Effective Properties of Fibre-Reinforced Elastomers: Parameter Calibration by Prototype Deformation Modes

Fibre reinforcement · effective properties · parameter calibration

The finite element method (FEM) has been established for the design of elastomer components and is today used in practice in the development process of such components. Similarly, fibre- and fabric-reinforced rubber components are designed to meet increased loads. The simulation of these components poses a challenge and is an object of research. This paper shows the parameter calibration of a model for reinforced elastomers by adjustment to experiments. For this purpose, we select the Holzapfel, Gasser & Ogden model developed for the description of human tissue. This model is also available in commercial FEM systems for the description of anisotropic behaviour. Unfortunately, there has not been any possibility of adjusting to experiments.

Effektive Eigenschaften von verstärkten Elastomeren: Parameter-Anpassung durch Prototyp-Deformationen

Faserverstärkung · effektive Eigenschaften · Parameterbestimmung

Für die Auslegung von Elastomerbauteilen ist die Finite-Elemente-Methode (FEM) etabliert und wird im Entwicklungsprozess eingesetzt. Heute werden faser- und gewebeverstärkte Gummibauteile konzipiert, um erhöhten Belastungen standzuhalten. Die Rechner-Abbildung solcher Bauteile stellt jedoch noch eine Herausforderung dar und ist aktueller Forschungsgegenstand. Hier zeigen wir, wie sich ein gegebenes Modell für faserverstärkte Elastomere an Experimente kalibrieren lässt. Wir wählen das Modell nach HOLZAPFEL, GASSER & OGDEN, das zur Beschreibung menschlichen Gewebes entwickelt worden ist. Inzwischen ist dies in kommerziellen FEM-Programmen für anisotropes Materialverhalten verfügbar. Leider mangelt es an einer Möglichkeit, die Kennwerte anzupassen. Wir zeigen hier dieses Modell für uniaxiale und biaxiale Deformationsmoden.

Figures and Tables:
By a kind approval of the authors.

In technical applications, fabric- and (long) fibre- or filament-reinforced elastomer components are used, for example, in membranes and tubes or also in automobile engineering as air-spring bellows. Fibres are the reinforcing material whereas elastomers are primarily responsible for sealing and formation. Thus, such elastomer components are suitable for withstanding considerably higher (tensile) stress levels than would be the case with purely rubber components.

Simulation models are also to be increasingly used for the development process of these components in order to establish the level of stress in the material prior to manufacturing and, where applicable, to allow conclusions for the design to be drawn. In this way, the respective properties may be predicted or even specifically set at a very early stage within the development phase of these products.

When implementing such models, the inclusion of fibre or fabric reinforcement within the elastomer matrix involves a special task. This is due, on the one hand, to the often considerable differences in size in the longitudinal scales of the individual constituents and, on the other hand, to the difficulties of adjusting suitable material parameters to experimental results.

In this paper, we wish to show how the parameters of the specially selected Holzapfel, Gasser & Ogden material model can be adjusted by means of suitable tests and subsequent calibration, see [1]. This model thus becomes accessible and effectively usable for computer simulations on the component scale using the finite element method (FEM). One of the core ideas of this material model is the formulation with additional terms in the strain energy potential function which take into account fibre orientation and stiffness. This results in simulation of the effective material properties on the macroscale, causing the information of the mesoscale or microscale to be blurred in the order of magnitude of the reinforcing material's dimensions and to be no longer accessible in this analysis. This multiscale modelling enables the material response to be calculated on the component level and can, for example, result anisotropically and thus become integrated into the FEM model. This then represents a "homogenisation" of the material properties in the order of magnitude of components.

Hence, the objective here is to perform basic experiments which induce the uniaxial stress and biaxial stress prototype deformations in specimens and thus include the parameter area of the (incompressible) elastomer material, see Baaser [2]. We assume that fibre or fabric reinforcement does not influence (macroscopic) volume constancy. The specific adjustment of a material model's parameters to the two above-mentioned deformation modes represents the formulation of this model in the respective form. From the authors' point of view, this procedure has not been explicitly applied for reinforced elastomer components, neither for the specified Holzapfel, Gasser & Ogden model nor for similar models [1] (see, for example, Fung [3]). This appears to be one of the reasons for a hardly known data base in literature or due to details in the examples of commercial FEM program systems.

