem of Bing's stated above: If G is a pointlike decomposition of E^3 such that $P[H_G]$ is 0-dimensional and E^3/G is homeomorphic to E^3 , then G is shrinkable. (Received June 23, 1967.)

648-118. D. L. PRULLAGE, University of Kentucky, Lexington, Kentucky 40506. Summability in topological groups. III. Preliminary report.

Let G be a group under addition and let d be a metric on G such that subtraction is continuous with respect to the topology induced by d . Limitation methods are defined, Toeplitz-type theorems are derived and several results concerning the convergence field of regular methods are given. For example, no regular method sums all bounded sequences, in fact, a regular method is given which transforms a two-valued sequence into an unbounded sequence. (Received June 23, 1967.)

648-119. R. H. CAMERON and D. A. STORVICK, University of Minnesota, Minneapolis, Minnesota. A Lindelöf theorem and analytic continuation for functions of several variables, with an application to the Feynman integral.

First an m -dimensional Lindelöf theorem is given which for $m = 2$ is as follows. Let $w = f(z_1, z_2)$ be analytic and bounded in the generalized polycylinder $T = \Delta_1 \times \Delta_2$, $\Delta_j = \{z_j: |\arg z_j| < \theta_j, 0 < |z_j| < R_j \cong \infty\}$. Let $f(z_1, z_2)$ tend to a unique limit a as $(z_1, z_2) \rightarrow (0, 0)$ along $L_1 \times L_2$, where L_j is a continuous path $z_j = z_j(t)$ lying in Δ_j and terminating at $z_j = 0$. Then $f(z_1, z_2)$ tends uniformly to a as $(z_1, z_2) \rightarrow (0, 0)$ inside any angular subdomain $V_1 \times V_2$, where $V_j = \Delta_j \cap \{z_j: |\arg z_j| < \theta_j - \delta_j\}$. Next the following analytic continuation theorem is proved. Let D_1, D_2 be simply connected regions in the z and w -planes respectively, and let l_2 be a line segment in D_2 . Let $f(z, w)$ be defined in $D_1 \times D_2$, let it be analytic in $w \in D_2$ for each $z \in D_1$, let it be analytic in $z \in D_1$ for each $w \in l_2 \subset D_2$, and let it be bounded on every compact subset of $D_1 \times D_2$. Then $f(z, w)$ is analytic in $D_1 \times D_2$. Finally, a translation theorem for Feynman integrals is proved, using the two preceding theorems. (Received June 23, 1967.)

648-120. C. F. OSGOOD, National Bureau of Standards, Room A-302, Building 101, Washington, D. C. 20234. A method in Diophantine approximation. II and III.

This continues part I (Acta Arithmetica XII (1966) 111-129). Let Q be the rationals. (II) Suppose that $y(z) = \sum_{i=1}^l g_i(z)y(z)$ where $l \geq 0$, each $g_i(z) \in Q[i, z]$, and each $\deg g_i(z) < i$. Suppose further $y(z)$ is analytic on a punctured disk about zero, $\widetilde{y}(z)$ is the analytic continuation of $y(z)$ about zero, and $\widetilde{y}(z) - y(z) \neq 0$. Then a result is shown about the simultaneous diophantine approximation of the Taylor coefficients of $\widetilde{y}(z) - y(z)$ at nonzero Gaussian-rational points. In part III results are shown about the algebraic structure of a class of functions which satisfy the conditions above. These functions form a module over a certain ring of functions under $*$, i.e. $f_1(z) * f_2(z) = \int_0^z f_1(z-t)f_2(t)dt$. (Received June 23, 1967.)

648-121. C. F. KENT, Case-Western Reserve University, Cleveland, Ohio 44106. Restricted ω -rule for arithmetic.

Shoenfield (Bull. Acad. Pol. Sci. VII, 7, (1959)) states the result that arithmetic with an ω -rule restricted by requiring that there exist a (general) recursive function ϕ , so that $\phi(x)$ is a proof number for the x th premise, is as strong as arithmetic with unrestricted ω -rule. There is a proof of this result using a semantic-tableaux type reduction in a natural deduction setting, which shows that the ω -rule can indeed be restricted by "small classes" of recursive functions, e.g. the primitive recursive functions, without impairing its deductive power. The same argument produces proofs of ordinals less than ω^2 , in contrast to ω^ω by Shoenfield's construction. (Received June 25, 1967.)

648-122. TIM ANDERSON, University of British Columbia, Vancouver 8, British Columbia, Canada. Hereditary radicals and derivations of algebras. Preliminary report.

Let $U(F)$ be any class of algebras (not necessarily associative or finite dimensional) over a field F of characteristic 0 which is universal in the sense that ideals and homomorphic images of algebras in $U(F)$ are again in $U(F)$. Theorem. Let π be a hereditary radical defined in $U(F)$. Then

for each $A \in U(F)$ which satisfies the descending chain condition (D.C.C.) on ideals, $(\pi(A))D \subseteq (A)$ for all derivations D of A . Corollary. If A is a Lie algebra over a field of characteristic 0 and π is a hereditary radical then $\pi(I) = \pi(A) \cap I$ for each ideal I of A which satisfies D. C. C. The corollary partially extends to Lie algebras a result of Amitsur (see A general theory of radicals, II, Amer. J. Math. 76 (1954), 100-125) for associative rings. (Received June 26, 1967.)

648-123. ERIK HEMMINGSEN, Syracuse University, Syracuse, New York 13210, and W. L. REDDY, State University of New York, Albany, New York. Expansive homeomorphisms on homogeneous spaces.

An expansive homeomorphism is constructed on each of a class of homogeneous spaces that are fibered by the cantor set over specially chosen compact manifolds. In the two dimensional case the manifolds are Klein Bottles; and, from these, higher dimensional cases are constructed by analogy. In previously published examples of expansive homeomorphisms on perfect, homogeneous, compact, metric spaces the space has been a group space and the homeomorphism has been the conjugate of an automorphism of the group. This is not the case in the examples now constructed. (Received June 26, 1967.)

648-124. CHESTER FELDMAN, Kent State University, Kent, Ohio 44240. Decomposition of operator norms.

A maximal vector of a bounded operator A on a Hilbert space \mathcal{H} is a unit vector m such that $\|A^*m\| = \|A\|$. Definition. A set of operators will be called projectors if any two of its members A and B have the property that $\|A + B\|^2 = \|A\|^2 + \|B\|^2$. Theorem. If $A + B$ has a maximal vector, the range of A is a one-dimensional space which coincides with the range of B , and the inner product $(A^*x, B^*y) = 0$ for all x and y in \mathcal{H} , then A and B are projectors. Using this result an analogue of Parseval's equality can be obtained. Theorem. If the operator A has a maximal vector, then there exists a sequence $\{A_i\}$ of projectors such that $\|A\|^2 = \sum \|A_i\|^2$. (Received June 25, 1967.)

648-125. JESŪS GIL DE LAMADRID, University of Minnesota, Minneapolis, Minnesota 55455. Representations of commutative algebras with involution. Preliminary report.

In an earlier announcement (Abstract 640-17, these *Notices* 13 (1966), 848), representations $\pi: \mathfrak{A} \rightarrow \mathfrak{A}_\pi$ of a complex algebra \mathfrak{A} with involution onto an algebra \mathfrak{A}_π consisting of closed (bounded or not) operators in a Hilbert space H were introduced. Here we examine representations of commutative \mathfrak{A} . Theorem. For every representation $\pi: \mathfrak{A} \rightarrow \mathfrak{A}_\pi$, \mathfrak{A} commutative, there exists a completely regular space S , a unique regular Borel spectral measure E on S , with projection-values in $\mathcal{L}(H)$, and an involution preserving mapping $x \rightarrow \hat{x}$ of \mathfrak{A} into the space \mathcal{K}_E of E -equivalence classes of E -measurable complex functions such that for every $x \in \mathfrak{A}$, $\pi(x) = \int_S \hat{x}(s) dE(s)$, where the integral is taken in the sense of the spectral theorem for unbounded operators. This theorem is well known for Banach algebras \mathfrak{A} , and includes the spectral theorem (for bounded and unbounded operators), Stone's theorem on representations of locally compact abelian groups, and Hille's theorem on the spectral decomposition of normal semigroups. (Received June 26, 1967.)

for each $A \in U(F)$ which satisfies the descending chain condition (D.C.C.) on ideals, $(\pi(A))D \subseteq (A)$ for all derivations D of A . Corollary. If A is a Lie algebra over a field of characteristic 0 and π is a hereditary radical then $\pi(I) = \pi(A) \cap I$ for each ideal I of A which satisfies D. C. C. The corollary partially extends to Lie algebras a result of Amitsur (see A general theory of radicals, II, Amer. J. Math. 76 (1954), 100-125) for associative rings. (Received June 26, 1967.)

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648-126. MURRAY EISENBERG, University of Massachusetts, Amherst, Massachusetts 01002. Expansive automorphisms of Banach space. Preliminary report.

In [Fund. Math. 59 (1966), 307-312], it was shown that an automorphism of a finite-dimensional euclidean or unitary space is expansive if and only if none of its characteristic roots has modulus one. Here the spectral decomposition theorem is used to obtain the following Theorem. Let B be a complex Banach space. If u is a bounded invertible linear operator on B whose spectrum is disjoint from the unit circle, then u is expansive. When B is actually a separable Hilbert space, then the converse holds only if B is finite-dimensional. When B is a real Banach space, similar results are obtained through complexification. (Received June 26, 1967.)

648-127. W. W. ARMSTRONG, Bell Telephone Laboratories, Incorporated, Holmdel, New Jersey 07733. Diffuse semigroups (systems with a stochastic product).

Let S be a locally compact Hausdorff space, \mathcal{B} be the σ -algebra of all Borel subsets of S , and \mathcal{P} be the set of all regular Borel probability measures on S . Suppose $P*Q$ is an associative binary operation defined for P, Q in \mathcal{P} with values in \mathcal{P} which is affine and continuous (in the topology of weak convergence) in each variable P, Q separately; then the structure so defined is termed a topological diffuse semigroup. S is embedded in \mathcal{P} as the set of all point-masses. Topological diffuse semigroups are generalizations of the "generalized convolutions" studied by K. Urbanik. Joint continuity of the latter class of operations is shown. Let EF be the closure of the union of the supports of the measures $x*y$ for x in E and y in F . A right ideal R is a subset of S such that RS is contained in R . Left ideals and two-sided ideals are defined similarly. S is a diffuse group if $\{x\}S - S\{x\} = S$ for all x in S . A necessary and sufficient condition is derived for the union of the minimal right ideals to be dense in the minimal two-sided ideal of a compact diffuse semigroup. "Haar measure" is shown to exist for compact diffuse groups, and a limit theorem for Cesàro averages of $*$ - powers is proved. (Received June 26, 1967.)

648-128. T. G. DANDEL, JR., 29 Wiggins Street, Princeton, New Jersey 08540. Toward a stochastic theory of spin. Preliminary report.

Methods of Edward Nelson are generalized to gain a continuous conceptualization of the ordinary, discrete quantum mechanical spin of a particle. Reasonable classical, nonrelativistic equations of motion of a charged sphere are quantized and Nelson's version of Hamilton-Jacobi theory is applied to yield the fundamental stochastic equations of motion. The stochastic acceleration defined by the second of these is the classical generalized force divided by the mass. It is possible to derive these equations from very simple assumptions about the infinitesimal generator of the Markovian semigroup describing the stochastic motion. The case of stationary states of the free electron is examined in detail. Four states arise naturally: the average of the sphere's precessional spin and that of its figure axis spin may be either up or down. The average of the sum of these two spins assumes only the values $k/2$ and $-k/2$. A form of the ergodic theorem is applied to show that the time average of figure axis spin is zero with probability one. Under mild additional assumptions the limit of the radius of the sphere approaching zero yields two copies of the Pauli equation, multiplicity due to observability of figure axis spin. (Received June 26, 1967.)

648-129. HOWARD SHERWOOD, Illinois Institute of Technology, Chicago, Illinois. A characterization of pseudometrically generated probabilistic metric spaces.

A probabilistic metric space (S, \mathcal{F}) is pseudometrically generated if and only if there is a probability space (D, \mathcal{B}, μ) satisfying (1) D is a collection of pseudometrics for S ; (2) For every real number x and for every pair p, q belonging to S , the set $\{d \in D : d(p, q) < x\}$ is a \mathcal{B} -measurable set; (3) For every real number x and for every pair p, q in S , $\mathcal{F}(p, q) = F_{pq}$ where F_{pq} is the distribution function defined by $F_{pq}(x) = \mu\{d \in D : d(p, q) < x\}$. The ordered pair (S, \mathcal{F}) is an E-space over the metric space (M, d) (briefly an E-space) if the elements of S are functions from a probability space (Ω, \mathcal{A}, P) into M such that for every pair p, q in S and every real number x , the set $\{t \in \Omega : d(p(t), q(t)) < x\}$ belongs to \mathcal{A} (i.e., the composite one-place function $d(p, q)$ is P -measurable), and \mathcal{F} is the mapping from $S \times S$ into the collection of distribution functions defined via $\mathcal{F}(p, q) = F_{pq}$, where $F_{pq}(x) = P\{t \in \Omega : d(p(t), q(t)) < x\}$ for every real number x . Theorem 1. If (S, \mathcal{F}) is an E-space then (S, \mathcal{F}, T_m) is a Menger space. Theorem 2. A probabilistic metric space is pseudometrically generated if and only if it is isometric to an E-space. (Received June 26, 1967.)

648-130. W. E. PARR, University of Maryland, Baltimore, Maryland 21228. Orthogonalization and the generalized inverse of a matrix.

Penrose has defined a generalized inverse (A^+) for an arbitrary $m \times n$ matrix (A) and various authors have given methods for computing A^+ . In this paper we discuss the relationship of the Gram-Schmidt orthogonalization procedure to the generalized inverse. In particular, we utilize orthogonalization to obtain the Theorem. $A^+ = XAY$ where $X = (LA)^* D^2 L$ and L and D are, respectively, lower triangular and diagonal matrices obtained via Gram-Schmidt, and where $Y = U D^2 (AU)^*$, similarly. Although this theorem applies regardless of the rank of A , a simplification is possible if A is of full column or full row rank. (Received June 26, 1967.)

648-131. W. J. SCHNEIDER, Syracuse University, Syracuse, New York 13219. Large Cantor sets with very small subsets of relative harmonic measure one.

Let $\omega[z, A, D]$ be the harmonic measure of a set $A \subset \partial D$ relative to a plane domain D . Theorem 1. There exists a Cantor S with the properties (i) $S = A \cup B$ with $A \cap B$ empty and B closed. (ii) $M_\alpha(A) = 0$ (all $\alpha > 0$), $\mu(B) > 0$ (where M_α and μ are Hausdorff α -measure and 2 - d Lebesgue measure respectively). (iii) $\omega[z, A, E^2 - S] \equiv 1$. The proof follows from: Lemma (The choke-off lemma). Let C_1 and C_2 be two disjoint Cantor sets. Let J be any analytic Jordan curve which separates C_1 from C_2 ($C_2 \subset \text{Int } J$) Then for any fixed $z_0 \notin C_1$ in $\text{Ext } J$ and any $\epsilon > 0$, there exists a Cantor set $R \subset J$ with $M_\alpha(R) = 0$ (all $\alpha > 0$) and $\omega[z_0, C_2, E^2 - (C_1 \cup C_2 \cup R)] < \epsilon$. As an immediate consequence of Theorem 1 and the Kline-Moore threading theorem one obtains: Corollary (Lohwater-Piranian). A homeomorphism of $|z| \leq 1$ onto a Jordan domain, which is conformal in $|z| < 1$ may take a set of measure zero on $|z| = 1$ onto a set of positive area. (Received June 27, 1967.)

648-132. M. S. BERGER, Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, New York 10012. A new bifurcation theory for a class of nonlinear elliptic partial differential equations.

We attempt to isolate the characteristic properties of that class of nonlinear elliptic partial differential equations analogous to Hamiltonian systems of ordinary differential equations. We show that the real solutions of a large class of problems obtainable as Euler-Lagrange equations of a continuous functional in the Sobolev space $\dot{W}_{m,2}(\Omega)$ can be partitioned into a countably infinite number of classes that are not destroyed under those small perturbations which are themselves Euler-Lagrange expressions. An example is given to show the relevance of the results obtained for equilibrium processes in nature. (Received June 27, 1967.)

648-133. ARUNAVA MUKHERJEA, Eastern Michigan University, Ypsilanti, Michigan 48197. On a random Uryson equation.

We consider the random Uryson equation $\lambda \int_{D_\omega} K(t,u,x(\omega,u))du - x(\omega,t) = y(t)$, where for every $\omega \in \Omega$, $(\Omega, \mathfrak{A}, \mu)$ being a probability space, (i) D_ω is a closed interval in $[0,1]$ (ii) $x(\omega,t)$, the unknown function from Ω into $C[0,1]$, λ is a scalar and the nonnegative kernel $K(t,u,s)$ is continuous with respect to t, u and s . (iii) $y(t) \in C[0,1]$ and $x(\omega,t)$ is a random variable $\forall t$. Let \mathfrak{F} be a σ -algebra generated by a sequence of pairwise disjoint sets. It is shown in this paper that (1) the above equation admits of a measurable solution if $\mathfrak{A} = \mathfrak{F}$ (2) if $\mathfrak{A} \supset \mathfrak{F}$ so that μ is regular relative to \mathfrak{F} , then for every $\delta > 0$, there exists an $A \in \mathfrak{A}$ with $\mu(A) > 1 - \delta$ and a random variable $x(\omega,t): \Omega \rightarrow C[0,1]$ which satisfies the above equation for every $\omega \in A$. This study has been motivated by an earlier consideration of a similar equation by Bharucha-Reid in his paper "On random solutions of nonlinear integral equations" in Non-linear problems of engineering, Academic Press Inc., New York, 1964, pp. 23-28. (Received June 28, 1967.)

648-134. WILLIAM RUCKLE, Lehigh University, Bethlehem, Pennsylvania 18015. Decompositions of operator spaces.

Let M_1, M_2, \dots be a sequence of closed subspaces of a Banach space X such that each element x of X has a unique expansion $x = \sum_{i=1}^{\infty} m_i$, $m_i \in M_i$. The topological dual, X^* , of X is shown to be isomorphic (= topologically isomorphic) to the space of all sequences (f_i) , $f_i \in M_i^*$ such that $\sup_n |\sum_{i=1}^{\infty} f_i(m_i)| < \infty$ for each sequence (m_i) with $\sum_{i=1}^{\infty} m_i \in X$. By means of this representation theorem it is proven that for E a Banach space having a Schauder basis: (a) The dual of the Banach space $\mathcal{N}(E)$ of nuclear operators on E is isomorphic to the Banach space of all bounded operators on E^* ; (b) The dual of the Banach space of compact operators on E is isomorphic to the space of all operators on E^* which are the limit in the w^* -operator topology of a sequence bounded in $\mathcal{N}(E^*)$. (Received June 27, 1967.)

648-135. GREGERS KRABBE, Purdue University, Lafayette, Indiana 47907. An integral representation theorem.

Let u be a linear mapping of the space BV (consisting of all left continuous functions of bounded variation on the closed interval $[-\infty, \infty]$): we shall give a necessary and sufficient condition to insure

the existence of a function F such that $u(g) = \int g dF$ for all g in BV . Let G be the family of all functions f in BV such that $|f(-\infty)| + \text{var } f \leq 1$ (where var = total variation). Definition. A G_0 -net is a generalized sequence in G that converges pointwise on the open interval $(-\infty, \infty)$ to the constant-function zero. Theorem. Let u be a linear mapping of BV into a Banach space X , and suppose that u transforms G_0 -nets into bounded nets that converge (in X) to 0: there exists one (and only one) right-continuous regulated function F on $[-\infty, \infty]$ with values in X such that $F(-\infty) = 0$, $F(\infty) = F(\infty -)$ and $u(g) = \int g dF$ for all g in BV ; the integral is a σ -integral (i.e., refinement type). This theorem is still valid when X is a quasi-complete locally convex Hausdorff linear space; it extends a result in T. H. Hildebrandt's paper [Proc. Amer. Math. Soc. 17 (1966), 658-664]. (Received June 27, 1967.)

648-136. JOHN FOURNIER, The University of Wisconsin, Madison, Wisconsin 53706. Bohr sets are finite unions of unrelated sets.

Consider dirichlet series $f(z) = \sum_{n=1}^{\infty} a(n)n^{-z}$ for which $|f(z)| \leq 1$ whenever $\text{Re } z > 0$. Rider has defined a Bohr set to be any subset B of the positive integers for which there is a constant K so that $\sum_{n \in B}^{\infty} |a(n)| \leq K$ for all such f (Bull. Amer. Math. Soc. 72 (1966), 558-560). All of Rider's examples of Bohr sets are sets in which no element divides any other. Call such a set unrelated. Theorem. Every Bohr set is a union of at most $4K^2$ unrelated sets, where K is the constant associated with B in the above. It is easy to construct examples of unrelated sets which are not Bohr sets but it may be that every infinite unrelated set contains an infinite Bohr set. (Received June 29, 1967.)

648-137. MURRAY SCHECHTER, Lehigh University, Bethlehem, Pennsylvania. Degeneracy in the discrete Tchebycheff problem.

Let A be a given $m \times n$ real matrix with $m > n$ and $\text{rank } (A) = n$, let $b \in R_m$ and let $\| \cdot \|$ denote Tchebycheff (or L_{∞}) norm. The discrete Tchebycheff problem is the following: let $0 < \epsilon = \min \{ \|Ax - b\|, x \in R_n \}$. Find all x such that $\|Ax - b\| = \epsilon$. It is known that a solution to this problem always exists and is unique if the rows of A satisfy the Haar condition. In this paper it is shown that R_m may be written as a union of finitely many disjoint subsets such that for all choices of b lying in the same subset the discrete Tchebycheff problem has the property that all solutions have a common set of maximal residuals and these residuals have the same signs; furthermore, uniqueness or non-uniqueness of the solution is common to all b in the same subset. These subsets are found by solving finitely many problems of the following form: let B and C be given matrices having the same number of rows. Find all diagonal matrices D with 1 and -1's on the diagonal such that the inequalities $DBx \geq 0$ has no nontrivial solutions and the inequalities $DCy \geq 0$ has a nontrivial solution. A method for solving problems of this type is given. (Received June 29, 1967.)

648-138. HENRYK MINC, University of California, Santa Barbara, California 93106. Bounds for permanents of nonnegative matrices.

In a recent paper Jurkat and Ryser obtained bounds for permanents of nonnegative matrices [J. Algebra 3 (1966), 1-27, inequalities (12.27)]. In the present paper the same inequalities are re-proved by elementary methods and the cases of equality are discussed. (Received June 29, 1967.)

Truncation error estimates for g-fractions.

Let σ be nondecreasing with infinitely many points of increase, moments $s_n = \int_0^1 t^n d\sigma(t)$, and generating function $f(z) = s_0 - s_1 z + s_2 z^2 - \dots, |z| < 1$. The analytic continuation $f(z) = \int_0^1 (1+zt)^{-1} d\sigma(t) = s_0 / (1 + g_1 z / 1 + (1 - g_1) g_2 z / 1 + (1 - g_2) g_3 z / 1 + \dots, |\arg(1+z)| < \pi$. Here $0 < g_n < 1$; see H. S. Wall, Trans. Amer. Math. Soc 48 (1940), 165-184. Let $w_n(z) = s_0 / 1 + \dots + (1 - g_{n-1}) g_n z / 1, w_n^*(z) = s_0 [1 + (1 - g_1) z / 1 + g_1 (1 - g_2) z / 1 + \dots + g_{n-1} (1 - g_n) z / 1] / (1+z)$. For complex z let $K_n(z), K_n^*(z)$ be the open disks whose boundaries contain $\{w_n(z), w_{n+1}(z), w_n^*(z)\}$ and $\{w_n(z), w_{n+1}^*(z), w_n^*(z)\}$, respectively ($0 \notin$ the half plane $K_0^*(z)$), and put $G_n(z) = K_n(z) \cap K_n^*(z)$. Theorem. Let z be complex, $|\arg(1+z)| < \pi$. Then $f(z) \in G_n(z) \subset G_{n-1}(z) \subset \dots \subset G_0(z)$, and every point of $G_n(z)$ is the value of a terminating g-fraction with leading approximants $w_0(z), w_1(z), \dots, w_{n+1}(z)$. $G_n(z)$ has interior angle $|\arg(1+z)|$; $G_n(z) - G_{n+1}(z)$ has alternate interior angles $|\arg(-1/z)|, |\arg z / (1+z)|$. For $x > 0, w_0^*(x) < w_1(x) < w_2^*(x) < \dots < f(x) < \dots < w_2(x) < w_1^*(x) < w_0(x)$, and for $-1 < x < 0, w_0(x) < w_1(x) < \dots < f(x) < \dots < w_1^*(x) < w_0^*(x)$. $\text{diam } G_n(z) \leq K(z) |(1 - (1+z)^{1/2}) / (1 + (1+z)^{1/2})|^n$, with $K(z) = s_0 \max[1, |\tan(1/2)\arg(1+z)|] |(1+z)^{1/2} - (1+z)^{-1/2} / \text{Re}(1+z)^{1/2}$. Examples include the hypergeometric function $F(a,b; c; -z), 0 < a, b \leq 1, a + b \leq c + 1$. (Received June 29, 1967.)

648-140. J. I. NIETO, University of Maryland, College Park, Maryland 20740. On the essential spectrum of multiplication operators, singular integral operators and symmetrizable operators.

Let T be a linear bounded operator in a Banach space X , with spectrum $\sigma(T)$. The essential spectrum $\sigma_{em}(T)$ of T is the complement of the set of all λ for which index of $T - \lambda I$ is zero. Theorem 1. Let f be a complex-valued function in $L^\infty(\mathbb{R}^n)$ ($n \geq 1$) and T the operator in $L^p(\mathbb{R}^n)$ ($1 \leq p \leq \infty$): $T\phi = f\phi$. Then $\sigma_{em}(T) = \sigma(T) = \text{Set of all } \lambda \text{ such that for any neighborhood } V \text{ of } \lambda \text{ its inverse image } f^{-1}(V) \text{ has positive measure.}$ Theorem 2. Let T be a singular integral operator with variable kernel in $L^p(\mathbb{R}^n)$ ($1 < p < \infty$) of the type considered by R. Seeley in J. Math. Anal. Appl. 7 (1963), 289-309. Then for $n \geq 2, \sigma_{em}(T) = S(T)(\dot{\mathbb{R}}^n \times \Sigma)$, where $S(T)$ is the symbol of $T, \dot{\mathbb{R}}^n$ the one-point compactification of \mathbb{R}^n and Σ the unit sphere in \mathbb{R}^n . For $n = 1$ only holds: $S(T)(\dot{\mathbb{R}}^n \times \Sigma) \subset \sigma_{em}(T)$. Theorem 3. Let X be a Banach space, H a Hilbert space so that: (1) $X \subset H$ and the injection from X into H being continuous; (2) X is dense in H . Assume further that T is symmetrizable over X , i.e., T is bounded over X and symmetric with respect to the inner product induced by H on X . Then $\sigma_{em}(T)$ consists precisely of those points $\lambda \in \sigma(T)$ which are not isolated eigenvalues of finite multiplicity. This theorem generalizes a classical result of H. Weyl. (Received June 30, 1967.)

648-141. P. S. MARCUS, 404 Main Street, Mt. Carroll, Illinois. Order induced by a transformation group. Preliminary report.

The Hilbert-Veblen reconstruction of Euclidean geometry is generally accepted as definitive. Professor Busemann, in his book Projective geometry and projective metrics indicates on pp. 149-150 how Euclid's superposition theory can be treated in a way which is more in the spirit of the Erlangen program than is the corresponding congruence theory of Hilbert. The following theorem shows that the Hilbert and Veblen theory of order can also be treated in accordance with the spirit of the Erlangen program. Definition. The ternary relation ABC for points A, B, C of an arbitrary set S is

satisfy the equation $\Delta^n f \equiv 0$. The argument relies very heavily on the family F_0 of polynomial elements of F , as well as approximations of elements of F by Bernstein polynomials. No use is made of potential theoretic or Hilbert space methods. (Received July 3, 1967.)

648-145. S. P. SINGH, University of Windsor, Windsor, Ontario, Canada. On commuting analytic functions with fixed points.

The following well-known conjecture--if f and g are two continuous functions that map a closed interval on the real line into itself and commute, must they have a common fixed point--has been disproved by Boyce and Huneke independently. In this note a theorem on commuting analytic functions has been proved. If f and g are two analytic functions in a domain D of the complex z -plane that map a compact and connected subset C of D into itself and if f and g commute then they have a common fixed point provided that $|f'(z)| < 1$ for all z in C . (Received July 3, 1967.)

648-146. W. B. JONES and W. J. THRON, University of Colorado, Boulder, Colorado 80302. Variable parabolic convergence regions for continued fractions.

