Pauli's "New Testament":

The 1933 *Handbuch* Article – *Die allgemeinen Prinzipien der Wellenmechanik*

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History of Science Society Annual Meeting
Phoenix, AZ
November 21, 2009

Pauli and Einstein, 1926

Wolfgang Pauli – Lebenslauf

1900 Born in Vienna
1921 Ph.D. with Sommerfeld in Munich
- 1921 *Relativitätstheorie* article for the *Enzyklopädie der mathematischen Wissenschaften*

1921-1922 *Assistent* with Born in Göttingen
1922-1923 Bohr Institute in Copenhagen
1923-1928 Hamburg
- 1925 Exclusion principle
- 1926 *Quantentheorie* article for the *Handbuch der Physik*

1928-1940 ETH Zürich
- 1931 University of Michigan
- 1931 Lorentz Medal
- 1933 *Die allgemeinen Prinzipien der Wellenmechanik* article for the *Handbuch der Physik*
- 1935-1936 IAS Princeton

1940-1946 Princeton University
- 1940 Spin-statistics theorem
- 1941 University of Michigan
- 1942 Purdue University
- 1945 Nobel Prize

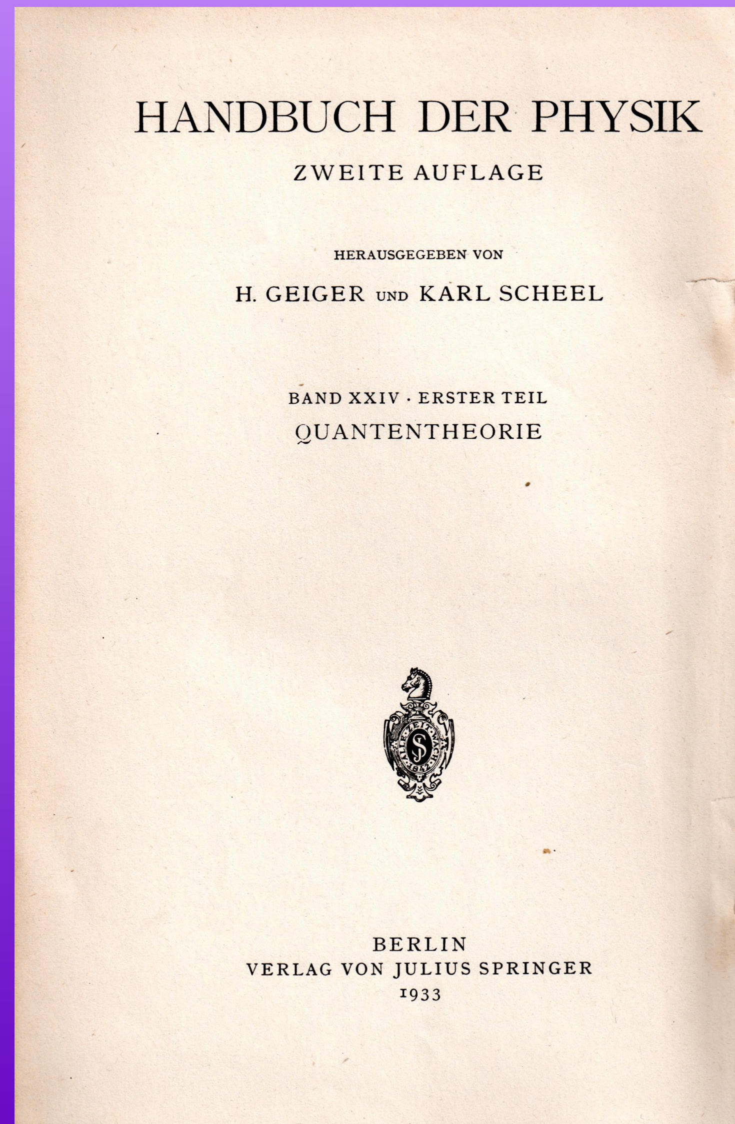
1946-1958 ETH Zürich
- 1958 Revised edition of *Handbuch* article

1958 Dies in Zürich

Pauli, 1924

Handbuch der Physik, 2nd ed. Hans Geiger and
Karl Scheel, eds. Berlin: Julius Springer, 1933.

Pauli, Heisenberg, and Fermi, Lake Como, 1927



HANDBUCH DER PHYSIK

ZWEITE AUFLAGE

HERAUSGEGEBEN VON

H. GEIGER UND KARL SCHEEL

BAND XXIV · ERSTER TEIL

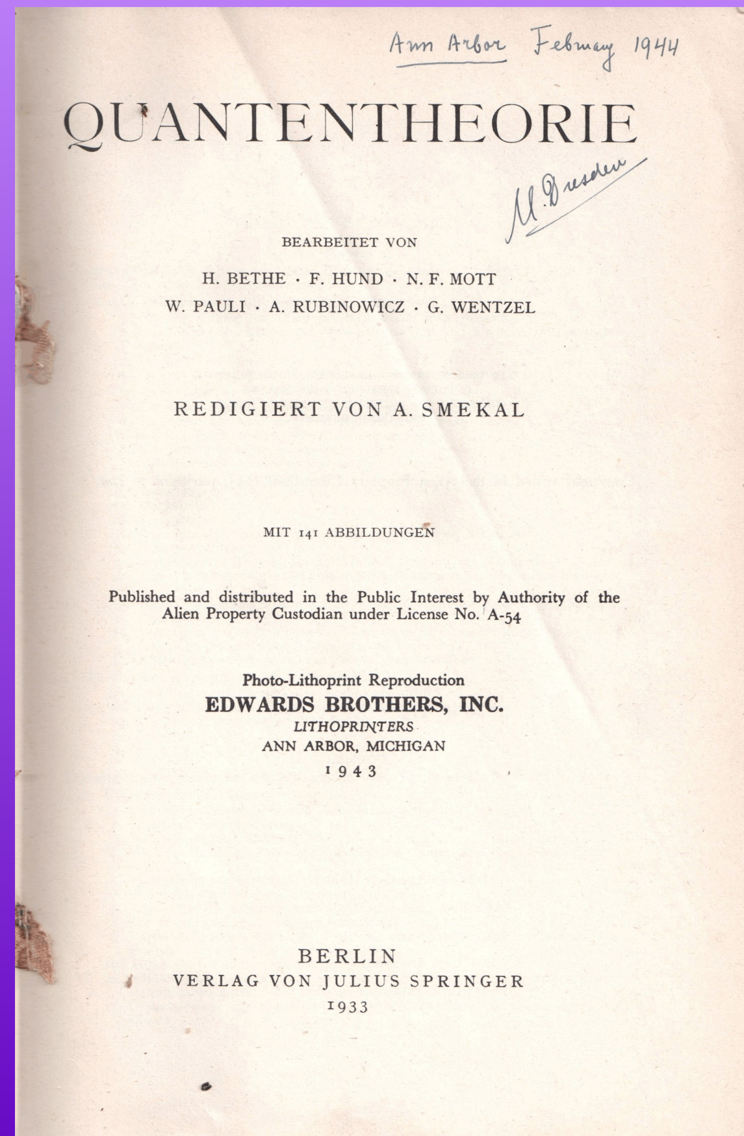
QUANTENTHEORIE

BERLIN
VERLAG VON JULIUS SPRINGER
1933

Handbuch der Physik, 2nd ed. Hans Geiger and Karl Scheel, eds. Vol. 24, Part 1, *Quantentheorie*. Adolf Smekal, ed. Berlin: Julius Springer, 1933.