Authors

V. Achenbach, H. Baaser, T. Cvjetkovic, Weinheim

Corresponding author:
Prof. Dr.-Ing. Herbert Baaser
Freudenberg R&D Services,
69465 Weinheim, Germany
E-mail:
herbert.baaser@freudenberg.de

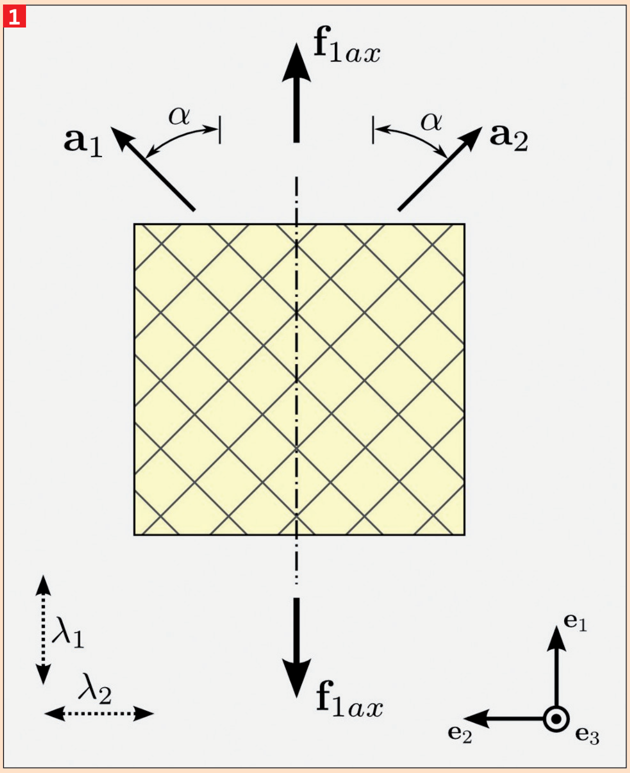

Fig. 1: Geometrical variables for the "uniaxial stress" prototype case in which the main fibre layers are oriented to the loading direction at an angle of +α und –α

In this paper, we would like to close this gap and present a suitable procedure by showing the formulations of the material model and a suitable parameter adjustment by means of an optimisation procedure in Matlab®. The relevant necessary, complex mathematical reformulations have been realised in Maple®. The parameter sets gained by means of this procedure can then be used directly for the purpose of simulating fibre- and fabric-reinforced components within FEM systems (e.g. ABAQUS) in which the model is available [1].

Holzapfel, Gasser & Ogden material model [1]

This hyperelastic material model can take into account several fabric layers. For purposes of simplification, in this paper we first consider a layer whose fibre orientation is set by the two standardised direction vectors $\vec{a}_1$ and $\vec{a}_2$. Deformation exists in the form of the deformation gradient **F** and the derived right Cauchy-Green tensor **C** = **F**^{T}**F**. Furthermore, a distinction is made between a change in form (isochoric part) and volumetric deformation using multiplicative splitting $\mathbf{F} = (J^{1/3}\mathbf{I})\overline{\mathbf{F}}$. Here, $\overline{\mathbf{F}}$ represents the purely isochoric deformation and $(J^{1/3}\mathbf{I})$ with $J = \det(\mathbf{F})$ the change in volume. Analogously, for the right Cauchy-Green deformation tensor, this yields $\mathbf{C} = (J^{2/3}\mathbf{I})\overline{\mathbf{C}}$, see Holzapfel [4]. The strain energy potential function function $\Psi = \Psi_{vol} + \overline{\Psi}_{isochor}$ is the starting point for the further procedure. The volumetric contribution is provided by $\psi_{vol}(J)$, whereas the isochoric part is additively divided into $\overline{\Psi}_{isochor} = \overline{\Psi}_{isotrop} + \overline{\Psi}_{anisotrop}$. For the purpose of modelling several layers, the strain energy potential function is expanded by the appropriate terms. In this way, the strain energy potential function, in line with [4], yields in isolated form

$$\Psi = \Psi_{vol} + \overline{\Psi}_{isotrop} + \overline{\Psi}_{anisotrop} \ .$$