It is well known that a continued fraction $K(a_n/1) = a_1/1 + a_2/1 + \dots$ converges only if $\sum_1^\infty |(a_2 a_4 \dots a_{2n}) / (a_3 a_5 \dots a_{2n+1})|$ or $\sum_1^\infty |(a_3 a_5 \dots a_{2n+1}) / (a_4 a_6 \dots a_{2n+2})|$ diverges. Refer to this as condition (A). If the elements a_n lie in a parabolic region $E = \{a_n : |a_n| - R e^{[a_n e^{-2\psi i}]} \leq 1/2 \cos^2 \psi, -\pi/2 < \psi < \pi/2\}$, then it is known that $K(a_n/1)$ converges if and only if (A) holds. The first proof of this result based on elementary methods was given by W. J. Thron [Convergence of sequences of linear fractional transformations and of continued fractions, J. Indian Math. Soc. 27 (1963), 103-127]. By modifying the methods of Thron, the authors have proved the following extension, which allows for variable parabolic element regions: Theorem 1. Let $\{P_n\}$ be a sequence of complex numbers $P_n = p_n e^{i\psi_n}$ satisfying $|P_n - 1/2| \leq M < 1/2$. Let the elements a_n lie in the parabolic regions $E_n = \{a_n : |a_n| - R e^{-i(\psi_n + \psi_{n-1})} \leq 2p_{n-1} (\cos \psi_n - p_n)\}$. Then $K(a_n/1)$ converges if and only if (A) holds. Note. The earlier result is obtained by taking $p_n = 1/2 \cos \psi$ and $\psi_n = \psi$. Along the same lines the authors have also proved: Theorem 2. Let $\{P_n\}$ be a sequence of complex numbers $P_n = p_n e^{i\psi_n}$ satisfying $|P_n - 1/2| < 1/2$. Let $a_n \in E_n$ where E_n is defined as in Theorem 1. Then $K(a_n/1)$ converges if the sequence $a_n/p_n p_{n-1}$ is bounded. (Received July 3, 1967.)

648-147. V. S. MANDREKAR and M. G. NADKARNI, University of Minnesota, Minneapolis, Minnesota 55455. Generalizations and applications of a theorem of Mackey.

Let G be a separable locally compact abelian group with G_0 being its subgroup. A hermitian projection valued measure E defined on the Borel subsets \mathcal{B} of G is called G_0 -stationary if there exists a group of unitary operators $\{U_g, g \in G_0\}$ on a separable Hilbert space H such that $U_g E(\sigma) U_{-g} = E(\sigma + g)$ for $\sigma \in \mathcal{B}$ and $g \in G_0$. Using Hellinger-Hahn theorem we give a representation of G_0 -stationary spectral measures in terms of quasi-invariant (under G_0) measures on \mathcal{B} . For $G = G_0$ and U_g strongly continuous in G , this gives Mackey's extension of Stone-Von Neumann theorem [Duke Math. J. (1949), 313]. The case $G_0 = G$ but U_g not necessarily continuous leads to a different kind of representation. Combined with the recent work of K. deLeeuw and I. Glicksberg on the decomposition of certain group representations [J. Analyse Math. 15 (1965), 135] this gives new results in the linear

prediction of not necessarily continuous stationary processes. Applications of the general situation are made to some problems in analytic measures and a simplified proof is offered to the main result of F. Forelli's Measures orthogonal to polydisc algebra [see Abstract 67T-352, these Notices 14 (1967), 523]. (Received July 3, 1967.)

648-148. R. D. MOYER, University of Kansas, Lawrence, Kansas 66044. A decomposition of flat functions.

Let M be a C^∞ manifold. A complex valued C^∞ function ϕ on M is said to be flat at $x_0 \in M$ if all differential operators on ϕ vanish at x_0 . It is shown that every flat function is the product of two flat functions, one of which is real and nonnegative. (Received July 3, 1967.)

648-149. D. W. HENDERSON, Cornell University, Ithaca, New York 14850. D-dimension. I. A new transfinite dimension.

In this paper, we introduce a new transfinite dimension called D-dimension. D-dimension as a topological function from all metric spaces to the ordinals (≥ 1), with an extra symbol, Δ , added, satisfies the following axioms: (We use the conventions that, for each ordinal α , $\Delta \alpha, \Delta + \alpha = \Delta + \Delta = \alpha + \Delta = \alpha \oplus \Delta = \Delta \oplus \Delta = \Delta$, where ' \oplus ' denotes the natural sum of ordinals.) (I) If either $D(X)$ or $\text{Ind}(X)$ is finite, then $D(X) = \text{Ind}(X)$. (II) $D(X) = \text{l.u.b. } \{D_p(X) | p \in X\}$, where $D_p(X) = \text{minimum } \{D(N) | N \text{ a neighborhood of } p \text{ in } X\}$. (III) If F is a closed subset of the space X , then $D(X) \leq D(X - F) + D(F)$. (IV) If Y is a subspace of X , then $D(Y) \leq D(X)$. (V) If there is a point $x \in X$, such that $D_x(X) = D(X)$, then $D(X \times I) = D(X) + 1$. (VI) If A and B are closed subsets of X , then $D(A \cup B) = \text{maximum } \{D(A), D(B)\}$. It is shown that these axioms together with a seventh one characterize D-dimension. In addition, the following properties are demonstrated: (VIII) $D(X \times Y) \leq D(X) \oplus D(Y)$. (IX) $D(Q^\alpha) = \alpha$, where Q^α are the compact metric spaces constructed by Smirnov such that $\text{Ind}(Q^\alpha) = \alpha$, for all countable ordinals, α . With the exceptions of (I) and (IX) none of these are known to be satisfied by the transfinite inductive dimension, Ind . (Received July 3, 1967.)

648-150. ROGER RIGELHOF, McMaster University, Hamilton, Ontario, Canada. A characterization of $M(G)$.

Let G be a locally compact group, and $M(G)$ the Banach algebra of bounded regular Borel measures on G . The following theorem characterizes those Banach algebras which are isometric and isomorphic to the algebra $M(G)$ for some locally compact group G . Theorem. Let A be a Banach algebra, S its unit sphere and S^ϵ the set of extreme points of S . Suppose that (1) there is a Banach space E such that A is the dual of E ; (2) multiplication is $\sigma(A, E)$ -continuous in each variable separately; (3) if $x \in A$ is such that $xy = 0$ for all y in A , then $x = 0$; (4) if $x \in S^\epsilon$, then the mapping $y \rightarrow xy$ is an isometry of A onto itself; (5) $S^\epsilon \cup \{0\}$ is $\sigma(A, E)$ -closed; (6) there is a nonzero multiplicative linear functional p on A ; (7) if $G = \{x \in S^\epsilon : p(x) = 1\}$ where p is as in (6), then (i) for $f \in E$, there is a $g \in E$ such that $\overline{x(f)} = x(g)$, and (ii) for $f, g \in E$, there is a $h \in E$ such that $x(h) = x(f)x(g)$. Then G is a locally compact group and A is isomorphic and isometric to $M(G)$. If S^ϵ is $\sigma(A, E)$ -closed, then G is compact. G is unique to within isomorphism and homeomorphism. Conversely if G is a locally compact group then $M(G)$ satisfies (1)-(7). (Received July 3, 1967.)

648-151. SOON-KYU KIM, University of Illinois, Urbana, Illinois. Local triviality of Hurewicz fiber maps.

Let (E, B, p) be a Hurewicz fibering, where the total space E is a connected separable metric ANR and the base B is a weakly locally contractible paracompact space. F. Raymond proved the local triviality of the map p under certain conditions in his paper which appeared in *Topology* 3 (1965). We generalize Raymond's result. Theorem. Suppose E is locally compact and each fiber is a connected compact orientable 2-manifold with nonempty boundary. Then $p' : E \times I \rightarrow B$, defined by $p'(x, t) = p(x)$ for each $(x, t) \in E \times I$, is a locally trivial fiber map. Theorem. Suppose E is a generalized manifold (gm) over a principal ideal domain L with boundary $\partial(E)$, which is also an ANR, and there exists a lifting function for (E, B, p) which is "stationary" in $\partial(E)$ and $\text{Int}(E)$. If there is a point b such that (each component of $p^{-1}(b) \cap$ (each component of $\partial(E)$) is connected, then all fibers are gms over L with boundary, moreover, if we also assume that $p^{-1}(b)$ is a compact space of dimension ≤ 2 , then the fibering (E, B, p) is locally trivial. Theorem. Suppose E is a η -gm over L and all fibers are homeomorphic to a space which is either the real line or a connected 2-manifold with finitely generated homology groups and exactly one end. Then the fibering (E, B, p) is locally trivial. (Received July 3, 1967.)

648-152. W. M. KINCAID, University of Michigan, Ann Arbor, Michigan. Secant methods for solving systems of nonlinear equations.

Generalizing the secant method of solving nonlinear equations to systems of the form $f_i(x_1, \dots, x_n) = 0, i = 1, \dots, n$, is complicated by the need for avoiding 'collapsed' configurations of iterative points. Workable methods in which the configurations are controlled have been devised. In using these methods a lower limit is set on the magnitude of the displacement in passing from one iterative point to the next to prevent function differences from being obscured by roundoff errors. A procedure for determining appropriate step lengths in different directions has been developed; it involves finding scaling factors for the f 's to reduce various function differences to the same order of magnitude. (Received July 3, 1967.)

648-153. P. S. GREEN and RICHARD HOLZSAGER, University of Maryland, College Park, Maryland 20740. The Samelson product and the J-homomorphism.

Let $h' : S^k \times S^{n-1} \rightarrow S^{r-1}$ be a map of type (ζ, η) ($\zeta \in \prod_k(S^{r-1}), \eta \in \prod_{n-1}(S^{r-1})$). Then one can form the Hopf construction $h \in \prod_{k+n}(S^r)$. Theorem. If $\rho \in \prod_p(X), \sigma \in \prod_r(X)$, then $[\sigma \circ h, \rho] = [\sigma, \rho] \circ S^{p-1}h + [\sigma \circ S \zeta, [\sigma \circ S \eta, \rho]]$. If $\alpha \in \prod_k(SO(n))$, then $J(\alpha) = [\partial^{-1}(\alpha), i_n]$, the mixed Whitehead product $(\prod_*(\text{total space}) \times \prod_*(\text{fiber}) \rightarrow \prod_*(\text{fiber}))$ in the fibration $S^n \rightarrow BO(n) \rightarrow BO(n+1)$, where ∂ is the boundary operator for the universal $SO(n)$ -bundle. The Jacobi identity, together with the above theorem, yields a formula for $J(\langle \alpha, \beta \rangle)$, where \langle , \rangle is the Samelson product in $SO(n)$. This answers a question raised by Michael Barratt. (Received July 3, 1967.)

648-154. S. KINOSHITA, Florida State University, Tallahassee, Florida 32306. Elementary ideals of linear graphs in a 3-sphere. Preliminary report.

Let L be a linear graph (1-dimensional polyhedron) in a 3-sphere S^3 and l a 1-cycle with integral coefficients on L . A homomorphism of the fundamental group $G = \pi(S^3 - L)$ into an infinite cyclic group is defined by using the linking number of $g \in G$ and l . Hence we can define elementary ideals $\{E_i\}$ of l on L . Suppose that each component of L is not a tree. Then we have $E_i = (0)$ for $i \leq \beta - \alpha$, $o(E_i) = (0)$ for $\beta - \alpha < i < \beta$ and $o(E_i) = (1)$ for $\beta \leq i$, where α is the number of components of L , β the 1-dimensional Betti number of L and o a trivializer. E_i may not be symmetric and $E_{\beta - \alpha + 1}$ may not be principal. (Received July 3, 1967.)

648-155. S. BARON, McGill University, Montreal 2, Province of Quebec, Canada. Reflectors as compositions of epi-reflectors.

Let R be a reflector from \mathcal{L} to \mathcal{A} . J. F. Kennison [Reflective subcategories, to appear in Illinois J. Math.] and the author [Abstract 67T-331, these *Notices* 14 (1967), 516] have independently shown that if certain conditions are imposed on \mathcal{L} , then $R = ST$, where T is a \mathcal{L} -epi reflector from \mathcal{L} to a subcategory, \mathcal{B} , that contains \mathcal{A} and S is a \mathcal{B} -epi reflector from \mathcal{B} to \mathcal{A} . When such a factorization exists, \mathcal{B} is called an intermediate category of the pair $(\mathcal{A}, \mathcal{L})$. If \mathcal{L} satisfies some additional conditions, there is a largest and a smallest intermediate category. Theorem. Let \mathcal{A} be a reflective subcategory of a locally and colocally small, left complete category, \mathcal{L} . (1) The full, replete subcategory, \mathcal{B}' , whose objects are the \mathcal{L} -extremal subobjects of \mathcal{A} -objects is the smallest intermediate category of $(\mathcal{A}, \mathcal{L})$. (2) Let \mathcal{B}'' be the full, replete subcategory such that B'' is a \mathcal{B}'' -object iff for every \mathcal{B}' -object, B' , having reflection map $r(B'): B' \rightarrow R(B')$ and every pair of maps $f, g: R(B') \rightarrow B''$, $f \circ r(B') = g \circ r(B')$ implies $f = g$. \mathcal{B}'' is the largest intermediate category of $(\mathcal{A}, \mathcal{L})$. (Received July 3, 1967.)

648-156. H. LeROY PETERSON, The University of Connecticut, Storrs, Connecticut 06268. The existence of irregular Borel measures on dyadic spaces.

Let X be a Hausdorff space. (1) If every open subset of X is σ -compact, then every Borel measure on X is regular (see Zakon, Canad. Math. Bull. 7 (1964), 41-44). A dyadic space is a Hausdorff space which is the image, under a continuous map, of $\{0,1\}^m$ for some cardinal m . Theorem. A dyadic space X is metric if and only if every Borel measure on X is regular. A fortiori, the converse of statement (1) is true for dyadic spaces; there is an example which shows that the converse is not true for arbitrary compact Hausdorff spaces. Reference is made to Efimov and Engelking (Coll. Math. 13 (1965), 181-197), Dieudonné (C. R. Acad. Sci. Paris 209 (1939), 145-147), and Ulam (Fund. Math. 16 (1930), 141-150). Corollary. A σ -compact, locally compact group G is metric if and only if every Borel measure on G is regular. The latter statement is a less precise version of a theorem of the author. (Received July 3, 1967.)

648-157. A. H. LACHLAN, Simon Fraser University, Burnaby 2, B. C. Canada. Initial segments of one-one degrees.

Define O to be the one-one degree of the odd numbers. We consider only one-one degrees $\geq O$ with their natural partial ordering. Theorem 1. A finite initial segment of one-one degrees with greatest member is a lattice. Theorem 2. A finite initial segment of r.e. one-one degrees with greatest member is a distributive lattice. The proofs use the notion of the disjoint union $\underline{a} \oplus \underline{b}$ of one-one degrees $\underline{a}, \underline{b}$, where if A, B represent $\underline{a}, \underline{b}$ respectively then $\{2x \mid x \in A\} \cup \{2x + 1 \mid x \in B\}$ represents $\underline{a} \oplus \underline{b}$. (Received July 3, 1967.)

648-158. J. J. H. MILLER, University of Massachusetts, 100 Arlington Street, Boston, Massachusetts 02115. On power-bounded operators and operators satisfying a resolvent condition.

By means of a classification of the spectrum of the operator and the estimation of its resolvent on certain contours within the unit ball, local estimates for holomorphic functions of a class of linear operators on a finite dimensional linear vector space are obtained. These methods are then applied to prove two theorems of Kreiss and Morton, and some new results are also obtained. In particular the following theorem is proved. Let $\mathcal{S}(C)$ be the family of all linear operators T on m -dimensional unitary space \mathcal{V}_m such that $|T^n| = \sup_{|v|=1} |T^n v| \leq C, v \in \mathcal{V}_m$, for all nonnegative integers n , and put $\mu(T) = \min_j |\kappa_j(T)|$ where $\{\kappa_1(T), \dots, \kappa_m(T)\}$ is the spectrum of T . Theorem. If $T \in \mathcal{S}(C)$ and T^{-1} exists, then for all nonnegative integers n $|(T/(1 - r/2m)\mu(T))^{-n}| \leq K(m)C/(r\mu(T))^{m-1}$ where $K(m) < e^{9m^2}$ and r is any number satisfying $0 < r \leq 1$. (Received July 3, 1967.)

648-159. MICHAEL GOLOMB, Purdue University, Lafayette, Indiana 47907. Splines, n-widths and optimal approximations.

Let \mathcal{R} be a subset of the seminormed space \mathcal{X}_0 . In the Kolmogorov concept of n -width one asks for an n -dimensional subspace \mathcal{M}_* of \mathcal{X}_0 ("optimal subspace") such that the distance $E(\mathcal{R}, \mathcal{M}) = \sup_{x \in \mathcal{R}} \inf_{y \in \mathcal{M}} \|x - y\|_0$ is minimized, this minimal distance being the n -width of \mathcal{R} . For this paper n -dimensional subspaces are replaced by n -flats (a flat is a translate of a subspace). The 0 -width of the set \mathcal{R} is then a meaningful question and if \mathcal{R} is a "disk" (the intersection of a flat μ^* and an ellipsoid $\{x: \|Rx\| \leq \rho\}$), then the optimal 0 -flat is the R -spline x_* that interpolates μ , and the 0 -width is the radius of \mathcal{R} . The optimal n -flat (for $n \geq 1$) is the translate by x_* of the n -dimensional subspace optimal for a modified set \mathcal{R}' . Characterization of the optimal n -flats and formulas for the n -widths are developed. Applications are indicated, in particular to the problem of extending a function given on a subset of \mathbb{R} to \mathbb{R} , so that the extended function is in the Sololev class \mathcal{S}^m . (Received July 3, 1967.)

648-160. PETER FALLEY, 100 Overlook Terrace, New York, New York 10040. Full ideals of operators. Preliminary report.

Let E_1, E_2 be Banach spaces, and $\mathcal{B}(E_1, E_2)$ the space of bounded operators from E_1 into E_2 . If \mathfrak{M} is a saturated (saturé, Bourbaki) class of bounded sets in E_2 , denote by $\mathfrak{A}\mathfrak{M}$ the set of operators in $\mathcal{B}(E_1, E_2)$ which map the full unit ball B_1 into elements of \mathfrak{M} . Also, given a subset $\mathcal{C} \subset \mathcal{B}(E_1, E_2)$,

denote by $\mathfrak{M}_{\mathfrak{S}}$ the saturated class of bounded sets in E_2 generated by the sets $A(B_1)$, $A \in \mathfrak{S}$. A right ideal \mathfrak{A} in $\mathfrak{B}(E_1, E_2)$ is defined to be full if $\mathfrak{A} = \mathfrak{A}\mathfrak{M}_{\mathfrak{A}}$. Every space of the form $\mathfrak{A}\mathfrak{M}$ is a full ideal in $\mathfrak{B}(E_1, E_2)$. Every right ideal of operators on a Hilbert space is a full ideal. There exist right ideals of operators on reflexive Banach spaces which are not full ideals. (Received July 5, 1967.)

648-161. D. J. FIELDHOUSE, Queen's University, P. O. Box 101, Kingston, Ontario, Canada. Pure projectivity.

P. M. Cohn calls a submodule P of the left A -module M pure iff $0 \rightarrow E \otimes P \rightarrow E \otimes M$ is exact for all right modules E . Theorem 1. P is pure in M iff all maps from fp (= finitely presented) modules F to M/P can be lifted to M . Following Maranda, a left module Q is called pure projective iff $\text{Hom}(Q, M) \rightarrow \text{Hom}(Q, M/P) \rightarrow 0$ is exact whenever P is pure in M . Theorem 2. For any module G there exists an exact sequence $0 \rightarrow K \rightarrow Q \rightarrow G \rightarrow 0$ with Q pure projective and K pure in Q . Theorem 3. Q is pure projective iff Q is a direct summand of a direct sum of fp modules. Corollary. If P is pure in M and M/P is a direct summand of a direct sum of fp modules, then P is a direct summand of M . Additional characterizations of purity and pure projectivity are given. (Received July 5, 1967.)

648-162. BASIL GORDON, University of California at Los Angeles, California and LORNE HOUTEN, 226 Maine Street, Brunswick, Maine. On plane partitions, strictly monotonic on rows.

In Abstract 642-38 of these *Notices* 14 (1967), 71-72, a k -rowed partition of n , strictly monotonic on rows, is defined. In this paper the authors prove that the generating functions $B_k(x)$ for such partitions are $B_k(x) = P^{[k/2]} Q^{2\{k/2\}} \prod_{\nu=1}^{k-2} (1-x^\nu)^{[(k-\nu)/2]}$ where $P = \prod_{\nu=1}^{\infty} (1-x^\nu)^{-1}$, $Q = \prod_{\nu=1}^{\infty} (1-x^{2\nu-1})^{-1}$ and where $[\theta]$ and $\{\theta\}$ denote respectively the integral and fractional parts of θ . Letting $k \rightarrow \infty$ the authors obtain an expression for $B(x) = \sum_{n=0}^{\infty} b(n)x^n$ where $b(n)$ is the number of plane partitions of n with strictly decreasing parts on each row, namely $B(x) = \prod_{\nu=1}^{\infty} (1-x^\nu)^{-[(\nu+1)/2]}$. (Received July 5, 1967.)

648-163. TORSTEN NORVIG, 125 Shornecliffe Road, Newton, Massachusetts 02158. A method of fictitious "play" for linear economic exchange models.

Consider a linear economic exchange model given by its $m \times n$ nonnegative utility matrix $A = (a_{ij})$ with positive row and column sums and by its positive endowment $B = (b_1, \dots, b_m)$ with $\sum_{i=1}^m b_i = 1$. It is known (David Gale, The theory of linear economic models, McGraw-Hill, New York, 1960) that given this model there exist a nonnegative $m \times n$ equilibrium distribution matrix $Y = (y_{ij})$ and a unique positive equilibrium price vector $P = (p_1, \dots, p_n)$ such that $\sum_{j=1}^n y_{ij} = 1$, all j , $\sum_{j=1}^n p_j = 1$, $\sum_{j=1}^n y_{ij} p_j = b_i$, all i , and such that $\sum_{j=1}^n a_{ij} y_{ij}$ is maximized for all i . Generate a "learning sequence" $P^k = (p_1^k, \dots, p_n^k)$, $k = 1, 2, \dots$, by the following rules. (i) P^1 is arbitrary except that $p_j^1 \geq 0$, all j , and $\sum_{j=1}^n p_j^1 = 1$. (ii) for $k = 1, 2, \dots$ and all j , $p_j^{k+1} = p_j^k + c_j^k$ where $c_j^k = \sum_{i \in C(j)} b_i$ with $C(j) = \{i | a_{ij}/p_j^k \geq a_{it}/p_t^k, \text{ all } t, \text{ and there does not exist } t < j \text{ with } a_{ij}/p_j^k = a_{it}/p_t^k\}$. Theorem. $\lim_{k \rightarrow \infty} p_j^k/k = p_j$, all j , where (p_1, \dots, p_n) is the unique equilibrium price vector of the given model. (Received July 5, 1967.)

648-164. JACK BRYANT, Texas A and M University, College Station, Texas 77843. On convolution and sequences of Banach spaces.

This paper generalizes and extends results of the author [On convolution and Fourier series, Duke Math. J. 34 (1967), 117-122] to the following situation: $\{B_n\}$ is a sequence of B-spaces contained in $L(0,2\pi)$ with the Fourier series of f summable $(C,1)$ in some of the spaces and convergent in others. Under very general conditions, we are able to obtain functions h and g with $f = g * h$ and $g \in L$, with h satisfying the same conditions f does. (Received July 5, 1967.)

648-165. WILLIAM LOPEZ, 522 West Johnson Street, Madison, Wisconsin 53703. Examples in the fixed point theory of finite polyhedra.

Consider the following questions in the fixed point theory for finite polyhedra K . (1) If K admits a map of Lefschetz no. = 0, does K fail to have the fixed point property (f.p.p.)? (2) If K and K' both have the f.p.p., does their union along an edge have the f.p.p.? (3) If K and K' are of the same homotopy type and K has the f.p.p., does K' have the f.p.p.? (4) If K has the f.p.p., does $K \times I$ (or $K \times K$) have the f.p.p.? (5) If K has the f.p.p., does the suspension of K have the f.p.p.? Finite polyhedra are constructed to show that all these questions are answered in the negative. The basic polyhedron from which the counterexamples are constructed is obtained from $X = P_2(C) \cup S_1 \times S_2 \cup P_4(C)$, where $P_2(C)$ and $P_4(C)$ are complex projective spaces, S_1 and S_2 are 2-spheres, and some simple identifications are made. X has the f.p.p. and Euler characteristic = 8. The fact that the Euler characteristic is even is what is essential. (Received July 5, 1967.)

648-166. H. B. ENDERTON, University of California, Berkeley, California 94720. A language of order $1 \frac{1}{2}$.

Let \mathcal{L} be the language obtained from the usual first-order language for number theory by adding the quantifier: If $\phi(v)$ is a wff then so is $\bigwedge F \forall v \phi(F|v)$. Here F is a function variable and $F|v$ is a term representing the course-of-values operation $\bar{a}(n)$. The set of true \mathcal{L} -sentences (in the standard interpretation) is Δ_2^1 , in fact it is recursive in the type 2 functional \bar{E}_1 (which is the type 2 object naturally associated with this quantifier). A set of natural numbers is \mathcal{L} -definable iff it is primitive recursive in \bar{E}_1 . We can also consider the infinitary rule of inference which, for any function α , permits $\bigvee F \wedge v \phi(F|v)$ to be derived from $\{\phi(\bar{n}) : n \text{ in range } \bar{\alpha}\}$. This is analogous to the ω -rule but with our strengthened quantifier. Any true \mathcal{L} -sentence is derivable with this rule. The derivable second-order sentences form a proper subset of the set of sentences true in all β -models. (Received July 5, 1967.)

648-167. R. G. VINSON, Huntingdon College, Montgomery, Alabama 36111. On order in geometry.

The order of points on a line is a fundamental concept that must be considered in any study of Euclidean, hyperbolic, or elliptic geometry. Theorems are given here that have as their result the axiomatic construction of these three geometries using the same basic concepts of order and continuity of points on a line. This result is accomplished by first considering arcs and building the required lines from special classes of these arcs. (Received July 5, 1967.)

648-168. SISTER MARION BEITER, Rosary Hill College, Buffalo, New York 14226. Coefficients of cyclotomic polynomials of index $3qr$.

Let $F_m(x) = \sum_{n=0}^{\phi(m)} c_n x^n$ be the m th cyclotomic polynomial with $m = pqr$, where p, q, r are odd primes and $p < q < r$. For $m = 3qr$, it is known that c_n satisfies the inequality $|c_n| \leq 2$. The following refinements of this case are obtained: (1) $F_m(x)$ may have a coefficient of 2 or of -2, but not of both. (2a) If $3|r - q$, then $-1 \leq c_n \leq 2$; (2b) If $3|r + q$, then $-2 \leq c_n \leq 1$. The case in which q and r are twin primes is a special instance of (2b) in which, for $n = r$, $c_n = -2$. (3) The relation $r = 3kq \pm 1$ is a sufficient but not a necessary condition that $-1 \leq c_n \leq 1$. These results follow from the fact that the value of c_n depends on the number of partitions of n of the form $a + 3\alpha q + 3\beta r + \delta_1 q + \delta_2 r$, with $\delta = 0, 1$; $0 \leq a < 3$; α, β nonnegative integers. Each such partition of n contributes $(-1)^{\delta_1 + \delta_2}$ to the value of c_n . (Received July 5, 1967.)

648-169. J. L. CHRISLOCK, University of California, Santa Cruz, California 95060. The structure of medial semigroups.

A semigroup S is said to be archimedean if for each $a, b \in S$ there is a positive integer n such that $a^n \in SbS$ and is said to be medial if it satisfies the identity $xaby = xbay$. Theorem. A medial semigroup S is a semilattice of archimedean semigroups. The classes of this decomposition will be called the archimedean components of S . Theorem. If a medial semigroup has an idempotent e , then (1) the collection E of all the idempotents of S is a rectangular band, (2) eSe is an abelian group, (3) S has a kernel which is isomorphic to $eSe \times E$, and (4) some power of each element of S lies in the kernel. Theorem. A medial semigroup is separative ($a^2 = b^2 = ab$ implies $a = b$) if and only if in each of its archimedean components $ax = bx$ and $ya = yb$ implies $a = b$. Theorem. A medial semigroup can be embedded in a union of groups if and only if it is separative. (Received July 5, 1967.)

648-170. K.-T. LIU, State University of New York at Buffalo, Buffalo, New York 14214. On the semiprimitivity of group rings.

Let RG be the group ring of a group G over an associative ring with unit R . A ring R is semiprimitive if its Jacobson radical $J(R)$ is zero, it is semisimple if its simplicial radical $\mathcal{L}(R)$, the intersection of maximal two sided ideals, is zero. An integer n is cancelable in a ring R if $nx = 0$ for $x \in R$ implies $x = 0$. We say a group G is cancelable in a ring R if every finite order of element in G is cancelable in R . Theorem 1. If R is semisimple and if G is finite such that the order of G is cancelable in R , then RG is semiprimitive. Theorem 2. Let G be an abelian torsion group. If R is semisimple and if G is cancelable in R , then RG is semiprimitive. Theorem 3. Let G be a free group. If R is semisimple and if every finite index of normal subgroup in G is cancelable in R , then RG is semiprimitive. Theorem 4. Let G be an abelian group. If R is semisimple and if every finite index of subgroup in G is cancelable in R , then RG is semiprimitive. Theorem 5. Let C be a central subgroup of G . Suppose G/C is locally finite. If R is semisimple and if every finite index of subgroup of C in G is cancelable in R , then RG is semiprimitive. (Received July 5, 1967.)

648-171. W. F. PFEFFER, University of California, Davis, California 95615. Singular integrals are Perron integrals of a certain type. Preliminary report.

