Lecture

„Internal Material Flow", winter term 2015/16

Exercise: „Reliability and availability"

Reliability *R* is a measure that the establishment within a period of time *T* extending into the future fulfills the required functions trouble-free and correctly under defined conditions.

The life period

The life period of a product t is the time till its failure. Time can mean precise time units (hours, months etc.) but time can also be specified in load changes, switching operations, mileage etc.

Failure

Failure can occur in two ways:

Sudden failure = sudden changeover to an inoperable state (a tire bursts ...)
Drift failure = definined end, e.g. tire reaches 1.6 mm of profil depth

Terms:

Failure	failure – End of functionality
Life period	TTF - time to failure
Average life period	MTTF - mean time to failure (for not repairable parts)
	MTBF - mean time between failure (for repairable parts)
Average period of repair	MTTR - mean time to repair

Weibull distribution

Any reliability prediction can only be done by means of approppriate statistical models. The most important statistical model is the Weibull distribution. According to *parameter b* the Weibull distribution can be an *exponential* or a *logarithmic normal distribution.*

The density funtion of Weibull distribution is:

$$h = \frac{b}{T} \cdot \left(\frac{t}{T}\right)^{b-1} \cdot e^{-\left(\frac{t}{T}\right)^{b}}$$

with

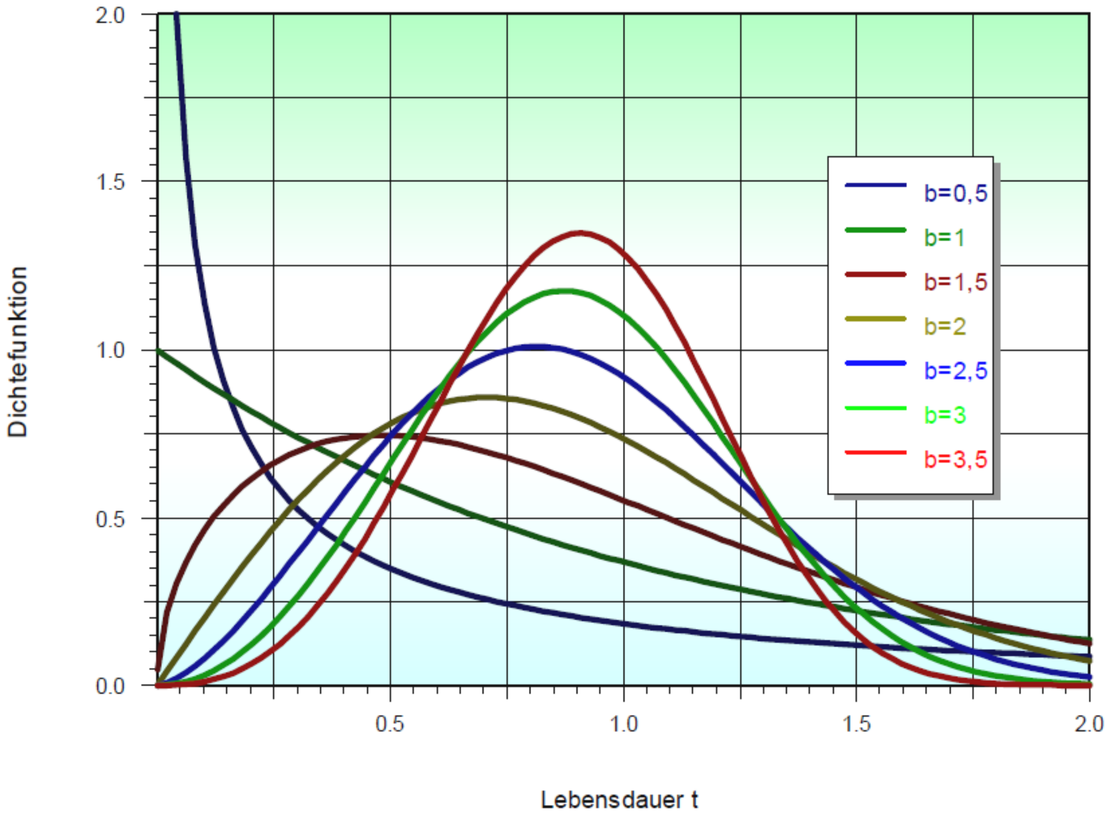
h = Probability density for time T

t = Life period variable (route, period of use, load changes etc.)

