Experiences and Observations on the Effectiveness of a Procedure for Vibration Treatment of String Instruments

(Translated by Henry Strobel and Lothar Tews)

Part l

The increasing interest in this procedure among working violin makers and musicians demands an answer to the question: why and how can this procedure result in improved sound in string instruments?

The violin is a complex construction for sound production that statically and dynamically must satisfy very different requirements - and this is true of course also for the viola, the violoncello, the contrabass and other string instruments. At the same time the particular sound production raises some problems.

The strings alone unfortunately don't suffice because of their poor sound power coupling to the environment. Therefore a resonance body is needed, which helps to eliminate this inadequacy. That is the careful handiwork of the violin maker, the violin body built and perfected with the traditional experience of a hundred years: “A refined, ingenious, and sensitive resonator system.” If for example you have put a new set of modern strings on your instrument, the string tension necessary for normal tuning has loaded the front of your violin at the bridge just as if you had placed an air travel suitcase of about 13 kg weight there instead! But - as you will know - there is inside the violin a bass bar and a sound post, which among other things also ensure that this load is distributed and the instrument is indeed in static equilibrium. This body, consisting of front, back, and sides is, from the standpoint of technical mechanics, actually a doubly arched, pre-stressed construction of domes supported on all sides, the static and dynamic considerations of which have long been well understood by specialists of this discipline - but of course much less so the action of the violin. For it is almost paradoxical that this to such an extent “prestressed construction” can, by the string excitation from bowing, coupled by the bridge, be stimulated to those various particular vibrations that characterize this system and make possible violin playing in its manifold tone colors.

In addition to the shaping and arrangement of the vibrating components however, the materials that they are made of is also important. We find four or five different kinds of wood, whose microscopic cellular structure provides for inner strength and good vibration characteristics. In addition, the behavior of the entire “resonator system” is affected by the strongly anisotropic, elastic characteristics of the wood. For example the sound velocity in the spruce plate parallel to the grain is four times as high as that perpendicular to it.

This construction is held together with glue, which is often done with preloading and at increased temperature of the parts. But what actually happens to the material during this process? After superficial penetration into the cell structure of the wood these glued connections consist actually of layers of a macromolecular, gel-like “compound material.” These have a viscous component which cannot be neglected - and thus inelastic characteristics - and that precisely in the vicinity of the ribs, where one finds the characteristically flexible “spring elements” of the front and back. Also the glued-in (under tension) bass bar affects significant regions of the spruce front inelastically.

Moreover, after completion the white violin is sealed and given a coat of varnish, or, more precisely: a macromolecular, hardened and sufficiently viscous, polymer surface coating in several layers, which, with its inelastic component, affects the originally good elastic qualities of the tonewood in its vibration response (Ref. 1, 2). Thus the complete, playable violin actually consists not only of select and well seasoned woods, but has a rather significant amount by volume of “compound material” that in part, by its inelasticity, results in local tensions that lead to losses in the applied vibration energy. Therefore it can happen that not all the modes of vibration will be activated, and the higher ones can be rendered ineffective through an acoustic “short circuit.” The violin sounds weaker and the sound spectrum contains holes.

Consequently one asks: what should be done to lessen the effects of these handicaps and thus achieve an improvement in tone?

According to the foregoing:
1) local stresses that are built in at the time of making or after a critical repair, which partially block vibration modes of the resonator system or inhibit several coupled modes (recovery and removal of localized stress conditions, i.e. relaxation), and
2) the inelastic, energy-absorbing proportions of the material's rigidity must both be reduced.

To l):
This problem has been known in industry for a long time, where the removal of deformation stresses (from rolling or drawing), especially the removal of thermally induced stresses (as in welded joints) is possible through a subsequent heat treating process. Here one uses the higher mobility (at higher temperatures) of elementary objects (molecules, radicals, atoms) to achieve a lower interference configuration in the structure of the material:

“eigenstress” reduction. Now of course one cannot subject a violin to a gross heat treatment. But the basic concept of this effect does come to mind: By raising the vibration amplitude of these elementary objects in a compound material their positions in the network (crystal or molecule lattice) are reordered, which macroscopically improves the homogeneity of the material. Through this the desirable elastic properties of the material can be regained. This is shown by results of research into the effect of moisture on the modulus of elasticity and the damping factor of tonewoods, which can be explained by the relocation of the adsorbed water molecules, especially the radicals at or in the surface of the wood.

To 2):
Here too a vibration relaxation is suggested. If a periodically varying stress is applied, the the viscous motion resistance in polymers causes a partial conversion of the mechanically applied work into heat energy. If the stress cycle amplitude of the vibration is great enough, the viscosity itself is altered - and in fact in the direction of a reduction in damping. This irreversible process can eventually result in a reduction of vibration damping in the modally excited compound material of the front, back, and ribs. The violin speaks more easily, the sound levels are comparatively greater for the same bowing and the sound spectrum is more even.

Description of the Process

The question remains, how, with which frequencies, and, most importantly, how strongly the instrument, or more precisely, its body must be vibrated to achieve the desired effect of the kind we have discussed. Since in this kind of treatment the vibration of the strings per se is of no interest to us - although the effective stress loading of the tuned strings on the body is - the strings and the tailpiece are thoroughly damped with sponge rubber padding. For the actual vibration relaxation treatment of the body an unbalanced vibrator is used, (specially adapted to each type of string instrument) which consists of a small direct current motor, on the shaft of which, and if necessary using an intermediate bearing, two eccentric disks (vibrating weights) are mounted. The relative position of both disks is adjustable with high accuracy, whereby the motor rotation results in a specified, predetermined imbalance. This imbalance vibrator is fixed to the bridge parallel to its plane so that the imbalance masses can rotate freely beside the left foot of the bridge (i.e. over the bass bar underneath). The effective amplitude of the alternating force, $F{\downarrow}_u$, with which the body is set into forced vibration can be determined by the setting of the imbalance and measurement of the rpm. Experience from preliminary tests teaches that for this value $F{\downarrow}_u$ only 5% of the above mentioned static bridge loading on the front plate is sufficient to obtain an effective stress relaxation effect with a sufficient number of alternating load cycles(duration of treatment). A calculation shows that this modulation of the string tension of a normally tuned instrument corresponds only to a periodic detuning of less than a quartertone. (See also the article on the calculation details in *Instrumentenbau-Zeitschrift*, number 7/8, (1997), page 31.) Thus it is virtually impossible that the instrument to be treated will be damaged. With a stroboscope the vibration condition (movement amplitude of the bridge, ribs, and f-holes) can be monitored during the treatment period. Interestingly, as was to be expected, in the stroboscopic view the forced vibration appears inharmonic, that is, several different frequencies are involved. Thus it is possible by choosing the vibrator frequency to apply the “overstressing” effect also to body modes lying higher in the spectrum. Nevertheless it is advisable in practice to conduct the treatment in several steps, that is, each step at a higher frequency and at a correspondingly reduced imbalance setting. In this manner the amplitude of the stress cycle remains constant, and can excite many modes of the resonator system.

The effectiveness of this procedure can be monitored and proven, in that in each treatment step, at a constant frequency, the power consumption of the direct current motor can be traced over time . It decreases as the number of load cycles increases (at constant rpm). This is an indicator that part of the mechanical energy absorbed by the resonator system with each load cycle was used to reduce the vibration hindering effect of any damping mechanisms that are present. (See also Note 1.) Also acoustic measurements before and after show that with this treatment an objectively observable relaxation effect can be obtained. (See the Results in Part II). The improvement observable after subjective evaluation in response, modulability, evenness, and playability - as voiced by many musicians - is an indication of the reduction in excessive local stresses available from such treatment, in other words - of an effective relaxation of stresses.

Finally it should be noted that the tone improving effect of this “overstressing vibration treatment” cannot be replaced by even a twenty or thirty year period of “playing-in” (Lit 2).The vibrational energy applied to the body even by fortissimo bowing is much too small for stress relief.

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Note 1:
A calculation has shown that in the measuring conditions given heating of the rotor windings, with its increase in resistance, also results in a decrease in power consumption. But this is too small to explain the measured reduction in power consumption solely by the temperature rise in the rotor winding. A significant part of the reduction in motor power consumption is attributable to a *relaxation of the vibration system of the instrument body*.

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Part II

In Part II an attempt was made to clarify to what extent an “overstressing-vibration treatment” - abbreviated in the following as Vibrelax - can affect the vibrating conditions of the resonator system to result in an improved tone and responsiveness in playing.

The desire also to be able to objectively prove this now, led to setting up a measuring system, that would be simple and clear, but also informative enough to quantitatively define the acoustic changes in the instrument.

Measurement Principles

If one examines the literature of recent years with respect to measurement techniques, two different measurement principles emerge. In one a continuous vibration is applied to the bridge so that the filtering function of the bridge and the building up of body vibrations are largely excluded. (Ref. 1) The frequency spectrum is as usual taken from a microphone and its associated electronics at a distance and presents a detailed analysis of the differences which should make possible a value classification of instruments according to the standard of old Italian instruments. The characteristic functionality of bowed instruments can however be investigated by a quite different method of excitation essentially detailed and more comprehensive. (Ref. 2 and 3). In this by means of a standardized impulse all possible vibration modes of the body are immediately excited “on the spot,” and the “answer signal” arriving at the foot of the bridge as well as its magnitude is recorded. This power impulse in the form of a “Dirac transfer function” presents to the instrument a very broad excitation spectrum at the moment of impact, from which it can derive the energy necessary to excite its eigenmodes. Here the body uses its own resonator system as a comb filter. Through computer analysis of the answer signal, among other things, a direct correlation of frequency and damping constants to the individual resonators of the whole system is possible. Results of this can be found in the literature under the keyword modal analysis. In this method one arrives at valuable quantitative results for the sound radiation of the vibrating body, which until recently, rather more qualitatively, was only possible with holographic interferometry - and that only at great expense. (Ref. 4). Unfortunately the above-mentioned procedure of modal analysis is extraordinarily tedious and time consuming, and its use may be reserved for research purposes in well equipped institutions.

Nevertheless, to quantitatively record the effect of Vibrelax treatment on bowed instruments and to objectivize a subjective evaluation, a measurement apparatus was constructed which is used each time, before and after treatment. The following basic principles apply to it:

the vibration excitation is to be as much as possible in the normal playing situation of the instrument, that is, with external damping influences only at the neck (the fingerboard, left hand) and at the lower back (shoulder rest),
the different and frequency dependent effective vibration modes are to be taken from sound measurement in the near field, just as they are presented to the player's ear. The ineffective (closely spaced, out of phase vibrating modes, hydrodynamically short-circuited) resonances are not recorded, which corresponds to the actual situation in performance,
the differently located modes are recorded by measuring the resonance peak as to position and width in a frequency diagram.

The excitation in the case described comes from a mid-range speaker (70 mm diameter) that is mounted in a highly damping speaker box (to make its own resonances completely ineffective), and on the circular mounting pad of which the violin lies horizontally near the endbutton. (Figure 1.). It is supported in a padded, open fork at the neck. The output force of a full size violin at the speaker amounts to only about 2.3 Newtons. Better said: the excitation of the body occurs through the pulsating pillow of air between the conical loudspeaker diaphragm and the back of the violin in the area of the lower block. In this way the vibration modes of the body which are only slightly damped can be built up. The sound radiated from the body in the near field is picked up with a spherical characteristic electret microphone, and the whole apparatus is placed in an anechoic cabinet.

The signal processing is shown schematically in fig. 2. The loudspeaker is fed from a low distortion sine wave generator via an output amplifier. The adjustable frequency can be measured, with a precision of +/- 1 hz, from the synchronous TTL signal of the generator by a digital frequency counter. The signal from the microphone is high impedance amplified to a level of 1 volt ac and is fed via a buffer stage to a filter with optional characteristics at one's disposal (low pass, high pass, bandpass and corner filter), which can be switched into the signal path as needed. Another output from this buffer amplifier can go to an oscilloscope and serves at the same time as an input for a spectrum analyzer, which is required for the resonance measurements. After this the signal goes to a logarithmic precision rectifier, from the output of which the value of the sound pressure level can be read in three dB-measurement regions (dB-meter). Furthermore the vibrations of the front plate can be brought through a bimorphic piezo-ceramic sensor of very small mass (less than a gram) via an electrometer amplifier to the vertical input of an oscilloscope, and, with the horizontal input from the excitation signal Lissajous figures can be used to monitor the resonance and phase of the vibrating front.

Making the Measurements

The frequency measurements were made pointwise at a distance of 200 cm in the range from 32 to 15 kHz, to get a general frequency curve for the sound pressure plot for the instruments arriving for investigation (The values for frequencies lower than 160 Hz were corrected according to Remark 1. (See Remarks.) A possible double excitation from sympathetically vibrating strings as well as the tailpiece was avoided. See Measurement Results, Fig.3, which merely give a general survey of the response of the instrument to the excitation of a defined speaker power (Standard level inside: 90 dB sound level at 1 kHz). Two artifacts bias the frequency curves made this way:

1) As known through other authors (Ref. 5), the directional characteristics of the instrument itself at neighboring frequencies prove to be not centrally symmetrical, so that in measuring with only a fixed microphone the frequency curves can exhibit small interruptions;

2) from about 1.5 to 2 kHz the sound wavelength in air is comparable with the body width measurements. The shadowing effect becomes more and more effective here, allowing the sound level measured value to fall (Remark 2). From both these influences there is data on the absolute values for the sound level and so a direct comparison with the frequency curves of other authors is not possible, however it in no way impairs a frequency curve comparison of the same instrument in the same measurement system before and after Vibrelax processing.

