Design of Partially Prestressed Concrete Structures Based on Swiss Experiences

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A characteristic feature of partially prestressed concrete is the fact that cracking is tolerated to a certain extent under working load conditions. This cracking is the consequence of precompressing the tension zone of a beam or a slab to a degree deliberately less than that required for full prestress. By doing so, the unfavorable side effects of full prestressing, such as severe creep, substantial loss of prestress and large upward deflection (camber), can be considerably reduced or avoided altogether. Fine and well distributed cracks, on the other hand, are usually harmless.

In 1939, Emperger[1] suggested that ordinary reinforced structures be provided with additional prestressing wires to attain finer cracks. In 1945, Abeles[2] approached the idea from the other direction: of the wires required for the ultimate moment of pretensioned elements, he would either tension only a part of them to their full capacity, or all of them to an initial prestress well below that normally utilized in prestressing. The design was verified by checking concrete stresses, obtained from the assumption of the section being uncracked, against fictitiously high permissible tensile stresses for the concrete.

Today, the term partially prestressed concrete denotes primarily post-tensioned structures with a more or less arbitrary ratio of prestressing steel and ordinary nonprestressed mild steel. Usually, the prestressing steel is tensioned to the same initial prestress as it would be for fully prestressed concrete. The stresses and strains are calculated for combined bending and axial load (the prestressing force) assuming the section to be cracked. However, pretensioned members may also be partially prestressed.

Papers on partial prestressing have become more numerous in recent years. Literature in English has been compiled in Ref. 3 and a survey of the state of the art is given in Refs. 4 and 5. Most of these papers present theoretical research work or deal with suggested design procedures for statically determinate structures.

Less well known in English speaking countries is that as early as 1968, partial prestressing became an official design practice in Switzerland. The Swiss Code SIA 162[6] provided efficient rules for the design of statically determinate as well as indeterminate structures in buildings, highway and railway bridges. This development was based, among other factors, on certain design concepts[7] as well as on theoretical and practical investigations[8,9] which had been completed or were in progress at the Institute of Structural Engineering of the Swiss Federal Institute of Technology in Zurich (ETH). Since that time, the majority of prestressed structures in Switzerland have been designed according to these rules. Full prestressing is now used only in exceptional cases.

Experience with partial prestressing has been overwhelmingly positive, and no cases of damage attributable to partial prestressing are known. On the contrary, repetition of some of the earlier cases of damage due to using too high a prestress has been avoided.

The following presentation covers both statically determinate and indeterminate post-tensioned structures with bonded tendons. Some of the principles can also be applied to precast pretensioned members (see later discussion).

Synopsis

This paper is based on the Swiss experiences with partial prestressing and reflects both research and design practice. This experience has been gained primarily with post-tensioned structures and began in 1968.

A simple method of design is proposed that allows a smooth transition from reinforced to fully prestressed concrete. Depending on the area of prestressing steel selected, the area of additional nonprestressed mild steel must be varied.

Major emphasis is given to the meticulous detailing of nonprestressed reinforcement (and especially to narrow bar spacings) to ensure serviceability and crack width limitations.

In addition, the desirability of including a certain minimum amount of nonprestressed reinforcement, even for fully prestressed structures, is emphasized and the influence of the degree of prestress on some key parameters is discussed. Finally, the design method is illustrated with an example of a three-span continuous highway bridge.

ENGINEER'S VIEWPOINT

For a better understanding and appreciation of the suggestions made later in this paper a few important preliminary remarks need to be made. To arrive at a good engineering structure the following requirements and priorities must be fulfilled:

1. Sound overall concept
2. Good detailing
3. High quality workmanship
4. Sufficient design calculations

Today (perhaps because of the advent of the computer), an unfortunate tendency prevails in overemphasizing design calculations and underrating the other aspects, in particular Points 2 and 1. Often there is an erroneous belief that sophisticated design methods will yield better structures. But quite the contrary, such methods often run the risk of blind faith in formulas without achieving a really sound structure and, consequently, lead to endless calculations with the increased likelihood of grave mistakes, especially in terms of structural detailing.

Moreover, complicated design methods may well hamper the progress of the art. An example of this is partially prestressed concrete. The author is convinced that the design procedure for partially prestressed concrete structures can and must be simple. Otherwise, partial prestressing is not likely to become accepted in practice.

With this in mind, the following simple design procedure for partially prestressed members and structures is suggested. It is oriented toward the needs of the practicing engineer. Several assumptions and approximations will be introduced, which are found to be sensible and justified with regard to the design task as a whole. For example:

1. Since the initial prestress may differ by as much as 10 percent from the design value (because of the scatter of anchorage and friction losses), there is no need to make a precise computation of the decompression force. (This term will be explained later in the paper.) It is sufficiently accurate to take the decompression force as the effective prestressing force, P_e. This is determined by considering concrete strain due to creep and shrinkage as would be done for a fully prestressed member.[12,17]

2. Similarly, the decompression moment can be calculated approximately and expressed in terms of P_e.

3. The analysis of indeterminate structures for dead and live load may reasonably be based on an uncracked section because, in general, under service loads, cracking does not significantly affect the moment distribution.

The same philosophy used in the design procedure also proves valuable in code regulations, which should give clearcut principles and be easy to apply. They should not try to replace engineering judgment but emphasize the responsibility of the designer.

The chapter treating partial prestressing in the Swiss Code[6] contains just 600 words and no formulas. Experience with it since 1968 has shown that simple and clearcut code provisions lead to a wide acceptance of partial prestressing.

DECOMPRESSION MOMENT

The applied moment capacity before tensile stresses would be developed in a prestressed cross section is equal to the so-called decompression moment M_{Dec} This can be practically defined as the bending moment which, combined with the action of the effective prestressing force after shrinkage and creep of concrete and relaxation of prestressing steel, P_e, produces zero concrete stress at the extreme fiber of a section at which tensile stresses are caused by applied loads (Fig. 1).

For statically indeterminate structures, the secondary moment due to prestressing, M_{Ps}, must be taken into account. This value depends on several parameters such as the variation of eccentricity and the magnitude of the prestressing force along the structure, its support conditions, and other situations.

DEGREE OF PRESTRESS

The degree of prestress provides a measure of the intensity to which a cross section is prestressed. Prestressed sections have a decisive advantage over nonprestressed sections in that their be-

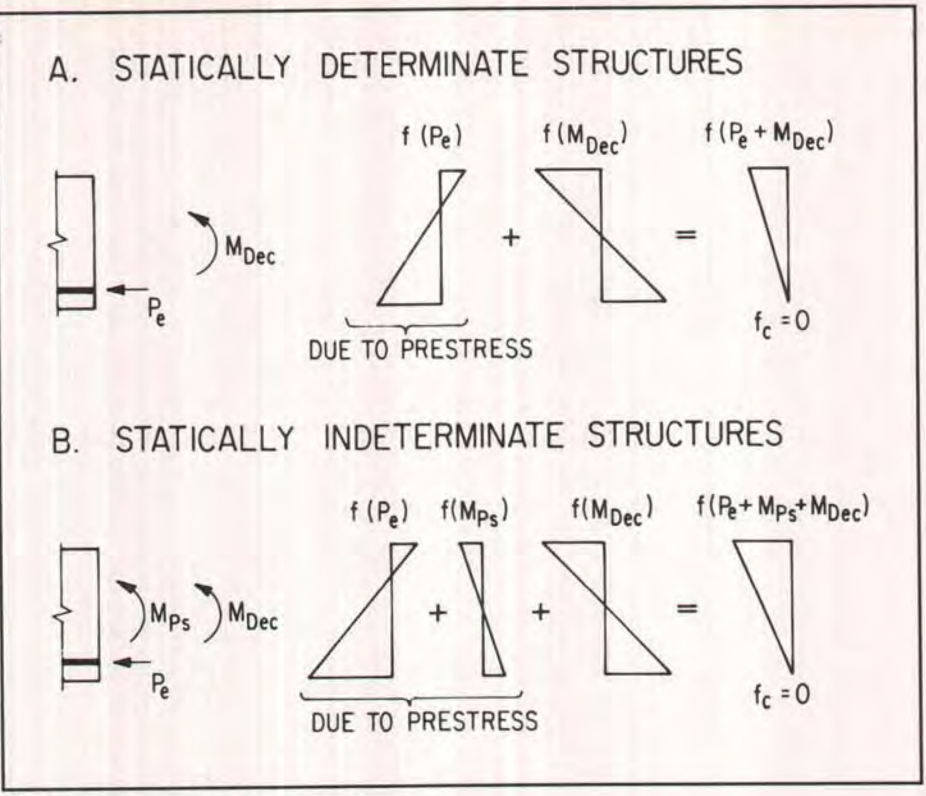

Fig. 1. Definition of decompression moment.

havior is more favorable under service conditions (especially crack width and deflections). At the ultimate limit state, however, no substantial difference exists. An appropriate definition of the degree of prestress should therefore take into account the effects of prestress under service conditions.

