MATH 150 MIDTERM 1 PRACTICE PROBLEMS

1. Give an inductive definition for the follwing:

(i) Number of occurences of connective $\wedge$ in a formula φ. Similarly do for other connectives.
(ii) Number of occurrences of letter A in a formula φ.
(iii) Numer of occurrences of a formula $A \to B$ as a subformula in φ. In general do the same with an arbitrary formula ψ in place of $A \to B$.
(iv) Numer of symbols in φ that are not parentheses.

2. We make the following definitions:

$A \perp B$	is always false, regardless of the values of A and B.
$A \triangle B$	is tautologically equivalent to $\neg(A \leftrightarrow B)$.
NAND	is tautologically equivalent to $\neg(A \wedge B)$.
NOR	is tautologically equivalent to $\neg(A \vee B)$.

Decide if the given set of connectives is complete or incomplete, and in each case justify your decision with an argument.

(a) $\{\neg, \leftrightarrow\}$.
(b) $\{\to, \vee, \wedge\}$.
(c) $\{\to, \leftrightarrow, \wedge$.
(d) $\{\perp, \to\}$.
(e) $\{\perp, \leftrightarrow\}$.
(f) $\{\perp, \vee, \wedge\}$.
(g) $\{\triangle, \to\}$.
(h) $\{\triangle, \leftrightarrow\}$.
(i) $\{\triangle, \neg\}$.
(j) {NAND}.
(k) NOR}.

3. Build the following circuits using the following units:

(A) Unit NAND only.
(B) Unit NOR only.
(C) Both units NAND and NOR, but try to use minimum number of units.

Description of circuits: There are three lights, A, B and C. Input signals at A, B and C sending by sensors into the circuit are as follows: If the light is on then input signal is "1", if the light is off then input signal is "0". Output signals:

(i) If one or more light is off, output signal is "1". If all of the lights are on, the output signal is "0".
(ii) If two or more light is off, output signal is "1". If all of the lights are on, the output signal is "0".
(iii) If precisely two lights are on then the output signal is "0". Otherwise the output signal is "1".
(iv) If at least one, but not all lighs are on then the output signal is "0". Otherwise the output signal is "1".
(v) If light A is on and precisely one of B, C is also on then the output signal is "1". Otherwise the output signal is "0".

4. Given is a set of formulas Σ. Decide if the following is correct, and justify by an argument.

(a) $\Sigma \models A \to B$ or $\Sigma \models A \wedge \neg B$.
(b) If $\Sigma \models A \vee B$ and $\Sigma \models \neg A$ then $\Sigma \models B$.
(c) If $\Sigma \cup \{A \wedge B\}$ is not satisfiable then $\Sigma \models \neg A$ or $\Sigma \models \neg B$.
(d) If $\Sigma \cup \{A \vee B\}$ is not satisfiable then $\Sigma \models \neg A$ and $\Sigma \models \neg B$.