Beyond the pointwise frequency curves particular worth was placed on measurements of the body resonances, which at slowly varying frequencies one after the other could be distinctly seen as signal peaks on the spectrum analyzer and dB meter. From these acoustic measurements the body resonances (vibration modes) with respect to their frequency range and their -3dB points with respect to resonance widths (corresponding to the logarithmic damping decrement) are known. These very reliable measured data form the base for an efficiency assessment of the differences from Vibrelax processing.

In the future it is anticipated that not only continuous excitation but optionally excitation by pulsed sine signal packets corresponding to frequency will be used to, in this way, enable close examination of the “one shot” behavior of the individual excited resonances on the oscilloscope. This offers the possibility to make objective statements about the changes in the “speaking (response) characteristics” of the instrument after Vibrelax treatment.

Measurement Results

In the time since 1996 altogether 10 violins, 6 violas and 9 cellos underwent a Vibrelax processing, in which - as already mentioned - the instrument was acoustically measured in the ready-to-play (or made ready-to-play) delivered condition and in the final condition after treatment. Furthermore there flowed from this additional valuable experience in the explanation and significance of the measurement results by comparative measurements on historical reference instruments with acoustic tone copies from renowned violin shops. In some cases these tone copies were additionally vibration treated and afterwards measured a second time. All these experiences allow the conclusion today, that the effectiveness of the applied Vibrelax process is dependent on the delivery condition of the instruments. Is it for example in the possession of an orchestra player and continuously in use, is it thus "well played-in, corresponding to an available sound improvement only in some select parameters. Or does the instrument come from the shop after a major repair (bar, front crack, or such) or newly built - then experience teaches that a comprehensive improvement in tone and best of all an improvement in responsiveness (speaking) - and those are qualities that formerly were accessible only through a (very) subjective determination by seasoned musicians - is available.

The literature on an objective approach to the marks of quality (see for example the dissertation of H. Dűnnwald (Ref. l) records that the characteristic body vibrations (resonances) in their frequency position with respect to height and breadth, as the peaks in the frequency diagram represent, can be a key for the objective classification of instruments. Therefore resonances were carefully considered in all measurement projects. In this it turned out that the total number of resonances following a Vibrelax treatment was far greater than before (fig 4 a.). The increase was especially pronounced at higher frequencies (approximately above 2 kHz) (fig. 4 b and remark 3.) But more modes per third interval at high frequencies means that for the same excitation before the Vibrelax treatment fewer modes were seen to be acoustically active. But this can only be explained by for example that through internal stresses (type 2.) fewer modes were previously actively radiating (direct mode blocking) or that previously the high, closely located modes were inactive because of acoustic short circuits and thus were not verified. Consequently the vibration treatments etc. lead to a reduction of the stresses introduced into the body in construction or repair, which one generally understands as stress relaxation. It is primarily this effect, that finally in practice leads to a reduction in damping in the vibrational movement or body sound wave spreading. For these reasons one speaks here not exclusively of “dedamping” but rather of relaxation and the process used is called “*vib*rations*relax*ation,” abbreviated Vibrelax.

Through the particularities of the measurement principles used, in recording resonances of the body sound emission in the near field, their measured values (for example the -3dB resonance width) can be distinguished clearly from those, which for example have been observed by the process of modal analysis. A simple energetic consideration makes clear that the energy required for sound radiation comes from the kinetic energy of the vibrating body, which is thus loaded by "vibration damping." Hence the modes arrive at a greater vibration amplitude through stress removal (relaxation) thanks to Vibrelax. and so the resonances must become broader through increasing radiation damping. And so paradoxically a "damping increase" in the measured values can be advantageous, if at the same time the sound level rises. An enlightening example of that can be seen in the measured analysis of the previously mentioned cello (fig.5). Here the variation in the measured sound pressure level over frequency is represented as the difference of a series of measurements before and after the Vibrelax treatment. After the level measurements at the 200 cm distance the resonances could be measured in their corresponding frequencies (up to approx. 2 kHz) before and after Vibrelax, from the -3 dB widths of which the Q values were determined.

$${}_{\text{before}}^{\text{after}}\Delta Q = (Q_{\text{after}} - Q_{\text{before)}}) / Q_{\text{before}}$$

Lining up the relative Q differences and the level differences in the marked frequency ranges shows that each rise in level is connected with a lowering of Q and vice versa - a strong argument for a significant effect of radiation damping, which after a Vibrelax treatment acts differently in different frequency ranges.

This general possible level increase through Vibrelax, which is noticeable most in a new instrument or after a repair, is of course of itself no indicator of a tonal improvement compared to a greater richness in overtones, which primarily offers the possibility of modulability by the player. Undoubtedly for that there are two suppositions: the mastery of a difficulty bow technique, and the capability of the instrument to radiate sound over a broad frequency range. As is well known the bowed string delivers not only the information of the stopped tones via the bridge to the resonance system of the body but, because of the non-sinusoidal string vibration, also the accompanying harmonics (fig. 6), which as overtone content build up the desired tone (for that see also the instructive presentation of K. Osse, Ref.9). Now the body has at its "fundamental resonance" in the playing range in the frequency intervals of the vibrating strings harmonics as well as natural vibrations (resonances), and these, by the particular bowing technique used (bow pressure, bow hold, bowing contact point, etc.), are simultaneously excited and can contribute to a full sound picture. (For details of this connection see likewise Ref. 9, page 105 ff.) According to that it is desirable that the body have, along with the resonances in its natural playing range, resonances in harmonic intervals up to much higher frequencies, which in the following will be designated as coupled resonances (fig. 7). From that it is reasonable to look through the entire pool of measured resonances (in the cello there are far more than a hundred) for such coupled resonances, which can best be quickly accomplished with a computer program.

As an example from numerous series of measurements there is shown in fig. 8 the number of coupled resonances per third interval for three series of measurements of an instrument (cello, F. Scheidt, 1997), which was computed from the peak measurements in the frequency range of 63 to 10 kHz. The evaluation program captured all resonances up to the 25th harmonic of every fundamental resonance. The cello was first in the white, then after grounding and varnishing, and later after eight months of UV drying was finally in playing condition at our disposal for Vibrelax treatment. It is interesting now that the computer calculation can provide information also about the tone improving influence of varnish. Harmonics (overtones) in the entirety of the coupled resonances (fig. 9). As one easily learns, the compelling conclusion is: the greater on the one hand the number of coupled resonances per third interval is, and on the other hand the greater the total occurrence of single harmonics in this reservoir is, the greater is the supply of resonator systems at the disposal of the player for tone color, and which he can call on for musical expression. As the diagrams show, a Vibrelax treatment has a marked, positive effect on the these key numbers, which indicate quantitatively the potential overtone richness of an instrument; and particularly in the frequency domain, in which an increased richness of overtones occurs especially (fig. 8) as also after the increased occurrence of single, possible overtones, which are reliable indicators for the timbre of the played instrument (fig. 9).

The improvement here in the overtone supply from the Vibrelax processing of string instruments could not be verified by an application of von Reumont's procedure (Ref. 12) by K. Leonhardt in Mittenwald - and by E. Meinel and D. Holz in the Vogtland (Zwota) (Ref.13) only with a practically negligible result. In both cases they relied only on merely taking the frequency curve and additionally subjective sound appraisals for assessing the effectiveness. While K. Leonhardt as supporter of a necessary, long playing-in phase for the new instrument by vibration treatment hoped for an abbreviation of this "incubation time" and (very probably) with too little excitation energy - even if active over a very long time - it did not come to the desired result; in the other case the technical requirements were not given for a sufficiently strong excitation of the instruments (essentially guitars) to economically put them into them into an industrial production process for quality improvement. What is crucial for the success of the procedure - as was previously stated in Part I - is a supervised, high power vibration with stepwise increases in rpm but at the same time with constant effective value of the imbalance energy.

To get an enhanced and independent assessment of the potential of the process, before and after a Vibrelax treatment, besides the described analog measurement process, a modal analysis of the instrument in beginning and ending condition was conducted each time. The results of the condition comparison, which likewise indicate a noticeable improvement in quality through the application of the Vibrelax process, (Cello, Opus 26; M. Schleske, 1997) remain reserved for a later publication.

In closing we must thank Prof. v. Reumont. As the "Nestor" for this successful treatment method his interest in our developments and his advice have been of especial value to us.

Likewise we are indebted to Dr.sc. D. Holz for his friendliness, placing at our disposal literature which is not easily accessible.

We have especially enjoyed the cooperation of Geigenbaumeister Dipl.Phys.-Ing (FH) M. Schleske in a very essential collaboration on this subject, and thank him in particular.

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Remarks:

1. Space considerations dictated using a small diameter speaker, the efficiency of which of course markedly fell off below 160 Hz. This was taken care of by taking the speaker frequency response in the empty sound cabinet below 160 Hz as a correction to the measured data.

2. The falling off of the level above about 1500 Hz is not instrument specific. Rather this phenomenon is conditioned by the sound shadowing effect of the instrument body, in which the sound waves in air approximate the instrument dimensions (average lower body width). Quantitatively this can be considered by a sound diffraction calculation, in which the body back near the speaker according to the Babinet principle is to be seen as circular disk shaped diffraction slot. The resulting maxima and minima superimpose the sound pressure measurements in this frequency range and for violins agree strikingly well with a value for the average obstacle diameter that complies with an average lower body width of 22 to 24 cm.

3. This obvious desirable effect can be determined quantitatively. For the front as an edge supported plate the calculable number of possible eigenfrequencies per bandwidth (here a third interval) Δf for large f is given by the equation:

$$_{f\rightarrow oo}(\Delta N/\Delta f) = 4Sm'/W_b$$

where S indicates surface; m': mass per unit surface; W_b: bending wave impedance.
(See also Ref. 7, page 244.)

An increase in the number of resonances from Vibrelax processing implies therefore a lessening of bending wave impedance and/or an increase in the plate surface S, which could only be possible if the previously blocked edge regions (linings and purfling) were no longer contributing to the whole vibrating surface. An influence of a change in modulus of elasticity is excluded according to the opinion of several authors. Only the frequency dependence of the E-modulus of spruce measured by D. Holz (Ref. 11) from 2 kHz up requires a slight correction (fig. 4 b).