The following definition of the degree of prestress κ has been used by the author since the late sixties.[9] Another differentiation can be made between the service load degree of prestress κ and the permanent load degree of prestress $\bar{\kappa}$.

Service Load Degree of Prestress

For both statically determinate and indeterminate structures the service load degree of prestress κ at a given cross section is defined as:

$$\kappa = \frac{M_{Dec}}{M_{D+L}} \tag{1}$$

where

M_{Dec} = decompression moment as defined above

M_{D+L} = moment due to total service load, i.e., dead load D plus live load L

Thus, the degree of prestress κ represents that portion of the total service load moment for which the section is prestressed. A value of $\kappa = 0$ means no prestressing, i.e., ordinary reinforced concrete, whereas a $\kappa = 1$ corresponds to full prestressing. The parameter κ also represents that portion of the effect of the total service load which is compensated by the effect of prestressing in the cross section considered.

Permanent Load Degree of Prestress

For certain purposes it may be convenient to use the permanent load degree of prestress $\bar{\kappa}$ which is defined as:

$$\bar{\kappa} = \frac{M_{Dec}}{M_D} \qquad (2)$$

where

M_{Dec} = decompression moment as defined above

M_D = moment due to dead or permanent load D

If the cross section is prestressed to the degree $\bar{\kappa} = 1$, zero stress in the extreme tension fiber is attained for the effect of dead load plus prestress. Then the cross section is just fully prestressed for permanent loads.

Obviously, the definitions of the degree of prestress κ and $\bar{\kappa}$ are primarily valuable for beam structures, where the decompression moment can be easily calculated. For slabs, such as flat slabs with column strip prestressing,[13] other definitions of the degree of prestress may be appropriate.

Relevance of the Degree of Prestress

The definitions of the decompression moment and of the degree of prestress are not only of theoretical significance but also of practical importance. The definition of the degree of prestress κ can be useful, for instance, in comparing sections with different proportions of prestressed and nonprestressed steel, or for other parametric studies.

From a practical viewpoint, for some types of structures a certain value of κ may be always most economical and lead to an acceptable service behavior. In such cases, κ (or $\bar{\kappa}$) serves as a proven value and gives the designer a very useful guide. Nevertheless, in many practical design situations the value of the decompression moment and the degree of prestress need not be calculated.

SMOOTH TRANSITION FROM REINFORCED TO FULLY PRESTRESSED CONCRETE

During the years when only ordinary reinforced concrete and fully prestressed concrete were in use, it appeared justifiable to use quite different design methods for the two methods of construction. With the introduction of partial prestressing, such a differentiation appears to serve no useful purpose. A modern design method must enable a smooth transition from nonprestressed through partially prestressed to fully prestressed concrete: a design method, in fact, for any degree of prestress.

Consequently, the method must present a unified approach to reinforced, partially prestressed, and fully prestressed concrete structures for both ultimate and serviceability limit states. The method developed by the author and presented below meets these requirements.

DESIGN METHOD

The aim of the design method is to determine the longitudinal prestressed and nonprestressed (mild steel) reinforcement in the governing cross sections of a partially prestressed concrete beam structure so as to fulfill the ultimate and serviceability requirements. The shear design and other aspects are not considered here.

It is important to note that the main ideas of the design method are independent of any particular code. Most of the basic data, such as load factors, resistance factor (or capacity reduction factor), material constants, etc., can be taken from any code. For the design example given in Appendix A, these basic values are taken from the Swiss Code.[6]

It is assumed that all concrete dimensions of the structure are known. In normal cases, as a first approach, the same concrete dimensions as for an analogous fully prestressed structure can be taken. Furthermore, it is assumed that all material constants of concrete, prestressed steel and nonprestressed (mild) steel are known, and that the moments due to vertical dead and live loads have been calculated.

The proper design of an engineered structure is usually not a straightforward process. Iteration loops may be necessary. This is also true for the design of partially prestressed concrete structures. However, to make the design procedure as clear as possible, only the main design steps and no eventual iteration loops are presented below.

Step 1 — Choice of moment for which section must be prestressed

In partially prestressed concrete structures the engineer can choose the prestressed and nonprestressed reinforcement in such a way that construction is made easier when compared with fully prestressed structures. The reason for this is the considerable freedom of design, allowed particularly for the choice of the size, number and location of the prestressing tendons.

In continuous beams, for instance, the size and length of the tendons can be chosen from a practical viewpoint. For example, the same tendons can be used for more than one span or even the entire length of the structure.

In addition, the choice of the moment for which a section must be prestressed, i.e., the decompression moment, is often highly influenced by engineering judgment for a given situation such as:

(a) Durability, e.g., no cracks under permanent load over intermediate supports of a continuous girder bridge.

(b) Economic conditions, e.g., the ratio of the price of prestressing steel to the price of nonprestressed steel.

(c) Deformation and fatigue considerations.

(d) Minimum required amount of nonprestressed mild steel reinforcement.

(e) Miscellaneous factors.

One or a combination of the above factors may be decisive in the choice of the decompression moment and thus to the degree of prestress in the governing cross sections.

In many cases, the first choice of the desired decompression moment M_{Dec} can be based on the bending moment due to permanent load, M_D. For buildings, $M_{Dec} \simeq M_D$ is a common choice. This corresponds to a permanent load degree of prestress of $\bar{\kappa} \simeq 1$. For highway bridges a similar or slightly higher value of M_{Dec} may be appropriate.

As mentioned above, the first chosen value of M_{Dec} can change during the subsequent design process. For instance in continuous beams, the resulting value of M_{Dec} in a certain cross section may be influenced by other practical considerations at the same section or at other governing cross sections.

In some cases, instead of choosing M_{Dec} it may be appropriate to start with the desired balancing radial forces u of parabolically curved tendons. They are commonly related to the permanent or dead load D. Often $u \simeq 0.8\,D$ is chosen, which corresponds to $\bar{\kappa} \simeq 1.0$.

Step 2 — Design of prestressed reinforcement

In the second step, the necessary initial prestressing force in the governing cross sections is calculated for the adapted moment M_{Dec} by the usual procedures for prestressed concrete, e.g.,

$$P_i = \frac{1}{\eta} \; \frac{M_{Dec} + M_{Ps}}{e + k} \qquad (3)$$

where

P_i = initial prestressing force in cross section considered (after deducting friction losses)

η = P_e / P_i = estimated reduction factor for prestressing force P_i, accounting for losses due to

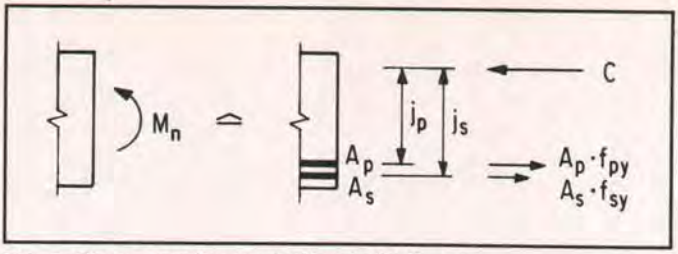

Fig. 2. Stress resultants at ultimate limit state.

shrinkage and creep of concrete and relaxation of prestressing steel: η may vary from 0.8 to 0.9, with $\eta = 0.9$ for $\bar{\kappa} \leq 1$ and $\eta = 0.8$ for $\bar{\kappa} >> 1$.

e = eccentricity of prestressing force measured from centroid of uncracked concrete section. The distance of the centroid of tendons from the extreme tension fiber must be estimated first; see design example in Appendix A.

k distance from centroid of uncracked section to the kern limit opposite to the line of action of the prestressing force, e.g., top kern limit when prestressing force is on bottom side of member.

M_{Ps} = secondary moment due to prestress as defined previously. For the initial calculation of P_i an estimate will be adequate (see design example in Appendix A).