Contents:

1. A. Rubinowicz (Lemburg)
   "Ursprung und Entwicklung der älteren Quantentheorie," 1-82.
2. W. Pauli (Zürich)
   "Die allgemeinen Prinzipien der Wellen-mechanik," 83-272.
3. H. Bethe (München)
   "Quantenmechanik der Ein- und Zwei-Elektronenprobleme," 273-560.
4. F. Hund (Leipzig)
   "Allgemeine Quantenmechanik des Atom- und Molekelbaues," 561-694.
5. G. Wentzel (Zürich)
   "Wellenmechanik der Stoß- und Strahlungs-vorgänge," 695-784.
6. N. F. Mott (Bristol)
   "Wellenmechanik und Kernphysik," 785-841.



Ann Arbor February 1944

QUANTENTHEORIE

BEARBEITET VON

H. BETHE · F. HUND · N. F. MOTT
W. PAULI · A. RUBINOWICZ · G. WENTZEL

REDIGIERT VON A. SMEKAL

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BERLIN
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1933

Max Dresden – Curriculum Vitae

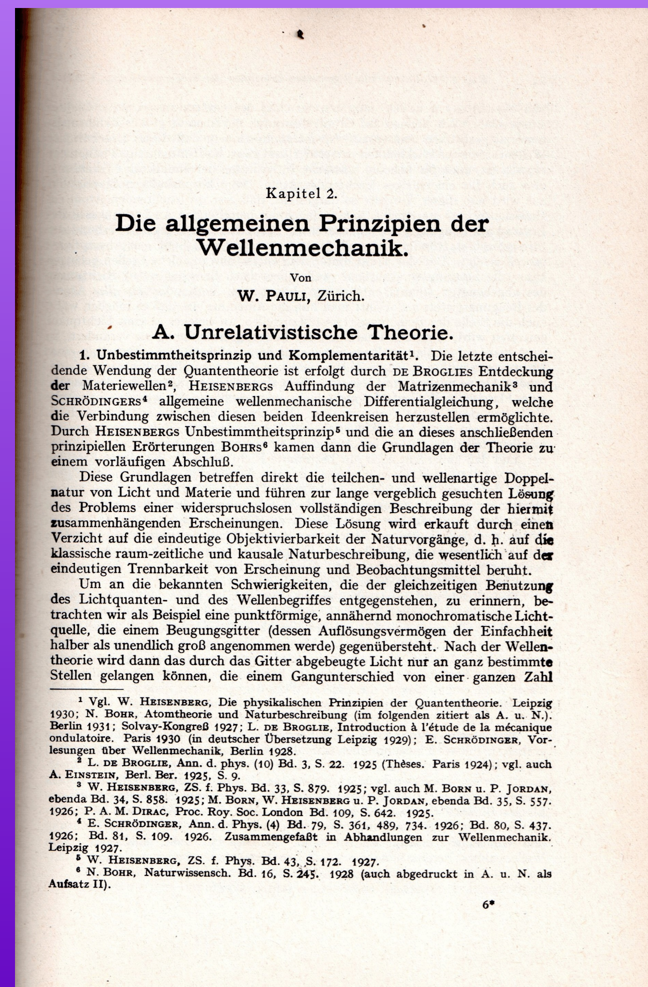
1918 Born in Amsterdam
1938 M.S. Amsterdam
1938-1939 Leiden
1946 Ph.D. University of Michigan
1946-1957 University of Kansas
1957-1960 Northwestern
1960-1964 University of Iowa
1964-1989 Stony Brook
1989-1997 Stanford
1997 Dies in Palo Alto

See: Peter Kahn. "In Appreciation: Remembering Max Dresden (1918-1997)." *Physics in Perspective* 5 (2003), 206-233.

Bertha Cummins and Max Dresden before their wedding, Lawrence, KS, 1948

Wolfgang Pauli. “Die allgemeinen Prinzipien der Wellenmechanik,” 83-272.

Part A, “Unrelativistische Theorie”



Kapitel 2.

Die allgemeinen Prinzipien der Wellenmechanik.

Von

W. PAULI, Zürich.

A. Unrelativistische Theorie.

1. Unbestimmtheitsprinzip und Komplementarität[1]. Die letzte entscheidende Wendung der Quantentheorie ist erfolgt durch DE BROGLIES Entdeckung der Materiewellen[2], HEISENBERGS Auffindung der Matrizenmechanik[3] und SCHRÖDINGERS[4] allgemeine wellenmechanische Differentialgleichung, welche die Verbindung zwischen diesen beiden Ideenkreisen herzustellen ermöglichte. Durch HEISENBERGS Unbestimmtheitsprinzip[5] und die an dieses anschließenden prinzipiellen Erörterungen BOHRS[6] kamen dann die Grundlagen der Theorie zu einem vorläufigen Abschluß.

Diese Grundlagen betreffen direkt die teilchen- und wellenartige Doppelnatur von Licht und Materie und führen zur lange vergeblich gesuchten Lösung des Problems einer widerspruchslosen vollständigen Beschreibung der hiermit zusammenhängenden Erscheinungen. Diese Lösung wird erkauft durch einen Verzicht auf die eindeutige Objektivierbarkeit der Naturvorgänge, d. h. auf die klassische raum-zeitliche und kausale Naturbeschreibung, die wesentlich auf der eindeutigen Trennbarkeit von Erscheinung und Beobachtungsmittel beruht.

Um an die bekannten Schwierigkeiten, die der gleichzeitigen Benutzung des Lichtquanten- und des Wellenbegriffes entgegenstehen, zu erinnern, betrachten wir als Beispiel eine punktförmige, annähernd monochromatische Lichtquelle, die einem Beugungsgitter (dessen Auflösungsvermögen der Einfachheit halber als unendlich groß angenommen werde) gegenübersteht. Nach der Wellentheorie wird dann das durch das Gitter abgebeugte Licht nur an ganz bestimmte Stellen gelangen können, die einem Gangunterschied von einer ganzen Zahl

[1] Vgl. W. HEISENBERG, Die physikalischen Prinzipien der Quantentheorie. Leipzig 1930; N. BOHR, Atomtheorie und Naturbeschreibung (im folgenden zitiert als A. u. N.). Berlin 1931; Solvay-Kongreß 1927; L. DE BROGLIE, Introduction à l'étude de la mécanique ondulatoire. Paris 1930 (in deutscher Übersetzung Leipzig 1929); E. SCHRÖDINGER, Vorlesungen über Wellenmechanik, Berlin 1928.

[2] L. DE BROGLIE, Ann. d. phys. (10) Bd. 3, S. 22. 1925 (Thèses. Paris 1924); vgl. auch A. EINSTEIN, Berl. Ber. 1925, S. 9.

[3] W. HEISENBERG, ZS. f. Phys. Bd. 33, S. 879. 1925; vgl. auch M. BORN u. P. JORDAN, ebenda Bd. 34, S. 858. 1925; M. BORN, W. HEISENBERG u. P. JORDAN, ebenda Bd. 35, S. 557. 1926; P. A. M. DIRAC, Proc. Roy. Soc. London Bd. 109, S. 642. 1925.