Let E_m , $m \geq 1$ an integer, be the m -dimensional Euclidean space, σ a pre-algebra of subsets of E_m , and let κ_x , $x \in E_m \cup \{\infty\}$, be a family of sequences $\{B_n\} \subset \sigma$. Under this setting, using majorants and minorants, a generalised Perron integral can be defined on σ (see Abstract 625-3, these *Notices* 12 (1965), 545 and 67T-184, these *Notices* 14 (1967), 270). Given a finite set $S \subset E_m$ a suitable choice of the system σ and families κ_x gives the following: Theorem. If f is a function on E_m which is locally Lebesgue integrable in $E_m - S$ and such that the singular integral of f over E_m exists, then f is Perron integrable. Here the singular integral is meant in the sense of S. G. Mikhlin, Multidimensional singular integrals and integral equations, Chapter II, §5, sec. 3. (Received July 5, 1967.)

648-172. D.-N. VERMA, New Mexico State University, Las Cruces, New Mexico 88001. Solution of the "basis problem" for the finite-dimensional representations of a complex simple Lie algebra of type A.

The notations defined and the Theorem announced in Abstract 625-140 (these *Notices* 12 (1965), 584) are used below. Also used is the truth of the conjecture there when σ is a reflection in any root, though that is false for general σ ; proofs of these appear in a Yale Ph.D. dissertation, 1966. Let \mathcal{L} be simple of type A_l and let the simple roots be so labelled that the positive roots are all of the form $\alpha_i + \dots + \alpha_j$, $1 \leq i \leq j \leq l$. Let \mathfrak{M}_Λ be an irreducible (right) \mathcal{L} -module with highest weight $\Lambda = \sum_1 m_i \lambda_i$, and ξ be a highest weight-vector. For any integer m let \bar{m} denote $m + 1$. For $1 \leq i \leq l$ and $0 \leq q \leq m_i$, there is a unique element $u_q^{(i)}$ in the universal associative envelope U of \mathcal{L} satisfying $u_q^{(i)} \cdot f_i^{-q + \bar{m}_i} \cdot f_{i+1}^{-q + \bar{m}_i + \bar{m}_{i+1}} \dots f_{l-1}^{-q + \bar{m}_i + \dots + \bar{m}_{l-1}} = f_i^{\bar{m}_i} \cdot f_{i+1}^{\bar{m}_i + \bar{m}_{i+1}} \dots f_{l-1}^{\bar{m}_i + \dots + \bar{m}_{l-1}} \cdot f_l^q$, where $f_i \in \mathcal{L}_{-\alpha_i}$. Theorem. The image of ξ under the set $\{u_{q_1}^{(1)} u_{q_2}^{(2)} \dots u_{q_l}^{(l)} \mid 0 \leq q_i \leq m_i\}$ is a basis of the space \mathcal{S} of those elements of \mathfrak{M}_Λ that are annihilated by \mathcal{L}_{α_i} , $1 \leq i \leq l - 1$. As a corollary we have the "Branching Law" due to H. Weyl: The space \mathcal{S} (which is clearly an \mathcal{L} -module) is the sum of $(m_1 + 1)(m_2 + 1) \dots (m_l + 1)$ distinct \mathcal{L} -weightspaces, each one-dimensional, for weights of the form $\Lambda - q_1 \beta_1 - q_2 \beta_2 - \dots - q_l \beta_l$, $0 \leq q_i \leq m_i$, where $\beta_i = \alpha_i + \dots + \alpha_l$. (Received July 5, 1967.)

648-173. A. R. ELCRAT, General Electric Corporation, Space Sciences Laboratory, King of Prussia, Pennsylvania. Existence theorems for a nonlinear partial differential equation of viscous incompressible flow.

Existence of a solution is shown for a boundary value problem for a nonlinear partial differential equation for the velocity in time dependent flow of a viscous, incompressible fluid in an elastic tube which is intended to represent blood flow in large vessels (Lieberstein, *Acta Biotheoretica* 17 (1965), 50-94). Since the only boundary condition imposed is that the fluid adheres to the wall of the tube, there may be many solutions; however, a theorem has been proven stating that any two regular solutions are time asymptotic in L^2 norm (Elcrat and Lieberstein, General Electric TIS R67SD9, accepted *J. of Math. Biosci.*). The problem is set so that the differential operator in the equation maps a Banach space X into a Banach space Y and is twice continuously Frechét differentiable. Solutions are obtained by applying the Kantorovich-Newton theorem or a generalization of it by Altman (*Bull.*

Acad. Polon. Sci. 4 (1955), 189-193). These solutions are defined by convergent sequences whose elements are solutions of boundary value problems for linear equations of parabolic type. The solutions of these linear problems and their derivatives through second order are uniformly Hölder continuous. Their existence is guaranteed by theorems of Friedman (Macmillan, 1964). (Received July 6, 1967.)

648-174. A. K. SNYDER, Lehigh University, Bethlehem, Pennsylvania 18015. Regular matrix summability and the Čech compactification of the rationals.

Call a point x in a topological space heavy if there exists a probability Borel measure μ and a sequence $\{U_n\}$ of open neighborhoods of x such that $\mu(x) = 0$ and for each open neighborhood U of x , $\mu(U_n \cap U)/\mu(U_n) \rightarrow 1$. In countable spaces this definition corresponds to that of Henriksen and Isbell, Averages of continuous functions on countable spaces, Bull. Amer. Math. Soc. 70 (1964), 287-290. They state that for X countable, $C^*(X)$ is summable by a regular matrix if and only if X contains a heavy point. Let Q denote the rationals, βQ the Čech compactification of Q . Theorem 1. $\beta Q \sim Q$ contains no heavy points. Theorem 2. There exists a countable subset $X = \{x_n\}$ of $\beta Q \sim Q$ such that $\{f(x_n)\}: f \in C^*(\beta Q \sim Q)$ is summable by a regular matrix. Rudin has shown that a result similar to Theorem 2 fails for the Čech compactification of the integers. By Theorems 1 and 2 the natural extension of the above result of Henriksen and Isbell to the setting of certain subalgebras of $C^*(X)$ for X countable and maximal ideal space is false. (Received July 6, 1967.)

648-175. S. J. FARLOW, National Institutes of Health, Building 12A, Room 2057, Bethesda, Maryland 20014. Periodic solutions of parabolic partial differential equations.

It is proved that: if the coefficients of a second order parabolic equation in an infinite space-time cylinder, $D \times (-\infty, \infty)$; the nonhomogeneous term; and the mixed data on the boundary of $D \times (-\infty, \infty)$, are periodic in t with period T , then there exists a unique solution in $D \times (-\infty, \infty)$ which is also periodic in t with period T . (The Dirichlet problem was solved by Šmulev.) This result is also extended to the case with nonlinear data on the boundary. In proving this nonlinear extension, the author uses a notion of a sequence $F_n = \{f_\alpha^n\}$ (of families of real-valued functions defined on a compact metric space) converging to a family $\{f_\alpha\}$ in an equiconvergent manner. The reason for introducing this device lies in the fact that if each family F_n is equicontinuous and if the sequence $\{F_n\}$ converges to $\{f_\alpha\}$ in an equiconvergent manner, then $\{f_\alpha\}$ is equicontinuous. Using this result, one can show (with the aid of the Arzela-Ascoli theorem) that a certain family of functions is compact in some space, and hence the conditions of the Schauder fixed point theorem are satisfied. Uniqueness is also obtained in the nonlinear case under certain monotone conditions. (Received July 6, 1967.)

648-176. R. K. WRIGHT, The University of Vermont, Burlington, Vermont 05401. On the nonrecurrence of the logarithmic rank jump.

Let \mathcal{F} be an r -constrained field, i.e., a logarithmic domain of rank r which contains all logarithmic monomials of rank $\leq r$ (see [Walter Strodt, Mem. Amer. Math. Soc. No. 13 (1954)] for definitions), and which is closed under multiplication, division, and differentiation. When P is a first-order differential polynomial with its coefficients in \mathcal{F} and m is a principal monomial of P , the

logarithmic rank of m may not be $\leq r$; Strodt has shown it is always $\leq r + 1$. Under certain natural quasilinearity hypotheses on P we obtain necessary and sufficient conditions that the "rank jump" occur. We show that when it occurs, if Q is given by $Q(Y) = P(m + Y)$, then any principal monomial n of Q is of logarithmic rank $\leq r + 1$, and moreover if g is a member of any $r + 1$ -constrained field containing \mathcal{F} , with g asymptotically equivalent to n , then any principal monomial of $Q(g + Y)$ is of logarithmic rank $\leq r + 1$. We indicate the function of this result in the theory developed by Strodt and his students. (Received July 6, 1967.)

648-177. P. C. MORRIS, Oklahoma State University, Stillwater, Oklahoma 74074. Singular extensions and cohomology of Lie algebras.

Let $E_{\mathcal{L}}(L, M)$ denote the singular extension classes of an R -Lie algebra L by a left L -module M , where R is a commutative ring with unity. Let $E_{\mathcal{L}}(L^e, M)$ be the singular extension classes of the enveloping algebra L^e by $M \in {}_L e \mathfrak{M}_e$, the category of L^e -bimodules with right operation defined by the augmentation $\epsilon: L^e \rightarrow R$. Let $E_{\mathcal{L}}^s(L, M)$ and $E_{\mathcal{L}}^s(L^e, M)$ be the corresponding R -split classes. Denote by \mathcal{E}_1 (respectively \mathcal{E}_0) the class of all exact (respectively R -split exact) sequences in ${}_L e \mathfrak{M}_e$. The relations $\text{Ext}_{\mathcal{E}_0}^2(R, M) \xrightarrow{\cong} E_{\mathcal{L}}^s(L^e, M) \xrightarrow{\subset} E_{\mathcal{L}}^s(L, M) \xrightarrow{\cong} H^2(V(L), M) \xrightarrow{\subset} E_{\mathcal{L}}(L^e, M) \xrightarrow{\cong} \text{Ext}_{\mathcal{E}_1}^2(R, M) \xrightarrow{\subset} E_{\mathcal{L}}(L, M)$ hold if $H_2(V(L)) = 0$, where the L^e -complex $V(L)$ is derived as usual from the exterior algebra of L . In general, the diagram holds with the omission of ν , replacing μ by an injection. A simple example with $H_2(V(L)) = 0$ shows that λ and ν are not bijections. (Received July 6, 1967.)

648-178. W. D. NATHAN, Syracuse University, Syracuse, New York 13210. Open mappings on manifolds. Preliminary report.

Let M^n, N^p be manifolds of dimension n and p , $n \geq p$, and let $f: M^n \rightarrow N^p$ be proper and open. The branch set, $B(f)$, is the subset of M^n defined by $p \in M^n - B(f)$ iff there is a neighborhood U of p on which f is topologically equivalent to the projection of E^n onto E^p . Theorem. Let $n = 2$, and let N^p denote the real line or S^1 . If $\dim B(f) = 0$, then the set of points p in M^2 at which $f^{-1}(f(p))$ is not a 1-manifold is discrete; if $B(f)$ is discrete, then f is locally topologically equivalent to the map $z \rightarrow \text{Re}(z^d)$ of the plane into the line, $d = 1, 2, \dots$. If $p = 1$, define the singular set, $S(f)$, as follows: let $y \in N^1$, and let C be a component of $f^{-1}(y)$. $C \cap S(f) = \emptyset$ if there is an interval I about y such that for any subinterval J about y , and any point $x \in J$, $i_*: H_*(f^{-1}(x) \cap U; \mathbb{Z}_2) \rightarrow H_*(U; \mathbb{Z}_2)$ is an isomorphism, where U is the component of $f^{-1}(J)$ containing C , and H is Cech homology. Otherwise, $C \subset S(f)$. Theorem. Let f be differentiable. If $f(S(f))$ is discrete, $N^1 = S^1$, and n is even, then $\mathfrak{X}(M^n) = \sum \mathfrak{X}(f^{-1}(y))$, where the summation is taken over all points y in $f(S(f))$; if $n = 2$, then $f(S(f))$ is discrete. (Received July 6, 1967.)

648-179. T. M. JENKINS, Oakland University, Rochester, Michigan 48063. Lipschitz functions on abstract metric spaces.

For every positive number $a \leq 1$ and for every bounded complex function f defined on a metric space (X, d) , define $\|f\|_a = \sup \{ |f(x) - f(y)| / d^a(x, y) : x, y \in X, x \neq y \}$. Let $\text{Lip}(X, d^a)$ denote the Banach space of functions f with $\|f\| = \max(\|f\|_{\infty}, \|f\|_a) < \infty$. Let $\text{lip}^0(X, d^a) = \{ f \in \text{Lip}(X, d^a) : f \text{ vanishes at } \infty \}$

and $|f(x) - f(y)|/d^{\alpha}(x,y) \rightarrow 0$ as $d^{\alpha}(x,y) \rightarrow 0$ }. Definition. A metric space (X,d) has the complex Lipschitz extension (CLE) property if, for every subset $Y \subset X$ and every $f \in \text{Lip}(Y,d)$, f has an extension $f^* \in \text{Lip}(X,d)$ with $\|f^*\|_1 = \|f\|_1$. Theorem 1. Let (X,d) be a metric space in which every closed bounded set is compact and such that every subset containing exactly four points has the CLE property. If $0 < \alpha < 1$, then $\text{Lip}(X,d^{\alpha})$ is isometrically isomorphic to the second dual space of $\text{lip}^0(X,d^{\alpha})$. A critical step in the proof of Theorem 1 is the following. Lemma. If every four-point subset of a separable metric space (X,d) has the CLE property, then so does (X,d) itself. For the space $R_1 = \text{reals modulo } 1$ with the usual metric, Theorem 1 was proved by K. de Leeuw [Banach spaces of Lipschitz functions, *Studia Math.* 21 (1961), 55-66]. (Received July 6, 1967.)

648-180. J. A. R. HOLBROOK, University of California, San Diego, P. O. Box 103, La Jolla, California 92037. Inequalities involving the numerical radius of an operator.

If T is an operator on a complex Hilbert space H , the numerical radius $w(T) = \sup \{ |(Th, h)| : h \in H \text{ and } \|h\| = 1 \}$. Halmos conjectured and Berger proved that $w(T^n) \leq (w(T))^n$, $n = 1, 2, 3, \dots$. Percy and others have since given simplified proofs of this "power inequality". Although it seems difficult to derive this inequality from any more general result concerning several operators (for example, the best one can say concerning commuting operators A and B is that $w(AB) \leq 2 w(A)w(B)$) one does have the following related inequality. Theorem. If A and B are double commuting operators on H (i.e., $AB = BA$ and $AB^* = B^*A$), then $w(AB) \leq w(A) \|B\|$. A number of proofs of this result have been found. By constructing appropriate operators on $H \oplus H$ one may assume that one of the operators is unitary, handling this case via the spectral theorem. Another approach has the interesting feature that it yields a new proof of the "power inequality" as a by-product. (Received July 6, 1967.)

648-181. F. M. C. CARROLL, Annursnac Hill Road, Concord, Massachusetts 01742. On the computation of a schedule.

If a finite set of events $\{E_m\}$ ($m = 1, M$) of a task, K , is to be performed in a finite time, T , with the expenditure of a finite set of resources $\{C_n\}$ ($n = 1, N$) it is possible to compute a schedule of $\{E_m\}$ through a linear mapping of subsets of the associated $\{E_m, C_n\}$ onto continuous subsets of T , say $\{\mathcal{T}(E_m, C_n)\}$, under the following conditions: (i) $RT_{mn} = T - \mathcal{T}(E_m, C_n)$ is technically equivalent to "time remaining" for every technically appropriate pair of subscripts m, n , and is computed according to the well-known "algebra of successors" for sets satisfying Peano's Postulates; (ii) the well-ordering postulate on subsets of $\{RT_{mn}\}$; (iii) $RT_{pi} + RT_{qj} > 0$ for all technically appropriate pairs p_i, q_j . In fact, $RT_{pi} + RT_{qi} \leq 0$ represents a schedule conflict for the events E_p, E_q using resources C_i and C_j . (Received July 6, 1967.)

648-182. M. K. BENNETT, University of Massachusetts, Amherst, Massachusetts 01002. A lattice characterization of convexity.

The lattice of convex subsets of a vector space over an ordered division ring can be characterized lattice theoretically by methods used in Abstract 642-7, these *Notices* (1967), 62. The imposition of two further conditions on the lattice forces the ring to be the real field. We call such

lattice a convexity lattice. A set is affine iff it forms a modular pair with every set in the convexity lattice. Theorem. Let L be a convexity lattice. Let $\bar{L} = \{A: A \in L \text{ and } \forall \{x_a\}_{a \in I} \text{ such that } \exists c, d \in L \text{ with } x_a \wedge (c \vee d) \wedge A = x_a \text{ then } \exists a, b \leq A \text{ such that } x_a \wedge (a \vee b) = x_a\}$, where lower case letters represent atoms of L . Then \bar{L} is a meet-sublattice of L and is the lattice of topologically closed convex subsets of a vector space over \mathbb{R} . (Received July 6, 1967.)

648-183. N. A. DERZKO, University of Toronto, Toronto, Canada. A solution of Baxter's equation in the algebra of complex matrices.

Let \mathcal{M} denote the algebra of complex $n \times n$ matrices and \mathcal{M}_0 be the subspace of matrices with zero diagonal. Suppose $A = \text{diag}(w_1, \dots, w_n)$, where the w_k are all different. Denote by D the inner derivation mapping $\mathcal{M} \rightarrow \mathcal{M}_0$ defined by $DX = AX - XA$. Let the linear mapping $\Gamma: \mathcal{M}_0 \rightarrow \mathcal{M}$ be one of the right inverses of D . The inverse relation to the derivation law is known as Baxter's equation and is $(\Gamma X_1)(\Gamma X_2) = \Gamma\{X_1(\Gamma X_2) + (\Gamma X_1)X_2\}$. Not every right inverse Γ , of course, will satisfy this equation and our theorem characterizes those which do. Preliminary to stating the theorem we introduce some notation. $X = \{x_{ij}\}$ denotes an $n \times n$ matrix with entries x_{ij} , and X has entries $x_{ij}/(w_i - w_j)$. The diagonal part of a matrix $Y = \{y_{ij}\}$ is denoted by $\text{dp}(Y) = \text{diag}(y_{11}, \dots, y_{nn})$. Theorem. Γ is a right inverse of D satisfying Baxter's equation if and only if Γ has one of the following two forms: (a) $\Gamma X = \Gamma_0 X + \text{dp}\{(\Gamma_0 X)G\}$ or (b) $\Gamma X = \Gamma_0 X + \text{dp}\{G(\Gamma_0 X)\}$, where $G = \{g_{ij}\}$ is constructed as follows. Let g_2, \dots, g_n be nonzero complex numbers. Then $g_{ii} = 0, i = 1, \dots, n; g_{ij} = (-1)^{j-i-1} g_{i+1} \dots g_j$, and $g_{ji} = 1/g_{ij}$ for $i < j$. This theorem has a straightforward generalization in the case of any matrix A with n different eigenvalues, based upon the fact that such a matrix is similar to a diagonal one of the type we have assumed. (Received July 6, 1967.)

648-184. H. L. TURRITTIN, 4046 Beard Avenue South, Minneapolis, Minnesota 55410. Convergent solutions of ordinary linear nonhomogeneous differential equations.

If the equation $dX/ds = s^{h-2} [\sum_{k=0}^{\infty} s^{-k} A_k X + \sum_{k=0}^{\infty} s^{-k} B_k]$ possesses a unique formal solution $X = \sum_{k=0}^{\infty} s^{-k} C_k$, when the integer $h \geq 2$ and the two indicated summations converge for sufficiently large $|s|$, where the A_k 's and B_k 's are given n by n square constant matrices, then the indicated formal solution will usually diverge. The author proves this formal solution is an asymptotic representation of a true solution in appropriate sectors of the s -plane as $s \rightarrow \infty$. Moreover these asymptotic series solutions are summed and the true solutions are then represented instead by convergent generalized factorial series. The method closely parallels his work in Acta Math. 93 (1955), 27-66. Indeed the present result shows the restrictive hypothesis in the Acta paper is superfluous and hence all formal solutions of the corresponding homogeneous equation can likewise be summed and replaced by convergent series. (Received July 6, 1967.)

648-185. W. A. BEYER, Los Alamos Scientific Laboratory, Los Alamos, New Mexico. Cantor-type interval dissections as random number tests.

Let μ be a computable probability measure on the Borel subsets of $[0, 1]$. A set $\mathcal{M} \subset [0, 1]$ is called μ -null computable if: (i) $\mu(\mathcal{M}) = 0$; (ii) $\mathcal{M} = \bigcap_{i=1}^{\infty} M_i$ where M_i is the union of a finite or countable number of binary subintervals of $[0, 1]$ with endpoints ρ_j^i ($j = 1, 2, \dots$) and $M_{i+1} \subset M_i$;

(iii) ρ_j^i (all i, j) is recursively enumerable. The construction of \mathcal{M} is called a random number test. The set $\mathcal{A} = [0, 1] - \bigcup_{j=1}^{\infty} \mathcal{M}^j$ is called the μ -random set where the \mathcal{M}^j are the enumerable number of μ -null computable subsets of $[0, 1]$. Each $x \in \mathcal{A}$ has a binary expansion, the sequence of binary digits forming a "collective", a concept fundamental to the von Mises frequency theory of probability. This definition of a "collective" is equivalent to one recently given by Martin-Löf (*Information and Control* 9 (1966), 602-619). The dissections of the unit interval (see Zygmund, *Trigonometric series*, I, p. 194) which yield computable Cantor-type perfect sets \mathcal{M} are examples of random number tests provided $\mu(\mathcal{M}) = 0$. Since $\mu(\mathcal{M}) = 0$, Hausdorff dimension theory is relevant to the study of random number tests. If μ is the probability measure obtained from a Bernoulli process with success probability p ($1 > p \geq 1/2$), p computable, then $\dim_{\mu} \mathcal{A} \geq \log 2 / \log(1 - p)$ and is conjectured to be 1. (Received July 6, 1967.)

648-186. A. C. SUGAR, P. O. Box 4266, North Hollywood, California 91607. Investigations of a class of gravitational theories.

A mathematical or physical theory is like a mechanic's wrench; it is a tool for dealing with the real world, not a mirror reflecting it. However, this does not imply that the mirror should be discarded. Much of modern physics is highly counterintuitive and in this sense has annihilated the mirror. In 1963, the writer proposed a modification of Newton's inverse square law in the form of $F = c_1 u^2 + c_2 u^4$. However, when c_2 is given a value such that we obtain an advance of the perihelion compatible with observation then c_2 is too large to fit the potential theory. The underlying identity of the 1963 result generates a nondenumerable number of gravitational theories. By selecting that one of these theories which best fits the astronomical and molecular facts it is herein predicted that a predictive nonrelativistic theory will emerge. Currently under consideration is the following expression for gravitational force: $F = c_1 u^2 + c_2 u^4 - c_3 u^{4.1}$. The minus sign is significant since it would account for the molecular phenomena of a sphere of action and molecular repulsion. Thus this theory of gravitation reaches down from the enormous magnitudes and infinite extent of the macroscopic universe to the infinitesimal magnitudes of the microscopic world. Furthermore, this writer predicts that within a decade we will see not only the collapse of general relativity but also special relativity and Maxwell's electromagnetism and finally quantum mechanics. (Received July 6, 1967.)

648-187. DAVID KAY, University of Oklahoma, Norman, Oklahoma 73069, and MERLE GUAY, University of New Hampshire, Durham, New Hampshire. Convexity and a certain property P_m .

A natural generalization of Valentine's property P_3 [*Pacific J. Math.* 7 (1957), 1227-1235] is the following property P_m relative to a set S in a topological linear space: Each m -tuple of points in S possesses at least one pair whose join lies in S . We say further that S is (m, n) -convex if instead of one pair, at least n pairs of the possible $C_{m,2}$ pairs have their joins in S . These properties are investigated as they pertain to sets which may be expressed as the union of a finite number of convex sets. Theorem 1. A closed (m, n) -convex set with $n > (1/4)(m - 1)^2$ is either convex, or consists of a closed convex set and k isolated points where $k \leq m - (1/2)(3 + (2m^2 - 4m + 3)^{1/2})$, the bound on k being best possible. Theorem 2. Every connected $(m, 1)$ -convex set in a finite-dimensional linear space is an L_{2m-3} set; a closed, connected $(m, 1)$ -convex set is an L_{m-1} set (see Horn and Valentine,

Some properties of L sets in the plane, Duke Math. J. 16 (1949), 131-140). Theorem 3. The topological suspension of a closed, starlike $(m,1)$ -convex set S over two points v_1 and v_2 such that the join of v_1 and v_2 meets the convex kernel of S is $(m,1)$ -convex. A graph-theoretic approach is possible which leads to an "easy" proof of Valentine's theorem. (Received July 7, 1967.)

648-188. H. H. WICKE and J. M. WORRELL, JR. Sandia Corporation, Sandia Base, Albuquerque, New Mexico. On a class of spaces containing Arhangel'skii's p-spaces.

A theory is developed for a class of topological spaces, called μ -spaces, which contains Arhangel'skii's p-spaces [Mat. Sbornik 69 (111) (1966)] and the completely regular spaces having bases of countable order. The treatment is analogous to that given by the authors for spaces having bases of countable order. In many respects the μ -spaces are to the p-spaces as essentially T_1 -spaces having bases of countable order are to the developable spaces. In this connection, Arhangel'skii's theorem characterizing the metrizable spaces as the T_2 -paracompact spaces having bases of countable order may be compared instructively with the following result: The T_2 -paracompact p-spaces are the T_2 -paracompact μ -spaces. (Received July 7, 1967.)

648-189. A. M. GARSIA, University of California, San Diego, La Jolla, California 92037. An interpolation method for positive operators.

A real variable method for obtaining interpolation inequalities will be presented. The method is akin to that of Marcinkiewicz, but it is stronger in the sense that it yields the M. Riesz interpolation theorem. The basic idea consists in comparing the given function with a power of itself rather than with a constant as it is done in the Marcinkiewicz case. Essentially, the given function is truncated when it crosses a suitable curve rather than when it crosses a straight line. The method yields the right constants but seems to be restricted to the case of positive operators. (Received July 7, 1967.)

648-190. R. F. MILLAR, National Research Council of Canada, Radio and Electronics Engineering Division, Building M50, Montreal Road, Ottawa, Ontario, Canada. On the Rayleigh assumption in scattering by a periodic surface. Preliminary report.

A two-dimensional, time-harmonic, scalar plane wave falls from $y > 0$ upon a periodic, analytic reflecting surface $S: y = b \cos \kappa x$. The total field vanishes on S . In $y > b$, the scattered field is represented by a series of outgoing and evanescent plane waves. Lord Rayleigh (The theory of sound, Dover, New York, 1945, Section 272a; Proc. Roy. Soc. Ser. A 79 (1907), 399-416) assumed the validity of this representation in $y \geq -b$. Petit and Cadilhac (C. R. Acad. Sci. Paris Sér. A-B 262 (1966), 468-471) have demonstrated its invalidity when $\kappa b > 0.448$. In the present paper the validity of the Rayleigh assumption is established when (i) $\kappa b < 0.448$. It is shown that the discrete plane wave representation may be continued analytically into $y \leq b$ as far as the singularities of the solution; these lie in $y < -b$ when (i) obtains. Conformal mapping of S on a closed cylinder C , and results relating to the analytic continuation of solutions of elliptic partial differential equations, reduce the problem to one in potential theory; the field singularities are in $y < -b$ when those of a harmonic function Φ are in a circle C' within C . The Fourier series for Φ converges everywhere outside C' when (i) obtains, and the desired conclusion follows. (Received July 7, 1967.)

648-191. E. A. MICHAEL, University of Washington, Seattle, Washington. On the product of k-spaces.

By an example of C. H. Dowker [Amer. J. Math. 74 (1952), 555-577], the product of two k-spaces need not be a k-space. D. E. Cohen, on the other hand, proved in [Quart. J. Math. 5 (1956), 77-80] that the product of a locally compact Hausdorff space and a k-space is always a k-space. It is now shown that Cohen's result is the best possible, in the sense that if a Hausdorff space is not locally compact, then its product with some k-space fails to be a k-space. (Received July 7, 1967.)

648-192. C. R. CASSITY, Environmental Research Corporation, P. I. Box 264, Alexandria, Virginia 22313. Hybrid formulas with variable time step.

The formulas developed by Butcher (J. Assoc. Comp. Mach. 14 (1967), 84-99) are generalized to include variable time steps. (Received July 7, 1967.)

648-193. K. M. GARG, University of Alberta, Edmonton, Alberta, Canada. On continuous singular functions.

Following two results are proved in this paper for continuous singular functions that are strictly increasing: Theorem 1. If a continuous singular function f is strictly increasing, then (i) f has a derivative $+\infty$ at a set of points which has the power c in every interval and which is mapped by f into a set covering almost all the values assumed by f ; (ii) there exists a residual set of points where $D^+f = D^-f = +\infty$, $D_+f = D_-f = 0$; (iii) each of the four derivatives of f assumes any prescribed real value ≥ 0 at a set of points which has the power c in every interval and (iv) for every real number $r > 0$ the sets $\{x; D^+f \leq r \leq D_-f\}$ and $\{x; D^-f \leq r \leq D_+f\}$ are both everywhere dense. Theorem 2. If a continuous singular function f is strictly increasing, then to every real number $m > 0$ there corresponds a residual set of real numbers c for which the line $y = mx + c$ intersects the curve $y = f(x)$ in a nondense perfect set (possibly void). Similar results are established for continuous singular functions that are nowhere monotone, and, in general, for those which have no lines of invariability. (Received July 7, 1967.)