Based on the prestressing force P_i, the percentage of friction losses and the permissible initial stress of the tendon steel at the jacking end, the required area of prestressed reinforcement A_p can be calculated and the appropriate tendon size can be chosen (see design example in Appendix A).

Step 3 — Design of nonprestressed reinforcement

In the third step, the required area of nonprestressed reinforcement A_s is determined from the nominal moment strength needed (Fig. 2):

$$M_n = A_p f_{py} j_p + A_s f_{sy} j_s \qquad (4)$$

and

$$A_s = \frac{M_n - A_p f_{py} j_p}{f_{sy} j_s} \qquad (5)$$

where

M_n = nominal moment strength at section needed to fulfill the basic requirements for strength design. Note that M_n must be calculated with the safety factors of the applied code (load factors and resistance factor or capacity reduction factor), taking into account the estimated or calculated value of M_{Ps} in statically indeterminate structures (see design example in Appendix A).

f_{py} = specified yield strength of prestressed reinforcement

f_{sy} = specified yield strength of nonprestressed reinforcement

j_p = internal lever arm from centroid of prestressed reinforcement to line of action of compression force

j_s = internal lever arm from centroid of nonprestressed reinforcement to line of action of compression force

With some experience, the location of the compression force:

$$C = A_p f_{py} + A_s f_{sy} \qquad (6)$$

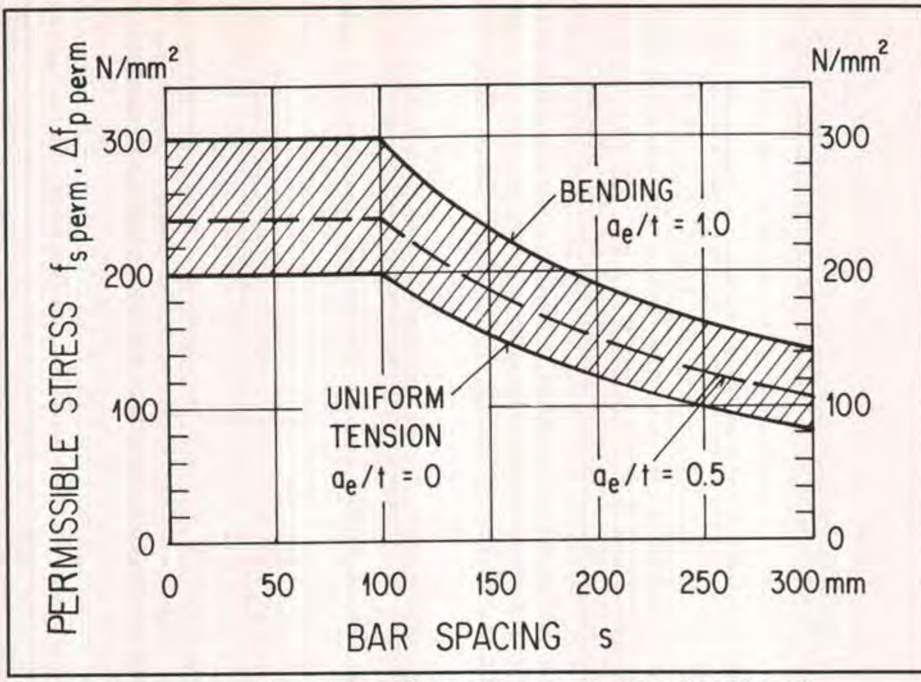

Fig. 3. Reference stress for crack width limitation as a function of spacing of reinforcement according to Draft Swiss Code SIA 162, August 1983. (Conversion factors: 100 mm ≃ 4 in., 200 N/mm² ≃ 30 ksi.)

and hence the magnitude of the lever arms of the internal forces, j_p and j_s, can be estimated with reasonable accuracy or may be calculated by iteration.

Step 4 — Meticulous detailing of nonprestressed reinforcement

The next, and often last, design step concerns the detailing of the nonprestressed reinforcement. The following factors must be considered:

(a) The closest possible spacing of bars compatible with the placing of concrete must be chosen so that under total service load, cracks which occur are only small and well distributed. Close spacing of bars (and in girders also close spacing of stirrups) leads to a narrow mesh of reinforcement near the surface, which significantly improves the quality of the structure.

(b) Even for high degrees of prestress, a minimum amount of nonprestressed reinforcement is necessary (see next main section in report).

Step 5 — Crack width limitation

An experienced designer will often be able to detail carefully the nonprestressed reinforcement without any further computation. However, for structures with stringent requirements some doubt concerning the maximum permissible bar spacing may arise. For such cases, the following procedure for determining the maximum bar spacing for sufficient crack width limitation may be followed.

This is based on the new draft of the Swiss Code SIA 162, dated August 1983. The chart in Fig. 3 has been reproduced from this draft code. The abscissa gives the bar spacing *s* (bonded tendons are to be included, i.e., one tendon corre-

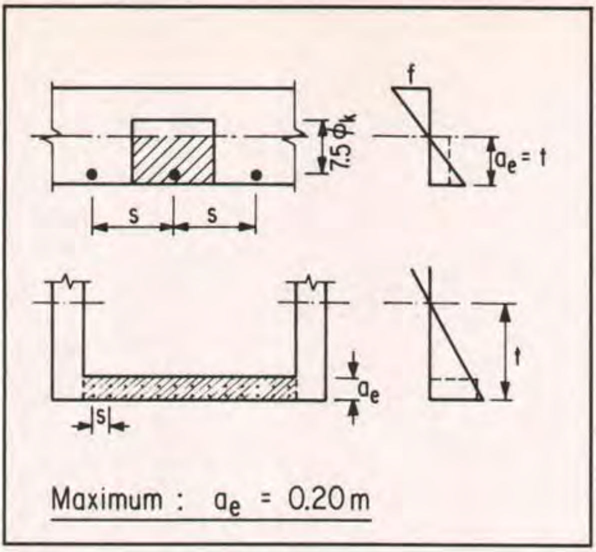

Fig. 4 Effective concrete tension zone for calculating the stress gradient index a_e/t (according to Draft Swiss Code SIA 162, August 1983).

sponds to one bar). The ordinate gives the permissible stress in the nonprestressed reinforcement $f_{s,perm}$, and the permissible stress increase in the prestressed reinforcement $\Delta f_{p,perm}$, respectively.

The diagram is intended for use in the case of "stringent requirements." These correspond to an average crack width of 0.15 mm (0.006 in.). For other values of crack widths similar diagrams can be developed. The upper curve in Fig. 3 is valid when the tensile flexural stress gradient across the section being considered is steep (e.g., through a solid slab). The lower curve is valid when the tensile flexural stress gradient is zero, or in other words, when the section is under practically uniform tension (e.g., a very thin tension flange of a deep section).

The stress gradient is represented by the ratio a_e/t defined in Fig. 4 (reproduced from the draft Swiss Code) for various cross section shapes, in which:

a_e = height of concrete tension zone effective for reinforcement considered [maximum a_e = 0.20 m (8 in.)]

t = height of concrete tension zone under relevant bending moment and axial load (see below) assuming an uncracked cross section

ϕ_k = bar diameter

The above proposition implies that the stress in the nonprestressed reinforcement f_s and the stress increase in the prestressed reinforcement Δf_p, calculated at the cracked cross section, should not exceed the permissible value given in Fig. 3. The values f_s and Δf_p in the cracked cross section can be calculated as for sections under combined bending and axial load using the modular ratio n (see Fig. 5).

If the crack width limitation is desired under the action of permanent and live load, the bending moment used for the calculation is M_{D+L} in the case of a statically determinate structure, whereas it is $M_{D+L} + M_{Ps}$ in a statically indeterminate structure. If the crack width limitation is only desired for permanent load

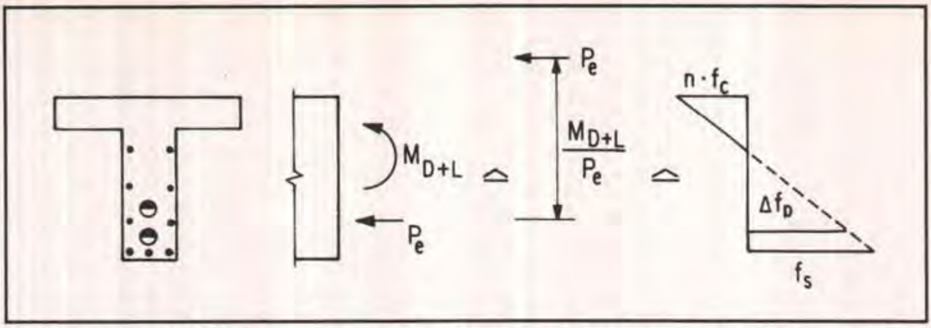

Fig. 5. Sectional forces and stress distribution in a cracked cross section (for statically indeterminate structures, M_{D+L} is substituted by $M_{D+L} + M_{Ps}$).

or for permanent load plus a part of live load, the bending moment is correspondingly reduced.