Thus, in the isotropic part, the influence of the matrix material can be established using the Neo-Hooke model as $\overline{\Psi}_{isotrop}(\overline{I}_1) = c(\overline{I}_1 - 3)$, the invariant $\overline{I}_1 = trace(\overline{\mathbf{C}})$ and $c = G/2$ as material parameter (and, in this case, half shear module of the matrix material). In line with this approach, the contribution of fibre reinforcement is considered in the anisotropic part by

$$\overline{\Psi}_{anisotrop}(\overline{I}_4, \overline{I}_6) = \frac{k_1}{2k_2} \sum_{i=4,6} \left[\exp\left(k_2 (\overline{I}_i - 1)^2\right) - 1 \right]$$
.

The fibre structure is therefore considered by the "pseudo-variants" in the model. They are derived from the structure tensor and the deformation parameter by $\overline{I}_4 = \overline{C} : \mathrm{A}_1$ and $\overline{I}_6 = \overline{C} : \mathrm{A}_2$. The structure tensors are defined by way of the dyadic product of the direction vectors, $A_i = \vec{a}_i \otimes \vec{a}_i$, i = 1,2. Parameters k_1 and k_2 are the fibre-specific variables mentioned in the introduction, whose adjustment is to be indicated below.

For the purpose of identifying these model parameters, the stresses are formulated in the form of the first Piola-Kirchhoff stress tensor **P** as "measuring or nominal stresses". This is done using the equation **P** = **F**·**S**. The second Piola-Kirchhoff stress tensor **S** is defined from derivation of the strain energy potential function **S** = 2∂ψ/∂**C**.

Formulation for the uniaxial case

The uniaxial stress case on a specimen strip as depicted in Figure 1 is determined by way of direction vectors and by the deformation gradient:

$$F_{1ax} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \text{ where } \vec{a}_1 = \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \\ 0 \end{bmatrix}$$

and
$$\vec{a}_2 = \begin{bmatrix} \cos(\alpha) \\ -\sin(\alpha) \\ 0 \end{bmatrix}$$

If the 1 direction is the stress direction, the deformation gradient is defined with the exception of transverse stretches λ_2 und λ_3 which, in turn, are influenced by the fibres. Insertion of these variables in P provides the uniaxial representation of the stress tensor:

$$P_1 = \lambda_1 \left(\frac{2c}{J^{2/3}} + \frac{4k_1 \left(\frac{\lambda_1^2 \cos(\alpha)^2}{J^{2/3}} + \frac{\lambda_2^2 \sin(\alpha)^2}{J^{2/3}} - 1 \right) \exp\left(k_2 \left(\frac{\lambda_1^2 \cos(\alpha)^2}{J^{2/3}} + \frac{\lambda_2^2 \sin(\alpha)^2}{J^{2/3}} - 1 \right)^2 \right) \cos(\alpha)^2}{J^{2/3}} + \frac{pJ}{\lambda_1^2} \right)$$

$$P_2 = \lambda_2 \left(\frac{2c}{J^{2/3}} \frac{4k_1 \left(\frac{\lambda_1^2 \cos(\alpha)^2}{J^{2/3}} + \frac{\lambda_2^2 \sin(\alpha)^2}{J^{2/3}} - 1 \right) \exp\left(k_2 \left(\frac{\lambda_1^2 \cos(\alpha)^2}{J^{2/3}} + \frac{\lambda_2^2 \sin(\alpha)^2}{J^{2/3}} - 1 \right)^2 \right) \sin(\alpha)^2}{J^{2/3}} + \frac{pJ}{\lambda_2^2} \right)$$