T = Characteristic life period in which 63,2% of the units have failed in total

b = Form parameter, gradient of the best-fit line in the Weibull net

The Weibull distribution is a two-parametric, continuous function. The both parameters are the so called **Form parameter b** and the **Location parameter T**. As a random variable *t* is usually used instead of x, because the Weibull distribution is often used in connection with life periods. Weibull distributions do not fulfill the criteron of being memoryless. Therefore, the Weibull distribution is suitable for modeling **early and wear-out failures** of components. The probability that a component failes in the next time interval is often higher with very old components than with new ones. Weibull distributions are only defined for positive values of t.



Quelle: weibull.de

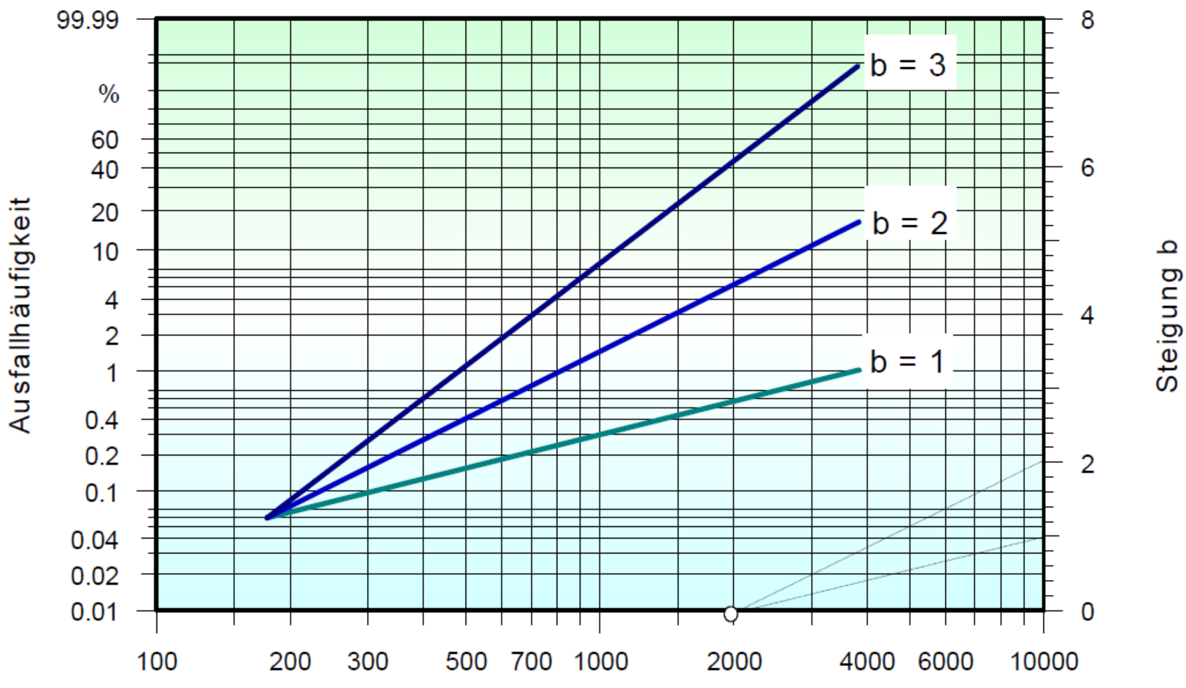
Interpretation of functions and parameters

According to the principal use of the Weibull distribution as a life period distribution is in the following argued with failures.The **form parameter b** can be used to model whether **early or wear-out failures** are more frequent. Is b<1 selected, increasingly early failures occur, with b>1 increasingly wear-out failures. **Is form parameter b=1 selected it follows exactly the exponential distribution. The locations parameter can be used to change the average life period.** But it doesn't generally specify the average lifer period.

