Optimal issuance across markets and over time[*]

Jesse Davis[†]

Northwestern University

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Abstract

A risk-averse agent can sell claims to an asset of uncertain value to investors who have private information. When investors can choose how much information to acquire, the agent optimally issues information-sensitive securities in each market (e.g., debt and equity). When the value of the asset varies over time, the agent chooses to retain and, at times, repurchase a portion of the claims for issuance at a later date. The agent's choice to smooth the information sensitivity of the claims issued, across markets and over time, has novel implications. First, the relative information insensitivity of debt can render it a suboptimal security for financing. Second, if the agent has private information about cash flows, he can signal that he has better information by selling, rather than retaining, a larger claim to the asset. Finally, while the sale of illiquid securities generates increased uncertainty at issuance, it can lower the agent's uncertainty when raising capital in the future.

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[†]Email: j-davis@northwestern.edu. To download the latest version, visit kellogg.northwestern.edu/faculty/davis.

1   Introduction

When an owner of an asset has decided to sell, he can do so multiple ways. He must decide to whom to sell, how much to issue, and over what time frame to liquidate his position. In practice, we observe owners availing themselves of this flexibility by issuing multiple securities, sometimes sequentially. Consider the exit strategy of a private equity firm. Such firms often utilize leveraged recapitalizations, selling debt and using the proceeds to return capital to equityholders before selling their equity position.[1] Furthermore, we observe private equity firms choosing partial exit, as in an initial public offering, where they generally retain a portion of their holdings for sale at a later date. Similarly, issuers of asset-backed securities typically sell a first-loss, equity tranche, along with multiple investment-grade and speculative-grade bonds. These issuers can also choose to retain different tranches: while some hold onto equity only, others hold a fraction of each security issued.

We develop a model in which the owner of the asset faces uncertainty about the information investors possess about the asset's value. This information varies endogenously, as investors choose how much information to acquire based on the securities sold, or exogenously, as information about the underlying value of the asset becomes available over time. We show that, in the face of this price uncertainty, a risk-averse owner optimally sells information-sensitive securities in multiple, imperfectly integrated markets (e.g., debt and equity) and retains a claim to the asset for future sale.[2] Notably, information-insensitive securities (e.g., riskless debt) play no role in the optimal issuance policy, and relatively information-insensitive securities, such as risky debt, have a diminished place. The choice to smooth the information sensitivity of his issuance generates situations in which the owner can signal his private information via issuance, instead of retention, and creates a role for illiquid securities in the optimal capital structure. While theoretically novel, our predictions are largely consistent with issuance patterns observed in practice in a variety of settings.

Section 2 introduces the basic framework. An uninformed, risk-averse agent owns a risky asset. The agent chooses how much debt to issue and sells the residual equity. Risk-neutral investors, who trade in imperfectly integrated markets, choose how much private information to acquire given the information-sensitivity of the securities available for trade. Information acquisition, in combination with information sensitivity, generates uncertainty about the price investors are willing to pay for each security.

The optimal capital structure minimizes the agent's uncertainty about the price he receives for selling claims to the asset.[3] We show that the agent always issues information-sensitive debt and equity. Consider how investors respond when this is not the case. If the debt issued is information-insensitive, debt investors acquire no information. Information sensitivity is concentrated in the residual equity,

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[1]Over the past decade, these loans have accounted for approximately $40 billion/year (approximately 10% of issuance volume) in the leveraged loan market.

[2]In our model, information sensitivity is defined as the difference in the expected value of the security across states. An information-insensitive security (e.g., risk-free debt) has the same expected value in each state. Imperfect integration implies that the beliefs of the marginal investor in each market are not perfectly correlated.

[3]In Appendix B.4, we consider a static setting in which information acquisition also affects the agent's expected proceeds.

which leads equity investors to maximize their information acquisition. But this is not optimal: the agent is issuing the security with the most information sensitivity to the investors who are acquiring the most information, thereby maximizing his uncertainty.

Notably, though issuing an information-insensitive security can minimize *aggregate* information acquisition, it is not optimal for the agent.[4] Instead, the agent's issuance decision is determined by the *relative* information possessed by investors in each market. The optimal capital structure smooths across both sources of uncertainty: the agent issues a more information-sensitive security to the market which acquires less information. Thus, the minimum-variance capital structure, optimally chosen by the agent, does not require a minimum-variance security, e.g., risk-free debt.[5] This result stands in contrast to settings, such as those considered by Dang, Gorton, and Holmström (2015) and Yang (2015), in which the agent optimally minimizes the costs of asymmetric information by issuing a minimally information-sensitive security.

In the static setting, we show that the beliefs of the marginal investor in each market are imperfectly correlated. This induces *variation across markets* in the price of the information-sensitive portion of each security, which in turn induces the agent to issue information-sensitive securities in each market.[6] In a multi-period setting, when information about the fundamental value of the asset is revealed in each period, the price *within* a market can exhibit *variation over time*. Consequently, rather than selling the entire asset all at once, we show that this variation leads the agent to retain a portion of his claim for issuance in future periods.

In Section 3, we characterize the optimal dynamic retention policy. At the beginning of each period, the agent chooses what fraction of the asset to retain for sale at a later date. To focus on the role of time-variation, we begin by restricting the agent to equity issuance and assume that the precision of investors' signals is exogenously specified. Prices in each period are imperfectly correlated due to the revelation of new information about fundamentals; smoothing his issuance over time allows the agent to lower the uncertainty he faces. We show that the non-linear relation between investor information and the value of the asset generates predictable changes in the *expected* price of the asset.[7] As a result, the liquidation path need not be monotonic — when the expected gains are sufficiently large, the agent repurchases shares for later sale.[8] Consistent with this prediction, Lakonishok and Lee (2001) find that purchases, not sales, have predictive power for future returns. This occurs despite the agent possessing

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[4]As we show in Proposition 6, if information-sensitivity is concentrated in one market, reducing the information sensitivity of that security leads to a small decline in the information obtained in that market, but a large increase in the other market, due to the convexity of the cost function.

[5]It is costless, though also without benefit, for the agent to issue risk-free debt, in addition to risky debt, in our setting. If additional issuance also generated any transaction costs, however, the agent would issue risky debt, only.

[6]This effect is not limited to debt and equity. If the agent had costless access to other distinct markets, he would find it beneficial to issue information-sensitive securities more broadly. Faulkender and Petersen (2006) find evidence that when firms are able to issue public debt, they increase their borrowing, and generally do so by accessing both the bank and bond markets.

[7]A wedge is generated between the expectation of the current period's price and fundamentals (i.e., the next period's price), which leads to predictability in expected returns. As in Albagli, Hellwig, and Tsyvinski (2015), this is due to (i) limits to arbitrage, (ii) asymmetric information, and (iii) the aforementioned non-linearity.

[8]Issuance can occur even when the agent expects the price to increase. As a result, issuance is a noisier predictor of expected future returns.

no private information about the underlying value of the asset.

Having established the agent's incentive to smooth his issuance of information-sensitive securities both across markets and over time, we then consider its implications.

We begin by assuming that the agent must make an initial investment in order to acquire the asset. In Section 4, the agent chooses how best to finance the investment, anticipating selling any remaining claims to the asset at a future date. The agent is not financially constrained — he has access to liquid assets — but he can raise no more initial capital from financial markets than is necessary to make the investment.[9] We show that, in contrast to the pecking order of Myers and Majluf (1984), the agent prefers to issue equity when the required level of investment is low, turning to debt only when equity issuance becomes too costly. Moreover, the agent only utilizes his liquid assets when the required outlay is sufficiently high, and then only in tandem with issuance in the capital markets.

In standard settings with asymmetric information, the relative information insensitivity of debt, and the cash flows to which it is sensitive, makes it optimal for raising capital. The agent in our model, however, is concerned not just with the optimal security with which to finance an investment, but with the *entire* capital structure. In particular, the agent must consider how his financing decision ultimately affects the residual piece he owns and later sells. While issuing debt minimizes the information sensitivity of current financing, it leaves the agent with a more information-sensitive claim to sell at a later date. For low levels of investment, the agent therefore prefers equity financing.[10] On the other hand, the agent faces more uncertainty about equity investors' beliefs due to their anticipation of future debt issuance.[11] As a result, when the required investment is sufficiently high, debt becomes preferable to equity. Finally, note that by utilizing his liquid assets, the agent reduces the amount of financing required from capital markets, which, in turn, reduces financing uncertainty. There exists a threshold level of investment at which point the agent can optimally smooth his issuance across time using capital markets; above this threshold, he begins to utilize his liquid assets so that his financing needs are reduced and this optimum is preserved. It is interesting to note that, in practice, private equity firms generally combine their liquid assets with capital raised via debt (a leveraged buyout) along with equity (as part of a syndicate) when investing in a new business. Furthermore, the empirical literature (e.g., Frank and Goyal (2003); Fama and French (2005)) provides evidence that, amongst young, high-growth firms, equity issuance is common.

In Section 5, the agent has private information about the underlying cash flow of the asset. As in Leland and Pyle (1977), his issuance decision can signal his type, i.e., his private information. We first show that, when the agent can credibly disclose his private information, equilibria exist in which higher types will optimally choose to retain a large claim to the risky asset.[12] This occurs because higher types must retain a larger claim to optimally smooth their issuance over time. In contrast, when we introduce private information, equilibria exist in which higher types signal through *increased issuance*,

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[9] Such a restriction could be motivated by moral hazard.

[10] Equity financing is always information-sensitive. When the level of required investment is sufficiently low, debt issuance can be information-*insensitive*.

[11] Equity investors account for the proceeds of future debt issuance, which are paid as dividends, in determining their valuation today. Debt holders, on the other hand, have no claim to any future issuance.

[12] In the setting considered, "higher" types have both higher expected cash flows and lower information sensitivity.

instead of through retention. In standard models of adverse selection and issuance (e.g., DeMarzo (2005)), the agent would like to sell his entire claim today. In signaling equilibria, agents take costly actions which reveal their private information to other agents — if agents of every type want to issue today, the only direction in which to signal is through retention.[13] In contrast, in our model, the agent would like to retain some of his claim for sale at a later date. As a result, equilibria exist in which excess issuance serves as a credible signal.

The analysis discussed thus far assumes the existence of cross-market learning, i.e., the ability to condition on prices in other markets. We consider its impact on price uncertainty in Section 6. Cross-market learning generates two countervailing effects. Holding investors' information acquisition decisions constant, cross-market learning increases the information available to investors. Moreover, this conditioning increases the correlation of investors' beliefs across markets. However, prices in other markets serve as free, public signals, to which investors respond by choosing to acquire less precise private signals. In our static setting, we show that when the marginal cost of information is low and fundamental uncertainty is high, this endogenous response can lower price uncertainty. When this is true, the agent prefers to be transparent at issuance, releasing information about interest in both markets.

When a previously-issued security is illiquid, i.e., does not trade, investors in other securities are unable to condition upon its price in the secondary market. This illiquidity, however, also affects the agent's uncertainty in the primary market. We begin by showing that investors form more precise expectations about the value of an illiquid security, leading to increased price uncertainty. This implies that issuing illiquid securities, such as bank loans and private equity, would generally be suboptimal in a static setting. This need not be true in a dynamic setting. If the agent anticipates future issuance, this illiquidity can be valuable — the inability to condition on previous issues can lower the agent's uncertainty about future issuance.[14] In such situations, bank loans and private equity may be preferable early in a firm's life.

We conclude, discussing directions for future research, in Section 7.

1.1 Related Literature

This paper builds on the noisy rational-expectations equilibrium literature (e.g.,Diamond and Verrecchia (1981); Hellwig (1980)) which focuses on the role of information acquisition (e.g., Grossman and Stiglitz (1980); Verrecchia (1982)), trade in multiple securities (e.g., Admati (1985)), and the informational role of derivatives (e.g., Brennan and Cao (1996)). While many of these early papers focused on securities with linear payoffs, there is now a growing literature (e.g., Barlevy and Veronesi (2000); Vanden (2008); Breon-Drish (2015)) which allows for and examines the effect of a non-linear relation between payoffs and investor information, as in our model.

Our model of financial markets is most closely related to Albagli et al. (2015) and Chabakauri, Yuan,

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[13]Other analyses explore alternative dimensions along which the agent can signal, including price (e.g., Grinblatt and Hwang (1989); Welch (1989)) or liquidity (e.g., Williams (2015)).

[14]In Section C.3, we argue that this illiquidity can also prevent investors from obtaining large stakes in the firm, which reduces their incentive to learn.

and Zachariadis (2015). We extend the setting of Albagli et al. (2015) with risk-neutral investors who trade on imperfect information. The information in our model, however, is about the likelihood of each underlying state, as in Chabakauri et al. (2015). This innovation provides a tractable setting in which to study both information acquisition and price dynamics, which are absent from both papers. Both Albagli et al. (2015) and Chabakauri et al. (2015) discuss the capital structure implications of asymmetric information, but focus on its impact on a security's expected value. In our setting, similar implications are considered, but the focus of our paper is on the uncertainty generated by investors' information acquisition.

Investors in the model extract non-redundant signals from both bond and stock prices. In Back and Crotty (2015), market makers learn from bond and stock order flow, but an investor is endowed with his information and free to trade in either market. Similarly, Albagli et al. (2015) take the informational characteristics of investors in each market as given. In contrast, we allow investors to choose how much information to obtain, given the security available for trade and the information obtained by investors in other markets. This is similar to Goldstein, Li, and Yang (2013), in which segmented investors choose whether to acquire a signal, given their beliefs about the information contained in prices across markets. That paper's focus is on correlated claims to the same cash flow (e.g., CDS and the underlying bond), and more importantly, does not consider how an issuer might optimally choose to structure claims/issuance, given the behavior of investors. Finally, in contrast to each of these papers, but similar to the market participation literature (e.g., Allen and Gale (1994)), we allow some investors to choose, ex-ante, in which market to trade.

From the issuance perspective, our static model most closely resembles Boot and Thakor (1993), in which a risk-neutral agent chooses how to split and subsequently sell claims to the entire firm to investors whose information acquisition depends upon the securities issued. We assume that the agent is risk-averse, however, and that the agent can sell claims in imperfectly integrated markets. Our paper is part of a large literature in which the information acquisition of investors drives firm issuance decisions, including Boot and Thakor (1993), Fulghieri and Lukin (2001), Axelson (2007) , and Yang (2015). In contrast to each of these papers, in our model, prices in each market are imperfectly correlated. This is similar to Rahi and Zigrand (2009), which considers how arbitrageurs, with access to all markets, might design securities for trade with investors who are restricted to trade a subset of the securities.

Tranching is optimal, as in DeMarzo (2005) and Riddiough (1997), but in our model this is due to differences in the information acquired in each market. The optimal division of claims is based on the information-sensitivity of each security and endogenous information acquisition; however, unlike Dang et al. (2015) and Yang (2015), in our setting, information-insensitivity in one tranche is undesirable, as it concentrates the information-sensitivity in the other tranche. This makes our result more similar to Farhi and Tirole (2015), though in their model the solution to this problem is to bundle the two components together.

In a dynamic setting, we allow the agent to sell his claim over time, and as in DeMarzo and Urošević (2006), Hennessy, Livdan, and Miranda (2010) and Bond and Zhong (2015), we allow for share repurchases. In each paper, the relevant friction is asymmetric information; however, in their

models, the issuer has private information, whereas in our setting, investors acquire private information.

Our paper is related to the literature on violations of the pecking order of Myers and Majluf (1984). Fulghieri and Lukin (2001) and Yang and Zeng (2015) argue that equity issuance is optimal when the agent wants to encourage information acquisition. Fulghieri, García, and Hackbarth (2015) and Chakraborty, Gervais, and Yilmaz (2011) show that equity-like securities can be *less* sensitive to the effects of adverse selection, and therefore optimal, depending upon the assumptions about the distributional nature of the asymmetric information. In Axelson (2007), equity is preferred when raising capital in good states of the world is more valuable to the issuer. In contrast, our results arise through a novel channel: the agent's desire to smooth his sources of uncertainty across markets.

We also consider separating equilibria, as in Leland and Pyle (1977) and DeMarzo (2005), however, we consider settings in which issuance, rather than retention, can serve as a credible signal of type. Along this dimension, our work is related to the IPO literature in which firms can signal their type by issuing at a lower price (e.g., Grinblatt and Hwang (1989); Welch (1989)). In both papers, issuers also choose how much to sell, but in Grinblatt and Hwang (1989), retention is a positive signal of firm value, and in Welch (1989), issuers sell only what is necessary to finance their investment in equilibrium. In our setting, however, the price is set by investors; furthermore, issuance can signal a higher asset value because it generates excess uncertainty.[15]

2 Static Issuance

We begin by describing the basic structure of the model, characterizing the financial market equilibrium, and solving for the optimal issuance policy in a static setting.

2.1 Model Setup

2.1.1 Assets and Timing

A risk-averse agent owns an asset with uncertain future payoffs.[16] There are three periods, $t \in \{0, 1, 2\}$, and two states of the world, $s \in \{L, H\}$.[17] At time-0, it is known that the asset payoff, x, will be drawn from either G_H (in the high state) or G_L (in the low state). Both G_s are known, non-degenerate distributions and G_H first-order stochastically dominates G_L. It is without loss of generality to allow for limited liability: we assume $G_s(x) = 0$ for all $x < 0$. We define the expected payoff of the asset in each state:

$$V_s \equiv \int_0^\infty x \, dG_s(x),$$

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[15] In Grinblatt and Hwang (1989), retention also serves as a signal of lower variance — in our model, lower variance and higher asset value are always positively related, and so retention can also signal higher variance.

[16] While we assume the agent is endowed with the asset, our analysis accommodates a setting in which the agent is allowed to sell his entire claim at the same time he is raising capital to acquire the asset.

[17] In Appendix B.1, we modify the asset payoff to allow for N > 2 states. Under analogous distributional assumptions, the tenor of our main results remains unchanged.

and assume that both V_L and V_H are finite.

The agent does not know $q \equiv \mathbb{P}[s = H]$ with certainty, but knows

$$q = \Phi[z], \quad z \sim \mathcal{N}(\mu_z, \tau_z^{-1}),$$

where Φ is the cumulative distribution function of the standard Gaussian probability distribution.[18]

At time-1, z is revealed. At time-2, the asset payoff is determined. There are no actions which can be taken to alter the distribution of the asset's payoff. There is a risk-free asset in perfectly elastic supply and the risk-free rate is normalized to zero.

2.1.2 Market Participants

In addition to the single, risk-averse agent there exists a unit-measure continuum of risk-neutral investors. The initial information set of the agent and investors is identical; they have symmetric beliefs about the distributions, G_s, and share a common prior about the distribution of z. Each investor, however, is endowed with the ability to generate a private signal about the expected value of the asset. More specifically, each investor, indexed by $i \in [0, 1]$, observes

$$s_i = z + \varepsilon_i \quad \varepsilon_i \sim \mathcal{N}(0, \tau_i^{-1}), \qquad \mathbb{E}[\varepsilon_i \varepsilon_j] = 0 \, \forall j \neq i, \, j \in [0, 1]$$

By observing a private signal which conditions on z, investors can more precisely estimate from which distribution the payoff will be drawn.[19] Each investor can choose the precision of his signal ($\tau_i \geq 0$), but his choice is subject to a cost function, $C(\tau_i)$. We assume only that the cost function possesses standard characteristics: C is continuous, $C(0) = C'(0) = 0$, and $C', C'' > 0$ for all τ. The cost function is identical across investors.

Market Segmentation: Investors are restricted in the type of securities they can purchase. While they will be able to observe security prices in other markets, and use those prices to inform their beliefs, they can only purchase the securities trading in their market. Our focus will be on two markets: debt and equity.[20] The total measure of debt (m_D) and equity (m_E) investors will be determined endogenously and sums to one ($m_D + m_E = 1$), i.e., every investor trades in one market or the other. Investors are restricted from shorting and are subject to position limits.[21]

The market segmentation extends to the liquidity shocks (noise trade) in each market. We assume there are liquidity shocks, distributed across markets, such that

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[18]This formulation implies that $q \in (0, 1)$; however, because z is not drawn from a standard Gaussian distribution, q is *not* uniformly-distributed.

[19]Many models of information acquisition allow agents to condition directly upon the future, realized payoff. One can think of the signal available to investors as an approximate coarsening of these technologies.

[20]In Section C.2 we discuss the motivation and evidence for market segmentation in more detail and in Section 2.4 consider the implications of broader segmentation, e.g., segmentation based on credit rating or exchange.

[21]The assumption that investors are unable to short is unnecessary. Allowing for (bounded) short positions would make learning more valuable for investors, but should not change the optimal capital structure. Furthermore, such a restriction is observed more generally in security issuance. Investors are risk-neutral, and so position limits are necessary to bound their demand.

$$\begin{pmatrix} u_d \\ u_e \end{pmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_n^{-1} & 0 \\ 0 & \tau_n^{-1} \end{bmatrix} \right)$$

and that these shocks generate price-independent demand for a fraction $\Phi(u_d)$, $\Phi(u_e)$ of any issuance in the debt and equity markets, respectively. For tractability, we assume that liquidity shocks are uncorrelated and have identical second moments.

In Appendix B.2, we allow investors to trade in both markets and allow the variance of liquidity shocks to differ across markets; doing so leaves our main results unchanged.

2.1.3 Issuance

Due to the difference in their risk preferences, the agent would like to sell the asset to investors. If he sells at time-1, he receives

$$\mathbb{E}_1[x] = V_L + q\Delta V \qquad \Delta V \equiv (V_H - V_L) \tag{1}$$

from investors.[22] At time-0, the only term in (1) about which the agent is uncertain is the realization of q. Instead of waiting for this uncertainty to be resolved, however, the agent could sell claims to the asset at time-0.

We assume that the agent has mean-variance preferences over the proceeds he receives for the asset. He optimally chooses a face value of debt, F, to sell in the debt market; the residual claim is sold in the equity market.[23] Investors choose (i) in what market to trade, so that ex-ante returns are equalized across markets, and (ii) the precisions of their private signals, to maximize their expected profits. These are determined jointly in equilibrium.

Importantly, the agent does *not* set the price of each security. Instead, prices are determined in the market using the rational expectations equilibrium of Albagli et al. (2015). Using their private information, as well as the information contained in the price of debt (p_D) and price of equity (p_E), each investor submits his demand schedule and markets clear.[24] The price-setting process is detailed in the next section.

2.2 Financial Market Equilibrium

To determine his optimal capital structure, the agent must form beliefs about the price he will receive for each claim. If the agent issues debt with face value F, then given the distribution of payoffs

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[22]The agent would never wait until time-2 to sell the asset. At time-1, the agent and the investors have the same expectation of the asset's payoff; however, due to the residual uncertainty in the asset's payoff, and the agent's risk-aversion, he always prefers to sell before the cash flow is realized.

[23]Under the assumption that liquidity shocks are identical *within* each market, the agent is indifferent between issuing (i) a single debt claim with face value F and (ii) multiple, potentially subordinated, debt claims with a *total* face value of F within the debt market.

[24]In Appendix B.3, we explore whether or not, upon observing (and conditioning upon) the market-clearing prices, the agent would choose to accept the offered prices. In short, with sufficient risk-aversion, the offer is always accepted, and so we focus on that case here.

in each state, and limited liability, we can write the expected value of debt in each state $D_s(F) \equiv \mathbb{E}[min(F, x)|x \sim G_s]$. Similarly, given F, the expected value of equity in each state can be written $E_s(F) \equiv \mathbb{E}[max(x - F, 0)|x \sim G_s]$. We can then solve for the price of debt and equity, taking investors' choice of signal precision and market as given.

In addition to their private signal, s_i, each investor conditions on the price of debt and the price of equity. Investors differ only in their beliefs about the likelihood of each state, and as a result, this will be the only source of variation in their valuation of each security.[25] An investor's expectation of the value of debt can be written:

$$\mathbb{E}[D_L + q(D_H - D_L)|s_i, p_D, p_E] = D_L + \mathbb{E}[\Phi(z)|s_i, p_D, p_E]\Delta D$$
$$\Delta D(F) \equiv D_H(F) - D_L(F)$$

Similarly, an investor's expectation of the value of equity can be written:

$$E_L + \mathbb{E}[\Phi(z)|s_i, p_D, p_E]\Delta E(F) \qquad \Delta E(F) \equiv E_H(F) - E_L(F)$$

We will refer to $\Delta D(F)$, $\Delta E(F)$ as the *information-sensitivity* of debt and equity, respectively. In our setting, information sensitivity measures the change in an investor's expected value of the asset to changes in his beliefs about q, e.g.,$\frac{\partial \mathbb{E}[min(F,x)|]}{\partial \mathbb{E}[q]} = \Delta D(F)$. It is straightforward to show that the information-sensitivity of any claims issued must equal the information-sensitivity of the underlying assets, i.e., $\Delta D(F) + \Delta E(F) = \Delta V$. Financial engineering cannot reduce the information sensitivity of the agent's holdings.

We will conjecture and verify that investors can construct signals, s_D and s_E, from p_D and p_E, and that these signals will be independent and normally-distributed, conditional upon the true value, z. Let τ_D, τ_E, denote their precisions, respectively, which are determined in equilibrium. As a result, each investor believes

$$z|s_i, s_D, s_E \sim \mathcal{N}\left(\frac{\tau_z \mu_z + \tau_i s_i + \tau_D s_D + \tau_E s_E}{\tau_z + \tau_i + \tau_D + \tau_E}, \frac{1}{\tau_z + \tau_i + \tau_D + \tau_E}\right).$$

Lemma 1. *If z is normally-distributed, then the expectation of q can be written:*

$$\mathbb{E}[q] = \mathbb{E}[\Phi(z)] = \Phi\left(\frac{\mathbb{E}[z]}{\sqrt{1 + \mathbb{V}[z]}}\right)$$

We will be looking for a symmetric equilibrium in which $\tau_i = \tau_{i,D}$ for all debt investors and $\tau_i = \tau_{i,E}$ for all equity investors. Applying Lemma 1, a debt investor observing private signal s_i believes the expected value of debt to be

$$D_L(F) + \Phi\left(\frac{\tau_z \mu_z + \tau_{i,D} s_i + \tau_D s_D + \tau_E s_E}{\sqrt{\psi^D(1 + \psi^D)}}\right)\Delta D(F),$$

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whereas the expected value of equity for an equity investor with signal s_i can be written:

$$E_L(F) + \Phi\left(\frac{\tau_z\mu_z + \tau_{i,E}s_i + \tau_D s_D + \tau_E s_E}{\sqrt{\psi^E(1+\psi^E)}}\right)\Delta E(F),$$

where $\psi_D \equiv \tau_z + \tau_{i,D} + \tau_D + \tau_E$, and $\psi_E \equiv \tau_z + \tau_{i,E} + \tau_D + \tau_E$.

Lemma 2. *If $G_H \succ_{FOSD} G_L$, ΔD, $\Delta E \geq 0$ for all $F \geq 0$. Furthermore, $\frac{\partial \Delta D}{\partial F} \geq 0 \geq \frac{\partial \Delta E}{\partial F}$.*

Investors are restricted to purchase no more than $\frac{1}{m_D}$ $\left(\frac{1}{m_E}\right)$ units of debt (equity); in combination with their risk-neutrality, this implies that debt (equity) investors' demand is an element of the set: $\{0, \frac{1}{m_D}\}(\{0, \frac{1}{m_E}\})$. Under the assumption of FOSD, the value of each security is increasing in each investor's conditional expectation of z. Investor beliefs in each market can be ordered by their private signals, s_i, and so we posit a threshold strategy: investors purchase $\frac{1}{m_D}$ $\left(\frac{1}{m_E}\right)$ units of debt (equity) if $s_i \geq x_D(z, u_d)$ $(s_i \geq x_E(z, u_e))$; otherwise, they hold only the risk-free security. The thresholds, x_D and x_E, are functions of fundamentals (z) and the realized liquidity shock in each market (u_d, u_e).

We normalize the supply of each security to one. If we impose market-clearing in the debt market:

$$1 \quad = \quad \underbrace{\left[1 - \Phi\left(\sqrt{\tau_{i,D}}\left(x_D(z, u_d) - z\right)\right)\right]}_{\text{total demand by debt investors}} + \underbrace{\Phi\left(u_d\right)}_{\text{liquidity demand}} \tag{2}$$

It is clear from (2) that markets clear if and only if $x_D(z, u_d) = z + \frac{u_d}{\sqrt{\tau_{i,D}}}$. Market clearing in the equity market yields $x_E(z, u_e) = z + \frac{u_e}{\sqrt{\tau_{i,E}}}$. Moreover, $x_D(z, u_d)$, $x_E(z, u_e)$ summarize the information contained in prices:

$$s_D = x_D(z, u_d) = z + \frac{u_d}{\sqrt{\tau_{i,D}}}; \quad s_E = x_E(z, u_e) = z + \frac{u_e}{\sqrt{\tau_{i,E}}} \tag{3}$$

The marginal debt (equity) investor, whose signal $s_i = s_D$ $(s_i = s_E)$ sets the price equal to his conditional expectation:

$$p_D = D_L + q_D\Delta D \quad \text{where } q_D = \Phi\left(\frac{\tau_z\mu_z + (\tau_{i,D} + \tau_n\tau_{i,D})s_D + \tau_n\tau_{i,E}s_E}{\sqrt{\psi^D(1+\psi^D)}}\right) \tag{4}$$

$$p_E = E_L + q_E\Delta E \quad \text{where } q_E = \Phi\left(\frac{\tau_z\mu_z + \tau_n\tau_{i,D}s_D + (\tau_{i,E} + \tau_n\tau_{i,E})s_E}{\sqrt{\psi^E(1+\psi^E)}}\right) \tag{5}$$

It is clear from (4) and (5) that prices are invertible; observing p_D, p_E is equivalent to observing s_D, s_E. Furthermore, as (3) makes clear, s_D and s_E are independent, normally-distributed signals, conditional on the true value, z. Taken together, this verifies our conjecture. Note that because $s_D \neq s_E$, the beliefs of the marginal investor in each market are imperfectly correlated.

Proposition 1. *There exists a unique equilibrium. Debt investors purchase $\frac{1}{m_D}$ units of debt if $s_i \geq s_D$ and equity investors purchase $\frac{1}{m_E}$ units of equity if $s_i \geq s_E$, where s_D and s_E are defined in (3). Otherwise, investors hold the risk-free asset. The price of debt, p_D, and price of equity, p_E , are given by (4) and (5), respectively.*

We now turn to the agent's beliefs about prices. We will focus on the price of debt; analogous statements hold with respect to equity. We will refer to the variance of the marginal investor's beliefs about the expected value of q, that is, $\mathbb{V}_0[q_D]$, as *information-driven uncertainty*. Note that price uncertainty, $\mathbb{V}_0[p_D] = \mathbb{V}_0[q_D]\Delta D(F)^2$, is a product of its information-driven uncertainty and its information sensitivity.

Lemma 3. *If τ_n is sufficiently high and $|\mu_z|$ sufficiently low, $\frac{\partial \mathbb{V}_0[q_D]}{\partial \tau_{i,D}}$, $\frac{\partial \mathbb{V}_0[q_D]}{\partial \tau_{i,E}} > 0$. Furthermore, when $\tau_{i,D} > \tau_{i,E}$, $\mathbb{V}_0[q] > \mathbb{V}_0[q_D] > \mathbb{V}_0[q_E]$.*

Learning by debt investors ($\uparrow \tau_{i,D}$) increases the information-driven uncertainty of debt ($\mathbb{V}_0[q_D]$). Moreover, debt investors also condition upon the price of equity — as a result, increased information acquisition by equity investors ($\uparrow \tau_{i,E}$) has a similar effect. Finally, if debt investors acquire more information than equity investors ($\tau_{i,D} > \tau_{i,E}$), their beliefs about fundamentals are more variable ($\mathbb{V}_0[q_D] > \mathbb{V}_0[q_E]$). Throughout the rest of the paper, we will assume that τ_n is sufficiently high such that the inequalities in Lemma 3 hold.[26]

Lemma 4. *If $\mu_z = 0$, $\mathbb{E}_0[q] = \mathbb{E}_0[q_D] = \mathbb{E}_0[q_E]$. If $\mu_z > 0$ ($\mu_z < 0$), then $\mathbb{E}_0[q] > E[q_D]$ ($\mathbb{E}_0[q] < E[q_D]$); if, in addition, $\tau_{i,D} > \tau_{i,E}$, $\mathbb{E}_0[q_E] > E[q_D]$ ($\mathbb{E}_0[q_E] < E[q_D]$) when τ_n is sufficiently high.*

In our model, there is a non-linear relation between investors' information and the fundamental value of the asset: investors obtain information about z, but form expectations about $\Phi(z)$. As emphasized in Albagli et al. (2015), this has the potential to generate a wedge between the expected price paid (q_D) and the expected value (q).[27] When $\mu_z = 0$, the value of the asset is affected symmetrically by information about z; information acquisition has no effect on the expected price. We discuss the intuition for Lemma 4 when $\mu_z \neq 0$ in Appendix B.4.

2.3 Optimal Issuance

In this section, we assume $\mu_z = 0$, which implies that the agent's objective is to minimize the variance of his proceeds (since $\mathbb{E}_0[q_E] = \mathbb{E}_0[q_D] = \mathbb{E}_0[q]$). In Appendix B.4, we consider the implications for optimal issuance in the more general case, $\mu_z \neq 0$.