Note that M_{Ps} must be calculated according to the usual procedures for analyzing prestressed concrete structures. As an axial load the prestressing force P_e (after all losses) in the tendon axis may be used, calculated in the usual manner for fully prestressed concrete, i.e., neglecting the time-dependent effects of the additional nonprestressed steel in the flexural tension zone. For a rigorous approach, the decompression force should be used instead, but the error involved in taking the more convenient value of P_e is very small (see also Refs. 12 and 17). The value of n employed in this calculation is not very critical; a value of $n = 10$, which is on the safe side with respect to the resulting steel stresses, may be used, but $n = 7$ may also be appropriate.

This procedure of using P_e to calculate f_s and Δf_p in partially prestressed concrete structures has been found to work well in Swiss design practice since 1968. Note that the current Swiss Code[6] limits f_s and Δf_p in buildings and highway bridges to 150 N/mm² (approx. 22 ksi) independent of bar spacing.

For calculating f_s and Δf_p in cracked partially prestressed cross sections, tables[14] have been used successfully in Switzerland during the last 15 years as a standard design aid in engineering offices. Both tables and corresponding computer programs are based on the principles described above (axial load = P_e). The stress calculation may also be done by using tables and computer programs as described in Ref. 15 (axial load = decompression force).

MINIMUM MILD STEEL REINFORCEMENT EVEN FOR FULL PRESTRESSING

In fully prestressed concrete structures containing post-tensioned bonded tendons, it has often been the practice to provide only a very limited amount of additional nonprestressed mild steel reinforcement, with the expectation that no substantial tensile stresses will occur. The fact, however, is that such structures are often subject to relatively high tensile stresses (for instance stresses caused by shrinkage and temperature, due to differences between the actual and calculated bending moments in statically indeterminate structures, or due to support settlement, overloading, and other conditions).

If in fully prestressed concrete structures sufficient additional nonprestressed reinforcement is not provided, they frequently exhibit a very unsatisfactory cracking behavior, e.g., cracks occur at large spacings and steel elongation concentrates at a few cracks. Furthermore, the bending stiffness of such a cracked zone is thus considerably reduced due to the low total content of reinforcement.

Partially prestressed structures containing a mixture of prestressed and nonprestressed reinforcement, by contrast, have a larger total area of reinforcement. This leads to a greater bending stiffness in the cracked state and, as a result of the crack distributing effect of the nonprestressed reinforcement, to an improved cracking behavior. Partially prestressed structures, when subject to tensile stresses, thus exhibit in general a more favorable service performance than analogous more highly prestressed structures containing relatively small amounts of nonprestressed reinforcement.

It is, therefore, usually preferable in prestressed structures to reduce the area of prestressed reinforcement, i.e., the decompression moment and degree of prestress, respectively, and to incorporate a minimum amount of nonprestressed reinforcement. This steel area may be calculated from the condition that the reinforcement before yielding must be able to resist the tensile force which would crack the concrete in the corresponding tension zone.

In general, the following rules, based on the author's experience, can also be used:

(a) The area of the nonprestressed reinforcement must be at least:

$\bar{\rho}_{min} =$ 0.3 to 0.4 percent for bending (plates and webs)

$\bar{\rho}_{min} =$ 0.6 to 0.8 percent for uniform tension (tension flanges)

where

$\bar{\rho}_{min} = A_{s,min}/\bar{A}_c$ = ratio of minimum nonprestressed reinforcement

$A_{s,min}$ = area of minimum nonprestressed reinforcement

$\bar{A}_c$ = area of concrete tension zone effective for $A_{s,min}$ (hatched in Fig. 4)

The lower values of $\bar{\rho}_{min}$ should be used for average quality of concrete with compressive strengths of $f'_c \simeq 30$ N/mm² (4350 psi) and the higher values for higher quality of concrete with compressive strengths of $f'_c \simeq 40$ to 50 N/mm² (5600 to 7250 psi).

(b) The minimum reinforcement must be provided as skin reinforcement using the closest practicable bar spacing.

(c) In the webs of T-beams and box girders, the minimum nonprestressed reinforcement may be concentrated to a certain extent close to the extreme tension fiber.

(d) In sections where the tendons are not located near the extreme tension fiber (inclined tendons), the nonprestressed longitudinal reinforcement in the outer tension zone should be designed for a tensile force equal to the shear force at the nearest support due to dead and live load.[19]

The above rules apply for post-tensioned structures using bonded tendons. For pretensioned members the need for nonprestressed reinforcement depends on the bond properties of the prestressing steel, provided that nonprestressed steel is not needed for ultimate strength. In the case of well distributed deformed thin wires with a diameter up to 7 mm (0.28 in.) as commonly used in Switzerland, additional nonprestressed reinforcement is unnecessary. Any cracks which may occur will be closely spaced, due to the favorable bond conditions of the deformed wires, and only small crack widths may be expected under service loads.

DEFLECTION CONTROL

For serviceability, in most cases the long-term deflection under permanent load is of primary interest, not the deflection under full live load. Partially prestressed structures show a far more favorable long-term deflection behavior than either fully prestressed or ordinary reinforced analogous concrete structures.

Fully prestressed structures often show considerable camber because they have been prestressed to compensate for tension due to full live loads. This camber will not develop in partially prestressed structures due to the lower overall prestress. With a judicious choice of the degree of prestress, the stress gradient under permanent load, i.e., the difference between top and bottom fiber stresses, can be kept very small. Consequently, the long-term deformations will also be small.

In the large majority of cases met in practice, the degree of prestress is chosen so as to prevent cracking under permanent load ($\bar{\kappa} \geq \approx 0.9$). Thus, the long-term deflections can be calculated using the homogeneous section as with fully prestressed concrete.

Only in the few cases of a small degree of prestress, when cracking is to be expected under permanent load, do the computational methods of reinforced concrete need to be employed, considering combined bending and axial force in the cracked state. The same applies in calculating elastic deformations under full design live load when cracking occurs.

FATIGUE LOADING

The stress increases in nonprestressed and prestressed steel due to live load can on principle be calculated in the same manner as described in Step 5. This calculation is presented in Ref. 10 for both $\bar{\kappa} > 1$ and $\bar{\kappa} < 1$. The same publication discusses the influence of the degree of prestress κ on the stress increase in the steel as a function of the ratio M_D/M_{D+L}.

In Switzerland since 1968 partially prestressed buildings and highway bridges were designed assuming $f_{s,perm} = \Delta f_{p,perm} = 150$ N/mm² (approx. 22 ksi), calculated for full live load at the cracked cross section. Among all types of these structures, deck slabs of highway bridges, partially prestressed in the transverse direction corresponding to $\bar{\kappa} \approx 1$, may be the type of structure most likely to be subject to fatigue loading. However, to date no damage due to fatigue is known.

Fatigue tests on concrete beams containing a mixture of nonprestressed and prestressed steel[15] have shown that the prestressed steel tends to behave better than the nonprestressed steel. The majority of fatigue failures occurred in the nonprestressed steel. This leads to the finding that the fatigue problem of partially prestressed concrete is not significantly different from that of ordinary reinforced concrete.

On the other hand, fatigue tests on post-tensioned beams with extremely curved bonded tendons have shown that fatigue failure of prestressing wires may occur due to friction between the prestressed steel and the metal duct prior to the fatigue failure of the nonprestressed steel.[18] More research on this subject seems to be needed, including also the definition of the fatigue loads.

INFLUENCE OF DEGREE OF PRESTRESS

The influence of the degree of prestress on some important parameters is of particular interest. The following example is taken from Ref. 10. For a rectangular cross section in a statically determinate structure subject to a total bending moment M_{D+L}, Fig. 6 shows the influence of the degree of prestress κ on the following parameters:

(a) Total ultimate safety factor ($\gamma = M_n/M_{D+L}$ according to SIA 162[6]).

(b) Areas of prestressed reinforcement A_p, nonprestressed reinforcement A_s, and total reinforcement $A_p + A_s$.