Applications

The dominant application of Weibull distibutions are the life period investigations. The distribition function — **contextually called failure probability** – can quickly be identified with a certain net, the *Weibull probability net*. On this paper the **x-axis** is scaled **logarithmically** and the **y-axis doubly logarithmically**. **Thus, the distribution function has the shape of a straight line.**

$H = 1 - e^{-\left(\frac{t}{T}\right)^b}$ with H = Sum of failure probability respectively failure frequency (normalized to 1, in % multiplied by 100)



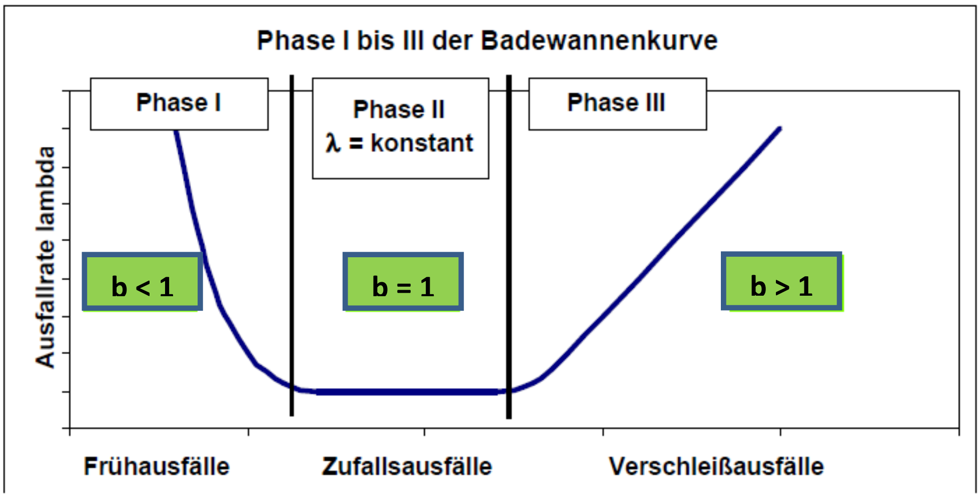
Source: weibull.de

The failure rate λ changes over the life period of many products. The typical development is shown in the so called *bath tub curve*.

There are three phases:

Ergebnis von Lebensdauerversuchen:

Der Verlauf der Ausfallrate λ (Badewannenkurve)



Phase I: Early failures

They come about manufacturing errors; the failure rate decreases for a short term.
In the lifer period net b < 1 (manufacturing errors, material defects, assembly faults, handling errors in the learning phase, „teething problems“ etc.)

Example:
Soft soldering joints that are “cold joints” because of lack of flux do indeed have contact at start (are o.k. in test bay) but break down after a short time (no contact).

Phase II: Randon failures (engaged state)

There is nearly no abrasion; the failure rate is constant.
In the lifer period net b = 1. The early failures are abated, usual wear parts are replaced at regular maintenance, fatigue failures don’t occur, yet, failure events are mainly determined by random failures.

Example:
The black-out of a new ran-in vehicle (totaled) only occurs as a consequence of a crash (by accident).

Phase III: wear-out failures

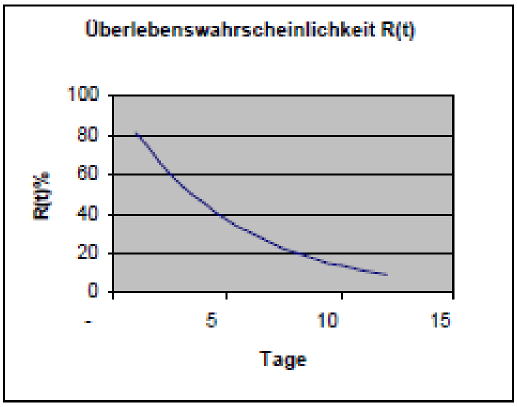
It comes about increasing aging wear-out; the failure rate rises again
In the life period net b > 1

Example:
Ignition plugs, corrosion damages, wear-out on bearings, rotting of foods and medicine.

The exponential distribution (phase II)

It extends in the random area of the bath tub curve (phase II). The failure rate is constant and der form factor is b = 1.

$$R(t) = e^{-\lambda \cdot t}$$



Here applies:

Failure rate	$\lambda = 1/T$
Life period	t
Characteristics of life period	T
Average life period	t-quer = T = $1/\lambda$ = MTBF = MTTF
Survivability	$R(t) = e^{-\lambda * t}$
Failure probability	G(t) = 1 - R(t)
Failure density	$g(t) = \lambda * R(t)$
Original totality	N
Number of still faultless parts	R = N * R(t)
Number of defect parts	G = N * G(t) = N – R

Example:

500 components of a certain type are tested in a life test. It is known that the life period of these components depend on chance, i.e. is exponentially distributed. After 1000 h 5 components have failed.