If the agent's uncertainty about p_D and p_E is sufficiently low, he is always better off selling his entire claim to the asset at time-0.

Proposition 2. *If τ_z^{-1} and τ_n are sufficiently high, retaining any claim to the asset for sale at time-1 increases the variance of the agent's proceeds.*

If the fundamental uncertainty is sufficiently high ($\downarrow \tau_z$) and liquidity shocks are sufficiently certain ($\uparrow \tau_n$), the agent prefers not to wait until time-1 to sell the asset. Retaining any portion for sale at time-1, after the uncertainty about z has resolved, simply increases his uncertainty. We leave for

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[26]While the variance of a given investor's beliefs about z is always increasing in the precision of his information, this does not necessarily imply that the same must be true with respect to the marginal investor's beliefs about $\Phi(z)$. The sufficient conditions ensure that this is the case.

[27]The existence of a wedge also requires limits to arbitrage and asymmetric information between investors.

Section 3 our analysis of how the agent would choose to sell his claim over time, and therefore assume that the conditions detailed in the proof of Proposition 2 hold. As a result, the agent's objective is to choose the face value of debt, $F \geq 0$, to minimize the variance of his total proceeds, i.e.,

$$\mathbb{V}_0[p_D + p_E] \;\; = \;\; \Delta D^2 \mathbb{V}_0[q_D] + \Delta E^2 V_0[q_E] + 2\Delta D \Delta E Cov(q_D, q_E). \tag{6}$$

2.3.1 Fixed Precision, Fixed Market

We begin by taking as given the market in which investors trade and the precision of their signals. Let F^* be the optimal face value of debt.

Proposition 3. *When the agent chooses F^* to minimize (6), there exists a unique $\Delta D(F^*) \, \forall \, \tau_{i,D}, \, \tau_{i,E}$.*

1. The agent issues a more information-sensitive security in the market with less precise information, e.g., $\tau_{i,D} > \tau_{i,E} \implies \Delta D(F^*) < \Delta E(F^*)$

2. Both securities are information-sensitive when the relative difference in investors' information is sufficiently small , e.g., $\downarrow |\tau_{i,D} - \tau_{i,E}| \implies \Delta D(F^*), \Delta E(F^*) > 0$

Price uncertainty is driven by (i) information-driven uncertainty (e.g., $\mathbb{V}_0[q_D]$) and (ii) information sensitivity. As Lemma 3 makes clear, the relative precision of the information in each market drives the relative difference in their information-driven uncertainty. To minimize his price risk, the agent issues a more information-sensitive security in the market with less information, that is, the market with less information-driven uncertainty. Notably, however, the agent doesn't exclusively issue in the market with the least information. By selling claims in multiple markets, the agent reduces the risk of low liquidity demand — q_D, q_E are imperfectly correlated. When this diversification benefit is sufficiently large, he issues information-sensitive debt *and* equity.

2.3.2 Endogenous Information, Fixed Market

We consider now the scenario in which, while restricted to trade in a given market, investors can choose the precision of their signal. The conditional expectation of $\Phi(z)$ of an investor with private signal precision τ_i can be written

$$q_i \equiv \mathbb{E}[\Phi(z)|s_i, p_D, p_E] = \frac{\tau_z \mu_z + \tau_i s_i + \tau_D s_D + \tau_E s_E}{\sqrt{\psi^i(1 + \psi^i)}},$$

where $\psi^i \equiv \tau_z + \tau_i + \tau_D + \tau_E$. Recall that investors (i) differ only in their beliefs about z and (ii) purchase the asset only if their beliefs about z exceed those of the marginal investor. Then the investor's expected utility from trading debt (EU_D) can be written

$$EU_D \equiv \frac{\Delta D}{m_D} \mathbb{E}_0 \left[(q_i - q_D)|(q_i > q_D) \right] \tag{7}$$

Using similar reasoning, we can write an equity investor's expected value from trading equity:

$$EU_E \equiv \frac{\Delta E}{m_E} \mathbb{E}_0 \left[(q_i - q_E)|(q_i > q_E) \right] \tag{8}$$

Proposition 4. *If fundamental uncertainty is sufficiently low (τ_z sufficiently high), an investor's expected trading gains (EU) are increasing and concave in the precision of the investor's private information, τ_i.*

Holding fixed the information acquisition of others, as τ_i increases, the investor becomes more (less) likely to purchase the security when it's undervalued (overvalued). Even if an investor were to learn z perfectly ($\tau_i \to \infty$), however, the benefit is limited: any mispricing is limited to the security's information sensitivity and an investor's potential holdings are bounded.

Each investor in market j chooses τ_i to maximize his expected profits, subject to the cost of information:

$$EU_j - C(\tau_i) \quad j \in \{D, E\} \tag{9}$$

Corollary 1. *Given a face value, F, there is a unique symmetric equilibrium in which $\tau_i^* = \tau_{i,D}$ ($\tau_i^* = \tau_{i,E}$) for all debt (equity) investors. As the face value of debt increases, debt (equity) investors acquire more (less) precise signals (i.e., $\frac{d\tau_{i,D}^*}{dF} \geq 0 \geq \frac{d\tau_{i,E}^*}{dF}$). Furthermore, $\frac{\partial \tau_{i,D}^*}{\partial \Delta D}, \frac{\partial \tau_{i,D}^*}{\partial \tau_z}, \frac{\partial \tau_{i,E}^*}{\partial m_D} > 0 > \frac{\partial \tau_{i,D}^*}{\partial m_D}$. Finally, the ability to observe prices in other markets decreases the information acquired.*

The optimal precisions solve equations (20) and (21) found in the Appendix. Investors always choose to acquire private information (i.e., $\tau_i^* \neq 0$) when the claim issued in their market is information-sensitive. Unsurprisingly, given that the expected value is proportional to $\frac{\Delta D}{m_D}$, when investors can purchase a more information-sensitive security, or acquire a larger position, a more precise signal is acquired. As Lemma 2 makes clear, as the face value of debt increases, the information-sensitivity of debt (equity) increases (decreases), which implies that debt (equity) investors acquire more (less) information. If fundamental uncertainty increases, then investors in both markets acquire more information. The ability to condition on prices in the other market "crowds out" investors' private information acquisition. We explore this final effect in more detail in Section 6.1.

Proposition 5. *When investors can choose how much information to acquire, the agent always issues information-sensitive securities in both markets. Suppose (i) the agent chooses F^* to minimize (6) and (ii) investors choose τ_i^* to maximize (9). Then there is a unique, optimal $\Delta D(F^*)$, with $\tau_i^* = \tau_{i,D}$ ($\tau_i^* = \tau_{i,E}$) for all debt (equity) investors:*

1. If $m_D \leq m_E$, (i) $0 < \Delta D(F^*) \leq \Delta E(F^*)$ (ii) $\mathbb{V}_0[q_D] \geq \mathbb{V}_0[q_E]$

2. If $m_D \geq m_E$, (i)$\Delta D(F^*) \geq \Delta E(F^*) > 0$ (ii) $\mathbb{V}_0[q_D] \leq \mathbb{V}_0[q_E]$

In a market with fewer investors, each can purchase a larger share of each security; this incentivizes more information acquisition, increasing the variance of the price. As a result, the agent issues the more information-sensitive claim to the market with the broader investor base.[28] Note, however, that

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[28]Similar results can be generated if the two markets differ in their information technology. Suppose, for instance, it is cheaper for debt investors to acquire information — $\frac{\partial C_D(\tau)}{\partial \tau} < \frac{\partial C_E(\tau)}{\partial \tau} \forall \tau$. Given the opportunity to purchase a security with the same information sensitivity, debt investors would acquire a more precise signal than equity investors. As a result, the agent optimally sets $\Delta D < \Delta E$. Note, however, that an important question remains: why do debt investors have superior information technology if the agent responds in this fashion? If acquiring the superior technology required some costly, upfront investment, an equilibrium in which only one type of investor makes the investment (so that $\frac{\partial C_D(\tau)}{\partial \tau} \neq \frac{\partial C_E(\tau)}{\partial \tau}$) may be difficult to construct.

the agent continues to balance both sources of uncertainty. For example, when there are more debt investors ($m_D > m_E$), the debt claim is more information-sensitive. However, the optimal face value encourages equity investors to learn more, so that the information-driven variance of equity ($\mathbb{V}_0[q_E]$) is higher.

An information-insensitive security plays no role in the optimal capital structure. Suppose instead that the agent issued risk-free debt, i.e., $\Delta D(F) = 0$. While the agent faces no uncertainty about the price at which he can sell debt, the information-sensitivity of the asset is now concentrated in the equity market ($\Delta E(F) = \Delta V$). When investors can choose how much information to acquire, this implies that the agent will face more uncertainty about the marginal equity investor's beliefs ($\mathbb{V}_0[q_E] > \mathbb{V}_0[q_D]$). This is not optimal — it would require the agent to sell the security with more information-sensitivity in the market with more information-driven uncertainty.

Proposition 6. *Aggregate information acquisition ($\tau_{i,D} + \tau_{i,E}$) is minimized when the agent issues an information-insensitive security in the market with fewer (e.g., $m_D < m_E$) investors when $C''' \geq 0$ and $\frac{C''}{C'} > 0$ is sufficiently high.*

Notably, the optimal capital structure does not necessarily minimize aggregate information acquisition. When the cost function is sufficiently convex, information acquisition is minimized when information sensitivity is concentrated in one market. However, the optimal capital structure is driven by the *relative* information acquired by investors in each market rather than by the *aggregate* information generated. We show in the proof of Proposition 6 that if $C(\tau) = \kappa \tau^a$ with $\kappa > 0$, $a \geq 2$, then the sufficient conditions are met.

2.3.3 Endogenous Information, Market Entry

Finally, we consider our benchmark model, in which investors can both choose in which market to invest and choose the precision of their signals. If all investors are able to freely choose in which market to trade, in equilibrium, ex-ante returns in both markets should be equal: $EU_D = EU_E$.

Proposition 7. *Expected returns in each market are only equalized if $\frac{\Delta D(F)}{m_D^*(F)} = \frac{\Delta E(F)}{m_E^*(F)} = \Delta V$.*

The intuition for the choice of market entry is clear from equations (7) and (8). Suppose, instead, that $\frac{\Delta D(F)}{\alpha_D(F)} > \frac{\Delta E(F)}{\alpha_E(F)}$. This would lead to more learning by debt investors ($\tau_{i,D} > \tau_{i,E}$), which in turn implies that $EU_D > EU_E$ (by Proposition 4). When the returns in each market are equalized ($EU_D = EU_E$) the optimal signal in each market is the the same ($\tau_{i,E}^* = \tau_{i,D}^*$) and so, therefore, is the information-driven uncertainty ($\mathbb{V}[q_D] = \mathbb{V}[q_E]$). Effectively, fully flexible market entry exactly undoes the effect of capital structure on information-driven uncertainty.

As we discuss in detail in Section C.2, some capital is unable to move freely between markets. To capture this friction, we assume that there exists some minimum measure of investors which must hold only debt ($m_{D,min}$) or equity ($m_{E,min}$), while the remaining investors ($1 - m_{D,min} - m_{E,min}$) can choose in which market to trade. When $1 - m_{E,min} \geq m_D^*(F) \geq m_{D,min}$, there are sufficient unconstrained investors to equalize the expected returns in each market. As a result, the precision is

fixed for all choices of F, and the agent can only control the information-sensitivity of each security. On the other hand, consider F sufficiently low such that $m_D^*(F) < m_{D,min}$. All unconstrained investors will trade the firm's equity ($m_E = 1 - m_{D,min}$) and equity investors will acquire more precise signals ($\tau_{i,E}^* > \tau_{i,D}^*$).[29] Furthermore, in this region the measure of investors in each market remains fixed, and so the agent regains the ability to affect how much information is acquired.[30]

Theorem 1. *When investors can choose how much information to acquire, and in which market to trade, the agent always issues information-sensitive securities in both markets. Suppose (i) the agent chooses F^* to minimize ([6]) and investors choose (ii) τ_i to maximize ([9]) and (iii) the optimal market in which to trade. Then there is a unique, optimal $\Delta D(F^*)$, with $\tau_i^* = \tau_{i,D}(\tau_{i,E})$ for all debt (equity) investors:*

1. *If $m_{D,min}$, $m_{E,min} \leq \frac{1}{2}$, (i)$\Delta D(F^*) = \Delta E(F^*)$ (ii) $\mathbb{V}_0[q_D] = \mathbb{V}_0[q_E]$*
2. *If $m_{E,min} > \frac{1}{2}$, (i)$\Delta D(F^*) < \Delta E(F^*)$ (ii) $\mathbb{V}_0[q_D] > \mathbb{V}_0[q_E]$*
3. *If $m_{D,min} > \frac{1}{2}$, (i)$\Delta D(F^*) > \Delta E(F^*)$ (ii) $\mathbb{V}_0[q_D] < \mathbb{V}_0[q_E]$*

When investors are sufficiently unconstrained, allowing for market entry can increase the agent's utility. Suppose that the majority of investors are constrained to trade equity, i.e., $m_{E,min} > \frac{1}{2}$. The only way for the agent to smooth across both sources of uncertainty is to choose a level of debt such that (i) all unconstrained investors choose to trade debt, so that $\mathbb{V}_0[q_D] > \mathbb{V}_0[q_E]$, but with (ii) more information-insensitivity in the equity claim, $\Delta E(F) > \Delta D(F)$. This is equivalent to the problem we considered above, in which investors are fixed in each market. On the other hand, when there is sufficient flexibility in investors' choice of markets (i.e., both $m_{D,min}$ and $m_{E,min}$ are less than $\frac{1}{2}$), the agent can perfectly balance his two sources of risk: $\Delta D(F) = \Delta E(F)$ and $\mathbb{V}_0[q_D] = \mathbb{V}_0[q_E]$.

2.4 Discussion

Our results stand in contrast to those of Boot and Thakor (1993), which argues that when selling the firm, information sensitivity should be concentrated in one security, equity. In their model, the agent wants to encourage information acquisition so as to increase the price paid. Surprisingly, we show in the proof of Proposition 6 that aggregate information ($\tau_{i,E} + \tau_{i,D}$) can be *maximized* in the *minimum-variance* capital structure.[31] As a result, when markets are segmented, the solution in our setting may also be the optimal capital structure when the agent wants to encourage information acquisition. We hope to explore this idea further in future work.

The recent literature on security design (e.g., Yang (2015); Dang et al. (2015)) has focused on the optimality of debt, due to its relative information-insensitivity. Such securities generally minimize the frictions generated by information acquisition. In our setting, an agent faces a similar friction but, importantly, wants to sell his entire claim to the asset. As a result, he must consider the risk of the entire capital structure, and so an information-insensitive security plays no role in the optimal capital

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[29]This is because the expected return to trading equity will exceed that of debt ($EU_E > EU_D$).

[30]Applying similar reasoning, for all face values of debt such that $\alpha_D \geq 1 - \alpha_{E,min}$, the measure of debt investors is fixed: $\alpha_D = 1 - \alpha_{E,min}$.

[31]This is true in case (i) of Theorem 1, or when debt and equity investors exist in equal measure.

structure.[32] This is similar to the result of Farhi and Tirole (2015), who show that by selling an information-insensitive tranche, a seller may be left with a too-risky claim which he finds difficult to sell.

As the intuition for the results above make clear, the agent generally benefits from the ability to sell in each segmented market to which he has access. It is important to consider, therefore, situations in which financial market segmentation extends beyond debt and equity markets. For instance, bank loans and publicly-traded debt, by definition, are held by different sets of investors. The latter market, however, is only accessible to firms which have a credit rating. It is straightforward to show that if the agent can costlessly borrow in both the bank loan and public debt markets, the firm will raise more capital via debt. This is consistent with the results of Faulkender and Petersen (2006), which argues that, all else equal, firms with credit ratings typically borrow more.[33] Within a given debt market, many investors face investment restrictions which depend upon a bond's rating.[34] At the same time, issuers of asset-backed securities typically issue bonds with ratings across the spectrum.[35] Our model suggests that one motivation for doing so is that by tapping into distinct markets, issuers lower the uncertainty they face.[36,37]

Many debt investors are exogenously restricted to hold securities with sufficiently high ratings. Insurance companies are restricted to hold debt of sufficient quality and regulation penalizes holding riskier bonds on bank balance sheets.[38] If default risk increases sufficiently, there may only be a relatively small number of distressed debt investors able to lend capital. In practice, it is generally observed that debt is less information-sensitive than equity.[39] Such restrictions provide one reason why this might be the case. Suppose that when debt is risk-free, $m_E = m_D$: equity and debt investors exist in equal measure. As the face value of debt rises, the measure of potential debt investors falls ($m_D < m_E$), and so in equilibrium, the agent will set $\Delta D(F^*) < \Delta E(F^*)$. Notably, this implies that

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[32]If it is feasible to sell multiple claims in each market, risk-free debt could be issued as well: such issuance leaves the agent's utility unaffected. If investors were willing to pay a premium for such securities (for unmodeled reasons), he could accommodate them.

[33]Our results are potentially consistent with the analysis of Rauh and Sufi (2010). One surprising result they highlight is that relative to investment-grade firms, high-yield issuers are more likely to issue to both banks and institutional lenders. If there is a fixed cost to issuance, firms with less information sensitivity (low ΔV) may choose to borrow in fewer markets; a shock to the expected value of the asset in the low state of the world would lead to (i) downgrade and (ii) firms tapping other markets for funding.

[34]Mutual funds are a prominent example — many funds are categorized by the types of bond in which they can invest. Cantor, Gwilym, and Thomas (2007) provides evidence that pension plan sponsors place similar restrictions on investment managers.

[35]For example, Benmelech and Dlugosz (2009) discuss the construction of CLO's, noting that 40% of the deals issued a tranche with each of the following ratings: AAA,AA,A,BBB,BB and NR.

[36]This explanation differs from, but is not inconsistent with, an oft-cited explanation: a clientele effect, in which investors demand (and sometimes pay a premium for) bonds with certain (typically, higher) ratings.

[37]For a given corporation, bond ratings are relatively homogenous; however, it is possible for firms to issue commercial paper with a rating which differs from its longer-maturity debt. As documented in Lemmon, Liu, Mao, and Nini (2014), a large number of firms have securitized their receivables and received an A-1 rating; the firms which did so typically had lower ratings, e.g., A or BBB, consistent with our story when credit risk and information sensitivity (ΔV) are positively correlated.

[38]Ellul, Jotikasthira, and Lundblad (2011) provide evidence that such restrictions can impact bond prices, for instance, when there are a limited number of potential buyers for downgraded bonds.

[39]There are, of course, many exceptions. Information about recovery value or downside risk will generally induce a larger change in debt, not equity, prices.

the agent does not necessarily issue to the *broadest* investor base — he may be willing to trade-off a more concentrated group of capital providers *within* a market against a higher concentration of risk in a given security when comparing *across* markets.

The agent in our model is a stand-in for several possible entities. It could be that, as is commonly assumed, the agent is an entrepreneur, or a large individual stakeholder in a privately-held firm. Private equity firms prioritize identification of the best "exit strategy," i.e., the optimal way to liquidate their position. Moreover, in practice, many private equity firms issue debt prior to sale and use the proceeds to pay equityholders — a leveraged recapitalization — which closely corresponds to the setting studied. Similarly, in structured finance, special purpose vehicles purchase assets (loans) with the explicit goal of structuring and selling claims — both debt and equity — to investors. Finally, it is possible to interpret our model in the context of executive compensation. If a risk-averse CEO is compensated as a function of the enterprise value of the firm, then his optimal capital structure will closely match the structure suggested by our main results.[40]

There is a growing literature (e.g., Axelson (2007); Dang, Gorton, and Holmström (2012); Yang (2015)) exploring the impact of buyers' ability to generate private information about an asset for sale. In our model, this assumption has several interpretations. It could be that, as professional investors with industry expertise, they have special expertise in determining the likelihood of success. Alternatively, after the agent has chosen how to sell his asset, it could be that further information about q becomes available, which investors are able to acquire before submitting their bids.

In Appendix C.1 we (i) consider a setting in which the agent has previously issued debt and/or equity, (ii) examine the implications of dividend recapitalizations when liquidation is costly, and (iii) provide an alternative payoff structure in which an increase in z makes "tail" realizations more likely.

3   Dynamic Issuance

When information about the probability of each state is revealed over time, the price of the asset will vary over time. This provides an incentive for the agent to retain some claim to the asset for sale at a later date. Retention, in combination with price variation, can help maximize his expectation of and reduce the uncertainty surrounding his final proceeds.

3.1   Time-varying asset value

For the remainder of the paper, we analyze the sale of the asset over multiple periods. We consider a setting in which there are $T + 2$ periods, i.e., $t \in \{0, 1, ..., T + 1\}$.

As before, $q \equiv \mathbb{P}[s = H]$, but we now set $q = \Phi[z_T]$. The state variable, z_t, is stochastic:

$$z_{t+1} = (1 - \rho)\mu_z + \rho z_t + e_{t+1}, \quad e_{t+1} \sim \mathcal{N}(0, \tau_z^{-1}),$$

At time-t, z_t is revealed and known by all agents. At $T + 1$, the true state is resolved. At $T + 2$, the

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[40]If the compensation is a linear function of the enterprise value, it will match exactly.

cash flow is realized.[41]

Throughout Section 3, we assume that the agent issues equity, only.[42] There exists a unit-measure continuum of investors; in each period, t, investor i observes a private signal $s_{i,t} = z_{t+1} + \varepsilon_{i,t}$, with $\varepsilon_{i,t} \sim \mathcal{N}(0, \tau_{i,t}^{-1})$. [43] We assume that $\tau_{i,t}$ is always non-zero and finite.

In each period, investors/traders are limited to purchase κ_t units, where κ_t denotes the fraction of the shares outstanding at time-t. Liquidity shocks, u_t, are i.i.d. and normally-distributed, $u_t \sim \mathcal{N}(0, \tau_n^{-1})$. A rational expectations equilibrium exists as established in Section 2; it is straightforward to show that prices reveal a signal $s_{E,t} \sim \mathcal{N}(z_{t+1}, \tau_{E,t}^{-1})$, where $\tau_{E,t} = \tau_n \tau_{i,t}$.

The price at time-t $(p_{E,t})$ is determined via recursion. At time-T, the expected value of the cash flow is $p_{E,T} = V_L + \Phi(z_T)\Delta V$. At $T-1$, the marginal (price-setting) investor with signal $s_{i,T-1} = s_{E,T-1}$ sets the price equal to his expectation of $p_{E,T}$: $p_{E,T-1} = V_L + q_{E,T-1}\Delta V$, where

$$q_{E,T-1} = \Phi\left(\frac{\tau_z[(1-\rho)\mu_z + \rho z_{T-1}] + (\tau_{i,T-1} + \tau_{E,T-1})s_{E,T-1}}{\sqrt{\psi_{T-1}^E(1+\psi_{T-1}^E)}}\right)$$

and $\psi_{T-1}^E = \tau_z + \tau_{i,T-1} + \tau_{E,T-1}$.

At $T-2$, investors have imperfect information about z_{T-1} and no information about z_T. To find $p_{E,T-2}$, we first take the expectation of $q_{E,T-1}$ given z_{T-1} *and* the information (denoted $\mathcal{P}$) that the marginal investor's signal must equal the signal contained in the price. To do so, we will utilize a more general version of Lemma 1.

Lemma 5. *Suppose $y|\mathcal{I} \sim \mathcal{N}(\mu_y, \sigma_y^2)$. Then, if a, c are constants, $\mathbb{E}\left[\Phi\left(\frac{a+y}{\sqrt{c}}\right)|\mathcal{I}\right] = \Phi\left(\frac{a+\mu_y}{\sqrt{c+\sigma_y^2}}\right)$.*

This allows us to write:[44]

$$\mathbb{E}[q_{E,T-1}|z_{T-1}, \mathcal{P}] = \Phi\left(\frac{(1-\rho)\mu_z + \rho z_{T-1}}{\sqrt{1 + \tau_z^{-1} + \psi_{T-1}^P}}\right), \tag{10}$$

where $\psi_{T-1}^P \equiv \frac{\tau_{i,T-1}\left(\frac{1+\tau_n}{\tau_n}\right)}{\left(\psi_{T-1}^E\right)^2}$. Using iterated expectations, it is clear that $p_{E,T-2} = V_L + q_{E,T-2}\Delta V$ and

$$q_{E,T-2} \equiv \mathbb{E}[\mathbb{E}[q_{E,T-1}|z_{T-1}, \mathcal{P}]|s_{i,T-2} = s_{E,T-2}, s_{E,T-2}, z_{T-2}, \mathcal{P}].$$

We define $p_{E,T-k} \equiv V_L + q_{E,T-k}\Delta V$; $q_{E,T-k}$ is fully characterized in equation (25) in the Appendix.

Lemma 6. *If $\mathbb{E}[z_T|z_{T-k}] > 0$ $(\mathbb{E}[z_T|z_{T-k}] < 0)$, $\mathbb{E}[q_{E,t}|z_{T-k}, \mathcal{P}]$ increases (decreases) with t.*

For intuition, we can compare (10), the agent's expectation of the price at $T-1$, to his expectation of $q_{E,T}$ (the price at time-T).

---

[41]When $T=1$ and $z_0 = \mu_z$ we return to the original, static setting.

[42]This is equivalent to assuming that (i) markets are no longer segmented and (ii) liquidity shocks across markets are perfectly correlated.

[43]Restricting the signal to condition on the state variable one period ahead ensures that all information sets are identical at the beginning of each period.

[44]For brevity, the steps required to reach (10) are relegated to the Appendix.

$$\mathbb{E}[q_{E,T}|z_{T-1}] = \mathbb{E}[\Phi(z_T)|z_{T-1}] = \Phi\left(\frac{(1-\rho)\mu_z + \rho z_{T-1}}{\sqrt{1+\tau_z^{-1}}}\right)$$

The additional term, $\psi_{T-1}^P > 0$, in the denominator of equation (10) reflects the additional variance generated in the determination of prices. It is the difference between the variance of the marginal investor's beliefs and the variance of a *given* investor's beliefs.[45] Of course, investors also anticipate this effect, and so the price at each point in time accounts for the excess variance generated by trade in the *future*. Given the non-linearity in Φ, this implies that the wedge between $\Phi\left(\mathbb{E}[z_T|z_{T-k}]\right)$ and the agent's expectation of the price shrinks as $t \to T$. When $\mathbb{E}[z_T|z_{T-k}] \neq 0$, this leads to a predictable component in the expected path of prices.

Lemma 7. *When τ_n is sufficiently large, and $|\mathbb{E}[z_T|z_{T-k}]|$ sufficiently small, $\mathbb{V}[q_{E,t}|z_{T-k}, \mathcal{P}]$ increases with t.*

It is straightforward to read this result as a modification of Lemma 3. First, when τ_n is sufficiently large, the variance of the price is less than the variance of the "fundamental", i.e., the price in the next period. Intuitively, the agent faces less uncertainty about the price one period ahead than he does about the price two periods ahead. On the other hand, the variance of prices falls as $|\mathbb{E}[z_T|z_{T-k}]|$ increases. As was just discussed, the wedge between prices and $\Phi\left(\mathbb{E}[z_T|z_{T-k}]\right)$ shrinks as $t \to T$; when $|\mathbb{E}[z_T|z_{T-k}]|$ is sufficiently high, the latter effect outweighs the former.

3.2 Optimal Retention

We now solve for the agent's optimal dynamic retention policy. We use the following notation: at time-t, the agent chooses to issue $1 - \alpha_t$ of his *remaining* claim. That is, he retains a fraction $\prod_{j=0}^{t} \alpha_j$ of the firm for sale in subsequent periods and sells $(1 - \alpha_t)\prod_{j=0}^{t-1}\alpha_j$ of the firm at time-t. The agent can repurchase or short shares, i.e., $\alpha_t > 1$, $\alpha_t < 0$ are feasible choices.

There are three state variables on which the agent conditions: the number of periods remaining $(T - t)$, the fraction of the firm remaining to be sold $(\prod_{j=0}^{t-1}\alpha_j)$, and the current value of the underlying fundamental process (z_t). The agent has mean-variance preferences — to insure that the solution is dynamically consistent, the objective function at each time-t maximizes his utility over his terminal wealth:

$$\max_{\alpha_t}\mathbb{E}[W_T|z_t] - \frac{\gamma}{2}\mathbb{V}[W_T|z_t] \quad W_T = \sum_{j=0}^{T}\left[\prod_{k=0}^{j-1}\alpha_k\right](1-\alpha_j)p_{E,j}.$$

At time-t, the price received for each prior issuance is known. We can rewrite the objective function to reflect that the agent's choice affects only his future proceeds:

$$\max_{\alpha_t}\mathbb{E}[W_{t,T}|z_t] - \frac{\gamma}{2}\mathbb{V}[W_{t,T}|z_t] \quad W_{t,T} = \sum_{j=t}^{T}\left[\prod_{k=0}^{j-1}\alpha_k\right](1-\alpha_j)p_{E,j}. \tag{11}$$

---

[45]For more detail, see the discussion of Lemma 4 in Appendix B.4.

We will solve for the optimal retention policy recursively, and define $\alpha_t(z_t, \prod_{j=0}^{t-1} \alpha_j)$ as the solution to equation (11).[46]

Time-T: The agent sells his remaining stake in its entirety: $\alpha_T = 0$. Because z_T is known by all, the agent and investors share the same expected value for the asset, i.e., there is no price uncertainty. Furthermore, there remains residual uncertainty regarding the true state of the world ($s \in \{L, H\}$) and the realization of the cash flow. Given his risk-aversion, it is always optimal for the agent to sell any remaining claim.

Time-T-1: We can rewrite the future proceeds from issuance given $\alpha_T = 0$:

$$W_{T-1,T} = \left[\prod_{k=0}^{T-2} \alpha_k\right] [V_L + \Delta V((1 - \alpha_{T-1})q_{E,T-1} + \alpha_{T-1}q_{E,T})]$$

and solve for the optimal retention policy:[47]

$$\alpha_{T-1}\left(z_{T-1}, \prod_{k=0}^{T-2} \alpha_k\right) = \frac{\chi \hat{T}_{-1}}{\gamma \Delta V \prod_{k=0}^{T-2} \alpha_k} + w_{T-1}$$

$$\chi \hat{T}_{-1} \equiv \underbrace{\frac{\mathbb{E}[q_{E,T} - q_{E,T-1}|z_{T-1}]}{V[q_{E,T} - q_{E,T-1}|z_{T-1}]}}_{market-timing} \quad w_{T-1} = \underbrace{\frac{\mathbb{V}[q_{E,T-1}|z_{T-1}] - Cov[q_{E,T-1}, q_{E,T}|z_{T-1}]}{\mathbb{V}[q_{E,T-1} - q_{E,T}|z_{T-1}]}}_{variance-minimizing},$$

Consider the case when $\mathbb{E}[q_{E,T} - q_{E,T-1}|z_{T-1}] = 0$: then, $\alpha_{T-1} = w_{T-1}$ minimizes the variance of future proceeds. When there is more uncertainty about future proceeds ($\mathbb{V}[q_{E,T}|z_{T-1}] > \mathbb{V}[q_{E,T-1}|z_{T-1}]$), the agent issues more than he retains ($\alpha_{T-1} < \frac{1}{2}$). As was the case when analyzing the issuance across debt and equity markets, however, when there is sufficient diversification ($\mathbb{V}[q_{E,T-1}|z_{T-1}] > Cov[q_{E,T-1}, q_{E,T}|z_{T-1}]$), the agent retains some portion of his claim ($\alpha_{T-1} > 0$).

When $z_{T-1} > (\rho - 1)\mu_z$, the agent expects prices to increase over time ($\mathbb{E}[q_{E,T} - q_{E,T-1}|z_{T-1}] > 0$) and so optimally adjusts his retention decision so that he can issue more in the following period ($\uparrow \alpha_{T-1}$); the opposite is true when $z_{T-1} < (\rho - 1)\mu_z$. Such price changes are expected, but not guaranteed: $\chi \hat{T}_{-1}$ captures the risk-return tradeoff. In addition, the magnitude of the adjustment accounts for the agent's risk-aversion (γ) and the remaining information-sensitivity of his holdings ($\Delta V \prod_{k=0}^{T-2} \alpha_k$). If either fall, the agent places less emphasis on his desire to reduce uncertainty.

Time-T-2: Given the optimal policy at $T - 1$, the agent's future proceeds can be written:

$$W_{T-2,T} = \left[\prod_{k=0}^{T-3} \alpha_k\right]\left[V_L + \Delta V\left[\frac{\chi \hat{T}_{-1}}{\prod_{k=0}^{T-3} \alpha_k}(q_{E,T} - q_{E,T-1}) + (1 - \alpha_{T-2})q_{E,T-2} + \alpha_{T-2}q_{E,T-1}^P\right]\right]$$

$$q_{E,T-1}^P = (1 - w_{T-1})q_{E,T-1} + w_{T-1}q_{E,T}$$

---

[46]The solution described below is well-defined when $\prod_{j=0}^{t-1} \alpha_j \neq 0$, i.e., the agent still owns some share of the firm.

[47]In the proof of Theorem 2, we verify that this is the unique, optimal solution.

The fraction retained by the agent (α_{T-2}) trades off (i) the price of equity in the current period ($q_{E,T-2}$) and (ii) the price of a *portfolio* ($q_{E,T-1}^P$) of future equity issuance.[48] Surprisingly, however, $q_{E,T-1}^P$ is not the actual, optimal portfolio chosen by the agent but instead represents the *minimum-variance* portfolio.