[4] E. SCHRÖDINGER, Ann. d. Phys. (4) Bd. 79, S. 361, 489, 734. 1926; Bd. 80, S. 437. 1926; Bd. 81, S. 109. 1926. Zusammengefaßt in Abhandlungen zur Wellenmechanik. Leipzig 1927.

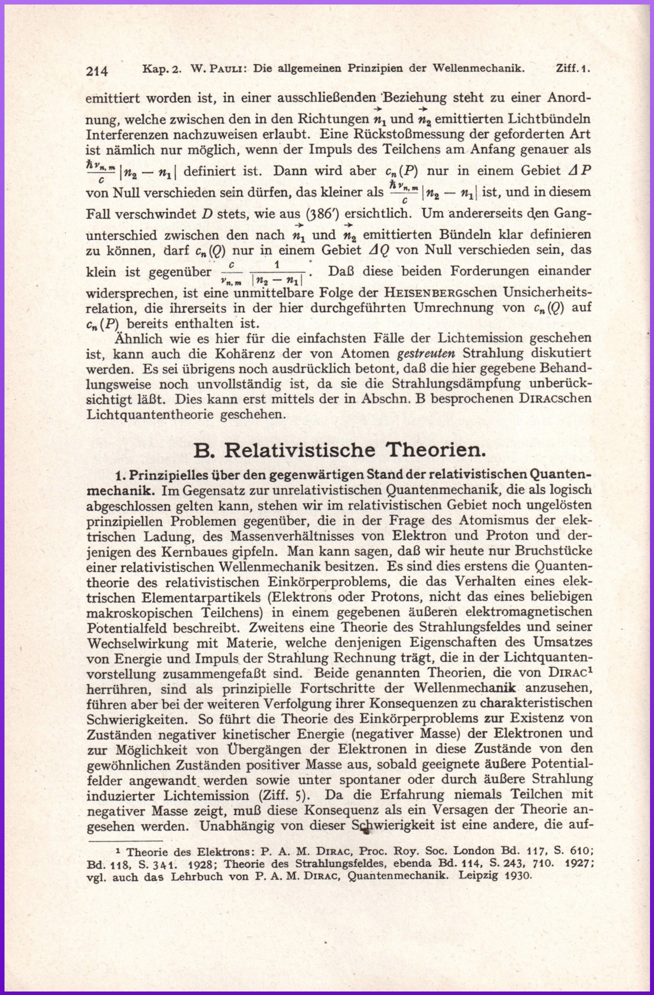
[5] W. HEISENBERG, ZS. f. Phys. Bd. 43, S. 172. 1927.

[6] N. BOHR, Naturwissensch. Bd. 16, S. 245. 1928 (auch abgedruckt in A. u. N. als Aufsatz II).

6*

Wolfgang Pauli. “Die allgemeinen Prinzipien der Wellenmechanik,” 83-272.

Part B, “Relativistische Theorie”



emittiert worden ist, in einer ausschließenden Beziehung steht zu einer Anordnung, welche zwischen den in den Richtungen $\vec{n}_1$ und $\vec{n}_2$ emittierten Lichtbündeln Interferenzen nachzuweisen erlaubt. Eine Rückstoßmessung der geforderten Art ist nämlich nur möglich, wenn der Impuls des Teilchens am Anfang genauer als $\frac{\hbar\nu_{n,m}}{c}|n_2 - n_1|$ definiert ist. Dann wird aber $c_n(P)$ nur in einem Gebiet ΔP von Null verschieden sein dürfen, das kleiner als $\frac{\hbar\nu_{n,m}}{c}|n_2 - n_1|$ ist, und in diesem Fall verschwindet D stets, wie aus (386′) ersichtlich. Um andererseits den Gangunterschied zwischen den nach $\vec{n}_1$ und $\vec{n}_2$ emittierten Bündeln klar definieren zu können, darf $c_n(Q)$ nur in einem Gebiet ΔQ von Null verschieden sein, das klein ist gegenüber $\frac{c}{\nu_{n,m}} \frac{1}{|n_2 - n_1|}$. Daß diese beiden Forderungen einander widersprechen, ist eine unmittelbare Folge der HEISENBERGschen Unsicherheitsrelation, die ihrerseits in der hier durchgeführten Umrechnung von $c_n(Q)$ auf $c_n(P)$ bereits enthalten ist.

Ähnlich wie es hier für die einfachsten Fälle der Lichtemission geschehen ist, kann auch die Kohärenz der von Atomen *gestreuten* Strahlung diskutiert werden. Es sei übrigens noch ausdrücklich betont, daß die hier gegebene Behandlungsweise noch unvollständig ist, da sie die Strahlungsdämpfung unberücksichtigt läßt. Dies kann erst mittels der in Abschn. B besprochenen DIRACschen Lichtquantentheorie geschehen.

B. Relativistische Theorien.

1. Prinzipielles über den gegenwärtigen Stand der relativistischen Quantenmechanik. Im Gegensatz zur unrelativistischen Quantenmechanik, die als logisch abgeschlossen gelten kann, stehen wir im relativistischen Gebiet noch ungelösten prinzipiellen Problemen gegenüber, die in der Frage des Atomismus der elektrischen Ladung, des Massenverhältnisses von Elektron und Proton und derjenigen des Kernbaues gipfeln. Man kann sagen, daß wir heute nur Bruchstücke einer relativistischen Wellenmechanik besitzen. Es sind dies erstens die Quantentheorie des relativistischen Einkörperproblems, die das Verhalten eines elektrischen Elementarpartikels (Elektrons oder Protons, nicht das eines beliebigen makroskopischen Teilchens) in einem gegebenen äußeren elektromagnetischen Potentialfeld beschreibt. Zweitens eine Theorie des Strahlungsfeldes und seiner Wechselwirkung mit Materie, welche denjenigen Eigenschaften des Umsatzes von Energie und Impuls der Strahlung Rechnung trägt, die in der Lichtquantenvorstellung zusammengefaßt sind. Beide genannten Theorien, die von DIRAC[1] herrühren, sind als prinzipielle Fortschritte der Wellenmechanik anzusehen, führen aber bei der weiteren Verfolgung ihrer Konsequenzen zu charakteristischen Schwierigkeiten. So führt die Theorie des Einkörperproblems zur Existenz von Zuständen negativer kinetischer Energie (negativer Masse) der Elektronen und zur Möglichkeit von Übergängen der Elektronen in diese Zustände von den gewöhnlichen Zuständen positiver Masse aus, sobald geeignete äußere Potentialfelder angewandt werden sowie unter spontaner oder durch äußere Strahlung induzierter Lichtemission (Ziff. 5). Da die Erfahrung niemals Teilchen mit negativer Masse zeigt, muß diese Konsequenz als ein Versagen der Theorie angesehen werden. Unabhängig von dieser Schwierigkeit ist eine andere, die auf-

[1] Theorie des Elektrons: P. A. M. DIRAC, Proc. Roy. Soc. London Bd. 117, S. 610; Bd. 118, S. 341. 1928; Theorie des Strahlungsfeldes, ebenda Bd. 114, S. 243, 710. 1927; vgl. auch das Lehrbuch von P. A. M. DIRAC, Quantenmechanik. Leipzig 1930.