How many components will survive 25.000 h?

The failure rate of 5/500 in 1000h can be used as an estimation value for the failure rate.

$$\lambda = \frac{5}{500 \cdot 1000h} = 10 \cdot 10^{-6}\ 1/h$$

How many components will survive 25.000 h?

<u>Solution:</u>

$$R = N \cdot R(t)$$

$$R = 500 \cdot e^{-10 \cdot 10^{-6} \cdot 25000}$$

$$R = 389 \text{ Teile}$$

The Weibull distribution (phase I and III)

It applies for areas I and III, because it is not memoryless. It describes the general case where the failure rate is not constant but increases or declines over the time.

$$\lambda(t) = \frac{b}{T} \cdot \left(\frac{t}{T}\right)^{b-1}$$

$$R(t) = e^{-\left(\frac{t}{T}\right)^{b}}$$

Failure rate	λ (t)
Life period	t
Characteristics of life period	T
Survivability	R(t)
Failure probability	G(t) = 1 - R(t)
Failure density	g(t) = λ * R(t)
Proportionate failure	b

Original totality	N
Number of still faultless parts	R = N * R(t)
Number of defect parts	G = N * G(t)

1000 gear shifts are built and delivered. The form factor b=2 and the life period are 140.000 load changes. How many gear shifts are already destructed after 26.000 load changes G(t)?

$$R(t) = e^{-\left(\frac{t}{T}\right)^b}$$

G(t) = 1 - R(t)

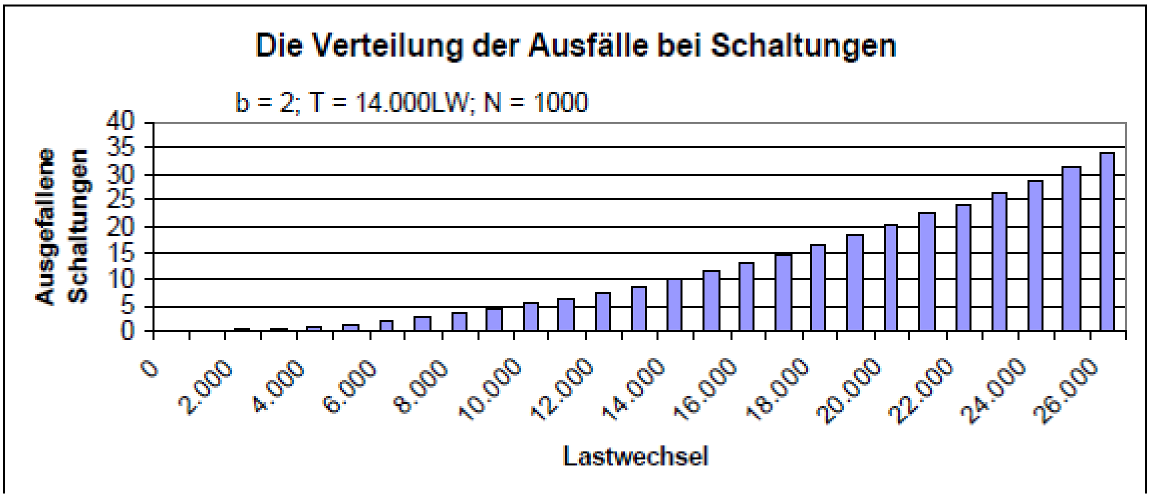
G(t) = 1 - e $^{-(t/T)b}$

G(t) = 1 - e $^{-(26000/140000)2}$

G(t) = 3,39%

G = N * G(26.000) = 1000 * 0,0339 = 34 parts

Result: After 26.000 load changes 34 gear shifts are destructed. In other words: The failure probability is G(t) = 3,39%. The average lift period is approx. 14000 load changes.