This arises because the expected gains from market-timing at $T-1$ ($\frac{\chi\hat{T}_{-1}}{\prod_{k=0}^{T-3}\alpha_k}(q_{E,T} - q_{E,T-1})$) are independent of the retention decision at $T-2$ (α_{T-2}). Retaining more in the current period makes the agent's future proceeds riskier. To compensate, he reduces his risk-taking at $T-1$, i.e., the market-timing component of his retention decision ($\alpha_{T-1} - w_{T-1}$). This is readily apparent by examination of the expression for α_{T-1}: the denominator of the market-timing component is $\prod_{k=0}^{T-2}\alpha_k$, and so any change in α_{T-2} leaves the market-timing component unchanged.

The optimal fraction retained, α_{T-2}, exists and is unique:

$$\alpha_{T-2}\left(z_{T-2}, \prod_{j=0}^{T-3} \alpha_j\right) = \frac{\chi\hat{T}_{-2}}{\gamma\Delta V \prod_{k=0}^{T-2}\alpha_k} + w_{T-2}$$

$$\chi\hat{T}_{-2} \equiv \frac{\mathbb{E}[q_{E,T-1}^P - q_{E,T-2}|z_{T-2}] - cov\left(q_{E,T-1}^P - q_{E,T-2}, \chi\hat{T}_{-1}\left(q_{E,T} - q_{E,T-1}\right)\right)}{\mathbb{V}[q_{E,T-1}^P - q_{E,T-2}|z_{T-2}]}$$

$$w_{T-2} = \frac{\mathbb{V}[q_{E,T-2}|z_{T-2}] - Cov[q_{E,T-1}^P, q_{E,T-2}|z_{T-2}]}{\mathbb{V}[q_{E,T-1}^P - q_{E,T-2}|z_{T-2}]}$$

As above, the fraction retained has two elements: w_{T-2}, which is variance-minimizing, and χ_{T-2}, which adjusts the retention decision for any expected changes in the price. Both terms, however, are modified to reflect the dynamic nature of the problem. The variance-minimizing component smooths across today's price and the variance-minimizing *portfolio* in the future. It also accounts for the uncertainty regarding the construction of that portfolio (w_{T-1} is unknown at $T-2$). Due to the agent's risk-aversion, the market-timing component now accounts for any *future* market-timing proceeds. When market-timing proceeds are positively correlated over time, the agent hedges his risk by reducing how much he retains (issues) to take advantage of expected increases (decreases) in the price.

We can now state the optimal retention decision at any point in time, summarized in the theorem below.

Theorem 2. *At time $T-k$, given z_{T-k}, and the fraction of the firm held by the agent, $\prod_{j=0}^{T-k-1}\alpha_j$, the agent optimally retains:*

$$\alpha_{T-k}\left(z_{T-k}, \prod_{j=0}^{T-k-1}\alpha_j\right) = \frac{\chi\hat{T}_{-k}}{\gamma\Delta V \prod_{k=0}^{T-k-1}\alpha_k} + w_{T-k} \tag{12}$$

and his future proceeds can always be written as a linear function of the retention decision:

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[48]The agent chooses, dynamically, how to allocate the sale of his claim across time, creating a "portfolio" of equity issuance.

$$W_{T-k,T} = \left[\prod_{k=0}^{T-k-1} \alpha_k\right]\left[V_L + \Delta V\left(q_{E,T-k} + \frac{\sum_{j=1}^{k-1}\chi_{T-k+j}}{(\gamma\Delta V)\prod_{k=0}^{T-k-1}\alpha_k} + \alpha_{T-k}\left(q_{E,T-k+1}^P - q_{E,T-k}\right)\right)\right] \quad (13)$$

where we define w_{T-k+1}, χ_{T-k} recursively as follows:

$$w_{T-k} = \frac{\mathbb{V}[q_{E,T-k+1}^P|z_{T-k}] - cov\left(q_{E,T-k+1}^P, q_{E,T-k}|z_{T-k}\right)}{\mathbb{V}[q_{E,T-k+1}^P - q_{E,T-k}|z_{T-k}]}$$

$$q_{E,T-k+1}^P = (1 - w_{T-k+1})q_{E,T-k+1} + w_{T-k+1}q_{E,T-k+2}^P$$

$$\hat{\chi_{T-k}} = \frac{\mathbb{E}[q_{E,T-k+1}^P - q_{E,T-k}|z_{T-k}] - Cov(q_{E,T-k+1}^P - q_{E,T-k}, \sum_{j=1}^{k-1}\chi_{T-k+j}|z_{T-k})}{\mathbb{V}[q_{E,T} - q_{E,T-1}|z_{T-1}]}$$

$$\chi_{T-k} = \hat{\chi_{T-k}}\left(q_{E,T-k+1}^P - q_{E,T-k}\right)$$

The agent's retention decision can always be broken into two components. The first component, w_{T-k}, is the variance-minimizing fraction retained, given the agent's beliefs about the future *variance-minimizing* portfolio. This component does not account for any future market-timing activity; it is the exact fraction the agent would retain if his expectation of $q_{E,t}$ was constant over all horizons. The other component of his retention decision, $\frac{\hat{\chi_{T-k}}}{\gamma\Delta V \prod_{k=0}^{T-k-1}\alpha_k}$, is how much more or less the agent retains, given his beliefs about the predictable path of prices, after accounting for how current market-timing proceeds covary with *future* market-timing retention decisions. The separability in the two components arises because future market-timing proceeds are always *independent* of the agent's retention decision in the given period.[49]

3.3 Discussion

By not realizing his gains from trade immediately, i.e., by retaining a share of the firm, the agent maximizes his utility. Information about the asset value is revealed over time — investors can only condition upon a noisy version of the fundamental value (z_T). As a result, prices in each period are imperfectly correlated: by retaining a claim to the firm and selling fractions of the asset at different points in time, the agent can lower the uncertainty he faces with respect to his entire claim.

As the agent's horizon increases, this portfolio effect becomes more pronounced, i.e., the benefits of diversification increase. In such cases, retention need not imply that the agent faces lower uncertainty about future prices, but rather, that he anticipates benefiting from this portfolio effect. Similarly, the size of the agent's remaining stake determines how willing he is to take advantage of market-timing opportunities. While still holding a large fraction of the firm, his uncertainty about future proceeds lowers how much weight he places on expected price changes. This implies that the agent's trading activity is less likely to predict future returns when the size of his stake is relatively large.

Finally, we note that in our model, there is an asymmetry in the predictability of returns after a repurchase versus an issuance. When $\mathbb{E}[z_T|z_{T-k}] > 0$, the agent expects prices to increase over time. In the absence of risk-aversion, this would lead the agent to repurchase shares. When market-timing

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[49]Interestingly, however, the market-timing component *is* sensitive to *past* retention decisions, i.e., $\frac{1}{\prod_{k=0}^{T-k-1}\alpha_k}$.

gains are positive but small, however, they are generally outweighed by the agent's incentive to lower his uncertainty by issuing in the current period. As a result, while repurchases indicate high expected future returns, issuance can be followed by price *increases* as well, making it a noisier signal of the sign of future returns.

In Appendix C.3, we use the dynamic setting to consider the implications when (i) the agent must retain a share of the firm to maintain control rights and (ii) the agent must issue debt and equity sequentially, instead of simultaneously.

4   Investment

The optimality of debt for raising capital when investors can acquire information has been shown in a variety of settings.[50] In our model, however, the agent must consider how the security he chooses for financing alters the structure of the residual claim he owns and later sells. To capture this tradeoff, we now assume that the agent must make an investment in order to acquire the asset.

4.1   Two-stage Financing

We return to the time-varying asset of Section 3 but consider issuance in two stages.

At $T - k$, the agent must invest I dollars to acquire the asset.[51] The agent's expected utility from holding the asset is such that the agent always wants to proceed with the purchase.[52,53] The agent has sufficient liquid assets (e.g., cash) to make the investment, but can also finance the purchase by issuing F dollars of debt or by selling $(1 - \alpha)$ of his equity.[54] He is constrained to raise no more capital than is necessary to guarantee that the investment is made.[55] His choice set, $\{F(I,c), \alpha(I,c)\}$, given the cash contributed $(c \geq 0)$, can be written:

$$D_L(F(I,c)) \equiv (1 - \alpha(I,c))V_L \equiv I - c$$

At $T - k + j$, $j \geq 1$, the agent sells his remaining stake by issuing publicly-traded, subordinated debt with face value F_J and selling any remaining equity.[56,57] For tractability, we assume that debt

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[50]Recent papers include Dang et al. (2012); Yang (2015).

[51]The analysis which follows holds for any $k \leq T$; we use the notation of Section 3 for consistency.

[52]While an important factor for real decision making (see Bond, Edmans, and Goldstein (2012) for a recent literature survey), we abstract away from any feedback effects in which the agent learns from the price to determine whether or not to make the investment.

[53]Examples include: a novel use for the asset; a private equity firm which positively alters the distribution of cash flows due to industry expertise or improved management; an SPV who can optimally sell structured claims to the underlying cash flow in a way which was infeasible for the original owner.

[54]In Appendix C.3, we allow the agent to raise capital from both markets.

[55]This restriction could be due to unmodeled moral hazard. In its absence, the agent might want to raise more capital at $T - k$, for the reasons analyzed in Section 3. This restriction allows us to focus on how the choice of financing interacts with the later issuance decision.

[56]An increase in the holding period, j, captures the relative uncertainty faced by the agent at each point in time as well as the effect of intervening trade.

[57]The assumption of subordination simplifies the expressions which follow but is not necessary to achieve our main results.

and equity investors exist in equal measure and have access to signals with equal precision, that any equity issued trades in all future periods, that senior debt is privately held and that $\mu_z = z_{T-k} = 0$.

If the agent chooses to finance the investment via debt, it will be placed on the balance sheet of the firm. If he raises $F(I,c)$ today to purchase the asset, and sells F_J upon acquisition, his proceeds can be written

$$
E_L(F(I,c) + F_J) + q_{E,T-k+j}\Delta E(F(I,c) + F_J) + D_J(F(I,c), F_J) + q_{D,T-k+j}\Delta D_J(F(I,c), F_J)
$$
$$
- [I - D_L(F(I,c)) - q_{D,T-k}\Delta D(F(I,c))]
$$

Under the assumption that $\mu_z = z_{T-k} = 0$, $\mathbb{E}[q_{E,T-k+j}] = \mathbb{E}[q_{D,T-k+j}] = \mathbb{E}[q_{D,T-k}]$. As a result, his expected proceeds are simply the expected value of the asset minus the purchase price: $V_L + \mathbb{E}[q]\Delta V - I$. For every dollar of debt raised to purchase the asset, the expected value of the agent's residual claim falls one-for-one: his capital structure decision does not change his expected proceeds from investing in the asset. Using similar steps, we reach the same conclusion when the agent finances his investment via equity. As a result, his objective is to minimize the variance of his proceeds.

As shown in Appendix C.1, the agent chooses F_J such that $\Delta E^*(F + F_J) = \Delta D_J^*(F, F_J)$. If the agent finances his investment using debt, $\Delta E^*(F + F_J) = \frac{\Delta V - \Delta D(F(I,c))}{2}$. As the level of investment increases, the equity and subordinated debt issued become less information-sensitive. We can write the variance of the agent's proceeds if he finances via debt as

$$
\mathbb{V}[P|Debt] = \left[\frac{\Delta V - \Delta D(F(I,c))}{2}\right]^2 \mathbb{V}[q_{E,T-k+j} + q_{D,T-k+j}] + \left[\frac{\Delta D(F(I,c))}{2}\right]^2 \mathbb{V}[2q_{D,T-k}]
$$
$$
+ 2\left[\frac{(\Delta V - \Delta D(F(I,c)))}{2}\right]\left[\frac{\Delta D(F(I,c))}{2}\right] Cov(2q_{D,T-k}, q_{E,T-k+j} + q_{D,T-k+j}).
$$

If he finances his investment through the issuance of equity, his choice of F_J will set $\Delta E^*(F + F_J) = \frac{\Delta V}{2}$. Equity investors at $T-k$ anticipate receiving the proceeds of the debt issued at $T-k+j$ and submit their demand accordingly. As a result, $p_{E,T-k} = V_L + (q_{E,T-k} + \hat{q}_J)\frac{\Delta V}{2}$, where $\hat{q}_J = \mathbb{E}[q_{D,T-k+j}|s_{E,T-k}, s_i = s_{E,T-k}]$. The variance of the agent's proceeds if he finances via equity can be written

$$
\mathbb{V}[P|Equity] = \left[\frac{\Delta V}{2}\right]^2 \left[(1 - \alpha(I,c))^2 \mathbb{V}[q_{E,T-k+j} + q_{D,T-k+j}] + \alpha(I,c)^2 \mathbb{V}[q_{E,T-k} + \hat{q}_J]\right]
$$
$$
+ 2\left[\frac{\Delta V}{2}\right]^2 \alpha(1 - \alpha)Cov(q_{E,T-k} + \hat{q}_J, q_{E,T-k+j} + q_{D,T-k+j})
$$

Proposition 8. *The agent finances his investment via equity if $I \leq \underline{I}$; it is financed via debt if $I \geq \bar{I}$. When financing via debt, the agent supplies no cash ($c = 0$) when $I \leq I_D$; when financing via equity, the agent supplies no cash when $I \leq I_E$.*

The thresholds $\underline{I}$, $\bar{I}$, I_D, I_E are described in the proof.

When the level of investment is sufficiently low, the information-insensitivity of debt makes it suboptimal for raising capital. For intuition, consider a value of investment sufficiently low so that

24

$\Delta D_L(F(I,c)) = 0$. If he finances via debt, the variance of his proceeds reduces to $\left[\frac{\Delta V}{2}\right]^2 \mathbb{V}[q_{E,T-k+j} + q_J]$. On the other hand, any equity sold at time $T - k$ must be information-sensitive — moreover, because $\mathbb{V}[q_{E,T-k} + \hat{q_J}] < \mathbb{V}[q_{E,T-k+j} + q_J]$, it is clear that equity is the preferred form of issuance. Using equity allows the agent to sell some of his information-sensitive claims at an earlier date when the information-driven uncertainty is lower. Even though the debt is information-insensitive, which in other settings makes it optimal for raising capital, it leaves a maximally-sensitive residual for sale at a later date, rendering it suboptimal.

As the level of investment increases, the agent must also consider the relative information-driven uncertainty across markets. As we show in Appendix C.3, an investor's expectation of future prices is more responsive to his information than his expectation of the current price.[58] As a result, $\mathbb{V}[q_{E,T-k} + \hat{q_J}] > \mathbb{V}[2q_{D,T-k}]$, and $Cov(q_{E,T-k} + \hat{q_J}, q_{E,T-k+j} + q_{D,T-k+j}) > Cov(2q_{D,T-k}, q_{E,T-k+j} + q_{D,T-k+j})$. For low levels of investment, when issuance at $T-k$ has very little information-sensitivity, this difference is irrelevant. As both I and the information-sensitivity of financing increases, however, this difference in information-driven uncertainty eventually makes equity issuance suboptimal.

If cash is used to offset the cost of the investment, it reduces the information-sensitivity of any securities used for financing. As argued above, when the level of investment is low, the securities used for financing are *insufficiently* information-sensitive, given that information-driven uncertainty at $T-k$ is lower than that of issuance j periods later. As a result, when I is low, the investment is financed entirely via capital markets. Increasing the investment level allows the agent to better balance his sources of risk. When $I = I_D$, the agent can perfectly smooth his uncertainty over time. As a result, for all $I > I_D$, the agent uses cash to reduce investment at a one-for-one rate so that his optimal level of financing is preserved.

4.2 Discussion

In our setting, the pecking order of Myers and Majluf (1984) is effectively flipped. For low levels of investment, the agent uses equity. He switches to using debt only when the level of investment becomes so large that issuing equity is too costly. Finally, his liquid assets are only used for very high levels of investment, and interestingly, only used in combination with either debt or equity.

Note that the mechanism which generates this reversal is *not* that investors, instead of the agent, possess private information. In fact, if after raising capital for investment, the agent could risklessly sell his remaining claims to the firm, the pecking order would be restored.[59] In that case, the agent's objective would be to minimize the variance of his financing proceedings — which, as argued above, could be done via internal capital, and if necessary, debt. Our results are driven by a novel channel: the agent's desire to smooth his sources of uncertainty across markets and over time. Interestingly, and in contrast to many other papers in the literature, the agent is not financially constrained; he has sufficient liquid assets to make the investment but optimally chooses to offload some of his risk through

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[58]When investors take expectations of future prices, they no longer need to account for the increased variance generated by trade in each period. As a result, their private information is more valuable in predicting the future price.

[59]For instance, if investors were unable to obtain any further information about z_T, the agent and investors would share a common valuation, allowing him to sell his remaining claim risklessly.

financing.

The empirical literature contains a number of purported violations of the pecking order. For example, Frank and Goyal (2003) and Fama and French (2005) find that equity issuance, especially among small, young, and high-growth firms, which are more likely to be subject to information asymmetry, is common. Such issuance, however, is potentially consistent with the predictions of our model. First, small and young firms are more likely to still be owned by a small group of individuals for whom such issuance is of first-order importance, given its relative weight in their overall asset portfolio.[60] Second, when a small, young firm is raising capital from professional investors, it is likely that the investors possess superior information about the firm's prospects — an oft-cited advantage of venture capital firms. As a result, when the agent retains a residual claim in the firm, he may find it optimal to sell more, rather than less, risky securities today.

In Appendix C.3, we allow the agent to issue both equity and debt to finance his investment. In this case, we show that when the level of investment is sufficiently *high*, the agent never chooses to finance his investment by accessing a single market (i.e., he issues debt and equity) This is due to his incentive to smooth his uncertainty *across markets* at a given point in time. Taken in combination with the result above implies that, in the setting considered, we should not expect the agent to rely solely on debt to finance his investments.

5   Adverse Selection

The literature on adverse selection and issuance (e.g., Leland and Pyle (1977)) has generally focused on settings in which the owner of an asset with private information can send a positive signal about the asset's value through retention. Such models have generally assumed that the owner knows, with certainty, the price he will receive for each claim he sells. In our model, however, because investors also have private information, the price received by the agent is uncertain. As a result, he generally chooses to retain a share of his claim *absent* the effects of adverse selection. To understand the impact of such retention on the possible signaling equilibria, we now relax the assumption that the agent and investors begin with the same information set and endow the agent with private information about the underlying cash flows.

5.1   Setup

We adopt the time-varying asset value framework from Section 3, with one modification: in the low state, the asset pays $x + y$, where it is known by all agents that $y \in [y_L, y_H] \equiv \mathcal{Y}$.[61] Moreover, we restrict the distribution of cash flows so that (i) $G_H(0) = G_L(0) = 0$, (ii) $G_L(x - y_H) \geq G_H(x)$, (iii) if $G_L(x' - y_H) > G_H(x')$ then $\forall x$ s.t. $x' \leq x < inf\{x \ : \ G_H(x) = 1\}$, $G_L(x' - y_H) > G_H(x')$, (iv)

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[60]If the firm is owned by diversified investors, the uncertainty generated by issuance would be a second-order concern.
[61]One way to interpret this assumption is that while investors might be able to acquire information about the likelihood of each state of the world (perhaps due to their valuation expertise), the agent better understands the cash flow generation of the asset, *conditional* upon the state.

$\exists \bar{x}$ s.t. $\forall x \geq \bar{x}$, and $\forall y \in \mathcal{Y}$, $g_L(x-y) < g_H(x)$.[62]

The expected value of debt and equity in the low state, conditional upon both y and F, are now $D_L(y,F) \equiv \int_0^\infty min(F, x+y)\, dG_L(x)$ and $E_L(y,F) \equiv \int_{F-y}^\infty (x+y-F)\, dG_L(x)$. While the information sensitivity of debt is still increasing in the face value of debt issued (and the sensitivity of equity is still falling in F), because the expected value of debt and equity are both *increasing* in y:

$$\frac{\partial D_L(y,F)}{\partial y} = G_L(F-y) \quad \frac{\partial E_L(y,F)}{\partial y} = 1 - G_L(F-y),$$

the information sensitivity of both debt and equity fall with y (i.e., $\frac{\partial \Delta D(y,F)}{\partial y}$, $\frac{\partial \Delta E(y,F)}{\partial y} < 0$). An increase in y is unambiguously valuable to the agent: the expected value of the asset increases, whereas the asset's information sensitivity falls ($\frac{\partial \Delta V(y)}{\partial y} < 0$).

At $T - k$, the agent knows y perfectly but cannot credibly convey this information to investors. Investors can observe y at T. The agent can signal his type (y) through his choice of capital structure. To facilitate comparison to the existing literature, we adopt the framework of DeMarzo (2005): the agent sells debt at $T - k$, and retains the equity for sale at T. If investors believe the agent is of type $\hat{y}$ but is really of type y, his proceeds can be written

$$P(y, \hat{y}, F) = D_L(\hat{y}, F) + q_{D,T-k}\Delta D(\hat{y}, F) + E_L(y, F) + q_{E,T}\Delta E(y, F),$$

allowing us to write his expected utility as $U(y, \hat{y}, F) = \mathbb{E}[P(y, \hat{y}, F)] - \frac{\gamma}{2}\mathbb{V}[P(y, \hat{y}, F)]$. We will show the existence of and solve for incentive-compatible separating equilibria in which there is a one-to-one mapping from y to F. As in Section 3, we assume that the agent takes as given the precision of the signals received by investors at $T - k$.[63]

5.2 Known Debt Price $(q_{D,T-k})$

We begin by assuming that q_D is known *before* F is chosen. The agent forms expectations about the price of equity using q_D: in general, $q_D \neq \mathbb{E}[q_{E.T}|q_{D,T-k}]$, as in Proposition 6. This closely corresponds to the setting studied by Leland and Pyle (1977) and DeMarzo (2005) — the price at which the agent can issue today is known and retention of any residual claim generates disutility for the agent.

Suppose $q_{D,T-k} \geq \mathbb{E}[q_{E.T}|q_{D,T-k}]$. In the absence of adverse selection, i.e., if the agent could credibly convey his type to investors, he would sell his entire claim as a passthrough $(F^*(y) = \bar{F})$: he expects to get a higher price issuing at $T - k$, and retention only serves to increase the uncertainty of his proceeds.

On the other hand, suppose $q_{D,T-k} < \mathbb{E}[q_{E,T}|q_{D,T-k}]$. Retaining some portion of his claim allows the agent to reap the a higher price, in expectation, for the information-sensitive portion of his cash flows. Though the agent is risk-averse, there exists some level of risk he is willing to take in order to engage in an activity — retention — with a positive expected return. Moreover, the agent directly

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[62]Assumption (i) simplifies the expressions below; (ii) is equivalent to FOSD, after accounting for y; (iii) and (iv) insure that there exists a unique solution to the agent's problem.

[63]This ensures the existence of a differentiable, incentive-compatible solution to the agent's problem.

controls how much risk he takes through his choice of F. As a result, in the absence of adverse selection, there always exists a finite $F^*(y)$ which maximizes the agent's utility and satisfies the following equation:

$$\frac{\mathbb{E}[q_{E,T}|q_{D,T-k}] - q_{D,T-k}}{\gamma \mathbb{V}[q_{E,T}|q_{D,T-k}]} = \Delta E(y, F^*(y)). \tag{14}$$

The left-hand side of (14) is the risk-return tradeoff to retaining any equity for sale at time-T; it determines the optimal level of information-sensitivity retained by agents. Notably, $\Delta E(y, F^*(y))$ is constant for all investors, and so given $F^*(y_L)$, it must be that[64]:

$$\frac{\partial F^*(y)}{\partial y} = \frac{1 - G_L(F^*(y) - y)}{G_H(F^*(y)) - G_L(F^*(y) - y)}$$

It is clear that $\frac{\partial F^*(y)}{\partial y} < 0$ for all values of y: in the absence of adverse selection, higher types retain a larger share of their claim for sale at a later date. Increasing y reduces the information-sensitivity of equity — thus, the higher the type, the more the agent must *decrease* how much debt he issues to preserve the desired variance of his residual (equity) cash flows.

When y is private information, the agent can signal his type through his choice of debt issuance. Let $f(y)$ be the proposed issuance schedule. We will solve for separating equilibria in which the agent's information is fully revealed, i.e., $f(y)$ must be one-to-one, so that $\forall y' \neq y, f(y') \neq f(y)$. Furthermore, the issuance schedule must be incentive-compatible:

$$f(y) \in \arg \max_{F \in f(\mathcal{Y})} U(y, f^{-1}(F), F) \tag{15}$$

If $f(y)$ is differentiable, then incentive-compatibility implies that

$$\frac{\partial f(y)}{\partial y} = \frac{-\frac{\partial U(y, \hat{y}=y, F=f(y))}{\partial \hat{y}}}{\frac{\partial U(y, \hat{y}=y, F=f(y))}{\partial F}}$$

Proposition 9. *There always exists a separating equilibrium described by the differential equation, $\frac{\partial f(y)}{\partial y}$, defined in (27) in the proof.*

1. If $q_{D,T-k} \geq \mathbb{E}[q_{E.T-k+j}|q_{D,T-k}]$, $f(y_L) = \bar{F}$, and $\frac{\partial f(y)}{\partial y} < 0$ (i.e., higher types signal through more retention).

2. If $q_{D,T-k} < \mathbb{E}[q_{E.T-k+j}|q_{D,T-k}]$, $f(y_L) = F^*(y_L)$ and there always exists an equilibrium in which $\frac{\partial f(y)}{\partial y} > 0$ (i.e., higher types signal through more issuance).

It is always the case that $\frac{\partial U(y, \hat{y}=y, F=f(y))}{\partial \hat{y}} > 0$ — the agent's utility is increasing in investors' beliefs about his type. As a result, whether the agent signals via retention ($\frac{\partial f(y)}{\partial y} < 0$) or issuance ($\frac{\partial f(y)}{\partial y} > 0$) is determined by the sign of $\frac{\partial U(y, \hat{y}=y, F=f(y))}{\partial F}$, i.e., whether or not issuing more debt would be beneficial to the agent along the equilibrium path.

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[64]This is shown formally in the proof of Proposition 9.

When $q_{D,T-k} \geq \mathbb{E}[q_{E.T-k+j}|q_{D,T-k}]$, raising the face value of debt *always* increases the agent's utility. As a result, $\frac{\partial f(y)}{\partial y}$ must be negative: increased retention signals to investors that the agent is a higher type. The signal is credible because the agent is (i) passing up the opportunity to sell for a higher price and (ii) holding onto a risky asset.

On the other hand, when $q_{D,T-k} < \mathbb{E}[q_{E.T-k+j}|q_{D,T-k}]$ the effect of increased debt issuance on the agent's utility is non-monotonic. If $f(y)$ exceeds $F^*(y)$, then the agent's marginal utility from issuing debt is negative ($\frac{\partial U(y,\hat{y}=y,F=f(y))}{\partial F} < 0$). The marginal reduction in uncertainty (by issuing at a known price) is exceeded by the lost opportunity to issue in the future at what is expected to be a higher price. If we conjecture an equilibrium in which higher types issue more debt ($\frac{\partial f(y)}{\partial y} > 0$), then because $f(y_L) = F^*(y_L)$ and $\frac{\partial F^*(y)}{\partial y} < 0$, it is clear that $f(y)$ exceeds $F^*(y)$. Along the equilibrium path, the agent issues more debt than is desired, verifying our conjecture: $\frac{\partial U(y,\hat{y}=y,F=f(y))}{\partial F} < 0 \implies \frac{\partial f(y)}{\partial y} > 0$. Higher types issue *more* debt in equilibrium, taking on lower risk but receiving a lower price for doing so.

5.3   Unknown Debt Price ($q_{D,T-k}$)

We return now to our benchmark setting in which $q_{D,T-k}$ is unknown before F is chosen. We assume that $\mu_z = z_{T-k} = 0$, so that $\mathbb{E}[q_{D,T-k}|z_{T-k}] = \mathbb{E}[q_{E,T}|z_{T-k}]$, i.e., there is no market-timing incentive and the agent's objective is to minimize the uncertainty of his proceeds.

Proposition 7 tell us that $\mathbb{V}[q_{D,T-k}|z_{T-k}] < \mathbb{V}[q_{E,T}|z_{T-k}]$: there is more information-driven uncertainty surrounding equity issuance, as it occurs further into the future. To smooth across his sources of uncertainty, the agent optimally issues debt which is more information-sensitive. As in Proposition 3, the agent only issues information-sensitive securities in both markets when the diversification benefit is sufficiently large.

If $\mathbb{V}[q_{D,T-k}|z_{T-k}] \leq Cov(q_{D,T-k}, q_{E,T}|z_{T-k})$, the lower variance of issuing debt outweighs the diversification benefit of issuing equity.[65] If y were publicly known, the agent would choose to sell his entire claim as a passthrough ($F^* = \bar{F}$). On the other hand, when $\mathbb{V}[q_{D,T-k}|z_{T-k}] > Cov(q_{D,T-k}, q_{E,T}|z_{T-k})$, the agent optimally smooths across both sources of uncertainty by issuing both debt and equity. In this case, in the absence of adverse selection, there exists a finite face value of debt, $F^*(y)$, which maximizes the agent's utility and satisfies the following equation:

$$\frac{\Delta E(y, F^*(y))}{\Delta D(y, F^*(y))} = \frac{\mathbb{V}[q_{D,T-k}|z_{T-k}] - Cov(q_{D,T-k}, q_{E,T}|z_{T-k})}{\mathbb{V}[q_{E,T}|z_{T-k}] - Cov(q_{D,T-k}, q_{E,T}|z_{T-k})} \tag{16}$$

$\frac{\Delta E(y,F^*(y))}{\Delta D(y,F^*(y))}$ is constant for all investors. Given $F^*(y_L)$, this implies that[66]:

$$\frac{\partial F^*(y)}{\partial y} = \frac{\frac{\Delta D(y,F^*(y))}{\Delta V(y)} - G_L(F^*(y) - y)}{G_H(F^*(y)) - G_L(F^*(y) - y)}.$$

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[65]Note that this is more likely to be true the longer the agent must wait to sell his equity (i.e., as $k \uparrow$).

[66]This is shown in the proof of Proposition 10.

Unlike the setting above, $\frac{\partial F^*(y)}{\partial y}$ may be non-monotonic. As y increases, the information-sensitivity of both debt and equity fall, though not necessarily proportionally. Preserving the optimal ratio requires the agent to account for the relative sensitivity of each security and adjust for any over-/under-response by altering his debt issuance. For low levels of debt, such that $\frac{\Delta D(y,F^*(y))}{\Delta V(y)} < G_H(F^*(y))$, the agent optimally increases his debt issuance at a more than one-to-one ratio $(\frac{\partial F^*(y)}{\partial y} > 1)$. On the other hand, if the debt is sufficiently risky, such that $\frac{\Delta D(y,F^*(y))}{\Delta V(y)} > G_L(F^*(y)-y)$, then the agent optimally issues less debt $(\frac{\partial F^*(y)}{\partial y} < 0)$.

As above, we now derive the separating equilibria, $f(y)$, when the agent's type is private information.

Proposition 10. *There always exists a separating equilibrium described by the differential equation, $\frac{\partial f(y)}{\partial y}$, found in (28) in the proof.*

1. If $\mathbb{V}[q_{D,T-k}|z_{T-k}] \leq Cov(q_{D,T-k}, q_{E,T}|z_{T-k})$, $f(y_L) = \bar{F}$, and $\frac{\partial f(y)}{\partial y} < 0$ (i.e., higher types signal through more retention).

2. If $\mathbb{V}[q_{D,T-k}|z_{T-k}] > Cov(q_{D,T-k}, q_{E,T}|z_{T-k})$, $f(y_L) = F^*(y_L)$ and if $\frac{\partial f(y)}{\partial y} > \frac{\partial F^*}{\partial y}$, there exists an equilibrium in which $\frac{\partial f(y)}{\partial y} > 0$ (i.e., higher types signal through more issuance).

Again, it is always the case that $\frac{\partial U(y, \hat{y}=y, F=f(y))}{\partial \hat{y}} > 0$ and so it is the sign of $\frac{\partial U(y, \hat{y}=y, F=f(y))}{\partial F}$ which determines whether the agent signals via retention $(\frac{\partial f(y)}{\partial y} < 0)$ or issuance $(\frac{\partial f(y)}{\partial y} > 0)$.

When $\mathbb{V}[q_{D,T-k}|z_{T-k}] \leq Cov(q_{D,T-k}, q_{E,T}|z_{T-k})$, raising the face value of debt *always* increases the agent's utility. As when $q_{D,T-k} \geq \mathbb{E}[q_{E.T-k+j}|q_{D,T-k}]$, this implies that $\frac{\partial f(y)}{\partial y}$ is *always* negative. Increased retention is an effective, costly signal because any retention increases the uncertainty of the agent's proceeds.