Wolfgang Pauli. “Die allgemeinen Prinzipien der Wellenmechanik,” 83-272.

Part A, “Unrelativistische Theorie”

The most influential presentation of “Copenhagen” orthodoxy?

- Complementarity
- Observables fixed by experience, not an operator algebra
- Measurement and the movable “cut”
- Wave-packet “reduction”

Bohr, Heisenberg, and Pauli, Copenhagen, 1936

But see also: Pascual Jordan. *Anschauliche Quantentheorie. Eine Einführung in die moderne Auffassung der Quantenerscheinungen*. Berlin: Julius Springer, 1936.

Wolfgang Pauli. “Die allgemeinen Prinzipien der Wellenmechanik,” 83-272.

Part A, “Unrelativistische Theorie”

5. Wechselwirkung mehrerer Teilchen. Operatorkalkül, 111-121.

The manner in which a composite system consisting of several component systems is described in the quantum theory is of fundamental importance for this theory and is its most characteristic feature. It demonstrates, on the one hand, the fruitfulness of Schrödinger’s idea of introducing a ψ-function that satisfies a linear equation, and, on the other hand, the purely symbolic character of this function, which differs in principle from the wave functions of classical theory (surface waves of fluids, elastic waves, electromagnetic waves).

If a system of several particles is present, one obtains *no* sufficient description of the system through the specification of the probability for *one* of the particles to be found at a specific place. Consider, e.g., a system consisting of two material particles that are located in a closed box. This box is divided into two parts by a dividing wall with a tiny, closable opening. By a sudden closing of the opening and the detaching of the two halves, then it can be determined for each particle in which half of the box it found itself at the moment in question. One can now investigate not only how large the probability is for each particle to be found in the one or the other half, but also how frequently the particles find themselves in the same or in different halves of the box. Instead of the dividing wall, one can employ “microscopes” with shortwave radiation, and instead of a division of a finite volume in only two parts, an arbitrarily fine partition of the space can be effected. Thus, let there be now N particles, and let their coordinates be $x_k^{(1)}, x_k^{(2)}, \ldots, x_k^{(N)}$, for which we could also write $q_1 \ldots q_f$, with $f = 3N$ designating the number of degrees of freedom of the system; furthermore, we can write simply dq for the multidimensional volume element $dq_1 dq_2 \ldots dq_f$.

Wolfgang Pauli. “Die allgemeinen Prinzipien der Wellenmechanik,” 83-272.

Part A, “Unrelativistische Theorie”

5. Wechselwirkung mehrerer Teilchen. Operatorkalkül, 111-121.

The fundamental assumption for the description of a system of several material particles can then be formulated in the following manner:

1. *In every moment of time t there exists a probability*

$$W(q_1 \ldots q_f; t)dq \tag{87}$$

for finding the coordinates of the first particle in the range $(q_k, q_k + dq_k)$ $(k = 1, 2, 3)$, *those of the second particle in* $(q_k, q_k + dq_k)$ $(k = 4, 5, 6)$, *and those of the* N^{th} *particle in* $(q_k, q_k + dq_k)$ $(k = f - 2, f - 1, f)$.

[Postpone questions about particle distinguishability.]

By integrating W over the coordinates of all but one particles one obtains N new functions

$$W_1(x_1, x_2, x_3),\ W_2(x_4, x_5, x_6), \ldots\ W_N(x_{3N-2}, x_{3N-1}, x_{3N}),$$

which give the probability for finding a specific one of the particles in a specific spatial location, without asking where the other particles are to be found. These functions say less about the system than did the original function of f arguments, in that the latter cannot be derived unambiguously from the former, rather only the converse holds.

Wolfgang Pauli. “Die allgemeinen Prinzipien der Wellenmechanik,” 83-272.

Part A, “Unrelativistische Theorie”

5. Wechselwirkung mehrerer Teilchen. Operatorkalkül, 111-121.

The existence of the probability $W(q_1 \ldots q_f; t)$ entails the assertion that or is possible only under the assumption that the position measurements of the different particles do not fundamentally disturb one another, in the sense that the usefulness of the knowledge of the position of one particle for predicting other measurements (e.g., the position of this particle at a later time) is not lost by coming to know the position of another particle. This situation is closely connected with the question to what extent the simultaneity of the position measurements of the different particles is essential for the existence of the probability. That is to say: under what circumstances does there exist a probability

$$W(x_k^{(1)}, t^{(1)}; x_k^{(2)}, t^{(2)}; \ldots x_k^{(N)}, t^{(N)})dq_1 \ldots dq_{3N} \tag{88}$$

for finding the first particle at time $t^{(1)}$ in space element $x_k^{(1)}, x_k^{(1)} + dx_k^{(1)}$, the second particle at time $t^{(2)}$ in space element $x_k^{(2)}, x_k^{(2)} + dx_k^{(2)}$, and the N^{th} particle at time $t^{(N)}$ in space element $x_k^{(N)}, x_k^{(N)} + dx_k^{(N)}$. In general, i.e., if any kind of interaction forces among the particles are present, the mutual freedom from disturbance of the measurements is guaranteed when and only when for separation r_{ab} of any pair of particles (a, b) and the corresponding times

$$|t_a - t_b| < r_{ab}/c. \tag{89}$$

The change in the effect of a force that particle a exerts on particle b brought about by the position measurement on a can, therefore, be propagated at most with the velocity of light c.

Wolfgang Pauli. “Die allgemeinen Prinzipien der Wellenmechanik,” 83-272.

Part A, “Unrelativistische Theorie”

5. Wechselwirkung mehrerer Teilchen. Operatorkalkül, 111-121.

As far as the choice of the Hamiltonian operator is concerned we first assume that, in the case where no interaction occurs between the particles but where these can be subjected to arbitrary external forces, the Hamiltonian operator decomposes into independent summands:

$$\boldsymbol{H} = \boldsymbol{H}^{(1)} + \boldsymbol{H}^{(2)} + \ldots + \boldsymbol{H}^{(N)}, \tag{96}$$

in such a way that $\boldsymbol{H}^{(1)}$ transforms only one function $\psi(x_k^{(1)})$ containing the coordinates of the first particle, but carries a function containing only the coordinates of the other particles over into itsself.

. . .

An additive decomposition of the Hamiltonian operator in independent summands thus corresponds to a product decomposition of the wave function in independent factors. This is in accord with the circumstance that, in the case of statistically independent particles, the probability $W(q_1 \ldots q_f; t)dq$ decomposes into a product.

Too many people today think that Schrödinger invented not only the term, “entanglement,” but also the concept in a series of papers published in 1935-1936 and triggered by the EPR paper:

Erwin Schrödinger. “Die gegenwärtige Situation in der Quantenmechanik.” *Die Naturwissenschaften* 23 (1935), 807-812, 823-828, 844-849.

Erwin Schrödinger. “Discussion of Probability Relations Between Separated Systems.” *Proceedings of the Cambridge Philosophical Society* 31 (1935), 555-662.

Erwin Schrödinger. “Probability Relations Between Separated Systems.” *Proceedings of the Cambridge Philosophical Society* 32 (1936), 446-452.