648-194. TAKAYUKI TAMURA, University of California, Davis, California, and JOHN SHAFER, 15 Webster Court, Amherst, Massachusetts 01002. Power semigroups.

If A and B are nonempty subsets of a semigroup S , the product of A and B is defined: $AB = \{ab; a \in A, b \in B\}$. It can be shown that under this operation the nonempty subsets of S form a semigroup which is called the power semigroup of S , $\mathcal{C}(S)$. Theorem 1. $\mathcal{C}(S)$ is a group iff S has one element. Theorem 2. If G_1 and G_2 are finite groups and their power semigroups are isomorphic, then G_1 and G_2 are isomorphic. The proofs of the above theorems follow from two lemmas: Lemma 1. If S has more than one element and is cancellative, then $\mathcal{C}(S)$ has a proper ideal. Lemma 2. If G is a finite group, then $\mathcal{C}(S)$ has a unique maximal subgroup which is isomorphic to G . Theorem 3. $\mathcal{C}(S)$ is a band iff every nonempty subset of S is a semigroup. Theorem 4. $\mathcal{C}(S)$ is a lattice iff S is a chain. Theorem 5. If $\mathcal{C}(S_1)$ and $\mathcal{C}(S_2)$ are finite lattices with the same cardinality, then $\mathcal{C}(S_1)$ and $\mathcal{C}(S_2)$ are isomorphic and so are S_1 and S_2 . (Received July 7, 1967.)

648-195. H. H. SNYDER, Southern Illinois University, Carbondale, Illinois 62901. Regular functions on a Banach algebra associated with Laplace's equation.

Let \mathfrak{A} be the set of elements $a = \sum_{-\infty}^{\infty} x_n e_n$, where x_n is real and the basis elements e_n of \mathfrak{A} satisfy a product law of the form $e_n e_k = (1/2)(\pm e_{\pm|n-k|} \pm e_{\pm(n+k)})$, the signs being determined by the signs of n and k . Define the norm $\|a\| = \sum_{-\infty}^{\infty} |x_n|$. Then the subset \mathfrak{F} of elements of \mathfrak{A} having finite norm is a Banach algebra over the real field. We study functions $f(\zeta)$, $f: \mathfrak{F} \rightarrow \mathfrak{F}$, where $\zeta = ze_{-1} + xe_0 + ye_1$ is an element of the 3-space \mathfrak{F} of \mathfrak{F} spanned by e_{-1}, e_0, e_1 . Call $f(\zeta)$ monogenic on a domain $\mathfrak{D} \subset \mathfrak{F}$ if it satisfies a differentiability condition and the series for $\|f(\zeta)\|$ converges almost uniformly on \mathfrak{D} . Theorem I. $f(\zeta)$ monogenic on $\mathfrak{D} \Rightarrow f(\zeta)$ is analytic on \mathfrak{D} . Theorem II. If Σ is a sufficiently smooth closed surface containing the constant $a \in \mathfrak{F}$, then $\iint_{\Sigma} f(\zeta)(\zeta - a)^{-1} d\sigma = Kf(a)$, where $K \in \mathfrak{F}$ is a constant depending on the trace C of Σ in the plane $x = \text{Re}a$. A detailed exposition of these results will appear in a forthcoming number of the Mathematische Forschungsberichte monograph series. (Received July 7, 1967.)

648-196. R. A. GUY, Queen's University, Kingston, Ontario, Canada. On certain maps of modules which are direct sums of sub modules.

We study and generalize the algebraic aspect of certain situations which appear in analysis and in differential geometry. Let E^p and F^p be two modules over a commutative ring A , each of which is a direct sum of p sub modules $E_{p\alpha}, F_{p\beta}$ ($\alpha, \beta = 1, \dots, p$). We define different classes of maps from $E_{p\lambda}^p$ to $F_{p\lambda}^p$. We let $u(x) = \sum_{\lambda=1}^p u^{\lambda}(x)$, where $x \in E_1^p$, $x = \sum_{\alpha=1}^p x_1^{\alpha}$, $u(x) = \sum_{\alpha=1}^p A_{p\alpha}^{\lambda}(x^{\alpha})$. The maps $A_{p\alpha}^{\lambda}$ can be members of different sets, a general case is the set $E_{p_1} \times \dots \times E_{p_{\lambda-p}}(E_{p\alpha}^{F_{p\lambda}})$ of arbitrary maps from $E_{p\alpha}$ to $F_{p\lambda}$, which depend parametrically on variables $x_1^{p_1}, \dots, x_{p\lambda-p}^{\lambda-p}$. It is then possible to introduce the matrices $(A_{p\alpha}^{\lambda})$, $(x^{p_1}, \dots, x^{p_p})$, $(u^{p_1}, \dots, u^{p_p})$, but in this general case the matrix calculus is impossible. It is possible however if we specialize this class of the maps $A_{p\alpha}^{\lambda}$. We do that in several ways. We show then, that the group of such invertible matrices is a direct product of p subgroups. It is also possible to introduce in certain splitting exact sequences classes of sections which generalize linear sections. (Received July 7, 1967.)

648-197. GASTON SMITH, William Carey College, Hattiesburg, Mississippi. Matrix transformations which preserve regularity.

For each nonnegative integer r let f_r denote a complex-valued function which is analytic on N , where N is a neighborhood of the origin in the complex plane with radius $R > 1$. Let (a_{rk}) be a non-negative matrix such that $\sum_{k=0}^{\infty} a_{rk}$ converges for each r and let $\sum_{k=0}^{\infty} e_{rk}(n)z^k$ denote the Taylor expansion of $[f_r(z)]^n$ about the origin. Theorem. For each r assume that the Taylor expansion of f_r about the origin has nonnegative coefficients, that $|f_r(z)| \leq 1$ on the disk $|z| \leq 1$, and that $f_r(1) = 1$. If $A = (a_{rk})$ is regular and if $k \leq s \leq t$ implies $e_{sk}(n) \geq e_{tk}(n)$ for each fixed k then $C = (c_{rk})$ is regular, where $c_{rk} = \sum_{n=0}^{\infty} b_{rk}(n)$ and $a_{rn}[f_r(z)]^n = \sum_{k=0}^{\infty} b_{rk}(n)z^k$. (Received July 7, 1967.)

648-198. E. C. POSNER, E. R. RODEMICH and HOWARD RUMSEY, JR., Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91103. Epsilon entropy of the normal distribution.

Consider normal distribution on the real line of mean 0 and variance 1. We wish to partition the line into consecutive intervals of length ϵ , $\epsilon > 0$, so as to minimize the entropy of the resulting partition; the entropy H of a partition into sets of probability p_i is the sum $H = \sum_i p_i \log(1/p_i)$. It is shown that for every positive ϵ , the minimizing partition is the one in which 0 is the midpoint of some interval of the partition. The proof proceeds in several stages. First, the case $\epsilon \geq 3$ is done by manipulation of various inequalities. The more difficult case $\epsilon \leq 3$ is done by considering the Fourier series of the periodic function which gives the entropy as a function of the midpoint of any interval in the partition. Various function-theoretic tools are needed in this case. The result of this abstract on minimum entropy is needed in the theory of product epsilon entropy of mean-continuous Gaussian processes on the unit interval. (Received July 7, 1967.)

648-199. D. R. COLNER and C. T. TAAM, Georgetown University, Washington, D. C. 20007. On the solutions of a differential equation in a Banach space. Preliminary report.

The equation $dx/dt = Ax + f(x,t)$ is considered in a Banach space with an unbounded closed linear operator A having a dense domain and generating an exponentially damped semigroup of class C_0 . Both $f(x,t)$ and $Df(x,t)$, its Frechet differential, satisfy a Lipschitz condition in x uniformly in t and are small in norm at $x=0$ for all t . When f is periodic in t , the existence of a solution of the same period is demonstrated and the uniqueness and stability of this solution are discussed. The existence is proven by applying Banach's fixed point theorem to a related integral equation in a Banach space of continuously differentiable, periodic functions and showing that the differentiability of the fixed point implies that its range lies in the domain of A . Analogous results are shown to hold in the almost periodic, bounded and compact cases. (This research was supported in part by N.S.F. grant 6707 and by U. S. Army Research Office-Durham, under contract no.: DA-31-124-ARO-D-271.) (Received July 7, 1967.)

648-200. B. J. TRAWINSKI, University of Alabama, Birmingham, Alabama 35233. Convergence of sequences of graphs with variable support. Preliminary report.

Let N stand for the set of natural numbers and let G_n^C ($n \in N$) be the graph of a function $G_n(x,c)$ with support X_{cn} ($c \geq 2$), consisting of the first $cn + 1$ nonnegative integers. Let p be a prime, π a product of primes and $\{\pi(p,k)(k \geq 0)\} = \{\pi p^k (k \geq 0)\}$ a partition of N . It is shown that for a certain type of sequence $\Lambda_G^C = \{G_n^C\}_{n \in N}$ there is a countable normalized support Y^C on which Λ_G^C has a limit, and that the structure of Y^C depends on the partition of N and Euler's function. (Received July 7, 1967.)

648-201. M. A. H. DEMPSTER, Balliol College, University of Oxford, Oxford, England. Convex cones and pseudo-inverse in Hilbert space. Preliminary report.

Denote by $R'_+(X)$ the closed convex cone generated by a set X in a real Hilbert space H . Let B be an orthonormal basis and suppose that H is such that $R'_+(B)$ has a nonempty interior. Theorem 1.

Let $\{M, N\}$ be a complementary pair of (closed) subspaces in H . The following situations are mutually exclusive: (i) $N \cap R_+(B) = \{0\}$, when M has a basis in $\text{int } R_+(B)$, (ii) $N \cap R_+(B) = R_+(N \cap B)$, when M has a basis in $\text{int } R_+(B \setminus M \cap B)$, (iii) N has a basis in $\text{int } R_+(G)$, where G is a proper subset of B , minimal with respect to this property, when $M \cap R_+(B) = R_+(M \cap B)$, (iv) $N \cap \text{int } R_+(B) \neq \emptyset$, when $M \cap R_+(B) = \{0\}$. This result extends finite-dimensional results of A. Ben-Israel (J. Math. Anal. Appl. 9 (1964), 303-314). If A is a bounded linear operator on H with closed range in the Hilbert space K , then the pseudo-inverse A^+ of A is a unique bounded linear operator on K to H (see e.g. F. J. Beutler, J. Math. Anal. Appl. 10 (1965), 451-493). Using Theorem 1 and the mapping properties of A^+ , the relations between the structure of A and the lineality space of the cone $R_+(A[B])$ are characterized Theorem 2. $\text{cl}\{y : y = Ax, x \in R_+(B) \in R(A^*)\} = \{y : A^+y \in R_+(B), y \in R(A)\} \subset K$. Corollary 3. If the projection A^+A leaves $R_+(B)$ invariant, then $R_+(A[B]) = \{y : A^+y \in R_+(B), y \in R(A)\}$. Similar results for more general operators A are indicated. Results for more general spaces will be treated in a future paper. (Received July 7, 1967.)

648-202. JOHN HAYS, 460 West 24 Street, New York, New York 10011. Introduction to hyper-boolean systems. II: Modules of N-tuples; hyperboolean determinants and matrices.

From the model of algebra of boolean differences, we construct rings and modules of ordered boolean pairs. This we now extend to n -tuples and form the module $\mathcal{M}_n(B)$ by combining j - to k -tuples ($j, k \leq n$), with intersection defined cyclically: $\bigcap \langle A_1, \dots, A_n \rangle \langle B_1, \dots, B_n \rangle = \langle \bigcup \bigcap A_1 B_1 \cap \dots \cap A_n B_n, \bigcup \bigcap A_1 B_2 \cap \dots \cap A_n B_1, \bigcup \bigcap A_1 B_n \cap \dots \cap A_n B_{n-1} \rangle$. Let \mathcal{G} be the ground set (ring or field - here a ring), \mathcal{V} the vector set. Define for $\mathcal{M}(B)$ inner product $i: \mathcal{V} \times \mathcal{V} \rightarrow \mathcal{V}$ as $i: \langle A_1, \dots, A_n \rangle \cdot \langle B_1, \dots, B_n \rangle = \bigcup \bigcap A_1 B_1 \cap A_2 B_2 \cap \dots \cap A_n B_n$. A boolean combination is $\bigcup_i \bigcap B_i V_i$, for boolean B_i , hyperboolean (vector) V_i . (Independence and basis conditions are similar to those for number n -tuples.)

Theorem 4. $\langle \mathcal{M}(B), \cdot \rangle$ is an inner (module) product space over a boolean ring. Use i to define determinants and matrices of booleans after the fashion for numbers ($\langle \cdot, +, - \rangle \rightarrow \langle \bigcap, \bigcup, \setminus \rangle$). Then we can define outer product $O: \mathcal{V} \times \mathcal{V} \rightarrow \mathcal{V}$ on $b, 3$ -vectors as $O: \langle A, B, C \rangle \otimes \langle E, F, G \rangle = \langle \begin{pmatrix} B & C \\ F & G \end{pmatrix}, \begin{pmatrix} C & A \\ G & E \end{pmatrix}, \begin{pmatrix} A & B \\ E & F \end{pmatrix} \rangle$. Theorem 5. Algebra $\langle \mathcal{M}(B), \cdot, \otimes \rangle$ is an "inner-outer" (module) product space; V_1, V_2 are totally independent (dependent) iff $V_1 \cdot V_2$ is the null boolean ($V_1 \otimes V_2$ is the null vector). Theorem 6. The set of n^2 hyperboolean matrices over a boolean ring forms a total matrix algebra with identity. (Received July 7, 1967.)

648-203. WITHDRAWN

648-204. PAUL FJELSTAD, University of Bern, Switzerland. Set theories as algebras.

An algebra with three operations $0, ', \cdot$, which operations are nullary, unary, and binary respectively, is defined to be a world just in case the following axioms hold, for elements p, q, r of the algebra $0, \cdot$ -axioms: $p(qr) = (pq)r, pq = qp, pp = p, 0p = 0$; $'$ -axioms: if $p' = q'$ then $p = q, q' \neq 0, pq' = q'$ or $pq' = 0$ for all p ; atomic axiom: if $pr = qr$ for all r which are atoms, then $p = q$, where, for the last axiom, an element r is defined to be an atom iff $r \neq 0$ and $pr = r$ or $pr = 0$ for all p . Some theorems for worlds are: no world is finite; if there is a world there are infinitely many of them; there is a world in the sense that one can be constructed by means of a word algebra; the homomorphic image of a world is a world iff the homomorphism is injective (which has as a

consequence that a world cannot be equationally defined and that there is no free world). A binary relation \in is defined for a world by $q \in p$ iff $pq' = q'$, and a world is said to be interpreted as a set-theory when its elements are interpreted as sets and \in is interpreted as the membership relation. Additional structure is easily postulated for a world by demanding that various partial operations for a world be full operations. In this way algebras can be arrived at which can be interpreted as specific set-theories, such as those of Zermelo, Fraenkel, Gödel, or Kelley, for example. (Received July 7, 1967.)

648-205. J. A. BAKER, University of Waterloo, Waterloo, Ontario, Canada. On quadratic functionals continuous along rays.

Let X be a complex vector space and let $f: X \rightarrow \mathbb{C}$, where \mathbb{C} is the field of complex numbers, be quadratic ($f(x + y) + f(x - y) = 2f(x) + 2f(y)$ for all $x, y \in X$). S. Kurepa has found (Glasnik Mat.-Fiz 20 (1965), 79-92) the general solution under the assumption $f(cx) = |c|^2 f(x)$ and suggested considering the more general hypothesis $|f(cx)| = |c|^2 |f(x)|$ ($c \in \mathbb{C}, x \in X$). Here Kurepa's problem is solved by showing that in this case one of (i) $f(cx) = c^2 f(x)$, (ii) $f(cx) = |c|^2 f(x)$, (iii) $f(cx) = \bar{c}^2 f(x)$ must hold and the general solution is given in cases, (i) and (iii). Further, the general solution is given under the more general assumption that for each fixed $x \in X$, the mapping $c \mapsto f(cx)$ is a continuous mapping of \mathbb{C} into \mathbb{C} . (Received July 3, 1967.)

Abstracts Presented by Title

67T-455. JOHN DECICCO, Illinois Institute of Technology, Chicago, Illinois and G. E. CULBERTSON, U. S. Naval Academy, Annapolis, Maryland. Some results on central and parallel fields of force in a Euclidean space of several dimensions.

In a Euclidean space E_n of dimension $n \geq 2$, a k -dimensional parallel field of force is one for which the force vector acting at any point of its region of definition determines a straight line parallel to a fixed k -flat L_k , and a $(k - 1)$ dimensional central field of force with center L_{k-1}^0 is one for which the force vector acting at any point of its region of definition determines a straight line which always intersects a given $(k - 1)$ dimensional flat L_{k-1}^0 , its center. The integer k is the smallest one such that $1 \leq k \leq n - 1$. Every dynamical trajectory of a k -dimensional parallel field of force with direction μ_k is contained in some $(k + 1)$ flat parallel to direction μ_k . Every dynamical trajectory of a $(k - 1)$ dimensional central field of force with center L_{k-1}^0 is contained in a $(k + 1)$ flat which passes through the center L_{k-1}^0 . (Received March 1, 1967.)

67T-456. P. C. C. WANG, Michigan State University, East Lansing, Michigan. On some sign reverse rule property of first passage probabilities.

Let $\{X(t), t \geq 0\}$ be a Markov process with state space E the set of integers and with transition probability function $P_t(i, j)$. In this note we postulate the total positivity of the Markov matrix associated with a given process and then deduce the sign reverse rule property of some first-passage probability distributions into a specified set of states. We also characterize the sign reverse rule of order 2 (written RR_2) matrices which is useful in the theory of reliability. A matrix $P_t(i, j)$ is said to be row sequential sign reverse (written RSSR) if $(P_t(k, \ell))$, $k = i, i + 1; \ell = \dots, 0, 1, 2, \dots$, is RR_2 for every fixed $t \geq 0$, and for all $i \in E$. Let $m_r(t) = \min \{j | P_t(r, j) > 0, j \in E\}$ $M_r(t) = \max \{j | P_t(r, j) > 0, j \in E\}$ $R_r(t) = \{0, \dots, 0, P_t(r, m_r), P_t(r, m_r + 1), \dots, P_t(r, M_r), 0, \dots\}$ $S_r(t) = (P_t(k, m))$, $k = r, r + 1, m = m_{r+1}, \dots, M_r$. We have established the following theorems. Theorem 1. For a regular Markov matrix P_t , the following conditions are equivalent (1) P_t is RR of all orders, (2) $m_r(t) \geq m_{r+1}(t)$, $M_r(t) \geq M_{r+1}(t)$, $S_r(t)$ is RSSR for each $r \in E$ and every fixed $t \geq 0$. Theorem 2. Let B and C be nonempty sets of states such that $B \subseteq C$; and define $F(n, j) = P_r\{X(m) \in B, \forall 1 \leq m \leq n - 1, X(n) = j | X(0) \in C\}$. Let P_t be totally positive of order 2 (written TP_2) of a regular Markov matrix associated with a Markov process. Then $F(n, j)$ satisfies the sign reverse rule of all orders in the variables $j(j \in E)$ and $n(n \geq 1)$. (Received March 8, 1967.)

67T-457. Y. C. WU and S. M. ROBINSON, Union College, Schenectady, New York 12308. A weaker separation axiom than T_1 implies a normal space is T_4 .

In their paper Separation axioms between T_0 and T_1 , Aull and Thron pose the question of whether any separation axiom weaker than T_1 implies a normal space is T_4 . This question has an affirmative answer. Definition. X is a strong T_0 space provided for each $x \in X$, $\{x\}'$ is a union of a family of closed sets, such that the intersection of the nonempty members is empty and at least one of the nonempty members is compact. Definition. X is a strong T_D space provided for each $x \in X$, $\{x\}'$ is a finite union of closed sets, the intersection of the nonempty members of which is empty.

Theorem. A normal space that is either strong T_0 or strong T_D is T_4 . (Received March 17, 1967.)

67T-458. C. C. CHEN, Queen's University, Kingston, Ontario, Canada. Almost uniquely complemented lattices.

An almost uniquely complemented lattice is a lattice with 0 and 1 each of whose element has at most one complement. Theorem. Every almost uniquely complemented lattice L can be embedded (with 0 and 1 preserved) into an almost uniquely complemented lattice N such that each element of L is complemented in N . The proof is based on the following simple construction which does not require the solution of any word problem other than that for free lattices generated by posets. (See [#] R. P. Dilworth, Trans. Amer. Math. Soc. 57 (1954), 123-154). Construction. Let S be the set of all noncomplemented elements of L . Associate to each x in S a new symbol x' and let S' denote the set of all such symbols. We turn $L \cup S'$ into a poset by defining $x \leq y$ in $L \cup S'$ if $x = y$ or $x \leq y$ in L . Let M be the free lattice generated by $L \cup S'$. By eliminating certain "singular elements" of M and adjoining a new 0 and 1 to M (as it was done in [#]), we obtain the required lattice N . Corollary 1. Every almost uniquely complemented lattice is a sublattice (with 0 and 1 preserved) of a lattice with unique complements. Corollary 2. Every lattice is a sublattice of a lattice with unique complements. (This was established in [#] through the concept of lattices with unary operator.) (Received March 13, 1967.)

67T-459. C. P. BRUTER, University of Waterloo, Waterloo, Ontario, Canada. Génération des matroïdes binaires et de leurs orthogonaux.

Soit M un matroïde de caractéristique 2 (ou binaire) sur un ensemble fini E dont les atomes sont tous de cardinal différent. On dit que les atomes C et C' de M sont des atomes générateurs minimaux ssi $C = (C' - C'') \cup (C'' - C')$ entraîne: (i) il n'existe pas C_1 et C_2 tels que $C = (C_1 - C_2) \cup (C_2 - C_1)$ avec $|C_1 \cup C_2| \leq |C' \cup C''|$ et à la fois $|C_1| \leq |C|$, $|C_2| \leq |C|$, (ii) C' vérifie les mêmes propriétés que C . Nous citons trois théorèmes: Théorème 1. Le matroïde binaire M de rang r est engendré par ses atomes générateurs minimaux en nombre égal à r . Théorème 2. Tout système S de r atomes tels que quelques soient les atomes C_1, C_2, C_3 appartenant à S , $C_1 \not\subset C_2 \cup C_3$, engendrent le matroïde binaire M de rang r . Théorème 3. L'orthogonal M^* du matroïde binaire M de rang r est engendré par $|E| - r$ atomes C^* tels que $|C \cap C^*| = 0$ ou 2. (Received April 4, 1967.)

67T-460. MARTIN ZUCKERMAN, Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, New York 10012. Sufficiency conditions for implication among the axioms of choice for finite sets. Preliminary report.

Let $P^\#(X) = \{\text{finite subsets of } X\}$. Let $I_k = \{\text{integers } \geq k\}$, $k \geq 0$, let $Z \in P^\#(I_1)$ and let $n \in I_1$.

If for any decomposition of n into a sum of (not necessarily distinct) primes $p_1 + \dots + p_r$, (i) some $p_i \in Z$, then Z and n satisfy Szmielew's condition (S), (ii) $kp_i \in Z$, some $k \in I_1$, some $i = 1, \dots, r$, then Z and n satisfy Mostowski's condition (S), (iii) $\exists k_1, \dots, k_r \in I_0$ s.t. $k_1 p_1 + \dots + k_r p_r \in Z$, then Z and n satisfy condition (M). Theorem. For each $n \in I_7 \cup \{5\}$, \exists infinitely many $Z \in P^\#(I_1)$ for which $[Z] \rightarrow [n]$ (see Abstract 636-3, these *Notices*) 13 (1966), 576) but condition (S), in both senses, fails. Definition. n is a Z -number provided either $n = 1$ or else (i) and (ii) hold: (i) $\exists z \in Z$ s.t. $(n, z) > 1$. (ii) Whenever $n = n_1 + n_2$, $n_1, n_2 \in I_2$, then $\exists k_1, k_2 \in I_0$, s.t. $k_1 n_1 + k_2 n_2 \in Z$. Z and n satisfy condition (Z) provided either n is a Z -number or else (i) (above) and (ii)': Whenever $n = n_1 + n_2$, $n_1, n_2 \in I_2$, then Z and n_i satisfy condition (Z), $i = 1$ or 2 . Theorem. Condition (Z) is sufficient for the implication $[Z] \rightarrow [n]$. Condition (Z) is intermediate in strength between (M) and Mostowski's (S); it provides a convenient unified method of obtaining many implications among these axioms. (Received April 4, 1967.)

67T-461. ISIDORE FLEISCHER, Fleischer Foundation, Brooklyn, New York. Generalized compactification for weak topologies.

All spaces are Hausdorff. X is a set equipped with separating maps P : a subset of a product. \bar{X} is the largest space to which P extends by continuity: P converges only on the filters converging in \bar{X} . To P may be added any map whose graph in the augmented product is closed. Morphisms are P preserving maps. $X = \bar{X}$ is preserved under products and closed subspaces. Now let every filter generated by preimages of neighborhood filters (of points) cluster in every factor. Morphisms preserve this: on closed domains they are closed maps. The maximal filters so generated are the traces of \bar{X} 's neighborhood filters. More generally, every filter belonging to a class admitting joins with preimages of neighborhood filters, whose members cluster in every factor, clusters, and if maximal, converges; \bar{X} may be obtained as their limits if the class is inductive; and in one-one fashion if and only if distinct maximal filters have disjoint closed elements in some P -image. The usual example is the class, containing the preimages of neighborhood filters, of filters generated by subsets of a certain type closed under finite intersection. Besides compact and realcompact theory there results a nonstandard treatment of uniform completion and some apparently new applications to linear compactness. (Received April 6, 1967.)

67T-462. J. K. ODDSON, Institute for Fluid Dynamics and Applied Mathematics, University of Maryland, College Park, Maryland. A Phragmen-Lindelöf theorem for a plane half strip.

Let D be an unbounded open subset of the plane half strip $y \geq 0$, $|x| \leq x_0$. Let $\mathcal{L}_a = \{L = \sum_1^2 a_{ij} D_{ij}; \sum_1^2 a_{ii} = 1; \sum_1^2 a_{ij} \xi_i \xi_j \geq a \sum_1^2 \xi_i^2; 0 < a \leq 1/2\}$. Define $\mu = \{2 \tan^{-1}(a/1 - a)^{1/2} - (1 - 2a)(4a(1 - a))^{1/2} / [4a(1 - a)x_0]\}$. Theorem. Let $u(x, y)$ be a real valued function, twice differentiable in D and continuous on $D \cup \partial D$, such that $u \geq 0$ on ∂D and, for some $L \in \mathcal{L}_a$, $Lu \leq 0$ in D . If, for some $\epsilon > 0$, $u^- = O\{\exp(\mu - \epsilon)y\}$ as $y \rightarrow +\infty$, $(x, y) \in D$, then $u \geq 0$ in \bar{D} . Here $u^-(x, y) = \max\{0, -u(x, y)\}$. A lower bound for μ is given by $\mu \geq (a/1 - a)(\pi/2x_0)$. (Received April 12, 1967.)

67T-463. NATHANIEL COBURN, University of Michigan, 347 West Engineering, Ann Arbor, Michigan 48104. Characteristic manifolds in nonequilibrium hydrodynamics.

Two topics are considered in paper: (1) the determination of the characteristic manifolds in Newtonian nonequilibrium hydrodynamics of a nonheat conducting perfect fluid by use of the concept of (C^n, C^m) functions due to A. Lichnerowicz (Theories relativistes de la gravitation et de l'electro-

magnetisme, Masson et Cie, Paris, 1955); (2) the correction of an erroneous statement in a previous paper (N. Coburn, General theory of simple waves in relaxation hydrodynamics, J. Math. Anal. Appl. 11 (1965), 102-130). By differentiating the basic equations of relaxation hydrodynamics, a quasi-linear system of third order partial differential equations is obtained. It is assumed that the density, entropy, relaxation variable, velocity vector are (C^1, C^3) ; the relaxation scalar, K , the rate of relaxation, q^* , and all other rate variables are (C^0, C^2) . Using standard techniques, the discontinuity relations can be reduced to a system of four linear homogeneous algebraic equations in five unknowns. By assuming that q^*/K is of class (C^1, C^2) , an additional relation is obtained. The condition that the determinant shall vanish leads to the same characteristic relation as was obtained previously (N. Coburn, loc. cit). (Received April 24, 1967.)

67T-464. ARUNAVA MUKHERJEA, Eastern Michigan University, Ypsilanti, Michigan 48197. A necessary and sufficient condition for a family of sample operators on a separable Banach space to be a random operator.