(c) Stress f_s in nonprestressed reinforcement.

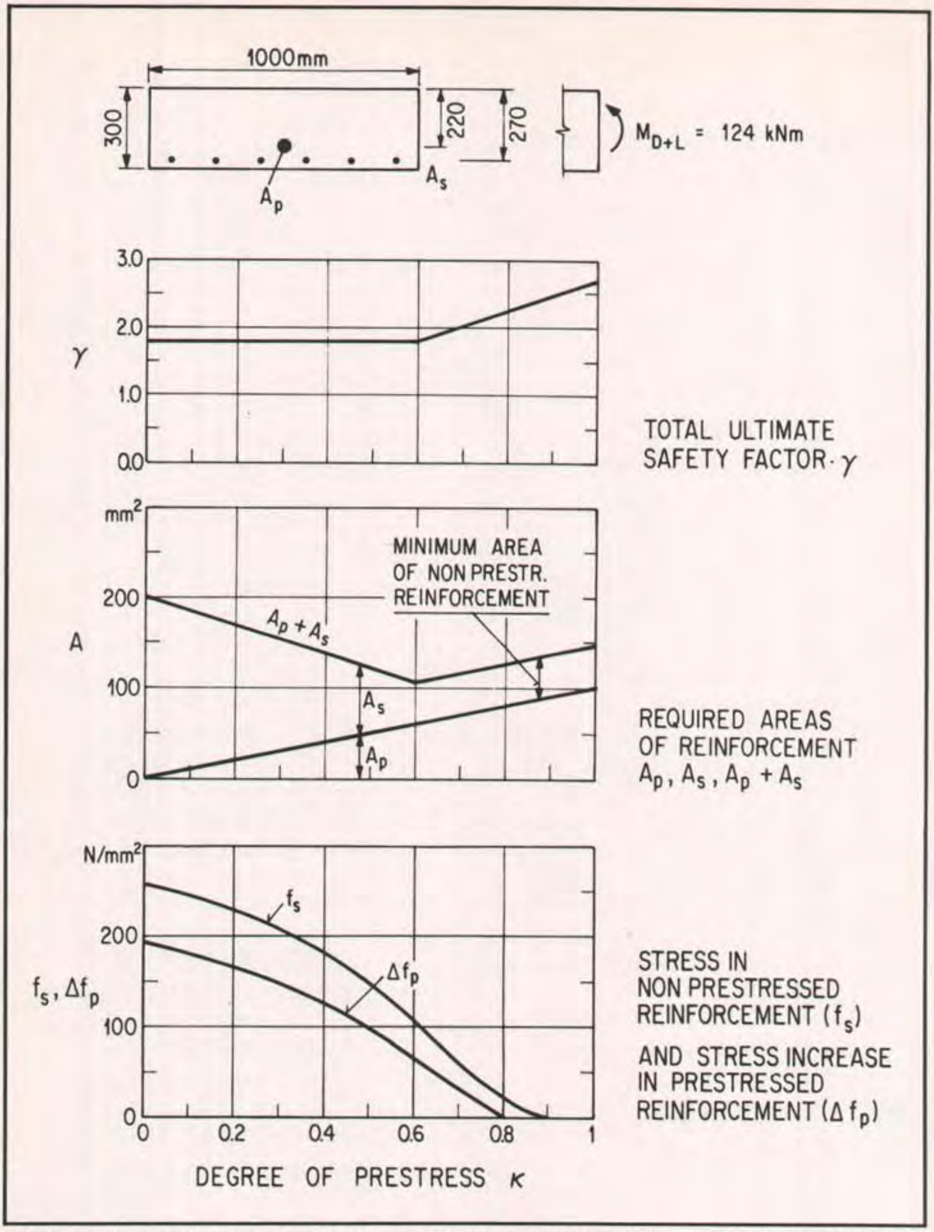

Fig. 6. Influence of degree of prestress on key parameters. (Conversion factors: 1000 mm ≃ 40 in., 200 mm² ≃ 0.3 sq in., 200 N/mm² ≃ 30 ksi, 124 kNm ≃ 1100 kips-in.)

(d) Stress increase Δf_p in prestressed reinforcement.

The cross section was at first designed to be fully prestressed (fulfilling the usual concrete stress conditions). The area of prestressing steel was then successively reduced, and the area of nonprestressed steel increased, if found necessary, to maintain the total ultimate safety factor at 1.8 (see also Appendix A, Step 3). The area of nonprestressed steel was not reduced below a certain minimum. Fatigue loading of the section was not considered.

It must be noted that the shape of the curves in Fig. 6 is influenced to a certain extent by the shape of the member cross section, e.g., for flanged cross sections slightly different curves appear. However, the following conclusions drawn from the rectangular cross section are in general valid for all cross sections commonly used:

(a) For high degrees of prestress, the value of the ultimate load safety factor is in excess of that required (due to the concrete stress conditions for $\kappa = 1$ and due to the minimum amount of nonprestressed steel).

(b) The total quantity of steel attains its minimum at a certain value of κ ($\kappa =$ 0.6 in this example).

(c) The stress in the nonprestressed steel and the stress increase in the prestressed reinforcement are extremely small for higher degrees of prestress ($\kappa > \simeq 0.7$). For medium values ($\kappa \simeq 0.4$ to 0.7), these stresses are still considerably lower than the steel stresses in ordinary reinforced concrete sections.

In many cases it may be appropriate (see previous section) to choose the area of prestressing steel A_p and the area of nonprestressed steel A_s so that:

(a) The ultimate safety is not in excess of that required.

(b) A_s is equal to the minimum area of nonprestressed reinforcement.

This leads to a degree of prestress κ, for which the total amount of steel, $A_p + A_s$, is a minimum ($\kappa = 0.6$ in this example). Higher degrees of prestress are mostly unsuitable.

SAME INITIAL TENDON STRESS FOR ALL DEGREES OF PRESTRESS

The behavior of partially prestressed concrete structures under service loads depends primarily on the additional strains caused by the stress in the nonprestressed mild steel reinforcement and the stress increase in the prestressed reinforcement after crack formation.

The absolute value of the stress in the prestressed reinforcement has no influence on the structural behavior under service loads as long as it is within the elastic limit, a condition usually fulfilled in practice. Safety against the ultimate limit state is also unaffected, because it is assured by designing the cross section as earlier described in Step 3.

The tendons in partially prestressed structures can therefore be tensioned independently of the degree of prestress to the same initial value f_{pi} or $f_{pi,perm}$, respectively, as for fully prestressed structures.

CONCLUSIONS

Compared to full prestressing, partial prestressing of post-tensioned structures offers considerably more freedom of design, especially in choosing the size, number, and location of prestressing tendons. Depending on the adopted area of prestressing steel, more or less nonprestressed mild steel reinforcement needs to be added to satisfy ultimate and serviceability requirements. It is imperative that a minimum amount of nonprestressed reinforcement, which does not fall below a certain value, is incorporated into all post-tensioned structures, even those which are fully prestressed.

The proposed design method applies well-known and established design procedures for reinforced or fully prestressed structures. The design method is simple and easily applicable for any degree of prestress. It allows a smooth transition from nonprestressed concrete through partially prestressed concrete to fully prestressed concrete.

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APPENDIX A — DESIGN EXAMPLE

The following design example illustrates the design method described earlier in this paper.

The structure selected is a three-span continuous highway bridge. The longitudinal dimensions and cross-sectional configuration at midspan and near the intermediate supports are given in Fig. A1. Shown also in the figure are the loads and material properties.

For illustrative purposes, only the longitudinal direction and only the three governing cross sections A, B, and C (Fig. A2) are considered. (For a final design, additional cross sections would have to be checked.)

Initially, the moments due to dead load and dead load plus live load (extreme values) have to be calculated using the bending stiffness of the uncracked sections. The moment diagram is shown in Fig. A2.

All the governing cross sections A, B, and C are considered within the five design steps of the design procedure. However, in each step only the essential points are covered.

Step 1 — Choice of moment for which section must be prestressed

The first choice is to make the approximate decompression moment in all three governing cross sections (see Fig. A2) equal to the dead load moment, i.e., $M_{Dec} = M_D$.

$M_{Dec,A} = +9{,}860$ kNm (87,260 kips-in.)

$M_{Dec,B} = -20{,}588$ kNm (182,200 kips-in.)

$M_{Dec,C} = +5{,}194$ kNm (54,820 kips-in.)