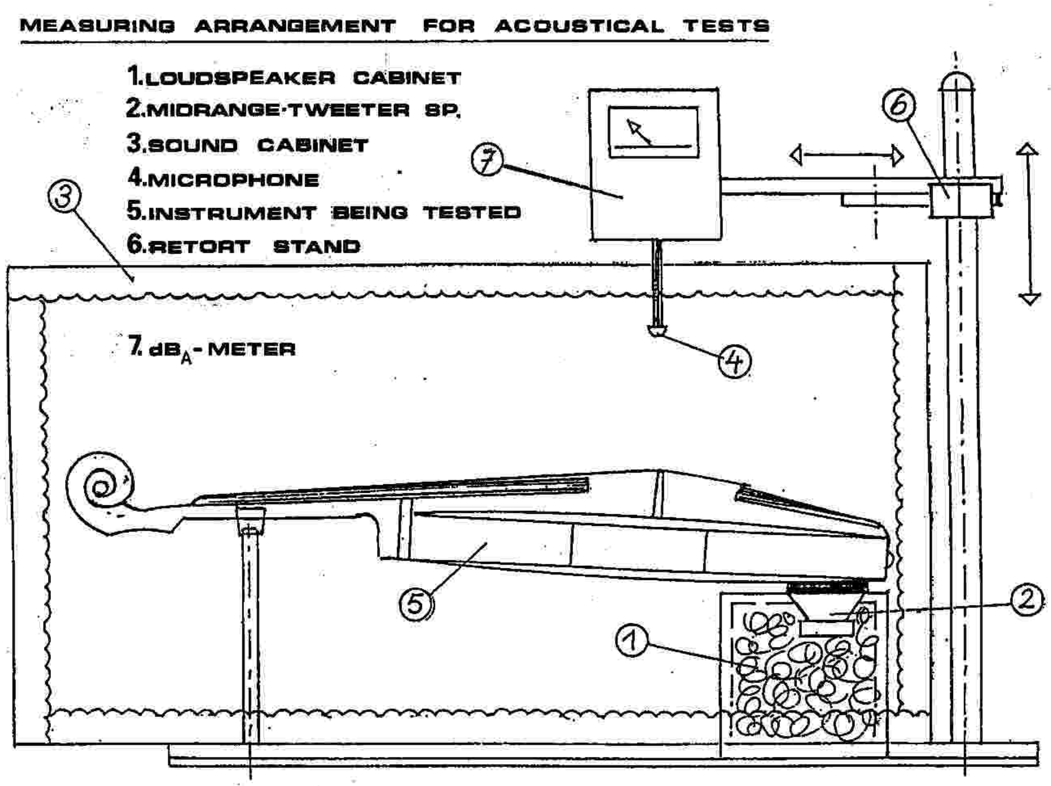


ILLUSTRATION 1

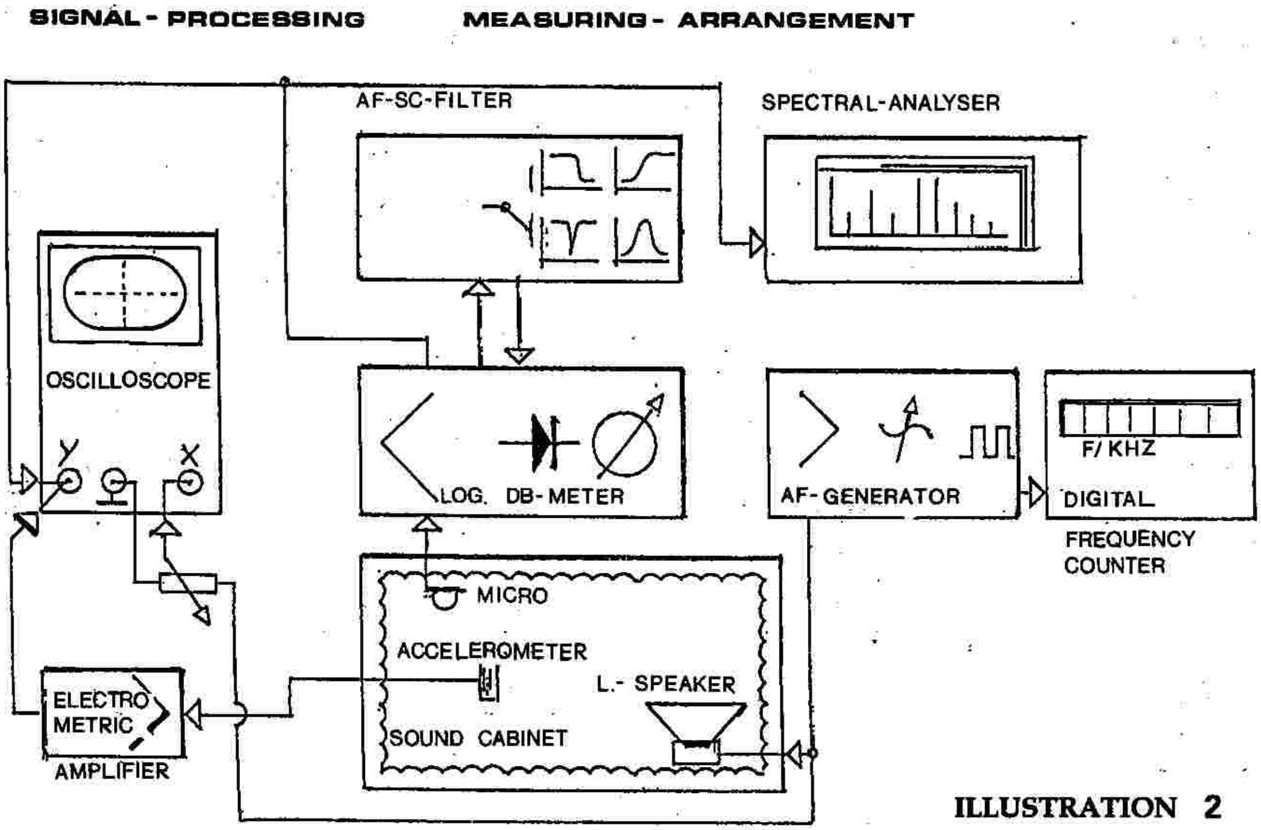
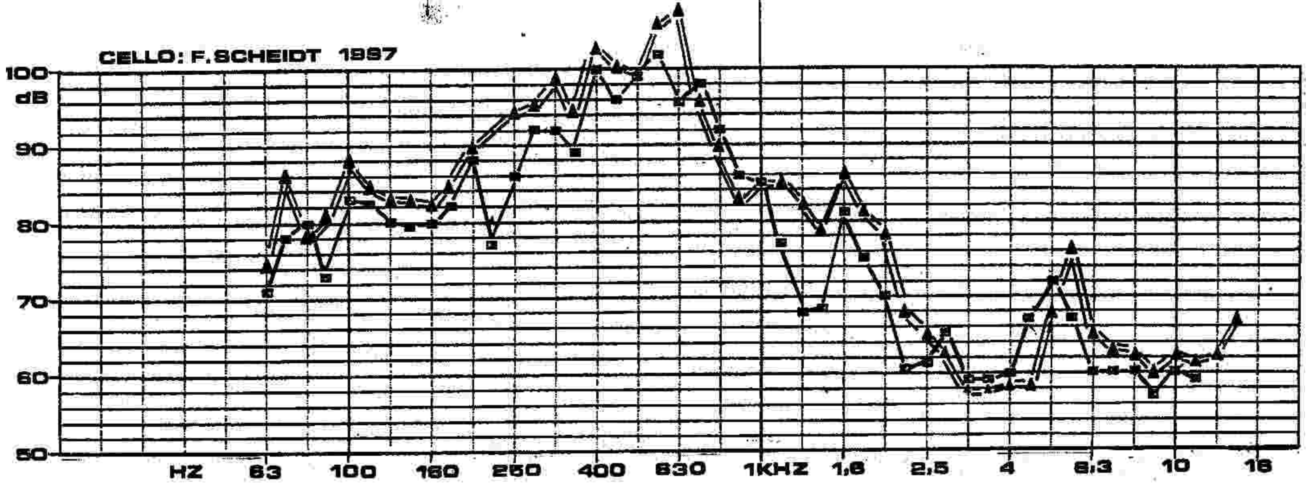


ILLUSTRATION 2

NUMBER OF RESONANCES PER THIRD

▲ ═══ ▲ AFTER VIBRELAX	1	1	1	2	3	2	3	3	3	4	5	4	4	5	7	8	9	15	18	14	21	21	Σ 154
BEFORE ——□	1	1	-	1	4	2	-	3	3	4	4	3	4	5	6	7	9	15	10	13	14	12	Σ 121



REMARKS: STRINGS and TAILPIECE FOAM-DAMPED

ILLUSTRATION 3

NUMBER OF RESONANCES PER THIRD

CELLO: F. SCHEIDT 1997

VIBRELAX	1	1	1	2	3	2	3	3	3	4	5	4	4	5	7	8	9	15	18	14	21	21	Σ 154
fully varnished	1	1	-	1	4	2	-	3	3	3	4	3	4	5	6	7	9	15	10	13	14	12	Σ 120
white	1	1	1	2	2	2	2	3	4	5	3	4	5	5	7	8	5	8	11	7	2	12	Σ 102

63HZ 100 160 250 400 630 1KH 1,6 2,5 4 6,3 10
=C

VIOLA: LANDAN

VIBRELAX	1	-	2	2	3	2	3	3	3	5	4	3	3	6	5	5	4	4	8	Σ66
fully varnished	-	1	1	1	1	1	2	2	1	4	2	3	2	4	5	4	5	3	6	Σ 48

125HZ 1KH 10
=C

VIOLINE: GÖTTING 1993

VIBRELAX	2	1	2	2	3	2	2	3	5	4	4	3	4	7	8	3	10	Σ 65
fully varnished	1	1	1	2	2	2	2	2	5	2	1	2	4	7	4	1	6	Σ 45