When some diversification benefit exists, that is, $\mathbb{V}[q_{D,T-k}|z_{T-k}] > Cov(q_{D,T-k}, q_{E,T}|z_{T-k})$, the effect of increased debt issuance can be negative. When $f(y)$ exceeds the solution to (16), then the agent is issuing more debt than he would have otherwise, leading to more variance in his debt proceeds. As before, we can conjecture an equilibrium in which good types issue more debt $(\frac{\partial f(y)}{\partial y} > 0)$. If $\frac{\partial F^*(y)}{\partial y}$ is negative, or if $\frac{\partial f(y)}{\partial y} > \frac{\partial F^*(y)}{\partial y} > 0$, then, following the same reasoning as above, it must be that $\frac{\partial f(y)}{\partial y} > 0$, that is, good types issue *more* debt in equilibrium.[67] By issuing more today, high types take on excess risk, credibly signaling their type.

5.4 Discussion

We find that agent retention, as in Leland and Pyle (1977), can serve as a signal of higher-quality *or lower-quality* firms, depending upon the conditions. Specifically, the latter scenario can arise when issuance is costly. For example, we show that when the price of equity (sold at T) is expected to exceed that of debt (sold at $T - k$), issuance is a credible signal because the agent is forgoing higher expected proceeds. Alternatively, when equity retention allows the agent to smooth across his sources

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[67] $\frac{\partial F^*(y)}{\partial y} < 0$ when the expectation of the cash flows in both the low and high state becomes sufficiently similar. By the same reasoning, it is sufficiently "flat" under similar conditions.

of issuance uncertainty, as in Section 3, excess debt issuance can be costly because the agent takes on excess risk.[68]

Note that, in the *absence* of adverse selection, we find equilibria in which higher types choose to retain larger shares of the asset. Thus, adverse selection need not imply retention by higher types, and moreover, retention by higher types need not imply the presence of adverse selection.

The standard result, that higher types retain larger claims, can be recovered in our model. For instance, if agents of every type would choose to sell their entire claim today in the absence of adverse selection, this is the *only possible* separating equilibrium. This setting closely matches that analyzed by much of the the adverse selection and capital structure literature (e.g., Leland and Pyle (1977); DeMarzo (2005)). Under these conditions, the only direction in which agents can signal their type is through retention. We show that when immediate issuance is no longer the dominant action for all types, we can reverse the direction of the possible signaling equilibria.

6   Cross-Market Learning

We have assumed throughout the paper that investors in one market are able to condition on the realized price of the security issued in the other market (i.e., cross-market learning). While cross-market learning increases uncertainty (investors can observe an additional public signal), it also reduces the information acquired by investors. In what follows, we consider under what conditions the latter dominates the former, and the relationship between liquidity and uncertainty.

6.1   Issuance Transparency and Cross-market Learning

We return to the static issuance setting found in Section 2. For simplicity, suppose investors exist in equal measure in both markets, and let $\tau_{i,CM}$ represent the optimal precision chosen with cross-market learning.[69] Then we can write the optimal precision chosen *without* cross-market-learning as $\tau_{i,CM} + \tau^+$, where τ^+ represents the additional precision chosen by investors, given their inability to observe the price in the other market.[70] With or without cross-market learning, the optimal capital structure in both scenarios sets $\Delta D = \Delta E$. As a result, the difference in the agent's utility is driven by the variance of $q_E + q_D$.

Proposition 11. *If $\tau^+ > \frac{\tau_n}{(1+\tau_n)}\tau_{i,CM}$ and τ_z is sufficiently low, then the agent's utility is higher (i.e., the variance of his proceeds is lower) with cross-market learning.*

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[68]This result bears similarities to Williams (2015), in which higher types can signal through issuance in more, not less liquid markets. In his model, however, this result arises only when liquidity is paired with retention in a multidimensional signaling framework.

[69]While investors are often able to express their demand conditional upon the price of the security they purchase, a limit order which conditions upon another security's price is not commonly observed. One interpretation of such observation is that, during the book-building process, the agent (or investment bank organizing the issuance) is transparent about the interest in the other market.

[70]As noted in Corollary 1, the ability to condition on prices in other markets decreases information acquisition due to substitutability in information.

Cross-market learning generates two countervailing effects. Holding precisions fixed, Lemma 3 states that the price volatility rises with cross-market learning. By increasing the information available to investors, cross-market learning increases the uncertainty faced by the agent at issuance. On the other hand, as Proposition 4 makes clear, the availability of a free, public signal induces investors to endogenously acquire less information, lowering the volatility. When the latter effect is sufficiently large ($\tau^+ > \frac{\tau_n}{(1+\tau_n)}\tau_{i,CM}$), the variance of each security ($\mathbb{V}[q_D] = \mathbb{V}[q_E]$) falls with cross-market learning.

Whether cross-market learning is optimal, however, also depends upon $Cov(q_D, q_E)$, and therefore the correlation between q_E, q_D. Consider the situation when one market is issued an information-insensitive security (e.g., $\Delta D = 0$). With cross-market learning, the marginal investors in each market are conditioning on a single signal. In the absence of cross-market learning, the beliefs in each market would be uncorrelated — in the information-insensitive market, the marginal investor acquires no information, and therefore his beliefs are constant.

As the information-sensitivity in each market equalizes (e.g., as $\Delta D \to \Delta E$), the correlations with and without cross-market learning move in opposite directions. With cross-market learning, the marginal investor in each market conditions on the price in the other market, but the relative weight each investor places on each signal differs.[71] As a result, the correlation in their beliefs falls. In the absence of cross-market learning, investor beliefs become increasingly correlated as $\Delta D \to \Delta E$. The signals in each market condition on the same random variable, z; as information acquisition in each market becomes more similar, the correlation of beliefs increases through this signal. For this effect to be relevant, however, the prior beliefs must be sufficiently uncertain, i.e., τ_z must be sufficiently low, so that the correlation is sufficiently sensitive to the signals acquired.[72]

6.2 Bank Loans and Private Equity: Effects of Illiquidity

Bank loans and private equity differ in many ways from their public, traded counterparts. For one, trade in these securities is typically less frequent, if possible at all. Further, as holders of private securities, banks and private equity investors may have access to information which would be unavailable to holders of public securities.

To understand the impact of these features, we return to the setting of Section 3. As in Section 3, we assume the investor can only issue equity, but now assume that it is privately held. For simplicity, we consider the implications of "perfect" illiquidity, i.e., investors are unable to trade their equity until time-T. At $T - k$, rather than taking an expectation of the price at $T - k + 1$, investors take an expectation of the price at T. We can write the marginal investor's expectation of q :

$$q_{PE,T-k} = \Phi\left(\frac{\mathbb{E}[z_T|z_{T-k}, s_i = s_p, s_p]}{\sqrt{1 + \mathbb{V}[z_T|z_{T-k}, s_i = s_p, s_p]}}\right).$$

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[71]When $\Delta D = 0$, investors in both markets update using only the price of equity. As ΔD grows, the marginal debt investor places more (relative) weight on s_D whereas the equity investor places more weight on s_E. This gap is maximized when $\Delta D = \Delta E$.

[72]When τ_z is high, even as the correlation falls, the variation in the beliefs in the two markets remains low.

Holders of private securities also often have access to information public investors do not: managerial forecasts, forthcoming product developments, etc. We capture this benefit by assuming that the information they receive allows them to be more forward-looking, i.e., able to obtain signals about the realization of z multiple ($l \geq 1$) periods in the future. Let each investor's private signal be written $s_{i,T-k} = z_{T-k+l} + \varepsilon_i$, where $l \geq 1$ and $\varepsilon_i \sim \mathcal{N}\left(0, \left(\tau^I_{i,T-k}\right)^{-1}\right)$. For tractability, we assume that the precision of investors' private signals is the same in both the public and private market, and constant over time.

Lemma 8. *Illiquidity generates increased uncertainty: (1)$\mathbb{V}[q_{PE,T-k}|z_{T-k}] > \mathbb{V}[q_{E,T-k}|z_{T-k}] \; \forall l \geq 1$, $k \geq 2$ and (2) $\mathbb{V}[q_{PE,T-k}|z_{T-k}]$ is increasing when investors obtain more forward-looking information ($\uparrow l$), when $|\mathbb{E}[z_T|z_{T-k}]|$ is sufficiently low.*

Because the security does not trade in the intervening periods, private equity investors are better able to forecast the price at which they can liquidate their position. This increases the agent's uncertainty about the price he will receive, given sufficient information acquisition.[73] This effect is amplified by investors' ability to acquire more forward-looking information. In a static setting, the use of illiquid securities such as bank debt and private equity may be limited — the agent would prefer to issue their publicly-traded counterparts, as they generally lower the uncertainty of his proceeds.[74]

If the agent could issue over multiple periods, however, this illiquidity could be valuable.[75] Within a given market, if previously issued securities aren't traded, the information investors acquire today is only valuable when considering new holdings, which lowers their incentive to acquire information. As a result, it may be valuable to issue private equity or a bank loan earlier in a firm's life-cycle. We explore this setting in Appendix C.3.

We consider, now, an example which highlights the cross-market effects of illiquidity. Consider the case when the agent is restricted to issuing no more than F_{max} in debt markets — due, perhaps, to unmodeled costs of financial distress or leverage restrictions required by potential lenders. To highlight the mechanism of interest, we assume that he borrows the entire amount at time-0 — this is optimal when F_{max} is sufficiently low, as discussed in Section 4. For simplicity, we assume that $T = 2$. The agent can choose what fraction of his equity to issue in the two subsequent periods and at time-0, he can choose to issue either a bank loan or borrow from the public debt market.

As in our dynamic model, the agent sells his remaining equity at time-2 ($\alpha_2 = 0$). At time-1, he optimally retains the fraction $\alpha_1(z_1, \alpha_0)$ specified in equation (33).[76] If the agent issued a bank loan at time-0, and the bank loan is illiquid, then investors at time-1 can only condition upon their own signal and the price of equity. This affects $\mathbb{V}[q_{E,1}|z_1]$ in two ways: it induces equity investors to learn

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[73]It is a sufficient, not necessary, condition that $\tau^I_{i,T-k} = \tau_i$, for $\mathbb{V}[q_{PE,T-k}|z_{T-k}] > \mathbb{V}[q_{E,T-k}|z_{T-k}]$.

[74]If $(1-\rho)\mu_z \sum_{j=1}^{k} \rho^{(j-1)} + \rho^{k-1} z_{T-k} \neq 0$, the non-linearity in Φ alters the agent's expected proceeds, as in Lemma 6. The illiquidity of private equity reduces the variance of the price at which investors expect to liquidate their positions, relative to publicly-traded equity, which reduces both the premium (discount) when $(1-\rho)\mu_z \sum_{j=1}^{k} \rho^{(j-1)} + \rho^{k-1} z_{T-k}$ is negative (positive). As in Appendix B.4, if the reduction in the discount is sufficiently large, it could compensate for the additional uncertainty, making private equity desirable in a static setting.

[75]This discussion ignores the potential costs of illiquidity to the investor. In a richer model, such considerations would be an important input in determining the optimal capital structure.

[76]Because we assume the agent can't issue equity at time-2, $\alpha_0 = 1$.

more, but it eliminates a source of uncertainty, the price of debt at time-1. As in Section 6.1, let τ^+ represent the additional precision chosen by equity investors when they're unable to condition upon the price of debt. If τ^+ is sufficiently small, both $\mathbb{V}[q_{E,1}|z_1]$ and $cov(q_{E,1}, q_{E,2}|z_1)$ fall. When this is the case, issuing a bank loan at time-0 instead of a traded bond lowers the variance of the agent's equity proceeds at time-1. As a result, even when the proceeds from issuing a bank loan at time-0 are more uncertain, the loan may still be preferable.

More generally, when the investor anticipates significant future issuance, he may find it optimal to issue a less-liquid, more-variable security today. While we have emphasized the difference between public and private issuance, it is worth noting that equity market liquidity greatly exceeds that found in bond markets. This variation in liquidity, in combination with cross-market learning, may generate a preference for equity or debt issuance which depends upon the expectation of future issuance and investors' ability to acquire information.

7   Conclusions

When deciding how to sell his claim to a risky asset, a risk-averse agent must account for the private information held by potential investors. We show how the precision of this information can vary (i) endogenously in response to the securities the agent chooses to sell (ii) and as information about the fundamental value of the asset is revealed. As a result, the agent optimally liquidates his position by selling information-sensitive claims across markets and over time. Issuing information-insensitive securities leaves the agent with excessively risky claims to sell in other markets and at later dates, and so play no role in the optimal capital structure. Surprisingly, we show that the agent can use increased issuance as a signal of better private information, and demonstrate that selling illiquid claims can make future issuance less risky.

We conclude by discussing several potentially fruitful directions for future research:

- **Learning along multiple dimensions**: In our model, debt and equity investors generally differ in how much information they choose to acquire. However, in practice, debt and equity investors may also differ in what type of information they choose to acquire. For example, in a setting with multiple shocks, debt investors may choose to allocate more resources to learning about shocks which affect those states of the world in which the firm is more likely to default.

- **Asset pricing**: As the analysis makes clear, security prices can be driven by information acquired by investors across markets, not just by the investors holding the security. Our model provides a tractable framework for studying how the market value of a firm, as well as the volatility and correlation of the securities within its capital structure, vary with the firm's financing decisions.

- **Encouraging information acquisition**: We consider a setting in which information acquisition by investors generates disutility for the agent. This may not always be the case. For instance, if the investor used the information in prices to make investment decisions, he may want to structure

his claims to incentivize information acquisition. While our results suggest that the minimum-variance capital structure does not minimize information acquisition, it would be interesting to analyze the robustness of these results in a richer setting.

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A   Proofs

Proof of Lemma 1. Let v and z be normally-distributed random variables: $v \sim \mathcal{N}(\mu_v, \sigma_v^2)$ and $z \sim \mathcal{N}(\mu_z, \sigma_z^2)$. Then

$$\Phi\left(\frac{\mu_z - \mu_v}{\sqrt{\sigma_z^2 + \sigma_v^2}}\right) = \mathbb{P}[v < z] = \mathbb{E}\left[\mathbb{P}[v < z \,|\, z] \,|\, z \sim \mathcal{N}(\mu_z, \sigma_z^2)\right]$$

$$= \mathbb{E}\left[\Phi\left(\frac{z - \mu_v}{\sigma_v}\right) \,|\, z \sim \mathcal{N}(\mu_z, \sigma_z^2)\right]$$

Substituting $\mu_v = 0$ and $\sigma_v = 1$, this implies that

$$\mathbb{E}\left[\Phi\left(z\right) \,|\, z \sim \mathcal{N}(\mu_z, \sigma_z^2)\right] = \Phi\left(\frac{\mu_z}{\sqrt{\sigma_z^2 + 1}}\right)$$

□

Proof of Lemma 2. Given any face value of debt, F, the payoff to both debt $(min(F, max(0, x)))$ and equity $(min(0, x - F))$ are weakly increasing in the cash flow, x. Moreover, both securities are piecewise differentiable with respect to x. As a result, as was first shown by Rothschild and Stiglitz (1970), the assumption of first-order stochastic dominance implies that $D_H \geq D_L$ and $E_H \geq E_L$. This implies that for any F, ΔD and $\Delta E \geq 0$.

Lastly,

$$\frac{\partial D_s}{\partial F} = 1 - G_s(F) \quad \implies \quad \frac{\partial \Delta D}{\partial F} = G_L(F) - G_H(F) \geq 0$$

$$\frac{\partial E_s}{\partial F} = G_s(F) - 1 \quad \implies \quad \frac{\partial \Delta E}{\partial F} = G_H(F) - G_L(F) \leq 0$$

where both inequalities follow from first-order stochastic dominance.   □

Proof of Proposition 1. The result follows from the risk-neutral/normal model of Albagli et al. (2015). Note that the payoffs of both debt and equity are strictly increasing in z and both payoffs are twice-differentiable. Investor positions are bound above and below and the asset supply is fixed. Moreover, the price of debt (equity) is monotone in investors beliefs as well as s_D (s_E).   □

Proof of Lemma 3. The price of debt can be written $p_D = D_L + q_D \Delta D$, where

$$q_D = \Phi\left(\frac{\tau_z \mu_z + (\tau_i^D + \tau_d \tau_i^D)s_D + \tau_E s_E}{\sqrt{\psi^D(1 + \psi^D)}}\right) \equiv \Phi(\tilde{q_D})$$

Both D_L and D_H are known. Using a first-order Taylor expansion, we can write
$$\mathbb{V}_0[p_D] \approx \Delta D^2 \left[\phi(\mathbb{E}_0[\tilde{q_D}])^2 \mathbb{V}_0[\tilde{q_D}]\right],$$

where

$$\mathbb{E}_0[\tilde{q_D}] = \left(1 + \frac{1}{\psi^D}\right)^{-\frac{1}{2}} \mu_z,$$

and ϕ is the standard Gaussian pdf, which is maximized at zero. As a result, an increase in $|\mu_z|$, which pushes $\mathbb{E}_0[\tilde{q_D}]$ away from zero, causes $\phi(\mathbb{E}_0[\tilde{q_D}])$, and therefore, $\mathbb{V}_0[p_D]$, to fall. It is also clear that $\mathbb{V}_0[p_D]$ will increase with information sensitivity, ΔD.

Knowing that the private signal of the marginal investor exactly equals the public signal obtained from the price of debt implies that

$$\mathbb{V}_0[\tilde{q_D}] = \frac{\frac{1}{\tau_z}\left(\tau_E + \tau_{i,D} + \tau_D\right)^2 + \left(\frac{\tau_{i,D}}{\tau_n} + 2\tau_{i,D} + \tau_D\right) + \tau_E}{\psi^D(1 + \psi^D)}$$

and so we can calculate the effect of increased within-market learning as:

$$\frac{\partial \mathbb{V}_0[\tilde{q_D}]}{\partial \tau_{i,D}} = (1 + \tau_n)\frac{\psi^D\left[\tau_{i,D}\left(\tau_n + \frac{1}{\tau_z}(1 + \tau_n) - \frac{1}{\tau_n}\right) + \left(\left(\frac{1}{\tau_n} + 1 + \frac{1}{\tau_z}\right)\tau_E + \left(\frac{1}{\tau_n} + 1\right)\tau_z\right)\right]}{\left[\psi^D(1 + \psi^D)\right]^2}$$

$$+ (1 + \tau_n)\frac{(\tau_{i,D} + \tau_D) + \left(\frac{1}{\tau_n} + 1\right)(\tau_z + \tau_E)}{\left[\psi^D(1 + \psi^D)\right]^2}$$

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Similarly, we can write

$$\phi(\mathbb{E}_0[\tilde{q_D}])^2 = \phi\left(\left(1 + \frac{1}{\psi^D}\right)^{-\frac{1}{2}}\mu_z\right)^2 = \frac{1}{2\pi}exp\left(-\mu_z^2\frac{\psi^D}{1+\psi^D}\right)$$

$$\frac{\partial\phi(\mathbb{E}_0[\tilde{q_D}])^2}{\partial\tau_{i,D}} = \frac{1}{2\pi}exp\left(-\mu_z^2\frac{\psi^D}{1+\psi^D}\right)\left[\frac{-\mu_z^2(1+\tau_n)}{(1+\psi^D)^2}\right]$$

Combining these allows us to write the effect of increased within-market learning:

$$\frac{\partial\mathbb{V}_0[p_D]}{\partial\tau_{i,D}} = (\Delta D)^2\frac{\phi(\mathbb{E}_0[\tilde{q_D}])^2\psi^D(1+\tau_n)}{[\psi^D(1+\psi^D)]^2}\{\tau_{i,D}\left(\tau_n + \frac{1}{\tau_z}(1+\tau_n) - \frac{1}{\tau_n}\right) + \left(\left(\frac{1}{\tau_n}+1+\frac{1}{\tau_z}\right)\tau_E + \left(\frac{1}{\tau_n}+1\right)\tau_z\right)$$
$$- \left[\frac{\mu_z^2\left[\frac{1}{\tau_z}\left(\tau_E+\tau_{i,D}+\tau_D\right)^2 + \left(\frac{\tau_{i,D}}{\tau_n}+2\tau_{i,D}+\tau_D\right)+\tau_E\right]}{1+\psi^D}\right] + Remainder\}$$

$$> (\Delta D)^2\frac{\phi(\mathbb{E}_0[\tilde{q_D}])^2\psi^D(1+\tau_n)}{[\psi^D(1+\psi^D)]^2}\{\tau_{i,D}\left(\tau_n + \frac{1}{\tau_z}(1+\tau_n) - \frac{1}{\tau_n}\right) + \left(\left(\frac{1}{\tau_n}+1+\frac{1}{\tau_z}\right)\tau_E + \left(\frac{1}{\tau_n}+1\right)\tau_z\right)$$
$$- \left[\mu_z^2\left[\frac{1}{\tau_z}\left(\tau_E+\tau_{i,D}+\tau_D\right)^2 + \left(\frac{\tau_{i,D}}{\tau_n}+2\tau_{i,D}+\tau_D\right)+\tau_E\right]\right] + Remainder\}$$

$$= (\Delta D)^2\frac{\phi(\mathbb{E}_0[\tilde{q_D}])^2\psi^D(1+\tau_n)}{[\psi^D(1+\psi^D)]^2}\{\tau_{i,D}\left(\tau_n + \frac{1}{\tau_z}(1+\tau_n) - \frac{1}{\tau_n} - \mu_z^2\left(\frac{1}{\tau_n}+2+\tau_n\right)\right) + \left(\frac{1}{\tau_n}+1+\frac{1}{\tau_z}-\mu_z^2\right)\tau_E$$
$$- \frac{\mu_z^2}{\tau_z}\left(\tau_E+\tau_{i,D}+\tau_D\right)^2 + \left(\frac{1}{\tau_n}+1\right)\tau_z + Remainder\}$$

Note that the remainder is always positive. As a result, when $|\mu_z|$ is sufficiently small, and if $\tau_z < \tau_n$(or ignoring τ_z, if τ_n is sufficiently large), $\frac{\partial\mathbb{V}_0[p_D]}{\partial\tau_{i,D}} > 0$.

Similarly,

$$\frac{\partial\mathbb{V}_0[\tilde{q_D}]}{\partial\tau_{i,E}} = \tau_n\frac{\psi^D\left[\tau_z + \tau_E\left(1+\frac{1}{\tau_z}\right) + \tau_{i,D}\left(\tau_n + \frac{1}{\tau_z}(1+\tau_n) - \frac{2}{\tau_n} - 1\right)\right] + \left[\tau_E + \tau_z + \tau_{i,D}\left(\tau_n - \frac{1}{\tau_n}\right)\right]}{[\psi^D(1+\psi^D)]^2}$$

Following steps similar to those above:

$$\frac{\partial\phi(\mathbb{E}_0[\tilde{q_D}])^2}{\partial\tau_{i,E}} = \frac{1}{2\pi}exp\left(-\mu_z^2\frac{\psi^D}{1+\psi^D}\right)\left[\frac{-\mu_z^2\tau_n}{(1+\psi^D)^2}\right]$$

which implies that

$$\frac{\partial\mathbb{V}_0[p_D]}{\partial\tau_{i,E}} = \frac{(\Delta D)^2\phi(\mathbb{E}_0[\tilde{q_D}])^2\tau_n\psi^D}{[\psi^D(1+\psi^D)]^2}\{\tau_z + \tau_E\left(1+\frac{1}{\tau_z}\right) + \tau_{i,D}\left(\tau_n + \frac{1}{\tau_z}(1+\tau_n) - \frac{2}{\tau_n} - 1\right)$$
$$- \mu_z^2\left[\frac{\frac{1}{\tau_z}\left(\tau_E+\tau_{i,D}+\tau_D\right)^2 + \left(\frac{\tau_{i,D}}{\tau_n}+2\tau_{i,D}+\tau_D\right)+\tau_E}{1+\psi^D}\right] + Remainder\}$$

$$> \frac{(\Delta D)^2\phi(\mathbb{E}_0[\tilde{q_D}])^2\tau_n\psi^D}{[\psi^D(1+\psi^D)]^2}\{\tau_z + \tau_E\left(1+\frac{1}{\tau_z}\right) + \tau_{i,D}\left(\tau_n + \frac{1}{\tau_z}(1+\tau_n) - \frac{2}{\tau_n} - 1\right)$$
$$- \mu_z^2\left[\frac{1}{\tau_z}\left(\tau_E+\tau_{i,D}+\tau_D\right)^2 + \left(\frac{\tau_{i,D}}{\tau_n}+2\tau_{i,D}+\tau_D\right)+\tau_E\right] + Remainder\}$$

$$= \frac{(\Delta D)^2\phi(\mathbb{E}_0[\tilde{q_D}])^2\tau_n\psi^D}{[\psi^D(1+\psi^D)]^2}\{\tau_z + \tau_E\left(1+\frac{1}{\tau_z}-\mu_z^2\right) + \tau_{i,D}\left(\tau_n + \frac{1}{\tau_z}(1+\tau_n) - \frac{2}{\tau_n} - 1 - \mu_z^2\left(\frac{1}{\tau_n}+2+\tau_n\right)\right)$$
$$- \frac{\mu_z^2}{\tau_z}\left(\tau_E+\tau_{i,D}+\tau_D\right)^2 + Remainder\}$$

When $|\mu_z|$ is sufficiently small, and if τ_z is sufficiently smaller than τ_n(or ignoring τ_z, if τ_n is sufficiently large), $\frac{\partial\mathbb{V}_0[p_D]}{\partial\tau_{i,E}} > 0$.

Using our work from above, we can also compare the sensitivity of $\mathbb{V}_0[\tilde{q_D}]$ with respect to learning in each market.

$$\frac{\partial \mathbb{V}_0[\tilde{q_D}]}{\partial \tau_{i,D}} = \frac{(1+\tau_n)}{\tau_n} \left[\frac{\partial \mathbb{V}_0[\tilde{q_D}]}{\partial \tau_{i,E}} + \frac{1}{\psi^D(1+\psi^D)} \right]$$

If $\frac{\partial \mathbb{V}_0[\tilde{q_D}]}{\partial \tau_{i,E}} > 0$, $\frac{\partial \mathbb{V}_0[\tilde{q_D}]}{\partial \tau_{i,D}} > \frac{\partial \mathbb{V}_0[\tilde{q_D}]}{\partial \tau_{i,E}}$.

We also need to show that if $\tau_{i,D} > \tau_{i,E}$ then $\mathbb{V}_0[q_D] > \mathbb{V}_0[q_E]$. We begin by defining γ as the difference between the precision in each market (i.e., $\gamma \equiv \tau_{i,D} - \tau_{i,E}$). Then,

$$\mathbb{V}_0[\tilde{q_E}] = \frac{\frac{1}{\tau_z}(\tau_E + \tau_{i,E} + \tau_D)^2 + \left(\frac{\tau_{i,E}}{\tau_n} + 2\tau_{i,E}\right) + (\tau_D + \tau_E)}{\psi^E(1+\psi^E)}$$

$$\mathbb{V}_0[\tilde{q_D}] = \frac{\frac{1}{\tau_z}(\tau_E + \tau_{i,E} + \tau_D)^2 + \left(\frac{\tau_{i,E}}{\tau_n} + 2\tau_{i,E}\right) + (\tau_D + \tau_E) + \gamma\left(\frac{1}{\tau_n} + 2\right) + \frac{1}{\tau_z}\left(2\gamma(\tau_E + \tau_{i,E} + \tau_D) + \gamma^2\right)}{\psi^E(1+\psi^E) + \gamma(1+2\psi^E) + \gamma^2}$$

We can write $\mathbb{V}_0[\tilde{q_E}] = \frac{a}{b}$ and $\mathbb{V}_0[\tilde{q_D}] = \frac{a+c}{b+d}$. In order for $\mathbb{V}_0[\tilde{q_D}] > \mathbb{V}_0[\tilde{q_E}]$, we want $ad < bc$:

$$\left[\frac{1}{\tau_z}(\tau_E + \tau_{i,E} + \tau_D)^2 + \left(\frac{\tau_{i,E}}{\tau_n} + 2\tau_{i,E}\right) + (\tau_D + \tau_E)\right]\left[\gamma(1+2\psi^E) + \gamma^2\right] < \left[\frac{1}{\tau_z}\left(2\gamma(\tau_E + \tau_{i,E} + \tau_D) + \gamma^2\right) + \gamma\left(\frac{1}{\tau_n} + 2\right)\right]\left[\psi^E(1+\psi^E)\right]$$

After some simplification, this reduces to

$$\left[\psi^E + \gamma\right]\left[\frac{1}{\tau_z}(\tau_E + \tau_{i,E} + \tau_D)^2 + \left(\frac{\tau_{i,E}}{\tau_n} + 2\tau_{i,E}\right) + (\tau_D + \tau_E)\right] < \left[\frac{\psi^E}{\tau_z}((\tau_E + \tau_{i,E} + \tau_D) + \gamma) + (\tau_z + \tau_E + \tau_D)\left(\frac{1}{\tau_n} + 2\right) + \tau_{i,E}\right]\left[(1+\psi^E)\right]$$

To insure that this inequality holds for any $\tau_{i,D}$, $\tau_{i,E}$, we can match on endogenous terms and define our condition in terms of exogenous variables.

$$\left[\psi^E + \gamma\right]\left[\left(\frac{\tau_{i,E}}{\tau_n}\right) - \tau_{i,E}\tau_n\right] < \left[\psi^E + \gamma\right](\tau_z + \tau_D) + \left[(\tau_z + \tau_E + \tau_D)\left(\frac{1}{\tau_n} + 1\right)\right]\psi^E$$
$$+ \left[\frac{\psi^E}{\tau_z}((\tau_E + \tau_{i,E} + \tau_D) + \gamma) + (\tau_z + \tau_E + \tau_D)\left(\frac{1}{\tau_n} + 2\right) + \tau_{i,E}\right]$$

If $\tau_n \geq 1$, then the LHS must be negative, and so the inequality will hold. That is, if $\tau_n \geq 1$, $\frac{\mathbb{V}_0[\tilde{q_D}]}{\mathbb{V}_0[\tilde{q_E}]} > 1$. To show that $\mathbb{V}_0[q_D] > \mathbb{V}_0[q_E]$, however, we must show that

$$\frac{\mathbb{V}_0[\tilde{q_D}]}{\mathbb{V}_0[\tilde{q_E}]} > \frac{\phi(\mathbb{E}_0[\tilde{q_E}])^2}{\phi(\mathbb{E}_0[\tilde{q_D}])^2}$$
$$\approx 1 + \mu_z^2\left(\frac{\gamma}{(1+\psi^E)(1+\psi^E+\gamma)}\right)$$

It is clear that for sufficiently small $|\mu_z|$, the inequality above must hold.