Let $(\Omega, \mathfrak{A}, \mu)$ be a probability space and \mathfrak{X} , a separable Banach space. Definition. A mapping $T: \Omega \times \mathfrak{X} \rightarrow \mathfrak{X}$ is a random operator if for every $x \in \mathfrak{X}$, the mapping $T(\omega)[x]: \Omega \rightarrow \mathfrak{X}$ is a generalized random variable, i.e., if $\{\omega: T(\omega)[x] \in B\} \in \mathfrak{A}$ for every Borel set B in \mathfrak{X} . T is called almost surely continuous if the mapping $T(\omega): \mathfrak{X} \rightarrow \mathfrak{X}$ is continuous for almost all $\omega \in \Omega$. Lemma. Let T be an almost surely uniformly continuous random operator. Then given $\delta > 0$, $\epsilon > 0$, there exists an $A \in \mathfrak{A}$ with $\mu(A) > 1 - \delta$ and an $\alpha > 0$, such that $\|x - y\| < \alpha \Rightarrow \|T(\omega)[x] - T(\omega)[y]\| < \epsilon$ for every $\omega \in A$. This A is independent of ϵ . Theorem. Let $T(\omega): \mathfrak{X} \rightarrow \mathfrak{X}$ be an uniformly continuous operator for almost all $\omega \in \Omega$. Then the family of operators $\{T(\omega)\}_{\omega \in \Omega}$ is a random operator if and only if there exists a countable subfamily $\{T(\omega_i)\}_{i=1}^{\infty}$ such that $\{\omega: \|T(\omega)[x] - T(\omega_i)[x]\| < \epsilon\} \in \mathfrak{A}$ and $\inf_{1 \leq i < \infty} \|T(\omega)[x] - T(\omega_i)[x]\| = 0$, for almost all $\omega \in \Omega$ and all $x \in \mathfrak{X}$. This subfamily is independent of ω and x . (Received April 27, 1967.)

67T-465. A. M. KRALL, The Pennsylvania State University, University Park, Pennsylvania 16802. The density of meteorite impacts on the moon.

It is shown that, if the velocities of meteors are assumed to be small relative to the earth moon system, so that, when falling toward the earth or moon, they follow streamlines in the earth-moon gravitational field, then the current density of meteorite impacts on the moon is essentially constant. This constancy remains in effect unless the distance from the earth to the moon is in the neighborhood of 15,000-20,000 miles, when the density becomes much greater on the backside of the moon. It is therefore conjectured that the apparent increased scarring on the backside of the moon occurred at that time when the earth and moon were much closer together--probably when the moon's surface solidified. (Received April 25, 1967.)

67T-466. A. E. HURD, University of California, Los Angeles, California 90024. Backward continuous dependence for mixed parabolic problems.

It is shown that backward solutions of (abstract) mixed parabolic problems, as considered by J. L. Lions and B. Malgrange (Math. Scand. 8 (1960), 277-286), depend continuously on their initial values in the following sense: There are positive numbers Λ and δ depending on the constants in the basic assumptions of the above paper, and a positive constant K depending on these constants and the number $M = \max |u(t)| (t \in [0, -T])$, such that if $|u(0)| \leq \Lambda$ then $|u(t)| < K \Lambda^\delta$ for all $t \in [0, -T]$. The solutions are assumed to be slightly more regular than those considered by Lions and Malgrange. The

proof is an extension of that used by Glagoleva (Soviet Math. Dokl. 4 (1)(1963), 13-17) who established similar results for the first boundary value problem for second order parabolic equations. (Received November 14, 1966.)

67T-467. CHING CHOU, University of Rochester, Rochester, New York 14627. The cardinality of the set of left invariant means on a semigroup.

If S is a semigroup, the set of left invariant means on S will be denoted by $M_l^*(S)$. Theorem (I). Let N be the additive semigroup of positive integers. $\text{Card}(M_l^*(N)) = 2^c$, where c is the cardinality of the real numbers. (II). If G is either an infinite abelian group or a left amenable nontorsion group then $\text{Card}(M_l^*(G)) \geq 2^c$. (III). Let G be an infinite abelian group. If, moreover, G is divisible or G is of bounded order then $\text{Card}(M_l^*(G)) = 2^{2^{\text{Card}(G)}}$. All the above cardinalities can in fact be realized by sets of means which, when considered as measures on the appropriate compactifications, have pairwise disjoint supports. (Received May 10, 1967.)

67T-468. R. PADMANABHAN, Madurai University, Madurai-2, India. Two identities for lattices.

In this paper it is shown that every equational class of lattices which can be defined by a finite number of identities can be characterized (among the class of algebras with two binary operations) by means of two identities. Let W denote such an equational class defined by, say, $I: f(y_1, y_2, \dots, y_n) = g(y_1, y_2, \dots, y_n)$, in addition to the lattice identities (it is known that in presence of the lattice axioms a finite set of lattice identities is equivalent to a single one). Theorem. The equational class of lattices W is characterized by the identities: (1) $(xf)z + (((a + u)u + v) + w) = (gz)x + ((v + w) + u)$ and (2) $xy + y = y$. Now, W becomes the class of all lattices if the identity I is thought of as the trivial identity $y = y$. Thus, in particular, the class of all lattices can be defined by two identities and this settles affirmatively a question raised by Iu. I. Sorkin in 1962. (Received May 1, 1967.)

67T-469. G. A. GRÄTZER, The University of Manitoba, Winnipeg, Canada and R. N. MCKENZIE University of California, Berkeley, California. Equational spectra and reduction of identities.

A unified treatment of the results of J. H. Bennett (J. Symbolic Logic 30 (1965), 264) and G. Grätzer (Abstract 632-22, these *Notices* 13 (1966), 331) is obtained by constructing a uniform scheme of axioms for the primal algebras. For $S \subseteq \omega \sim 1$, let $S^* = \cap \{T \supseteq S: 1 \in T \ \& \ (\forall x, y \in T)(x \cdot y \in T)\}$. $\text{Sp}(\Sigma)$ denotes the set of cardinalities of finite models of Σ . Theorem 1. There exist a recursive set Σ of identities in five function symbols, and a one-one recursive function $f: \omega \sim 1 \rightarrow \Sigma$ such that whenever $S \subseteq \omega \sim 1$, $S^* = \text{Sp}(f(\omega \sim 1 \sim S))$. Theorem 2. For every first order sentence θ , there is a first order sentence ψ such that $\text{Sp}(\theta) \sim I_r(\text{Sp}(\theta)) = \text{Sp}(\psi)$, where $I_r(S) = \{X \in S: (\forall y, z \in S)(x = y \cdot z \rightarrow x = 1 \vee z = 1)\}$. Now let θ be the sentence $\forall x \forall y (0 \cdot x = x \cdot 0 = h(x, x) = 0, 1 \cdot x = 0 + x = x + 0 = h(y, h(y, x)) = x)$. Rosenbloom (Amer. J. Math. 64 (1942), 167-188) has shown that the identities of a primal algebra have a finite basis. Theorem 3. If Δ is a finite set of identities, then $\Delta \cup \{\theta\} \equiv \{e\}$ for some identity e . Corollary. The identities of a primal algebra have a one-element basis. Another corollary is the result of B. H. Neumann that for every finite set Δ of identities, $\text{Sp}(\Delta) = \text{Sp}(e)$ for some identity e . Theorem 3 has been obtained independently by Alfred Tarski. (Received May 1, 1967.)

67T-470. ROBERT GILMER, Florida State University, Tallahassee, Florida and WILLIAM HEINZER, Louisiana State University, Baton Rouge, Louisiana 70803. Irredundant intersections of valuation rings.

If there exists a set $\{V_\alpha\}$ of valuation overrings of an integral domain D such that $D = \bigcap_\alpha V_\alpha$ and such that this representation is irredundant, D is said to have property (S) and the representation $D = \bigcap_\alpha V_\alpha$ is called an S-representation of D . Theorem 1. A Prüfer domain D has property (S) if and only if $D = \bigcap_\alpha D_{M_\alpha}$ where $\{M_\alpha\}$ is the set of maximal ideals M of D which have the following property: there exists a finitely generated ideal A such that M is the unique maximal ideal of D containing A . Moreover, if the Prüfer domain D has property (S), then $D = \bigcap_\alpha D_{M_\alpha}$ is the unique S-representation of D . Theorem 2. If D is a Bezout domain with group of divisibility G , then D has property (S) if and only if G has an irreducible coréticulé realization (see P. Jaffard's Les systèmes d'ideaux, p. 38). By example it is shown that in general an integral domain with property (S) need not have a unique S-representation. (Received May 1, 1967.)

67T-471. C. S. HOLMES, Miami University, Oxford, Ohio 45056. Projectivities of free products.

Let $G = A *_{\mathcal{C}} B$ ($A, B \neq C$) be a free product of its subgroups A and B with amalgamated subgroup $C = A \cap B$. Suppose $[A:C] > 2$ or $[B:C] > 2$. Theorem 1. If C is normal in G , then G is determined by its subgroup lattice. Theorem 2. If C is the center of G , then any projectivity of G is induced by a group isomorphism. Corollary. Any projectivity of a free product $A * B$ ($A, B \neq 1$) is induced by an isomorphism. The proofs depend upon the fact that $A * B$ ($A, B \neq 1$) contains a noncyclic free group unless A and B are both of order two. The methods used are primarily extensions and refinements of those used by E. L. Sadovskii (Mat. Sbornik (63) 21 (1947), 63-82). (Received May 1, 1967.)

67T-472. SAMI BERAHA, Ball State University, Muncie, Indiana 47306. Powers of nonnegative matrices with special symmetries.

Given the $n \times n$ matrix C defined (for $i, j = 1, 2, 3, \dots, n$) by (1) $c_{ii} > 0$; (2) $c_{ij}c_{ji} = 0, i \neq j$; (3) $c_{ij} + c_{ji} > 0$; (4) for all $i \prod_{x=1}^n c_{ix} = 0$. Then C^2 will have at least three strictly positive rows (i.e. that contain no zero element); and three will be the true minimum. Replacing (4) by its opposite (4*) there exists an "i" such that $\prod_{x=1}^n c_{ix} > 0$ would result in C^2 having one and only one strictly positive row. Adding to (1), (2), (3), and (4) one more condition: (5) for all $i \prod_{y=1}^n c_{yi} = 0$, results in C^2 having a true maximum of n strictly positive rows when n is odd. In other words C^2 can be a strictly positive matrix when n is odd whereas, (5*) the existence of an i such that $\prod_{y=1}^n c_{yi} > 0$ makes it impossible for any countable power of C, C^n , to become a strictly positive matrix. (Received May 1, 1967.)

67T-473. T. W. RICHEL, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. A class of spaces determined by sequences with their cluster points. Preliminary report.

Suppose R is a topological space. Let U be nonopen in R iff there is a $p \in U$ and a set $A \subseteq R - U$ such that cardinality of $A \leq \aleph_0$ and $p \in \text{Cl } A$. Call such a space a c -space. Then every sequential space is a c -space (see S. P. Franklin, Fund. Math. 57 (1963), 107-115). We characterize c -spaces by Theorem. R is a c -space iff R is the quotient of a disjoint topological sum of countable spaces

$\{C_\alpha: \alpha \in A\}$. Further, each C_α may be chosen homeomorphic to $\mathbb{N} \cup \{a\}$, where \mathbb{N} is the set of natural numbers with the discrete topology, and a is one point of its Stone-Ćech compactification. We can extend the definition of c -space by choosing, instead of \aleph_0 , an arbitrary cardinal number τ . For such a space we can establish a theorem similar to the above theorem for c -spaces. (Received May 1, 1967.)

67T-474. C. K. GUPTA, University of Manitoba, Winnipeg, Canada. A faithful matrix representation for certain soluble groups. Preliminary report.

Let F be the free group of countable infinite rank and let V be its verbal subgroup generated by the words (a) $[x,y; u,v; z]$ and (b) $[x^{-1}, y^{-1}; u,v] [x^{-1}, v^{-1}; y,u][x^{-1}, u^{-1}; v,y][v^{-1}, y^{-1}; x,u][y^{-1}, u^{-1}; x,v][u^{-1}, v^{-1}; x,y]$. It is shown that F/V is isomorphic to a certain group of 3×3 matrices over a certain ring. There is a four generator group in which all values of the word (a) are trivial but some values of the word (b) are nontrivial. Further if F is of rank 3, then the matrix representation is faithful for F/U , where U denotes the verbal subgroup of F generated by the word (a) only. (Received May 2, 1967.)

67T-475. V. A. NICHOLSON, University of Iowa, Iowa City, Iowa 52240. Surfaces with local mapping cylinder neighborhoods are tame.

A weaker collaring condition implies tameness for 2-manifolds-with-boundary. Let $Y \subset E^3$, C a complementary domain of Y , then Y has a mapping cylinder neighborhood (MCN) , $U = f(M \times [0,1])$, in \bar{C} if there exists a space M and a map f from $M \times [0,1]$ into \bar{C} such that $f|_{M \times [0,1]}$ is a homeomorphism into $E^3 - Y$, $f_1(M) = f|_{M \times \{1\}} = Y$, and U is a neighborhood of Y in \bar{C} . It was shown by R. H. Bing and A. Kirkor (Fund. Math. 55 (1964), 175-180), with the correction $f_1(M) = Y$, that Y is tame if Y has a MCN where Y is an arc and M is the 2-sphere. The above correction was noticed by L. Howell and the result extended by him in an unpublished paper for Y a 3-cell. If Y is a compact connected 2-manifold-with-boundary, Y is shown to be tame if Y has a MCN in each complementary domain, where M is a compact connected 2-manifold. Let M be a disk, $U = f(M \times [0,1])$ a mapping cylinder such that $f_1(M) \subset Y$ and U be a one-sided neighborhood of x , then $f_1^{-1}(x)$ is cellular and $E^3 - Y$ is locally simply connected on one side of Y on a neighborhood of x in Y . (Received May 3, 1967.)

67T-476. HUNG-HSI WU, University of California, Berkeley, California 94720. Generalized Bloch theorem.

Let $X \subseteq \mathbb{R}^n$ be open, and $\mathcal{F}: X \rightarrow \mathbb{R}^n$ is a family of C^∞ maps. We say \mathcal{F} is a quasi-conformal family (of order $K > 0$) iff for each $x \in X$ and for each $F \in \mathcal{F}$, the ratio of the maximum axis to the minimum axis of the ellipsoid $dF_x(S)$ (where S is the unit sphere in the tangent space of x) is less than K . Theorem. Let B be the unit ball in \mathbb{R}^n and let $\mathcal{F}: B \rightarrow \mathbb{R}^n$ be a family of solutions of $P(D)f = 0$ where $P(D)$ is an elliptic differential operator with C^∞ coefficients and contains only partials of a fixed order m . If \mathcal{F} is a quasi-conformal family and if $|JF(0)| = 1$ for each $F \in \mathcal{F}$, then there is a universal constant β such that: for each $F \in \mathcal{F}$, there is an open ball of radius β in $F(B)$ onto which F maps some open set diffeomorphically. Bochner proved this for the case $P(D) = \text{Laplacian}$ (Bull. Amer. Math. Soc., 1946), which already implies the classical Bloch Theorem of holomorphic functions.

Our proofs differ from Bochner's and rests squarely on the Lemma. If $\mathcal{F}: X \rightarrow \mathbb{R}^n$ is a family of C^1 mappings which is compact in the weak (or coarse) C^1 -topology and for all $F \in \mathcal{F}$, $|JF(x_0)| \geq 1$ ($x_0 \in X$), then each F maps a neighborhood of x_0 diffeomorphically onto an open ball of radius $\gamma > 0$, where γ is independent of F . (Received May 3, 1967.)

67T-477. O. L. MANGASARIAN, Shell Development Company, Emeryville, California 94608.

Lower bound for constrained minimization problems.

Let $\theta(x): E^n \rightarrow E^1$, $g(x): E^n \rightarrow E^m$, and $a \in E^n$, $a \geq 0$. Let $\theta(x)$ and the m -components of $g(x)$ be differentiable and convex on some open set containing $\{x \mid -a \leq x \leq a\}$. Let $\theta(\bar{x}) = \min_{x \in R} \theta(x)$, $\bar{x} \in R = \{x \mid g(x) \leq 0, -a \leq x \leq a\}$. Theorem. $\theta(\bar{x}) \geq \theta(x) + ug(x) - (\nabla\theta(x) + u\nabla g(x))x - |\nabla\theta(x) + u\nabla g(x)|a$, for any $x \in E^n$, $u \in E^m$, $u \geq 0$, where $\nabla = (\partial/\partial x_1, \dots, \partial/\partial x_n)$, and $|x| = (|x_1|, \dots, |x_n|)$. Proof. By Wolfe's weak duality theorem [Theorem 1, Quart. Appl. Math. 19 (1961), 239-244] $\theta(\bar{x}) \geq \theta(x) + ug(x) - (\nabla\theta(x) + u\nabla g(x))x - (v + w)a$, for each x , u , v , w such that $x \in E^n$, $u \in E^m$, $u \geq 0$, $v \in E^n$, $v \geq 0$, $w \in E^n$, $w \geq 0$, and $v - w = -(\nabla\theta(x) + u\nabla g(x))$. In particular, for any x and $u \geq 0$, let: $v_i = -(\nabla\theta(x) + u\nabla g(x))_i$, $w_i = 0$ for $(\nabla\theta(x) + u\nabla g(x))_i \leq 0$; and $v_j = 0$, $w_j = (\nabla\theta(x) + u\nabla g(x))_j$ for $(\nabla\theta(x) + u\nabla g(x))_j > 0$. Hence $(v + w)a = |\nabla\theta(x) + u\nabla g(x)|a$ and the theorem is established. (Received May 5, 1967.)

67T-478. H. W. GUGGENHEIMER, University of Minnesota, Minneapolis, Minnesota 55455.

Hill equations with coexisting periodic solutions.

For a Hill equation $x'' + \lambda p(t)x = 0$, let $[\lambda_n, \Lambda_n]$ be the n th interval of instability. Let ω be the period of the nonnegative function $p(t)$. It is known that $I(\lambda_n) = \lambda_n \omega \int_0^\omega p \, dt \geq 4n^2$. Theorem. If $\lambda_n = \Lambda_n$, then $I(\lambda_n) \geq 4(n+1)^2 \cos^2 \pi/2(n+1)$. The result is best possible. The proof uses the methods of unimodular centro-affine differential geometry. (Received May 8, 1967.)

67T-479. B. E. JOHNSON and A. M. SINCLAIR, The University, Newcastle upon Tyne, England.

Continuity of derivations and a problem of Kaplansky.

Let \mathfrak{A} be a semisimple Banach algebra over the real or complex field and let D be a ring derivation on \mathfrak{A} . Then there is a central idempotent e in \mathfrak{A} such that $e\mathfrak{A}$ and $(1-e)\mathfrak{A}$ are closed under D , $e\mathfrak{A}$ is finite dimensional, and D is continuous and linear on $(1-e)\mathfrak{A}$. In particular if D is linear then D is continuous on \mathfrak{A} . Left and right centralisers on \mathfrak{A} are continuous. (Received May 9, 1967.)

67T-480. R. D. M. ACCOLA, Brown University, Providence, Rhode Island. On the problem of characterizing hyperelliptic Riemann surfaces in terms of vanishing theta-nulls. Preliminary report.

Let W be a Riemann surface of genus $p \geq 2$ and let $\theta[\mu](u)$ be the associated theta function with half integer theta characteristic $[\mu]$. The following theorems generalize the known cases of the "p - 2 conjecture" in a way that loses the correct count on the codimension of the hyperelliptic locus in Teichmüller space. The converses of these theorems are well known. Theorem 1. Let W have odd genus p . Suppose that $\theta[\mu](u)$ vanishes at $u = 0$ together with all its derivatives up to and including order $(p-1)/2$ for one theta characteristic $[\mu]$. Then W is hyperelliptic. Theorem 2. Let W have even genus p . Suppose that $\theta[\mu](u)$ vanishes at $u = 0$ together with all its derivatives up to and including

order $(p - 2)/2$ for two distinct theta characteristics $[\mu]$. Then W is hyperelliptic. Theorem 1 follows easily from classical results. Theorem 2 is made to follow from theorem 1 by a device introduced by H. M. Farkas (Automorphisms of compact Riemann surfaces and the vanishing of theta constants, Bull. Amer. Math. Soc. 73 (1967), 231-232). (It is believed that the two theorems above are new, but the author would appreciate anyone who would authoritatively confirm this or inform him to the contrary.) (Received May 11, 1967.)

67T-481. D. E. RAMIREZ, University of Washington, Seattle, Washington 98105. Weakly almost periodic functions on discrete groups.

Let G be an infinite compact abelian group; Γ the (discrete) dual group; $M(G)^\wedge$ the algebra of Fourier-Stieltjes transforms; $M(G)^\wedge^-$ the completion of $M(G)^\wedge$ in the sup-norm topology on Γ ; and $WAP(\Gamma)$ the (continuous) bounded weakly almost periodic functions on Γ . Theorem. Let Γ be an infinite discrete abelian group. Then $M(G)^\wedge^- \neq WAP(\Gamma)$. The case for Γ not of bounded order is due to W. Rudin [Weak almost periodic functions and Fourier-Stieltjes transforms, Duke Math. J. 26 (1959), 215-220]. This theorem is based on the author's characterization of $M(G)^\wedge^-$ [Uniform approximation by Fourier-Stieltjes transforms, to appear]: for f a (continuous) bounded function on Γ , $f \in M(G)^\wedge^-$ if and only if $\{\lambda_n\} \subset M(\Gamma)$, $\|\lambda_n\| \leq 1$, $\|\lambda_n^\wedge\|_\infty \rightarrow 0$ implies $\int_\Gamma f d\lambda_n \xrightarrow{n} 0$. (Received May 11, 1967.)

67T-482. WILLIAM STENGER, Institute for Fluid Dynamics, University of Maryland, College Park, Maryland. An improvement of Aronszajn's inequality.

Let A be a negative-definite compact operator on a separable Hilbert space H . Let H' be a closed subspace of H and let $H'' = H \ominus H'$. Denote by P' and P'' the projection operators onto H' and H'' respectively. Let $\lambda_1 \leq \lambda_2 \leq \dots$, $\lambda_1' \leq \lambda_2' \leq \dots$, and $\lambda_1'' \leq \lambda_2'' \leq \dots$ be the eigenvalues of the operators A , $P'AP'$, and $P''AP''$. Aronszajn [Proc. Nat. Acad. Sci. U.S.A. 34 (1948), 474-480] has shown that $\lambda_i' + \lambda_j'' \leq \lambda_{i+j-1}$ for $i, j = 1, 2, \dots$. In the present paper the following result is proved. Theorem. For given indices i, j , a necessary condition for the equality $\lambda_i' + \lambda_j'' = \lambda_{i+j-1}$ to hold is that the dimension r of the subspace H'' (or by symmetry H' , in which case i and j are interchanged) satisfies the inequality $m - i \leq r \leq j - 1$, where $m = \min \{h | \lambda_h = \lambda_{i+j-1}\}$. If this necessary condition is satisfied, then a necessary and sufficient condition for the above equality to hold is that for any sufficiently small $\epsilon > 0$ the quadratic form with the symmetric matrix of the determinant $W(\lambda_{i+j-1} - \epsilon)$ has in canonical coordinates the diagonal form $-x_1^2 - x_2^2 - \dots - x_{m-i}^2 \pm x_{m-i+1}^2 \pm \dots \pm x_{m-1}^2 + x_m^2 + \dots \pm x_r^2$, where $W(\lambda) = \det\{([A - \lambda I]^{-1} v_h, v_k)\}$, $h, k = 1, 2, \dots, r$ is the Weinstein determinant for the basis $\{v_1, v_2, \dots, v_r\}$ of H'' (or H'). (Received May 12, 1967.)

67T-483. G. C. NELSON, Case-Western Reserve University, Cleveland, Ohio 44106. Minimal elements in the many-one degrees of the predicates $H_a(x)$. Preliminary report.

For background see Y. Moschovakis [Pacific J. Math. 18 (1966), 329-342] where the following question was asked. For each $\xi < \omega_1$, ξ principle for addition, is there an $a \in 0$, $|a| = \xi$, such that a is minimum with respect to $<$? It is known that for $\xi = \omega$ and $\xi = \omega^2$ that there are such minimum elements. The following result gives a negative answer for all other ξ . Theorem. There exists a partial recursive function g such that whenever $a \in 0$, $|a| \geq \omega^3$, and $|a|$ is principle for addition, then

$g(a) \in 0$, $|g(a)| = |a|$, and $g(a) \lesssim a$. (Received May 12, 1967.)

67T-484. R. L. WILSON, Jr., University of Wisconsin, Madison, Wisconsin 53706. Isotopisms of loops of prime order. Preliminary report.

There are several known sufficient conditions that a loop be isomorphic to all of its loop isotopes. Associativity is a well known example of such a condition, but is not in general necessary. Using results of Schneider and Bryant (Canad. J. Math. 18 (1966), 120-125) the present paper shows that for loops of prime order associativity is actually necessary. (Received May 12, 1967.)

67T-485. T. E. HARRIS, University of Southern California, Los Angeles, California 90007. Topologies for certain integer-valued measures.

Let X be a Hausdorff space with topology T , X^{∞} the Cartesian product space $X \times X \times X \times \dots$, and $Q: X^{\infty} \rightarrow Z$ the quotient map of X^{∞} onto Z , where Q identifies two points of X^{∞} if the coordinates of one are a permutation (possibly infinite) of the coordinates of the other. If $y \in X^{\infty}$, $z = Qy$, and $A \subset X$, let $z(A) \leq \infty$ be the number of coordinates of y in A . A basis for the topology T_1 of Z consists of all sets of the form $\{z: z \in Z, z(A_i) \geq 1, i = 1, 2, \dots, n; Z(A_i \cup A_j) \geq 2, 1 \leq i < j \leq n; \dots; Z(A_1 \cup A_2 \cup \dots \cup A_n) \geq n\}$, $A_1, A_2, \dots, A_n \in T$; $n = 1, 2, \dots$. This is deducible from P. Hall's theorem. T_1 is not Hausdorff, and all continuous functions on it are constant. Fix $x^* \in X$, let $Z^* = \{z: z \in Z, z(X - U) < \infty \text{ if } x^* \in U \in T\}$. Take as a subbase for a topology T^* in Z^* all sets in $T_1 \cap Z^*$ plus all sets $\{z: z \in Z^*, z(\bar{U}) \leq k\}$, $k = 1, 2, \dots$, $U \in T$, $x^* \notin \bar{U}$. Then T^* is Hausdorff. If X is complete separable metric, then T^* is equivalent to the topology induced by a generalization to certain infinite measures of Prohorov's metric. (Received May 11, 1967.)

67T-486. B. GERSHUNI, Singel 270, Amsterdam, The Netherlands. Some properties of the concept of class.

The following properties of the new concept of class (excepting property 1) follow from its way of writing in the form: $C = a_1, a_2, \dots$. (1) A class may have on principle singulars as well as proper plurals as elements. Remark. A proper plural or a proper class has more than one element. (2) Any class admits besides its usual description as having such and such elements also the description as having itself as its unique element. (3) In the case that the class has a singular as its unique element, both descriptions coincide. (4) A class C which has the object D as its unique element is identical with this object: $C = D$. "Has... as its unique elements" and "is..." coincide. (5) Two classes such that every has the other as element coincide: $a \in b \wedge b \in a \Rightarrow a = b$. Nevertheless "being an element" and "being a subclass" do not coincide. So a, b is a subclass but not an element of a, b, c . (6) A class having itself as element has no other element in this description. (7) The extension-axiom for classes: Two classes are equal iff they have the same elements in one of their descriptions (2). (8) Acceptable axiom: Only singulars are elements of a class. (Received May 15, 1967.)

67T-487. N. D. GUPTA, University of Manitoba, Winnipeg, Canada. Periodicity of the commutator subgroup of a certain group.

Let G be a group in which every commutator is of order dividing n . If G is locally soluble then it is easily seen that the commutator subgroup G' is periodic. But without any such restriction on G , the periodicity of G' is known only for the case $n = 2$ (I. D. MacDonald, Math. Z. 76 (1961), 270-282). Here the periodicity of G' is shown for the case $n = 3$. Let c be a commutator and let x be an arbitrary element of G . Using the assumption $c^3 = 1$, one gets $(cx)^3 = (x^2c^2x^{-1})^{-1}(xc^2xcx^{-2})^{-1}(x^2c^2x^{-1}) \cdot (xc^2xcx^{-2})x^3 = c*x^3$, where $c*$ is a commutator and so is of order dividing 3. Now writing an arbitrary element of G' as $c_1c_2\dots c_m$ (c_i being commutators), one gets $(c_1c_2\dots c_m)^{3^m} = 1$ by repeated application of $(cx)^3 = c*x^3$. (Received May 15, 1967.)

67T-488. L. K. ARNOLD, Daniel H. Wagner, Associates, Station Square One, Paoli, Pennsylvania. Sigma-finite invariant measures. Preliminary report.

Let (X, \mathcal{B}, m) be a totally finite measure space. Let T be a one to one, measurability-preserving nonsingular transformation of X onto X . Assume any subset of X mentioned is measurable. A subset E of X satisfies (*) iff for any decomposition $\{E_n\}$ of E and integers $\{k_n\}$ such that $T^{k_n}(E_n) \subset E$ for each n we have $\sum m(T^{k_n}(E_n))$ is finite. Theorem. There is a sigma-finite, invariant (i.e. $m'(B) = m'(T(B))$ for all B in \mathcal{B}) measure defined on \mathcal{B} and equivalent to m if and only if X is the countable union of subsets which satisfy (*). Necessity follows from a previous result of the author. Sufficiency follows from the fact that condition (*) implies E is bounded (as defined by P. Halmos in Lectures on ergodic theory). (In the above, nonsingular means $m(B) = 0$ iff $m'(T(B)) = 0$, and equivalent means m and m' have precisely the same sets of measure zero.) (Received May 15, 1967.)

67T-489. R. P. RIGELHOF, McMaster University, Hamilton, Ontario, Canada. Isomorphisms of measure algebras.