200HZ 1KH 10
=g

ILLUSTRATION 4A

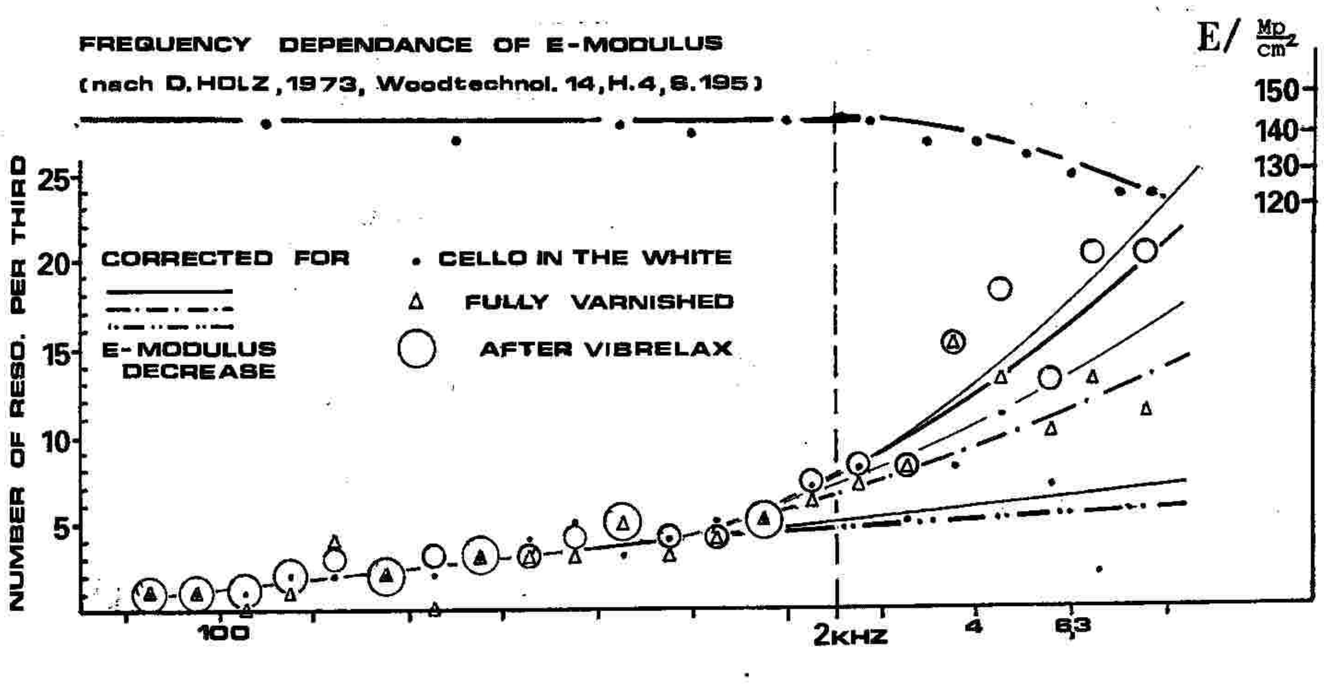


ILLUSTRATION 4B

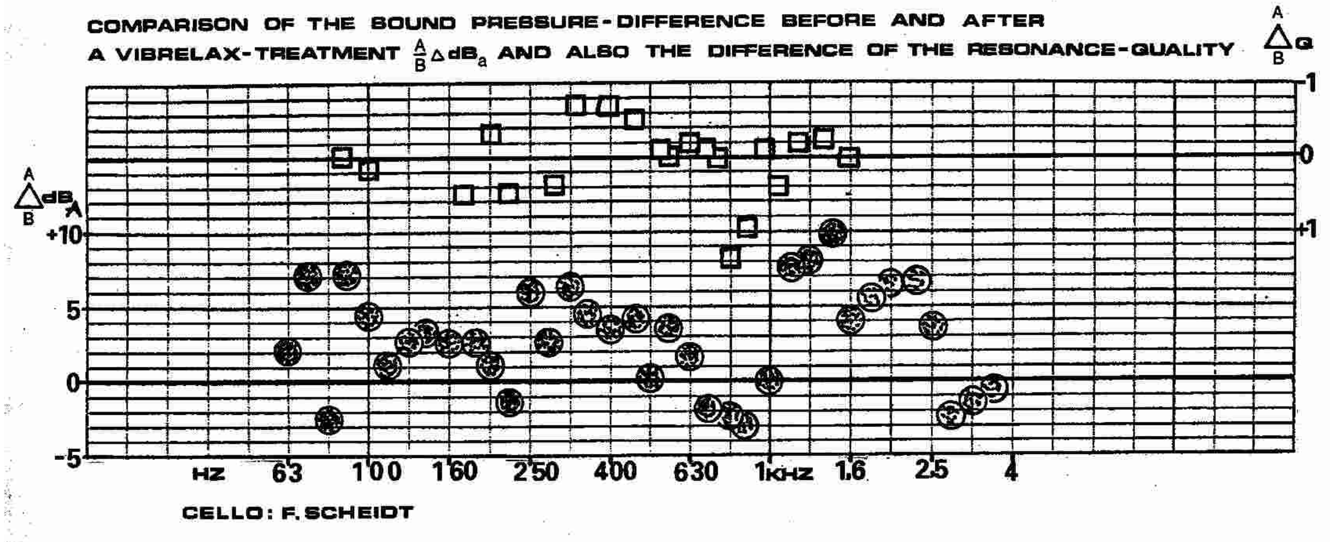


ILLUSTRATION 5

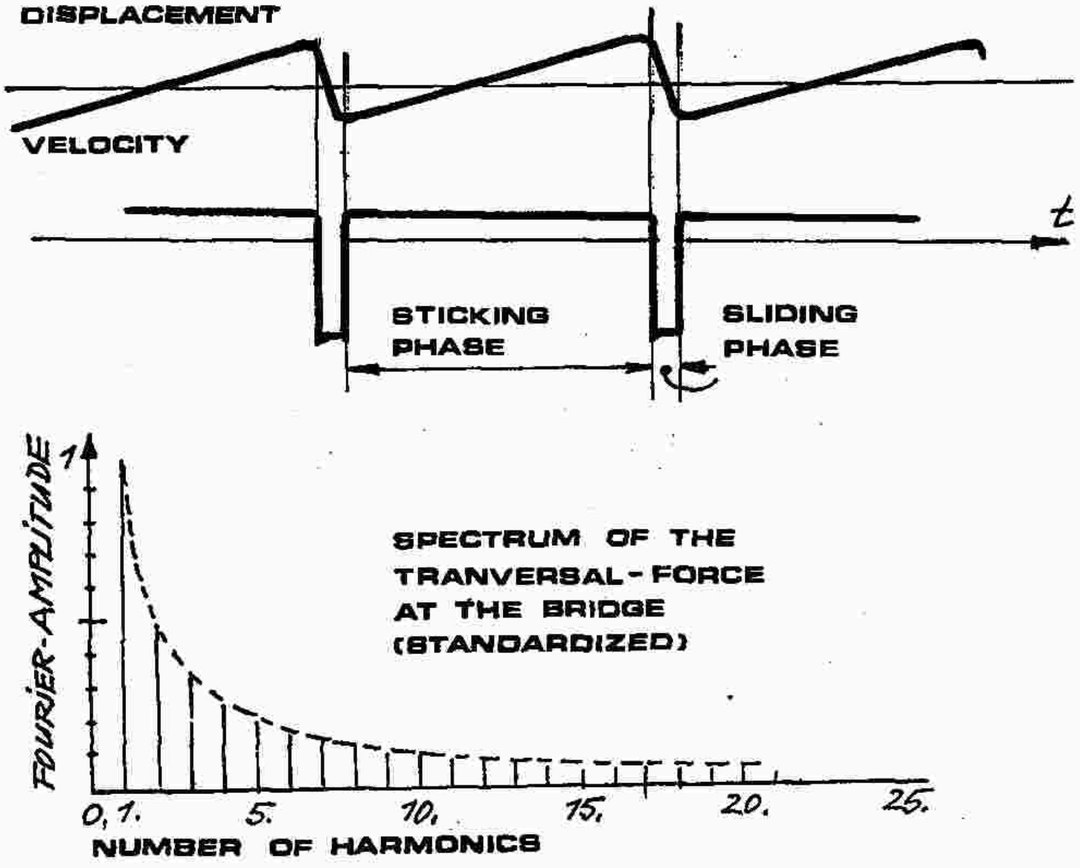
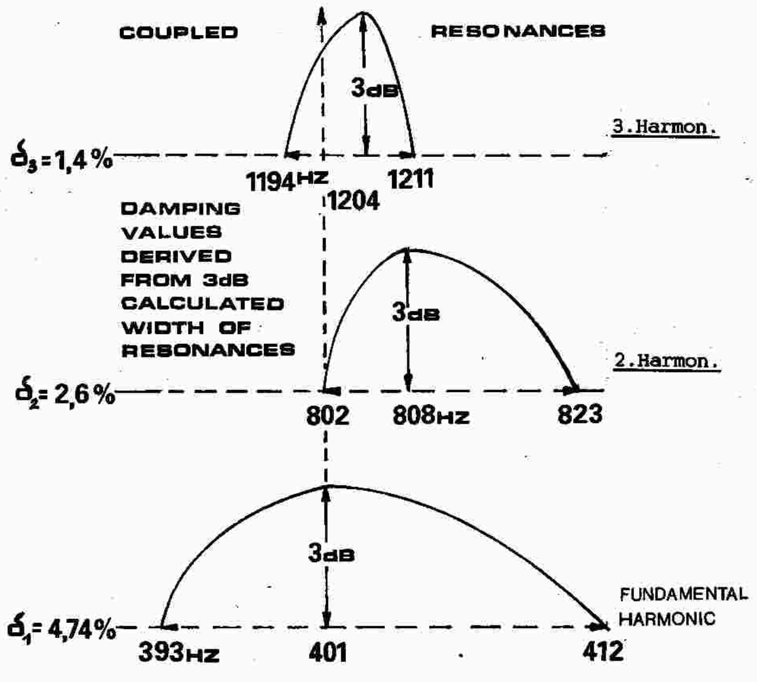


ILLUSTRATION 6



(coupled is more familiar and better stilistic)

ILLUSTRATION 7

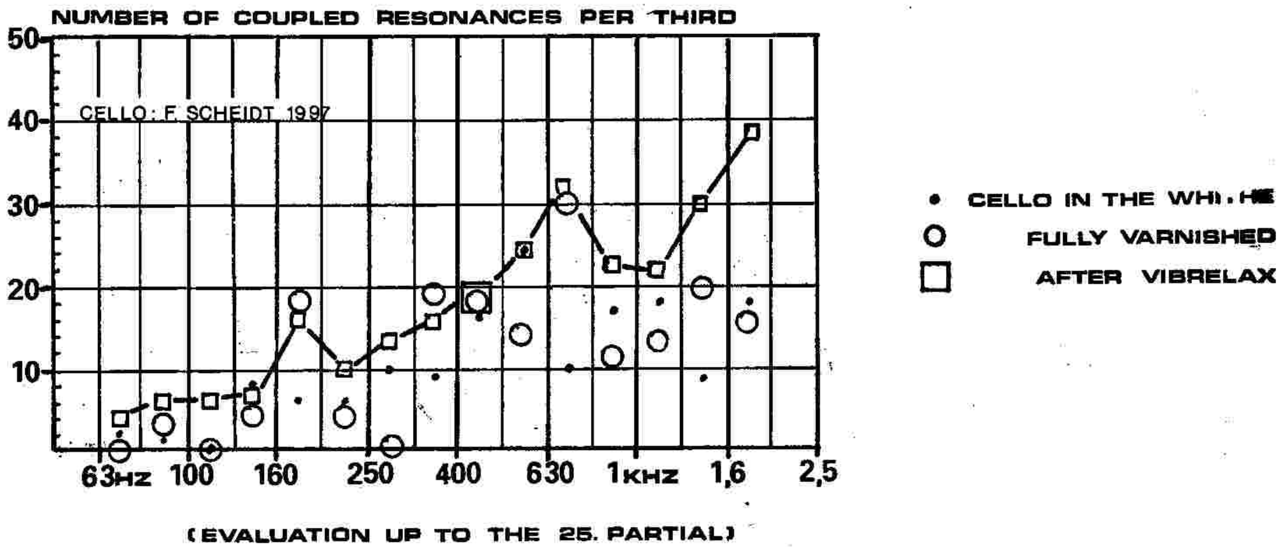


ILLUSTRATION 8

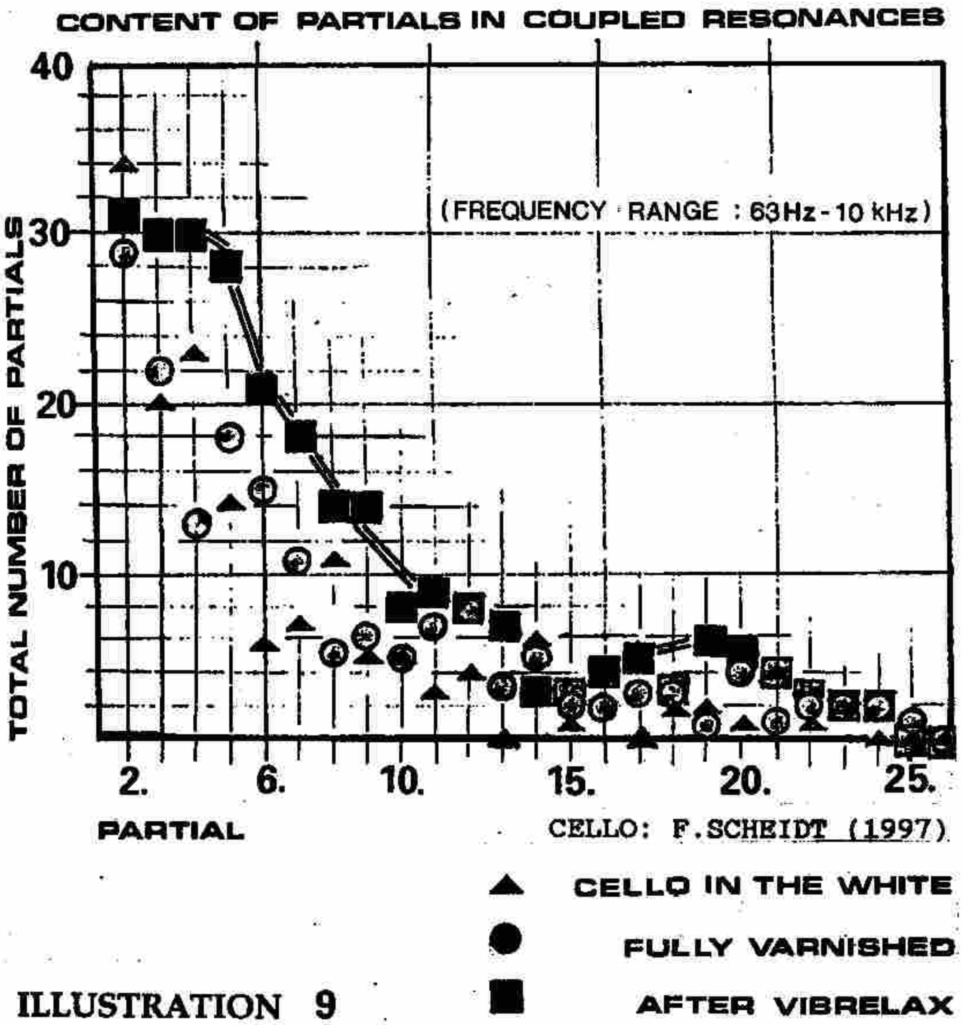


ILLUSTRATION 9

Experiences and Observations on the Effectiveness of a Procedure for Vibration Treatment of String Instruments

Part III

In the two previously presented sections (Part I and Part II) we tried to present in detail the procedures used on numerous string instruments to improve their tonal qualities and response. We also tried to establish proof of the achieved desirable changes in playing and tonal qualities through discussion of the results of acoustic measurements, before and after the treatment. The principal objective was to supersede and supplement the until now only subjective evaluation by the player and listener (often the same person), documented by many letters, by quantifiable analyses of these acoustic measurements making an objective and thus generally accepted evaluation possible.

The actual subject of our investigation is the vibration response of the instrument body, that is, how this complex resonator system reacts to forced vibrations. So far, because of the chosen test and measuring set-up (see Part II), it was only possible to examine the steady state vibrational behavior; which limited what could be concluded about the resonance behavior of the body, and some possible and expected vibrational phenomena were not accessible to investigation. To determine the resonant behavior of the body from acoustic measurements taken in the surrounding sound field can likewise be subject to limitations which, among others, result from the well-known frequency dependent direction of sound radiation. Thus it was appropriate to consider a different measuring method, one which allows direct conclusions about the actual vibration behavior of the body.

- Modal Analysis Investigations -

The oscillation energy transferred from the vibrating strings through the bridge into the plates of the body generates among other things flexible bending waves in these plates. The behavior of this wave type can be presented mathematically by solution of the related boundary value problem, whereby the geometrical dimensions of the vibrating body - (the *boundary values*) - determine the formation of the different wavelengths. At certain frequencies (*eigenfrequencies*, or resonant frequencies), systems of standing waves are formed by reflection at the edges. These deform (*bend*) the plates elastically and produce steady *eigenforms* (mathematically speaking: *eigenvalues*) corresponding to the eigenfrequencies. Now if one knows these " vibrational bending patterns " and their frequency dependent distribution on the plates, then direct conclusions can be drawn about the acoustic effectiveness of such vibrating plates (that is to say: the *front* and the *back*). Unfortunately, because of the complicated shape of the plates (actually double curved shells) and their flexible coupling with the ribs and the sound post, a definite mathematical solution of this boundary value problem is virtually impossible.

Therefore the task is, to determine the eigenfrequencies and eigenmodes through measurements. One advantage here is the linearity of the problem. In such a case the reciprocity law holds; i.e. the exciter (transmitter) and the signal recorder (receiver) are mutually interchangeable without distortion of the signal. That makes it possible to place a bending wave receiver (i.e. an accelerometer) at the previous drive point (the bridge), during excitation by a (standardized) tap (i.e. a mechanical impulse) at given coordinate points, serially in time. Each bending wave created in this way provides information, via the accelerometer, which is stored and available for computer analysis. If there is a sufficiently dense grid of fixed measuring points distributed over the plates of the body, the effect of the individually triggered bending waves can be integrated and we can, thanks to the reciprocity law, get complete information about the effect of a vibrational excitation of the body introduced by the strings at the bridge.

Since the tap has the character of an impulse function (i.e. a very steep rise in the time domain) a Fourier analysis of the received signals gives us a dense frequency spectrum, in which of course the boundary value eigenfrequencies, the problem we are trying to solve, are also to be found. Since the applied impulse has a uniform magnitude, the damping for selected frequencies can be determined from the decay curve; and with suitable software the computer offers a representation of the eigenforms (also called *“modes”*) in a suitable format, where the plate deformations from their initial position are shown as “deflection peaks” or “deflection valleys.” Also, in those places where the front or back plates remained at rest we have the nodal regions mapped, or in the two-dimensional plots simply the “nodal lines”. This leads to the conclusion that these pictures of the eigenforms (mode diagrams) can give detailed information about the vibrational response of the whole body under varied conditions. The results of the mode diagrams of violins with their plates at various stages of graduation are very informative and illustrative in this connection. (see Lit.1)

The basic idea of applying this methodology to investigate the effect of a vibration treatment on the vibration response of string instruments is most of all founded on the fact that the measurement signal here directly reflects the oscillations of the vibrated body. With the earlier measuring method it was the resulting sound field which was picked up by a microphone, and only via this detour could the vibrational behavior of the instrument body be determined. Of course here too the surrounding atmosphere comes into play; because with this research method the air in the body is likewise excited and influences, with its mass and pressure, the flexible coupling of the front and back plates. (see Lit.3 and Lit. 4 on this).