Finally, we can compare the variance of the price received to the variance of the "fundamental" value. As before, we begin by examining $\mathbb{V}_0[\tilde{q}]$ and $\mathbb{V}_0[\tilde{q_D}]$:

$$\mathbb{V}_0[\tilde{q}] = \frac{1}{\tau_z} > \mathbb{V}_0[\tilde{q_D}] \quad \Longleftrightarrow$$
$$\psi^D > \tau_z\left(\tau_{i,D}\left(\frac{1}{\tau_n} - \tau_n\right) - (\tau_z + \tau_E)\right)$$

As long as $\tau_n \geq 1$, the inequality above holds. To show that $\mathbb{V}_0[q_D] > \mathbb{V}_0[q]$, however, we must show that

$$\frac{\mathbb{V}_0[\tilde{q_D}]}{\mathbb{V}_0[\tilde{q}]} > \frac{\phi(\mathbb{E}_0[\tilde{q}])^2}{\phi(\mathbb{E}_0[\tilde{q_D}])^2}$$
$$\approx 1 + \mu_z^2\left(\frac{1}{\psi^D}\right)$$

It is clear that for sufficiently small $|\mu_z|$, the inequality above must hold. $\qquad \square$

Proof of Lemma 4. Using Lemma 5, we can write the agent's expectation of q_D :

$$\mathbb{E}_0[q_D] = \Phi\left(\frac{\psi^D\mu_z}{\sqrt{\psi^D(1+\psi^D) + (\tau_{i,D}+\tau_D+\tau_E)^2\tau_z^{-1} + (\tau_{i,D}+\tau_D)^2\tau_D^{-1} + \tau_E}}\right)$$

$$\equiv \Phi(\bar{q_D}\mu_z)$$

which implies that the expected *price* of debt is simply $\mathbb{E}_0[p_D] = D_L + \mathbb{E}_0[q_D]\Delta D$. The agent's expectation of the fundamental value of the asset, i.e., the price he'd receive by waiting until time-1 to sell it can be written $D_L + \mathbb{E}_0[q]\Delta D$, where

$$\mathbb{E}_0[q] = \Phi\left(\frac{\mu_z}{\sqrt{1+\tau_z^{-1}}}\right).$$

It is straightforward to compare the expected price to the expected fundamental value of the asset:

$$\bar{q_D} = \frac{1}{\sqrt{1+\tau_z^{-1} + \frac{\frac{\tau_{i,D}+\tau_{i,D}}{\tau_n}}{(\psi^D)^2}}} < \frac{1}{\sqrt{1+\tau_z^{-1}}}$$

If $\mu_z = 0$, $\mathbb{E}_0[q_D] = \mathbb{E}_0[q]$. Otherwise, given that Φ is a strictly increasing function, this implies that

$$\implies \mathbb{E}_0[q] \quad \begin{cases} > \mathbb{E}_0[q_D] & \text{if } \mu_z > 0 \\ < \mathbb{E}_0[q_D] & \text{if } \mu_z < 0 \end{cases}$$

Finally, we must compare $\bar{q_D}$ to its similarly-defined counterpart, $\bar{q_E}$. Suppose $\tau_{i,D} > \tau_{i,E}$. Then $\bar{q_D} < \bar{q_E}$ iff $\frac{\tau_{i,D}}{(\psi^D)^2} > \frac{\tau_{i,E}}{(\psi^E)^2}$ which is true iff

$$(\tau_{i,D} - \tau_{i,E})\left[\tau_z^2 + 2\tau_z(\tau_D+\tau_E) + \tau_D^2 + \tau_E^2 + 2\tau_{i,E}\tau_{i,D}\left(2\tau_n^2 - 1\right)\right] > 0$$

If τ_n is sufficiently large, $\bar{q_D} < \bar{q_E}$. When these conditions hold,

$$\implies \mathbb{E}_0[q_D] \quad \begin{cases} < \mathbb{E}_0[q_E] & \text{if } \mu_z > 0 \\ > \mathbb{E}_0[q_E] & \text{if } \mu_z < 0 \end{cases}$$

In combination with Lemma 2 ($\Delta D \geq 0$), this completes the proof. $\qquad\square$

Proof of Proposition 2 Because $\mathbb{V}_0[q_D], \mathbb{V}_0[q_E] < \mathbb{V}[q]$, it is sufficient to show that if $Cov(q_D, q) > \mathbb{V}_0[q_D]$ and $Cov(q_E, q) > \mathbb{V}_0[q_E]$ the agent will sell everything at time-0.

$$Cov(\tilde{q_D}, \tilde{q}) > \mathbb{V}[\tilde{q_D}] \quad \text{if}$$

$$[\tau_{i,D} + \tau_d\tau_{i,D} + \tau_E]^2 > \tau_z\left(\frac{\tau_{i,D}}{\tau_n} + \tau_{i,D}\right) + (\tau_{i,D} + \tau_d\tau_{i,D} + \tau_E)$$

This inequality holds if τ_z is sufficiently low, and τ_n is sufficiently high. A similar argument holds with respect to $Cov(\tilde{q_E}, \tilde{q})$. Given $\mu_z = 0$, the $\phi(\mathbb{E}[\cdot])^2$ is constant, which completes the proof. $\qquad\square$

Proof of Proposition 3. The agent's objective is to choose F to minimize

$$\mathbb{V}_0[p_D + p_E] = \Delta D^2\mathbb{V}_0[q_D] + \Delta E^2 V_0[q_E] + 2\Delta D\Delta E\sqrt{V[q_D]\mathbb{V}[q_E]}Corr(q_D, q_E).$$

The marginal value of increasing F can be written:

$$\frac{\partial\mathbb{V}_0[p_D + p_E]}{\partial F} = 2\left[G_L(F) - G_H(F)\right]\left[\Delta D(F)\left(\mathbb{V}_0[q_D] - Cov(q_D, q_E)\right) - \Delta E(F)\left(V_0[q_E] - Cov(q_D, q_E)\right)\right]$$

We define $\underline{F} \equiv sup\{F : \Delta D(F) = 0\}$ and $\bar{F} \equiv inf\{F : \Delta E(F) = 0\}$. It is straightforward to show that (i) $\forall F \leq \underline{F}$, debt is information-insensitive, (ii) $\forall F \geq \bar{F}$, equity is information-insensitive, and (iii) if $\underline{F} < F < \bar{F}$, then both debt and equity are information-sensitive. Note that it is sufficient to consider F such that $\underline{F} \leq F \leq \bar{F}$. For any value of F outside of this interval, $\frac{\partial\mathbb{V}_0[p_D+p_E]}{\partial F} = 0$.

By Lemma 3, if $\tau_{i,D} > \tau_{i,E}$, $\mathbb{V}_0[q_D] > \mathbb{V}_0[q_E]$ and therefore $\mathbb{V}_0[q_D] > Cov(q_D, q_E)$. If $\mathbb{V}[q_E] < Cov(q_D, q_E)$, it must be that $\frac{\partial\mathbb{V}_0[p_D+p_E]}{\partial F} \geq 0$ (because $G_L(F) - G_H(F)$ is always (weakly) positive). As a result, to *minimize* the uncertainty of his proceeds, the agent sets $F^* = \underline{F}$, which implies that $\Delta E(F^*) = \Delta V$ and $\Delta D(F^*) = 0$.

On the other hand, if $\mathbb{V}[q_E] > Cov(q_D, q_E)$, then it must be that

$$\frac{\Delta D(F^*)}{\Delta E(F^*)} = \frac{\mathbb{V}_0[q_E] - Cov(q_D, q_E)}{\mathbb{V}_0[q_D] - Cov(q_D, q_E)} \tag{17}$$

and because $\mathbb{V}_0[q_D] > \mathbb{V}_0[q_E]$, this implies that $0 < \Delta D(F^*) < \Delta E(F^*) < \Delta V$.

We show that this is the only solution to the agent's minimization problem in two steps. First, we define $\frac{\partial\mathbb{V}_0[p_D+p_E]}{\partial F} \equiv 2\left[G_L(F) - G_H(F)\right]\eta(F)$, so that

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$$\frac{\partial \eta(F)}{\partial F} = [G_L(F) - G_H(F)] \left[\mathbb{V}_0[q_D] + V_0[q_E] - 2Cov(q_D, q_E)\right] \geq 0$$

If $G_L(F^*) \neq G_H(F^*)$, then F^* is unique. Otherwise, due to the weak monotonicity of $\eta(F)$, $F^* \in [F_l, F_h]$ where $F_l \equiv min\{F : \eta(F) = 0\}$, $F_h \equiv max\{F : \eta(F) = 0\}$. Note that it must be that $\underline{F} < F^* < \bar{F}$.

Second, we show that for all other points in which $\frac{\partial \mathbb{V}_0[\mathbf{P_D} + \mathbf{P_E}]}{\partial \mathbf{F}} = 0$, the variance must be higher. Note that at any other such F such that $\frac{\partial \mathbb{V}_0[\mathbf{P_D} + \mathbf{P_E}]}{\partial \mathbf{F}} = 0$, it must be that $G_L(F) = G_H(F)$. There are two cases to consider:

1. Suppose $G_L(F) = G_H(F)$ and $F < F_l$. Then there exists some $F' > F$ such that $G_L(F') > G_H(F')$ and because $F' < F_l$ it must be that $\eta(F) < 0$. As a result, the agent could lower his uncertainty by increasing F.

2. Suppose $G_L(F) = G_H(F)$ and $F > F_h$. Then there exists some $F' < F$ such that $G_L(F') > G_H(F')$ and because $F' > F_h$ it must be that $\eta(F) > 0$. As a result, the agent could lower his uncertainty by decreasing F.

Following similar steps yield the counterpart results when $\tau_{i,E} > \tau_{i,D}$.

Conditions under which $\mathbb{V}[q_E] > Cov(q_D, q_E)$:

When $\mu_z = 0$, $\mathbb{V}_0[\tilde{q_E}] > Cov(\tilde{q_E}, \tilde{q_D}) \implies \mathbb{V}_0[q_E] > Cov(q_E, q_D)$. As $\tau_{i,E} \to \tau_{i,D}$

$$Cov(\tilde{q_E}, \tilde{q_D}) = \frac{\frac{1}{\tau_z}(\tau_{i,D} + \tau_D + \tau_E)(\tau_{i,E} + \tau_D + \tau_E) + (\tau_{i,D} + \tau_D) + (\tau_{i,E} + \tau_E)}{\sqrt{\psi^D(1 + \psi^D)\psi^E(1 + \psi^E)}}$$

$$\to \frac{\frac{1}{\tau_z}(\tau_{i,D} + 2\tau_D)^2 + 2(\tau_{i,D} + \tau_D)}{\psi^D(1 + \psi^D)}$$

$$\mathbb{V}_0[\tilde{q_E}] = \frac{\frac{1}{\tau_z}(\tau_E + \tau_{i,E} + \tau_D)^2 + (\tau_{i,E} + \tau_D) + (\tau_{i,E} + \tau_E) + \frac{\tau_{i,E}}{\tau_n}}{\psi^E(1 + \psi^E)}$$

$$\to \frac{\frac{1}{\tau_z}(\tau_{i,D} + 2\tau_D)^2 + 2(\tau_{i,D} + \tau_D) + \frac{\tau_{i,D}}{\tau_n}}{\psi^D(1 + \psi^D)}$$

It's clear that in this case, $\mathbb{V}_0[q_E] > Cov(q_E, q_D)$, and by the continuity of both functions, must hold when $|\tau_{i,E} - \tau_{i,D}|$ is sufficiently small. $\qquad\square$

Proof of Proposition 4. Each investor chooses the precision of his signal, taking the precision of all other investors as given. We will represent the individual investor's precision as τ_i and his beliefs about the precisions chosen by others as $\tau_{i,D}$ and $\tau_{i,E}$, as before. Increasing or decreasing τ_i has no effect on the precision of the signal obtained from prices - each individual is part of a continuum of investors. q_D remains the same and we now write $q_{i,D}$:

$$q_{i,D} = \Phi\left(\frac{\tau_z\mu_z + \tau_i s_i + \tau_D s_D + \tau_E s_E}{\sqrt{\psi^{i,D}(1 + \psi^{i,D})}}\right) \equiv \Phi(\tilde{q_{i,D}})$$

We approximate $q_{i,D}$ and q_D using a first-order Taylor expansion, e.g., $q_{i,D} \approx \Phi\left(\mathbb{E}[\tilde{q_{i,D}}]\right) + \phi\left(\mathbb{E}[\tilde{q_{i,D}}]\right)\left(\tilde{q_{i,D}} - \mathbb{E}[\tilde{q_{i,D}}]\right)$, where $\mathbb{E}[\tilde{q_D}]$ is defined above and

$$\mathbb{E}[\tilde{q_{i,D}}] = \left(1 + \frac{1}{\tau_z + \tau_i + \tau_D + \tau_E}\right)^{-\frac{1}{2}}\mu_z.$$

With $\mu_z = 0$, we can simplify symmetrically: $q_{i,D} \approx \frac{1}{2} + \phi(0)\tilde{q_{i,D}}$ and $q_{i,D} \approx \frac{1}{2} + \phi(0)\tilde{q_D}$. As a result, we can rewrite each investor's expected utility from trading the asset as

$$EU_D = \frac{\Delta D}{m_D}\phi(0)\mathbb{E}_0\left[(\tilde{q_{i,D}} - \tilde{q_D})|\tilde{q_{i,D}} > \tilde{q_D}\right].$$

The expectation of both $\tilde{q_{i,D}}$ and $\tilde{q_D}$ is zero and they are both normally-distributed variables (with different variances, depending upon the precisions chosen). As a result, we can use the properties of the folded normal distribution to write

$$\mathbb{E}_0\left[(\tilde{q_{i,D}} - \tilde{q_D})|\tilde{q_{i,D}} - \tilde{q_D} > 0\right] = \sqrt{\frac{\mathbb{V}[(\tilde{q_{i,D}} - \tilde{q_D})]}{2\pi}}$$

$$\implies EU_D = \left[\frac{\Delta D\phi(0)}{m_D\sqrt{2\pi}}\right]\sqrt{\mathbb{V}[(\tilde{q_{i,D}} - \tilde{q_D})]} \tag{18}$$

The information acquisition of investors has no impact on the term in brackets in equation (18).

$$\frac{\partial EU_D}{\partial \tau_i} = \left[\frac{\Delta D}{m_D 2\pi}\right]\frac{1}{2}\frac{\partial \mathbb{V}[(\tilde{q_{i,D}} - \tilde{q_D})]}{\tau_i}[\mathbb{V}[(\tilde{q_{i,D}} - \tilde{q_D})]]^{-\frac{1}{2}} > 0 \iff \frac{\partial \mathbb{V}[(\tilde{q_{i,D}} - \tilde{q_D})]}{\tau_i} > 0$$

$$\frac{\partial^2 EU_D}{\partial \tau_i^2} = \left[\frac{\Delta D}{m_D 2\pi} \right] \left[\frac{1}{2} \frac{\partial^2 \mathbb{V}[(q_{\tilde{i},D} - \tilde{q_D})]}{\tau_i^2} \left[\mathbb{V}[(q_{\tilde{i},D} - \tilde{q_D})] \right]^{-\frac{1}{2}} - \frac{1}{4} \left[\frac{\partial \mathbb{V}[(q_{\tilde{i},D} - \tilde{q_D})]}{\tau_i} \right]^2 \left[\mathbb{V}[(q_{\tilde{i},D} - \tilde{q_D})] \right]^{-\frac{3}{2}} \right]$$

This gives us sufficient conditions for $\frac{\partial EU_D}{\partial \tau_i} > 0$ and $\frac{\partial^2 EU_D}{\partial \tau_i^2} < 0$:

$$\frac{\partial \mathbb{V}[(q_{\tilde{i},D} - \tilde{q_D})]}{\tau_i} > 0 \quad \text{and} \quad \frac{\partial^2 \mathbb{V}[(q_{\tilde{i},D} - \tilde{q_D})]}{\tau_i^2} < 0 \quad \Longrightarrow \quad \frac{\partial^2 EU_D}{\partial \tau_i^2} < 0$$

The random variance $q_{\tilde{i},D} - \tilde{q_D}$ can be rewritten as

$$q_{\tilde{i},D} - \tilde{q_D} = \frac{(\tau_i + \tau_D + \tau_E)\, z + \tau_i \varepsilon_i + \tau_D u_D + \tau_E u_E}{\sqrt{\psi^{i,D}(1 + \psi^{i,D})}} - \frac{(\tau_{i,D} + \tau_D + \tau_E)\, z + (\tau_{i,D} + \tau_D)\, u_D + \tau_E u_E}{\sqrt{\psi^D(1 + \psi^D)}}$$

Note that $z, \varepsilon_i, u_D, u_E$ are all independent. As a result,

$$\mathbb{V}[(q_{\tilde{i},D} - \tilde{q_D})] = \left[\frac{\tau_i + \tau_D + \tau_E}{\sqrt{\psi^{i,D}(1 + \psi^{i,D})}} - \frac{(\tau_{i,D} + \tau_D + \tau_E)}{\sqrt{\psi^D(1 + \psi^D)}} \right]^2 \frac{1}{\tau_z} + \left[\frac{\tau_i}{\psi^{i,D}(1 + \psi^{i,D})} \right]$$
$$+ \left[\frac{\tau_D}{\sqrt{\psi^{i,D}(1 + \psi^{i,D})}} - \frac{\tau_{i,D} + \tau_D}{\sqrt{\psi^D(1 + \psi^D)}} \right]^2 \frac{1}{\tau_D} + \left[\frac{\tau_E}{\sqrt{\psi^{i,D}(1 + \psi^{i,D})}} - \frac{\tau_E}{\sqrt{\psi^D(1 + \psi^D)}} \right]^2 \frac{1}{\tau_E}$$

First, we establish under what conditions $\mathbb{V}[(q_{\tilde{i},D} - \tilde{q_D})]$ is increasing in τ_i.

$$\frac{\partial \mathbb{V}[(q_{\tilde{i},D} - \tilde{q_D})]}{\partial \tau_i} = \frac{\frac{1}{\tau_z}(\tau_i + \tau_D + \tau_E)\, \psi^{i,D} + \psi^{i,D}(1 + \psi^{i,D})}{[\psi^{i,D}(1 + \psi^{i,D})]^2} - \frac{\frac{1}{\tau_z}(\tau_{i,D} + \tau_D + \tau_E)\, \psi^{i,D}}{\sqrt{\psi^D(1 + \psi^D)}\, [\psi^{i,D}(1 + \psi^{i,D})]^{\frac{3}{2}}}$$
$$= \frac{\psi^{i,D}}{\tau_z\, [\psi^{i,D}(1 + \psi^{i,D})]^{\frac{3}{2}}} \left[\frac{(\tau_i + \tau_D + \tau_E) + \tau_z(1 + \psi^{i,D})}{\sqrt{\psi^{i,D}(1 + \psi^{i,D})}} - \frac{(\tau_{i,D} + \tau_D + \tau_E)}{\sqrt{\psi^D(1 + \psi^D)}} \right]$$

This implies that $\frac{\partial \mathbb{V}[(q_{\tilde{i},D} - \tilde{q_D})]}{\partial \tau_i} > 0 \iff$

$$\left[(1 + \tau_z)\psi^{i,D} \right]^2 \psi^D(1 + \psi^D) > (\tau_{i,D} + \tau_D + \tau_E)^2\, \psi^{i,D}(1 + \psi^{i,D})$$
$$\left[(1 + \tau_z)^2 \psi^{i,D} \right] \psi^D + \left[(1 + \tau_z)^2 \psi^{i,D} \right] (\tau_{i,D} + \tau_D + \tau_E + \tau_z)^2 > (\tau_{i,D} + \tau_D + \tau_E)^2 (1 + \psi^{i,D})$$
$$\left[(1 + \tau_z)^2 \psi^{i,D} \right] \psi^D + \left[(1 + \tau_z)^2 \psi^{i,D} \right] \left[2\tau_z(\tau_{i,D} + \tau_D + \tau_E) + \tau_z^2 \right] > (\tau_{i,D} + \tau_D + \tau_E)^2 (1 - (2\tau_z + \tau_z^2)\psi^{i,D})$$

It is clear that as τ_z increases, the RHS falls and the LHS increases. If τ_z is sufficiently large, then $\frac{\partial \mathbb{V}[(q_{\tilde{i},D} - \tilde{q_D})]}{\partial \tau_i} > 0$. Turning our attention to establishing concavity, we can write

$$\frac{(\partial \mathbb{V}[(q_{\tilde{i},D} - \tilde{q_D})])^2}{\partial^2 \tau_i} = \frac{\psi^{i,D}}{\tau_z\, [\psi^{i,D}(1 + \psi^{i,D})]^{\frac{5}{2}}} \left[\frac{-2\psi^{i,D}(1 + \tau_z)\, \psi^{i,D}}{(\psi^{i,D}(1 + \psi^{i,D}))^{\frac{1}{2}}} + \frac{(1 + 4\psi^{i,D})(\tau_{i,D} + \tau_D + \tau_E)}{2\sqrt{\psi^D(1 + \psi^D)}} \right]$$

And so $\frac{(\partial \mathbb{V}[(q_{\tilde{i},D} - \tilde{q_D})])^2}{\partial^2 \tau_i} < 0 \iff$

$$\psi^{i,D} \left(4\psi^{i,D}(1 + \tau_z) \right)^2 \psi^D(1 + \psi^D) > \left((1 + 4\psi^{i,D})(\tau_{i,D} + \tau_D + \tau_E) \right)^2 (1 + \psi^{i,D})$$
$$\psi^{i,D} \left(4\psi^{i,D}(1 + \tau_z) \right)^2 \left((1 + \tau_z)(\psi^D) + \tau_z(\tau_{i,D} + \tau_D + \tau_E) \right) > (\tau_{i,D} + \tau_D + \tau_E)^2 \left[(1 + \psi^{i,D})(1 + 4\psi^{i,D})^2 - \psi^{i,D}(4\psi^{i,D}(1 + \tau_z))^2 \right]$$

As above, the LHS is positive and increasing in τ_z whereas for sufficiently large τ_z the RHS must be negative, due to the term in brackets. As a result, if τ_z is sufficiently large, then $\frac{\partial^2 \mathbb{V}[(q_{\tilde{i},D} - \tilde{q_D})]}{\partial \tau_{2_i}} < 0$. □

Proof of Corollary 1. The agent chooses τ_i to maximize $EU_D - C(\tau_i)$. Under the sufficient conditions established in the proof of Proposition 4, and the assumptions regarding the cost function, this implies that the optimal τ_i sets $\frac{\partial EU_D}{\partial \tau_i} = C'(\tau_i)$:

$$\frac{\partial \mathbb{V}[(q_{\tilde{i,D}} - \tilde{q_D})]}{\tau_i} \left[\mathbb{V}[(q_{\tilde{i,D}} - \tilde{q_D})]\right]^{-\frac{1}{2}} = \frac{C'(\tau_i)m_D(4\pi)}{\Delta D}$$

We are looking for a symmetric equilibrium, and so we can substitute $\tau_i = \tau_{i,D}$ into the expressions from the proof of Proposition 4. This yields:

$$\frac{\partial \mathbb{V}[(q_{\tilde{i,D}} - \tilde{q_D})]}{\tau_i} = \frac{1}{\psi^D(1+\psi^D)}$$

$$\mathbb{V}[(q_{\tilde{i,D}} - \tilde{q_D})] = \left[\frac{\tau_{i,D}\left(1 + \frac{1}{\tau_n}\right)}{\psi^D(1+\psi^D)}\right] \tag{19}$$

Taken together:

$$\sqrt{\tau_{i,D}\left(1 + \frac{1}{\tau_n}\right)\psi^D(1+\psi^D)} = \frac{\Delta D}{m_D C'(\tau_{i,D})(4\pi)} \tag{20}$$

Equity investors choose the precision of their signals in a similar fashion, so that $\tau_{i,E}$ solves

$$\sqrt{\tau_{i,E}\left(1 + \frac{1}{\tau_n}\right)\psi^E(1+\psi^E)} = \frac{\Delta E}{m_E C'(\tau_{i,E})(4\pi)} \tag{21}$$

Inspection of equations (20) and (21) yield the following comparative statics. Suppose τ_z increases - then both $\tau_{i,E}$ and $\tau_{i,D}$ must increase. On the other hand, suppose m_D falls - while $\tau_{i,D}$ must increase, in equilibrium, this causes $\tau_{i,E}$ to fall. It is straightforward to see that an increase in ΔD — which implies a decrease in ΔE — must result in $\tau_{i,D}$ rising and $\tau_{i,E}$ falling. In equilibrium, if $\frac{\Delta E}{m_E} > \frac{\Delta D}{m_D}$, it must be that $\tau_{i,E} > \tau_{i,D}$. Finally, equation (20) makes clear that if $\tau_E = 0$, which is equivalent to debt investors being unable to observe equity prices, $\tau_{i,D}$ must increase in equilibrium. $\square$

Proof of Proposition 5. As before, the agent's objective is to choose F to minimize the variance of his proceeds. The agent takes as given the precision choice of debt and equity investors, and so as before,

$$\frac{\partial \mathbb{V}_0[p_D + p_E]}{\partial F} = 2\left[G_L(F) - G_H(F)\right]\left[\Delta D(F)\left(\mathbb{V}_0[q_D] - Cov(q_D, q_E)\right) - \Delta E(F)\left(V_0[q_E] - Cov(q_D, q_E)\right)\right]$$

As before, in equilibrium $\frac{\partial \mathbb{V}_0[p_D + p_E]}{\partial F} = 0$, but now equations (20) and (21), i.e., investors will take the agent's choice as given in choosing their own precisions. As in Proposition 5, it is sufficient to consider F such that $\underline{F} \leq F \leq \bar{F}$. For any value of F outside of this interval, $\frac{\partial \mathbb{V}_0[p_D + p_E]}{\partial F} = 0$.

Suppose $m_D < m_E$. We begin our analysis at F_{eq} — the face value at which $\frac{\Delta D}{m_D} = \frac{\Delta E}{m_E}$ which implies that $\Delta E > \Delta D$. At F_{eq}, the investors' optimization problem implies that $\tau_{i,D} = \tau_{i,E}$ and so $\mathbb{V}_0[q_D] = V_0[q_E]$. As a result,

$$\frac{\partial \mathbb{V}_0[p_D + p_E]}{\partial F} = 2\left[G_L(F) - G_H(F)\right]\left(\mathbb{V}_0[q_D] - Cov(q_D, q_E)\right)\left(\Delta D - \Delta E\right) < 0.$$

On the other hand, consider F such that $\Delta D = \Delta E$. Then the investor's optimization problem implies that $\tau_{i,D} > \tau_{i,E}$, which in turn implies that $\mathbb{V}_0[q_D] > V_0[q_E]$ and

$$\frac{\partial \mathbb{V}_0[p_D + p_E]}{\partial F} = 2\left[G_L(F) - G_H(F)\right]\left[\Delta D(F)\left(\mathbb{V}_0[q_D] - V_0[q_E]\right)\right] > 0$$

It is clear, then, that at the optimal the optimal $\bar{F} > F^* > F_{eq} > \underline{F}$ and that in equilibrium, $\Delta D(F^*) < \Delta E(F^*)$ and $\mathbb{V}_0[q_D] > V_0[q_E]$. As before,

$$\frac{\Delta D(F^*)}{\Delta E(F^*)} = \frac{\mathbb{V}_0[q_E] - Cov(q_D, q_E)}{\mathbb{V}_0[q_D] - Cov(q_D, q_E)} \tag{22}$$

Showing that this is the only solution to the agent's problem uses the same steps as in Proposition 5, and so they are omitted here. Following similar steps yield the counterpart results when $m_E < m_D$. $\square$

Proof of Proposition 6.

Given that $\Delta D + \Delta E = \Delta V$ and adding equations (20), (21), in equilibrium it is always the case that:

$$\left[m_D C'(\tau_{i,D})\sqrt{\tau_{i,D}\psi^D(1+\psi^D)} + (1-m_D)C'(\tau_{i,E})\sqrt{\tau_{i,E}\psi^E(1+\psi^E)}\right] = \frac{\Delta V}{\left[4\pi\sqrt{\left(1 + \frac{1}{\tau_n}\right)}\right]} \tag{23}$$

We want to show under what conditions $\tau_a \equiv \tau_{i,E} + \tau_{i,D}$ is minimized by issuing an information-insensitive security. We rewrite the LHS of the equation above by defining $h(x, y) \equiv \sqrt{x\left(\tau_z + x + y\tau_n\right)\left(1 + \left(\tau_z + x + y\tau_n\right)\right)}$:

$$m_D C'(\tau_{i,D})h(\tau_{i,D}, \tau_a) + (1-m_D)C'(\tau_a - \tau_{i,D})h(\tau_a - \tau_{i,D}, \tau_a) \equiv g(\tau_{i,D}, \tau_a)$$

We have assumed that $C'(\tau_{i,D}), C''(\tau_{i,D}) > 0$. Note that $h(\tau_{i,D}) > 0$ and we can write

$$\frac{\partial h(x,y)}{\partial x} \equiv h'(\tau_{i,D}, \tau_a) = \frac{1}{2}\left[\left(\psi^D\right)\left(1+\psi^D\right)+\tau_{i,D}\left(1+2\left(\psi^D\right)\right)\right]\left(\tau_{i,D}\left(\psi^D\right)\left(1+\psi^D\right)\right)^{-\frac{1}{2}} > 0$$

and

$$\frac{\partial^2 h(x,y)}{\partial x^2} \equiv h''(\tau_{i,D}, \tau_a) = \left[\left(1+2\psi^D+\tau_{i,D}\right)\right]\left(\tau_{i,D}\left(\psi^D\right)\left(1+\psi^D\right)\right)^{-\frac{1}{2}}$$
$$- \frac{1}{4}\left[\left(\psi^D\right)\left(1+\psi^D\right)+\tau_{i,D}\left(1+2\left(\psi^D\right)\right)\right]^2\left(\tau_{i,D}\left(\psi^D\right)\left(1+\psi^D\right)\right)^{-\frac{3}{2}}$$

Then $\frac{\partial^2 g(\tau_{i,D},\tau_a)}{\partial \tau_{i,D}^2} \equiv g''(\tau_{i,D},\tau_a) > 0$ iff:

$$\left[\underbrace{C'''(\tau_a-\tau_{i,D})h(\tau_a-\tau_{i,D},\tau_a)+C''(\tau_a-\tau_{i,D})h'(\tau_a-\tau_{i,D},\tau_a)+C''(\tau_a-\tau_{i,D})h'(\tau_a-\tau_{i,D},\tau_a)+C'(\tau_a-\tau_{i,D})h''(\tau_a-\tau_{i,D},\tau_a)}_{\hat{E}}\right.$$
$$\left.+\frac{m_D}{1-m_D}\underbrace{\left[C'''(\tau_{i,D})h(\tau_{i,D}\tau_a)+C''(\tau_{i,D})h'(\tau_{i,D},\tau_a)+C''(\tau_{i,D})h'(\tau_{i,D},\tau_a)+C'(\tau_{i,D})h''(\tau_{i,D},\tau_a)\right]}_{\hat{D}}\right] > 0 \quad (24)$$

It can be shown that $h''(\tau_{i,D})$ is positive for sufficiently *low* τ_z, τ_n, but the latter is generally required to be sufficiently *high* for earlier Lemmas to hold. On the other hand, if $C''' \geq 0$ and C''/C' is sufficiently large, then $g''(\tau_{i,D}) > 0$.

If $g''(\tau_{i,D},\tau_a) > 0$, then *given* τ_a, it is maximized at either $\tau_{i,D} = \tau_a$ or $\tau_{i,D} = 0$ — the two "corners". If $m_D \geq \frac{1}{2}$ it is the former; if $m_D \leq \frac{1}{2}$ it is the latter. Suppose that $m_D < \frac{1}{2}$ (the argument follows the same structure when $m_D > \frac{1}{2}$). Then $g(0,\tau_a)$ is the maximum value, given τ_a. Let $\underline{\tau_a}$ be defined as the solution to

$$g(0,\underline{\tau_a}) = \frac{\Delta V}{\left[4\pi\sqrt{\left(1+\frac{1}{\tau_n}\right)}\right]}.$$

In this case, $\underline{\tau_a} = \tau_{i,E}$, and $\tau_{i,D} = 0$ satisfy equations (20), (21), that is they are the optimal solution to investors' information acquisition problems when $\Delta D = 0$. Moreover, because $g(0,\underline{\tau_a})$ is the maximum value, for all $\tau_{i,D} \in (0,\tau_a]$, it must be that

$$g(\tau_{i,D},\underline{\tau_a}) < \frac{\Delta V}{\left[4\pi\sqrt{\left(1+\frac{1}{\tau_n}\right)}\right]}.$$

Finally, it is straightforward to see that $\frac{\partial g(\tau_{i,D},\tau_a)}{\partial \tau_a} > 0$. This implies that equation (23) can only be satisfied if $\tau_a > \underline{\tau_a}$, that is, if ΔD, $\Delta E > 0$, then aggregate information must increase.

Note that if $m_D = \frac{1}{2}$, then the optimal precisions are equal: $\tau_{i,D} = \tau_{i,E}$. Moreover, note that if $m_D = \frac{1}{2}$ and $g''(\tau_{i,D},\tau_a) > 0$, then, given τ_a, the minimum value of $g(\tau_{i,D},\tau_a)$ solves:

$$C''(\tau_{i,D})h(\tau_{i,D},\tau_a)+C'(\tau_{i,D})h'(\tau_{i,D},\tau_a) = C''(\tau_a-\tau_{i,D})h(\tau_a-\tau_{i,D},\tau_a)+C'(\tau_a-\tau_{i,D})h'(\tau_a-\tau_{i,D},\tau_a).$$

which of course holds if $\tau_{i,D} = \tau_{i,E}$. As a result, when $\tau_{i,D} = \tau_{i,E}$, then τ_a must be maximized for equation (23) to hold. Finally, consider the case when $C(\tau) = \kappa\tau^a$ with $\kappa > 0$, $a \geq 2$. Then, turning to the term $\hat{D}$ from equation (24):

$$2\kappa(a)(a-1)\tau_{i,D}^{a-2}h'(\tau_{i,D})+\kappa a\tau_{i,D}^{a-1}h''(\tau_{i,D}) > 0 \text{ if}$$
$$2(a-1)h'(\tau_{i,D})+\tau_{i,D}h''(\tau_{i,D}) > 0 \text{ which holds if}$$
$$2h'(\tau_{i,D})+\tau_{i,D}h''(\tau_{i,D}) > 0$$

The last inequality can be written:

$$4\left[\left(\psi^D\right)\left(1+\psi^D\right)+\tau_{i,D}\left(1+2\left(\psi^D\right)\right)+\tau_{i,D}\left[\left(1+2\psi^D+\tau_{i,D}\right)\right]\right]\left(\left(\psi^D\right)\left(1+\psi^D\right)\right) > \left[\left(\psi^D\right)\left(1+\psi^D\right)+\tau_{i,D}\left(1+2\left(\psi^D\right)\right)\right]^2$$

which holds as long as

$$4\left[\tau_{i,D}\left[\left(1+2\psi^D+\tau_{i,D}\right)\right]\right]\left(\left(\psi^D\right)\left(1+\psi^D\right)\right)+3\left[\left(\psi^D\right)\left(1+\psi^D\right)\right]^2+\tau_{i,D}\left(1+2\left(\psi^D\right)\right)\left[2\left(\psi^D-\tau_{i,D}\right)\left(1+\psi^D\right)+\tau_{i,D}\right] > 0,$$

which is always true. Similar logic applies to the corresponding equity market expression, $\hat{E}$ from equation (24). Taken together, this implies that $g''(\tau_{i,D},\tau_a) > 0$. $\qquad\square$

Proof of Proposition 7. It is clear that if $\tau_{i,D} > \tau_{i,E}$, and $\gamma \equiv \tau_{i,D} - \tau_{i,E}$:

$$\mathbb{V}[(\tilde{q_{i,D}} - \tilde{q_D})] > \mathbb{V}[(\tilde{q_{i,E}} - \tilde{q_E})] \iff$$
$$\tau_{iE}\gamma\psi^E(2\tau_n - 1) + \gamma\psi^E[\tau_z + \gamma\tau_n] > \gamma^2(1 - \tau_n) - \gamma(\tau_z + 2\tau_{i,E}\tau_n)$$

When τ_n is sufficiently high, this must hold. In combination with Corollary 1, this implies that when $\frac{\Delta D}{m_D} > \frac{\Delta E}{m_E}$, the expected utility in the two markets cannot be the same. As a result, when investors can freely choose in which market to trade, m_D and m_E will be set in equilibrium such that $\frac{\Delta D}{m_D} = \frac{\Delta E}{m_E}$. $\qquad\square$

Proof of Theorem 1. Consider the agent's problem when both $m_{D,min}$ and $m_{E,min}$ are less than $\frac{1}{2}$. There are three regions to consider:

1. $m_D^*(F) \geq m_{D,min}$ and $m_E^* \geq m_{E,min}$. Then $\tau_{i,D} = \tau_{i,E}$ and is constant in this interval. As a result, by Proposition 4, in this region the optimal F sets $\Delta D = \Delta E$.