Let G be a locally compact Hausdorff group. By the measure algebra of G we mean the Banach *-algebra $M(G)$ of bounded regular Borel measures on G . It is well known that $M(G)$ is (isomorphic to) the dual of $C_0(G)$, the Banach space of all continuous complex-valued functions on G which "vanish at infinity". Among other results, we have the following: Lemma. Let F and G be locally compact Hausdorff groups, α a homeomorphism and isomorphism of F onto G and γ a continuous character on F . For μ in $M(F)$ and f in $C_0(G)$, let $T\mu(f) = \mu(\gamma(f \circ \alpha))$. Then the mapping $\mu \rightarrow T\mu$ is an isometric *-isomorphism of $M(F)$ onto $M(G)$. Theorem. Let F and G be locally compact Hausdorff groups and let T be a norm decreasing isomorphism of $M(F)$ onto $M(G)$. Then there is a homeomorphism and isomorphism α of F onto G , and a continuous character γ on F such that $T\mu(f) = \mu(\gamma(f \circ \alpha))$ for all μ in $M(F)$ and f in $C_0(G)$. Corollary. Every norm decreasing isomorphism of $M(F)$ onto $M(G)$ is an isometric *-isomorphism. The above theorem can be used to prove a theorem of Wendel (Pacific J. Math. 2 (1952), 251-261) concerning norm decreasing isomorphisms of $L^1(F)$ onto $L^1(G)$. (Received May 15, 1967.)

Multiplications on additive groups.

Let $G = F/N$, F free Abelian, $N \triangleleft F$. Define the group operation in G as $(b_\alpha + N) + (b_\beta + N) = (b_{\alpha,\beta} + N)$, where the b_α are a fixed set of coset representatives of the cosets in G . Let X_F be a multiplication on F . An operation X_G on G is said to be induced by X_F if $(b_\alpha + N)X_G(b_\beta + N) = \phi(b_\alpha X_F b_\beta)$.

Theorem 1. Necessary and sufficient conditions are given for an induced operation X_G to be a multiplication. Theorem 2. The following conditions are equivalent: (1) An induced operation X_G is independent of the coset representatives b_α . (2) X_F transforms N into an ideal $R(N)$ in $R(F)$, (X_F transforms F into $R(F)$). (3) For arbitrary words $W_1, W_2 \in F$, $\phi(W_1 X_F W_2) = \phi(W_1) X_G \phi(W_2)$. (4) X_G transforms G into a ring $R(G)$ which is a homomorphic image of $R(F)$ under ϕ , the canonical homomorphism mapping F onto G . (5) The group presentation of G in terms of generators of F , and relators the generators of N , is a ring presentation of $R(G)$. (6) A certain diagram is commutative. (Received May 16, 1967.)

67T-491. BJARNI JONSSON, Vanderbilt University, Nashville, Tennessee 37203. Equational classes of lattices.

Let M be the modular lattice obtained by amalgamating two diamonds (five-element modular nondistributive lattices) with an upper edge of one identified with a lower edge of the other. Theorem. For any equational class U of lattices the following three conditions are equivalent: (a) $M \in U$, (b) The inclusion $a(b + cd)(c + d) \leq c + ab + ad$ holds in U , (c) Every member of U is a subdirect product of lattices of dimension one and two. The equational classes that satisfy these conditions form a strictly increasing sequence of type $\omega + 1$, $U_0 \subset U_1 \subset \dots \subset U_\omega$. U_0 and U_1 consist of all one-element lattices and of all distributive lattices, respectively, while for $n > 1$, U_n is generated by the two-dimensional lattice of order $n + 3$. U_n is characterized by the additional condition $a \cap (x_i + x_j, 0 \leq i < j \leq n) \leq \sum (ax_i, 0 \leq i \leq n)$. The case $n = 2$ of this inclusion implies the inclusion in (b) above. Hence these results include a solution to Problem 45 in Birkhoff's Lattice theory, 3rd ed., Amer. Math. Soc. Colloq. Publ., Vol. 25, Amer. Math. Soc., Providence, R.I., 1966, p. 157. Some of the methods are borrowed from G. Grätzer, Equational classes of lattices, Duke Math. J. 33 (1966), where related but more special results are proved. (Received May 19, 1967.)

67T-492. F. M. YAQUB, American University of Beirut, Beirut, Lebanon. Generalized free α -products of Boolean algebras.

Let $\{B_t\}_{t \in T}$ be a family of α -complete Boolean algebras and let A be an α -complete Boolean algebra such that for every $t \in T$ there exists an α -monomorphism $f_t: A \rightarrow B_t$. The generalized free α -product of $\{B_t\}_{t \in T}$ with α -amalgamated α -subalgebra A is a pair $(\{g_t\}_{t \in T}, B)$ such that: (i) B is an α -complete B.A. (ii) For every $t \in T$, $g_t: B_t \rightarrow B$ is an α -monomorphism such that for every $t', t'' \in T$, $g_t \cdot f_{t'} = g_{t''} \cdot f_{t''}$. (iii) B is α -generated by $\bigcup_{t \in T} g_t(B_t)$. (iv) If D is an α -complete Boolean algebra and if $\{h_t: B_t \rightarrow D\}_{t \in T}$ is a family of α -homomorphisms such that for every $t', t'' \in T$, $h_{t'} \cdot f_{t'} = h_{t''} \cdot f_{t''}$, then there exists an α -homomorphism $h: B \rightarrow D$ such that $h g_t = h_t$ for every $t \in T$. Theorem. Let $\{B_t\}_{t \in T}$ be a family of α -complete Boolean algebras and let A be an α -complete, atomic Boolean algebra. Then the generalized free α -product of $\{B_t\}_{t \in T}$ with α -amalgamated α -subalgebra A exists and is unique (up to isomorphisms). (Received May 15, 1967.)

67T-493. D. A. EDWARDS, University of Oxford, Oxford, England and G. VINCENT-SMITH, University of Copenhagen, Copenhagen, Denmark. A Weierstrass-Stone theorem for compact convex sets.

Let X be a nonempty compact convex subset of a locally convex Hausdorff real topological vector space, let $A(X)$ be the space of real continuous affine functions on X , and let L be a linear subspace of $A(X)$ that contains the constant functions. Then L is dense (for the supremum norm) in $A(X)$ if and only if (i) L separates the extreme points of X , and (ii) for each f in $A(X)$ the set $\{g \in L : g > f\}$ is downward filtering. A characterization of the elements of \bar{L} , given that (ii) alone holds, has been obtained. (Received May 15, 1967.)

67T-494. H. B. COHEN, University of Pittsburgh, Pittsburgh, Pennsylvania 15213 and H. E. LACEY, University of Texas, Austin, Texas. On injective envelopes of Banach spaces.

It is known (Bull. Amer. Math. Soc. 70 (1964), 723-726) that in the category of Banach spaces and linear contraction maps, every Banach space B has an essentially unique injective envelope $(I, C(M))$. Two types of results are herein announced: facts about the envelopes of specific spaces; and results that hold for any B . For instance, Theorem. A compact Hausdorff space X is dispersed if and only if every subspace of $C(X)$ has an injective envelope of the form $(I, m(S))$ for a set S that depends on the subspace. Definitions. A subset W of a real or complex linear topological space is circled if for each w in W , the entire circle $\{cw : |c| = 1\}$ of vectors is in W . A compact subset K of a circle set W is called irredible in W if it meets every circle of W but no proper closed subset of K does. The letter e denotes the evaluation homeomorphism $X \rightarrow C(X)^*$. Theorem. Suppose $I: B \rightarrow C(X)$ is a map. Then $(I, C(X))$ is an injective envelope of B if and only if $I^* \circ e$ is Gleason's map onto an irreducible subset K of the weak $*$ closure of the set of extreme points of the unit ball of B^* . (Received May 15, 1967.)

67T-495. W. P. ZIEMER, Indiana University, Bloomington, Indiana 47401. The area and variation of linearly continuous functions on E^n .

Let f be a real-valued function defined on the unit n -cube Q whose distributional partial derivatives are measures. In addition assume that f is linearly continuous, c.f. Casper Goffman, Acta Math. 103 (1960) and let α and β be the associated area and gradient measures. Theorem. There is a Borel set $A \subset Q$ having the following properties: (i) A projects into a set of Lebesgue measure zero in each coordinate direction, (ii) if $g = f|_A$, then the graph of g is Hausdorff rectifiable, (iii) $\alpha(Q)$ equals the n -dimensional Hausdorff measure of the graph of g , (iv) for every Borel set $E \subset Q$, $\beta(E) = \int_{-\infty}^{\infty} H^{n-1}[g^{-1}(t) \cap E] dt$ where H^{n-1} denotes $n-1$ dimensional Hausdorff measure. (Received May 18, 1967.)

Descendents of strongly regular graphs.

Let G be a finite connected regular graph without multiple edges or loops. It is defined to be strongly regular if each vertex is adjacent to n_1 vertices, any two adjacent vertices are both adjacent to exactly p_{11}^1 vertices, and any two nonadjacent vertices are both adjacent to p_{11}^2 vertices (Pacific J. Math. 13 (1963), 391-418). Let O be a particular vertex of G , V_1 the set of vertices adjacent to O , and V_2 the set of vertices nonadjacent to O . From G we may derive another graph G^* by the following process: (i) Delete O and the edges of G incident with O , (ii) Delete all edges of G joining a vertex in V_1 to a vertex of V_2 , (iii) If a vertex in V_1 is nonadjacent to a vertex in V_2 , join them by a new edge. It is proved that the necessary and sufficient condition for G^* to be strongly regular is $2(p_{11}^1 + p_{11}^2) = 3n_1 - n_2 - 1$. In this case G^* will be called a descendent of G . The parameters of G^* are $n_1 = 2(n_1 - p_{11}^2)$, $p_{11}^{1*} = (n_1 + p_{11}^1 - 2p_{11}^2)$, $p_{11}^{2*} = n_1 - p_{11}^2$. Many new classes of strongly regular graphs can be derived by this process, from known classes. For example, if G is pseudogeometric with characteristics $(r, 2t, t)$, then G^* is pseudogeometric with characteristics $(r, 2t - 1, t - 1)$. (Received May 18, 1967.)

67T-497. M. C. FITTING, 611 West 112th Street, New York, New York 10025. Intuitionistic models and independence results in set theory.

P. Cohen's method of forcing is replaced by the use of a transfinite sequence of S. Kripke's intuitionistic models ("Semantical analysis of intuitionistic logic. I," in Formal systems and recursive functions, Proceedings of the Eighth Logic Colloquium, Oxford, July 1963, North Holland, Amsterdam, 1965). The resulting structure furnishes an intuitionistic counter-model to $\sim\sim(Z. F. \supset$ axiom of choice), expressed with no universal quantification. This, by a theorem in Kleene (Introduction to metamathematics, Van Nostrand, Princeton, N. J., 1952, p. 492, Theorem 59b) implies the classical unprovability of $(Z. F. \supset$ axiom of choice). The notion of complete sequences and the existence of a countable model is not needed. A similar result is obtained for the axiom of constructibility, although using the idea of complete sequences in the intuitionistic models. Moreover, the notion of complete sequences may be adapted to a single Kripke model to furnish a model-theoretic proof of the Kleene theorem cited above. (Received May 18, 1967.)

67T-498. ALBERT BAERNSTEIN, II, Wisconsin State University, Whitewater, Wisconsin 53190. A Tauberian theorem for a class of entire functions. Preliminary report.

Let f be an entire function of order ρ ($0 < \rho < 1$), all of whose zeroes lie on the negative real axis. Let $T(r) = T(r, f)$ and $N(r) = N(r, 0; f)$ be the quantities introduced in Nevanlinna's theory. Edrei and Fuchs [Duke Math. J. 27 (1960), 239] have shown that $T(r) = \sup \int_0^\infty N(t) K_\theta(rt^{-1}) t^{-1} dt$ where $k_\theta(t) = \pi^{-1} \sin \theta (t + t^{-1} + 2 \cos \theta)^{-1}$, the sup being taken over $0 < \theta < \pi$, so that T is a nonlinear integral transform of N . The Abelian result $N(r) \sim Ar^\rho$ ($r \rightarrow \infty$) implies $T(r) \sim AC_\rho^{-1} r^\rho$ ($C_\rho = 1$ for $0 < \rho \leq 1/2$ and $C_\rho = \sin \pi\rho$ for $1/2 \leq \rho < 1$) is well known. It is proved by the author that the converse also holds: If $T(r) \sim Br^\rho$, then $N(r) \sim BC_\rho r^\rho$. The proof combines a harmonic measure argument with an appeal to Wiener's General Tauberian Theorem. (Received May 18, 1967.)

67T-500. C. J. R. BORGES, University of California, Davis, California 95616. On continuously semimetrizable spaces and stratifiable spaces.

Recently Professor J. Nagata asked us a question which we will phrase as follows: Is it true that a topological space X is stratifiable and first countable (i.e. a Nagata space) if and only if X is semimetrizable by a semimetric which is continuous in one variable? Unfortunately, it turns out that neither implication is valid even for regular cosmic spaces (i.e. continuous images of separable metric spaces). (Received May 19, 1967.)

67T-501. W. D. L. APPLING, North Texas State University, Denton, Texas. Concerning a class of linear transformations.

U, F, R, R_A, R_A^+ and the notion of integral are as in Abstract 65T-320, these *Notices* 12 (1965), 609. For each (g, V) in $R_A \times F$, let $B(g)(V) = 1$ if $0 \leq g(V)$ and $B(g)(V) = -1$ if $g(V) < 0$; and let $\|g\| = \int_U |g(I)|$. Let C denote the set of all linear transformations T from R_A into R_A such that if V is in F , then $T(h)(V) \rightarrow 0$ as $\int_V |h(I)| \rightarrow 0$. Theorem 1. If T is in C , then T is bounded. Suppose m is in R_A^+ . For each (g, V) in $R_A \times F$, let $S(g)(V) = \sup \{z \mid z = \int_V \min\{|g(I)|, Km(I)\}, 0 < K\}$, and let $A(g)(V) = \int_V B(g)(I)S(g)(I)$. Theorem 2. If each of T_1 and T_2 is in C and $AT_1 = T_1$, then for each (g, V) in $R_A \times F$, $T_1(g)(V) = \int_V [A(g)(I)T_1(m)(I)/m(I)]$, and $T_1T_2 = T_2T_1$. Theorem 3. If T is in C , then T is reversible iff for each g in R_A g is absolutely continuous with respect to $T(g)$. (Received May 19, 1967.)

67T-502. PHILIP NANZETTA, Case Western Reserve University, Cleveland, Ohio 44106. On the lattice $D(X)$.

For a compact space X , $D(X)$ is the set of all continuous functions on X to the two-point compactification of the real line which are real-valued on a dense subset of X . Under the pointwise order, $D(X)$ is a distributive lattice. We have the following results: (1) X can be recovered from the lattice structure of $D(X)$, so that $D(X)$ and $D(Y)$ are isomorphic iff X and Y are homeomorphic. (2) The Dedekind completion of $D(X)$ is isomorphic to $D(Y)$ iff Y is the minimal projective extension of X . (3) The Dedekind completion of $D(X)$ is isomorphic to $D(Y)$ via a map taking bounded functions on X to bounded functions iff X is z-thin (i.e., if $(U_n)_{n \in \mathbb{N}}$ is a decreasing sequence of closed sets each of

which is a union of regular closed sets and if the intersection $\bigcap U_n$ is nowhere dense, then it is contained in a nowhere dense zero-set). (Received May 22, 1967.)

67T-503. YU-LEE LEE, Kansas State University, Manhattan, Kansas. On completion of measure spaces.

Let (X, \mathcal{A}, μ) be a measure space, and let (X, \mathcal{A}', μ') be its completion. For each subset T of X , let $\bar{\mu}(T) = \inf\{\sum_{n=1}^{\infty} \mu(A_n) \mid T \subset \bigcup_{n=1}^{\infty} A_n \text{ and } A_1, A_2, \dots \in \mathcal{A}\}$. Then by Hopf extension theorem we know that $\bar{\mu}$ is an outer measure on the family of all subsets of X , $\bar{\mu}$ is equal to μ on \mathcal{A} and $\mathcal{A} \subset \bar{\mathcal{A}}$, where $\bar{\mathcal{A}}$ is the family of all $\bar{\mu}$ -measurable subsets of X . **Theorem.** If (X, \mathcal{A}, μ) is a decomposable measure space, then $\bar{\mathcal{A}} = \mathcal{A}'$ and $\bar{\mu} = \mu'$ for sets in $\bar{\mathcal{A}}$. And there exist nondecomposable measure spaces such that $\bar{\mathcal{A}} \neq \mathcal{A}'$. (Received May 22, 1967.)

67T-504. TATSUJI KAMBAYASHI, University of Pisa, Pisa, Italy. Semiderivation is a representable functor. Preliminary report.

Let p be a prime, and R a commutative ring with 1 having characteristic p . For each integer $\nu > 0$, denote by R_ν the subring of all p^ν th powers of elements of R . By definition, a ν -special semiderivation of R into an R -module M , originally introduced by J. Dieudonné [Comm. Math. Helv. 28 (1954), 87-118], is an R_ν -linear mapping $R \rightarrow M$ which vanishes on $1 \in R$. Let $D_\nu(R, M)$ be the set of all such ν -special semiderivations, and set $D(R, M) = \bigcup_{\nu=1}^{\infty} D_\nu(R, M) =_{\text{def.}}$ all semiderivations from R to M . Both sets are made naturally into R -modules. **Theorem.** For every R as above one can construct an inverse system $\{J_\nu \leftarrow J_{\nu+1}; \nu > 0\}$ of R -modules and countably many semiderivations $j_\nu: R \rightarrow J_\nu$ in such a manner that every semiderivation $d: R \rightarrow M$ factors uniquely through $j = \text{proj} \lim j_\nu$ by a continuous R -linear mapping $f_M(d): J = \text{proj} \lim J_\nu \rightarrow M$ (thus, $d = f_M(d)j$), resulting in a canonical isomorphism, with M variable, $f_M: D(R, M) \xrightarrow{\sim} \text{Com}_R(J, M) =_{\text{def.}}$ all continuous R -linear mappings $J \rightarrow M$, where the topology on J is the natural inverse-limit topology. By analogy with the ordinary derivation-differential relationship, J may be called the module of semidifferentials of R . (Received May 22, 1967.)

67T-505. PIERRE BERTHIAUME, University of Montreal, Montreal, Canada. On adjoints of functors between functor categories.

In Adjoint functors (Trans. Amer. Math. Soc. 87 (1958), 294-329), Kan showed that if a functor $S: \underline{X} \times \underline{Y} \rightarrow \underline{Z}$ has a right adjoint $\underline{T}: \underline{Y}^* \times \underline{Z} \rightarrow \underline{X}$, then the corresponding "lifted" functor $T^V: (\underline{V}, \underline{Y})^* \times \underline{Z} \rightarrow (\underline{V}^*, \underline{X})$, \underline{V} any small category, has a left adjoint $(\underline{V}^*, \underline{X}) \times (\underline{V}, \underline{Y}) \rightarrow \underline{Z}$. This article shows that a criterion for the existence of the right adjoint to a functor between functor categories, and its construction, recently published by M. André (Categories of functors and adjoint functors, Am. J. Math. 88 (1966)) can be derived from the above theorem of Kan, and is in fact equivalent to it. A simpler criterion is also obtained together with various representations for functors. (Received May 25, 1967.)

67T-507. J. E. HUMPHREYS, University of Oregon, Eugene, Oregon. Ordinary representations of algebraic linear groups.

Let K be an algebraically closed field of prime characteristic, G a connected affine algebraic group defined over K , $f: G \rightarrow GL(n, K)$ an ordinary (not necessarily rational) matrix representation of G . Then if $x \in G$ is semisimple (resp. unipotent), $f(x)$ is a semisimple (resp. unipotent) matrix. As a corollary, the matrix group $f(G)$ is closed under taking semisimple and unipotent parts. The proof of our assertion is trivial when x has finite order and follows from elementary properties of tori when x has infinite order (see Lemma 8.3, p. 47, in A. Borel, Groupes linéaires algébriques, Ann. of Math. (2) 64 (1956)). The result fails, however, at characteristic 0. (Received May 26, 1967.)

67T-508. C. M. CHAMBERS and G. L. FINCH, University of Alabama, University, Alabama 35486. Fourier-Bessel series of generalized functions.

For each real number $\mu \geq -1/2$, consider the space \mathcal{S}_μ of infinitely differentiable, complex-valued functions on $(0, \infty)$ topologized by the countable family of separating seminorms $\Lambda_{n, l}^\mu(\phi) = \sup_{x \in (0, \infty)} |x^n ((1/x)(d/dx))^l (x^{-\mu-1/2} \phi(x))|$. \mathcal{S}_μ is a first axiom, Hausdorff, locally convex, sequentially complete, linear topological space. Let \mathcal{S}'_μ be the dual space with the weakest topology in which the linear functionals are continuous. Zemanian (J. Siam Appl. Math. 14 (1966), 561 and 678) has shown that (1) the ordinary Hankel transform of order μ is an automorphism of \mathcal{S}_μ . (2) The classical Parseval formula induces an automorphism (Hankel transform) of the generalized functions \mathcal{S}'_μ . (3) If $f \in \mathcal{S}'_\mu$ has compact support, then its Hankel transform is a regular distribution given by: $F(y) = \langle f, \phi_y \rangle$, where $\phi_y(x) = (xy)^{1/2} J_\mu(xy) \in \mathcal{S}_\mu$. Several miscellaneous theorems have been obtained and Fourier-Bessel series of generalized functions have been examined. A further table of transform pairs of generalized functions has been developed and some interesting distributional boundary-value problems have been solved in closed form. (Received May 26, 1967.)

67T-509. F. D. SENTILLES, Louisiana State University, Baton Rouge, Louisiana 70803. Compact and weakly compact operators on $C(S)_\beta$.

Let S and T be locally compact Hausdorff spaces. The β or strict topology on $C(S)$ is defined by the seminorms $P\phi(f) = \|\phi f\|$ for $\phi \in C_0(S)$ and $C(S)_\beta$ has strong dual the space $M(S)$ of bounded regular Borel measures on S with variation norm. Consequently, a continuous linear operator A from $C(S)_\beta$ to $C(T)_\beta$ has the integral representation $[Af](x) = \int_S f(y)\lambda(x, dy)$ where $\lambda(x, \cdot) \in M(S)$ for $x \in T$. Using this we obtain the Theorem. There is a β -neighborhood of O, V , in $C(S)$ such that $A(V)$ is weakly relatively compact (β relatively compact) in $C(T)_\beta$ if and only if A is continuous with the norm topology on $C(T)$ and A takes the unit ball in $C(S)$ into a weakly relatively compact (β -relatively compact) set in $C(T)_\beta$. (Received May 26, 1967.)

67T-510. CHARLES WELLS, Case Western Reserve University, Cleveland, Ohio 44106. A generalization of the regular representation of finite Abelian groups. II.

$M(d)$, A , α , β , $K(d)$, m and ϕ are as defined in Abstract 67T-271, these *Notices* 14 (1967), 409. Let $A(d)$ be the subgroup of A of d th roots of unity. Then $M(d) \cong A(d) \text{ wr } \text{Sym}(\alpha)$. Such a group is isomorphic to Ore's "symmetry of degree α of $A(d)$ " (Theory of monomial groups, Trans. Amer. Math. Soc. 51 (1942), 15-64) and the following theorems follow from his work. I. $Z(M(d)) \cong A(d)$. II. All complements of $K(d)$ in $M(d)$ are conjugate if and only if d is odd. III. If $\alpha > 2$, or if $\alpha = 2$ and $m = 2k$, k odd, then $K(d)$ is a characteristic subgroup of $M(d)$. IV. The derived group of $M(d)$ is the set of mappings ϕ for which $a_1 \dots a_\alpha = 1$ and whose canonical image in $M(d)/K(d) \cong \text{Sym}(\alpha)$ is in $\text{Alt}(\alpha)$. (Received June 3, 1967.)

67T-511. MARTIN FRIED and GEORGE BACHMAN, 88-12 151 Avenue, Howard Beach, New York 11414. Measure and integration in nonarchimedean Banach spaces.

Measure and integration theory is first developed for the case where μ is a set function on a σ -algebra \mathcal{S} of subsets of an abstract space Ω and where both μ and the integrable functions assume values in F , a field with nonarchimedean valuation. Definitions are similar but not identical to those considered by V. Celeste (Brooklyn Polytechnic Inst., Ph.D. Dissertation 1966) and results obtained represent a generalization of his work. Integration theory is next considered for functions defined on $(\Omega, \mathcal{S}, \mu)$ assuming values in a Banach space X over F , with F complete. Results analogous to those obtained for the classical situation are developed for both Bochner and Pettis type integrals. Lastly, an integral is introduced for functions defined on $(\Omega, \mathcal{S}, \mu)$ and assuming values in a Banach algebra X over F , with F complete. $x: \Omega \rightarrow X$ is called homomorphism integrable over Ω if for every $E \in \mathcal{S}$, there exists a vector $y_E \in X$ such that $h(y_E) = \int_E h x d\mu$ for all homomorphisms h . The integral is meaningful if F is the complex field and most results obtained apply to the classical setting. Important results include sufficient conditions for uniqueness and a characterization of the spectral radius for vectors y_E . (Received May 29, 1967.)

Using Seidel's result [Strongly regular graphs of L_2 -type and of triangular type, Indag. Math. (to appear)] it is shown that necessary conditions in terms of norm-residue symbols for existence of partially balanced incomplete block designs with non- $L_2(4)$ and non-T(8) schemes are the same as those for corresponding designs with $L_2(4)$ and T(8) schemes. Another result follows. Theorem. Existence of a two-classes association scheme A on a set V of v vertices with parameters $v, n_1, p_{11}^1, p_{11}^2$ such that (i) $v = 3n_1 - 2p_{11}^1 - 1; n_1 = 2p_{11}^2$ and $m^2 + m(p_{11}^1 - p_{11}^2 - 2n_1) + vp_{11}^2 = 0$ has an integral solution $\geq n_1 - p_{11}^1$, (ii) a partition (V_1, V_2) of V exists with m and $v - m$ elements respectively such that every vertex in $V_1(V_2)$ is adjacent to $m - n_1 + p_{11}^1$ (respectively p_{11}^2) vertices in V_1 (respectively V_2), implies the existence of an association scheme A^* with vertex set $x \cup V$ and parameters $v^* = v + 1, n_1^* = m, p_{11}^{1*} = m - n_1 + p_{11}^1, p_{11}^{2*} = m - n_1 + p_{11}^2$. Adjacency in A^* is given by (i) x is adjacent (nonadjacent) to all vertices in V_1 (respectively V_2); (ii) two vertices both in V_1 or V_2 are adjacent if and only if they are adjacent in A, (iii) v_1 in V_1 and v_2 in V_2 are adjacent if and only if they are nonadjacent in A. (Received May 29, 1967.)

67T-513. MAURICE CHACRON, University of Sherbrooke, Quebec, Canada. On the lattice of partitions of finite degree.

Let \mathbb{L} denote the lattice of partitions of a finite set E and let K be a subset of \mathbb{L} such that: (i) $(\forall \pi) (\pi \in K \Rightarrow \|E/\pi\| \leq r)$; (ii) $\prod_{\pi \in K} \pi = 0$; (iii) $\|K\|$ is minimum for (i) and (ii) ($\| \cdot \|$ denote the cardinality, E/π the set of classes module π , r a fixed natural integer). In the case $r = 2$, R. M. Karp stated that \prod -expressibility on at least one K with properties (i) - (iii) could be characterized by means of certain numerical conditions, for example, π is \prod -expressible iff $\lfloor \log_2 \|E/\pi\| \rfloor + \lfloor \log_2 (\text{Max}_{x \in E} \|\pi(x)\|) \rfloor = \lfloor \log_2 \|E\| \rfloor$. (For this result, see Some techniques of state assignment for synchronous sequential machines, IEE Trans. on electronic computer, Vol ECB, pp 507-158, Oct. 64.) It is shown that his result does not depend on the choice of r. Moreover, three new cases of \prod -expressibility are given by means of such numerical conditions. (Received June 2, 1967.)

67T-514. MORRIS MARDEN, University of Wisconsin, Milwaukee, Wisconsin. On the derivative of canonical products.

Let $f(z) = \prod_{j=1}^{\infty} G(z/a_j, p)$ where $G(u, p) = (1 - u) \exp[u + (1/2)u^2 + \dots + (1/p)u^p]$, $0 < |a_1| \leq |a_2| \leq \dots$, $\sum_{j=1}^{\infty} |a_j|^{-p-1} < \infty$. We show that if f is real and if $a_j = A_j \exp(i\alpha_j) = b_j + ic_j$, no nonreal zero $\zeta = \xi + i\eta$ of the derivative f' of f lies in all the circular regions $\Gamma(\zeta, a_j) = \{(\xi - b + c \tan p\alpha)^2 + \eta^2 - c^2 \sec^2 p\alpha\} \cdot \cos p\alpha > 0$. This is analogous to Jensen's theorem for f' when f is a real polynomial. A study is also made of the zeros of f' when $2\pi(k/p) \leq \arg a_j \leq \alpha + 2\pi(k/p)$ with $\alpha < \pi/p$, for $k = 0, 1, \dots, p - 1$ and all j. When all but one of the a_j are known to lie in a section $0 \leq \arg z < \pi/p(p + 1)$, the location of all, except at most one, zero of f' is determined. (Received June 2, 1967.)

67T-515. N. C. A. daCOSTA, University of Paraná, Caixa Postal 1963, Curitiba, Paraná, Brazil. Universes of Ehresmann-Dedecker.