The measuring approach used had similarities to our earlier method. A violoncello was available from the workshop of Martin Schleske, Munich, *(Op. 26 after Andrea Guarneri)*. The instrument was new, ready to play, and had already been used occasionally for tests. Its condition was measured by the method described in Part II, and afterwards it underwent a modal analysis in the violin research laboratory of Martin Schleske in Munich. After these tests it was, under our careful control and based on our experience, subjected to several days of a vibration treatment *(Vibrelax)* in Berlin, and then measured again acoustically under similar climatic conditions. Finally a second modal analysis took place in Martin Schleske's laboratory in Munich.

In the following the results of the two modal analyses are presented and discussed *before* and *after* the *vibrelax* treatment. From these two series of measurements (*before* and *after*) 52 eigenforms (modes) were calculated for both the front and back plates, viewed from the outside.

The data records for each mode contain:
- the mode number # (however the numbering is different "*before*" and "*after*" - therefore in the later discussion of the corresponding mode pairs, two numbers (before and after) are used; refer to fig. 3 to 7)
- the eigenfrequency (resonance) in Hz
- the damping in %
- the max. negative magnitude, in [Hz·m/N·s]
- the max. positive magnitude. in [Hz·m/N·s]

A preliminary comparative inspection of the eigenforms showed that they obviously fall into two different classes:
1.) those that do not at all correspond to the *before* measurements and
2.) those that correspond, with only slight deviations from the original eigenforms, so that pairs of identical eigenforms can be identified from both series of measurements, *before* and *after*.

Initially, in reviewing the second class, one had the impression that the vibration treatment had had no effect, and that only the modes of the first class would show that a *"dedamping"* of the vibrating body had occurred.

If we limit ourselves to these, then a comparison of the damping values δ (see illustration 2) shows that the spread of the *before* values became smaller (cf. also the statistical average values with their deviations), but that both series of measurements are within overlapping margins of error and thus rule out a binding conclusion.

Therefore our attention was directed toward the eigenforms of the second class. Some selected representatives are shown in illustrations 3 to 7. The pair [10/9][1] provides an example of the fact that for this mode a vibration treatment produces practically no change in the in-phase areas of the front and back plates. In the mode diagrams [17/15] (here especially for the front), [21/19], and [27/25], and also [26/24] it is quite noticeable, that the in-phase areas of the front{ [15];[19] and [25] } and of the back { [24];[34] and [46] }are increased, which should make a more effective sound radiation possible. For higher frequency mode diagrams {e.g. [48/46] } a finer partitioning of the plates can be clearly seen, whereby in fact in this example the front only in the upper region, and the back only in the lower are seen to be noticeably affected.

Here *vibrelax* affords an anisotropic modification of the vibration response of the body, which should show up in the directionality of the sound field for such frequencies.

Since in the case of the second class, a definite paired correlation of the eigenforms from both series of measurements, *before* and *after*, was possible, the use of these data records presented itself for a comparative review, so that the achieved changes could be recorded quantitatively. Here was an opportunity to inquire about an influence on the eigenfrequencies of the modes and to examine their resonance quality values Q. This quality factor is, as is well known, directly linked with the energy required for the maintenance of a particular oscillation:

$$Q = 2\cdot\pi\cdot(\text{energy})/(\text{energy loss per period})$$

and is connected with the damping δ (in %): $Q = \omega_o / (2\cdot\delta)$

A system with high Q requires therefore less energy, since the loss per period through friction (among other things converted into heat) is smaller. In illustration 8 the relative changes in Q (in %)

---

[1] The assignment of the notation is from the above-mentioned illustrations 3 to 7.

$\Delta Q = 100 \cdot (Q^{after} - Q^{before}) / Q^{before} = {}_{before}^{after}\Delta Q/Q^{before}$ of the modes

are shown above their respective eigenfrequencies. As can be seen, they lie (nearly all) in the positive region; which means a *vibrelax* treatment leads to an increase in Q (a reduction of damping in the vibrating system). An interesting exception is given by two mode pairs [9/8] and [10/9], which clearly exhibit a widening of resonance (an increase in damping). In this region of the lowest frequency body vibrations in cellos the air cushion coupling between front and back plates plays a significant role.(Lit. 2.1, pages 268 ff,; Lit 3.)

This leads in the end to a marked effect on the bending vibrations of front and back, which lets mechanical vibration energy of the body transfer into pressure fluctuations of the surrounding air. This effect, which favors sound radiation and is desired, is called radiation damping. So here an " increase " of the damping (paradoxically) is desired, which is also favored by the *vibrelax* treatment, as the values in illustration 8 show.

Incidentally, this effect led us to no longer speak of "*vibration dedamping*" but of predominantly stress reduction by relaxation, therefore of "*vibration relaxation*:" or, more briefly, of *VIBRELAX*.

The other diagram in illustration 8 again reflects the effect of a *vibrelax* treatment on modifications of the eigenfrequencies:

$({}_{before}^{after}\Delta f_{res}) / f_{res}^{before} = (f_{res}^{after} - f_{res}^{before}) / f_{res}^{before}$ in %.

The increase of the eigenfrequencies (up to 2,5%), which can be noted with all the modes, and which was recorded also with the previous acoustic measurements (sound field measurements), indicates that the friction losses of the flexible material vibrations became smaller. As is well known, the maximum attainable amplitude of forced vibrations sinks with an increase of the relative damping and at the same time lies at lower frequencies. But since a *vibrelax* treatment has precisely the opposite effect, that is f_{res} moves to higher values, we can conclude that through such treatment the friction losses are reduced and thus less energy is required to produce and maintain the elastic vibrations of the material "wood"; in other words, the "bowing effort" needed for playing should be less, and consequently the instrument's response should also be better. As for the question of in or at which components the friction in a vibrating instrument is principally reducible, this is not at all uniformly viewed and remains a still open question. To what extent the determining material characteristics of the wood itself undergo a favorable change remains unresolved, and it seems very unlikely. (See also Lit. 4, part 9.) It is much more likely that the "composite places" of the instrument (glue joints, linings etc.) can be influenced with respect to an accelerated aging treatment with VIBRELAX. (cf. also Part I.)

Further parameters, which are to be found in the data records from the modal analyses, are values for the magnitudes (deflections) (see illustration l). The question here was, what are the max. occurring level differences of the bending wave amplitudes; i.e. the peak-to-peak values for the respective magnitudes of the indicated pairs *before* and *after* a vibration treatment. These values are as usual placed above the (averaged) pair resonant frequency in illustration 9. While for the low modes hardly any influence on these parameters is registered from a *vibrelax* treatment (the picture of the differences at the top margin clarifies this), but with the higher modes, starting approximately from 600 Hz, a clear effect can be seen.

Here the peak-to-peak differences are dispersed and are clearly negative; i.e. the *before* values are larger than the *after* values in this frequency range of $f_{res} > 600$ Hz. Therefore *Vibrelax* in these cases smooths the "mountains of the bending waves" on the front and back substantially. The question remains open whether this is disadvantageous to the vibration response of the body and whether this frequency range is also significant in another context.

For this purpose the ratios of peak-to-peak magnitudes to the respective resonance quality values Q of the modes were calculated. The diagram in illustration 10 shows these ratios, as customarily represented, above the resonant frequencies. Here it is noticeable that the relatively wide spread scattering in the differences of the peak-peak magnitudes above 600 Hz in illustration 9, was obviously substantially "flattened" by calculating their ratios to the respective resonance quality values Q. The frequency response is homogeneous here up to the measuring limit, but again with the exception of the mode affected by radiation damping [9/8]. But since the *after* values are throughout now smaller here than the *before* values ("difference of measured values *before* minus *after*" in illustration 10 above), the Q of the modes must have increased as a result of the *vibrelax* treatment which, because of a reduction in damping, is equivalent to a better utilization of the applied vibration energy.

Thus

- The volume of sound is greater.
- The instrument “speaks” more easily.
- The sound is richer in overtones.
- A skillful bowing technique can be more effectively applied, and with that
- The modulability of the instrument can be controlled more completely and with less effort during playing.

Our work in developing the *VIBRELAX* procedure, which we have presented in three Parts, is based on statements about the “*vibration dedamping*” procedure by its author, Professor G. v. Reumont, to whom we owe particular thanks for many attendant consultations, particularly in the initial phases.

The interest in problems of sound improvement in string instruments and their
scientific background led to an advantageous cooperation with master violin maker Martin Schleske, Munich, without which the work of Part III would not have been possible. For that he is due our special thanks.

The authors are grateful to Mr. Henry Strobel and Mr. Lothar Tews for their agreement to translate the German Text into English. We owe the English version to that generous collaboration which is highly appreciated.