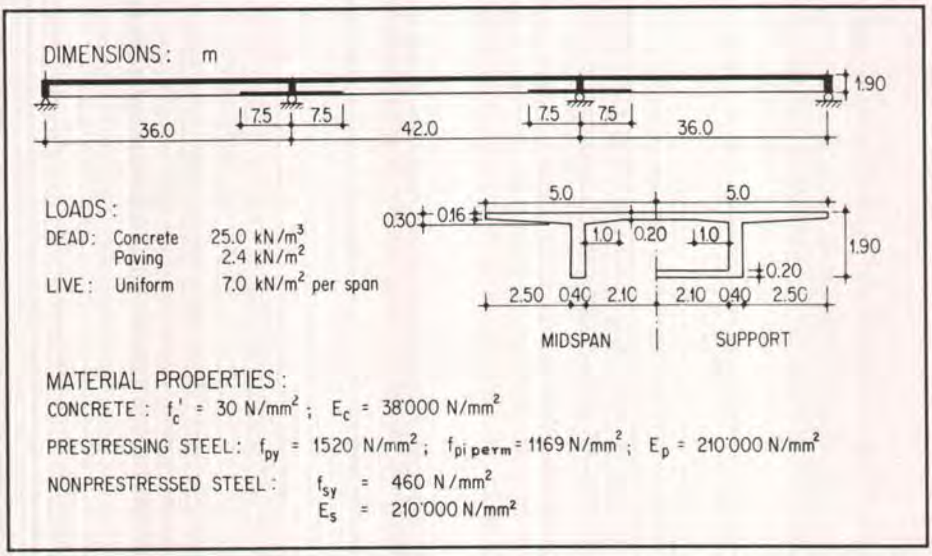

Fig. A1. Design example: Dimensions, loads, and material properties. (Conversion factors: 30 m ≃ 100 ft, 1 m ≃ 40 in., 25 kN/m³ ≃ 160 lb/ft³, 1 kN/m² ≃ 20 lb per sq ft, 1000 N/mm² ≃ 145 ksi).

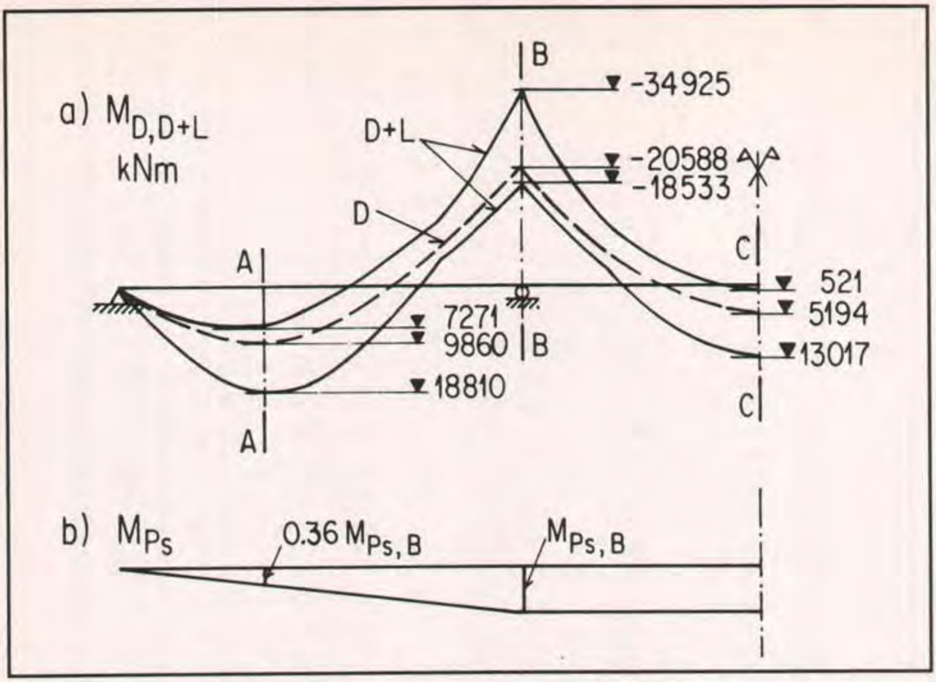

Fig. A2. Design example: Moment diagrams. (Note: 10,000 kNm ≃ 90,000 kips-in.)

Thus, all three cross sections would be fully prestressed for permanent loads ($\bar{\kappa} = 1.0$).

Step 2 — Design of prestressed reinforcement

When applying Eq. (3), the secondary moment due to prestress, M_{Ps}, of which the general shape is shown in Fig. 8b, must be estimated.

For a continuous beam a proven initial approximation is achieved by taking M_{Ps} to (minus) 30 to 50 percent of the decompression moment in the support section. In this example choose 35 percent:

$$M_{Ps,B} = M_{Ps,C} \simeq -0.35 \cdot M_{Dec,B} = 0.35 \cdot 20{,}588 \text{ kNm} = 7{,}206 \text{ kNm}$$
(63,770 kips-in.)

$$M_{Ps,A} = 0.36\, M_{Ps,B} \simeq 0.36 \cdot 7{,}206 \text{ kNm} = 2{,}594 \text{ kNm}$$
(22,960 kips-in.)

The cross-sectional properties such as location of the centroid of section and of the kern limit points are given in Fig. A3.

The distance of the centroid of tendons in all three sections A, B, C from the extreme tension fiber is estimated to be 0.20 m (8 in.).

Hence, the eccentricity (see Fig. A4) of the prestressing force (absolute value) is:

$e_A = 1.23$ m (48.4 in.)
$e_B = 0.52$ m (24.4 in.)
$e_C = 1.23$ m (48.4 in.)

With $\eta = 0.9$ the required initial prestressing forces become:

$$P_{i,A} = \frac{1}{0.9} \cdot \frac{9{,}860 + 2{,}594}{1.23 + 0.21} = 9{,}610 \text{ kN } (2{,}160 \text{ kips})$$

$$P_{i,B} = \frac{1}{0.9} \cdot \frac{-20{,}588 + 7{,}206}{0.52 + 0.72} = 11{,}991 \text{ kN } (2{,}695 \text{ kips})$$

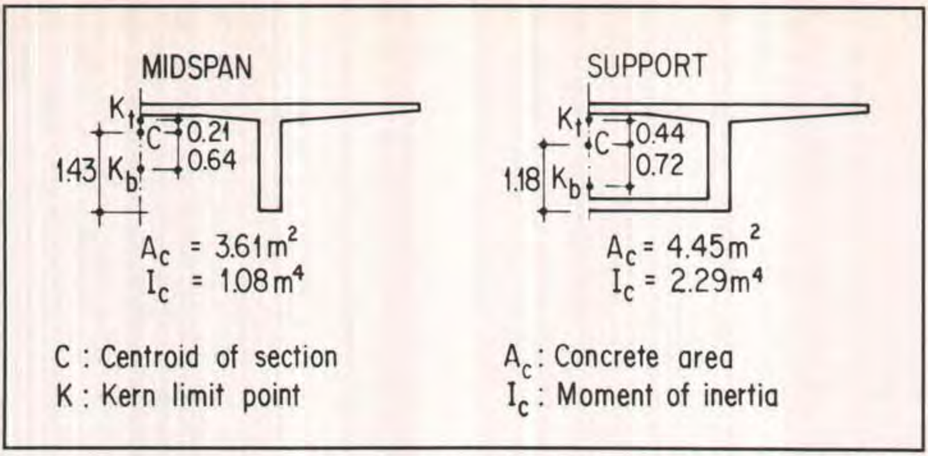

Fig. A3. Design example: Cross section properties. (Conversion factors: 1 m ≃ 40 in. ≃ 3.3 ft, 1 m² ≃ 11 sq ft, 1 m⁴ ≃ 115 ft⁴.)

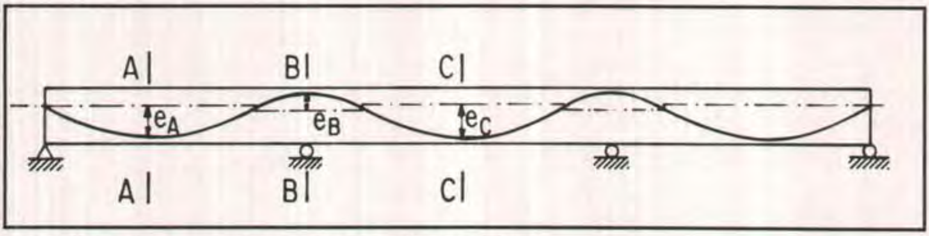

Fig. A4. Design example: Tendon profile (centroid).

$$P_{i,C} = \frac{1}{0.9} \cdot \frac{5{,}194 + 7{,}206}{1.23 + 0.21} = 9{,}568 \text{ kN (2,150 kips)}$$

It can be seen that approximately the same prestressing force is required in all governing cross sections. The tendons are arranged as shown in Fig. A4. and tensioned from both ends.