$$P_3 = \lambda_3 \left(\frac{2c}{J^{2/3}} + \frac{pJ}{\lambda_3^2} \right),$$

where, due to the disappearing of the transverse stresses, conditions $P_3 = 0$ and $P_3 = 0$ have to be met in this case. From the further condition for incompressibility, $J = \det(\mathbf{F}) = \lambda_1 \lambda_2 \lambda_3 = 1$ applies. This means that the variable p which is interpreted as hydrostatic pressure is eliminated from the equations. For this deformation mode, the still unknown stretches λ_2 and λ_3 have to be iteratively established from the resulting non-linear equation system. This directly results from the above-referenced conditions $P_2 = 0$ and $J = 1$ in

$$\begin{Bmatrix} P_2 = 0 \\ D_1\left(\frac{J^2-1}{2} - \ln(J)\right) = 0 \end{Bmatrix}$$

Based on ABAQUS, we here use the second equation as a "penalty term" against a change in volume. In this way, the model is also formulated for the uniaxial stress status and is accessible for parameter adjustment [1].

Formulation for biaxial case

The formulation is considerably simpler for this mode due to the structure of the equations since the deformation gradient is now fully known with

$$\vec{a_1} = \begin{bmatrix} 0 \\ \cos(\alpha) \\ \sin(\alpha) \end{bmatrix} \text{ and } \vec{a_2} = \begin{bmatrix} 0 \\ \cos(\alpha) \\ -\sin(\alpha) \end{bmatrix} \text{ as}$$

$$F_{1ax} = \begin{bmatrix} \frac{1}{\lambda^2} & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} .$$

Thus, after inserting all correlations, the components of the stress tensor result from the model [1]

$$P_1 = 0$$

$$P_2 = \lambda\Big(2c + 4k_1\left(-1+\lambda^2\cos(\alpha)^2 + \lambda^2\sin(\alpha)^2\right)\cdot \exp\Big(k_2\left(-1+\lambda^2\cos(\alpha)^2 + \lambda^2\sin(\alpha)^2\right)^2\Big)\cdot\cos(\alpha)^2 - \frac{2c}{\lambda^6}\Big)$$

$$P_3 = \lambda\Big(2c + 4k_1\left(-1+\lambda^2\cos(\alpha)^2 + \lambda^2\sin(\alpha)^2\right)\cdot \exp\Big(k_2\left(-1+\lambda^2\cos(\alpha)^2 + \lambda^2\sin(\alpha)^2\right)^2\Big)\cdot\sin(\alpha)^2 - \frac{2c}{\lambda^6}\Big)$$

Here, as depicted in Figure 2, the 2 and 3 directions represent the stress directions whereas a flat stress status is applied in thickness direction 1

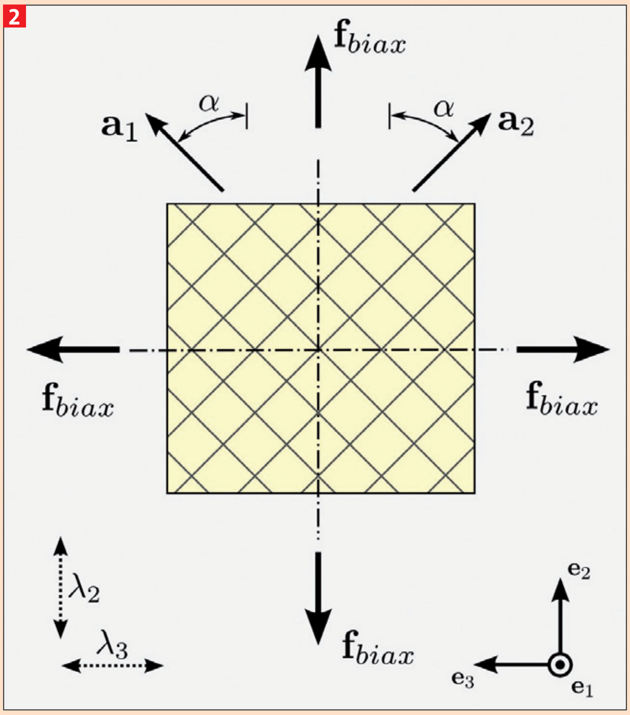

Fig. 2: Geometrical variables for the "biaxial stress" prototype case on a fibre-reinforced disc with orientation of the main fibre layers at an angle +α und −α