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Illustration III/1

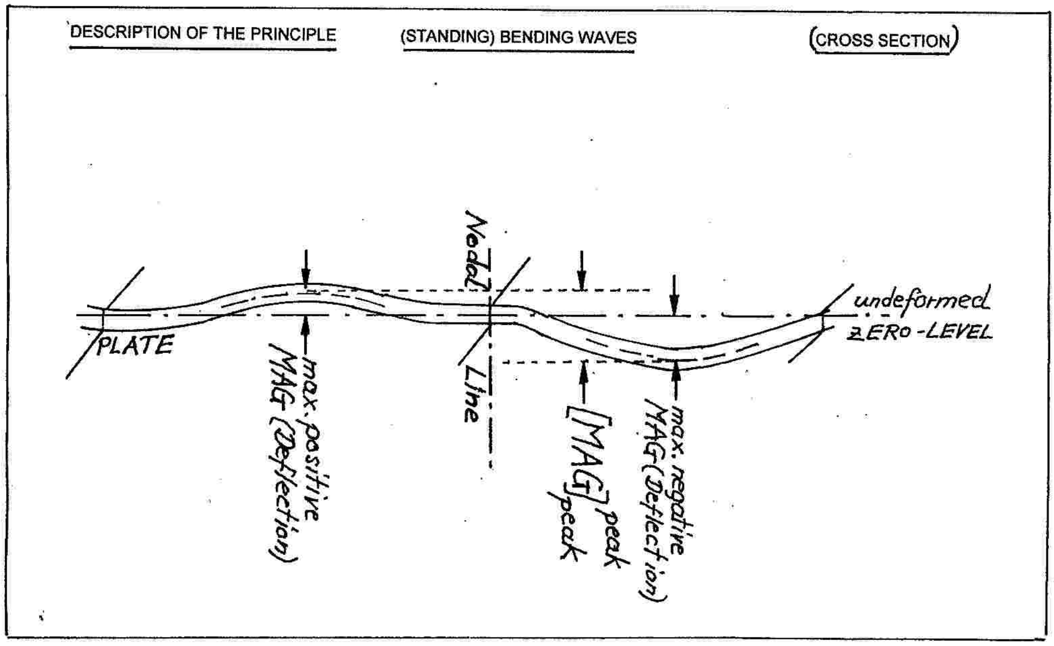
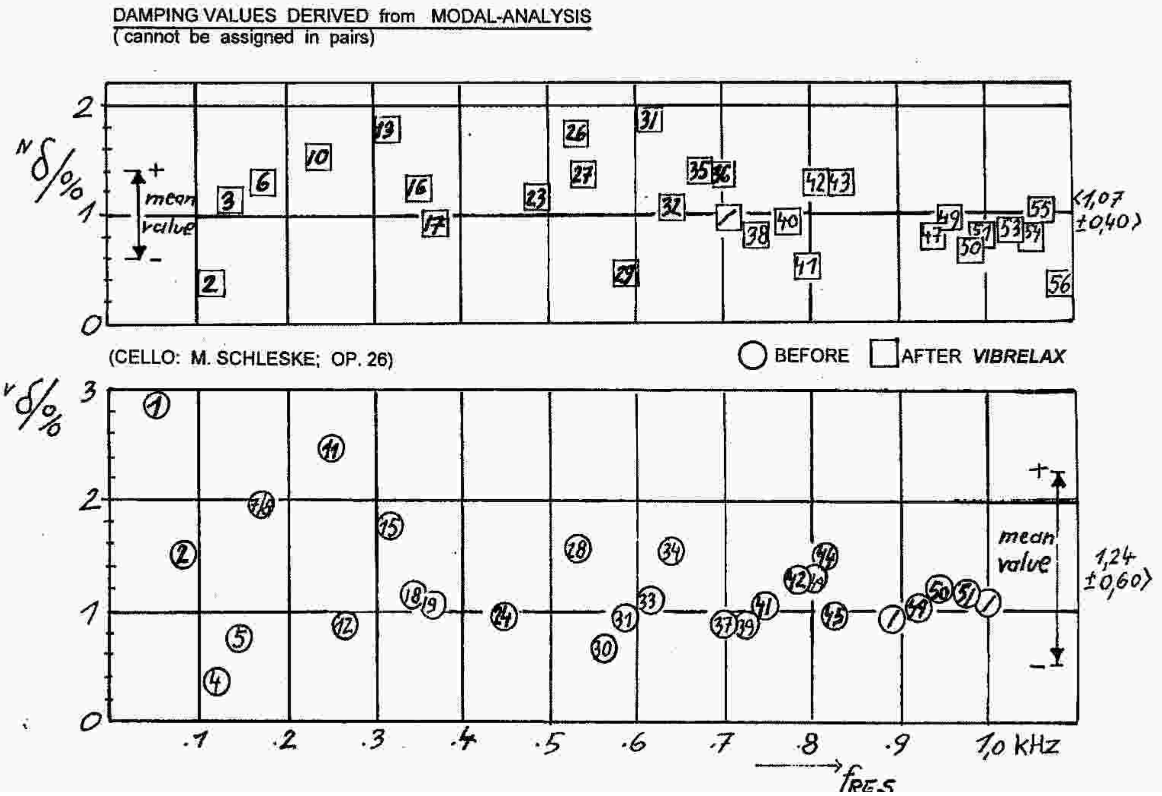
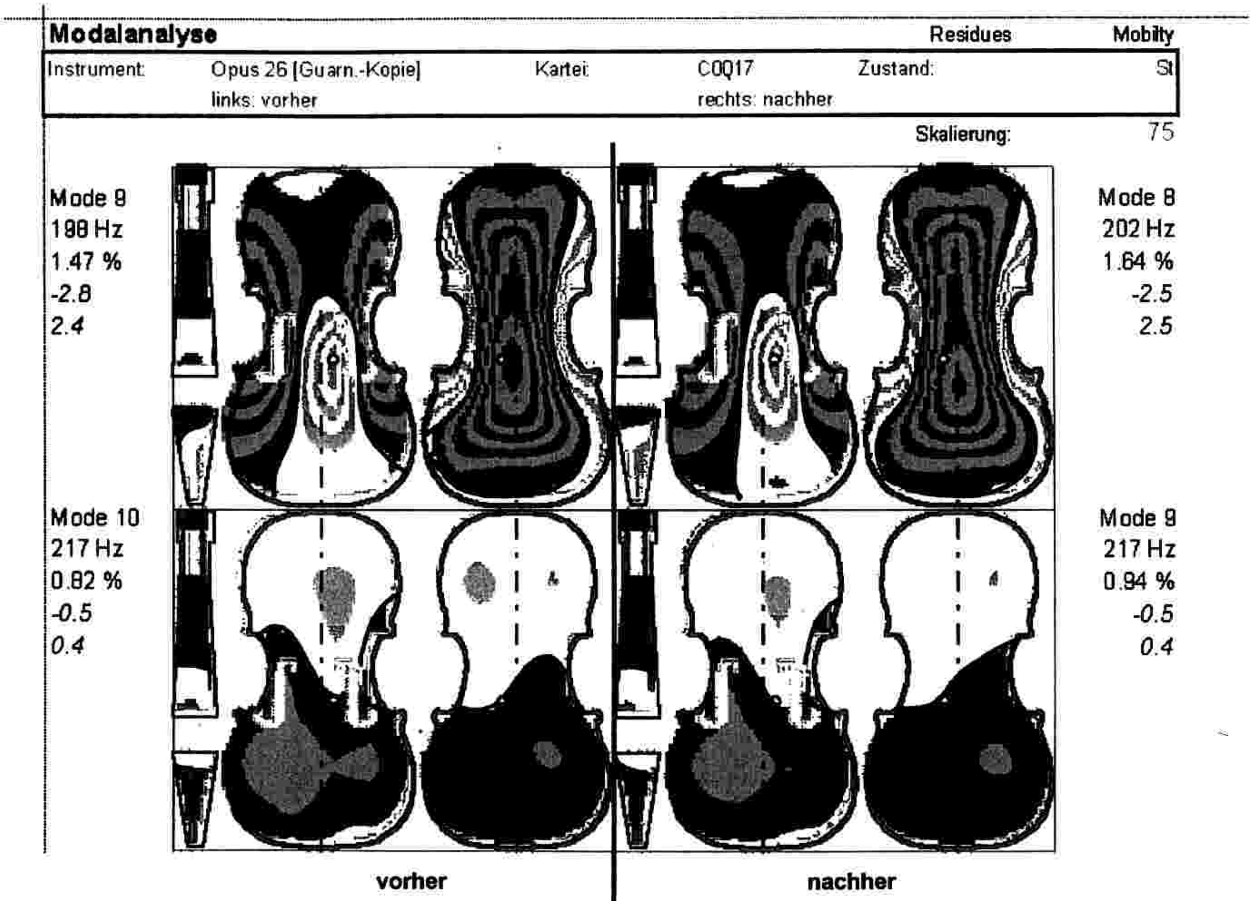


Illustration III/2



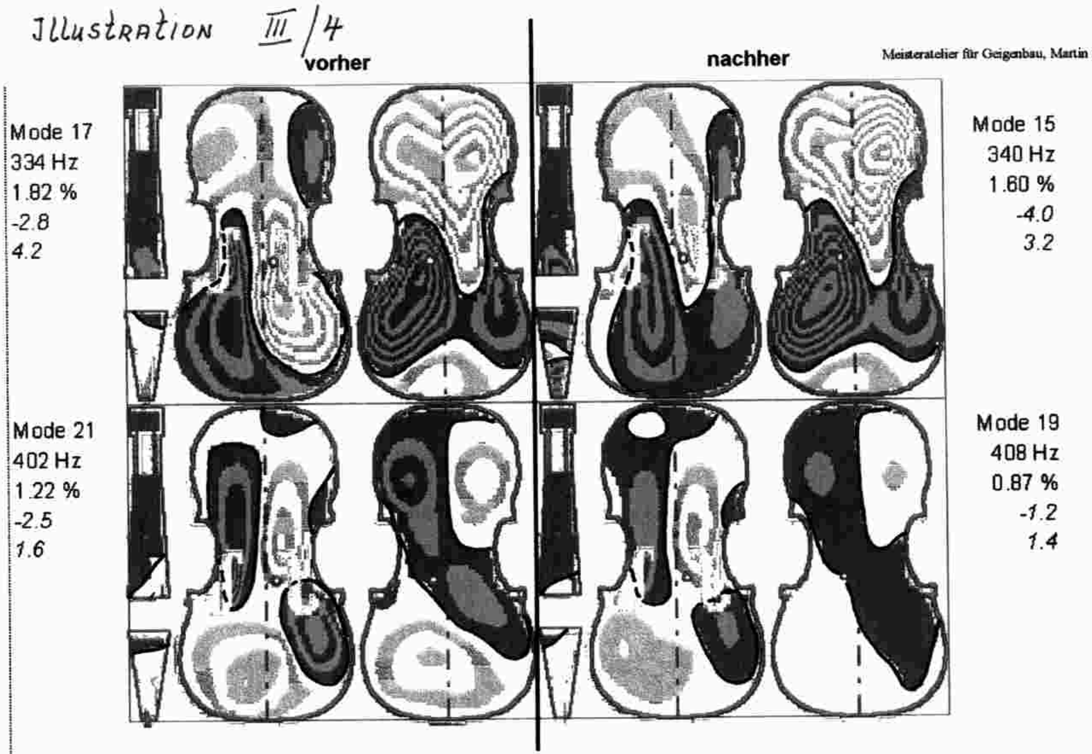
JIlustration III/3

Meisteratelier für Geigenbau, Martin Schleske München 2001



JIlustration III/4

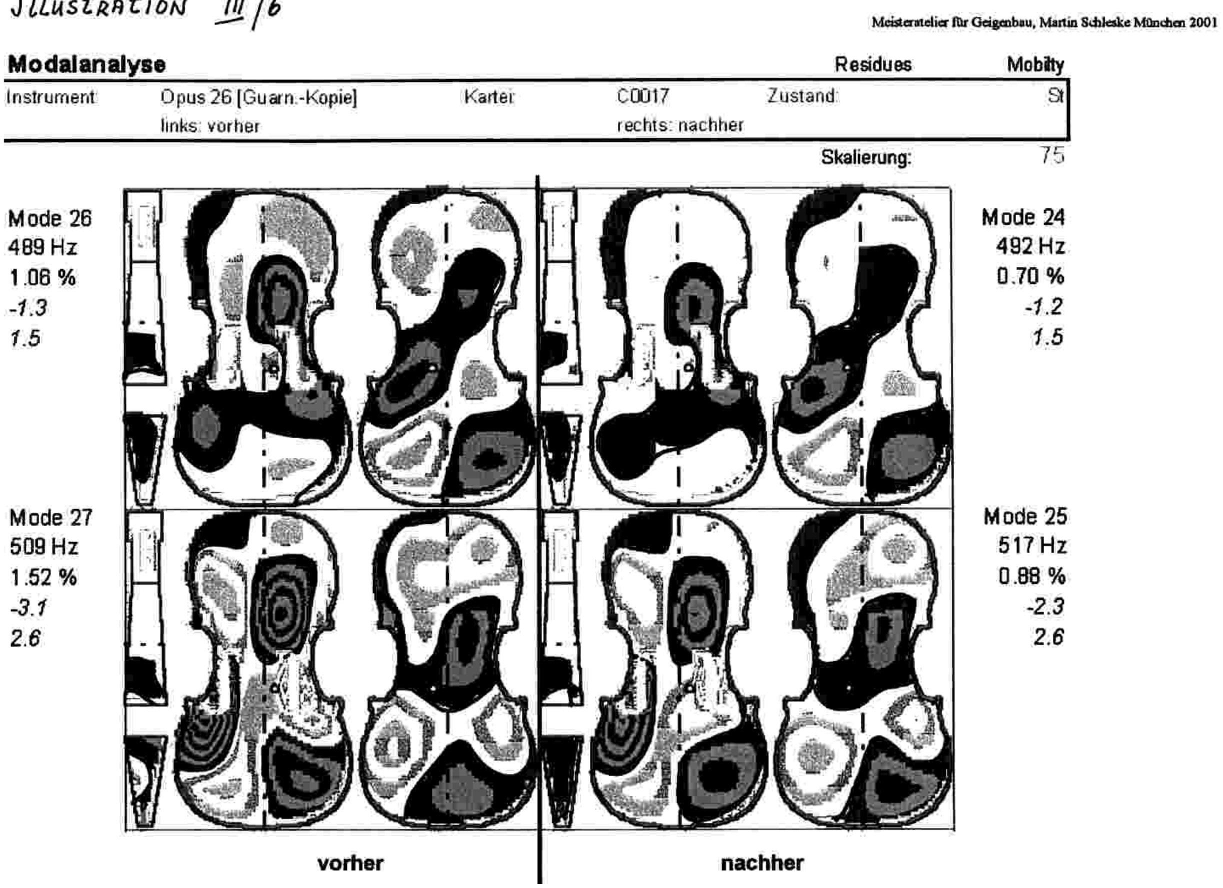
Meisteratelier für Geigenbau, Martin Schleske München 2001



Linienabstand (Abstand zwischen gleichartig gerasterten Linien) = 50 (Hz*m) / s*N*Skalierung. Phasenkonvention: Jeweils expandierende bzw. komprimierende Plattenbereiche des Corpus' gelten als gleichphasig und sind entsprechend mit gleichartigem Raster dargestellt. Decke jeweils links, Boden rechts. Ansicht beider Platten von außen.
Bezeichnung neben den Eigenformen (von oben nach unten): Mode-#, Frequenz (Hz); Dämpfung (%); *max. neg. Residue-Mag; max pos. Residue-Mag (Hz*m) / (s*N).*

JIlustration III/5

Illustration III/6



Meisteratelier für Geigenbau, Martin Schleske München 2001

Modalanalyse Residues Mobility

Instrument:	Opus 26 [Guarn.-Kopie]	Karte:	C0017	Zustand:	St
	links: vorher		rechts: nachher		

Skalierung: 75

Mode 26
489 Hz
1.06 %
-1.3
1.5

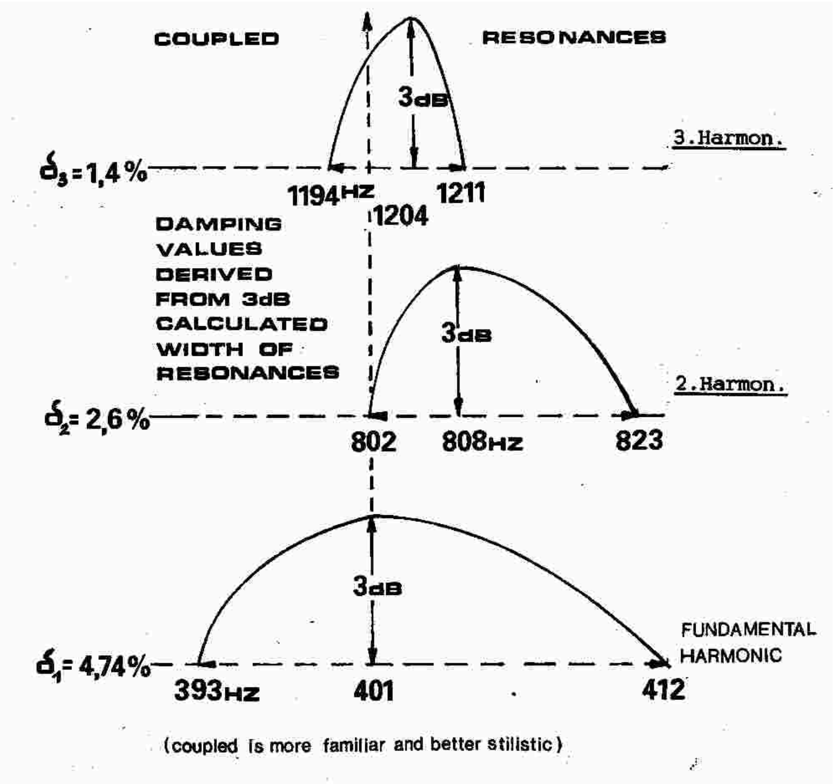
Mode 24
492 Hz
0.70 %
-1.2
1.5

Mode 27
509 Hz
1.52 %
-3.1
2.6

Mode 25
517 Hz
0.88 %
-2.3
2.6

vorher

nachher



(coupled is more familiar and better stilistic)