After calculating the percentage of friction loss according to the assumed tendon profile [defined by ordinates at distances of 1.875 m (73.8 in.)], the initial stresses f_{pi} in the prestressing tendons in the three cross sections can be determined from the permissible initial tendon stress $f_{pi,perm}$ at the jacking end:

$$f_{pi,A} = 0.96 \cdot f_{pi,perm} = 0.96 \cdot 1{,}169 = 1{,}122 \text{ N/mm}^2 \text{ (163 ksi)}$$

$$f_{pi,B} = 0.89 \cdot f_{pi,perm} = 0.89 \cdot 1{,}169 = 1{,}040 \text{ N/mm}^2 \text{ (151 ksi)}$$

$$f_{pi,C} = 0.82 \cdot f_{pi,perm} = 0.82 \cdot 1{,}169 = 959 \text{ N/mm}^2 \text{ (139 ksi)}$$

The above stresses give the area of prestressing steel in each cross section:

$$A_{p,A} \geq \frac{P_{i,A}}{f_{pi,A}} = \frac{9{,}610 \cdot 10^3}{1{,}122} = 8{,}665 \text{ mm}^2 \text{ (13.3 sq in.)}$$

$$A_{p,B} \geq \frac{P_{i,B}}{f_{pi,B}} = \frac{11{,}991 \cdot 10^3}{1{,}040} = 11{,}530 \text{ mm}^2 \text{ (17.9 sq in.)}$$

$$A_{p,C} \geq \frac{P_{i,C}}{f_{pi,C}} = \frac{9{,}568 \cdot 10^3}{959} = 9{,}977 \text{ mm}^2 \text{ (15.5 sq in.)}$$

For illustrative purposes assume that

the BBRV prestressing system is to be used. In this system a tendon comprises 38 wires with a diameter of 7 mm (0.276 in.) and a total steel area of 1462 mm² (2.27 sq in.).

With reference to the tendon layout in Fig. A5, use four tendons in each web comprising an area of:

$$A_{p,A} = A_{p,B} = A_{p,C} = 11{,}696 \text{ mm}^2 \text{ (18.1 sq in.)}$$

This area will provide the following prestress forces in the three governing sections:

$$P_{i,A} = 13{,}123 \text{ kN (2,950 kips)}$$
$$P_{i,B} = 12{,}164 \text{ kN (2,735 kips)}$$
$$P_{i,C} = 11{,}216 \text{ kN (2,520 kips)}$$

Step 3 — Design of nonprestressed reinforcement

In this numerical example the required nominal moment strength M_n is derived from the general design equation according to the Swiss Code:[6]

$$\frac{M_n}{1.3} \geq 1.4\,M_D + 1.4\,M_L + 1.0\,M_{Ps}$$

$$M_n = 1.8\,M_{D+L} + 1.3\,M_{Ps}$$

For the secondary moment due to prestress, M_{Ps}, in general the estimated value from Step 2 may be used. If this value appears unduly approximate, M_{Ps} can be calculated using standard methods for analyzing indeterminate prestressed concrete structures. In this example the following values were calculated with $P_e \simeq 0.9\,P_i$:

$$M_{Ps,A} = 3{,}056 \text{ kNm (27,050 kips-in.)}$$
$$M_{Ps,B} = M_{Ps,C} = 8{,}490 \text{ kNm (75,140 kips-in.)}$$

yielding:

$$M_{n,A} = 37{,}831 \text{ kNm (334,800 kips-in.)}$$
$$M_{n,B} = 51{,}828 \text{ kNm (458,700 kips-in.)}$$
$$M_{n,C} = 34{,}468 \text{ kNm (305,000 kips-in.)}$$

Assuming that the location of the compression force C for bending is in the middle of the compression flanges, and estimating that the centroid of the nonprestressed reinforcement to be in the tension zone, the internal lever arms become:

$$j_{p,A} \simeq j_{p,B} \simeq j_{p,C} \simeq 1.6 \text{ m (63 in.)}$$
$$j_{s,A} \simeq j_{s,C} \simeq 1.6 \text{ m (63 in.)}$$
$$j_{s,B} \simeq 1.7 \text{ m (67 in.)}$$

The required area of nonprestressed steel is calculated from Eq. (5):

$$A_{s,A} = 12{,}753 \text{ mm}^2 \text{ (19.8 sq in.)}$$
$$A_{s,B} = 29{,}530 \text{ mm}^2 \text{ (45.8 sq in.)}$$
$$A_{s,C} = 8{,}184 \text{ mm}^2 \text{ (12.7 sq in.)}$$

To check the assumed internal lever arms, the balancing compression forces are calculated from Eq. (6):

$$C_A = A_{p,A} f_{py} + A_{s,A} f_{sy} = 11{,}696 \cdot 1{,}520 + 12{,}753 \cdot 460 = 23{,}644 \text{ kN (5,315 kips)}$$

$$C_B = A_{p,B} f_{py} + A_{s,B} f_{sy} = 11{,}696 \cdot 1{,}520 + 29{,}530 \cdot 460 = 31{,}362 \text{ kN (7,050 kips)}$$

$$C_C = A_{p,C} f_{py} + A_{s,C} f_{sy} = 11{,}696 \cdot 1{,}520 + 8{,}184 \cdot 460 = 21{,}543 \text{ kN (4,845 kips)}$$

Assume that the full width of the compression flange in the span and above the support is effective in the ultimate state. For this to be true the shear connection between the web and projecting flange must be secured by transverse reinforcement. Then by taking a rectangular stress block, the distance of the neutral axis from the extreme compression fiber becomes:

$$x_A = 0.080 \text{ m (3.15 in.)}$$
$$x_B = 0.210 \text{ m (8.26 in.)}$$
$$x_C = 0.072 \text{ m (2.83 in.)}$$

Note that the first approximation, namely the assumption that the location of the compression force lies in the central plane of the slab turns out to be correct in Case B but slightly conservative in Cases A and B. The 0.2 m (8 in.)

thickness of the slab in compression above the support is appropriate.

Step 4 — Meticulous detailing of nonprestressed reinforcement

A possible arrangement of the nonprestressed reinforcement in the three governing sections is given in Fig. A5. The reinforcement is at even and narrow spacing distributed along the circumference of the section. In the tension flange above the support the reinforcement required for moment strength is concentrated towards the web. To secure the participation of the longitudinal reinforcement in the tension flange (by longitudinal shear), transverse reinforcement is needed.

The actual chosen nonprestressed reinforcement for moment strength amounts to:

$A_{s,A} = 12{,}744$ mm² (19.8 sq in.)
$A_{s,B} = 29{,}966$ mm² (46.4 sq in.)
$A_{s,C} = 8{,}338$ mm² (12.9 sq in.)

In the remaining areas of the sections bars ϕ_k 12, s = 150 mm (6.15 in.), are provided in accordance with the requirements for minimum reinforcement. This yields percentages of:

$\bar{\rho}_{min} = 0.38$ percent in the web
$\bar{\rho}_{min} = 0.75$ percent in the tension flange

Step 5 — Crack width limitation

In this example it is assumed that stringent requirements for crack width limitation prevail. Therefore, the method suggested earlier must be followed including a check for total service load.

Using accepted design procedures and the calculated values of M_{Ps} from Step 3, the reduction of the prestressing forces due to losses of creep and shrinkage and relaxation of prestressing steel are calculated with the following results:

$P_{e,A} = 0.85\,P_{i,A}$
$= 11{,}155$ kN (2,510 kips)
$P_{e,B} = 0.91\,P_{i,B}$
$= 11{,}069$ kN (2,490 kips)
$P_{e,C} = 0.93\,P_{i,C}$
$= 10{,}431$ kN (2,345 kips)

The stress in the extreme layer of bars of the nonprestressed reinforcement, f_s, and the stress increase in the extreme layer of prestressed reinforcement, Δf_p, under the action of M_{D+L}, M_{Ps} and P_e at the cracked section, are calculated with the aid of tables.[14] Therefore, it is assumed again that the total width of the compression slab is effective in all three sections. In the tension slab above the support (Section B), only the steel area required for moment strength is taken into account.