Erwin Schrödinger, mid-1920s

Erwin Schrödinger. “Die gegenwärtige Situation in der Quantenmechanik.” *Die Naturwissenschaften* 23 (1935), 807-812, 823-828, 844-849.

If two separated bodies, about which, individually, we have maximal knowledge, come into a situation in which they influence one another and then again separate themselves, then there regularly arises that which I just called *entanglement* [*Verschränkung*] of our knowledge of the two bodies. At the outset, the joint catalogue of expectations consists of a logical sum of the individual catalogues; during the process the joint catalogue develops necessarily according to the known law [linear Schrödinger evolution] . . . . Our knowledge remains maximal, but at the end, if the bodies have again separated themselves, that knowledge does not again decompose into a logical sum of knowledge of the individual bodies.

Erwin Schrödinger, mid-1930s

But nothing could be further from the truth. On the contrary, by the early 1930s, the concept of entanglement was a commonplace in the literature.

Consider one of many examples:

Hermann Weyl. *Gruppentheorie und Quantenmechanik*, 2nd. ed. Leipzig: S. Hirzel, 1931.
Ch. II, § 10, "The Problem of Several Bodies. Product Space."

Conditions that insure a maximum of homogeneity within $\mathfrak{c}$ [a composite system] *need not require a maximum in this respect within the partial system* $\mathfrak{a}$. Furthermore: *if the state of* $\mathfrak{a}$ *and the state of* $\mathfrak{b}$ *are known, the state of* $\mathfrak{c}$ *is in general not uniquely specified*, for a positive definite Hermitian form $\|a_{i,k,\, i'k'}\|$ in the product space, which describes a statistical aggregate of states $\mathfrak{c}$, is not uniquely determined by the Hermitian forms

to which it gives rise in the spaces $\mathfrak{R}$, $\mathfrak{S}$. In this significant sense quantum theory subscribes to the view that "*the whole is greater than the sum of its parts*," which has recently been raised to the status of a philosophical creed by the Vitalists and the Gestalt psychologists.

Hermann Weyl

Moreover, the failure of classical notions of particle independence in the quantum theory had been a focus of investigation since at least Einstein's 1905 photon hypothesis paper.

See:

Don Howard. "'Nicht sein kann was nicht sein darf,' or the Prehistory of EPR, 1909-1935: Einstein's Early Worries about the Quantum Mechanics of Composite Systems." In *Sixty-Two Years of Uncertainty: Historical, Philosophical, and Physical Inquiries into the Foundations of Quantum Mechanics.* Arthur Miller, ed. New York: Plenum, 1990, 61-111.

Einstein and Bohr ca. 1927

Pauli’s discussion of multiparticle quantum mechanics, especially his balls-in-a-box thought experiment, should be compared with Einstein’s discussion of his own ball-in-a-box thought experiment in the June 1935, post-EPR correspondence with Schrödinger, in which Einstein famously repudiates the EPR paper, noting that “the main point was buried by the erudition” and introduces what he, Einstein, terms the “separation principle” [“Trennungsprinzip”].

DESCRIPTION OF PHYSICAL REALITY 777

of lanthanum is 7/2, hence the nuclear magnetic moment as determined by this analysis is 2.5 nuclear magnetons. This is in fair agreement with the value 2.8 nuclear magnetons determined from La III hyperfine structures by the writer and N. S. Grace.[9]

[9] M. F. Crawford and N. S. Grace, Phys. Rev. **47**, 536 (1935).

This investigation was carried out under the supervision of Professor G. Breit, and I wish to thank him for the invaluable advice and assistance so freely given. I also take this opportunity to acknowledge the award of a Fellowship by the Royal Society of Canada, and to thank the University of Wisconsin and the Department of Physics for the privilege of working here.

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

1.

ANY serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These concepts are intended to correspond with the objective reality, and by means of these concepts we picture this reality to ourselves.

In attempting to judge the success of a physical theory, we may ask ourselves two questions: (1) "Is the theory correct?" and (2) "Is the description given by the theory complete?" It is only in the case in which positive answers may be given to both of these questions, that the concepts of the theory may be said to be satisfactory. The correctness of the theory is judged by the degree of agreement between the conclusions of the theory and human experience. This experience, which alone enables us to make inferences about reality, in physics takes the form of experiment and measurement. It is the second question that we wish to consider here, as applied to quantum mechanics.

Whatever the meaning assigned to the term *complete*, the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory.* We shall call this the condition of completeness. The second question is thus easily answered, as soon as we are able to decide what are the elements of the physical reality.

The elements of the physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements. A comprehensive definition of reality is, however, unnecessary for our purpose. We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.* It seems to us that this criterion, while far from exhausting all possible ways of recognizing a physical reality, at least provides us with one

See:

Don Howard. “Einstein on Locality and Separability.” *Studies in History and Philosophy of Science* 16 (1985),171-201.

Einstein to Schrödinger, 19 June 1935

I was very pleased with your detailed letter, which speaks about the little essay. For reasons of language, this was written by Podolsky after many discussions. But still it has not come out as well as I really wanted; on the contrary, the main point was, so to speak, buried by the erudition [die Hauptsache ist sozusagen durch Gelehrsamkeit verschüttet].

. . .

My way of thinking is now this: properly considered, one cannot get at the talmudist if one does not make use of a supplementary principle: the “separation principle.” That is to say: “the second box, along with everything having to do with its contents, is independent of what happens with regard to the first box (separated partial systems).” If one adheres to the separation principle, then one thereby excludes the second point of view, and only the Born point of view remains, according to which the above state description is an *incomplete* description of reality, or of the real states.

. . .

After the collision, the real state of (AB) consists precisely of the real state A and the real state of B, which two states have nothing to do with one another. *The real state of B thus cannot depend upon the kind of measurement I carry out on A*. (“Separation hypothesis” from above.) But then for the same state of B there are two (in general arbitrarily many) equally justified Ψ_B, which contradicts the hypothesis of a one-to-one or complete description of the real states.

Pauli's discussion of entanglement in Part A of the article, bears comparison with his own reaction to EPR in June of 1935.

Four days before Einstein wrote the previously quoted letter to Schrödinger, Pauli wrote to Heisenberg to complain about the EPR paper, and he put the emphasis on Einstein's failure to understand the physics of entanglement, about which he says that such matters "are, for us, trivialities."

Pauli and Heisenberg

Pauli to Heisenberg, 15 June 1935

[Einstein] now understands this much, that one cannot simultaneously measure two quantities corresponding to non-commuting operators and that one cannot simultaneously ascribe numerical values to them. But where he runs into trouble in this connection is the way in which, in quantum mechanics, two systems are joined to form a composite system. . . .

A pedagogical reply to [this] train of thought must, I believe, clarify the following concepts. The difference between the following statements:

a) Two systems 1 and 2 are not in interaction with one another (= absence of any interaction energy).

Definition. This is the case if, after a maximal observation on 1, the expectation values of all quantities of 1 have the *same temporal evolution* as if 2 were not present. (NB. Anyhow, for sufficiently short times the concept of an interaction plays no role.)

b) The composite system is in a state where the subsystems 1 and 2 are *independent*. (Decomposition of the eigenfunction into a product.)