As it is well known, to solve the difficulties of category theory, it is convenient to introduce in set theory the notion of universe of Sonner-Grothendieck (cf., for instance, our Abstract 65T-256, these *Notices* 12 (1965), 472). A set \underline{u} is a universe of Sonner Grothendieck if it satisfies certain conditions; as Ehresmann and Dedecker have pointed out, it seems interesting to suppress the condition $\forall x (x \in \underline{u} \supset x \subset \underline{u})$ from the definition of a universe \underline{u} . We study these new sets, called here universes of Ehresmann-Dedecker, and describe new forms of set theory where these universes exist. (Received June 7, 1967.)

67T-516. N. C. A. da COSTA, University of Paraná, Caixa Postal 1963, Curitiba, Paraná, Brazil and L. DUBIKAJTIS, University N. Copernicus, Toruń, Poland. On Jaśkowski's discursive propositional calculus.

Jaśkowski has described in (*Studia Societatis Scientiarum Torunensis* A (5) I (1948), 57-77, and (8) I (1949), 171-172) a propositional calculus, called by him discursive calculus; in this work the following questions are considered: (1) we give axiomatics for Jaśkowski's and related calculi; (2) we treat the problem of obtaining characteristic matrices for the discursive calculus; (3) we show how the discursive propositional calculus may be extended to a discursive predicate calculus of first order (or of higher order). Several problems are formulated. (Received June 7, 1967.)

67T-517. JAMES SERRIN, University of Minnesota, Minneapolis, Minnesota 55455. The Dirichlet problem for nonuniformly elliptic partial differential equations.

We consider the quasi-linear second order differential equation $\mathcal{A}(Du)D^2u = \mathcal{B}(u,Du)$, where Du and D^2u denote the sets of first and second partial derivatives of the dependent variable $u(x)$, and $x \in E^n$. The coefficient matrix \mathcal{A} obeys $\xi\mathcal{A}\xi > 0$ for all real vectors ξ , but is not assumed to be uniformly elliptic. With the equation normalized so that $\text{Trace } \mathcal{A} = 1$, we have the following typical result: Suppose the functions $\mathcal{A}(p)$ and $\mathcal{B}(u,p)$ are continuously differentiable for all real values of u and $p = (p_1, \dots, p_n)$, and that $\mathcal{B}(0,0) = 0$, $\partial\mathcal{B}/\partial u \geq 0$. Assume also that $(1 + |\mathcal{B}|)/\mathcal{L} \leq C \log |p|$ for large p , where $\mathcal{L} = (p - p_0)\mathcal{A}(p)(p - p_0)$ and C depends only on bounds for $|u|$ and $|p_0|$. Then the Dirichlet problem is uniquely solvable for arbitrary C^2 boundary data in any smoothly bounded convex domain Ω . The condition on \mathcal{B} can be relaxed if Ω is suitably restricted. Thus if $|\mathcal{B}| \leq \gamma|p| + \mathcal{B}'$ where γ is a positive constant and $\mathcal{B}'/\mathcal{L} \leq C \log |p|$, then the Dirichlet problem is uniquely solvable whenever $\kappa \geq \gamma$ on the boundary of Ω (here κ denotes the minimum normal curvature of the boundary). Similar results can be proved when \mathcal{A}, \mathcal{B} , and γ all depend on x, u, p , though here additional complications necessarily arise. (Received June 1, 1967.)

67T-518. MARTIN SCHECHTER, Belfer Graduate School of Science, Yeshiva University, New York, New York 10033. On intermediate extensions. III.

The notation and terminology follow Abstract 67T-200, these *Notices* 14 (1967), 275, and Abstract 66T-532, these *Notices* 13 (1966), 862. Let $a(u,v)$ be the bilinear form corresponding to the $W - Z$ extension A of A_0 . For each $u \in W$ there is an $f \in Z^*$ such that $a(u,v) = (f,v)$, $v \in Z$. Define the operator \hat{A} from W to Z^* by $\hat{A}u = f$. Theorem 1. If Y is dense in Z^* and $\hat{A} \in \Phi(W, Z^*)$, then $A \in \Phi(X, Y)$ and $i(A) = i(\hat{A})$. Theorem 2. If in addition Z is reflexive, then $v \in D(A^*)$ and $A^*v = g$ iff $v \in Z$, $g \in X^*$ and $a(u,v) = (u,g)$, $u \in W$. Theorem 3. Let K be a compact operator from W to Z^* which maps $D(A_0)$ into Y . Let B_0 be the operator $A_0 + K$ on $D(A_0)$. Then B_0 has a $W - Z$ extension B . Under the hypotheses of Theorem 2, $B \in \Phi(X, Y)$ and $i(B) = i(A)$. (Received June 7, 1967.)

67T-519. TILLA KLOTZ, University of California, Los Angeles, California 90024. A complete R_Λ -harmonically immersed surface in E^3 on which $H \neq 0$.

An example is given of a complete surface immersed in E^3 harmonically with respect to the conformal structure determined upon it by the positive definite form II' defined by $(H^2 - K)^{1/2} II' = HII - KI$, where $K < 0$ is Gauss curvature, and $H < 0$ is mean curvature. It had previously been shown (T. Klotz, Pacific J. Math. 21 (1967), 79-87) that H cannot be bounded away from zero on any complete surface in E^3 which is harmonically immersed with respect to the conformal structure determined upon it by any fixed positive definite linear combination $fI + gII$ of the fundamental forms with f and g smooth real valued functions. (Received June 5, 1967.)

67T-520. E. R. FERNHOLZ, Columbia University, New York, New York 10027. Envelopes of holomorphy on Stein spaces.

Let X be a Stein space. A connected analytic space Y is spread over X if there exists an analytic local homeomorphism $\pi: Y \rightarrow X$. Y is a weak domain of holomorphy if for any other connected analytic space Y' spread over X such that $Y \subset Y'$ then $H(Y) \neq H(Y')$. An envelope of holomorphy of Y is a weak domain of holomorphy Y' spread over X such that $Y \subset Y'$ and $H(Y) = H(Y')$. Theorem 1. If X has no locally reducible singularities then an envelope of holomorphy exists for any Y spread over X . Theorem 2. Any weak domain of holomorphy Y spread over X is biholomorphic to an open subset D of some Stein space X' and D is also a weak domain of holomorphy in X' . Theorem 3. If X has only isolated singularities then a weak domain of holomorphy spread over X is a Stein space. (Received June 5, 1967.)

67T-521. JOHN De CICCIO, Illinois Institute of Technology, Chicago, Illinois, and ARUN WALVEKAR, 7, Motilal Mansion, Parel, B, Bombay 12 (DD), India. The Dubois-Reymond and Euler-Lagrange equations for higher order derivatives.

The opulence Ω_{2n+3} of dimension $(2n+3) \geq 5$, is composed of the totality of differential elements of order $n \geq 1$, in a Euclidean space E_3 . An admissible arc is a union in this opulence Ω_{2n+3} , such that it possesses a finite number of corners of order $n \geq 1$. The first and second variations $I_1(\eta, \epsilon)$, and $I_2(\eta, \epsilon)$, are studied for the variation problem: $I(E) = \int_1^2 f dx = \text{minimum}$, where f is of at least

class $(n + 1) \geq +2$ in an open region of the opulence Ω_{2n+3} . By an extension of a fundamental lemma, a series of Dubois-Reymond integral equations are obtained. Thus $\sum_{a=0}^k (-1)^a (d^{k-a}/dx^{k-a})(\partial f/\partial y^{n-a}) + \sum_{a=k+1}^n (-1)^a \int_{x_0}^x dx_{a-k} \int_{x_0}^x dx_{a-k-1} \dots \int_{x_0}^x (\partial f/\partial y^{n-a}) dx_1$, is a polynomial in x alone whose degree does not exceed $(n - k - 1)$, for $0 \leq k \leq (n - 1)$. There is a similar set of n integral equations obtained with y replaced by z . In particular, with suitable restrictions on the admissible arcs, the system of two Euler-Lagrange differential equations, each of order $2n \geq 2$, are obtained. If a corresponding Hilbert determinant is not zero, there is found a system of ω^{4n} extremals. Applications to geometry and physics are studied. (Received June 5, 1967.)

67T-522. T. P. WHALEY, 1409 C 25th Avenue South, Nashville, Tennessee 37203. Infinite algebras satisfying the descending chain condition. Preliminary report.

Let m^+ denote the cardinal successor of the cardinal m . Theorem. Any algebra of power $m \geq \omega$ having at least one operation of rank two or more and satisfying the descending chain condition for subalgebras can be embedded in an algebra of power m^+ satisfying the descending chain condition. Corollary. If there is an algebra of power $m \geq \omega$ having finitely many operations and satisfying the descending chain condition then there is an algebra of power m^+ having finitely many operations and satisfying the descending chain condition. Corollary. For any $n \in \omega$ there is an algebra of power ω_n having one binary operation and satisfying the descending chain condition. Theorem. If $m \geq \omega$ is a regular cardinal, then any lattice of power m has a proper sublattice of power m . Corollary. No infinite lattice satisfies the descending chain condition for sublattices. The proof of the first theorem uses methods borrowed from unpublished work of C. C. Chang. (Received June 5, 1967.)

67T-523. ISRAEL NAVOT, Technion, Israel Institute of Technology, Haifa, Israel. A new representation of Hurwitz's determinants in the expansion of certain continued fractions.

Given a real strictly Hurwitz polynomial $H_n(s) = a_0 \prod_{\nu=1}^n (s - s_\nu)$, $n = 3, 4, \dots$, the standard method of calculating the continued fraction expansion (c.f.e.) of (1) $\{[\text{Odd } H_n(s)]/[\text{Even } H_n(s)]\}^{\pm 1}$ about its pole at infinity uses Routh's scheme, or Hurwitz's determinants (the two are equivalent in a certain matrix sense, see [1] Gantmacher, The theory of matrices, Vol. 2, New York, 1959), associated with $H_n(s)$ in testing it for zeros with negative real parts. In network theory cases are encountered where knowledge of the zeros of $H_n(s)$ precedes that of its coefficients and one would then prefer to have formulas for the coefficients in the c.f.e. of (1) directly in terms of the former rather than the latter. This is achieved by expressing Hurwitz's determinants Δ_r , $r = 0, 1, \dots, n$, in [1] as bi-alternants in the zeros of $H_n(s)$ and reads: (2) $\Delta_r = (-)^r r^{(r+1)/2} a_0^r A(0, 1, \dots, n - r - 1, n - r + 1, \dots, n + r - 1) / A(0, 1, \dots, n - 1)$ where the alternant in the denominator is the Vandermonde in s_1, s_2, \dots, s_n whereas the one in the numerator is obtained from it on replacing the exponents $0, 1, \dots, n - 1$ by $0, 1, \dots, n - r - 1, n - r + 1, \dots, n + r - 1$. (2) is an identity in a_0 and s_1, s_2, \dots, s_n independently of $H_n(s)$ being Hurwitz and it contains Orlando's formula ($r = n - 1, n$) in [1] as the simplest special case. Examples include $H_n(s) = \prod_{\nu=1}^n [s + i \exp(2\nu - 1) i \pi / 2n]$ and $(s + 1)^n$. (Received June 5, 1967.)

67T-524. M. B. POUR-EL, University of Minnesota, Minneapolis, Minnesota 55414. Hyper-
simplicity as a necessary and sufficient condition for nonindependent axiomatization. I.

All theories discussed here will be assumed to be both consistent and axiomatizable. Except for a brief excursion into intuitionistic logic, we also assume that all theories are formulated as applied predicate calculi. Theorem 1. The following three conditions are equivalent (a) \mathcal{T} is not independently axiomatizable. (b) Given any recursively enumerable axiomatization $\alpha_0, \alpha_1, \dots$ for \mathcal{T} the set A defined by (1) $A = \{i \mid \alpha_i \text{ is deducible in the predicate calculus from the } \alpha_j\text{'s for } j < i\}$ is hyper-simple. (c) There exists an axiomatization $\alpha_0, \alpha_1, \alpha_2, \dots$ for \mathcal{T} such that the set A defined by (1) is hypersimple. Remark. Theorem 1 holds for intuitionistic logics (in fact it holds for much weaker systems). (Received June 7, 1967.)

67T-525. C. C. CHEN, Queen's University, Kingston, Ontario, Canada and G. A. GRÄTZER, University of Manitoba, Winnipeg, Manitoba, Canada. Construction of Stone lattices. II.

For terminologies used, see Abstract 67T-364, these *Notices*, 14 (1967), 527. Let L, M be Stone algebras. Denote by ρ_b^L (or simply by ρ_b) the mapping from D_L into $b\phi_L$ ($b \in B_L$) determined by $[x\rho_b^L, 1] = [x, 1] \cap b\phi_L$ for each x in D_L ; and let $L(B, D, \phi)$ denote the Stone algebra with skeleton B , dense set D and structure mapping ϕ . Then: (1) L is a subalgebra of M iff B_L is a subalgebra of B_M , D_L is a sublattice of D_M ($1 \in D_L$), and $b\phi_L = b\phi_M \cap D_L$ for all b in B_M ; (2) Let B be a subalgebra of B_L , D a sublattice of D_L ($1 \in D$). Then there exists a homomorphism $\phi: D \rightarrow I(D)$ such that $L(B, D, \phi)$ is a subalgebra of L iff $d\rho_b^L \in D$ for all d in D , a in B ; (3) Let $f: L \rightarrow M$ be a homomorphism, $f_1 = f|_{B_L}$, $f_2 = f|_{D_L}$. Then $f_1: B_L \rightarrow B_M$, $f_2: D_L \rightarrow D_M$ are homomorphisms, f is one:one (resp. onto) iff f_1, f_2 are one:one (resp. onto); (4) Let $f_1: B_L \rightarrow B_M$, $f_2: D_L \rightarrow D_M$ be homomorphisms ($f_2 = 1$). Then there exists a homomorphism $f: L \rightarrow M$ extending f_1, f_2 iff $\rho_a^L f_2 = f_2 \rho_{af_1}^M$ for all a in B_L (or equivalently, $a\phi_L f_1 \subseteq af_1\phi_M$ for all a in B_L); (5) Let f_1 be a homomorphism from B_L onto a Boolean algebra B , f_2 a homomorphism from D_L onto a distributive lattice D . Then there exists a homomorphism $\phi: B \rightarrow I(D)$ and a homomorphism f from L onto $L(B, D, \phi)$ extending f_1, f_2 iff $b\phi_L f_2 = \{1\}$ for all b in Of_1^{-1} ; (6) $B_L \times M \cong B_L \times B_M$, $D_L \times M \cong D_L \times D_M$ and $b\phi_{L \times M} = b\phi_L \times b\phi_M$ for each $b = (b_1, b_2)$ in $B_L \times M$ (Received March 13, 1967.)

67T-526. R. I. SOARE, Cornell University, Ithaca, New York 14850. Recursion theory and Dedekind cuts.

Theorem. There is a semicreative Dedekind cut of every r.e. Turing degree. This strengthens a result of C. E. M. Yates (Duke J. Math. 32 (1965)), and has the following corollary which generalizes a result of C. G. Jockusch (Doctoral Dissertation, Massachusetts Institute of Technology, 1966).

Corollary. In every r.e. Turing degree there is a positive degree consisting of a single one-degree. Finally, in abstract 67T-52, these *Notices* 14 (1967), 144, Theorem 2 should be corrected to read: If a cut $L(A)$ is an increasing cylinder then A is not hyper-immune (but not conversely). (This material forms part of the author's doctoral dissertation written under Professor Anil Nerode at Cornell University.) (Received June 9, 1967.)

67T-527. W. A. KIRK, University of California, Riverside, California. On mappings with diminishing orbital diameters.

Let f be a mapping of a metric space M into itself. For $x \in M$, let $\mathcal{O}(f^n(x)) = \bigcup_{i=n}^{\infty} \{f^i(x)\}$, $n = 0, 1, \dots$, (where $f^0(x) = x$). If for each $x \in M$, $\lim_{n \rightarrow \infty} \delta(\mathcal{O}(f^n(x))) < \delta(\mathcal{O}(x))$ when $\delta(\mathcal{O}(x)) > 0$, then f is said to have diminishing orbital diameters. Theorem 1. Suppose M is compact and $f: M \rightarrow M$ is continuous with diminishing orbital diameters. Then for each $x \in M$, some subsequence of the sequence $\{f^n(x)\}$ has limit which is a fixed point of f . We next consider mappings which satisfy: (i) there is a constant C such that for each positive integer k and for each $x, y \in M$, $d(f^k(x), f^k(y)) \leq Cd(x, y)$. Theorem 2. Suppose f satisfies (i) and has diminishing orbital diameters. If for some $x \in M$ a subsequence of $\{f^n(x)\}$ has limit z , then $\lim_{n \rightarrow \infty} f^n(x) = z$, and $f(z) = z$. Theorem 2 generalizes a theorem of the author and Belluce [Abstract 67T-428, these *Notices* 14 (1967), 547]. An example is given showing that if (i) is deleted Theorem 2 becomes false. (Received June 9, 1967.)

67T-528. C. F. DUNKL, Princeton University, Princeton, New Jersey 08540, and J. F. FOURNIER, University of Wisconsin, Madison, Wisconsin 53706. Some extremal properties not possessed by regular polygons.

Let C and D be concentric closed discs with radii 1 and $r > 1$ respectively. Consider n -gons P circumscribed about C and let Q be the figure $P \cap D$. Fejes Toth showed that the regular n -gon is the P for which the area of Q is a minimum (Mathematikai Lapok 10 (1959), 23-25). It depends on r , however, whether the regular n -gon minimizes the perimeter of Q . Theorem 1. There are numbers r_1 and r_2 with $\sec(\pi/2(n-1)) < r_1 < \sec(\pi/n) < r_2$ such that: for $r > r_2$ the regular n -gon yields the Q of minimal perimeter, for $r_1 < r < r_2$ the minimal perimeter for Q occurs for a P with one vertex outside D and equal angles at the other vertices, and for $r < r_1$ the Q of minimal perimeter occurs for a P with two vertices outside D and equal angles at the other vertices. If $r = r_1$ or $r = r_2$ there are two figures Q of minimal perimeter. Consider now the dual situation with $r < 1$, P inscribed in C , and $Q = \text{convex hull}(P \cup D)$. Fejes Toth showed that the regular n -gon maximizes the perimeter of Q . Theorem 2. There is a number $r_3 < \cos(\pi/n)$ so that for $r < r_3$ the regular n -gon maximizes the area of Q and for $r > r_3$ the Q of maximal area occurs for a P with one side intersecting D and the others exterior to D and of equal length. Again there are two extremal figures Q if $r = r_3$. (Received June 12, 1967.)

67T-529. T. B. MUENZENBERGER and R. E. SMITHSON, University of Florida, Gainesville, Florida 32603. Fixed points and proximate fixed points.

Definitions of ϵ -continuity and proximate fixed points are extended to multifunctions in uniform spaces. Let X be a compact, Hausdorff space. Theorem 1. If X has the p.f.p.p., then X has the F.p.p. Now for a given property P of sets let $P(X)$ be the subsets of X which have property P . Suppose that X has a symmetric base \mathcal{B} for its uniformity and that there is a function $K: \mathcal{P}(X) \rightarrow \mathcal{P}(X)$ (where $\mathcal{P}(X)$ denotes the collection of all subsets of the set X) such that \mathcal{B} , K , and P satisfy the following conditions: (1) If $\square \neq A \in \mathcal{P}(X)$, then $\square \neq K(A) \in P(X)$; (2) if $A \subset B \in \mathcal{P}(X)$, then $K(A) \subset K(B)$; (3) if $A \in P(X)$, then $K(A) = A$ and $A^* \in P(X)$; (4) if $A \in P(X)$ and if $U \in \mathcal{B}$, then

$U[A] \in P(X)$. Theorem 2. If, in addition to the above assumptions, X has the F.p.p. for point P , u.s.c. multifunctions, then: (i) X has the p.F.p.p. for point P , uniformly upper V -continuous multifunctions; (ii) X has the p.F.p.p. for point P , V -continuous multifunctions. Corollary 1. Each nonempty, compact, convex subset of a locally convex, Hausdorff topological vector space has the p.F.p.p. both for point convex, uniformly upper V -continuous multifunctions and for point convex, V -continuous multifunctions. Corollary 2. Each tree has the p.F.p.p. for point connected, uniformly upper V -continuous multifunctions. (Received June 12, 1967.)

67T-530. T. B. MUENZENBERGER, 125 North East 129th Street, North Miami, Florida 33161.

A proximate fixed-point theorem for multifunctions.

Suppose that X is a compact, Hausdorff space which has a symmetric base \mathcal{D} for its uniformity. For a given property P of sets, let $P(X)$ be the subsets of X which have property P . Suppose further that there is a function $K: \mathcal{P}(X) \rightarrow \mathcal{P}(X)$ (where $\mathcal{P}(X)$ denotes the collection of all subsets of the set X) such that P and K satisfy the following conditions: (1) If $A \in \mathcal{P}(X)$, then $A \subset K(A) \in P(X)$; (2) if $A \subset B \in \mathcal{P}(X)$, then $K(A) \subset K(B)$; (3) if $A \in P(X)$, then $K(A) = A$ and $A^* \in P(X)$; (4) $\{\{x\} | x \in X\} \cup \{\emptyset\} \subset P(X)$; (5) if $A \in P(X)$ and if $U \in \mathcal{D}$, then $U[A] \in P(X)$. If, in addition to the above assumptions, X has the F.p.p. for point closed, point P , u.s.c. multifunctions, then for each open cover \mathcal{U} of X there exists a finite open cover \mathcal{V} of X which has the following property: if $G: X \rightarrow X$ is any multifunction such that for each $x \in X$ there exists a neighborhood N of x satisfying $G(N) \subset V$ for some $V \in \mathcal{V}$, then there is a point $x \in X$ such that $x \in U$ and $G(x) \subset U$ for some $U \in \mathcal{U}$. Two examples of spaces X with the aforementioned properties are nonempty, compact, convex subsets of locally convex, Hausdorff linear topological spaces and hereditarily unicoherent, arcwise connected, locally connected continua (trees). (Received June 12, 1967.)

67T-531. K. E. GUSTAFSON, Battelle Institute, Advanced Studies Center, 1227 Geneva,

Switzerland. Positive operator products.

Theorem. Let $0 < m_A \leq A \leq M_A$ and $0 < m_B \leq B \leq M_B$ be self adjoint operators with the indicated semibounds. Consider all B such that $(M_A - m_A) \cdot (M_A + m_A)^{-1} \leq m_B M_B^{-1}$ for a given A (or, equivalently, vice versa). Then $\text{Re}(B A x, x) \geq 0$. Remarks. There are no commutativity conditions assumed. The above theorem actually holds, with more notation necessary, for operators on a Banach space, positiveness in the (Lumer) semi-inner product sense, and therefore is a bona-fide numerical range, rather than spectral, conclusion. (Received June 12, 1967.)

67T-532. J. P. JONES, University of Washington, Seattle, Washington 98105. Degrees of

indecidability of number theories. Preliminary report.

Notions of theory and subtheory as well as the theories R , Q , and N are those of Tarski, Mostowski, and Robinson (Undecidable theories, Amsterdam, 1953). The subtheories of the complete theory N are referred to here as number theories. By the degree of undecidability of a theory we mean the Turing degree of recursive unsolvability associated with the set of Gödel numbers of its theorems. For example, the degree of undecidability of R is $0'$ and that of N is 0^ω . Feferman (Degrees of unsolvability associated with classes of formalized theories, J. Symbolic Logic 22 (1957),

161) has shown that every degree of unsolvability is the degree of undecidability of some first order theory. Our theorem characterizes the degrees of unsolvability associated in this way with number theories. Recall that a degree is said to be complete if it is the jump of some other degree. Theorem. A degree d is a degree of undecidability of some number theory if and only if d is complete. It is further proven that the number theory T associated with a given complete degree may always be chosen so that R is a subtheory of T and T is a subtheory of Q . Corollary. There is a continuum of theories between R and Q . (Received June 13, 1967.)

67T-533. WITHDRAWN.

67T-534. V. MARIC, University of Kentucky, Lexington, Kentucky. An application of the Bergman-Whittaker operator.

Let $\vec{x} = (x, y, z)$, $u = ((x + iy + z)/2)\zeta + ((iy - z)/2)\zeta^{-1}$ and let $H(\vec{x})$ be an axially symmetric harmonic function regular in the unit sphere, with the associate function $f(u)$. Theorem. If the expansion of $H(\vec{x})$ in spherical harmonics has Hadamard's gaps, i.e. $H(\vec{x}) = \sum_{k=0}^{\infty} a_k R_{n_k}^{n_k} P_{n_k, 0}(\cos \theta)$ ($R < 1$) where $n_{k+1}/n_k \geq q > 1$, $k \geq 1$, and if the coefficients a_k satisfy in addition $\sum_{k=0}^{\infty} |a_k| = \infty$, then the harmonic function, reciprocal to $H(\vec{x}) - a$, in the sense of the composition introduced in Duke Math. J. 30 (1963), 447-460, has infinitely many branch lines situated on the spheres $x^2 + y^2 + z^2 = |A_\nu(a)|^2$, where $A_\nu(a)$, $\nu = 1, 2, \dots$, are zeros of the equation $f(u) - a = 0$. The proof uses a result of W.H.J. Fuchs (Nagoya Math. J. 29 (1967), 167-175) and the Bergman-Whittaker operator. (Received June 12, 1967.)

67T-535. D. W. KUEKER, University of California, Los Angeles, California 90024. A generalization of Beth's Theorem on definability.

Let L be a first-order predicate language, and let $L(P)$ be the language formed by adding a new k -place predicate P to L . We use \underline{A} to denote structures for L and $(\underline{A}, \underline{P})$ to denote structures for $L(P)$, where \underline{P} is a k -place relation on \underline{A} . Theorem. Let T be a theory in $L(P)$ and let \underline{M} be the class of models of T . Let $n \geq 1$. Then the following are equivalent: (i) For every \underline{A} there are at most n relations \underline{P} such that $(\underline{A}, \underline{P})$ belongs to \underline{M} . (ii) There are formulas $G(v_1, \dots, v_m)$, $F_i(x_1, \dots, x_k, v_1, \dots, v_m)$, $1 \leq i \leq n$, of the language L such that $T \vdash \exists v_1, \dots, v_m G$ and $T \vdash \forall v_1, \dots, v_m (G \rightarrow \bigvee_{1 \leq i \leq n} \forall x_1, \dots, x_k (P(x_1, \dots, x_k) \leftrightarrow F_i))$. For $n = 1$ this is Beth's Theorem (Indag. Math. 15 (1953), 330-339). Our proof shows that the Theorem also holds if either (1) T is a theory in any language containing $L(P)$ and \underline{M} is the class of $L(P)$ -reducts of models of T , or (2) T is given by a single sentence of L_{ω_1, ω_0} , the infinitary language which allows countable conjunctions and disjunctions (see Lopez-Escobar, Fund. Math. 57 (1965), 253-272). This improves some unpublished results obtained by Craig and Daigneault by methods different from ours. Generalizations to theories involving several new predicates P_i are also obtained. (Received June 14, 1967.)

67T-536. M. BHASKARAN, University of Illinois, Urbana, Illinois 61801. Diagonal forms over P-adic rings. Preliminary report.

Let A be a P-adic ring and let p be the rational prime above P . Let J_d be the subring of A generated by the d th powers of elements of A . Theorem 1. If A is unramified, then every diagonal form of degree m in more than $4m^4$ variables represents zero nontrivially. Theorem 2. Let d be any rational prime. Then every element in J_d can be represented by any diagonal form over A of degree d in more than $d + 1$ variables with coefficients rational integers prime to p . To prove Theorem 1, it is observed that if there are pm^2 coefficients in the given form, coefficients a_1, a_2, \dots, a_p can be found from those of the given form such that $a_1 x_1^m + \dots + a_p x_p^m \equiv 0 \pmod{p}$ with not all $x_i \equiv 0 \pmod{p}$ and consequently, if there are $4m^4$ variables in the diagonal form, $[(4m^4 - pm^2)/p] + 1$ diagonal forms can be found with coefficients taken from the given diagonal form such that these forms represent zero mod p nontrivially. To prove Theorem 2, it is observed that if D is a prime ideal factor of d , every element in A is a d th power mod D and every element prime to p in J_p is a p th power mod p .

(Received June 15, 1967.)

67T-537. L. D. LIPNER, University of California, Berkeley, California 94720. Generalized quantifiers and weak direct products.

Notation is that of Abstract 67T-410, these *Notices* 14 (1967), 542. If $\mu(0)$ is an individual parameter 0 (where μ is the fixed similarity type of Q_a) we may define weak direct products with respect to 0 as usual: $P^W \langle \mathcal{A}_i : i \in I \rangle = P \langle \mathcal{A}_i : i \in I \rangle \uparrow \{f \in P \langle A_i : i \in I \rangle \mid \{i \in I \mid f(i) \neq 0_i\} \text{ is finite}\}$. $\text{Pow}^W(K)$ and $\text{Prod}^W(K)$ then have the obvious meaning. Weakening the restriction on \mathcal{K}_a by assuming merely that it is regular, and with no use of GCH at all, we have the Theorem. All the results 1-5 of 67T-410 remain true for weak direct products in place of direct products. (Received June 16, 1967.)

67T-538. ALEXANDER ABIAN, Iowa State University, Ames, Iowa 50010. On the cofinality of ordinal numbers.