2. $m_D^*(F) \leq m_{D,min}$. Then $m_D = m_{D,min} < 1 - m_{E,min} = m_E$. From the proof of Proposition 5, $\frac{\partial \mathbb{V}_0[p_D + p_E]}{\partial F} < 0$ in this interval, and so the optimal F sets $m_D^* = m_{D,min}$. But as this point overlaps with the fixed precision region, this choice of F is suboptimal.

3. $m_E^*(F) \leq m_{E,min}$. Then $m_E = m_{E,min} < 1 - m_{D,min} = m_D$. From the proof of Proposition 5, $\frac{\partial \mathbb{V}_0[p_D + p_E]}{\partial F} > 0$ in this interval, and so the optimal F sets $m_E^* = m_{E,min}$. But this point overlaps with the fixed precision interval, and so this choice of F is suboptimal.

Now consider the agent's problem when $m_{E,min} > \frac{1}{2}$.

1. $m_D^*(F) \geq m_{D,min}$ and $m_E^*(F) \geq m_{E,min}$. The agent would like to choose F such that $\Delta D = \Delta E$, but this isn't feasible. Instead he optimally minimizes $|\Delta D - \Delta E|$, which implies setting $m_E^*(F) = m_{E,min}$.

2. $m_D^*(F) \leq m_{D,min}$. Then the reasoning above holds: the optimal F sets $m_D^* = m_{D,min}$ but this is suboptimal.

3. $m_E^*(F) \leq m_{E,min}$. Then $m_E = m_{E,min} > 1 - m_{D,min} = m_D$. By Proposition 5, the optimal F sets $\mathbb{V}_0[q_E] < \mathbb{V}_0[q_D]$ which requires $m_E^*(F) < m_{E,min}$ and as a result $\Delta E > \Delta D$. This is the optimal level of debt, as the optimal choice of debt in the fixed precision interval overlapped with this interval and so is clearly suboptimal.

Similar steps yield our result when $m_{D,min} > \frac{1}{2}$. $\qquad\square$

Proof of Lemma 5. Using the proof of Lemma 1, substitute $\mu_v = -a$ and $\sigma_v = \sqrt{c}$, and under the assumption that both a and c are constants,
$$\mathbb{E}\left[\Phi\left(\frac{z + a}{\sqrt{c}}\right) \mid z \sim \mathcal{N}(\mu_z, \sigma_z^2)\right] = \Phi\left(\frac{\mu_z + a}{\sqrt{\sigma_z^2 + c}}\right)$$

$\qquad\square$

Verification of Equation (10):

We begin by noting that given z_{T-1}, we can write $s_{E,T-1}$:
$$\mathbb{E}[s_{E,T-1}|z_{T-1}] = (1 - \rho)\mu_z + \rho z_{T-1} \quad \mathbb{V}[s_{E,T-1}|z_{T-1}] = (\tau_z^{-1} + \tau_{E,T-1}^{-1}).$$

This allows us to write:
$$\mathbb{E}[q_{E,T-1}|z_{T-1}, \mathcal{P}] = \Phi\left(\frac{(1 - \rho)\mu_z + \rho z_{T-1}}{\sqrt{1 + \left(\psi_{T-1}^E\right)^{-1} + \left(\frac{\tau_{i,T-1} + \tau_{E,T-1}}{\psi_{T-1}^E}\right)^2 \mathbb{V}[s_{E,T-1}|z_{T-1}]}}\right)$$

$$\left(\psi_{T-1}^E\right)^{-1} + \left(\frac{\tau_{i,T-1} + \tau_{E,T-1}}{\psi_{T-1}^E}\right)^2 \mathbb{V}[s_{E,T-1}|z_{T-1}] = \frac{\psi_{T-1}^E + (\tau_{i,T-1} + \tau_{E,T-1})^2\left(\frac{1}{\tau_z} + \frac{1}{\tau_{E,T-1}}\right)}{\left(\psi_{T-1}^E\right)^2}$$

$$= \frac{1}{\tau_z} + \frac{-\tau_z(\tau_{i,T-1} + \tau_{E,T-1})}{\tau_z\left(\psi_{T-1}^E\right)^2} + \frac{(\tau_{i,T-1} + \tau_{E,T-1})^2\left(\frac{1}{\tau_{E,T-1}}\right)}{\left(\psi_{T-1}^E\right)^2}$$

$$= \frac{1}{\tau_z} + \frac{\tau_{i,T-1}\left(\frac{1 + \tau_n}{\tau_n}\right)}{\left(\psi_{T-1}^E\right)^2} \equiv \tau_z^{-1} + \psi_{T-1}^P$$

Proof of Lemma 6. The price at time-$T - k$ is $p_{E,T-k} = V_L + q_{E,T-k}\Delta V$ where

$$q_{E,T-k} \equiv \Phi \left(\frac{(1-\rho)\mu_z \sum_{j=1}^{k} \rho^{(j-1)} + \rho^k z_{T-k} + \rho^{k-1} \left(\frac{(\tau_{i,T-k}+\tau_{E,T-k})(e_{T-k+1}+u_{T-k}/\sqrt{\tau_{i,T-k}})}{\psi_{T-k}^E} \right)}{\sqrt{1 + \tau_z^{-1} \sum_{j=1}^{k-1} \rho^{2(j-1)} + \sum_{j=1}^{k-1} \rho^{2(j-1)} \psi_{T-j}^P + \rho^{2(k-1)} \left(\psi_{T-k}^E \right)^{-1}}} \right), \tag{25}$$

$$\psi_{T-j}^E \equiv (\tau_z + \tau_{i,T-j} + \tau_{E,T-j}), \qquad \psi_{T-j}^P \equiv \frac{\tau_{i,T-j} \left(\frac{1+\tau_n}{\tau_n} \right)}{\left(\psi_{T-j}^E \right)^2}.$$

Taking expectations of each object yields:

$$\mathbb{E}[q_{E,T-k}|z_{T-k},\mathcal{P}] = \Phi \left(\frac{(1-\rho)\mu_z \sum_{j=1}^{k} \rho^{(j-1)} + \rho^{k-1} z_{T-k}}{\sqrt{1 + \tau_z^{-1} \sum_{j=1}^{k} \rho^{2(j-1)} + \sum_{j=1}^{k} \rho^{2(j-1)} \psi_{T-j}^P}} \right)$$

$$\mathbb{E}[q_{E,T-k+n}|z_{T-k},\mathcal{P}] = \Phi \left(\frac{(1-\rho)\mu_z \sum_{j=1}^{k} \rho^{(j-1)} + \rho^{k-1} z_{T-k}}{\sqrt{1 + \tau_z^{-1} \sum_{j=1}^{k} \rho^{2(j-1)} + \sum_{j=1}^{k-n} \rho^{2(j-1)} \psi_{T-j}^P}} \right)$$

ψ_{T-j}^P is always positive when $\tau_{i,T-j} > 0$. Taken in combination with the fact that Φ is strictly increasing, the proof is completed. $\qquad\square$

Proof of Lemma 7. We use our notation from earlier and define $q_{E,\tilde{T}-k}$ as $q_{E,T-k} \equiv \Phi(q_{E,\tilde{T}-k})$. Then we can write:

$$\mathbb{V}[q_{E,T-k}|z_{T-k},\mathcal{P}] = \phi \left(\mathbb{E}[q_{E,\tilde{T}-k}|z_{T-k},\mathcal{P}] \right)^2 \mathbb{V}[q_{E,\tilde{T}-k}|z_{T-k},\mathcal{P}]$$

$$\phi \left(\mathbb{E}[q_{E,\tilde{T}-k}|z_{T-k},\mathcal{P}] \right)^2 = \frac{1}{2\pi} exp \left(- \frac{\left[(1-\rho)\mu_z \sum_{j=1}^{k} \rho^{(j-1)} + \rho^k z_{T-k} \right]^2}{1 + \tau_z^{-1} \sum_{j=1}^{k-1} \rho^{2(j-1)} + \sum_{j=1}^{k-1} \rho^{2(j-1)} \psi_{T-j}^P + \rho^{2(k-1)} \left(\psi_{T-k}^E \right)^{-1}} \right)$$

$$\mathbb{V}[q_{E,\tilde{T}-k}|z_{T-k},\mathcal{P}] = \frac{\rho^{2(k-1)} \left[\frac{(\tau_{i,T-k}+\tau_{E,T-k})}{\psi_{T-k}^E} \right]^2 \left[\frac{1}{\tau_z} + \frac{1}{\tau_{E,T-k}} \right]}{1 + \tau_z^{-1} \sum_{j=1}^{k-1} \rho^{2(j-1)} + \sum_{j=1}^{k-1} \rho^{2(j-1)} \psi_{T-j}^P + \rho^{2(k-1)} \left(\psi_{T-k}^E \right)^{-1}}$$

Similarly, we can write the agent's uncertainty about prices $n \le k-1$ periods ahead:

$$\mathbb{V}[q_{E,T-k+n}|z_{T-k},\mathcal{P}] = \phi \left(\mathbb{E}[q_{E,\tilde{T-k+n}}|z_{T-k},\mathcal{P}] \right)^2 \mathbb{V}[q_{E,\tilde{T-k+n}}|z_{T-k},\mathcal{P}]$$

$$\phi \left(\mathbb{E}[q_{E,\tilde{T-k+n}}|z_{T-k},\mathcal{P}] \right)^2 = \frac{1}{2\pi} exp \left(- \frac{\left[(1-\rho)\mu_z \sum_{j=1}^{k} \rho^{(j-1)} + \rho^k z_{T-k} \right]^2}{1 + \tau_z^{-1} \sum_{j=1}^{k-n-1} \rho^{2(j-1)} + \sum_{j=1}^{k-n-1} \rho^{2(j-1)} \psi_{T-j}^P + \rho^{2(k-n-1)} \left(\psi_{T-k+n}^E \right)^{-1}} \right)$$

$$\mathbb{V}[q_{E,\tilde{T-k+n}}|z_{T-k},\mathcal{P}] = \frac{\frac{1}{\tau_z} \sum_{j=1}^{n} \rho^{2(k-j)} + \rho^{2(k-n-1)} \left[\frac{(\tau_{i,T-k+n}+\tau_{E,T-k+n})}{\psi_{T-k+n}^E} \right]^2 \left[\frac{1}{\tau_z} + \frac{1}{\tau_{E,T-k+n}} \right]}{1 + \tau_z^{-1} \sum_{j=1}^{k-n-1} \rho^{2(j-1)} + \sum_{j=1}^{k-n-1} \rho^{2(j-1)} \psi_{T-j}^P + \rho^{2(k-n-1)} \left(\psi_{T-k+n}^E \right)^{-1}}$$

For all n, the denominator of $\mathbb{V}[q_{E,\tilde{T}-k}|z_{T-k},\mathcal{P}]$ exceeds the denominator of $\mathbb{V}[q_{E,\tilde{T-k+n}}|z_{T-k},\mathcal{P}]$ — the former always contains the latter. It is therefore sufficient to show that the numerator of $\mathbb{V}[q_{E,\tilde{T-k+n}}|z_{T-k},\mathcal{P}]$ exceeds the numerator $\mathbb{V}[q_{E,\tilde{T}-k}|z_{T-k},\mathcal{P}]$ to show that $\mathbb{V}[q_{E,\tilde{T-k+n}}|z_{T-k},\mathcal{P}] > \mathbb{V}[q_{E,\tilde{T}-k}|z_{T-k},\mathcal{P}]$. This is true if

$$\left[\frac{(\tau_{i,T-k}+\tau_{E,T-k})}{\psi_{T-k}^E} \right]^2 \left[\frac{1}{\tau_z} + \frac{1}{\tau_{E,T-k}} \right] < \frac{1}{\tau_z}$$

$$\tau_{i,T-k} \tau_z \left(1 - \tau_n^2 \right) < \tau_n \tau_z^2$$

This inequality holds for sufficiently large τ_z, τ_n.

On the other hand, as long as $\left[(1-\rho)\mu_z \sum_{j=1}^{k} \rho^{(j-1)} + \rho^k z_{T-k} \right]^2 \ne 0$, then $\phi \left(\mathbb{E}[q_{E,\tilde{T}-k}] \right)^2 < \phi \left(\mathbb{E}[q_{E,\tilde{T-k+n}}] \right)^2$. Thus, if $|(1-\rho)\mu_z \sum_{j=1}^{k} \rho^{(j-1)} + \rho^k z_{T-k}|$ is sufficiently small, $\mathbb{V}[q_{E,T-k+n}|z_{T-k},\mathcal{P}] > \mathbb{V}[q_{E,T-k}|z_{T-k},\mathcal{P}]$. $\qquad\square$

Proof of Theorem 2. Suppose the random variable w can be written as $w = c(a + bx)$, where a and b are random variables and c is a constant. If the objective is $U \equiv \mathbb{E}[w] - \frac{\gamma}{2}\mathbb{V}[w]$, and the choice variable is x, then it is straightforward to see that

$$\frac{\partial U}{\partial x} = c\mathbb{E}[b] - \gamma c^2 \left[Cov(a,b) + x\mathbb{V}[b] \right]$$

which implies that there is a unique critical point when

$$x^* = \frac{\frac{1}{c\gamma}\mathbb{E}[b] - Cov(a,b)}{\mathbb{V}[b]} \tag{26}$$

Moreover, U is maximized at x^*, as $\frac{\partial^2 U}{\partial x^2} = -\gamma c^2 \mathbb{V}[b]$, which is always less than zero. The future proceedings at each point in time can be written as a linear function of the agent's current retention decision, and so the optimal fraction retained can be written as in (26). We will prove the statement of the theorem by induction. We begin by showing that the recursion holds from $T-1$ to $T-2$:

$$W_{T-1,T} = \left[\prod_{k=0}^{T-2} \alpha_k\right] [V_L + \Delta V (q_{E,T-1} + \alpha_{T-1} (q_{E,T} - q_{E,T-1}))]$$

As a result, we can write

$$\alpha_{T-1}(z_{T-1}, \prod_{k=0}^{T-2} \alpha_k) = \frac{\frac{\mathbb{E}[q_{E,T}-q_{E,T-1}|z_{T-1}]}{\gamma\Delta V \prod_{k=0}^{T-2}\alpha_k} - Cov(q_{E,T} - q_{E,T-1}, q_{E,T-1}|z_{T-1})}{\mathbb{V}[q_{E,T} - q_{E,T-1}|z_{T-1}]}$$

If we define w_{T-1} as the retention decision which minimizes U at $T-1$, then we can rewrite

$$\alpha_{T-1}(z_{T-1}, \prod_{k=0}^{T-2} \alpha_k) = \frac{\frac{\mathbb{E}[q_{E,T}-q_{E,T-1}|z_{T-1}]}{(\gamma\Delta V) \prod_{k=0}^{T-2}\alpha_k}}{\mathbb{V}[q_{E,T} - q_{E,T-1}|z_{T-1}]} + w_{T-1}.$$

$$\equiv \frac{\hat{\chi_{T-1}}}{(\gamma\Delta V) \prod_{k=0}^{T-2} \alpha_k} + w_{T-1}$$

We can substitute this optimal policy into the agent's future proceeds at $T-2$, which are now linear in α_{T-2}:

$$W_{T-2,T} = \left[\prod_{k=0}^{T-3} \alpha_k\right] \left[V_L + \Delta V \left(q_{E,T-2} + \frac{\chi_{T-1}}{(\gamma\Delta V) \prod_{k=0}^{T-3}\alpha_k} + \alpha_{T-2} \left(q_{E,T-1}^P - q_{E,T-2}\right)\right)\right]$$

$$q_{E,T-k}^P \equiv q_{E,T-k} + w_{T-k} \left(q_{E,T-k+1}^P - q_{E,T-k}\right)$$

$$\chi_{T-k} \equiv \hat{\chi_{T-k}} \left(q_{E,T-k+1}^P - q_{E,T-k}\right)$$

To complete the proof, we will conjecture that we can write:

$$W_{T-k,T} = \left[\prod_{k=0}^{T-k-1} \alpha_k\right] \left[V_L + \Delta V \left(q_{E,T-k} + \frac{\sum_{j=1}^{k-1} \chi_{T-k+j}}{(\gamma\Delta V) \prod_{k=0}^{T-k-1}\alpha_k} + \alpha_{T-k} \left(q_{E,T-k+1}^P - q_{E,T-k}\right)\right)\right]$$

This implies that the optimal retention decision can be written:

$$\alpha_{T-k}(z_{T-k}, \prod_{k=0}^{T-k-1} \alpha_k) = \frac{\frac{\mathbb{E}[q_{E,T-k+1}^P - q_{E,T-k}|z_{T-k}]}{\gamma\Delta V \prod_{k=0}^{T-k-1}\alpha_k} - Cov(q_{E,T-k+1}^P - q_{E,T-k}, q_{E,T-k} + \frac{\sum_{j=1}^{k-1}\chi_{T-k+j}}{(\gamma\Delta V)\prod_{k=0}^{T-k-1}\alpha_k}|z_{T-k})}{\mathbb{V}[q_{E,T} - q_{E,T-1}|z_{T-1}]}$$

$$= \frac{\frac{\mathbb{E}[q_{E,T-k+1}^P - q_{E,T-k}|z_{T-k}] - Cov(q_{E,T-k+1}^P - q_{E,T-k}, \sum_{j=1}^{k-1}\chi_{T-k+j}|z_{T-k})}{\gamma\Delta V \prod_{k=0}^{T-k-1}\alpha_k}}{\mathbb{V}[q_{E,T} - q_{E,T-1}|z_{T-1}]} + w_{T-k}$$

$$= \frac{\hat{\chi_{T-k}}}{\gamma\Delta V \prod_{k=0}^{T-k-1} \alpha_k} + w_{T-k}$$

where w_{T-k} is the minimum-variance retention decision *ignoring* any future market-timing activity. Substituting this into the agent's future proceeds at $T-k-1$:

$$W_{T-k,T} = \left[\prod_{k=0}^{T-k-2} \alpha_k\right] \left[V_L + \Delta V \left(\frac{\sum_{j=1}^{k} \chi_{T-k+j}}{(\gamma\Delta V) \prod_{k=0}^{T-k-1}\alpha_k} + q_{E,T-k}^P\right)\right] \implies$$

$$W_{T-k-1,T} = \left[\prod_{k=0}^{T-k-2} \alpha_k\right] \left[V_L + \Delta V \left(q_{E,T-k-1} + \frac{\sum_{j=1}^{k} \chi_{T-k+j}}{(\gamma\Delta V) \prod_{k=0}^{T-k-2}\alpha_k} + \alpha_{T-k-1} \left(q_{E,T-k}^P - q_{E,T-k-1}\right)\right)\right]$$

which is of the conjectured form, completing the proof. $\qquad\square$

Proof of Proposition 8.

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First, we examine whether or not the agent wants to contribute cash when financing via debt:

$$\frac{\partial \mathbb{V}[P|Debt]}{\partial c} = \left[\frac{\partial \Delta D(F(I,c))}{\partial c}\right]\left[\frac{\Delta D(F(I,c))}{2}\right]\left[\mathbb{V}[2q_{D,T-k}] - Cov(2q_{D,T-k}, q_{E,T-k+j} + q_{D,T-k+j})\right]$$
$$- \left[\frac{\partial \Delta D(F(I,c))}{\partial c}\right]\left[\frac{\Delta V - \Delta D(F(I,c))}{2}\right]\left[\mathbb{V}[q_{E,T-k+j} + q_{D,T-k+j}] - Cov(2q_{D,T-k}, q_{E,T-k+j} + q_{D,T-k+j})\right]$$

Note that because the face value of debt required falls with c, $\frac{\partial \Delta D(F(I,c))}{\partial c} < 0$. There are two cases to consider: (1)$\mathbb{V}[q_{E,T-k+j} + q_{D,T-k+j}] > 2\mathbb{V}[q_{D,T-k}] > Cov(q_{D,T-k}, q_{E,T-k+j} + q_{D,T-k+j})$ and (2) $\mathbb{V}[q_{E,T-k+j} + q_{D,T-k+j}] > Cov(q_{D,T-k}, q_{E,T-k+j} + q_{D,T-k+j}) > 2\mathbb{V}[q_{D,T-k}]$. As a result, $\frac{\partial \mathbb{V}[P|Debt]}{\partial c} < 0$ only when $\Delta D(F(I,c))$ is sufficiently large, relative to $\Delta V - \Delta D(F(I,c))$, which implies that there is a threshold investment level below which $c = 0$. Let $I_D \equiv inf\{I : \frac{\partial \mathbb{V}[P|Debt]}{\partial c} \geq 0$ when $c = 0\}$. Note that in case (2), $\frac{\partial \mathbb{V}[P|Debt]}{\partial c} > 0$ for all c and so cash is never used.

We can do the same when the agent is financing via equity:

$$\frac{\partial \mathbb{V}[P|Equity]}{\partial c} = \Delta V^2 \frac{\partial \alpha(I,c)}{\partial c}[\alpha(I,c)]\left[\mathbb{V}[q_{E,T-k} + \hat{q_J}] - Cov(q_{E,T-k} + \hat{q_J}, q_{E,T-k+j} + q_{D,T-k+j})\right]$$
$$- \Delta V^2 \frac{\partial \alpha(I,c)}{\partial c}[1 - \alpha(I,c)]\left[\mathbb{V}[q_{E,T-k+j} + q_{D,T-k+j}] - Cov(q_{E,T-k} + \hat{q_J}, q_{E,T-k+j} + q_{D,T-k+j})\right]$$

We can use similar reasoning as above. The fraction of equity sold falls with c, which implies$\frac{\partial \alpha(I,c)}{\partial c} < 0$. There are two cases to consider: (1) $\mathbb{V}[q_{E,T-k+j} + q_{D,T-k+j}] > \mathbb{V}[q_{E,T-k} + \hat{q_J}] > Cov(q_{E,T-k} + \hat{q_J}, q_{E,T-k+j} + q_{D,T-k+j})$ and (2) $\mathbb{V}[q_{E,T-k+j} + q_{D,T-k+j}] > Cov(q_{E,T-k} + \hat{q_J}, q_{E,T-k+j} + q_{D,T-k+j}) > \mathbb{V}[q_{E,T-k} + \hat{q_J}]$.As a result $\frac{\partial \mathbb{V}[P|Equity]}{\partial c} < 0$ only when $\alpha(I,c)$ is sufficiently large, which implies that there is a threshold investment level below which $c = 0$. Let $I_E \equiv inf\{I : \frac{\partial \mathbb{V}[P|Equity]}{\partial c} \geq 0$ when $c = 0\}$. Note that in case (2), $\frac{\partial \mathbb{V}[P|Equity]}{\partial c} > 0$ for all c and so cash is never used.

For low levels of I, the agent finances the entire investment using debt or equity. Suppose I is low enough so that $\Delta D(F(I,0)) = 0$. Then the agent's proceeds if he finances via debt is simply $\left[\frac{\Delta V}{2}\right]^2 \mathbb{V}[q_{E,T-k+j} + q_{D,T-k+j}]$. On the other hand, $\alpha(I,0) > 0$ and $\mathbb{V}[q_{E,T-k} + \hat{q_J}] < \mathbb{V}[q_{E,T-k+j} + q_{D,T-k+j}]$. Equity financing is clearly preferable; moreover, as both functions are continuous (i.e., $\mathbb{V}[P|Debt]$ and $\mathbb{V}[P|Equity]$) and monotonic (both are increasing until $I \geq I_D$, $I \geq I_E$, respectively), there exists some $\underline{I}$ such that $\Delta D(F(\underline{I},0)) > 0$ and equity remains preferable.

In case (1), when $I \geq I_D$ ($I \geq I_E$), , $\frac{\partial \mathbb{V}[P|Debt]}{\partial c} = 0$ ($\frac{\partial \mathbb{V}[P|Equity]}{\partial c} = 0$), and the variance is constant, given the choice of financing – the agent uses cash to preserve the optimal smoothing across time. Moreover, the variance at this point $\{I, c(I)\}$is the minimum variance in the financing problem without investment; the agent has optimally smoothed his issuance across both time periods. That is, the variance given debt issuance is the

$$min_\delta \quad \mathbb{V}[\delta(q_{E,T-k+j} + q_{D,T-k+j}) + (1-\delta)(2q_{D,T-k})],$$

whereas, in the case of equity issuance, the variance when $c \geq 0$ can be written

$$min_\delta \quad \mathbb{V}[\delta(q_{E,T-k+j} + q_{D,T-k+j}) + (1-\delta)(q_{E,T-k} + \hat{q_J})]$$

Because $\mathbb{V}[q_{E,T-k} + \hat{q_J}] > \mathbb{V}[2q_{D,T-k}]$, and $Cov(q_{E,T-k} + \hat{q_J}, q_{E,T-k+j} + q_J) > Cov(q_{D,T-k}, q_{E,T-k+j} + q_{D,T-k+j})$, it is clear that when δ is optimally chosen, the variance of the latter exceeds the former. As a result, there exists a threshold, $\bar{I}$, beyond which financing via debt is optimal.

In case (2), when cash is never utilized, as $I \to \infty$, both $\Delta D, \Delta E \to \Delta V$. Moreover,

$$\mathbb{V}[P|Debt] \to \left[\frac{\Delta V}{2}\right]^2 \mathbb{V}[2q_{D,T-k}] \quad \mathbb{V}[P|Equity] \to \left[\frac{\Delta V}{2}\right]^2 \mathbb{V}[q_{E,T-k} + \hat{q_J}]$$

Due to the differnce in information-driven uncertainty, when I is sufficiently high, $\mathbb{V}[P|Debt] < \mathbb{V}[P|Equity]$. Due to the continuity and monotonicity of both functions, there exists a threshold beyond which debt is preferable when no cash is utilized. $\qquad \square$

Proof of Proposition 9. We begin by verifying the first-best policy for the agent, given $q_{D,T-k}$. The agent's expected utility can be written:

$$U(y, \hat{y}, F) \equiv D_L(\hat{y}, F) + q_{D,T-k}\Delta D(\hat{y}, F) + E_L(y, F) + \mathbb{E}[q_{E,T}|q_{D,T-k}]\Delta E(y, F) - \frac{\gamma}{2}\mathbb{V}[q_{E,T}|q_{D,T-k}](\Delta E(y, F))^2$$

which implies that

$$\frac{\partial U(y, \hat{y}F)}{\partial F} = (1 - G_L(F - \hat{y})) + q_{D,T-k}(G_L(F - \hat{y}) - G_H(F - \hat{y})) + (G_L(F - y) - 1)$$
$$+ \mathbb{E}[q_{E,T}|q_{D,T-k}](G_H(F) - G_L(F - y)) - \gamma\mathbb{V}[q_{E,T}|q_{D,T-k}]\Delta E(y, F)(G_H(F) - G_L(F - y))$$

Under perfect information, investors know the agent's type, i.e., $\hat{y} = y$:

$$\frac{\partial U(y, \hat{y} = y, F)}{\partial F} = [G_L(F - y) - G_H(F)][q_{D,T-k} - \mathbb{E}[q_{E,T}|q_{D,T-k}] + \gamma\mathbb{V}[q_{E,T}|q_{D,T-k}]\Delta E(y, F)].$$

If $q_{D,T-k} \geq \mathbb{E}[q_{E,T}|q_{D,T-k}]$, then $\frac{\partial U(y,F)}{\partial F}$ is always positive, and so $F^* = \bar{F}$ for all y. On the other hand, if $q_{D,T-k} < \mathbb{E}[q_{E,T}|q_{D,T-k}]$, then the agent optimally sets $\frac{\partial U(y,y,F)}{\partial F} = 0$ as shown in equation (14). As y increases, the agent wants to hold $\Delta E(y,F)$ constant, which requires:

$$\frac{\partial \Delta E(y,F)}{\partial y} + \frac{\partial \Delta E(y,F)}{\partial F}\frac{\partial F}{\partial y} = 0 \quad \implies$$
$$\frac{\partial F}{\partial y} = \frac{1 - G_L(F-y)}{G_H(F) - G_L(F-y)} < 0$$

Next, we establish conditions for differentiability using Mailath and von Thadden (2013). $\mathcal{Y}$, $\mathcal{F} \equiv [0, \bar{F}]$ if $\bar{F} < \infty$, $\mathcal{F} \equiv [0, \bar{F})$ if $\bar{F} = \infty$ are intervals on $\mathbb{R}$. The first-best contracting problem has a unique solution in both cases. The utility function is concave along the first-best path:

$$\frac{\partial^2 U(y, \hat{y} = y, F)}{\partial F^2} = -[G_L(F-y) - G_H(F)]^2 \left[\gamma \mathbb{V}[q_{E,T}|q_{D,T-k}]\right]$$
$$+ [q_{D,T-k} - \mathbb{E}[q_{E,T}|q_{D,T-k}] + \gamma \mathbb{V}[q_{E,T}|q_{D,T-k}]\Delta E(y,F)] [g_L(F-y) - g_H(F)]$$

If $q_{D,T-k} < \mathbb{E}[q_{E,T}|q_{D,T-k}]$, then along the first-best path $\frac{\partial U}{\partial F} = 0$ and so this simplifies to
$$\frac{\partial^2 U(y,y,F(y))}{\partial F^2} = -[G_L(F-y) - G_H(F)]^2 \left[\gamma \mathbb{V}[q_{E,T}|q_{D,T-k}]\right] < 0.$$

Consider now the case when $q_{D,T-k} > \mathbb{E}[q_{E,T}|q_{D,T-k}]$, If $\bar{F} = \infty$, then because in the limit as $F \to \infty$, $g_L(F-y) < g_H(F)$, then $\frac{\partial^2 U(y,y,F(y))}{\partial F^2} < 0$ in the limit as well. Alternatively, when $\bar{F}$ is finite, by FOSD, it must be that $g_L(\bar{F}-y) < g_H(\bar{F})$. Moreover, in both cases, if $\frac{\partial^2 U(y,y,F(y))}{\partial F^2} \geq 0$, then it must be that

$$\left[q_{D,T-k} - \mathbb{E}[q_{E,T}|q_{D,T-k}] + \frac{\gamma}{2}\mathbb{V}[q_{E,T}|q_{D,T-k}]\Delta E(y,F)\right] \neq 0$$
$$\text{and } \exists k \ s.t. \ |\frac{\partial U(y,y,F)}{\partial F}| > k > 0$$

Finally, we can apply **Theorem 3.5** of Mailath and von Thadden (2013):

$$\frac{\partial^2 U(y, \hat{y} = y, F(y))}{\partial F \partial y} = -g_L(F-y)\left[1 - \mathbb{E}[q_{E,T}|q_{D,T-k}] + \mathbb{V}[q_{E,T}|q_{D,T-k}]\Delta E(y,F)\right]$$
$$+ (G_H(F) - G_L(F-y))(1 - G_L(F-y))\gamma \mathbb{V}[q_{E,T}|q_{D,T-k}]$$
$$\implies \frac{\partial^2 U(y, \hat{y} = y, F(y))}{\partial F \partial y} < 0$$

$$\frac{\partial U(y, \hat{y}, F)}{\partial \hat{y}} = G_L(F - \hat{y})(1 - q_{D,T-k}) > 0$$

and $\frac{\partial^2 U(y, \hat{y}=y, F(y))}{\partial F \partial \hat{y}} = 0$, which implies that the separating equilibrium follows
$$\frac{\partial f(y)}{\partial y} = \frac{-[1 - G_L(f(y) - y)][1 - q_D]}{[G_L(f(y) - y) - G_H(f(y))][(q_D - \mathbb{E}[q_{E,T}|q_{D,T-k}]) + \gamma \Delta E(y, F^{FB})\mathbb{V}[q_{E,T}|z_{T-k}]]} \tag{27}$$
$\square$

Proof of Proposition 10. We begin by verifying the first-best policy for the agent. When $\mu_z = z_{T-k} = 0$, $\mathbb{E}[q_{D,T-k}|z_{T-k}] = \mathbb{E}[q_{E,T}|z_{T-k}]$, and so the agent maximizes:
$$U(y, \hat{y}, F) \equiv -\frac{\gamma}{2}\left[\mathbb{V}[q_{E,T}|z_{T-k}]\Delta E(y,F)^2 + \mathbb{V}[q_{D,T-k}|z_{T-k}]\Delta D(\hat{y},F)^2 + 2\Delta E(y,F)\Delta D(\hat{y},F)Cov(q_{E,T}, q_{D,T-k}|z_{T-k})\right]$$
which implies that

$$\frac{\partial U(y, \hat{y}, F)}{\partial F} = -\gamma(G_L(F - \hat{y}) - G_H(F))[\Delta D(\hat{y},F)\mathbb{V}[q_{D,T-k}|z_{T-k}] + \Delta E(y,F)Cov(q_{E,T}, q_{D,T-k}|z_{T-k})]$$
$$-\gamma(G_H(F) - G_L(F - y))[\Delta E(y,F)\mathbb{V}[q_{E,T}|z_{T-k}] + \Delta D(\hat{y},F)Cov(q_{E,T}, q_{D,T-k}|z_{T-k})]$$

Under perfect information, investors know the agent's type, i.e., $\hat{y} = y$, and can be written:

$$\frac{\partial U(y, \hat{y}=y, F)}{\partial F} = -\gamma[G_L(F - y) - G_H(F)]\left[\Delta D(y,F)\left(\mathbb{V}[q_{D,T-k}|z_{T-k}] - Cov(q_{E,T}, q_{D,T-k}|z_{T-k})\right)\right]$$
$$+ \gamma[G_L(F - y) - G_H(F)]\left[\Delta E(y,F)\left(\mathbb{V}[q_{E,T}|z_{T-k}] - Cov(q_{E,T}, q_{D,T-k}|z_{T-k})\right)\right]$$

We know that $\mathbb{V}[q_{E,T}|z_{T-k}] > \mathbb{V}[q_{D,T-k}|z_{T-k}]$. As a result, when $\mathbb{V}[q_{D,T-k}|z_{T-k}] < Cov(q_{E,T}, q_{D,T-k}|z_{T-k})$, then $\frac{\partial U(y,F)}{\partial F}$ is always positive, and so $F^* = \bar{F}$ for all y. On the other hand, if $\mathbb{V}[q_{D,T-k}|z_{T-k}] < Cov(q_{E,T}, q_{D,T-k}|z_{T-k})$, then the agent optimally sets $\frac{\partial U(y,y,F)}{\partial F} = 0$ as shown in equation (16). As y increases, the agent wants to hold the ratio $\frac{\Delta E(y,F)}{\Delta D(y,F)}$ constant, which requires:

$$\Delta D(y,F)\left[\frac{\partial \Delta E(y,F)}{\partial y} + \frac{\partial \Delta E(y,F)}{\partial F}\frac{\partial F}{\partial y}\right] - \Delta E(y,F)\left[\frac{\partial \Delta D(y,F)}{\partial y} + \frac{\partial \Delta D(y,F)}{\partial F}\frac{\partial F}{\partial y}\right] = 0$$

$$\frac{\partial F}{\partial y} = \frac{\frac{\Delta D(y,F)}{\Delta V(y)} - G_L(F-y)}{G_H(F) - G_L(F-y)}$$

Next, we establish conditions for differentiability using Mailath and von Thadden (2013). $\mathcal{Y}$, $\mathcal{F} \equiv [0, \bar{F}]$ if $\bar{F} < \infty$, $\mathcal{F} \equiv [0, \bar{F})$ if $\bar{F} = \infty$ are intervals on $\mathbb{R}$. The first-best contracting problem has a unique solution in both cases. The utility function is concave along the first-best path:

$$\frac{\partial^2 U(y, \hat{y} = y, F)}{\partial F^2} = -\gamma [G_L(F-y) - G_H(F)]^2 \left[\mathbb{V}[q_{D,T-k}|z_{T-k}] + \mathbb{V}[q_{E,T}|z_{T-k}] - 2Cov(q_{E,T}, q_{D,T-k}|z_{T-k})\right]$$
$$- [g_L(F-y) - g_H(F)]\gamma\left[\Delta D(\tilde{y}, F)\left(\mathbb{V}[q_{D,T-k}|z_{T-k}] - Cov(q_{E,T}, q_{D,T-k}|z_{T-k})\right)\right]$$
$$+ [g_L(F-y) - g_H(F)]\gamma\left[\Delta E(y, F)\left(\mathbb{V}[q_{E,T}|z_{T-k}] - Cov(q_{E,T}, q_{D,T-k}|z_{T-k})\right)\right]$$

If $\mathbb{V}[q_{D,T-k}|z_{T-k}] > Cov(q_{E,T}, q_{D,T-k}|z_{T-k})$, then along the first-best path $\frac{\partial U}{\partial F} = 0$ and so this simplifies to
$$\frac{\partial^2 U(y, y, F(y))}{\partial F^2} = -\gamma [G_L(F-y) - G_H(F)]^2 \left[\mathbb{V}[q_{D,T-k}|z_{T-k}] + \mathbb{V}[q_{E,T}|z_{T-k}] - 2Cov(q_{E,T}, q_{D,T-k}|z_{T-k})\right] < 0.$$

Consider now the case when $\mathbb{V}[q_{D,T-k}|z_{T-k}] < Cov(q_{E,T}, q_{D,T-k}|z_{T-k})$, If $\bar{F} = \infty$, then because in the limit as $F \to \infty$, $g_L(F-y) < g_H(F)$, then $\frac{\partial^2 U(y,y,F(y))}{\partial F^2} < 0$ in the limit as well. Alternatively, when $\bar{F}$ is finite, by FOSD, it must be that $g_L(\bar{F} - y) < g_H(\bar{F})$.

