Algebra II — exercise sheet 1

Hand in solutions on 16.10.2014

1. Let N be the set of all matrices in $\mathrm{GL}_n(K)$ with exactly one non-zero entry in every row and every column. Show that N is a closed subgroup of $\mathrm{GL}_n(K)$, that its identity component $N^\circ = \mathrm{D}_n$ is the subgroup of diagonal matrices, that N has $n!$ connected components and that N is the normaliser of D_n.

2. Give examples of non-closed subgroups of $\mathrm{GL}_2(\mathbb{C})$ and compute their closures.

3. Describe the Hopf algebra structures on the coordinate rings of $\mathbb{G}_a$ and GL_n.

4. Prove that a T_0 topological group is already T_2. Show that an infinite linear algebraic group is always T_0 but never T_2. Explain the discrepancy!

5. Show that the product of irreducible affine K-varieties is again irreducible. This fails for non-algebraically closed fields K: exhibit zero divisors in $\mathbb{C} \otimes_{\mathbb{R}} \mathbb{C}$.

Textbooks

A. Borel:	*Linear Algebraic Groups*, Springer GTM 126 (1969, 1997).
J.E. Humphreys:	*Linear Algebraic Groups*, Springer GTM 21 (1975).
T.A. Springer:	*Linear Algebraic Groups*, Birkhäuser (1981, 1998).
P. Tauvel, R.W. Yu:	*Lie Algebras and Algebraic Groups*, Springer (2005).

Scripts

Florian Herzig:	Toronto 2013.
Tamás Szamuely:	Budapest 2006.
Nicolas Perrin:	Bonn 2004.

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Lectures:	Monday, 12.15, large lecture hall Wegelerstraße 10
	Thursday, 14.15, small lecture hall Wegelerstraße 10
Tutorials:	Wednesday, 16.15 (Orlando); Friday, 12.15 (Tomasz)