As usual, an ordinal number w is called cofinal with an ordinal number r if $w = \lim_{h < r} (F_h + 1)$ where $(F_h)_{h < r}$ is an increasing sequence of ordinals. Also, the smallest ordinal (which turns out to be a cardinal number) that is cofinal with w is denoted by $\text{cf}(w)$. Theorem 1. For every ordinal w and r it is the case that ω^w is cofinal with ω^r if and only if $w \geq r$ and $\text{cf}(\omega^w) = \text{cf}(\omega^r)$. Theorem 2. For every ordinal w and r it is the case that ω_w is cofinal with ω_r if and only if $w \geq r$ and $\text{cf}(\omega_w) = \text{cf}(\omega_r)$. (Received June 19, 1965.)

67T-539. M. S. ROBERTSON, 104 Hullihen Court, Newark, Delaware, 19711. Quasi-subordinate functions. Preliminary report.

Let $f(z) = \sum_0^\infty a_n z^n$ and $F(z) = \sum_1^\infty A_n z^n$ be analytic in $|z| < 1$. Let $\phi(z)$ and $w(z)$ be two bounded analytic functions, $|\phi(z)| \leq 1$, $|w(z)| \leq 1$ for $|z| < 1$. If $f(z) = \phi(z)F\{zw(z)\}$ in the unit disc we say that $f(z)$ is quasi-subordinate to $F(z)$. If $\phi(z) \equiv 1$, then $f(z)$ is subordinate to $F(z)$. If $w(z) \equiv 1$ then $f(z)$ is majorized by $F(z)$. Let $A_1 = 1$, $F(z) \neq 0$ for $0 < |z| < 1$ and $[F(z^2)]^{1/2} = \sum_0^\infty D_{2k+1} z^{2k+1}$. Then $|a_n| \leq \sum_0^{n-q} |D_{2k+1}|^2$, $n = q, q+1, \dots$. If $F(z)$ is univalent and spiral-like for $|z| < 1$ then $|a_n|$

$\leq n - q + 1, n = q, q + 1, \dots$. If the conjecture $\sum_1^n |D_{2k-1}|^2 \leq n, n = 1, 2, \dots$ is true when $F(z)$ is univalent in $|z| < 1$, then the generalized Bieberbach conjecture $|a_n| \leq n, n = 1, 2, \dots$ follows. In any case, if $F(z)$ is univalent in $|z| < 1$ then $|a_n| \leq n$ for $n = 1, 2$ and 3 and $\limsup_{n \rightarrow \infty} |a_n/n| < 1$ except when $F(z) = z(1 - \epsilon z)^{-2}, |\epsilon| = 1$, in which case $|a_n| \leq n$ for all n . If $F(z) = \int_0^{2\pi} \mu(z, \phi) d\alpha(\phi)$ where $\mu(z, \phi)$ is regular for $|z| < 1$ and continuous in $\phi, 0 \leq \phi \leq 2\pi, \mu(z, \phi) \neq 0$ in $0 < |z| < 1$ for all ϕ , and where $\alpha(\phi)$ is a normalized function of bounded variation in $[0, 2\pi]$ then $|a_n| \leq \sum_{k=1}^n \int_0^{2\pi} |c_{2k-1}(\phi)|^2 |d\alpha(\phi)|, n = 1, 2, \dots$, where $[\mu(z^2, \phi)]^{1/2} = \sum_0^\infty c_{2k+1}(\phi) z^{2k+1}$. In particular if $A_1 = 1$ and if $F(z)$ is typically real for $|z| < 1$ then $|a_n| \leq n, n = 1, 2, \dots$. (Received June 19, 1967.)

67T-540. CHARLES HALLAHAN, Rutgers University, New Brunswick, New Jersey. On Lie algebras of derivations. Preliminary report.

Let A be a finite-dimensional Lie algebra over a field of characteristic 0, L a completely reducible Lie algebra of derivations of A . It is shown that A has an L -invariant Levi decomposition, and that any two L -invariant factors of A are conjugate by an automorphism $\exp(\text{Ad}x)$, where x is an L -constant (i.e., $xD = 0$, all D in L) in the nil radical of A . Analogous results hold for associative algebras, and for Cartan subalgebras of solvable Lie algebras. (Received June 19, 1967.)

67T-541. GARY MILLER, University of Missouri, 5100 Rockhill Road, Kansas City, Missouri 64110. A countable locally connected quasimetric space.

We construct a countable quasimetric space S which is both connected and locally connected. This gives still another example of a countable connected Hausdorff space and apparently the first which is locally connected. The quasimetric given agrees with the usual metric when restricted to the rationals which are dense in S . Thus there is a completion of the metric space of the rationals which is a quasi-pseudo-metric space and contains S as a dense subspace. (Received June 19, 1967.)

67T-542. WALTER RUDIN, University of Wisconsin, Madison, Wisconsin 53706. Geometric conditions for algebraic varieties.

Theorem 1. If f is an entire function of n complex variables and if every point of the zero-set of f has at least one coordinate whose absolute value is less than 1, then f is the product of a polynomial and a zero-free entire function; in particular, the zero-set of f is an algebraic variety. Here are two generalizations: Theorem 2 (Main Result). An analytic variety of pure dimension k in the space C^n of n complex variables is algebraic if and only if a vector space basis for C^n can be chosen in such a way that the coordinates of every point (z_1, \dots, z_n) of V satisfy the inequality $|z_{k+1}| + \dots + |z_n| < A(1 + |z_1| + \dots + |z_k|)^B$, where A, B are positive constants. Theorem 3. If V is an algebraic variety in C^n , if Ω is a domain of holomorphy in C^n , if f is holomorphic in Ω and vanishes on $V \cap \Omega$, then $f = P_1 g_1 + \dots + P_r g_r$, where each P_i is a polynomial vanishing on V and each g_i is holomorphic in Ω . (Received June 21, 1967.)

67T-543. N. J. BEZAK, and M. EISEN, Gulf Research and Development Company, Pittsburgh, Pennsylvania 15230. Continuity properties of operator spectra. Preliminary report.

Let X and Y be Banach spaces and let \mathcal{T} be the set of linear operators T defined on domains $D(T) \subseteq X$ and having ranges $R(T) \subseteq Y$. Consider \mathcal{T} topologized by the pseudo-metric G of Gokhburg and Markus. Theorem 1. The conjugate mapping c , which assigns to $T \in \mathcal{T}$ (with strongly dense domain in X) its conjugate $c(T) = T'$, is continuous with respect to G . The spectrum mapping σ_e maps $T \in \mathcal{T}$ into its extended spectrum $\sigma_e(T)$, a nonvoid closed subset of the extended complex plane C_∞^1 topologized by the chordal metric χ . The class of nonvoid closed subsets of C_∞^1 has the metric topology of the Hausdorff distance D induced by χ . Theorem 2. σ_e is upper semicontinuous on \mathcal{T} . Theorem 3. If $T \in \mathcal{T}$ is such that T is either closed or has strongly dense domain in X and $\sigma_e(T)$ is totally disconnected, then σ_e is continuous at T . Theorem 4. Let $T \in \mathcal{T}$ be closed with nonvoid resolvent set. Suppose there is a neighborhood of closed operators \mathcal{U} of T which satisfies either (a) if $S \in \mathcal{U}$ then $D(ST) = D(TS)$ and $ST = TS$, or (b) \mathcal{U} is Newburgh commutative. Then σ_e is continuous at T . Theorems 2,3 and 4 are generalizations of results of J. D. Newburgh. (J. D. Newburgh, The variation of spectra, Duke Math. J. 18 (1951), 165-176). (Received June 21, 1967.)

67T-544. C. S. BALLANTINE, Oregon State University, Corvallis, Oregon 97331. Products of positive definite matrices. III.

For given positive integers m and n , this paper presents necessary and sufficient conditions (nasc) on a real $n \times n$ matrix S of positive determinant that S be the product of m positive definite real (symmetric $n \times n$) matrices. (No essentially new nasc are given for $m \leq 2$.) For $m = 3$, one nasc is: S is itself positive definite, or else S is nonsymmetric and its symmetric part, $(S + S')/2$, is not nonpositive definite. For $m = 4$, a nasc is: S is not a negative scalar matrix. Every real $n \times n$ matrix of positive determinant is the product of five (or more) positive definite real matrices. Also, when n is odd, every real $n \times n$ matrix of positive determinant is the product of four positive definite real matrices, and is the product of three if and only if it is not an indefinite symmetric matrix. The 2×2 results and fragments of the $n \times n$ results were given in the first two papers of this series (to appear). (Received June 20, 1967.)

67T-545. A. A. MULLIN, Department of Defense, SCC, HHC, EADC/EAR, APO 96212. Applications of number theory to aesthetics. II.

This note continues G. C. Birkhoff's program (Aesthetic measure, Cambridge, 1933) in relation to additive number-theory. For terminology see Abstract 67T-358, these *Notices* 14 (1967), 525. Definition 1. A SUGETANKA is a GETANKA in which if only the odd-numbered lines are selected, in order, forms a GETANKA. Lemma. There exists (in a constructive sense) a 13-line GETANKA with the syllable structure (3 5 3 7 5 7 5 11 7 13 11 13 13) which is a SUGETANKA. Definition 2. A HUGETANKA is a GETANKA in which each line has an odd prime number of symbols (alphabetic letters; strokes for ideographs) such that the total number of symbols in the poem is an odd prime number. Note. A HUGETANKA has a complex interaction of semantical, syntactical, and number-theoretic constraints. Lemma. There exists, constructively, a TANKA which is HUGETANKA. Problem. Estimate the number of essentially different SUGETANKA structures with a prescribed total number of syllables. (Received June 26, 1967.)

67T-546. E. J. ECKERT, California State College at Los Angeles, 5151 State College Drive, Los Angeles, California 90032. Continuity of polynomial operators in Banach spaces. Preliminary report.

Let K be a normal cone with interior in a real Banach space E . Let P be an operator from E to E satisfying $\Delta_n^{m+1}P(x) = 0$ for all x, h in E . If P is continuous it is called a polynomial of degree at most m . Theorem. If P is monotonic on K (i.e. $y \geq x \geq 0$ implies $P(y) \geq P(x) \geq 0$) then P is a polynomial of degree at most m . The example (on the real line) $P(x) = f^2(x)$, where f is a discontinuous solution to $f(x + y) = f(x) + f(y)$, shows that positivity of P (i.e. $x \geq 0$ implies $P(x) \geq 0$) is not sufficient to guarantee the continuity of P . (Received June 26, 1967.)

67T-547. M. C. McCORD, University of Georgia, Athens, Georgia. On Poincaré duality for generalized homology theories. Preliminary report.

Let $\mu: (\underline{A}, \underline{B}) \rightarrow \underline{B}$ be a pairing of spectra in the sense of G. W. Whitehead (Trans. Amer. Math. Soc. 102 (1962), 227-283). Assume there exists a "unit element" 1 in $\pi_0(\underline{A})$ such that the map $\pi_q(\underline{B}) \rightarrow \pi_q(\underline{B})$ given by $z \rightarrow \mu_*(1 \otimes z)$ is an isomorphism. By the method of Whitehead and by use of Milnor's realization of the singular complex of a space, one gets generalized cohomology and homology theories $h^*(\cdot; \underline{A})$, $h_*(\cdot; \underline{B})$, and $h^*(\cdot; \underline{B})$ defined on the category of all pairs of topological spaces. The pairing μ gives rise to a slant product $h^n((X, Y) \times (Z, W); \underline{A}) \otimes h_q(Z, W; \underline{B}) \rightarrow h^{n-q}(X, Y; \underline{B})$ for arbitrary pairs (X, Y) and (Z, W) , which is shown to have suitable properties. The element 1 in $\pi_0(\underline{A})$ defines canonical classes i_n in $h^n(\mathbb{R}^n, \mathbb{R}^n - 0; \underline{A})$. A topological n -manifold M is \underline{A} -orientable if there exists a class u in $h^n(M \times M, M \times M - \Delta; \underline{A})$ suitably related to i_n . Along the lines of Spanier (Algebraic topology, McGraw-Hill, New York, 1966), it is shown by use of the slant product that such a class u induces isomorphisms $h_q(M - L, M - K; \underline{B}) \approx h^{n-q}(K, L; \underline{B})$ for any compact triangulable pair (K, L) in M . Whitehead (in the paper quoted above) has a similar theorem for compact triangulable homology manifolds. (Received June 26, 1967.)

67T-548. MAX SHIFFMAN, Stanford University, Stanford, California. On the existence of subsonic flows.

The existence of subsonic flows of a compressible fluid can be obtained from a variational problem $\delta \iint F(p^2 + q^2) dx dy = 0$ where $p = \phi_x$, $q = \phi_y$ and $\phi(x, y)$ is the velocity potential. The stream function $\psi(x, y)$ is introduced, and the variational problem converted into $\delta \iint G(\psi_x^2 + \psi_y^2) dx dy = 0$ for a suitably defined integrand G . The existence of subsonic flows satisfying prescribed conditions have been obtained by the author in previous publications, in a range (or rather, open set) of prescribed conditions. They satisfy a certain 2nd order elliptic quasi-linear partial differential equation, (or 1st order system). This determines all such subsonic flows. As one approaches the boundary of the range (or rather, open set), the flows are nearly sonic somewhere. The determination of $G(\psi_x^2 + \psi_y^2)$ can be done neatly and conveniently, and the integrand G is extended beyond certain values of $\psi_x^2 + \psi_y^2$ conveniently and linearly for larger $\psi_x^2 + \psi_y^2$. Similar existence and uniqueness theorems can be established for any elliptic variational problem $\delta \iint G(p, q) dx dy = 0$ with $G_{pp} G_{qq} - G_{pq}^2 > 0$ for all p, q ; or for the corresponding 2nd order elliptic partial differential equation, or for the elliptic system of first order partial differential equations with the conjugate function. Likewise, when the integrand is $G(x, y, p, q)$ with suitable restrictions. (Received June 26, 1967.)

67T-549. GIACOMO DELLA RICCIA, Indiana University, Bloomington, Indiana 47401. Complex analysis methods applied to the study of branching processes.

Let $n = 0, 1, 2, \dots$ be discrete time and S the space of probability distributions over N states. Consider a stochastic process $T: S \rightarrow S$ defined by $\vec{x}_{n+1} = \vec{x}_n P(\vec{x}_n)$, $n = 0, 1, \dots$, $\vec{x}_0 \in S$, where $P(\vec{x}_n)$ is a row stochastic matrix whose entries are homogeneous polynomials of the components of \vec{x}_n , all polynomials having the same degree. This is a type of branching process which has been studied on electronic computers by S. Ulam and P. Stein. The main problem is the study of the limit set $L(T)$ of a sequence of iterated points $\{T^n(\vec{x})\}$, $\vec{x} \in S$. Mathematical results are obtained by replacing real variables (x^1, \dots, x^N) by complex variables (z^1, \dots, z^N) and using the concept of normal families in the sense of Montel applied to the family of holomorphic functions $\{T^n(\cdot)\}$. It can be shown that if $S_\lambda = S \cap D_\lambda$ where D_λ is a maximal domain on which the family is normal, the limit set $L_\lambda(T)$ is the same for all sequences of iterates starting with any $\vec{x} \in S_\lambda$. The topological boundary of S_λ consists of singularities of any limit function $T_\alpha = \lim_{i \rightarrow \infty} T^{n_i^a}$ where $\{n_i^a\}$ is a subsequence for which the convergence is uniform on every compact $K \subset S_\lambda$. $L_\lambda(T)$ is the image of S_λ under a T_α and furthermore interesting ergodic properties can be established for T when $\vec{x} \in L_\lambda(T)$. (Received June 26, 1967.)

67T-550. B. N. SAHNEY, Queen's University, Kingston, Ontario, Canada. Nörlund summability of derived Fourier series.

If $f(u)$ be even, $f(u) \in L(-\pi, \pi)$ and defined by periodicity outside this range. Nörlund summability of Fourier series has been discussed by Rajgopal (Proc. Cambridge Philos. Soc. 59 (1963), 47-53) and author (Pacific J. Math. 13 (1963), 251-262). Let a function $P(u)$ tending to ∞ with u , and a sequence $\{p_n\}$ be defined as follows in terms of: $p(u)$, monotonic decreasing and strictly positive for $u \geq 0$, $P(u) \equiv \int_0^u p(x) dx$, $p_n \equiv p(n)$. Rajgopal (loc. cit.) has indicated that the Nörlund method of summation is regular since $\{p_n/P(n)\} \rightarrow 0$ as $n \rightarrow \infty$. Nörlund summability of derived Fourier series has been discussed by Astrachan (Duke Math. J. 2 (1936), 543-568) and Prasad and Siddiqi (J. Ind. Math. Soc. 14 (1950), 159-170). The result proved here is independent of the previous ones. Let $\psi(u) = (1/2)\{f(x+u) - f(x-u)\}$ and $h(u) = ((\psi(u)/(2 \sin u/2)) - s)$. Theorem. If $h(u) = o(1)$ as $u \rightarrow 0$, $\int_0^t |dh(u)| = O(t)$ then the derived Fourier series is (N, p_n) summable to the value zero at the origin, provided that $\int_0^u p(x)/x dx = O(P(u)/u) \int_0^u |dp(u)| = o(1)$. (Received June 26, 1967.)

67T-551. NICK METAS, Queen's College, Flushing, New York 11367. A necessary condition for a Banach space to be projective.

Definition. Let $\{x_n\}_{n=1,2,\dots}$ be a sequence of elements in a Banach space X and let x be an element of X . The sequence $\{x_n\}_{n=1,2,\dots}$ is said to converge weakly to x if $\lim_{n \rightarrow \infty} f(x_n) = f(x)$ for each continuous linear functional f defined on X . Theorem. Let P be a projective Banach space. (For the definition of projective Banach space, see Abstract 64T-308, these *Notices* 11 (1964), 458.) Suppose the sequence $\{x_n\}_{n=1,2,\dots}$ in P converges weakly to the element x in P . Then the sequence $\{x_n\}_{n=1,2,\dots}$ converges strongly to x , i.e. $\lim_{n \rightarrow \infty} \|x_n - x\| = 0$ where $\| \cdot \|$ denotes the norm in P . (Received June 28, 1967.)

67T-552. R. N. GUPTA, Ramjas College, Delhi-7, India. On f-injective modules and semi-hereditary rings.

Let M be a unital right R -module over a ring R with unity. Let I be a right ideal of R . M is said to be I -injective if for every R -homomorphism $f: I \rightarrow M$ there exists $m \in M$ such that $f(x) = mx \forall x \in I$. A module M which is I -injective \forall finitely generated right ideal I is called f -injective. The following results are proved: (a) A module M is f -injective iff (i) $(p^v)^\perp = Mp \forall p \in R$. (ii) $(I \cap J)^\perp = I^\perp + J^\perp \forall$ finitely generated right ideals I and J , where $S^\perp \equiv \{m \in M : mS = 0\}$, for subsets S of R . (b) A ring R is right noetherian iff every f -injective module over R is injective. (c) A ring R is semihereditary iff each quotient module of every injective module is f -injective. (d) A ring R is semihereditary self f -injective iff it is regular. (Received June 27, 1967.)

67T-553. R. H. HERMAN, University of Rochester, River Campus Station, Rochester, New York 14627. Generalizations of weakly compact operators.

An operator is almost weakly compact (has property R), if whenever T has a bounded inverse on M , an infinite dimensional subspace, M is reflexive (contains an infinite dimensional reflexive subspace). These classes of operators contain both the strictly singular and weakly compact operators, but this containment is proper in general. Theorem 1. T has property R if and only if whenever T has a bounded inverse on M , infinite dimensional, it follows that there exists $N \subset M$, N infinite dimensional such that $T|N$ is weakly compact. Hence we can show that these operators form a closed ideal in $[X]$. The almost weakly compact operators are seen to have weakly compact restrictions and are closed under the addition of arbitrary compact operators. Further, we give conditions under which these new operators agree with strictly singular or weakly compact operators. Examples are given to show that the properties of these operators do not, in general, carry over to or from conjugates. But we do have, Theorem 2. If $T: X \rightarrow Y$ where Y has an unconditional basis, then T^* almost weakly compact implies T is. Further, if the basis is shrinking, then T^* has property R implies T is almost weakly compact. (Received June 28, 1967.)

67T-554. M. G. CRANDALL, Stanford University, Stanford, California 94305. Norm preserving extensions of linear operators on Hilbert spaces. Preliminary report.

Let H be a Hilbert space. Let J be a linear contraction mapping, D a closed proper subspace of H , into H . Let P be the orthogonal projection of H on D and let $J_0 = JP$, regarded as defined on all of H . Let T be the positive square root of $(1 - J_0 J_0^*)$. Theorem. The contractions mapping H into H and which extend J are precisely the operators of the form $J_0 + T J_1$, where J_1 is any contraction mapping H into H and satisfying $J_1 P = 0$. Applications to the problem of characterizing maximal dissipative extensions of dissipative operators are immediate. The Kreĭn extension of a symmetric contraction to a self-adjoint contraction is easily obtained as a corollary. (Received June 29, 1967.)

67T-555. J. L. WALSH, University of Maryland, College Park, Maryland 20740. Inequalities expressing degree of convergence of rational functions.

Let E be a closed bounded set whose complement K is connected and possesses a Green's function $G_1(z)$ with pole at ∞ . Let C_ρ denote the locus $G_1(z) = \log \rho (>0)$ in K and E_ρ its

interior. If $f(z)$ is analytic in E , meromorphic with precisely ν ($0 < \nu < \infty$) poles in E_ρ , then (as is known) rational functions $R_{n,\nu}(z)$ of types (n, ν) exist such that with uniform norm we have $\limsup_{n \rightarrow \infty} \|f(z) - R_{n,\nu}(z)\|_E^{1/n} \leq 1/\rho$. This note gives an example of a function $F(z)$ with an infinity of poles and a Weierstrassian natural boundary in E_ρ , and rational functions $R_{nM_n}(z)$ of types (n, M_n) with $\limsup_{n \rightarrow \infty} \|F(z) - R_{nM_n}(z)\|_E^{1/n} \leq 1/\rho$ where $M_n/n \rightarrow 0$, $M_n \rightarrow \infty$. (Received June 29, 1967.)

67T-556. J. KATO, Tohoku University, Sendai, Japan, and AARON STRAUSS, University of Maryland, College Park, Maryland. On the global existence of solutions and Liapunov functions.

Liapunov functions have been constructed for $x' = f(t, x)$ under various assumptions, including (1) the zero solution θ is stable, (2) θ is uniformly stable, and (3) all solutions are bounded in the future. We construct them under the sole assumption of the "global" existence of solutions. In Theorem I we characterize the existence as a solution of the zero function in terms of Liapunov functions. We then use these Liapunov functions (constructed from the existence of θ) to determine the occurrence of (1) and (2). In Theorem 2 we characterize the existence of all solutions for all time using a Liapunov function. We then investigate the occurrence of (1), (2), and (3). Theorem 3 is the "in the future" analog of Theorem 2. These theorems generalize results of Yoshizawa (Stability theory by Liapunov's second method, J. Math. Soc. Japan (1966), Section 10) and Strauss (Liapunov functions and L^p solutions of differential equations, Trans. Amer. Math. Soc. 119 (1965), 37-50). (Received June 29, 1967.)

67T-557. J. R. BLUM, University of New Mexico, Albuquerque, New Mexico 87106, and J. A. SCHATZ, Sandia Corporation, Sandia, Base, Albuquerque, New Mexico. On orthogonal arrays of odd index.

Let $m(\lambda, t)$ be the maximum number of rows in an orthogonal array of strength t and index λ . Theorem. If λ is odd and $t \geq \lambda + 1$, then $m(\lambda, t) = t + 1$. (Received June 30, 1967.)

67T-558. CHARLES HIMMELBERG, University of Kansas, Lawrence, Kansas. Quotient uniformities. II.

Let f be a function from a set X onto a set Y , and define (f, f) from $X \times X$ to $Y \times Y$ by $(f, f)(u, v) = (f(u), f(v))$. An f -chain in X is a finite sequence $(u_0, v_0), \dots, (u_n, v_n)$ of ordered pairs of points of X such that $f(u_i) = f(v_{i-1})$, $i = 1, \dots, n$. Theorem 1. Let \mathcal{U}, \mathcal{V} be uniformities (entourage definition) for X, Y , respectively, such that f is a uniform quotient map. Then there exists a uniform space (Z, \mathcal{H}) containing (X, \mathcal{U}) as a uniform subspace, and there exists a uniformly continuous extension $g: Z \rightarrow Y$ of f such that $(g, g)[\mathcal{H}] = \mathcal{V}$. Moreover, \mathcal{H} will be such that, if Z is given the topology of \mathcal{H} , then the quotient topology on Y relative to g is the uniform topology of \mathcal{V} . Theorem 2. Let \mathcal{U} be a pseudo-metrizable uniformity (respectively, topology) for X , and give Y the corresponding quotient uniformity (topology) \mathcal{V} relative to f . Then \mathcal{V} is pseudo-metrizable if and only if there exist a pseudo-metric p for X and a family Q of pseudo-metrics for Y such that p generates \mathcal{U} , Q generates \mathcal{V} , and for each $q \in Q$ and $\epsilon > 0$ there exists $\delta > 0$ such that $\sum\{p(u_i, v_i) | 0 \leq i \leq n\} < \delta$ implies $\sum\{q(f(u_i), f(v_i)) | 0 \leq i \leq n\} < \epsilon$ for every f -chain $(u_0, v_0), \dots, (u_n, v_n)$ in X . (Received June 30, 1967.)

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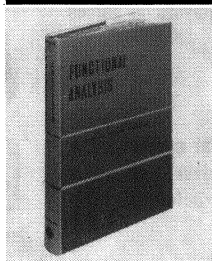
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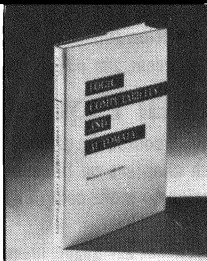


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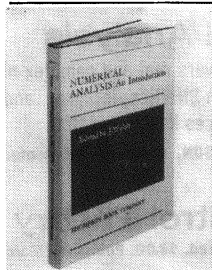


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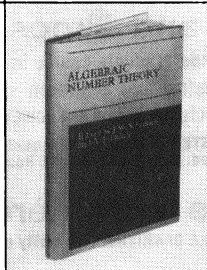


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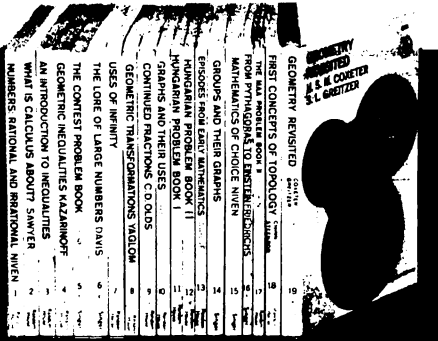
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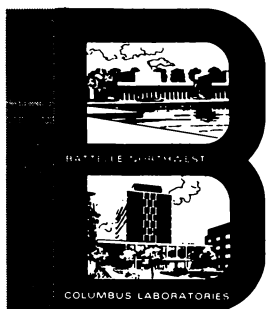
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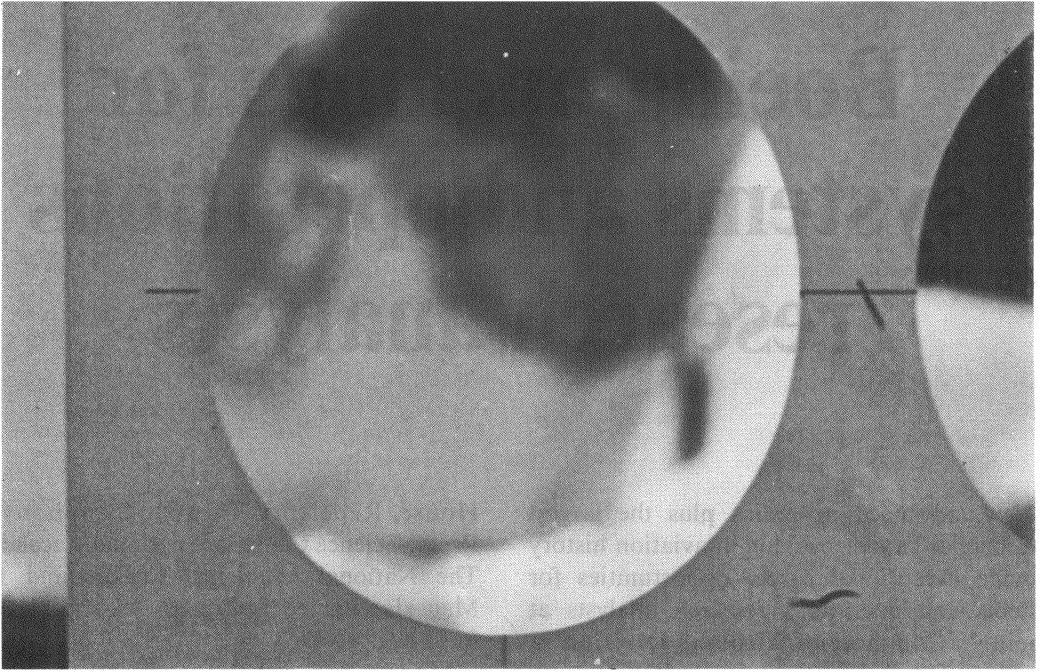
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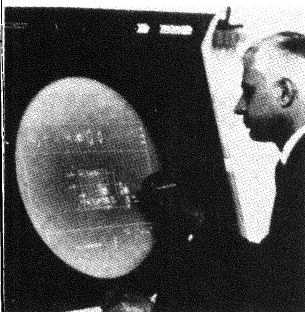
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