ILLUSTRATION 7

Illustration III/8

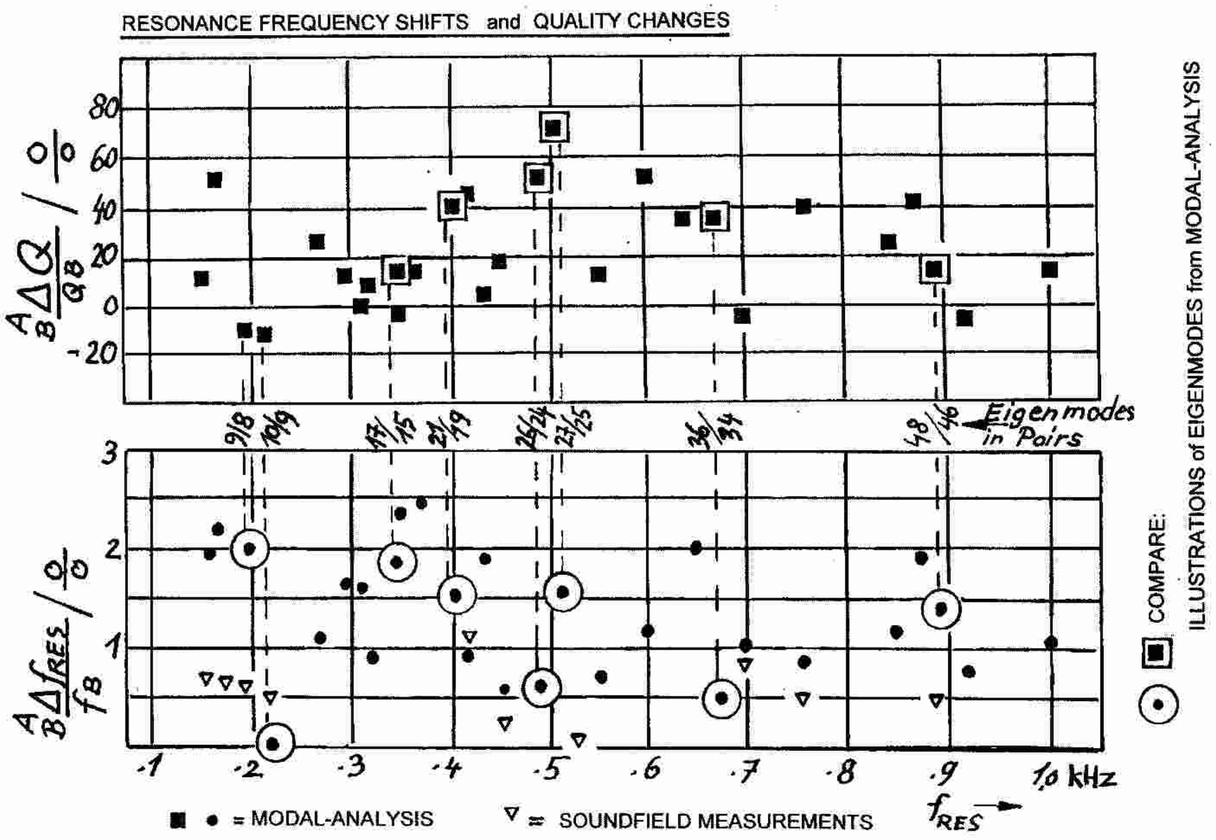


Illustration III/9

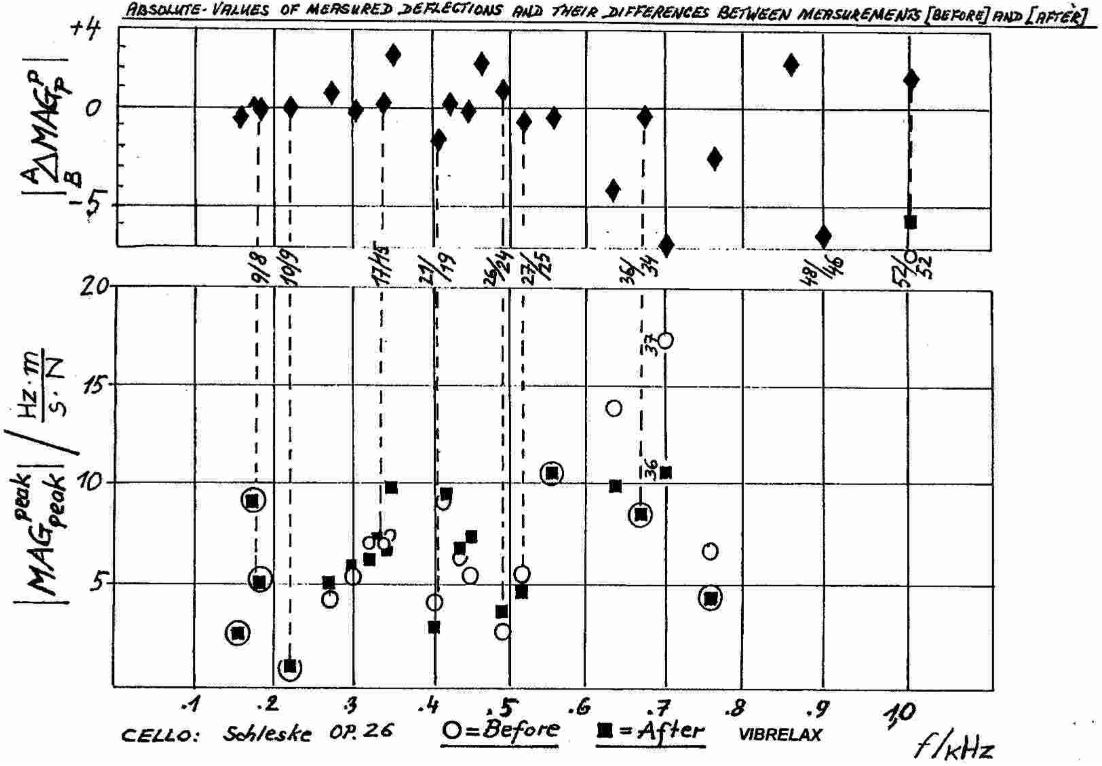
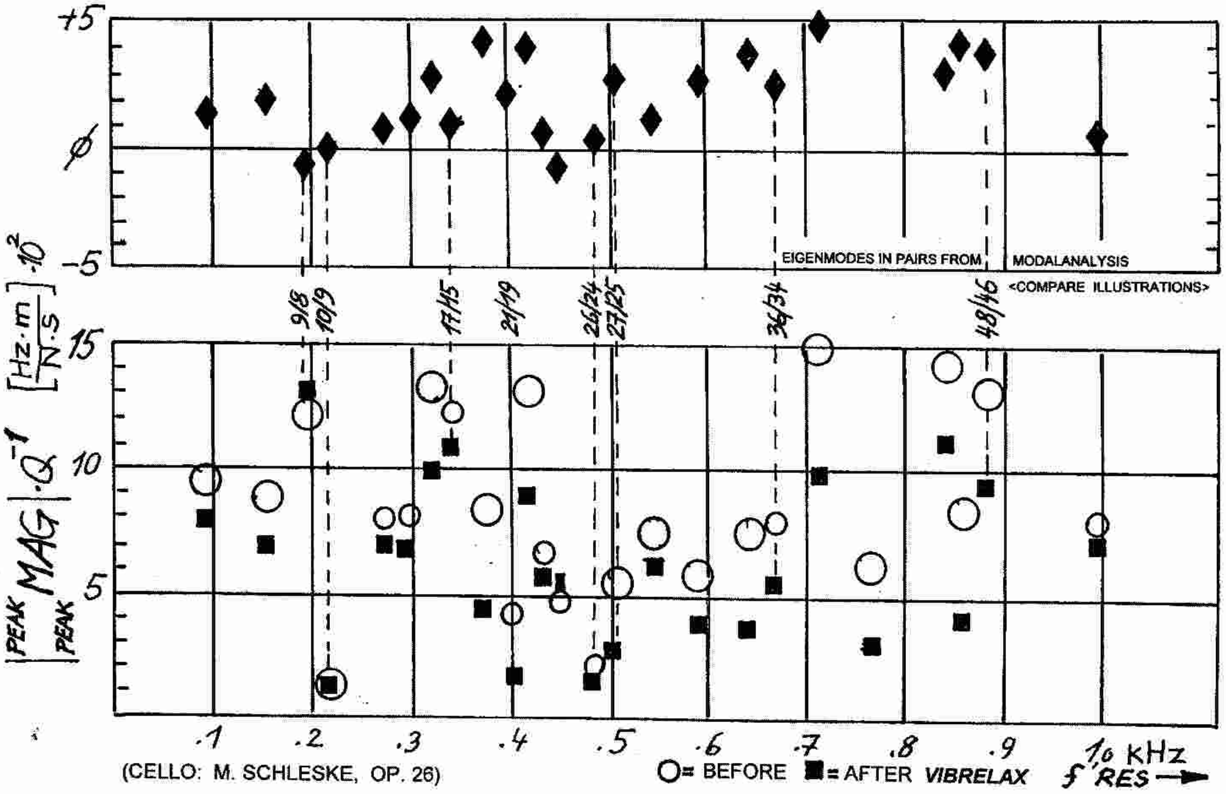


Illustration III/10

COMPARISON of NORMALIZED SOUND VELOCITY ($v_S = \omega \cdot A$) WITH RESONATOR-QUALITY Q



(Unpublished analysis by Dr. Gottfried Lehmann, August 6, 2000. *Draft* translation by Henry Strobel , August 15, 2000, to file, author, and L. Tews) Illustrations omitted!

Vibrelax - how can the effectiveness of this special overstressing vibration treatment be made convincingly plausible?

Thanks to the setting of the imbalance vibrator and the treatment frequency, the vibration conditions for the whole oscillating system (consisting of the instrument with the bridge-mounted motor assembly) can be determined.

Then the system (the instrument and the attached imbalance vibrator) can be modeled as a one-dimensional oscillator, in which the overall system is steady state driven for a period of vibration, where the available amplitude x from the system characteristics (mass m, coefficient of friction r, stiffness s) and the selected excitation angular frequency ϖ follow the differential equation:

$M\bullet x'' + r\bullet x' + s\bullet x = F(t)$

The damping constant δ is then defined by r/2M. In the reduced form of the above differential equation (after substitution of t by $t = \varpi_0\bullet t/\varpi$) we get the reduced damping constant $\Lambda_0 = 2\pi\delta/\varpi_0$. With
$\varpi_0^2 = s/M$ the force will be:
$F(t) = F_0 \sin \varpi t$ (sinusoidal) and may have a constant effective value F↓ during the whole treatment. The treatment frequency should be chosen so as not to coincide with one of the body resonance frequencies, which were identified in the earlier measurements. (Although the strings and tailpiece below the bridge are strongly damped from vibrating with foam rubber, there is in the frequency range from about 40 to 200 Hz a difficult-to-avoid resonant vibration of the tailpiece, which often appears in the measurement plots.)

Discussion of the power requirements:

An oscillator with constant amplitude x_0 and angular frequency ϖ needs for its excitation an energy per period of
$E_{mech} = M\bullet x_0^2\bullet\varpi\bullet\varpi_0\Lambda_0$

The energy required to maintain the oscillation is $N_{mech} = (1/T)E_{mech}$
or $N_{mech} = (1/2\pi)M\bullet x_0^2\bullet\varpi^2\bullet\varpi_0\bullet\Lambda_0$
respectively using the formerly given
$\varpi\Lambda_0 = 2\pi\delta$,
$N_{mech} = M\bullet x_0^2\bullet\varpi^2\bullet\delta$

The available oscillation amplitude x_0 is in this model proportional to the amplitude of the exciting force F_0 and
$x_0 = \text{constant}(1/s)F_0$
Thus the required mechanical energy is $N_{mech} = M(F_0/s)^{2\bullet}4\pi^2\bullet f^2\bullet\delta$

The electrical energy that must be supplied to the motor is $N_{el} = \eta^{-1}\bullet M(F_0/s)^2\bullet 4\pi^2\bullet f^2\bullet\delta$
(where η is the given electrical/mechanical efficiency)

It then follows that, commensurate with the foregoing M, F_0/s, η were not changed during the treatment, so
$(1/f^2)\ N_{el} = \text{const}\bullet\delta$,
a value that is directly proportional to the damping constant. (Note: for this discussion it is imperative to take care in setting the imbalance value and the frequency for a constant effective value of F↓.