Moreover, in both cases, if $\frac{\partial^2 U(y,y,F(y))}{\partial F^2} \geq 0$, then it must be that

$$[\Delta D(y, F)\left(\mathbb{V}[q_{D,T-k}|z_{T-k}] - Cov(q_{E,T}, q_{D,T-k}|z_{T-k})\right) - \Delta E(y, F)\left(\mathbb{V}[q_{E,T}|z_{T-k}] - Cov(q_{E,T}, q_{D,T-k}|z_{T-k})\right)] \neq 0$$

$$\text{and } \exists k \; s.t. \; |\frac{\partial U(y,y,F)}{\partial F}| > k > 0$$

Finally, we can apply **Theorem 3.5** of Mailath and von Thadden (2013):

$$\frac{\partial^2 U(y, \hat{y} = y, F)}{\partial F \partial y} = -\gamma \left(G_L(F-y) - G_H(F)\right)\left(1 - G_L(F-y)\right)\left[\mathbb{V}[q_{E,T}|z_{T-k}] - Cov(q_{E,T}, q_{D,T-k}|z_{T-k})\right]$$
$$- \gamma \left(g_L(F-y)\right)\left[\Delta E(y, F)\mathbb{V}[q_{E,T}|z_{T-k}] + \Delta D(\hat{y}, F)Cov(q_{E,T}, q_{D,T-k}|z_{T-k})\right] < 0$$

$$\frac{\partial U(y, \hat{y}, F)}{\partial \hat{y}} = \gamma \left(G_L(F-\hat{y})\right)\left[\mathbb{V}[q_{D,T-k}|z_{T-k}]\Delta D(\hat{y}, F) + \Delta E(y, F)Cov(q_{E,T}, q_{D,T-k}|z_{T-k})\right] > 0$$

$$\frac{\partial^2 U(y, \hat{y}, F)}{\partial \hat{y} \partial y} = -\gamma \left(G_L(F-\hat{y})\right)Cov(q_{E,T}, q_{D,T-k}|z_{T-k})\left(1 - G_L(F-y)\right) < 0$$

Taken together, this implies that the separating equilibrium follows

$$\frac{\partial f(y)}{\partial y} = \frac{\frac{-G_L(f(y)-y)}{[G_L(f(y)-y)-G_H(f(y))]}\left[\mathbb{E}[q_{D,T-k}|z_{T-k}] + \gamma\left(\mathbb{V}[q_{D,T-k}|z_{T-k}]\Delta D(y, f(y)) + Cov(q_{D,T-k}, q_{E,T}|z_{T-k})\Delta E(y, f(y))\right)\right]}{\left[\Delta E(y, f(y))\left(\mathbb{V}[q_{E,T}|z_{T-k}] - Cov(q_{D,T-k}, q_{E,T}|z_{T-k})\right) - \Delta D(y, f(y))\left(\mathbb{V}[q_{D,T-k}|z_{T-k}] - Cov(q_{D,T-k}, q_{E,T}|z_{T-k})\right)\right]}$$
(28)
□

Proof of Proposition 11. We can write the variance of $\tilde{q_D}$ when investors are unable to observe the price of equity in terms of τ^+, the difference in the two precisions:

$$\mathbb{V}_0^{EL}[\tilde{q_D}] = \frac{\frac{1}{\tau_z}(\tau_{i,D}(1+\tau_n))^2 + \tau_{i,D}\left(2 + \frac{1}{\tau_n} + \tau_n\right) + \tau^+(1+\tau_n)^2\left[\frac{1}{\tau_n} + \frac{\tau^+ + 2\tau_{i,D}}{\tau_z}\right]}{(\tau_z + (\tau_{i,D} + \tau^+)(1+\tau_n))(1 + \tau_z + (\tau_{i,D} + \tau^+)(1+\tau_n))}$$

In Proposition 3, we showed that $\frac{\partial \mathbb{V}_0[\tilde{q_D}]}{\partial \tau_{i,D}} > 0$, which implies that if we can find a γ such that $\mathbb{V}_0[\tilde{q_D}]$ is higher without cross-market learning, it will also be higher for any γ above this threshold.

$$\mathbb{V}_0^{CM}[\tilde{q_D}] = \frac{\frac{1}{\tau_z}\left(\tau_{i,D}(1+\tau_n)\right)^2 + \left(\frac{\tau_{i,D}}{\tau_n} + 2\tau_{i,D} + \tau_D\right) + \tau_E\left[\frac{2\tau_{i,D}(1+\tau_n)+\tau_E}{\tau_z} + 1\right]}{\left(1 + \tau_z + \tau_{i.D}(1+\tau_n) + \tau_E\right)\left(\tau_z + \tau_{i.D}(1+\tau_n) + \tau_E\right)}$$

If $\tau^+ = \tau_{i,E}\frac{\tau_n}{(1+\tau_n)}$, then the denominator in both expressions is the same. As a result, if $\tau^+ = \tau_{i,E}\frac{\tau_n}{(1+\tau_n)}$ (and using the observation that $\tau_{i,E} = \tau_{i,D}$ in equilibrium), $\mathbb{V}_0^{EL}[\tilde{q_D}] > \mathbb{V}_0^{CM}[\tilde{q_D}]$ if

$$\tau_{i,D}\tau_n\left[\frac{2\tau_{i,D}(1+\tau_n) + \tau_{i,D}\tau_n}{\tau_z} + 1\right] < \tau_{i,D}\frac{\tau_n}{(1+\tau_n)}(1+\tau_n)^2\left[\frac{1}{\tau_n} + \frac{\tau^+ + 2\tau_{i,D}}{\tau_z}\right]$$

$$\tau_{i,D}\tau_n < \tau_{i,D}(1+\tau_n)$$

As this always holds, it is clear that τ^+ is a sufficient lower bound, not a necessary one. Note that the same threshold holds for both the debt and equity markets (which are symmetric in this equilibrium).

We can write the covariance of $\tilde{q_D}$, $\tilde{q_E}$ in the absence of cross-market learning, with the knowledge that in both cases, investors in each market will obtain signals of the same precision:

$$Cov^{EL}(\tilde{q_D}, \tilde{q_E}) = \frac{\frac{1}{\tau_z}\left(\tau_i(1+\tau_n)\right)^2 + \frac{\tau^+(1+\tau_n)^2}{\tau_z}\left(2\tau_i + \tau^+\right)}{\left(\tau_z + (\tau_i + \tau^+)(1+\tau_n)\right)\left(1 + \tau_z + (\tau_i + \tau^+)(1+\tau_n)\right)}$$

Note that, as before, we can equate the denominators if $\tau^+ = \tau_i\frac{\tau_n}{1+\tau_n}$,

$$Cov^{CM}(\tilde{q_D}, \tilde{q_E}) = \frac{\frac{1}{\tau_z}\left(\tau_i(1+\tau_n)\right)^2 + \frac{\tau_i\tau_n}{\tau_z}\left(2\tau_i(1+\tau_n) + \tau_i\tau_n\right) + 2\tau_i(1+\tau_n)}{\psi(1+\psi)},$$

and so after substitution, we can compare the numerators:

$$\tau_i\tau_n\left(2\tau_i(1+\tau_n) + \tau_i\tau_n\right) < \tau_i\tau_n\left(2\tau_i(1+\tau_n) + \tau_i\tau_n\right) + 2\tau_i(1+\tau_n)\tau_z$$

At this level of τ^+, the covariance *with* cross-market learning must be higher. However, combining everything, we can write:

$$\mathbb{V}^{EL}[p_E + p_D] - \mathbb{V}^{CM}[p_E + p_D] = \left(\frac{\Delta V^2}{8\pi}\right)\frac{2\tau_i\left(1 - (1+\tau_n)\tau_z\right)}{\psi(1+\psi)}$$

For sufficiently low τ_z, therefore, $\mathbb{V}^{EL}[p_E + p_D] > \mathbb{V}^{CM}[p_E + p_D]$. $\qquad\square$

Proof of Lemma 8 Let $s_{i,T-k} = z_{T-k+l} + \varepsilon_i$ where $l \geq 1$ and $\varepsilon_i \sim \mathcal{N}\left(0, \left(\tau_{i,T-k}^I\right)^{-1}\right)$. Then $\psi_{T-k}^{PE} \equiv \tau_z\left[\sum_{j=1}^l \rho^{2(j-1)}\right]^{-1} + \tau_{i,T-k}^I + \tau_{PE,T-k}$ and

$$q_{PE,T-k} = \Phi\left(\frac{(1-\rho)\mu_z\sum_{j=1}^k \rho^{(j-1)} + \rho^k z_{T-k} + \rho^{(k-l)}\left(\frac{(\tau_{i,T-k}^I + \tau_{PE,T-k})(\sum_{j=1}^l \rho^{j-1}e_{T-k+l-j+1} + u_{T-k}/\sqrt{\tau_{i,T-k}})}{\psi_{T-k}^{PE}}\right)}{\sqrt{1 + \frac{1}{\tau_z}\sum_{j=1}^{k-l}\rho^{2(j-1)} + \rho^{2(k-l)}(\psi_{T-k}^{PE})^{-1}}}\right).$$

$$\mathbb{V}[q_{PE,T-k}|z_{T-k},\mathcal{P}] = \phi\left(\mathbb{E}[q_{PE,\tilde{T}-k}|z_{T-k},\mathcal{P}]\right)^2 \mathbb{V}[q_{PE,\tilde{T}-k}|z_{T-k},\mathcal{P}]$$

$$\phi\left(\mathbb{E}[q_{PE,\tilde{T}-k}|z_{T-k},\mathcal{P}]\right)^2 = \frac{1}{2\pi}exp\left(-\frac{\left[(1-\rho)\mu_z\sum_{j=1}^k \rho^{(j-1)} + \rho^k z_{T-k}\right]^2}{1 + \frac{1}{\tau_z}\sum_{j=1}^{k-l}\rho^{2(j-1)} + \rho^{2(k-l)}(\psi_{T-k}^{PE})^{-1}}\right)$$

$$\mathbb{V}[q_{PE,\tilde{T}-k}|z_{T-k},\mathcal{P}] = \frac{\rho^{2(k-l)}\left[\frac{(\tau_{i,T-k} + \tau_{E,T-k})}{\psi_{T-k}^{PE}}\right]^2\left[\frac{\sum_{j=1}^l \rho^{2(j-1)}}{\tau_z} + \frac{1}{\tau_{E,T-k}}\right]}{1 + \frac{1}{\tau_z}\sum_{j=1}^{k-l}\rho^{2(j-1)} + \rho^{2(k-l)}(\psi_{T-k}^{PE})^{-1}}$$

From the proof of Lemma 7, we know that the variance in the liquid market can be written:

$$\mathbb{V}[q_{E,T-k}|z_{T-k},\mathcal{P}] = \phi\left(\mathbb{E}[q_{E,\tilde{T}-k}|z_{T-k},\mathcal{P}]\right)^2 \mathbb{V}[q_{E,\tilde{T}-k}|z_{T-k},\mathcal{P}]$$

$$\phi\left(\mathbb{E}[q_{E,\tilde{T}-k}|z_{T-k},\mathcal{P}]\right)^2 = \frac{1}{2\pi}exp\left(-\frac{\left[(1-\rho)\mu_z\sum_{j=1}^k \rho^{(j-1)} + \rho^k z_{T-k}\right]^2}{1 + \tau_z^{-1}\sum_{j=1}^{k-1}\rho^{2(j-1)} + \sum_{j=1}^{k-1}\rho^{2(j-1)}\psi_{T-j}^P + \rho^{2(k-1)}\left(\psi_{T-k}^E\right)^{-1}}\right)$$

$$\mathbb{V}[q_{E,\tilde{T}-k}|z_{T-k},\mathcal{P}] = \frac{\rho^{2(k-1)}\left[\frac{(\tau_{i,T-k} + \tau_{E,T-k})}{\psi_{T-k}^E}\right]^2\left[\frac{1}{\tau_z} + \frac{1}{\tau_{E,T-k}}\right]}{1 + \tau_z^{-1}\sum_{j=1}^{k-1}\rho^{2(j-1)} + \sum_{j=1}^{k-1}\rho^{2(j-1)}\psi_{T-j}^P + \rho^{2(k-1)}\left(\psi_{T-k}^E\right)^{-1}}$$

We begin by showing that the denominator of $\mathbb{V}[q_{E,\widetilde{T-k}}|z_{T-k},\mathcal{P}]$ exceeds the denominator of $\mathbb{V}[q_{PE,\widetilde{T-k}}|z_{T-k},\mathcal{P}]$:

$$\tau_z^{-1}\sum_{j=k-l+1}^{k-1}\rho^{2(j-1)}+\sum_{j=k-l+1}^{k-1}\rho^{2(j-1)}\psi_{T-j}^P+\rho^{2(k-1)}\left(\psi_{T-k}^E\right)^{-1}+\sum_{j=1}^{k-l}\rho^{2(j-1)}\psi_{T-j}^P >$$

$$\tau_z^{-1}\sum_{j=k-l+1}^{k-1}\rho^{2(j-1)}+\sum_{j=k-l+1}^{k-1}\rho^{2(j-1)}\psi_{T-j}^P+\rho^{2(k-1)}\left(\psi_{T-k}^E\right)^{-1} >$$

$$\sum_{j=k-l+1}^{k}\rho^{2(j-1)}\left(\psi_{T-j}^E\right)^{-1} =$$

$$\frac{\left[\sum_{j=k-l+1}^{k}\rho^{2(j-1)}\right]}{\tau_z+\tau_i+\tau_E} \geq$$

$$\frac{\left[\sum_{j=k-l+1}^{k}\rho^{2(j-1)}\right]}{\tau_z+\left[\sum_{j=1}^{l}\rho^{2(j-1)}\right]\left[\tau_{i,T-k}^I+\tau_{PE,T-k}\right]}=\rho^{2(k-l)}(\psi_{T-k}^{PE})^{-1}$$

The second inequality arises because $\tau_z^{-1}+\psi_{T-j}^P>\left(\psi_{T-j}^E\right)^{-1}$. Next we show that the numerator of $\mathbb{V}[q_{PE,\widetilde{T-k}}|z_{T-k},\mathcal{P}]$ exceeds the numerator of $\mathbb{V}[q_{E,\widetilde{T-k}}|z_{T-k},\mathcal{P}]$:

$$\rho^{2(k-l)}\left[\frac{(\tau_{i,T-k}+\tau_{E,T-k})}{\psi_{T-k}^{PE}}\right]^2\left[\frac{\sum_{j=1}^{l}\rho^{2(j-1)}}{\tau_z}+\frac{1}{\tau_{E,T-k}}\right] \geq$$

$$\rho^{2(k-1)}\left[\frac{(\tau_{i,T-k}+\tau_{E,T-k})}{\psi_{T-k}^E}\right]^2\left[\frac{1}{\tau_z}+\frac{1}{\tau_{E,T-k}}\right]$$

because for all $l\geq 1$ $\psi_{T-k}^{PE}\leq\psi_{T-k}^E$, $\rho^{2(k-l)}\geq\rho^{2(k-1)}$ (as long as $0<\rho\leq 1$), and $\sum_{j=1}^{l}\rho^{2(j-1)}\geq 1$, where the inequalities are strict if $l>1$.

Note however, that if $|\mathbb{E}[z_T|z_{T-k}]|>0$, then $\phi\left(\mathbb{E}[q_{PE,\widetilde{T-k}}|z_{T-k},\mathcal{P}]\right)^2<\phi\left(\mathbb{E}[q_{E,\widetilde{T-k}}|z_{T-k},\mathcal{P}]\right)^2$. Thus, as long as $|\mathbb{E}[z_T|z_{T-k}]|$ is sufficiently low, it is clear that $\mathbb{V}[q_{PE,T-k}|z_{T-k},\mathcal{P}]>\mathbb{V}[q_{E,T-k}|z_{T-k},\mathcal{P}]$.

Finally, note that the numerator of $\mathbb{V}[q_{PE,\widetilde{T-k}}|z_{T-k},\mathcal{P}]$ is increasing in l. It is straightforward to show that the denominator is falling in l. Let $\hat{l}>l$. Then,

$$1+\frac{1}{\tau_z}\sum_{j=1}^{k-l}\rho^{2(j-1)}+\frac{\left[\sum_{j=k-l+1}^{k}\rho^{2(j-1)}\right]}{\tau_z+\left[\sum_{j=1}^{l}\rho^{2(j-1)}\right]\left[\tau_{i,T-k}^I+\tau_{PE,T-k}\right]} >$$

$$1+\frac{1}{\tau_z}\sum_{j=1}^{k-\hat{l}}\rho^{2(j-1)}+\frac{1}{\tau_z}\sum_{j=k-\hat{l}+1}^{k-l}\rho^{2(j-1)}+\frac{\left[\sum_{j=k-l+1}^{k}\rho^{2(j-1)}\right]}{\tau_z+\left[\sum_{j=1}^{\hat{l}}\rho^{2(j-1)}\right]\left[\tau_{i,T-k}^I+\tau_{PE,T-k}\right]} =$$

$$1+\frac{1}{\tau_z}\sum_{j=1}^{k-\hat{l}}\rho^{2(j-1)}+\left[\frac{1}{\tau_z}-\frac{1}{\tau_z+\left[\sum_{j=1}^{\hat{l}}\rho^{2(j-1)}\right]\left[\tau_{i,T-k}^I+\tau_{PE,T-k}\right]}\right]\sum_{j=k-\hat{l}+1}^{k-l}\rho^{2(j-1)}+\frac{\left[\sum_{j=k-\hat{l}+1}^{k}\rho^{2(j-1)}\right]}{\tau_z+\left[\sum_{j=1}^{\hat{l}}\rho^{2(j-1)}\right]\left[\tau_{i,T-k}^I+\tau_{PE,T-k}\right]} >$$

$$1+\frac{1}{\tau_z}\sum_{j=1}^{k-\hat{l}}\rho^{2(j-1)}+\frac{\left[\sum_{j=k-\hat{l}+1}^{k}\rho^{2(j-1)}\right]}{\tau_z+\left[\sum_{j=1}^{\hat{l}}\rho^{2(j-1)}\right]\left[\tau_{i,T-k}^I+\tau_{PE,T-k}\right]}$$

As a result, when investors' signals are more forward-looking, they generate more price uncertainty as long as $|\mathbb{E}[z_T|z_{T-k}]|$ is sufficiently low. $\qquad\square$

B Generalizations

B.1 N States

We generalize the distribution of cash flows to accommodate $N>2$ states as follows. In each state, the cash flow, x, is drawn from a known distribution, G_j, where $j\in 1,2,...,N$. Let the probability of each state, s, be written as

$$\mathbb{P}[s=j]=a_j+b_j q$$

where $q\equiv\Phi(z)$, $\Phi(\cdot)$ is the standard Gaussian CDF, and $z\sim\mathcal{N}(\mu_z,\tau_z^{-1})$. The sets of high ($H$) and low ($L$) states, $H\equiv\{j:b_j\geq 0\}$ $L\equiv\{j:b_j<0\}$, are defined in a manner which makes them analogous to the two-state setup in the main text; the probability of each state $j\in H$ is increasing in the realization of z, while the likelihood of each state $j\in L$

is falling. Insuring that this is a well-defined probability measure (i.e., $0 \leq \mathbb{P}[s = j] \leq 1 \; \forall j$, and $\sum_j \mathbb{P}[s = j] = 1$), requires

$$\sum_j a_j = 1, \quad \sum b_j = 0, \quad \sum_{j \in H} b_j \leq 1, \quad \text{and } a \geq 0 \text{ , } 0 \leq a_j + b_j \leq 1 \; \forall j.$$

Let $V_j \equiv \mathbb{E}[x|x \sim G_j]$, $V_H \equiv \sum_{j \in H} b_j V_j$, and $V_L \equiv \sum_{j \in L} b_j V_j$. Then the expected value of the asset at time-1 can be written

$$\mathbb{E}_1[x] = \sum_j a_j V_j + q \left[(V_H - V_L) \right], \tag{29}$$

Note the similarities between equation (29) and (1).[77] As $\mathbb{E}[q]$ tends to zero, the expected value of the asset tends to $\sum_j a_j V_j$ instead of V_L. Unlike the model analyzed in the main text, the likelihood of the high states do not necessarily tend to zero with $\mathbb{E}[q]$, and so cannot be neglected. However, because all agents know this (and agree on the value of $\sum_j a_j V_j$), this modification leaves unchanged the incentive to acquire information and does not alter the variance of the price received by the agent. If we further assume an equivalent statement about first-order stochastic dominance with respect to the aggregate "high" and "low" states, this generalization leaves unchanged our analysis.

B.2   Integrated Markets

We assume now that investors can freely trade in both markets simultaneously — they do not need to choose in which market to trade. Liquidity shocks remain normally-distributed and independent,

$$\begin{pmatrix} u_d \\ u_e \end{pmatrix} \sim \mathcal{N} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_d^{-1} & 0 \\ 0 & \tau_e^{-1} \end{bmatrix} \right),$$

but may have different variances, i.e., $\tau_d \neq \tau_e$. We begin by assuming that the precision of investors' private signal, τ_i, is exogenously given and $\mu_z = 0$. Given that $\tau_{i,E} = \tau_{i,D}$ (the investors in both markets are the same), $\psi^E = \psi^D = \psi$, and

$$\phi(\mathbb{E}_0[\tilde{q_D}])^2 = \frac{1}{2\pi} exp \left(-\mu_z^2 \frac{\psi}{1+\psi} \right) = \phi(\mathbb{E}_0[\tilde{q_E}])^2$$

$$\mathbb{V}_0[q_D] = \phi(\mathbb{E}_0[\tilde{q_D}])^2 \mathbb{V}_0[\tilde{q_D}] \qquad \mathbb{V}_0[q_E] = \phi(\mathbb{E}_0[\tilde{q_E}])^2 \mathbb{V}_0[\tilde{q_E}]$$

$$\mathbb{V}_0[\tilde{q_D}] = \mathbb{V}_0[\tilde{q_E}] + \frac{\tau_i \left(\frac{1}{\tau_d} - \frac{1}{\tau_e} \right)}{\psi(1+\psi)}$$

Thus, if $\tau_d > \tau_e$, there is less uncertainty about liquidity demand for debt and $\mathbb{V}_0[q_D] < \mathbb{V}_0[q_E]$. The inequality flips if $\tau_d < \tau_e$. Moreover,

$$Cov(q_E, q_D) = \phi(\mathbb{E}_0[\tilde{q_D}]) \phi(\mathbb{E}_0[\tilde{q_E}]) Cov(\tilde{q_E}, \tilde{q_D})$$

$$Cov(\tilde{q_E}, \tilde{q_D}) = \mathbb{V}_0[\tilde{q_E}] - \frac{\frac{\tau_i}{\tau_e}}{\psi(1+\psi)}$$

$$= \mathbb{V}_0[\tilde{q_D}] - \frac{\frac{\tau_i}{\tau_d}}{\psi(1+\psi)}$$

which implies that there is always a diversification benefit to issuing in both markets for the agent: $\mathbb{V}_0[q_D], \mathbb{V}_0[q_D] > Cov(q_E, q_D)$. From the proof of Proposition 3, this implies that the optimal face value of debt, F^* solves the following equation:

$$\frac{\Delta D(F^*)}{\Delta E(F^*)} = \frac{\mathbb{V}_0[q_E] - Cov(q_D, q_E)}{\mathbb{V}_0[q_D] - Cov(q_D, q_E)} \tag{30}$$

If $\tau_d < \tau_e$, then the agent issues less information-sensitive debt than equity, i.e., $0 < \Delta D(F^*) < \Delta E(F^*)$, and if $\tau_d > \tau_e$, $\Delta D(F^*) > \Delta E(F^*) > 0$. The intuition matches that presented in the main model. Taking the debt market as an example, more private learning ($\uparrow \tau_i$) generates uncertainty, which induces the agent to issue less information-sensitive debt. Similarly, more certainty about liquidity demand for debt ($\uparrow \tau_d$), which reduces price uncertainty, induces the agent to issue more information-sensitive debt.

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[77] If we match coefficients and set $b_H = 1 = -b_L$, $a_H = 0$, $a_L = 1$, Equation (29) collapses to (1).

Suppose instead that $\mu_z \neq 0$ which, as discussed in Appendix B.4, implies that, in general, $\mathbb{E}[q_D] \neq \mathbb{E}[q_E]$. Consequently, the agent can alter his expected revenue through his choice of capital structure. As a result, he is no longer just concerned with minimizing the variance of his proceeds. From the proof of Lemma 4,

$$\mathbb{E}_0[q_D] = \Phi\left(\frac{\psi\mu_z}{\sqrt{\psi(1+\psi) + (\psi - \tau_z)^2\tau_z^{-1} + (\tau_i + \tau_D)^2\tau_D^{-1} + \tau_E}}\right) \equiv \Phi(\bar{q_D}\mu_z)$$

$$\mathbb{E}_0[q_E] = \Phi\left(\frac{\psi\mu_z}{\sqrt{\psi(1+\psi) + (\psi - \tau_z)^2\tau_z^{-1} + (\tau_i + \tau_E)^2\tau_E^{-1} + \tau_D}}\right) \equiv \Phi(\bar{q_E}\mu_z)$$

As before, if $\tau_d > \tau_e$, there is more certainty about the price of debt. With it a little simplification, it is easy to see that this implies $\bar{q_D} > \bar{q_E}$. As a result, when $\mu_z > (<) 0$, then $\mathbb{E}_0[q_D] > (<)\mathbb{E}_0[q_E]$.[78] Of course, the inequalities flip when $\tau_d < \tau_e$.

From Appendix B.4, the agent's first-order condition can be written:

$$\mathbb{E}_0[q_D] - \mathbb{E}_0[q_E] = \gamma \left[\Delta D(F^*)\left(\mathbb{V}_0[q_D] - Cov(q_D, q_E)\right) - \Delta E(F^*)\left(V_0[q_E] - Cov(q_D, q_E)\right)\right]$$

Suppose $\tau_d > \tau_e$, and let F_0 be the solution to equation (30) so that $\Delta D(F_0) > \Delta E(F_0)$. When $\mu_z > 0$, issuing debt becomes even more attractive: it has lower variance *and* a higher expected price, and so $F^* > F^0$. [79] On the other hand, when $\mu_z < 0$, the agent is torn between choosing a security with low price/low uncertainty (debt) and high price/high uncertainty (equity). As a result, he tilts his issuance towards equity, i.e., $F^* < F^0$. Effectively, when $\mu_z > 0$, he decreases the discount he receives for the asset (and lowers the uncertainty of his proceeds), whereas when $\mu_z > 0$, he increases the premium he receives (while increasing the uncertainty of his proceeds).

Finally, we consider the effect of information acquisition; as in the main text, we return to our assumption that $\mu_z = 0$. When investors can trade both securities, they choose τ_i to maximize:

$$\phi(0)\left\{\Delta D\mathbb{E}_0\left[(\tilde{q}_i - \bar{q_D})|\tilde{q}_i > \bar{q_D}\right] + \Delta E\mathbb{E}_0\left[(\tilde{q}_i - \bar{q_E})|\tilde{q}_i > \bar{q_E}\right]\right\} - C(\tau_i)$$

In equilibrium, the optimal τ_i sets

$$\phi(0)\left\{\Delta D\frac{\partial\mathbb{E}_0\left[(\tilde{q}_i - \bar{q_D})|\tilde{q}_i > \bar{q_D}\right]}{\tau_i} + \Delta E\frac{\partial\mathbb{E}_0\left[(\tilde{q}_i - \bar{q_E})|\tilde{q}_i > \bar{q_E}\right]}{\partial\tau_i}\right\} = C'(\tau_i)$$

As equation (19) shows,

$$\frac{\partial\mathbb{E}_0\left[(\tilde{q}_i - \bar{q_D})|\tilde{q}_i > \bar{q_D}\right]}{\tau_i} = \frac{\partial\mathbb{E}_0\left[(\tilde{q}_i - \bar{q_E})|\tilde{q}_i > \bar{q_E}\right]}{\partial\tau_i}\sqrt{\frac{(1 + \frac{1}{\tau_e})}{(1 + \frac{1}{\tau_d})}}$$

When $\tau_d = \tau_e$, $\mathbb{E}_0\left[(\tilde{q}_i - \bar{q_D})|\tilde{q}_i > \bar{q_D}\right] = \mathbb{E}_0\left[(\tilde{q}_i - \bar{q_E})|\tilde{q}_i > \bar{q_E}\right]$. As a result, the precision chosen is independent of the capital structure. When $\tau_d > \tau_e$, $\mathbb{V}_0[q_D] < \mathbb{V}_0[q_E]$ and so the agent sets $\Delta D > \Delta E$. As F increases, however, this induces agents to choose a more precise signal (because $\frac{\partial\mathbb{E}_0[(\tilde{q}_i - \bar{q_D})|\tilde{q}_i > \bar{q_D}]}{\tau_i} > \frac{\partial\mathbb{E}_0[(\tilde{q}_i - \bar{q_E})|\tilde{q}_i > \bar{q_E}]}{\partial\tau_i}$); this offsets some of the benefit from setting $\Delta D > \Delta E$, however, because both $\mathbb{V}_0[q_D]$ and $\mathbb{V}_0[q_E]$ are increasing in τ_i. As a result, the agent must trade-off the two effects when choosing the optimal level of debt.