Definition. This is the case if, after a measurement of an arbitrary quantity F_2 is carried out on 2, with a known result $F_2 = (F_2)_0$ (number), the expectation values of the quantities F_1 of 1 remain the same as without a measurement on 2 having been carried out.

Quite independently of Einstein, it appears to me that, in providing a systematic foundation for quantum mechanics, one should start more from the composition and separation of systems than has until now (with Dirac, e.g.) been the case. — This is indeed—as Einstein has *correctly* felt—a very fundamental point in quantum mechanics, which has, moreover, a direct connection with your reflections about the cut and the possibility of its being shifted to an arbitrary place.

Note [DH]: Pauli here invents the concept later independently reasserted by Jon Jarrett in his 1983 University of Chicago Ph.D. and subsequently (1986) dubbed by Shimony “outcome independence.”

Wolfgang Pauli. “Die allgemeinen Prinzipien der Wellenmechanik,” 83-272.

Part B, “Relativistische Theorie”

Written at a stage of rapid transition, so at best a snapshot of the discussion at that time

- Dirac equation
- QED
- Continuing worries about negative energy solutions
- Serious worries about infinities

Pauli and Dirac, Oxford, 1938

Wolfgang Pauli. “Die allgemeinen Prinzipien der Wellenmechanik.” *Handbuch der Physik*, 2nd ed. Hans Geiger and Karl Scheel, eds. Vol. 24, Part 1, *Quantentheorie*. Adolf Smekal, ed. Berlin: Julius Springer, 1933, 83-272.

Subsequent publishing history:

- “Published and distributed in the Public Interest by Authority of the Alien Property Custodian under License No. A-54.” Ann Arbor, MI: Edwards Brothers, 1943, et seq.

- Revised edition. Wolfgang Pauli and Gunnar Källén. “Die allgemeinen Prinzipien der Wellenmechanik.” In *Handbuch der Physik*. Siegfried Flügge, ed. Vol. 5, Part 1, *Prinzipien der Quantentheorie*. Berlin: Springer-Verlag, 1958, 1-168. Omits sections 6-8 of Part B.

- Reprinted in: Wolfgang Pauli. *Collected Scientific Papers*, vol. 1. Ralph Kronig and Victor Weisskopf, eds. New York: Wiley-Interscience, 1964, 771-938.

- English translation: Wolfgang Pauli. *General Principles of Quantum Mechanics*. P. Achuthan and K. Venkatesan, trans. Berlin and New York: Springer-Verlag, 1980. Sections 6-8 from Part B of the original restored.

- Wolfgang Pauli. *Die allgemeinen Prinzipien der Wellenmechanik*. Norbert Straumann, ed. Berlin and New York: Springer-Verlag, 1990. Reprint of the 1958 edition with section 1, part of section 5, and sections 6-8 of Part B from the 1933 edition included as appendices.

Wolfgang Pauli. “Die allgemeinen Prinzipien der Wellenmechanik.” *Handbuch der Physik*, 2nd ed. Hans Geiger and Karl Scheel, eds. Vol. 24, Part 1, *Quantentheorie*. Adolf Smekal, ed. Berlin: Julius Springer, 1933, 83-272.

Subsequent publishing history:

But almost unknown is this . . .

Wolfgang Pauli. *The General Principles of Wave Mechanics*. James Alexander, Geoffrey Chew, Walter Selove, and Chen Yang, trans. Mimeograph. November 1946.

Variously titled. Also *Relativistic Wave Mechanics*, as with the example in the University of Notre Dame library.

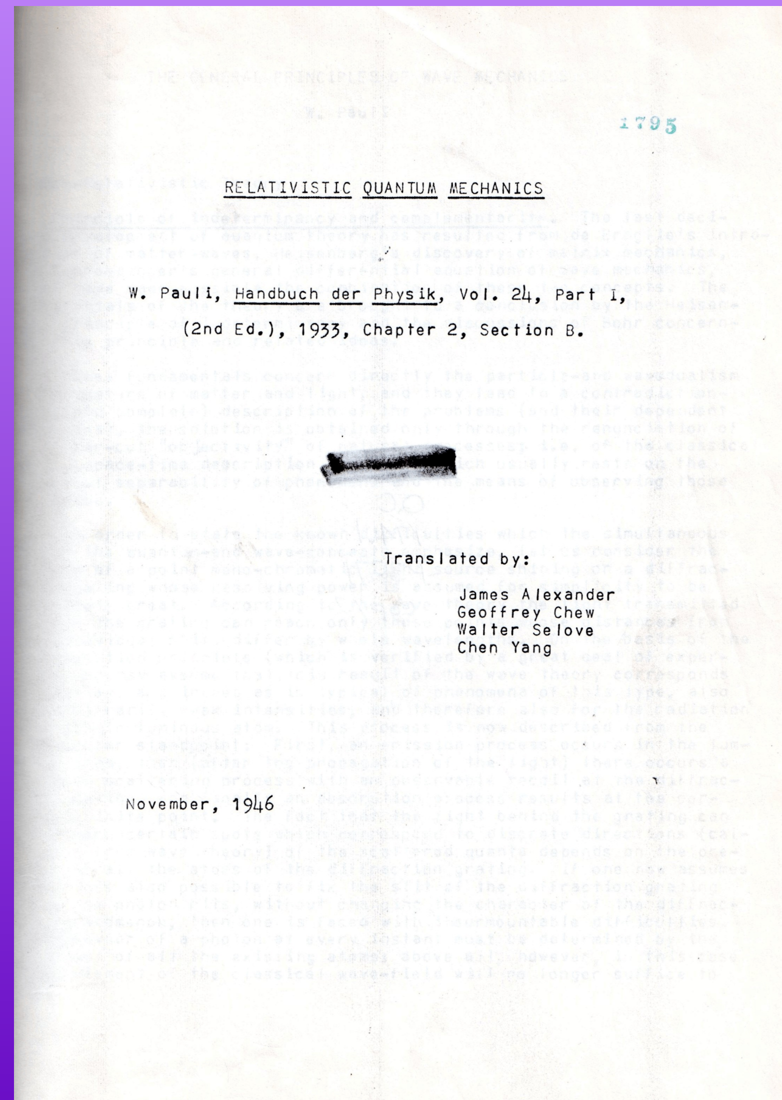
No more than twelve surviving copies worldwide as determined via WorldCat. No location given on 1946 copies.

1950 copies give as location:
"Urbana, Ill.: University of Illinois."

And:

"Assembled, edited and hectographed by members of the Physics Dept., University of Illinois."

1952 copies give as location:
"Berkeley, Calif."



1795

RELATIVISTIC QUANTUM MECHANICS

W. Pauli, Handbuch der Physik, Vol. 24, Part I,
(2nd Ed.), 1933, Chapter 2, Section B.

[redacted]

Translated by:

James Alexander
Geoffrey Chew
Walter Selove
Chen Yang

November, 1946

But all copies contain both

Part A “Non-Relativistic Theory”

and

Part B “Relativistic Theory”

of the 1933 *Handbuch* version of the article.

Parts A and B are typed on the same typewriter and the equations are entered by hand in the same hand. But the parts are separately numbered, Part A running to 151 pages, Part B to 104 pages, and Part A is single spaced, whereas Part B is double spaced.