λ - Ausfallrate

$$R(t) = e^{-\lambda t} = e^{-t/MTBF}$$

Task 1:

In a warehouse **overload protection devices** are need for efficient rackfeeders which switch off operation with a make contact and in fact with a MTBF of better than **3000 hours**.

a) In a test of **50 overload protection devices over 100 hours** of an available type **2** of them didn't operate any more. **What is the failure rate respectively the MTBF?**

$$\lambda = \frac{r}{n \cdot t} = \frac{2}{50 \cdot 100} = 4 \cdot 10^{-4}\ h^{-1}$$ **Failures per hour**

$$MTBF = \frac{1}{\lambda} = 2500\ h$$

b) How could one achieve a MTBF **of > 3000 hours** with the existing overload protection devices?

Parallel curcuit of two overload protection devices

c) What is the **MTBF** on this measure?

$$MTBF = \sum_{i=1}^{n} \frac{1}{i \cdot \lambda} = \frac{1}{1 \cdot 10^{-4}} + \frac{1}{2 \cdot 10^{-4}} = 2500 + 1250 = 3750\ h > 3000\ h$$

Source: DHBW

Reliability of the function η^{zuv} : $\boxed{\eta^{zuv} = \frac{n_r}{n_r + n_f}}$

n_r - Number of correct functional quality

n_f - Number of faulty functional quality

Probability of disfunction respectively functional reliability:

$\eta^{unz} = 1 - \eta^{zuv}$ oder $\boxed{\eta^{unz} = \frac{n_f}{n_r + n_f}}$

Example:

After the run-in period of 6 months for a rackfeeder that takes pallets from a roll conveyor at a transfer point and stores them in a high bay racking within an investigation of reliability the following numbers were recorded:

Number of correctly executed storing procedures: 12748

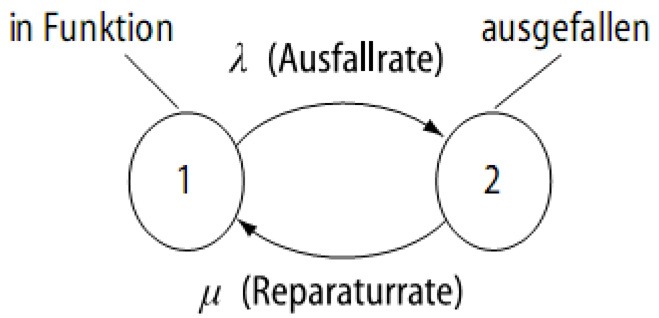
Number of disfunctions: 34

$$\eta^{zuv} = \frac{12748}{12748 + 34} = 0{,}9973 \Rightarrow 99{,}73\%$$

$$\eta^{unz} = 1 - \eta^{zuv} = 1 - 0{,}9973 = 0{,}0027 \Rightarrow 0{,}27\%$$

Availability

Availability records the relation between the sum of expected failures and the total theoretically usable operating time. The random sequence of the states functioning (1) and failed (2) can be modeled as Markov Process simplified.



It follows: $\eta = \lim_{t \to \infty} P_1(t) = \frac{\mu}{\mu + \lambda}$

The availability calculated according to this equation thus characterizes a process that extends over a very long time in a state of balance between degradation (due to failures), and improving (due to repairs).

$E(t_E) = MTBF$:Expected value of failure-free operating times (Mean Time Between Failures)

$E(t_A) = MTTR$: Expected value of downtimes (Mean Time To Repair)

With $MTBF = \frac{1}{\lambda}$ and $MTTR = \frac{1}{\mu}$ it follows:

$$\eta = \frac{MTBF}{MTBF + MTTR}$$

Task 2:

A bottling plant runs **5 days a week with 4 hours each day**. In the recent years the labelling machine failed after aprox. every **200 operating houres** by mechanical wear-out. The **reparair requires 6 hours** and can be executed outside normal operating time.

a) What is the **availability** relating to operationg time when is calculated for the worst case with a failure at the start of the daily operating time, i.e. that the plant can't run the planned 4 hours at one day?

$$\eta = \frac{200}{200+4} = 0{,}98$$

b) Assumed that the plant would be serviced **every 100 hours (outside scheduled operating time)** and the wear-out parts would be replaced in order to prevent production downtimes. Which **availability** could the plant achieve?

MTBF is assumed to 100 h, MTTR to 0 h, because it is outside operating time

$$\eta = \frac{100}{100+0} = 1$$

c) A repair with replacement of parts costs **1000 €**, production downtime in case **Fall a) only costs 200 €,** because only some deliveries have to be rearranged. For case a) the rapair after **every 200 hours on average** has of course to be considered. Would it be worth to service the machine **regularly after 100 hours**?