Conclusion
If there were no change at all (from the imbalance motor working on the treated instrument) during the particular treatments (from about 40 to 200 Hz) and if the successive treatments at the selected frequencies were ineffective, then with invariant δ the value $(1/f^2)N_{el}$, that is, in the diagram $(N_{el}/f^2)/f$ would be horizontal.

But as the figures show, the values of $(1/f^2)N_{el}$, being proportional to δ, become stepwise smaller, which means the motor power required diminishes - the energy requirement per oscillation period becomes smaller - in other words forced oscillations go more easily through (desirable)modifications in the system discussed. This can only be described as a relaxation or dedamping effect.

In the figures following, the key values of the measured power data of the actual DC motors at the beginning and end of each treatment step are N_b/f^2 or N_e/f^2 at the frequency applied. Included in that are influences which were conclusively assignable only to the motor and its bearing block in the course of the measurement period. Irregularities in the damping reduction can be explained through the swinging vibrations of the tailpiece, which were clearly visible with the stroboscope, and because of their essentially greater mass were quite remarkable. Also a motor replacement, which became necessary during a viola measurement, had no influence on the sort of relaxation recorded. The absolute value of the power measured was, as would be expected, displaced upward, since the second motor evidenced a higher power requirement at the same treatment frequency. Most significant was the desireable relaxation effect to be seen in the cello measurements, which were at different times (was there an influence from humidity differences?) but which were conducted with the same imbalance motor. Indeed it is not to be expected that a bare measurement of electrical power parameters and and treatment frequency allows conclusions as to the condition of the treated instrument.

To be able to make a statement on that, one first has to make acoustical measurements before and after Vibrelax conditioning.

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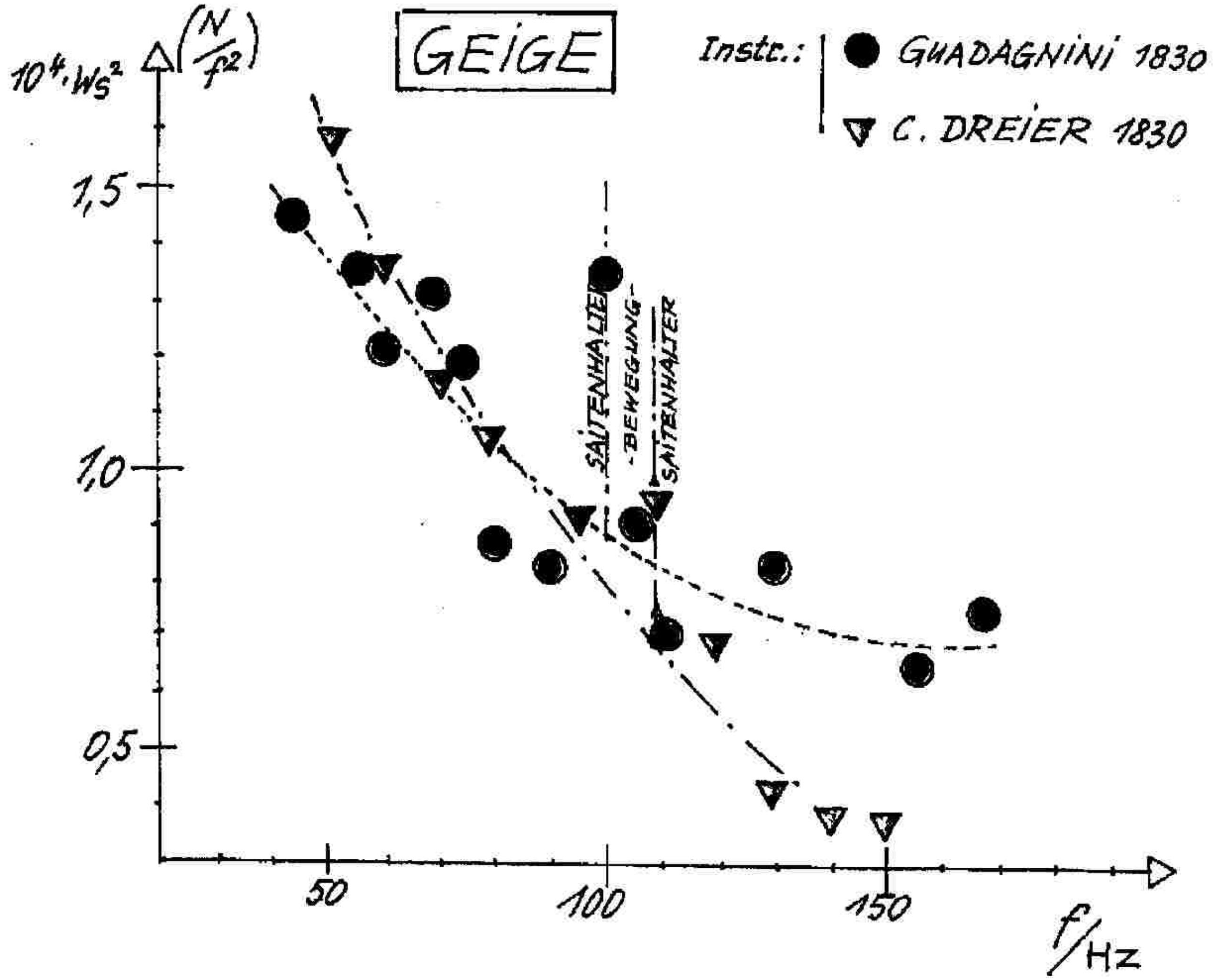
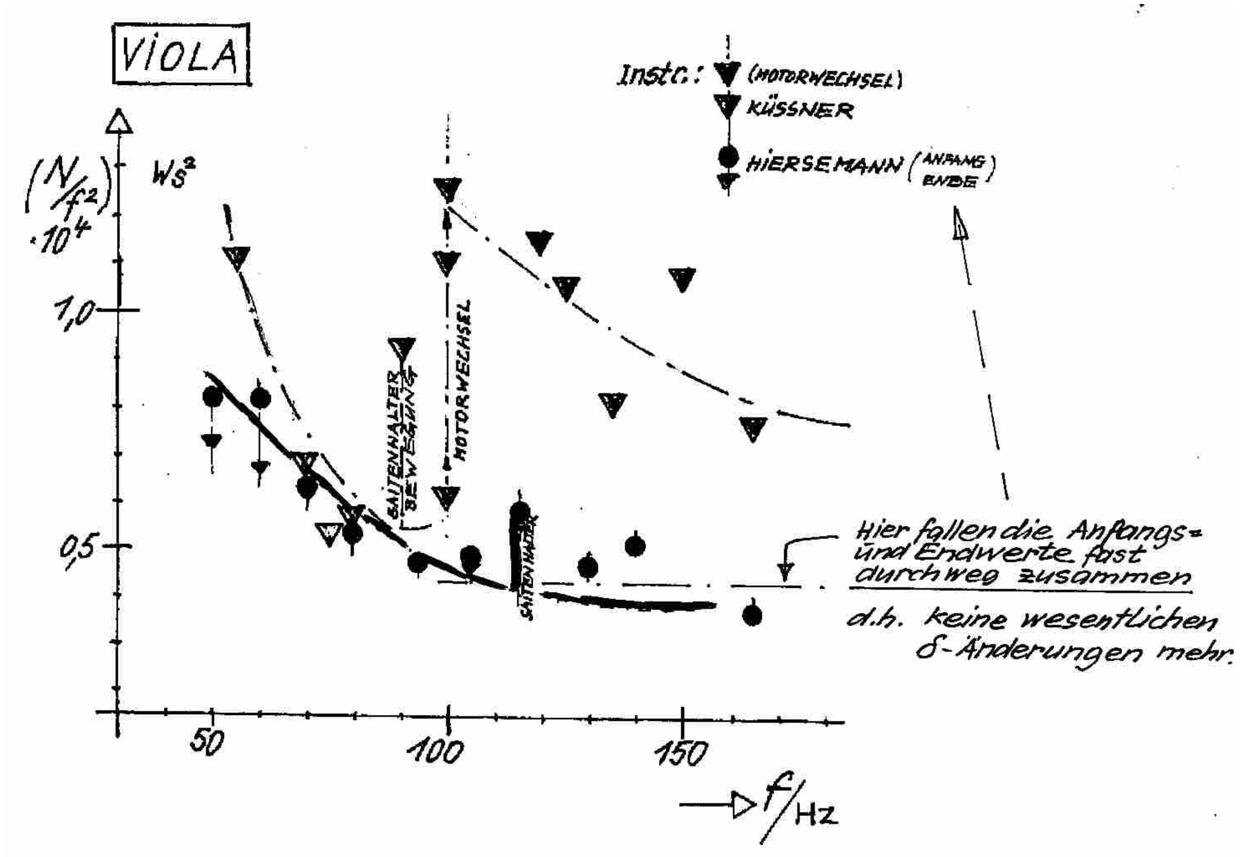
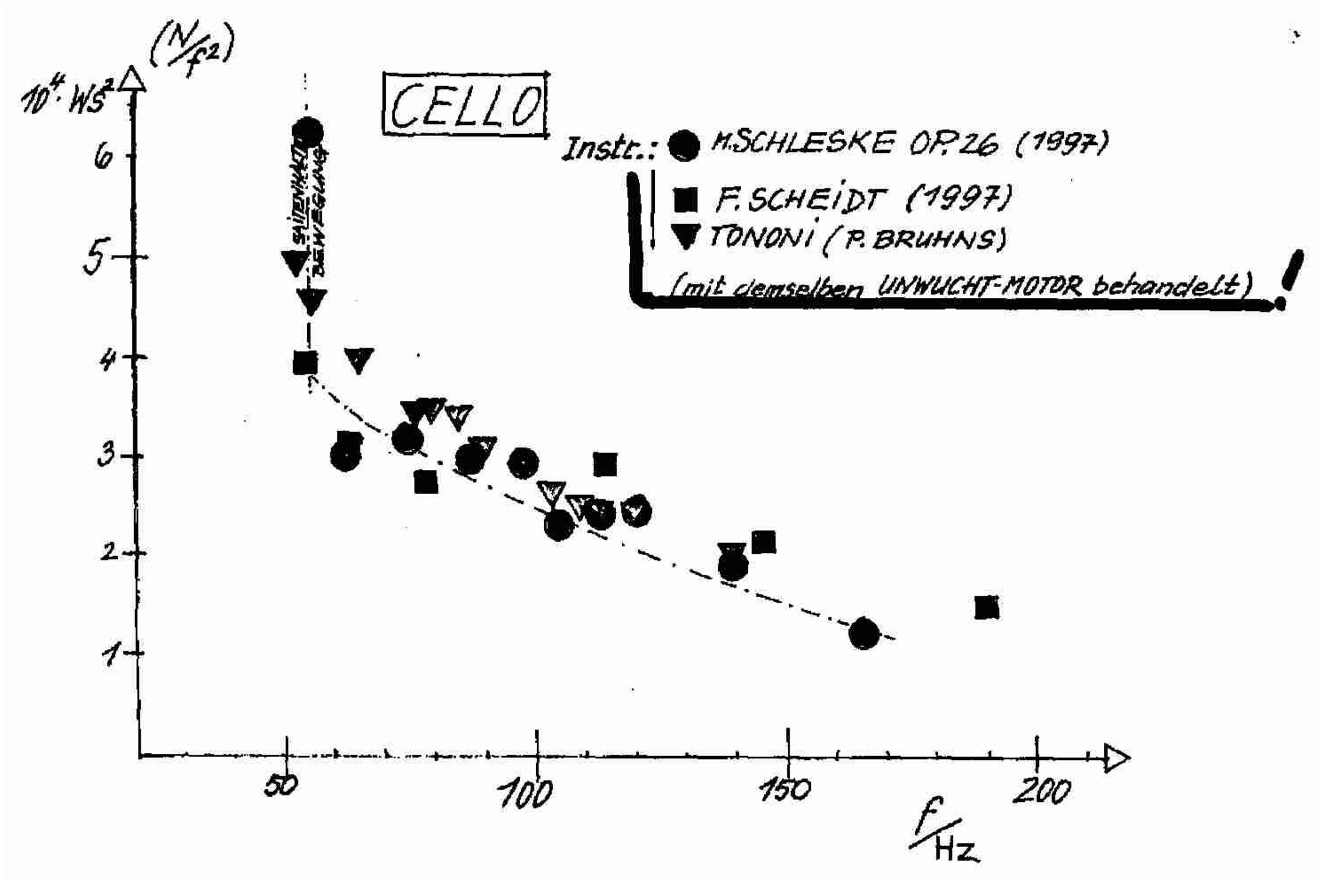



VIOLA
Instr.: (MOTORWECHSEL)
KÜSSNER
HIERSEMANN (ANFANG / ENDE)
$(N/f^2) \cdot 10^4$
Ws^2
1,0
0,5
SAITENHALTER BEWEGUNG
MOTORWECHSEL
SAITENHALTER
50
100
150
f/Hz
Hier fallen die Anfangs- und Endwerte fast durchweg zusammen
d.h. keine wesentlichen δ-Änderungen mehr.


(N/f^2)
$10^4 \cdot Ws^2$
CELLO
Instr.: M. SCHLESKE OP. 26 (1997)
F. SCHEIDT (1997)
TONONI (P. BRUHNS)
(mit demselben UNWUCHT-MOTOR behandelt) !
SAITENHALTER BEWEGUNG
6
5
4
3
2
1
50
100
150
200
f/Hz