B.3   No Commitment

We now allow the agent to decide whether or not to sell the firm, after observing the market-clearing prices. He has two options: sell the entire firm for $p_E + p_D$, or hold onto the firm and sell it at time-1 for $\mathbb{E}_1[x]$. His expected utility from waiting to sell until time-1, given p_E, p_D:

$$\mathbb{E}[\mathbb{E}_1[x]|p_D, p_E] - \frac{\gamma}{2}\mathbb{V}[\mathbb{E}_1[x]|p_D, p_E] - (p_E + p_D)$$

The agent is able to update his beliefs about z by observing the price of equity and the price of debt. We can compare the sales price, $p_E + p_D$, to $\mathbb{E}[\mathbb{E}_1[x]|p_D, p_E]$:

---

[78]The discount and premium effects are discussed in detail in Appendix B.4.

[79]With integrated markets, the RHS does not depend upon μ_z. The effect of μ_z on the LHS is non-monotonic. In the limit, as $\mu_z \to \infty$, $\mathbb{E}_0[q_D] - E[q_E] \to 0$. As a result, while F^* is initially increasing in μ_z it eventually returns to the same level as when $\mu_z = 0$.

$$
\begin{aligned}
p_E + p_D &= V_l + \Phi\left(\frac{\tau_z\mu_z + (\tau_{i,D} + \tau_D)s_D + \tau_E s_E}{\sqrt{\psi^D(1+\psi^D)}}\right)\Delta D + \Phi\left(\frac{\tau_z\mu_z + \tau_D s_D + (\tau_{i,E} + \tau_E)s_E}{\sqrt{\psi^E(1+\psi^E)}}\right)\Delta E \\
&\neq V_L + \Phi\left(\frac{\tau_z\mu_z + \tau_D s_D + \tau_E s_E}{\sqrt{\psi(1+\psi)}}\right)\Delta V = \mathbb{E}[\mathbb{E}_1[x]|p_D, p_E]
\end{aligned}
$$

The agent's expected value of the asset will not in general, equal, the market value of the firm. First, the agent's information set contains only the two public signals; investors in each market have a private signal as well. As a result, the agent places more weight on the public signals in forming his expectation of the asset's value.[80] Second, the marginal investor in the debt market will not, in general, agree with the marginal investor in the equity market about the likelihood of each state. Finally, though he updates his beliefs using prices, the agent knows that the market value of the firm is a noisy signal of the true z.

As a result, it is insufficient to assume that the agent will always sell at time-1 simply based on his expected valuation. However, the agent is risk-averse, and so still needs to consider the remaining uncertainty if he waits until time-1 to sell:

$$
\mathbb{V}[\mathbb{E}_1[x]|p_D, p_E] = \Delta V^2 \mathbb{V}\left(\Phi\left(q\right)|p_D, p_E\right).
$$

Since the variance of $\Phi(q) \neq 0$, with sufficiently high risk-aversion (γ), the investor will always choose to sell at time-0, even after observing prices.[81] Furthermore, when $\mu_z = 0$, increasing γ does not affect the agent's optimal issuance decision.

B.4 Optimal Issuance when $\mu_z \neq 0$

We return to the static issuance model of Section 2 and assume now that $\mu_z \neq 0$. We take the precisions of debt and equity investors' private signals as given. We assume that $|\mu_z|$ is sufficiently small and τ_n is sufficiently large, so that the conditions specified in the proofs of Lemmas 3 and 4 hold.

We begin by providing intuition for the results of Lemma 4 when $\mu_z \neq 0$. When the agent forms expectations about fundamentals (q), he impounds his prior uncertainty ($\mathbb{V}_0[z]$) into his expectation. On the other hand, when the agent forms expectations about the price (q_D), he must impound both (i) the marginal investor's posterior uncertainty about fundamentals ($\mathbb{V}[z|s_i, s_D, s_E]$) as well as (ii) his uncertainty about the beliefs of the marginal investor ($\mathbb{V}_0[\mathbb{E}[z|s_i = s_D, s_D, s_E]]$).[82] Notably, the identity of the marginal investor is not fixed. Instead, he is identified by the relationship between his private signal and the price signal ($s_i = s_D$). This generates additional variance about the marginal investor's beliefs when compared to the beliefs of a given investor: $\mathbb{V}_0[\mathbb{E}[z|s_i = s_D, s_D, s_E]] > \mathbb{V}_0[\mathbb{E}[z|s_i, s_D, s_E]]$.[83]

Taken together, and by the law of total variance, $\mathbb{V}[z|s_i, s_D, s_E] + \mathbb{V}_0[\mathbb{E}[z|s_i = s_D, s_D, s_E]] > \mathbb{V}_0[z]$. Suppose $\mu_z > 0$. This creates what Albagli et al. (2015) refers to as "downside risk" due to the asymmetric effect of information about z: bad news ($z < \mu_z$) has a larger effect on the value of the asset than good news ($z > \mu_z$). Downside risk in combination with the additional variance implies that $\mathbb{E}_0[q_D] < \mathbb{E}_0[q]$. Moreover, when $\tau_{i,D} > \tau_{i,E}$, the additional variance generated is larger in the debt market. As a result, $\mathbb{E}_0[q_D] < \mathbb{E}_0[q_E] < \mathbb{E}_0[q]$.

We assume that the agent has chosen to sell the firm at time-0. As a result, his objective is to choose the face value of debt, F, to maximize:

$$
U(F) = \mathbb{E}[p_D(F) + p_E(F)] - \frac{\gamma}{2}\mathbb{V}_0[p_E(F) + p_D(F)]
$$

As was argued in the proof of Proposition 3, it is sufficient for the agent to consider values of F which satisfy $\underline{F} \leq F \leq \bar{F}$. It is straightforward to show that

$$
\frac{\partial U(F)}{\partial F} = (G_H(F) - G_L(F))\{\mathbb{E}_0[q_D] - \mathbb{E}_0[q_E] - \gamma\left[\Delta D(F)\left(\mathbb{V}_0[q_D] - Cov(q_D, q_E)\right) - \Delta E(F)\left(V_0[q_E] - Cov(q_D, q_E)\right)\right]\} \tag{31}
$$

---

[80] In equilibrium, the marginal investor's private signal is the same as his own-market public signal and so *he* places more weight on his own-market public signal than the agent.

[81] The variance is non-zero as long as investors in neither market learn z perfectly.

[82] He must account for the former as well because the marginal investor will have already accounted for it in setting the price.

[83] While a similar effect (i.e., the additional variance generated by market clearing) is present in standard REE models (e.g., CARA-normal), the relationship between information and fundamentals is generally linear, and therefore symmetric. As a result, there is no wedge. This is similar to the case when $\mu_z = 0$.

Assume that $\tau_{i,D} > \tau_{i,E}$.[84] Let F_0 be the face value of debt which minimizes the variance of the agent's proceeds, i.e., the optimal issuance when $\mu_z = 0$. Because $\mathbb{V}_0[q_D] > \mathbb{V}_0[q_E]$, the optimal level of debt sets $\Delta E(F_0) > \Delta D(F_0)$.

When $\mu_z \neq 0$, the agent must also account for the expected difference in prices: $\mathbb{E}_0[q_D] - \mathbb{E}_0[q_E]$. If $\mu_z > 0$, there's "downside risk", and so when $\tau_{i,D} > \tau_{i,E}$, $\mathbb{E}_0[q_D] < \mathbb{E}_0[q_E]$. Equity is expected to sell for a higher price *and* there's lower uncertainty about what that price will be. The more debt issued, the lower the agent's expected revenue, and so the agent optimally sets $F^* < F_0$. On the other hand, when $\mu_z > 0$, there's "upside risk" which implies that the debt market, which has more uncertain beliefs about z, also has the higher expected price: $\mathbb{E}_0[q_D] > \mathbb{E}_0[q_E]$. As a result, the agent tilts his portfolio towards debt, i.e., $F^* > F_0$.

The extent to which the agent shifts his debt issuance as a result of any difference in expected revenue is, of course, a function of his risk-aversion. If the agent were risk-neutral, he would only issue an information-sensitive security in the market with the highest price. When $\tau_{i,D} > \tau_{i,E}$, this implies that he would set $\Delta V = \Delta D$ when $\mu_z < 0$, and $\Delta V = \Delta E$ when $\mu_z > 0$. With sufficient risk-aversion, this is no longer be the case, as the agent becomes willing to reduce his expected proceeds in order to minimize his uncertainty.

Albagli et al. (2015) consider a closely-related problem as an application of their model. A risk-neutral agent sells claims to a future cash flow into two segmented markets. Investors in each market have imperfect information about the underlying cash flow. They find that the optimal policy is to issue debt, which is exposed to "downside risk" to the market with less information, and equity, which is exposed to "upside risk", to the market with more information. When F is chosen optimally, this allows the agent to maximize the premium received for equity while minimizing the discount he receives for his debt.

In our setting, such a solution is no longer possible. Here, investors have imperfect information about the probability of each state, and therefore differ in their beliefs about the *expected* cash flow. Because the non-linearity is found in the probability of each state, any contingent claim on the underlying cash flow (including debt and equity) is exposed to the same asymmetry. For instance, when $\mu_z > 0$, *both* debt and equity are always exposed to downside risk, as long as both are information-sensitive. As a result, both are discounted, relative to the agent's expectation of fundamentals. On the other hand, when $\mu_z < 0$, he expects both debt and equity to sell at a premium.

However, the agent can still take advantage of differences in the informational characteristics of each market. When $\mu_z > 0$ and $\tau_{i,D} > \tau_{i,E}$, for instance, he can *reduce* the discount he expects to receive by issuing more information-sensitive equity. On the flip side, if $\mu_z < 0$, he increases his expected premium by issuing more information-sensitive debt.

Moreover, these premia/discounts also impact the agent's *original* decision as to whether or not to sell the firm at time-0. For example, as discussed in Section 2, if there's too much uncertainty about the price he'll receive from issuing debt and equity, the agent will optimally wait until time-1 to sell the asset. Now, in addition, the agent expects to receive either a premium ($\mu_z < 0$) or a discount ($\mu_z > 0$) if he sells at time-0. For example, when $\mu_z > 0$, even if the agent would like to sell because it lowers his uncertainty, he may choose not to do so because he expects to receive a lower price for the firm than if he waited until time-1.

C   Extensions

C.1   Extensions - Static Issuance

C.1.1   Static Issuance given initial capital structure

We assume that, prior to time-0, the agent has (i) sold a fraction $1 - \alpha$ of the firm's equity and (ii) issued debt with face value F. If we assume he has maintained control rights, the agent will choose how much additional debt (F_0) to issue to maximize his utility from selling *his* claim to the firm's cash flow:

$$\max_{F_0} \quad E_0[\alpha(p_E + p_{D,0})] - \frac{\gamma}{2}\mathbb{V}_0[\alpha(p_E + p_{D,0})].$$

If we let p_E represent the price of one share, then the agent would receive αp_E for his stake. Similarly, the agent receives α of the new debt proceeds (sold at $p_{D,0}$) — the remaining cash is distributed to the additional equityholders. Because α is a known constant at time-0, and under the assumption that $\mu_z = 0$, this reduces to minimizing $\mathbb{V}_0[p_E + p_{D,0}]$ — which is seemingly identical to the problem solved above. Now, however, we must consider whether (i) other owners of the firm would like to sell their stake and (ii) whether the previously issued debt is traded or privately held. We assume the new debt is subordinated to any existing debt. We will focus on the setting in which equity and debt investors exist in equal measure and cannot choose in which market to invest.[85]

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[84]When $\tau_{i,D} < \tau_{i,E}$, the argument below is flipped, with debt replacing equity and vice versa.

[85]It is reasonable that investors choose their market prior to the *initial* issuance of equity and/or debt, and so remain fixed at this time.

Consider the simplest case — when the agent *and* all existing owners choose to sell their equity. If the existing debt is privately held and senior to any new debt, then Proposition 5 holds. The agent chooses F_0^* to set the information sensitivity of the *new* debt, not the total debt, equal to the information sensitivity of equity:

$$\Delta D_0(F, F_0^*) = \Delta E(F + F_0^*) < \Delta D_0(F, F_0^*) + \Delta D(F, F_0^*). \tag{32}$$

Consider the expected utility of debt and equity investors:

$$EU_D = (\Delta D_0)E\left[(q_i - q_D)|(q_i > q_D)\right] \quad EU_E = (\Delta E)\,\mathbb{E}_0\left[(q_i - q_E)|(q_i > q_E)\right]$$

Setting the information-sensitivity in each market equal also perfectly balances the information-driven uncertainty, because in equilibrium $\tau_{i,D} = \tau_{i,E}$. On the other hand, suppose debt investors can also trade in the previously issued debt. If their signal exceeds that of the marginal debt investor, they will want to purchase *both* bonds. As a result,

$$EU_D = (\Delta D + \Delta D_0)E\left[(q_i - q_D)|(q_i > q_D)\right],$$

leading debt investors to acquire a more precise signal than otherwise. This causes $\mathbb{V}_0[q_{D,0}]$ to exceed $\mathbb{V}_0[q_E]$; to compensate, the agent will issue less information-sensitive debt at time-0. Similarly, if the other owners choose not to participate in the equity issuance, investors will have the opportunity to purchase a smaller stake:

$$EU_E = (\alpha \Delta E)\,\mathbb{E}_0\left[(q_i - q_E)|(q_i > q_E)\right],$$

Since equity investors choose to learn less, the volatility of the equity price falls. To take advantage of this, the agent will issue more information-sensitive equity. We summarize these results in the following corollary:

Corollary 2. *The optimal value of debt issued at time-0 is decreasing in (i) the fraction of the existing debt issue which is actively traded and (ii) the fraction of equity owners who choose not to sell their shares.*

C.1.2   Costly Liquidation/Dividend Recapitalization

The model implicitly assumes that equity owners will receive the proceeds from any debt issuance — any capital raised does not stay within the firm — which closely matches the process of dividend recapitalization. The economic motivation for a dividend recap in the model is similar to the justification provided by private equity firms: issuing debt and using the proceeds to pay a special dividend lowers the uncertainty faced by equityholders. As has been pointed out in discussions of dividend recaps, issuing more debt increases the likelihood of default. In the model, however, this leaves the expected value of the *entire* firm unaffected.

Missing, however, are any potential costs in bankruptcy or costs of financial distress, a point of emphasis when the procedure's social value is discussed. It is straightforward to modify the assumptions of Section 2 so that debtholders receive only a fraction $\delta < 1$ of the cash flow if $x < F$, akin to partial recovery.[86] Issuing more debt increases the likelihood of default, which lowers the expected value of the firm.

Suppose the asset is owned entirely by the agent, as in our static model. Critically, the agent maximizes the expected value of the asset subject to the cost of uncertainty, i.e., the owner of the asset fully internalizes the cost of partial recovery. If a dividend recapitalization is chosen, it is because it maximizes his utility, even as it lowers the expected value of the firm. Under the simplifying assumption that any trading gains/losses by financial market participants are welfare-neutral, a social planner, constrained to choose the optimal value of debt, would make an identical choice: it maximizes the utility of the risk-averse agent.[87]

On the other hand, if the firm already has debt (F_S) in its capital structure, the agent will *not* consider the impact of his decision on the value of those existing claims; he considers only the reduction in expected value of the *new* debt issuance. Suppose the new debt issued, F_J, is subordinate. If the firm defaults, senior debtholders are only made whole if the cash flow, x, exceeds F_S/δ. While the agent's optimal decision, F_J, *may* increase welfare relative to selling the entire claim in the equity market, it will not match the decision of the social planner, who must also account for the loss in expected value imposed on senior debtholders.[88]

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[86]Alternatively, the discount can apply to all cash flows below some threshold greater than $F + F_0$ to accommodate potential costs of financial distress.

[87]This is, admittedly, a strong assumption, since the preferences of noise traders are unmodeled. Moreover, it does not account for the potential cost of information acquisition - if this generates disutility for investors (and is not simply a financial transfer to other agents in the economy), a social planner would want to discourage such information acquisition.

[88]If senior debtholders are rational and forward-looking, they would (i) expect the new issuance and (ii) pay less for their claim in the first place. If the existing debt is *also* chosen by the owner of the asset (as is done when the debt was issued by a private equity firm in a leveraged buyout), then the owner of the asset would internalize this cost at the earlier date, returning us to the socially-optimal capital structure.

C.1.3 Tail Risk

While the main analysis of the paper focused on a two-state distribution, as we show in Appendix B.1, the payoff distribution is flexible enough to accommodate $N > 2$ states. An interesting application of this more general setup is to consider how issuing debt can help mitigate the effect of what we will term "tail risk".

For example, suppose there are three states: $s \in \{L, M, H\}$, where $G_H \succ_{FOSD} G_M \succ_{FOSD} G_L$, and

$$\mathbb{P}[s = H] = b_H q \quad \mathbb{P}[s = L] = b_L q \quad \mathbb{P}[s = M] = 1 - (b_H + b_L)q,$$

$$0 \leq b_H, b_L \leq 1,\ b_H + b_L \leq 1.$$

As z increases, "tail risk" increases — draws from the high and low state become more likely, while a draw from the "middle" state becomes less probable. The expected value of the asset is now written

$$\mathbb{E}[x] = V_M + \mathbb{E}[q](b_H V_H + b_L V_L - V_M),$$

and we can redefine $\Delta V \equiv b_H V_H + b_L V_L - V_M$.

Tail risk, or increased cash flow uncertainty, is often proxied by the volatility of cash flows. An increase in volatility leaves the expected value of the asset unchanged; for an analogous result in our setup, it must be that $b_H V_H + b_L V_L = V_M$, which implies that $\Delta V = 0$. In this case, the optimal capital structure *will* consist of information-insensitive debt. Note, however, that the equity in this setup is also information-insensitive. Such a capital structure provides no incentive for investors in *either* market to learn, which is the first-best result for the agent.[89] Because $\Delta E + \Delta D = \Delta V = 0$ must always hold, if information-sensitive debt is issued, it must be that $\Delta E \neq 0$ and $\Delta D \neq 0$. Of course, this implies that $|\Delta E| + |\Delta D| > \Delta V$. Issuing information-sensitive debt creates financial securities which are more information-sensitive than the asset itself![90]

Note, however, that in our model, this is a special case of "tail risk"; in particular, the symmetry of the shock is essential for the agent to issue information-insensitive debt. Suppose that b_H increases so that $\Delta V > 0$, and an increase in z causes the expected value of the asset to increase — the cash flow distribution becomes more positively-skewed. For low values of debt, the intuition from above generally holds: ΔD will fall, ΔE will rise, and capital structure sensitivity exceeds ΔV. However, if the face value increases sufficiently, debt will become positively exposed to the shock, and issuing non-linear securities can actually reduce price uncertainty.

C.2 Market Segmentation

In Section 2, we assume that investors are, or can be, restricted to invest in either debt or equity. In delegated asset management, fund managers are often provided with strict mandates about the securities in which they can invest; debt- and equity-only funds are common.[91] Even when the financial institution supports both equity and bond funds, at the research level, segmentation can remain — for instance, there exists both an equity and a credit analyst for the technology sector — consistent with the assumption that how much investors learn can be specific to a *security's* characteristics.

As we show in Appendix B.2, however, our results do not require that informed investors be segmented. Instead, it must be that markets are *imperfectly* integrated, as defined by Chen and Knez (1995). In our model, debt and equity markets are imperfectly integrated when the marginal investor in each market differs — note that this can occur even when investors trade both securities, as long as the liquidity shocks across markets are imperfectly correlated.

We end by considering some of the evidence for imperfect integration across corporate debt and equity markets.[92] In analyzing the corporate debt market, Collin-Dufresne, Goldstein, and Martin (2001) note that (i) the standard pricing factors proposed by structural models explain little of the changes in corporate bond spreads and (ii) there is a common factor across all bonds which can explain much of the residual variation. Kapadia and Pu (2012) argue that the credit and equity market securities for a given firm move in the wrong direction too frequently over short horizons for the markets to be integrated and propose limits to arbitrage as an explanation. Longstaff, Mithal, and Neis (2005) find evidence that bond-specific liquidity, as well as a bond-market liquidity factor, can explain corporate bond spreads; the existence of such pricing factors implies that the debt and equity markets are not perfectly integrated. Schaefer and Strebulaev (2008) show that, while bonds are exposed to both the Fama-French SMB and HML factors, this sensitivity

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[89] Investors are risk-neutral; any information acquired leaves unchanged the price they are willing to pay for the asset.

[90] The intuition is similar in other frameworks. If the asset's payoff is linear, increasing the volatility has no effect on the value of the asset, but any non-linear claims on the asset (e.g., debt, equity) will be sensitive to volatility shocks.

[91] As shown in He and Xiong (2013), such restrictions can be optimal in the presence of moral hazard.

[92] There is also empirical evidence confirming investor segmentation/specialization across a wide range of markets including Kim and Stulz (1988) (domestic and foreign equity) and Gabaix, Krishnamurthy, and Vigneron (2007) (mortgage-backed securities).

is not solely driven by the bond's credit exposure, suggesting that a component of the bond's returns are driven by a factor which extends beyond structural model fundamentals. Finally, note that the excess returns of capital structure arbitrage trading strategies, as documented by Duarte, Longstaff, and Yu (2007), is consistent with the existence of arbitrage across debt and equity markets.[93]

C.3   Extensions - Dynamic Issuance

C.3.1   Control Rights

In practice, the rights associated with equity ownership are negotiable - these including voting rights, for instance, or the ability to veto proposed mergers and acquisitions. To maintain control over the financing decisions of the firm, however, the agent must retain ownership of some portion of the equity. Furthermore, some retention is generally required of the owner after a firm's initial public offering. When the agent retains some equity for future sale, it affects his decision about how best to issue securities today.

Suppose, at $T - k$, that the agent has been hit with a liquidity shock — he would like to sell his entire ownership claim immediately. We assume that he is required, however, to hold onto a fraction α of the firm's equity until $T - k + j$. The variance of his proceeds can be written

$$\Delta E^2 \mathbb{V}[(1 - \alpha)q_{E,T-k} + \alpha q_{E,T-k+j}] + \Delta D^2 \mathbb{V}[q_D] + 2\Delta E \Delta D Cov(q_D, (1 - \alpha)q_{E,T-k} + \alpha q_{E,T-k+j}).$$

Note how closely this resembles the static issuance problem: the agent now considers a "portfolio" of equity issuance, however, split across multiple periods. As the required retention period increases ($\uparrow j$), or the required share retained grows ($\uparrow \alpha$), the agent's uncertainty about his liquidation price grows. Equity issuance becomes less desirable. It is straightforward to show that the optimal face value of debt sets $\Delta D(F^*) > \Delta E(F^*)$, and that F^* increases with both the retention period and retention share.[94]

C.3.2   Timing

In a multi-period setting, equity issued today receives a portion of any dividend paid out from the proceeds of future debt issuance. On the other hand, debt issued today is unaffected by the sale of equity in the future, assuming the proceeds are not retained within the firm. When agents must issue sequentially, this feature can generate a preference for which security to issue first.

For simplicity, suppose the agent is restricted to issue at time-T and $T - k$, and that at either point in time, he is restricted to issue either publicly-traded debt or equity, but cannot issue both. To focus on the role of risk reduction, we set $\mu_z = z_{T-k} = 0$. There are three possible ways for the agent to structure his claims. (1) He can issue debt with face value F at $T - k$, and sell his equity share at time-T. (2) He can sell a fraction, $1 - \alpha$ of his equity stake at $T - k$ and sell the remainder α at time-T. (3) He can sell the entire equity stake at $T - k$, and commit to issuing debt at time-T.[95]

If the agent issues debt at T-k, he receives $p_{D,T-k} = D_L(F) + q_{D,T-k}\Delta D$, where $q_{D,T-k} = q_{E,T-k}$ as defined in (25). At time-T, he receives $p_{E,T} = E_L(F) + q\Delta E(F)$ for selling his equity stake. As a result, he chooses F to minimize:

$$\Delta D(F)^2 \mathbb{V}[q_{D,T-k}|z_{T-k}] + \Delta E(F)^2 V[q|z_{T-k}] + 2\Delta D(F)\Delta E(F)Cov(q_{D,T-k}, q|z_{T-k}).$$

If, instead, the agent sells $(1 - \alpha)$ of the firm's equity at T-k, his choice of α minimizes:

$$((1 - \alpha)\Delta E)^2 V[q_{E,T-k}|z_{T-k}] + (\alpha\Delta E)^2 V[q|z_{T-k}] + 2\alpha(1 - \alpha)\Delta E^2 Cov(q_{E,T-k}, q|z_{T-k}).$$

As is clear from the two expressions, when the two markets are otherwise identical, there is a direct correspondence between the optimal decisions in both problems: $(1 - \alpha^*)\Delta E = \Delta D(F^*)$. As a result the agent is indifferent between the first two options, assuming no restrictions on the choice of α and F.

On the other hand, if the firm sells debt at time-T, equityholders receive $p_{D,T} = D_L(F) + q\Delta D(F)$; the agent, however, having sold his entire stake, receives nothing. Instead, at $T - k$, he receives

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[93]They also note that following a capital structure arbitrage strategy does expose investors to systemic risk, a feature which is absent in our model.

[94]Given the proposed liquidity shock, the agent may value future proceeds at a lower rate than any cash received today. This will only increase his incentive to sell today, pushing him to issue more debt.

[95]Given that he has sold his entire equity claim at time-T-k, and therefore the control rights, he commits to issuing debt prior to selling the equity.

$$p_{E,T-k} = E_L(F) + q_{E,T-k}\Delta E(F) + \mathbb{E}[p_{D,T}|z_{T-k}]$$
$$= V_L + q_{E,T-k}\Delta V.$$

By selling the equity first, he is effectively selling the entire firm at $T - k$; equityholders pay him now for the dividend they expect to receive from the debt proceeds at time-T. On the one hand, this is beneficial, in that the agent's uncertainty about q exceeds his uncertainty about $q_{E,T-k}$. On the other hand, he has lost the opportunity to diversify - q and $q_{E,T-k}$ are imperfectly correlated. For small k, the latter outweighs the former, and the agent prefers one of the first two options. That is, issuing equity before debt is suboptimal. When k is sufficiently high, the agent is indifferent between all three scenarios — he sets $F^* \to \infty$ in the first setting and $\alpha = 0$ in the second.

C.4   Investment Financing: Debt and Equity

We now modify the setting of Section 4 to allow for mixed capital markets financing. Specifically, the agent must finance his investment at $T - k$ using a mix of equity and debt. We will ignore the role of liquid assets here — their use will closely follow what was shown in the main text. Our objective is to show under what conditions the agent opts to use equity, as well as debt, for financing.

For tractability, we assume the agent cannot issue debt at $T - k + j$ and so only sells any remaining equity; this restriction tilts the terms in favor of debt issuance at $T-k$. As before, the agent is constrained to raise no more funds via financing than are necessary for investment; to insure that he raises sufficient capital, we assume that the agent chooses how much debt $F(I)$ to issue, which in turn sets the fraction of equity sold $\alpha(I,F)$ as the solution to

$$D_L(F(I)) + (1 - \alpha(I,F))E_L(F) = I.$$

It is straightforward to show that this implies $\frac{\partial \alpha(I,F)}{\partial F} > 0$, i.e., the fraction of equity sold at $T - k$ is decreasing in the face value of debt used for financing. On the other hand, increasing F causes the information-sensitivity of equity to fall ($\frac{\partial \Delta E}{\partial F} < 0$). As a result, given that equity is also sold at $T - k + j$ with a more uncertain price, this can make debt issuance more valuable than selling equity. On the other hand, if the agent finances the investment solely with debt, he forfeits the opportunity to reduce the uncertainty of his financing terms by accessing (and diversifying with) the equity market at $T - k$.

As before, when the level of investment is sufficiently low, e.g., if $\Delta D(F(I)) = 0$, equity is preferable. Furthermore, when the level of investment is sufficiently *high*, and the information-driven uncertainty at $T - k + j$ is sufficiently low, we can also show that the agent wants to finance using debt *and* equity. We examine the underlying mechanism for this result by considering the marginal value of reducing $\alpha(I,F)$ (i.e., reducing F) when the agent finances his entire investment through debt ($\alpha = 1$). If doing so reduces the uncertainty of the proceeds, on the margin, then it must be that at least some fraction of the investment is financed via equity.

First, we consider the marginal change in the variance of the agent's proceeds due to the shift in equity issuance from time $T - k + j$ to $T - k$. Of course, any shift in the issuance of equity to an earlier date reduces uncertainty:

$$-2\Delta E \frac{\partial \alpha(I,F)}{\partial F} \left[\Delta E(F(I))\mathbb{V}[q_{E,T-k+1}] + \Delta D(F(I))Cov(q_{D,T-k}, q_{E,T-k+j})\right].$$

Next, we consider the marginal change in uncertainty from lowering F due to the change in the information sensitivity of each security:

$$-2\left[G_L(F(I)) - G_H(F(I))\right]\left[\Delta D(F(I))\left(\mathbb{V}[q_{D,T-k}] - Cov(q_{D,T-k}, q_{E,T-k+j})\right) - \Delta E(F(I))\left(\mathbb{V}[q_{E,T-k+j}] - Cov(q_{D,T-k}, q_{E,T-k+j})\right)\right].$$

Under the assumption of FOSD, $G_L(F) > G_H(F)$. Holding fixed each security's information sensitivity, deferring issuance until $T - k + j$ leads to more volatile proceeds: $\mathbb{V}[q_{E,T-k+j}] > \mathbb{V}[q_{D,T-k}]$. However, when $\mathbb{V}[q_{D,T-k}] > Cov(q_{D,T-k}, q_{E,T-k+j})$ (i.e., the information-driven uncertainty at $T - k + j$ is sufficiently low), the agent also wants to diversify over time. When $\Delta D(F(I))$ is sufficiently large (i.e., when the level of investment is sufficiently high), the agent lowers the uncertainty of his proceeds by issuing equity along with debt when financing.

C.5   Illiquidity: Private then Public Issuance

We focus here on the *within-market* effect of illiquidity. We assume that at each date, the firm has a choice to issue private or public equity. If the firm chooses to go public, it will remain public in all following periods. Moreover, if it goes public, all shares issued freely trade in all following periods. If it remains private, the only shares which trade are those which are issued in that period. For simplicity, we set $T = 2$.

We work backwards, starting at $t = 2$. At this point, z is known by all agents, and no information is acquired — the agent is indifferent between issuing private or public equity, and $\alpha_2 = 0$. At $t = 1$, the agent optimally retains:

$$\alpha_1(z_1, \alpha_0) = \frac{\frac{\mathbb{E}[q_2 - q_1 | z_1]}{(\gamma \Delta V) \alpha_0} + \mathbb{V}[q_1 | z_1] - Cov[q_1, q_2 | z_1]}{\mathbb{V}[q_2 | z_1] + \mathbb{V}[q_1 | z_1] - 2Cov[q_1, q_2 | z_1]}. \tag{33}$$

Both investors form expectations about $\Phi(z)$; holding the number of shares for purchase fixed, the information acquired is of equal value to both private and public investors. However, if the firm remains private, investors can only acquire $\alpha_0(1 - \alpha_1)$ shares, whereas going public allows investors to purchase a fraction $1 - \alpha_0\alpha_1 > \alpha_0(1 - \alpha_1)$ of the firm. As a result, private equity investors choose to acquire less information, lowering the variance of q_1.

The agent must also consider the effect of information acquisition on his ability to take advantage of predictable changes in the price: $\mathbb{E}[q_2 - q_1 | z_1]$. If $\tau_z > \tau_i(1 + \tau_n)$, then the denominator of q_1 is increasing in the precision of the investors' private information. As a result, when $(1 - \rho)\mu_z + \rho z_1 > 0$, issuing publicly-traded securities lowers the price today relative to the price in the private market. Furthermore, this induces the agent to retain more of his holdings for issuance at the more uncertain q_2. For both reasons, this makes issuing privately more attractive. On the other hand, when $(1 - \rho)\mu_z + \rho z_1 < 0$, issuing publicly-traded securities raises the price today relative to the price in the private market, opening up the possibility that public issuance is preferable.

At $t = 0$, whether issued privately or publicly, investors will be able to purchase $1 - \alpha_0$ shares. Private investors, however, anticipate selling at $t = 2$, whereas public investors can trade in the following period. As shown in Section 6, this implies that the variance of q_0 is *higher* if the agent issues privately, assuming investors in the illiquid market obtain sufficient information. If this were his only opportunity to issue, the agent would optimally do so in the public markets. Private issuance now, however, preserves the option to issue privately at $t = 1$. If that option is sufficiently valuable, the agent may optimally choose to sell his stake in the private markets.