Case a): every 200 h: Costs of repair + breakdown costs = 1000 + 200 = 1200 €
Case b): every 100 h: Costs of repair = 1000 €
after 200 h: = 2000 €
Conclustion: Every 200 h case b) would be more expensive by 800 € – It's not worth it!

d) How would the result from c) change when **breakdown costs would be 1100 €**?

Case a): every 200 h: Costs of repair + breakdown costs = 1000 + 1100 = 2100 €, here service would be cheaper!

e) When would it be worth to assemble a better labeling machine for **11.000** that needs to be serviced only **every 1000 hours for 500 €?**

Service costs after every 1000 h:	500 €
Costs case a) after 1000 h:	(1000/200)*1200 = 6000 €
Savings after every 1000 h:	5500 €

*Amortization after (asset cost/savings)*MTBF*$_{neu}$ *= (11000/5500)*1000 = 2000 h*

Source: DHBW

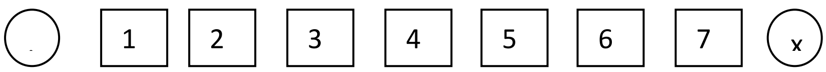
Series connection of elements

If the function of each element is required for the function of a system, then this corresponds to in <u>series connection</u> of all system elements. In this case the overall availability is::

$$\eta_{ges} = \eta_1 \cdot \eta_2 \cdots \eta_n = \prod_{i=1}^{n} \eta_i$$

Task 3:

The following system is given:



$\eta_{ges} = 70\%$ All stations have the same availability!

- **What has the single availability to be?**

$$\eta_{ges} = \eta_1 \cdot \eta_2 \cdots \eta_7 = \eta^7 = 0{,}7$$
$$\eta_i = \sqrt[7]{0{,}7} = 0{,}95$$
$$\Rightarrow \eta_i = 0{,}95 \Rightarrow \text{95\%-ge Verfügbarkeit.}$$

Technical throughput:

Technical throughput results from the „target performance" divided by overall availability. Each element of the systems has to enbable this throughput.

$\lambda_{tech} = \frac{\lambda_{soll}}{\eta_{ges}}$ λ_{soll} - required throughput of a plant.

$\lambda_{res} = \lambda_{tech} - \lambda_{soll}$ λ_{res} - Throughput reserve for the compensation of disfunctions.

Task 4:

λ_{tech} = 125 St./h; λ_{soll} = 100 St./h

Given is: 10 stations in series connection.

- **What is the single availability?**

$$\eta_{ges} = \frac{\lambda_{soll}}{\lambda_{tech}} = \frac{100}{125} = 0{,}8$$

$$\eta_i = \sqrt[10]{\eta_{ges}} = \sqrt[10]{0{,}8}$$

$\eta_i = 0{,}98 \Rightarrow$ 98%-ge Verfügbarkeit.

If the function of one element is sufficient for the function of a system then this corresponds to a parallel arrangement.

The overall availability is calculated as follows:

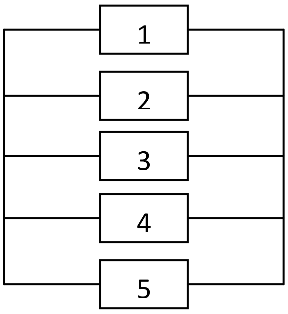
$$\eta_{ges} = 1 - (1-\eta_1)\cdot(1-\eta_2)\cdots(1-\eta_n)$$

$$\eta_{ges} = 1 - \prod_{i=1}^{n}(1-\eta_i)$$

Task 5: Parallel arrangement (Redundant system!)

Full redundancy (parallel circuit): Each of these elements can take 100% of the throughput.

Given is the following system:



$\eta_{ges} = 0{,}99$

All elements have the same availability.

- **What is the single availability?**

$$\eta_{ges} = 1 - (1 - \eta_i)^5$$
$$(1 - \eta_i)^5 = 1 - \eta_{ges}$$
$$\eta_i = 1 - \sqrt[5]{1 - \eta_{ges}}$$
$$\eta_i = 1 - \sqrt[5]{0{,}01}$$
$$\boxed{\eta_i \cong 0{,}6}$$

Small single availability but huge overall availability!