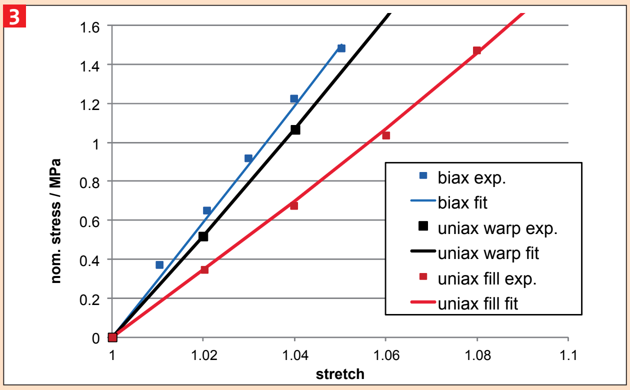

Fig. 3: Comparison of adjustment of the model on test data (dots) for one uniaxial stress in the warp direction (black) and the fill direction (red) and biaxial test (blue)

Parameter optimisation

The equations achieved here for the uniaxial and biaxial deformation mode are now used to determine free model parameters k_1 and k_2 with given fibre orientation using measurement data from the relevant tests. Hence, if test data is available in the form of stretches and stress responses, the free model variables can be established from the specified equations by adjustment theory using the „least square" method. We have used Matlab® for this purpose. However, as described above, a non-linear equation system has to be resolved for the uniaxial case in each iteration step. In the form specified here, this proves to be very robustly resolvable and in no way is an obstacle in the depicted procedure.

Experiments and results of adjustment

In this paper, we show the functionality of this adjustment procedure by a simultaneous calibration of the model on test results from one uniaxial stress in the warp and fill direction (α=0° and α=90°) and by an (bubble) inflation test of a fabric-reinforced elastomer plate, depicting a biaxial deformation status. Within the context of the above-referenced modelling operation, we here apply a two-layer structure of the fabric layers so that we may refer to an adjustment of two parameters k_1 and k_2 for each fabric layer. Thus, four model parameters are calibrated for this test. The basic elasticity of the material is represented by the neo-Hooke model; for this purpose, a shear model of G=1.5240 MPa has been determined in this case in the matrix material by means of a uniaxial tensile test.

Figure 3 depicts the results of parameter adjustment to the existing test results. Hence, for adjusted parameters k_1^{L1} = 1.4087 MPa and k_2^{L1} = 1.0255 for fabric layer 1 and k_1^{L2} = 2.4952 MPa, k_2^{L2} = 1.0636 for layer 2, very good concurrence of the model results in the considered deformation range up to approx. 8% elongation of the reinforced material.

Conclusion and Outlook

From the authors' point of view, for the first time, the Holzapfel, Gasser & Ogden material model has been made accessible by means of a representation for the uniaxial and biaxial deformation mode of a standard parameter adjustment [1]. Specifically for the uniaxial case, the model equations cannot be resolved analytically, necessitating, for this mode, an iterative resolution of the evolving non-linear equation system. The integration of this representation into an optimisation strategy within Matlab thus enables explicit calibration for the four free model parameters assumed here for the purpose of describing fibre-reinforced elastomer components within FEM simulations.

Literature

[1] G. Holzapfel, T. Gasser, R. Ogden, Journal of Elasticity **61** 2000 1.
[2] H. Baaser, Simulationsmodelle für Elastomere, ATZ-Automobiltechnische Zeitschrift 05/2010, S. 364.
[3] Y.-C. Fung: Biomechanics: Mechanical Properties of Living Tissues, New York (1993), Springer-Verlag, pp. 568. ISBN 0-387-97947-6.
[4] G. Holzapfel: Nonlinear Solid Mechanics, WILEY (2000).