The translation is complete, accurate, and highly readable.

THE GENERAL PRINCIPLES OF WAVE MECHANICS

W. Pauli

A. Non-Relativistic Theory

1. Principle of indeterminancy and complementarity. The last decisive development of quantum theory has resulted from de Broglie's introduction of matter-waves, Heisenberg's discovery of matrix mechanics, and Schroedinger's general differential equation of wave mechanics, which have made possible the combination of these two concepts. The fundamentals of the theory are brought to a conclusion by the Heisenberg principle of indeterminancy and the discussions of Bohr concerning this principle and related ideas.

These fundamentals concern directly the particle-and wavedualism of the nature of matter and light, and they lead to a contradiction-free (and complete) description of the problems (and their dependent phenomena), the solution is obtained only through the renunciation of the clear-cut "objectivity" of natural processes; i.e. of the classical causal space-time description of nature which usually rests on the clear-cut separability of phenomena and the means of observing those phenomena.

In order to state the known difficulties which the simultaneous use of the quantum-and wave-concepts emphasize, let us consider the example of a point mono-chromatic light source shining on a diffraction grating whose resolving power is assumed for simplicity to be infinitely great. According to the wave theory the light transmitted through the grating can reach only those points whose distances from the individual slits differ by whole wavelengths. On the basis of the superposition principle (which is verified by a great deal of experience) we may assume that this result of the wave theory corresponds to reality, and indeed as is typical of phenomena of this type, also for arbitrarily weak intensities, and therefore also for the radiation of a single luminous atom. This process is now described from the corpuscular standpoint: First, an emission process occurs in the luminous atom, then (after the propagation of the light) there occurs a combined scattering process with an observable recoil at the diffraction grating, and finally an absorption process results at the certain definite point. The fact that the light behind the grating can only reach certain spots which correspond to discrete directions (calculable from wave theory) of the scattered quanta depends on the presence of all the atoms of the diffraction grating. If one now assumes that it is also possible to fix the slit of the diffraction grating which the photon hits, without changing the character of the diffraction phenomenon, then one is faced with insurmountable difficulties. The behavior of a photon at every instant must be determined by the positions of all the existing atoms: above all, however, in this case the statement of the classical wave-field will no longer suffice to

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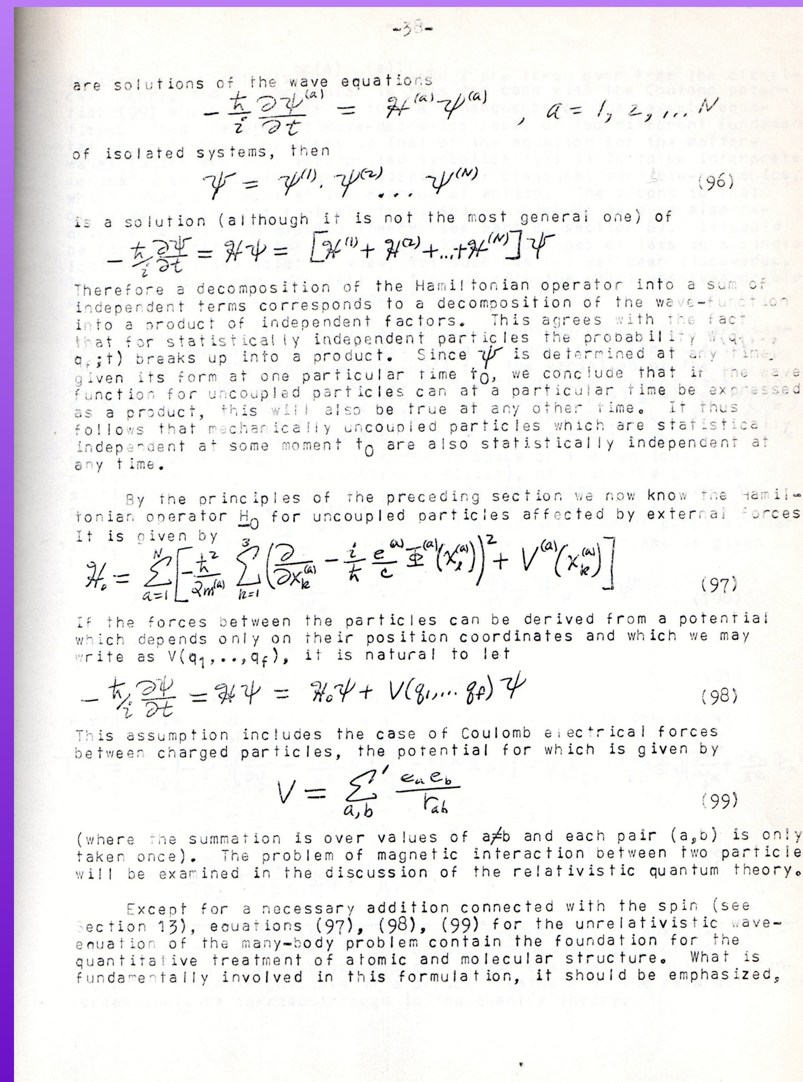

-38-

are solutions of the wave equations

$$-\frac{\hbar}{i}\frac{\partial \psi^{(a)}}{\partial t} = \mathcal{H}^{(a)}\psi^{(a)}, \quad a = 1, 2, \ldots N$$

of isolated systems, then

$$\psi = \psi^{(1)} \cdot \psi^{(2)} \cdots \psi^{(N)} \tag{96}$$

is a solution (although it is not the most general one) of

$$-\frac{\hbar}{i}\frac{\partial \psi}{\partial t} = \mathcal{H}\psi = \left[\mathcal{H}^{(1)} + \mathcal{H}^{(2)} + \ldots + \mathcal{H}^{(N)}\right]\psi$$

Therefore a decomposition of the Hamiltonian operator into a sum of independent terms corresponds to a decomposition of the wave-function into a product of independent factors. This agrees with the fact that for statistically independent particles the probability $W(q_1, \ldots q_f; t)$ breaks up into a product. Since ψ is determined at any time, given its form at one particular time t_0, we conclude that if the wave-function for uncoupled particles can at a particular time be expressed as a product, this will also be true at any other time. It thus follows that mechanically uncoupled particles which are statistically independent at some moment t_0 are also statistically independent at any time.

By the principles of the preceding section we now know the Hamiltonian operator $\underline{H}_0$ for uncoupled particles affected by external forces. It is given by

$$\mathcal{H}_0 = \sum_{a=1}^{N}\left[-\frac{\hbar^2}{2m^{(a)}}\sum_{k=1}^{3}\left(\frac{\partial}{\partial x_k^{(a)}} - \frac{i}{\hbar}\frac{e^{(a)}}{c}\Phi^{(a)}(x_i^{(a)})\right)^2 + V^{(a)}(x_k^{(a)})\right] \tag{97}$$

If the forces between the particles can be derived from a potential which depends only on their position coordinates and which we may write as $V(q_1, \ldots, q_f)$, it is natural to let

$$-\frac{\hbar}{i}\frac{\partial \psi}{\partial t} = \mathcal{H}\psi = \mathcal{H}_0\psi + V(q_1, \ldots q_f)\psi \tag{98}$$

This assumption includes the case of Coulomb electrical forces between charged particles, the potential for which is given by

$$V = \sum_{a,b}{}' \frac{e_a e_b}{r_{ab}} \tag{99}$$

(where the summation is over values of $a \neq b$ and each pair (a, b) is only taken once). The problem of magnetic interaction between two particles will be examined in the discussion of the relativistic quantum theory.