Partial redundancy:

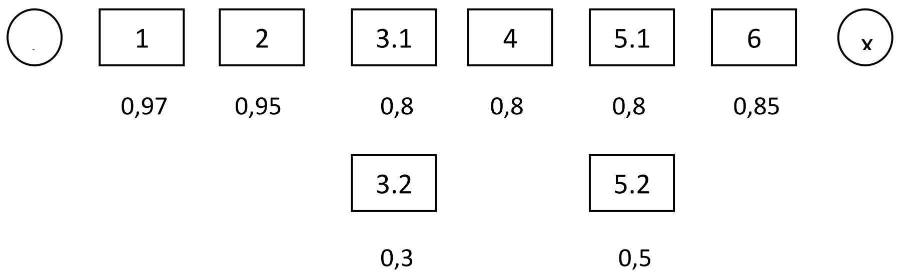
$P_{tech,i}$: Portion that the element i can take from the total throughput, when it is operating without disturbance.

With real redundancy is $P_{tech,i} = 1$.

$$\eta_i = 1 - \left[\underbrace{(1-\eta_{i.1}) \cdot (1-P_{tech,i.2})}_{\text{Failure probability element i.1 multiplied by throughput reserve El. i.2}} + \underbrace{(1-\eta_{i.2}) \cdot (1-P_{tech,i.1})}_{\text{Failure probability Element i.2 multiplied by throughput reserve El. i.1}} - \underbrace{(1-\eta_{i.1}) \cdot (1-\eta_{i.2}) \cdot (1-P_{tech,i.1}-P_{tech,i.2})}_{\text{Failure probability both simultaneously}} \right]$$

Task 6:

The following system is given:



$P_{tech,3.1} = 1 \qquad P_{tech,5.1} = 0{,}9$

$P_{tech,3.2} = 0{,}3 \quad P_{tech,5.2} = 0{,}6$

λ_{soll} = 100 St./h

λ_{tech} = ?

$$\eta_3 = 1 - \left[(1-0,8)\cdot(1-0,3) + (1-0,3)\cdot(1-1) - (1-0,8)\cdot(1-0,3)\cdot(1-1-0,3) \right]$$
$$\eta_3 = 1 - [0,2\cdot 0,7 + 0,7\cdot 0 - 0,2\cdot 0,7\cdot(-0,3)]$$
$$\eta_3 = 1 - [0,14 + 0 - 0,14\cdot(-0,3)]$$
$$\eta_3 = 1 - [0,14 + 0,042]$$
$$\eta_3 = 1 - 0,182$$
$$\eta_3 = 0,818$$

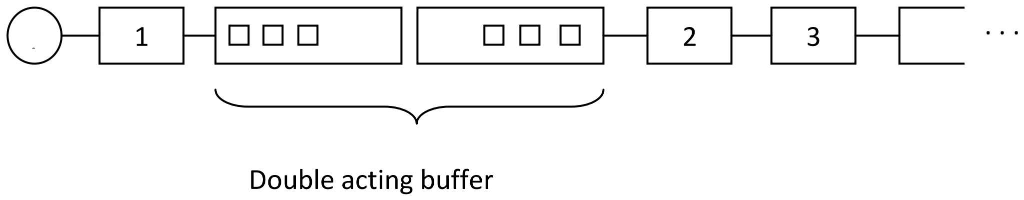
$$\eta_5 = 1 - \left[(1-0,8)\cdot(1-0,6) + (1-0,5)\cdot(1-0,9) - (1-0,8)\cdot(1-0,5)\cdot(1-0,9-0,6) \right]$$
$$\eta_5 = 1 - [0,2\cdot 0,4 + 0,5\cdot 0,1 - 0,2\cdot 0,5\cdot(-0,5)]$$
$$\eta_5 = 1 - [0,08 + 0,05 - 0,1\cdot(-0,5)]$$
$$\eta_5 = 1 - [0,08 + 0,05 + 0,05]$$
$$\eta_5 = 1 - 0,18$$
$$\eta_5 = 0,82$$

$$\eta_{ges} = 0,97\cdot 095\cdot 0,818\cdot 0,8\cdot 0,82\cdot 0,85 = 0,42$$

λ_{soll} = 100 pieces/h (given)

$$\lambda_{tech} = \frac{\lambda_{soll}}{\eta_{ges}} = \frac{100}{0,42} = 238 \text{ St./h}$$

Assignment with buffer



Buffer is used for bridging faults and acts as a parallel system!