Authors

Volker Achenbach is computational engineer at CAE dept. of Freudenberg R&D Services. He is working at his PhD thesis in cooperation with Semperit R&D, Wimpassing, Austria.
Prof. Dr.-Ing. Herbert Baaser is Senior Engineer for ‚Physical Material Properties and Life Cycle' at Freudenberg R&D Services. Tom Cvjetkovic is a student at Bingen University of Applied Sciences and, within the context of his bachelor thesis, has contributed to this subject.

Schutz gegen extreme Einflüsse

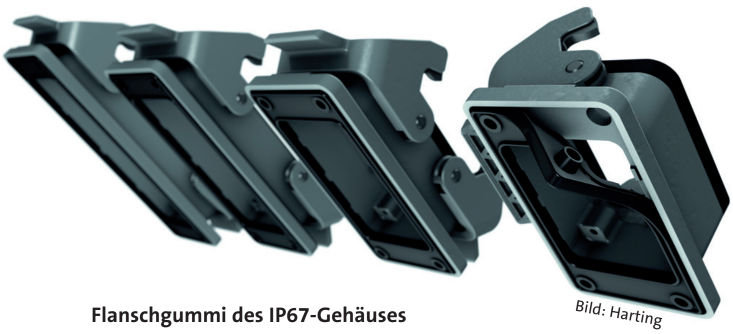
Flanschgummi des IP67-Gehäuses

Bild: Harting

GEHÄUSEPROGRAMM Han B von Harting, Minden, gibt es neben der Schutzart IP 65 nun auch mit IP 67. Dieses Anbaugehäuse verhindert auch unter Wasser das Eindringen von Feuchtigkeit und steigert so die Sicherheit für die Kontaktstellen im Inneren. Verantwortlich dafür ist eine effektivere Dichtung aus Nitrilkautschuk (NBR) am Flansch. Der umlaufende Kragen verhindert das Wegrutschen der Dichtung und kann zudem Unebenheiten ausgleichen. NBR hat eine hohe Beständigkeit gegenüber Ölen, Fetten und Kohlenwasserstoffen. Anbaugehäuse mit der Schutzart IP 67 sind neben Zuverlässigkeit, Sicherheit und Langlebigkeit durch eine einfache und schnelle Handhabung gekennzeichnet. Verriegelt werden sie mit den Bügeln Han-Easy Lock des Herstellers. Der hohe Anpressdruck bei geringem Krafteinsatz bewirkt beim Verschließen eine gute Abdichtung und Undurchlässigkeit zwischen Haube und Untergehäuse. Der Montageausschnitt des Gehäuses in vier Standardgrößen entspricht den Maßen des bereits vorhandenen Programms.

KONTAKT
Harting Deutschland, Minden,
Tel. +49 571 8896-260

Individuelle Formulierung wählbar

SILIKONELASTOMER Um den Verarbeitungsspielraum zu erhöhen, gibt es von Dow Corning, Midland, Michigan, USA, nun Silikonelastomere QP 1 für Festkautschuke HCR (High Consistency Rubbers). Sie eignen sich für medizinische Schläuche und andere extrudierte Anwendungen wie Kurzzeitimplantate. Die Materialien liegen als einkomponentige, nicht-katalysierte Basiskomponenten vor. Sie geben mehr Verarbeitungsspielraum, da die bestgeeignete Formulierung für die jeweilige Anwendung gewählt werden kann. Die Materialien erreichen eine hohe Festigkeit und sind mit Masterbatches des Herstellers auf Platinbasis katalysierbar. Alternativ ist auch die Härtung mit peroxidinitiierten Systemen möglich. Die Silikonelastomere entsprechen in ihrer Formulierung den FDA-Vorgaben für Anwendungen mit Lebensmittelkontakt und sind nach USP Klasse VI geprüft. Sie liegen in vier unterschiedlichen Härtegraden vor. Die HCR-Basisprodukte stehen in den Härtegraden Shore A 30, 50, 60 sowie 70 bereit und enthalten keine Lösemittel, Phthalate oder Latexadditive.

KONTAKT
Dow Corning, Wiesbaden,
Tel. +33 494 786 170


Produkte aus Silkonelastomeren QP 1

Bild: DOW Corning