$f_{s,A} = 139$ N/mm² (20.2 ksi)
$f_{s,B} = 143$ N/mm² (20.7 ksi)
$f_{s,C} = 189$ N/mm² (27.4 ksi)
$f_{p,A} = 130$ N/mm² (18.9 ksi)
$f_{p,B} = 136$ N/mm² (19.7 ksi)
$f_{p,C} = 179$ N/mm² (26.0 ksi)

The permissible stress can be determined using Figs. 3 and 4, and the bar spacings given in Fig. A5, from which the parameters:

$(a_e/t)_A = 0.20$ $s_A = 93$ mm (3.7 in.)
$(a_e/t)_B = 0.45$ $s_B = 150$ mm (5.9 in.)
$(a_e/t)_C = 0.19$ $s_C = 140$ mm (5.5 in.)

yield the following stresses:

$f_{s,perm,A} = 220$ N/mm² (31.9 ksi)
$f_{s,perm,B} = 175$ N/mm² (25.4 ksi)
$f_{s,perm,C} = 170$ N/mm² (24.7 ksi)

Note that when comparing f_s with $f_{s,perm}$ in Section C, the permissible steel stress is slightly exceeded. This minor overstress could be removed by providing $4\phi_k 26$ bars instead of $3\phi_k 26$ bars in the bottom layer of the reinforcement, resulting in a smaller bar spacing with a corresponding increase of $f_{s,perm}$.

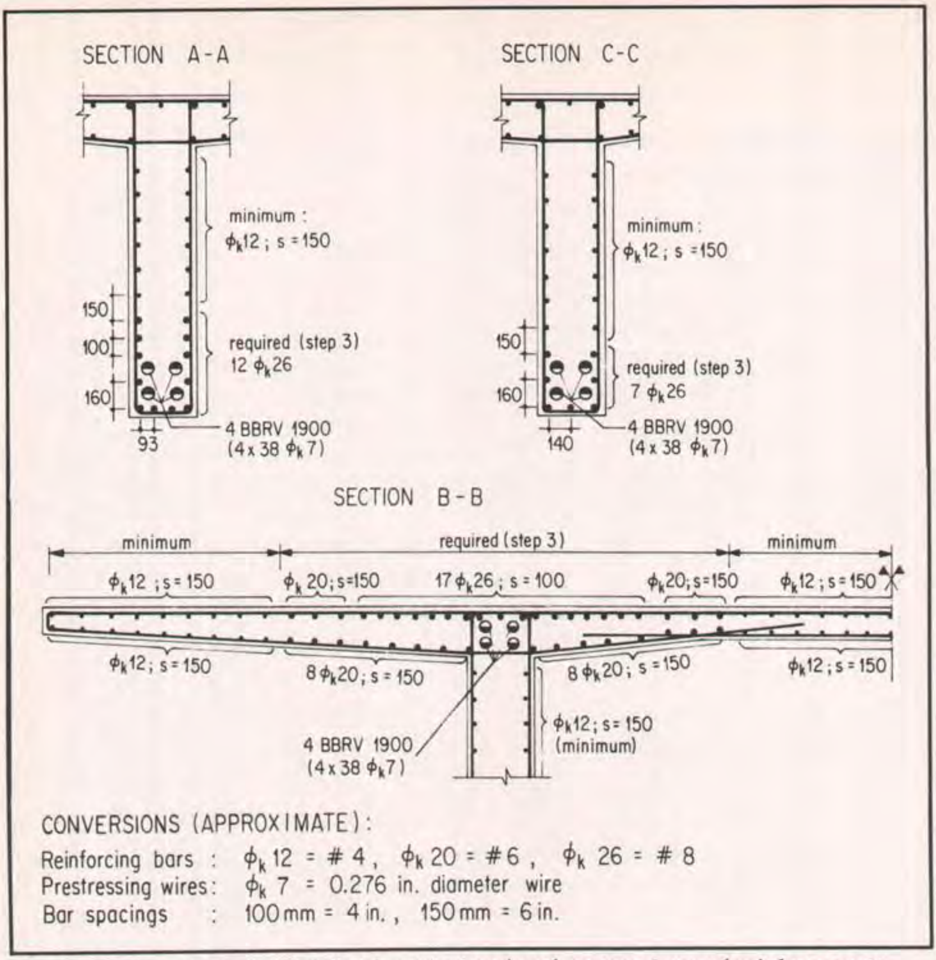

Fig. A5. Design example: Details of prestressed and nonprestressed reinforcement.

Resulting degrees of prestress

Knowing the prestressing forces from Step 5 and the calculated secondary moments M_{Ps} from Step 3, the resulting degrees of prestress can be determined:

$\kappa_A = 0.69$ $\kappa_B = 0.63$ $\kappa_C = 0.60$

$\bar{\kappa}_A = 1.32$ $\bar{\kappa}_B = 1.08$ $\bar{\kappa}_C = 1.26$

Thus, under the action of permanent loads in the considered cross sections no tensile stresses are expected.

Deflection control

The deflection in the middle of the side span is computed using standard procedures. Note that this deflection is higher than that in the center span. Using an assumed creep factor of 2.0 results in:

—Deflection due to permanent load ....+ 58 mm (2.3 in.)

—Deflection due to prestressing ....– 87 mm (3.4 in.)

Thus, under permanent load conditions a camber of 29 mm (1.1 in.) is to be expected. This is less than 1/1000 of the span length (36 mm), which may be a limit for bridges with regard to aesthetic requirements.

APPENDIX B — NOTATION

A_c = area of concrete cross section
$\overline{A}_c$ = area of part of concrete tension zone (shown hatched in Fig. 4)
A_p = area of prestressed reinforcement
A_s = area of nonprestressed tension reinforcement
$A_{s,min}$ = area of minimum nonprestressed tension reinforcement
a_e = height of the concrete tension zone effective for the reinforcement considered as defined in Fig. 4 [maximum a_e = 0.20 m (8 in.)]
C = concrete compression force
E_c = modulus of elasticity of concrete (short time)
E_p = modulus of elasticity of prestressed reinforcement
E_s = modulus of elasticity of nonprestressed reinforcement
e = eccentricity of prestressing force with respect to centroid of uncracked section
f_c = concrete stress
f'_c = specified compressive strength of concrete
f_{pi} = initial stress in prestressed reinforcement at section considered
$f_{pi,perm}$ = permissible initial stress in prestressing tendons at jacking end after tendon anchorage
f_{py} = yield stress of prestressed reinforcement (0.002 offset)
Δf_p = stress increase in prestressed reinforcement
$\Delta f_{p,perm}$ = permissible stress increase in prestressed reinforcement
f_s = stress in nonprestressed reinforcement
$f_{s,perm}$ = permissible stress in nonprestressed reinforcement
f_{sy} = yield stress of nonprestressed reinforcement (0.002 offset)
I_c = moment of inertia of concrete cross section
j_p = internal lever arm from centroid of prestressed reinforcement to line of action of compression force
j_s = internal lever arm from centroid of nonprestressed reinforcement to line of action of compression force
k = distance from centroid of uncracked section to kern limit
M_D = dead load moment
M_{Dec} = decompression moment, produces zero stress at extreme tension fiber
M_{D+L} = total service load moment
M_L = live load moment
M_{Ps} = secondary moment due to prestressing
M_n = nominal moment strength
n = modular ratio E_s/E_c and E_p/E_c
P_e = effective prestressing force in cross section considered
P_i = initial prestressing force in cross section considered
s = bar spacing
t = height of concrete tension zone of uncracked cross section under relevant bending moment and axial load
x = distance of the neutral axis from the extreme compression fiber
η = P_e/P_i = (estimated) reduction factor to the prestressing force P_i, due to shrinkage and creep of concrete and steel relaxation
κ = service load degree of prestress
$\overline{\kappa}$ = permanent load degree of prestress
$\overline{\rho}_{min}$ = $A_{s,min}/\overline{A}_c$ = ratio of minimum nonprestressed tension reinforcement
ϕ_k = bar diameter (mm)
u = radial forces of curved tendons
γ = total ultimate safety factor $\gamma = M_n/M_{D+L}$ (according to SIA 162[6]).

* * *