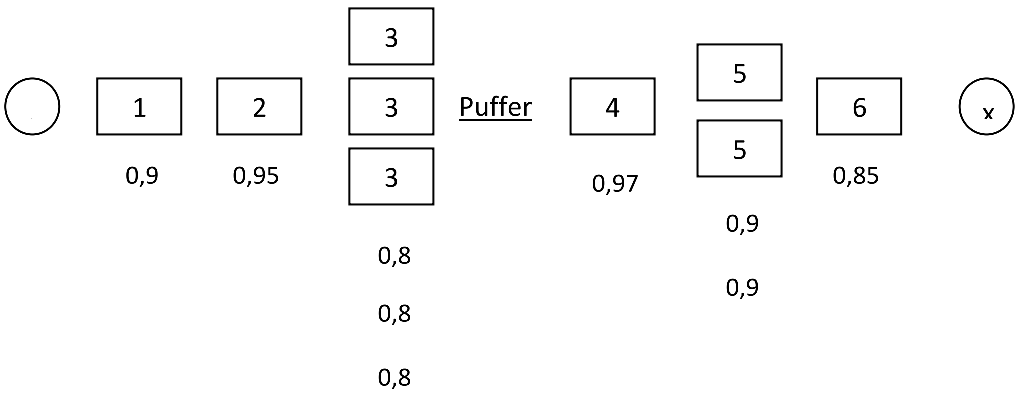
$\eta_p = \eta_o + f(1-\eta_o)$ $\quad\eta_o$ - upstream side

A part f of the malfunction period is covered by the buffer

$f = \frac{2}{3}$ with exponential malfunction distribution (typical malfunction period distribution).

Task 7:

The following system is given:



$P_{tech,5.1} = 0{,}75 \qquad P_{tech,5.2} = 0{,}8$

λ_{soll} = 100 St./h $\qquad \lambda_{tech}$ = ?

$$\eta_3 = 1-\left[(1-0{,}8)\cdot(1-0{,}8)\cdot(1-0{,}8)\right] = 0{,}992$$

$$\eta_5 = 1-\left[(1-0{,}9)\cdot(1-0{,}8)+(1-0{,}9)\cdot(1-0{,}75)-(1-0{,}9)\cdot(1-0{,}9)\cdot(1-0{,}75-0{,}8)\right]$$
$$\eta_5 = 1-\left[0{,}1\cdot 0{,}2+0{,}1\cdot 0{,}25-0{,}1\cdot 0{,}1\cdot(-0{,}55)\right]$$
$$\eta_5 = 1-\left[0{,}02+0{,}025+0{,}0055\right]$$
$$\eta_5 = 0{,}95$$

$$f = \frac{2}{3}$$

$$\eta_p = \eta_o + f(1 - \eta_o)$$

$$\eta_{p_{1,2,3}} = 0{,}9 \cdot 0{,}95 \cdot 0{,}0992 + \frac{2}{3} \cdot (1 - 0{,}9 \cdot 0{,}95 \cdot 0{,}992)$$

$$\eta_{p_{1,2,3}} = 0{,}949$$

$$\eta_{p_{4,5,6}} = 0{,}97 \cdot 0{,}95 \cdot 0{,}85 = 0{,}783$$

$$\eta_{ges} = \eta_{p_{1,2,3}} \cdot \eta_{p_{4,5,6}} = 0{,}949 \cdot 0{,}783 = 0{,}743$$

$$\lambda_{tech} = \frac{\lambda_{soll}}{\eta_{ges}} = \frac{100}{0{,}743} \cong 135 \text{ St./h}$$

Literature:

1. VDI 3581: *Verfügbarkeit von Transport- und Lageranlagen sowie deren Teilsysteme und Elemente*, 2004

2. VDI 4004: *Zuverlässigkeitskenngrößen – Übersicht*, 1986

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4. Peter S. Weygant: *Clusters for High Availability.* Hewlett-Packard Professional Books / Prentice Hall PTR, 1996

5. Gerhard H. Schlick: *Sicherheit, Zuverlässigkeit und Verfügbarkeit von Maschinen, Geräten und Anlagen mit Ventilen,* Expert Verlag, 2001