Except for a necessary addition connected with the spin (see Section 13), equations (97), (98), (99) for the unrelativistic wave-equation of the many-body problem contain the foundation for the quantitative treatment of atomic and molecular structure. What is fundamentally involved in this formulation, it should be emphasized,

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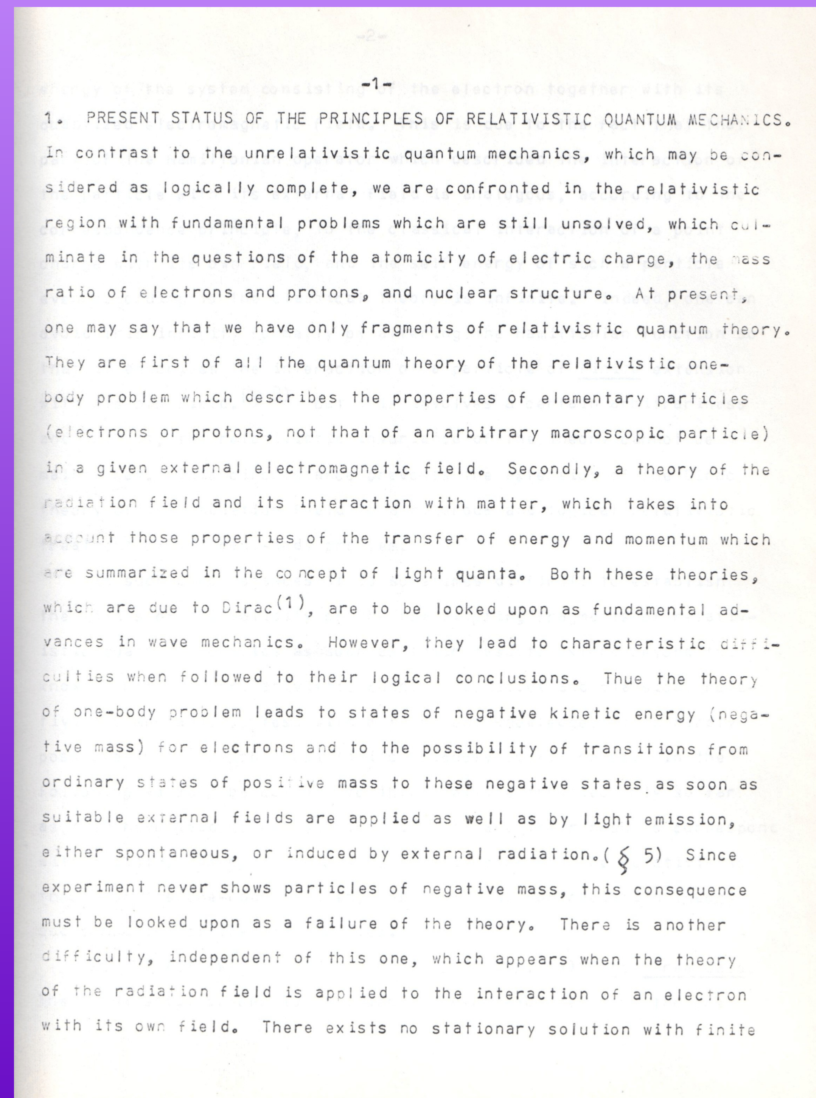
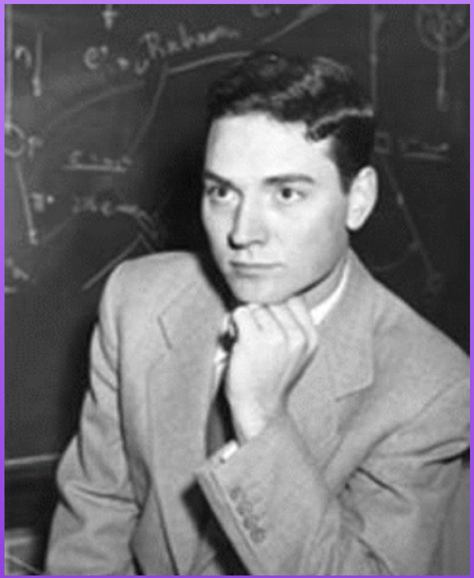


-1-

1. PRESENT STATUS OF THE PRINCIPLES OF RELATIVISTIC QUANTUM MECHANICS.

In contrast to the unrelativistic quantum mechanics, which may be considered as logically complete, we are confronted in the relativistic region with fundamental problems which are still unsolved, which culminate in the questions of the atomicity of electric charge, the mass ratio of electrons and protons, and nuclear structure. At present, one may say that we have only fragments of relativistic quantum theory. They are first of all the quantum theory of the relativistic one-body problem which describes the properties of elementary particles (electrons or protons, not that of an arbitrary macroscopic particle) in a given external electromagnetic field. Secondly, a theory of the radiation field and its interaction with matter, which takes into account those properties of the transfer of energy and momentum which are summarized in the concept of light quanta. Both these theories, which are due to Dirac(1), are to be looked upon as fundamental advances in wave mechanics. However, they lead to characteristic difficulties when followed to their logical conclusions. Thue the theory of one-body problem leads to states of negative kinetic energy (negative mass) for electrons and to the possibility of transitions from ordinary states of positive mass to these negative states as soon as suitable external fields are applied as well as by light emission, either spontaneous, or induced by external radiation. (§ 5) Since experiment never shows particles of negative mass, this consequence must be looked upon as a failure of the theory. There is another difficulty, independent of this one, which appears when the theory of the radiation field is applied to the interaction of an electron with its own field. There exists no stationary solution with finite



Geoffrey Chew, 1955
Ph.D., University of Chicago, 1946

Chen Ning Yang, 1963
Ph.D., University of Chicago, 1948

Walter Selove, 1950
Ph.D., University of Chicago, 1949

The translators.

But who was James Alexander?

I can place him at Argonne in the early 1950s doing work on computer design, but otherwise I can find nothing about him.

Here is what Geoff Chew reports about the translation:

Dear Don,

You are sorely taxing my memory. The translation occurred in my first year at Chicago, I believe because there then existed no English-language textbooks in quantum theory, other than Dirac's first two editions which were considered impenetrable by beginning students. I myself was introduced to the subject by a Fermi course in nuclear physics at Los Alamos. At Chicago I took a course by Teller and then joined some other Los Alamos transferees at a stupendously effective series of informal evening presentations by Fermi. Neither Fermi nor Teller used a text. I believe Fermi never opened a book. He had private notes that contained all of the physics then known. I recall that Yang was not invited to join Fermi's evening sessions because he had not been at Los Alamos. I am less sure about Selove and Alexander but have a feeling they also were not part of the favored group. I remember that, although I contributed to the translation, I benefited relatively little from it because of my access to Fermi. I have a vague recollection that my Pauli translation effort bore some relation to a foreign-language requirement that still existed at Chicago when I entered.

Sorry!

Geoff